UNIVERSITY OF IOWA STUDIES IN ENGINEERING

BULLETIN NO. 14

A STUDY IN FLOOD WAVES

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I. INTRODUCTION

1. *Synopsis.*—The elementary equations of a simple flood wave in a wide, rectangular channel, are developed and discussed in this study. It is assumed that as the wave is propagated down stream it is not affected by flow in tributaries nor by changing channel conditions, including roughness, slope, and shape of conduit. The equations discussed include the profile of backwater curves, the profile of the roll wave with unchanging front, the velocity of the flood wave in general, and the velocity and profile of the rising flood wave with fixed maximum stage.

Tests in a laboratory flume 52 ft. long, 8 in. wide, and 19 in. deep showed that the velocity of a wave near its genesis closely approximated the velocity of J. Scott Russell's "wave of translation."

2. *Acknowledgments.*—The writer is indebted to S. M. WOODWARD, professor of mechanics and hydraulics, for guidance in the preparation of the thesis[2] which is the substance of this study, and to DR. H. L. RIETZ, professor and head of mathematics, for his suggestions and criticisms.

The tests reported herein were conducted in the hydraulics laboratory of the State University of Iowa under the direction of PROFESSOR WOODWARD and the late DR. FLOYD A. NAGLER, professor of hydraulic engineering. The late DAVID L. YARNELL, senior engineer, Bureau of Agricultural Engineering, and members of his staff assisted with the experimental work.

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[2]"A study in flood waves," by Elmer E. Moots. Thesis submitted in partial fulfillment of the requirements for the degree Doctor of Philosophy in the Department of Mechanics and Hydraulics, in the Graduate College of the State University of Iowa, February 1927.

3. *Terminology.*—Steady flow exists in a conduit when the same volume of water passes each section per unit of time and the flow at any chosen section does not vary with the time.

Uniform flow is that particular case of steady flow for which the slope of the water surface is the same as the slope of the channel bed. In uniform flow all transverse sections of conduit have the same area and shape and these do not vary with the time.

A simple flood wave is a long, positive wave superposed upon a river initially in a state of steady flow. During the progression of the wave the river is in flood condition. When the maximum stage has been reached at a given station, the water level subsides slowly, and after some time the river is again in its initial state. During the wave movement the area of the transverse section at the gaging station varies with time, as does also the mean velocity of flow in that section.

The following symbols are used frequently throughout this paper.

h = depth of the water flowing in the channel. The depth is equivalent to the hydraulic radius if the channel is wide in comparison to its depth, and it is numerically equal to cross-sectional area per unit width of channel.

K = a coefficient determined by the roughness of the channel.

p = pressure at any point in the liquid; usually p represents the pressure at the bed of the channel.

q = rate of discharge per unit width of conduit.

ρ = density of liquid.

S = slope of channel bed.

S' = the neutral slope which is just sufficient to overcome channel friction and maintain uniform flow.

t = time in seconds.

u = velocity of flow, assumed to be uniform and normal to a transverse section.

x = the abscissa of a point measured along the channel bed from some fixed point x_o. The h-axis is normal to the x-axis and passes through the point x_o.

4. *Assumptions.*—

(a) The Equation of Continuity

It is assumed that water flows along the channel in a compact manner free from appreciable breaks on the surface and appreciable voids in the volume mass. This leads us to the equation of continuity which, for steady flow, states that the same quantity of water is passing each cross-section of the stream in a unit of time. This is equivalent to the equation

$$A_1u_1 = A_2u_2 \tag{1}$$

where A_1, A_2 represent the cross-sectional areas at sections 1 and 2, and u_1, u_2 represent the corresponding average velocities at those sections.

The equation of continuity for unsteady flow states that in a given interval of time the difference between inflow to and outflow from a given reach of conduit is equal to the change in storage in the reach.

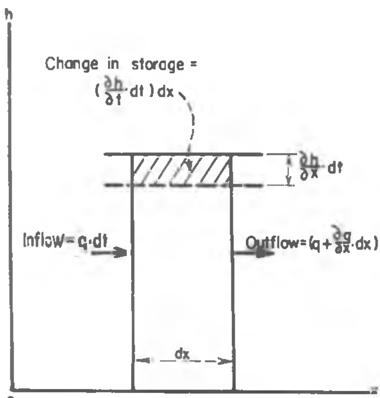

Fig. 1. Basis for Equation of Continuity

Let Fig. 1 represent a reach of conduit whose width is unity and whose length is dx. The rate of inflow is q, cu. ft. per sec., and in a short time interval dt, sec., the total volume of inflow is $q \, . \, dt$, cu. ft. The rate of outflow is $(q + \frac{\delta q}{\delta x} \cdot dx)$, cu. ft. per sec., and the total volume of outflow in the time interval dt, is $(q + \frac{\delta q}{\delta x} dx) \, dt$. In the same time interval the elevation of the water surface in the reach, dx in length, has changed an amount equal to $(\frac{\delta h}{\delta t} dt)$ ft., and the corresponding change in storage in the reach is $(\frac{\delta h}{\delta t} dt \, . \, dx)$ cu. ft. Equating the change in storage to the difference between inflow and outflow, dividing through by the common factor $(dx \,.\, dt)$, and using proper algebraic signs, the equation of continuity becomes

$$\frac{\delta q}{\delta x} + \frac{\delta h}{\delta t} = 0 \tag{2}$$

If $q = hu$, equation (2) may also be written

$$\frac{\delta h}{\delta t} + h\frac{\delta u}{\delta x} + u\frac{\delta h}{\delta x} = 0 \tag{3}$$

(b) The Equation of Motion

For a flood wave in a wide rectangular channel the usual equations of motion reduce to one equation. It is assumed that the filaments of flow are to a first approximation parallel to the bed and sides of the channel.

Referring to Fig. 2 and choosing rectangular axes h and x, where the latter is measured along the bed of the channel, we take an element $ABCD$ with dimensions h, $h + \frac{\delta h}{\delta x}\,dx$, dx, and unity.

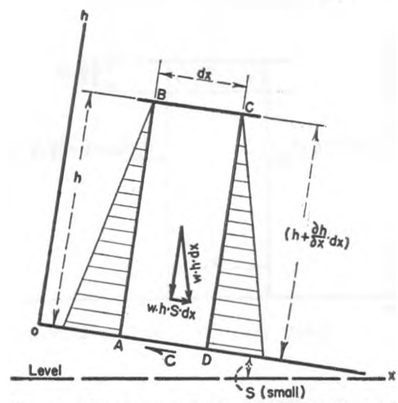

FIG 2. BASIS FOR EQUATION OF MOTION

The rate of change of momentum of the element $ABCD$ in the x-direction is equivalent to the product of its mass and its rate of change of velocity. Neglecting infinitesimals of a higher order than the first, the product of mass and acceleration is

$$\rho h\, dx\, \frac{du}{dt} \tag{4}$$

The total liquid pressure on the face AB is $\frac{wh^2}{2}$, and neglecting higher ordered infinitesimals the total pressure on the face CD is

$$\frac{wh^2}{2} + wh\,\frac{\delta h}{\delta x}\,dx$$

The difference of the two expressions gives the resultant force

$$-wh\frac{\delta h}{\delta x}dx \tag{5}$$

Introducing the symbols S and S' we may express the net force along the channel-bed acting on the element $ABCD$ as,

$$wh\,(S - S')\,dx \tag{6}$$

It has been found experimentally that the friction on the channel surface varies nearly as the square of the velocity and inversely as the mean depth, h,

$$S' = \frac{Ku^2}{h} \tag{7}$$

Writing Chezy's formula $u = C\sqrt{hS'}$ and solving for S',

$$S' = \frac{u^2}{C^2h}$$

we obtain the relation between the constants K and C,

$$K = \frac{1}{C^2}$$

Substituting (7) in (6) and combining with (5) and (4) we now have,

$$\rho\, h\, dx\frac{du}{dt} = -wh\frac{\delta h}{\delta x}dx + wh\left(S - \frac{Ku^2}{h}\right)dx \tag{8}$$

Simplifying and substituting the relation $\frac{du}{dt} = \frac{\delta u}{\delta t} + u\frac{\delta u}{\delta x}$ we obtain the equation of motion for a simple flood wave in the form,

$$\frac{\delta u}{\delta t} + u\frac{\delta u}{\delta x} + g\frac{\delta h}{\delta x} = g\left(S - \frac{Ku^2}{h}\right) \tag{9}$$

5. *Physical and Geometrical Concepts.*—For steady flow the mean velocity, u, at a transverse section is independent of the time and depends entirely on the location of that section as determined by the coordinate x.

For unsteady flow in a flood wave the velocity at a fixed section, $x =$ constant, does not remain constant and independent of time. However, at a given instant t_1, we can visualize the entire wave by an instantaneous "snap-shot" of the profile. At any given instant as x varies both h and u vary as in the case of steady flow. When the flow is unsteady the mean velocity at any given section depends on both the abscissa x and the time t.

Similarly, we can show that for unsteady flow, h, q, and p also depend on the two variables x and t.

The variables x and t are not entirely independent of each other, for if x varies so as to be always the abscissa of a transverse section moving with the liquid, the derivative of x with respect to t is the mean velocity u:

$$\frac{dx}{dt} = u \tag{10}$$

Let u be expressed as a function of two variables in the form,

$$u = f(x, t) \tag{11}$$

The total differential of u is

$$du = \frac{\delta u}{\delta x}\,dx + \frac{\delta u}{\delta t}\,dt \tag{12}$$

This equation holds true for any choice of independent variables and furthermore, it is valid when a functional relation exists between the two variables, x and t. Dividing (12) by dt and substituting Equation (10) we obtain

$$\frac{du}{dt} = \frac{\delta u}{\delta x}\,u + \frac{\delta u}{\delta t} \tag{13}$$

which is the total derivative of u with respect to t.

In Equation (13) the term $\frac{\delta u}{\delta t}$ is the time rate of change of velocity u at some fixed transverse section where $x = x_1$; the term $\frac{\delta u}{\delta x}$ is the change in the velocity due to a variation in x along the fixed profile obtained as a "snap-shot" at the instant $t = t_1$; and the term $u\frac{\delta u}{\delta x}$ is that part of the total derivative contributed by the changing velocity of the element of fluid in its onward course. The total derivative, du/dt, is the time rate of change of velocity when both x and t vary so as to satisfy the relation $u = dx/dt$.

Fig. 3 (a) shows the geometrical significance of Equation (13). Choose a point, P, with coordinates t, x, and u, on the surface

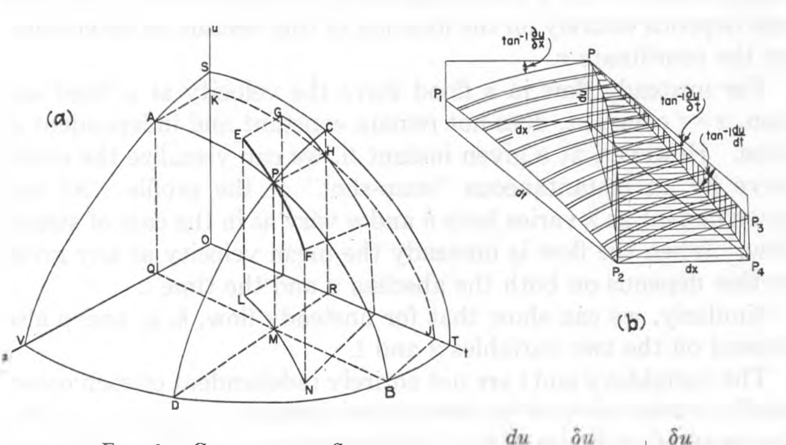

Fig. 3. Geometrical Significance of $\frac{du}{dt} = \frac{\delta u}{\delta x}\,u + \frac{\delta u}{\delta t}$

ABCDP. Let the equation of this surface be $u = f(x, t)$, referred to three mutually perpendicular axes. Represent the traces of the surface on the coordinate planes by the lines *SCT*, *TDV*, and *VAS*; let the curve *LMN* whose equation is $x = \phi(t)$ qualify a relation between x and t such that $u = \frac{dx}{dt}$; the mean velocity is now represented by either the slope of the tangent to the curve *LMN*, or the coordinate *MP*.

By passing a plane through *P* parallel to the *ut*-coordinate plane we obtain *APB*, as the curve of intersection with the given surface $u = f(x, t)$. As *P* moves along this curve x is fixed and u is a function of t alone; it follows that the slope of the tangent to *APB* at *P*, is $\frac{\delta u}{\delta t}$, the partial derivative of u with respect to t.

In a similar manner the slope of the tangent to the curve *CPD* at the point *P* is $\frac{\delta u}{\delta x}$, the partial derivative of u with respect to x.

If we erect a cylindrical surface perpendicular to the *xt*-plane, through the curve *LMN* which is defined by $x = \phi(t)$, the cylinder will intersect the given surface, $u = f(x, t)$, in the curve *EPF*. The point *P* must move so that it is always on the curve *EPF*, since it is only for points here that the relations $x = \phi(t)$ and $u = \frac{dx}{dt}$ are satisfied.

The curve *GHI* in the *ut*-plane corresponds to the curve *LMN* in the *xt*-plane, and is obtained as the trace of a cylinder parallel to the *ox*-axis through *EPF*. The curve *KHJ* is the *ut*-trace of a cylinder through *APB* parallel to the *ox*-axis. These two traces in the *ut*-plane intersect at the point *H*.

Finally, the total derivative du/dt at the point *P* is represented by the slope of the tangent line to the curve *GHI* at the point *H*, just as the dx/dt, the total derivative of x with respect to t, was represented by the slope of the tangent to the curve *LMN* at the point *M*.

Fig. 3 (*b*) shows an enlarged view of an element in the neighborhood of *P*. Here $PP_1P_2P_3$ is an elementary part of the surface $u = f(x, t)$. The curve PP_2 is an element of the space curve *EPF*. The surface PP_2P_4 represents a small part of the surface of the projecting cylinder whose trace is the curve *GHI*. The rest of the diagram is self-explanatory.

In the foregoing discussion, Fig. 3 was used to interpret the different parts of Equation (13) in which the velocity, u, is a function of x and t. Since the depth of flow, h, the rate of discharge per unit width, q, and the bottom pressure, p are also functions of x and t for unsteady flow accompanying a flood wave, we may similarly obtain the following equations by a simple change of notation:

$$\left.\begin{aligned} \frac{dq}{dt} &= \frac{\delta q}{\delta x} u + \frac{\delta q}{\delta t} \\ \frac{dp}{dt} &= \frac{\delta p}{\delta x} u + \frac{\delta p}{\delta t} \\ \frac{dh}{dt} &= \frac{\delta h}{\delta x} u + \frac{\delta h}{\delta t} \end{aligned}\right\} \qquad (14)$$

We may interpret the last equation of (14) by means of a simple two-dimensional vector diagram as shown in Fig. 4. In a short interval of time, dt, consider a wave front to have moved forward without appreciable change in shape, from 1-1 to 2-2. In this in-

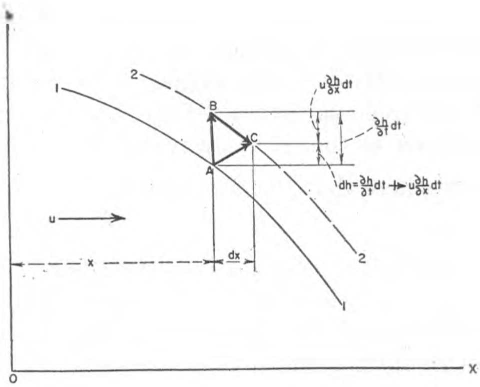

FIG. 4. GEOMETRICAL SIGNIFICANCE OF $\frac{dh}{dt} = u\frac{\delta h}{\delta x} + \frac{\delta h}{\delta t}$

terval of time a particle on the wave front has moved from A to C. The displacement AC is equal to the sum of the two vector displacements AB and BC. Let us restrict our attention to the vertical component of this displacement, AC, which is equal to dh. In the time interval dt, the water surface at station x rises from A to B, an amount which is represented by $\frac{\delta h}{\delta t} dt$. At station x also, the slope of the water surface (negative) at any instant, is $\frac{\delta h}{\delta x}$. The horizontal component of the displacement of a point on the water surface is $u \cdot dt$ in the time interval under consideration, and its vertical component is $\frac{\delta h}{\delta x}\,dx$. Adding vertical components of displacement from A to C with proper regard to sign, it is evident that

$$dh = \frac{\delta h}{\delta x}\,dx + \frac{\delta h}{\delta t}\,dt \qquad (15)$$

Dividing both sides by dt, we obtain

$$\frac{dh}{dt} = \frac{\delta h}{\delta x}\,u + \frac{\delta h}{\delta t} \qquad (16)$$

This is the differential equation for the vertical component of velocity of a particle A on a wave front.

II. Solutions of the Differential Equations

6. *Profiles and Stages.—*

(a) Backwater Curves — Steady Flow

An interesting determination of the backwater function is obtained from the equations of the simple flood wave. The channel is assumed to be wide and rectangular in section. Backwater is a case of steady flow, which, by definition, is equivalent to

$$hu = q = \text{constant.} \qquad (17)$$

Differentiating,

$$h \cdot du + u \cdot dh = 0 \qquad (18)$$

Since x is the only independent variable, Equation (9) becomes

$$u \cdot du + g \cdot dh = g\,(S - \frac{Ku^2}{h})\,dx \qquad (19)$$

Substituting in Equation (19) expressions for u and du obtained from Equation (17) we can now write,

$$\frac{q}{h}\,(\frac{-q \cdot dh}{h^2}) + g \cdot dh = g\,(S - \frac{Kq^2}{h^3})\,dx \qquad (20)$$

or, after simplifying and rearranging,

$$\frac{dh}{dx} = (S - \frac{Kq^2}{h^3})/(1 - \frac{q^2}{gh^3}) \qquad (21)$$

Let

$$q^2 = C^2S\,h_n{}^3 \text{ and } \frac{h}{h_n} = z \qquad (22)$$

where C is the coefficient in Chezy's formula and h_n is the neutral depth.

Equation (21) now takes the form,

$$dx = \frac{h_n}{S}\,(1 + \frac{1 - C^2S}{g\,(z^3 - 1)})\,dz \qquad (23)$$

which integrates into the backwater function,

$$x = \frac{h_n z}{S} - h_n \left(\frac{1}{S} - \frac{C^2}{g}\right)\left[\frac{1}{6} \log_e \frac{z^2 + z + 1}{(z-1)^2} - \frac{1}{\sqrt{3}} \cot^{-1} \frac{2z+1}{\sqrt{3}}\right] + C_1 \qquad (24)$$

in which C_1 is a constant of integration.

(b) The Roll Wave — Unsteady Flow

When a flood moves down a dry channel, the wave front moves in a steeply inclined wall of water whose profile is apparently unchanging so long as the channel conditions remain fixed and the source of supply is constant. This kind of wave, which is called the roll wave, may be generated by the failure of a dam which suddenly releases the impounded water from its reservoir into a dry channel.

In this discussion it is assumed that the profile of the roll wave does not change its form and that the motion of the water approximates that of a sliding solid body without relative motion between any of its parts.

Accordingly,

$$\frac{\delta u}{\delta x} = \frac{\delta u}{\delta t} = 0 \qquad (25)$$

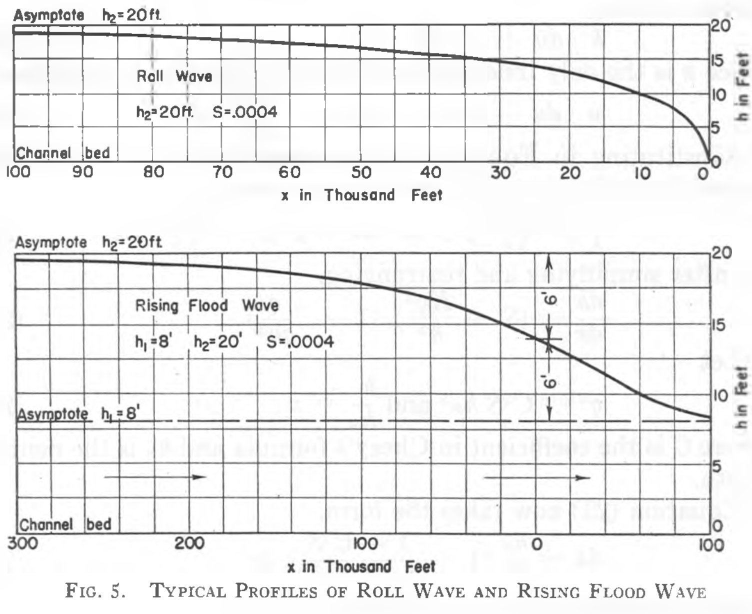

FIG. 5. TYPICAL PROFILES OF ROLL WAVE AND RISING FLOOD WAVE

and the Equation of Continuity (3) becomes

$$\frac{\delta h}{\delta t} + u \frac{\delta h}{\delta x} = 0 \qquad (26)$$

The Equation of Motion (9) may be simplified into the form,

$$\frac{\delta h}{\delta x} = (S - \frac{Ku^2}{h}) \qquad (27)$$

Rearrange and write in differential notation with the understanding that $t = t_1$.

$$\frac{dh}{S} + \frac{K \cdot u^2 \cdot dh}{S\,(Sh - Ku^2)} = dx \qquad (28)$$

The integral of Equation (28) is

$$\frac{h}{S} + \frac{Ku^2}{S^2} \log\,(Ku^2 - Sh) = x + C_1 \qquad (29)$$

Substituting from Chezy's formula,

$$u = C\sqrt{h_2 S} = \sqrt{\frac{h_2 S}{K}} \qquad (30)$$

where h_2 is the depth at the crest of the wave where the flow is substantially uniform, Equation (29) may now be written,

$$\frac{h}{S} + \frac{h_2}{S} \log S\,(h_2 - h) = x + C_1 \qquad (31)$$

Let $x = 0$ where $h = 0$. It follows that the constant of integration is,

$$C_1 = \frac{h_2}{S} \log S\, h_2$$

and

$$x = \frac{1}{S}[h + h_2 \log\,(1 - \frac{h}{h_2})] \qquad (32)$$

A profile of the roll wave obtained from this equation is shown in Fig. 5 (a).

7. *Sequence of Maximum Velocity, Discharge, and Stage*[3].—In this discussion as heretofore it is assumed that the river channel is rectangular in cross-section, and that the width is large in comparison with the depth. It is assumed also that the flood rises gradually to a unique maximum and then recedes slowly to uniform flow at neutral depth.

During the flood period the flow is unsteady, q varies with both x and t, and we can plot the discharge per unit width, q, at a given station, x_1, as a function of time, t. In Fig. 6 (a), the curve OMN is the flood hydrograph for one station and $O'M'N'$ is the hydrograph for the same flood at another station. No water is added or

[3]In the presentation of this section as far as Equation (36) substantial use has been made of a study compiled by M. Kleitz, "Note sur la theorie du movement non permanent des liquides et sur application a la propagation des crues des rivieres," *Annales des Ponts et Chaussees*, 1877, 2 Semestre, pp. 133-196.

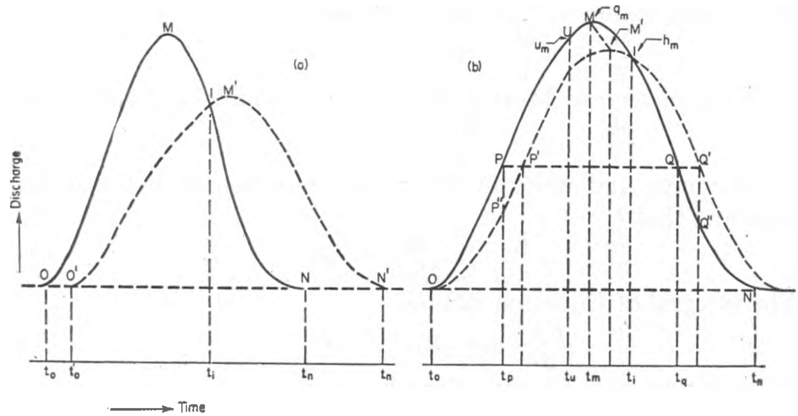

FIG. 6. SKETCHES OF FLOOD HYDROGRAPHS

withdrawn between the two stations. The ordinate of the line $OO'NN'$ represents the normal flow before and after the flood.

Subtract the area $O'INO'$, from the two equal areas $OMNO$ and $O'M'N'O'$. The areas $OMIO'O$ and $N'NIM'N'$ are equal and either represents the maximum storage between the two stations for the given flood.

The curves in Fig. 6 (*a*) show hydrographs for two stations separated by a differential distance, dx.

Consider the discharge Pt_p observed when the flood is rising, and an equal discharge, Qt_q, when the flood is receding. The time interval PP' is less than QQ', since for a given rate of discharge the mean velocity u is greater when the river is rising than when it is falling. Since the areas $OPIP'O$ and $NQIQ'N$ are equal, and since PP' is less than QQ', the point of intersection I will fall to the right of the two maxima M and M' and *simultaneous equal rates* of discharge at the two stations occur *after* the peak rate has occurred at both stations.

The discharge, q, at the time, t_i, is the same at the two stations and at this instant q does not change with x, or in symbols,

$$\frac{\delta q}{\delta x} = 0 \tag{33}$$

Substituting Equation (33) in the Equation of Continuity (2), we obtain, $\frac{\delta h}{\delta t} = 0$, which indicates that at a given station the flood has reached its maximum height. This maximum occurs at the time t_i which is later than the time of maximum flow, represented in the figure by t_m.

At a given station, differentiating the equation $q = hu$ with respect to the time, we obtain

$$\frac{\delta q}{\delta t} = u \frac{\delta h}{\delta t} + h \frac{\delta y}{\delta t} \qquad (34)$$

where u and h are positive.

In Equation (34), if $\frac{\delta q}{\delta t} = 0$, it is clear that both $\frac{\delta h}{\delta t}$ and $\frac{\delta u}{\delta t}$ must be zero or they must be of opposite signs. But we have shown that the maximum flow at a given station precedes the maximum stage. When $\frac{\delta u}{\delta t} = 0$, the stage h is still increasing and therefore $\frac{\delta h}{\delta t}$ must be positive. Since $\frac{\delta u}{\delta t}$ is negative, it follows that the velocity is decreasing and that it has therefore already taken on its maximum value. Hence the maximum velocity precedes the maximum discharge.

Finally for the simple flood wave under consideration, the maxima occur in the order: (1) maximum average velocity; (2) maximum discharge; and (3) maximum stage.

Evidently the relation between stage and discharge is single valued only in the case of steady flow. During a flood the discharge for a given gage height has two different values depending on whether the stage is rising or falling.

In some rivers it is often observed that the maximum stage of the flood remains almost stationary over a considerable period of time. In this case it is apparent that the maximum values of q and h occur simultaneously and $\frac{\delta h}{\delta t} = 0$ when $\frac{\delta q}{\delta t} = 0$. Also from Equation (34) $\frac{\delta u}{\delta t} = 0$. It follows that the maxima of h, q, and u may be regarded as occurring simultaneously.

8. *Velocity of a Flood Wave.*—There are at least three interpretations of the velocity of a flood wave:

(1) The velocity interpreted as the rate of movement of the discontinuities in the first derivative at either the beginning or the end point of the wave;

(2) The velocity as the rate of progression of the wave's peak or maximum height;

(3) The velocity as the virtual movement of a constant flow or stage height. This interpretation will be considered in greater detail.

(a) Virtual Velocity of a Constant Flow

The differential of q is

$$dq = \frac{\delta q}{\delta x}\, dx + \frac{\delta q}{\delta t}\, dt \tag{35}$$

If q now be regarded as constant we may obtain from Equation (35) a relation between x and t satisfying the condition $dq = 0$. We may solve for $\frac{dx}{dt}$ and obtain a virtual velocity representing the time rate of displacement of the cross-section at which the discharge is the constant q.

Substituting Equation (2) into Equation (35) it follows that,

$$-\frac{\delta h}{\delta t}\, dx = \frac{\delta q}{\delta t} dt$$

and the virtual velocity V_q of the section of constant flow is,

$$V_q = \frac{\delta x}{\delta t} = (\frac{\delta q}{\delta t})/(\frac{\delta h}{\delta t}) = u + h\, \frac{\delta u}{\delta t}/\frac{\delta h}{\delta t} \tag{36}$$

in which the last step was the substitution of Equation (34).

We note from Equation (36) that the speed of propagation, V_q, of a given flow, q, is greater than the mean speed, u, throughout the first period of the flood when both u and h are increasing and when their derivatives are therefore positive; V_q becomes equal to the mean speed when the latter is a maximum, since at this time, $\frac{\delta u}{\delta t} = 0$; then $\frac{\delta u}{\delta t}$ and $\frac{\delta h}{\delta t}$ take on opposite signs, V_q becomes smaller than the mean speed and approaches zero when q is a maximum, where $\frac{\delta q}{\delta t} = 0$. That V_q is here zero, simply indicates that the maximum flow then produced does not have its equivalent farther downstream. During the period between the maximum flow and the maximum height, V_q is negative. At the time of maximum stage, when $\frac{\delta h}{\delta t} = 0$, V_q is mathematically discontinuous and changes sign by passing through infinity. Physically this means simply that for an instant the same flow occurs at two transverse sections situated very near to each other.

(b) Virtual Velocity of the Maximum Flow

If q is a function of x and t, it follows that $\frac{\delta q}{\delta t}$ will in general also be a function of x and t, as expressed by,

$$\frac{\delta q}{\delta t} = \phi\ (x, t) \tag{37}$$

The flow is a maximum when $\phi\ (x, t) = 0$. Differentiating:

$$\frac{\delta\phi}{\delta x}\,dx + \frac{\delta\phi}{\delta t}\,dt = 0 \tag{38}$$

The virtual velocity of the maximum flow is

$$V_m = \frac{dx}{dt} = -\frac{\dfrac{\delta\phi}{\delta t}}{\dfrac{\delta\phi}{\delta x}} = -\frac{\dfrac{\delta^2 q}{\delta t^2}}{\dfrac{\delta^2 q}{\delta t\,.\,\delta x}} = \frac{\dfrac{\delta^2 q}{\delta t^2}}{\dfrac{\delta^2 h}{\delta t^2}} \tag{39}$$

since from the Equation of Continuity,

$$\frac{\delta q}{\delta x} = -\frac{\delta h}{\delta t}$$

The speed expressed by Equation (39) differs essentially from the speed of propagation of an assumed flow as given by Equation (36). In the former we consider successively the same rate of flow at different sections, while in the latter we consider at each section only the maximum rate of flow which may not be the same at all stations along the stream.

(c) Wave With Constant Velocity

Assuming the virtual velocity V_q is constant and equal to V_c for any value of x,

$$V_c = \frac{\dfrac{\delta q}{\delta t}\,dt}{\dfrac{\delta h}{\delta t}\,dt} = \frac{dq}{dh} \tag{40}$$

and replacing the differentials by finite differences

$$V_c = \frac{q_2 - q_1}{h_2 - h_1} = \frac{Q_2 - Q_1}{A_2 - A_1} \tag{41}$$

where A_1 and A_2 represent the areas of the transverse sections at normal and the highest stage, and where Q_1 and Q_2 represent the corresponding flows.

(d) Wave with Small Amplitude

The velocity of a small flood wave may be obtained from Equation (40), by substituting $q = hu$ as follows:

$$V_c = \frac{h\,du + u\,dh}{dh} = u + h\,\frac{du}{dh} \tag{42}$$

The river is assumed to be in an initial state of uniform flow with discharge q_1 and stage h_1. The discharge is then increased by a small amount which causes a correspondingly small increase in stage height.

Assuming the velocity u varies as the square root of the depth of flow, h, as given by the Chezy formula, it can be shown that

$$h\frac{du}{dh}=\frac{u}{2} \tag{43}$$

and

$$V_c=u+\frac{u}{2}=\frac{3}{2}u \tag{44}$$ [4]

where u and h may to a first approximation be replaced by u_1 and h_1 determined from low stage. Hence, it would appear that the velocity for a small wave is considerably greater than the mean velocity.

(e) The Roll Wave

The velocity V_q for the roll wave may be obtained very simply from Kleitz's Equation (36) where $u = K$ and $\frac{\delta u}{\delta t} = 0$. It follows that V_q is equivalent to the mean channel velocity, a result that might easily have been anticipated.

9. *Russell's Solitary Wave of Translation.*—The velocity of Russell's solitary wave of translation[5] corresponds closely to the velocity of the flood wave near its genesis. Concerning his wave Mr. Russell wrote as follows:

"I believe I shall best introduce this phenomenon by describing the circumstances of my own first acquaintance with it. I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped — not so the mass of water in the channel which it had put in motion; it accumulated around the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth, and well defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed."[6]

Russell's wave of translation is positive, consisting wholly of an elevation without the association of any trough or depression whatever. It lies entirely above the normal water surface and progresses smoothly and quietly without turbulence at any place along its profile.

The solitary wave of translation may be generated experimentally by quickly pouring water in one end of a rectangular trough

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[4]Assuming Manning's formula, in which u varies as $h^{\frac{2}{3}}$, the wave velocity $V_c=\frac{5}{3}u$.

This phase has also been treated by PHILIP FORCHHEIMER, Hydraulik, Third Edition, p. 298 (1930).

[5]Discovered by J. Scott Russell in 1834.

[6]Russell, J. Scott, "Report on Waves," *Report* of the British Association for the Advancement of Science, pp. 311-390, 1844.

containing some quiescent water, by dropping into the water a weight built so as to fit snugly into the flume, or by other means. The water, displaced from under the weight, forms a wave which moves rapidly away.

Mr. Russell found on the basis of experiments that the velocity, V, of the solitary wave of translation, could be computed by the formula,

$$V = \sqrt{gh_2} \qquad (45)$$

in which

g = acceleration of gravity

h_2 = height of wave crest above bottom of channel.

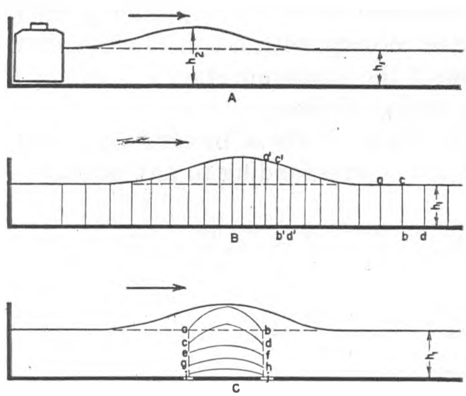

Fig. 7. The Solitary Wave of Translation (After Russell)

The solitary wave of translation derives its name from the horizontal translatory motion of the water below the normal water level and not because of the motion of the visible wave on the surface. Fig. 7, adapted from drawings in Mr. Russell's report, will serve to illustrate the action of this wave. Fig. 7 (*a*) represents the wave just after its genesis by lowering a weight into one end of the flume. Fig. 7 (*b*) illustrates the crowding or squeezing of the elements *ab*, *cd*, etc., situated below the plane of repose. As the wave proceeds these elements rise into thinner columns and then fall as the wave passes, always changing in such a manner as to retain an approximately constant volume. Fig. 7 (*c*) shows the actual paths, *ab*, *cd*, etc., of the water elements during the wave's passage, which results in a constant horizontal displacement of each element.

Russell found that the highest wave which could be formed was one whose height above the normal water surface was equal to the normal depth h_1, and that any additional height imposed w uld invariably result in a breaking up of the profile of the wave.

Bazin's experiments[7] on the propagation of Russell's wave of translation in a canal containing flowing water demonstrated that the velocity of the peak of the wave was closely represented by the formula,

$$V = \sqrt{gh_2} \pm u \tag{46}$$

where u represents the velocity of the water in the canal. The plus sign is to be used when the wave travels with the current and the minus sign when it travels against the current.

10. *The Monoclinal Wave.*—The profile of the rising or monoclinal flood wave moving with a constant velocity can be determined as follows if the maximum stage is assumed to be fixed over a considerable period of time.

Replacing the lower limits of integration of Equation (41) by general values q, h, corresponding to any position along the wave front,

$$V_c = \frac{q_2 - q}{h_2 - h} = \frac{h_2 u_2 - hu}{h_2 - h} \tag{47}$$

from which,

$$u = V_c - \frac{h_2}{h}(V_c - u_2) \tag{48}$$

But from Chezy's formula,

$$u_2{}^2 = C^2 h_2 S \text{ and } u_1{}^2 = C^2 h_1 S \tag{49}$$

or, substantially,

$$h_1 = \frac{u_1{}^2}{u_2{}^2} h_2 \tag{50}$$

since the two values of C will be nearly the same. Hence, from Equation (47),

$$V_c = \frac{h_2 u_2 - u_1{}^3 h_2 / u_2{}^2}{h_2 - h_1} = \frac{(1 - y_1{}^{\frac{3}{2}})\, u_2}{1 - y_1} = K_1 u_2 \tag{51}$$

where

$$y_1 = \frac{h_1}{h_2} = \frac{u_1{}^2}{u_2{}^2}$$

The mean velocity, u, may now be expressed,

$$u = V_c - \frac{h_2}{h}(V_c - u_2) = [K_1 - (K_1 - 1)\frac{h_2}{h}]\, u_2 \tag{52}$$

The wave under consideration is long, and it is evident that the

[7]Bazin, H., Experiments on the Solitary Wave in Running Water, *Comptes Rendus des Seance de l'Academie des Sciences*, 15, Juin 1885, Tome C, p. 1492.

mean velocity, u, will change very slowly with respect to both the time, t, and the abscissa, x. We may regard the expression,

$$\frac{du}{dt} = \frac{\delta u}{\delta t} + u\frac{\delta u}{\delta x} \tag{53}$$

as representing a negligibly small quantity. Hence the equation of motion for a simple flood wave (Equation 15) becomes

$$\frac{\delta h}{\delta x} = S - \frac{K\,u^2}{h} \tag{54}$$

Substituting Equation (52) into Equation (54)

$$\frac{\delta h}{\delta x} = S - \frac{K}{h}\,[K_1 - (K_1 - 1)\,\frac{h_2}{h}]^2 u_2{}^2 \tag{55}$$

But,

$$K = \frac{1}{C^2} = \frac{h_2\,S}{u_2{}^2} \tag{56}$$

Then,

$$\frac{\delta h}{\delta x} = S - \frac{h_2 S}{h}\,[K_1 - (K_1 - 1)\,\frac{h_2}{h}]^2 \tag{57}$$

To arrange for integration, let $t = t_1$, and use differentials

$$dx = \frac{dh}{S\left\{1 - \frac{h_2}{h}\,[K_1 - (K_1 - 1)\,\frac{h_2}{h}]^2\right\}} \tag{58}$$

If $h = h_2 z$ it can be shown that

$$dx = \frac{h_2}{S}\left\{1 + \frac{K_1{}^2\,z^2 - 2\,K_1\,(K_1 - 1)\,z + (K_1 - 1)^2}{(z - 1)\,[z^2 - (K_1 - 1)\,z + (K_1 - 1)^2]}\right\}dz \tag{59}$$

This equation, integrated after expanding into partial fractions, becomes the following equation of the profile of the wave

$$x + C_1 = \frac{h_2}{S}\left\{z + \frac{1}{3 - 2\,K_1}\log\,(1 - z) - \frac{(K_1 - 1)^2(2\,K_1 + 1)}{2\,(3 - 2\,K_1)}\log\,[z^2 - (K_1 - 1)\,z + (K_1 - 1)^2] - \frac{(K_1 - 1)^2\,[(2\,K_1 + 1)\,(K_1 + 1) - 4}{2\,(3 - 2\,K_1)\sqrt{(K_1 + 1)^2 - 4}}\log\frac{2z - (K_1{}^2 - 1) - (K_1 - 1)\sqrt{(K_1 + 1)^2 - 4}}{2z - (K_1{}^2 - 1) + (K_1 - 1)\sqrt{(K_1 + 1)^2 - 4}}\right\}$$

(60)[8]

[8]For another method of developing Equation (60), see "Prediction of Floods," by P. Y. Lin, Master's thesis in Mechanics and Hydraulics, State University of Iowa, 1925.

in which

$z = h/h_2$

h_2 = the depth at high stage

h = the depth at any point along the wave front

$K_1 = V_c/u_2$

u_2 = the mean velocity of a section at high stage, and

V_c = the virtual velocity of a constant flow moving at a constant rate.

The constant of integration, C_1 is determined from the condition that $x = 0$ when $h = \frac{h_1 + h_2}{2}$.

The profile of this wave, plotted from Equation (60), is shown in Fig. 5(*b*).

III. Hydraulic Laboratory Tests

11. *Laboratory Observations of Wave Velocities.*—The purpose of the author's experiments was to determine the maximum velocity of a flood wave just after it is formed and before it becomes stable.

The apparatus consisted of a rectangular wood flume 52 ft. long, 8-1/16 in. wide, and 19 in. deep. The upper end of the flume projected into a rectangular tank 7 ft. 3 in. long, 3 ft. 8-1/2 in. wide, and 3 ft. 2 in. deep which was divided into two compartments by 3/4 in. by 1-3/4 in. wooden baffles placed vertically at 1-1/4 in. centers. The upper compartment served as a stilling chamber which received its water supply from a 6 in. pipe, while the lower compartment was a forebay to the flume.

The tank and flume were supported on jacks which could be adjusted to change the gradient of the flume.

An enameled metal gage graduated to hundredths of a foot with its zero at the bottom of the tank was attached to the side of the forebay and was used to measure the stage of the water just back of a sliding headgate across the upper end of the flume. Similar gages were placed in the flume at 10 ft. intervals at stations designated by 0, 1, 2, 3, 4. Station 0 was 6 ft. 9 in. from the headgate, and station 4 was 5 ft. 1 in. from the outfall.

The headgate was arranged so it could be opened quickly by pulling a rope which was attached to the top of the gate. Guided by vertical wooden cleats, the gate could be moved up or down in small increments.

Water used in the tests was drawn from the laboratory's circulating system under a constant head of about 40 ft. and was regulated manually by a 6-in. gate valve. The gate valve was calibrated before each experiment by use of a venturi meter placed in the supply line which was read by a mercury gage.

The services of five men were necessary to produce and observe a flood wave. During a test these men occupied positions as follows: one observer at station 0; one observer at station 4; one operator for the gate valve in supply pipe; one assistant to read the gage in the forebay and to regulate the sliding gate when necessary; and one timekeeper who also read the mercury gage of the venturi meter.

The observer at station 0 read gage heights before and after passage of the wave. He signaled the timekeeper the instant the toe of the flood wave reached his station, indicating the beginning of the time interval for the wave to traverse the 40-ft. reach. The observer at station 4 signaled the timekeeper as the toe of the wave passed his station, closing the time interval for the wave travel. The observer at station 4 also read gage heights which were used as checks mainly, since stages at this station were affected appreciably by the drop-down curve.

At the beginning of a test the depth, h_1, was read at station 0 and at station 4 for a steady initial flow, q_1, indicated by the venturi meter. Next the gate valve was slowly opened further until the venturi meter indicated that a suitable increment of flow, Δq had been added. The setting of the valve was noted and after the flow became steady the staff gages in the forebay and at stations 0 and 4 were read. The latter reading of stage at station 0 was taken as the flood height and designated as h_2.

The head-gate, which up to this time had been entirely open, was then placed across the upper end of the flume to form a submerged orifice control. The gate valve was opened intermediate to its first and second positions, previously determined, and the flow into the forebay was somewhat greater than that necessary to produce in the flume a normal stage, h_1. As the forebay filled due to the excess inflow over outflow, the headgate was adjusted to maintain constantly at station 0 the normal stage, h_1. When the stage in the forebay reached the height previously recorded corresponding to flood stage h_2, the headgate was quickly raised and simultaneously the gate valve in the supply line was opened to the second position corresponding to flood stage supply. The wave moved forward with rapidly changing profile and the velocity of its toe was determined by observing the elapsed time in traversing a 40-foot reach as previously described.

After the wave had passed and the flow had become steady, the stage at station 0 was read and checked with the recorded flood height h_2.

The wave was quite smooth excepting at its toe where foaming occurred on the breaker. The slope of the wave at its toe was nearly vertical. When the wave reached station 4, however, its height was small, varying perhaps from one inch for the smaller waves to a maximum of three inches for the largest waves generated. The turbulent region was less pronounced as the wave progressed.

The slope of the wave, which was nearly vertical at the headgate, quickly flattened as it moved along the flume. At station 4 the slope was still changing appreciably and had not yet reached a condition of equilibrium. The profile of the wave was convex upwards in every instance.

The tests included variations of flow from 100 to 1200 gallons per minute, flood wave increments of 300 to 1100 gallons per minute, and having velocities from 5.9 to 10.5 feet per second. The slope of the test flume in Series A and B was .35 per cent, in Series C, 1.0 per cent, in Series D, 0.0 per cent, and in Series E, 3.0 per cent.

The laboratory data are summarized in Table 1 and observed wave velocities are compared with velocities computed by Bazin's formula

$$V = \sqrt{gh_2} + u_1 \qquad (61)$$

in which

V = velocity of wave, ft. per sec.
u_1 = velocity of initial flow, ft. per sec.
h_2 = depth of flow at wave crest, ft.
g = acceleration of gravity, ft. per sec.^{2}

Although the observed and computed velocities are in close agreement especially when the flume was set at the flatter gradients, it should be observed that the laboratory waves differ from Russell's wave in several important points:

1. The laboratory wave is a half positive wave with a fixed maximum stage h_2, while Russell's wave is a complete positive wave whose maximum stage is slowly attenuating.

TABLE 1. SUMMARY OF TEST DATA. WAVE VELOCITIES IN LABORATORY CHANNEL 8-1/16 IN. WIDE

Series A and B; 0.35 per cent Gradient

Test No.	h_1 ft.	u_1 fps	q_1 gpm	q_2 gpm	$\Delta q = q_2 - q_1$	h_2 ft.	V_o fps	V' fps
1	.11	3.02	100	875	775	.60	7.14	7.42
2	.12	2.76	100	875	775	.65	7.14	7.33
3	.11	3.02	100	875	775	.71	7.14	7.80
4	.12	2.76	100	1200±	1100±	.81	8.68	7.86
5	.13	2.55	100	1200±	1100±	.78	8.68	7.56
6	.14	2.37	100	1200±	1100±	.79	8.34	7.41
7	.11	3.02	100	600	500	.44	6.45	6.78
8	.14	2.48	105	600	495	.44	7.14	6.24
9	.18	3.68	200	700	500	.51	7.68	7.74
10	.19	9.48	200	700	500	.50	7.68	7.49
11	.22	4.52	300	800	500	.55	8.34	8.73
12	.22	4.52	300	800	500	.56	8.00	8.77
13	.25	5.12	385	885	500	.60	9.10	9.51
14	.28	4.57	385	885	500	.74	8.68	9.45
15	.45	4.31	585	885	500	.67	8.34	8.96
16	.45	4.12	585	885	500	.68	8.00	8.80
17	.42	3.97	500	800	300	.58	8.00	8.29
18	.44	3.92	500	800	300	.58	8.00	8.24
19	.37	3.60	400	700	300	.58	7.41	7.92
20	.38	3.48	400	700	300	.62	7.14	7.95
21	.29	3.43	300	600	300	.47	7.14	7.32
22	.32	2.79	300	600	300	.52	7.41	6.88
23	.24	2.77	200	500	300	.43	6.90	6.49
24	.24	2.77	200	500	300	.44	6.67	6.53
25	.14	2.37	100	400	300	.39	6.06	5.91
26	.14	2.37	100	400	300	.38	5.88	5.87

Series D; Level Gradient

Test No.	h1 ft.	u1 fps	q1 gpm	q2 gpm	$\Delta q = q_2 - q_1$	h2 ft.	V_o fps	V' fps
1	.27	1.23	100	605	505	.71	6.67	6.02
2	.27	1.23	100	610	510	.67	6.45	5.87
3	.41	1.62	200	710	510	.81	7.14	6.74
4	.41	1.62	200	715	515	.83	7.14	6.79
5	.46	2.16	300	790	490	.87	7.41	7.45
6	.23	1.45	100	790	690	.83	7.68	6.62
7	.27	1.23	100	425	325	.59	6.06	5.59
8	.23	1.45	100	425	325	.59	6.06	5.81
9	.39	1.70	200	510	310	.67	7.14	5.88
10	.39	1.70	200	510	310	.67	6.90	5.88
11	.49	2.03	300	615	315	.73	8.00	6.88
12	.49	2.03	300	620	320	.75	7.68	6.95
13	.53	2.44	395	720	325	.81	7.41	7.54
14	.59	2.23	395	720	325	.83	8.00	7.40

Series C; 1.0 per cent Gradient

Test No.	h_1 ft.	u_1 fps	q_1 gpm	q_2 gpm	$\Delta q = q_2 - q_1$	h_2 ft.	V_o fps	V' fps
1	.11	3.02	100	650	550	.42	7.14	6.70
2	.10	3.32	100	640	540	.42	7.68	7.00
3	.16	4.13	200	700	500	.44	8.00	7.89
4	.16	4.13	200	705	505	.43	8.34	7.85
5	.20	4.99	300	810	510	.49	8.68	8.97
6	.20	4.99	300	800	500	.47	9.10	8.88
7	.24	5.13	370	875	505	.51	9.50	9.18
8	.24	5.19	375	875	500	.50	9.50	9.21
9	.10	3.32	100	875	775	.51	8.34	7.37
10	.11	3.02	100	875	775	.51	9.10	7.07
11	.10	3.32	100	415	315	.33	7.14	6.58
12	.10	3.32	100	425	325	.32	7.14	6.53
13	.16	4.13	200	485	285	.32	7.68	7.34
14	.16	4.13	200	430	230	.35	8.00	7.49
15	.21	4.74	300	625	325	.40	8.68	8.33
16	.21	4.74	300	625	325	.39	8.34	8.28
17	.28	4.68	395	695	300	.44	9.10	8.44
18	.28	4.68	395	695	300	.44	9.10	8.44
19	.31	5.25	490	795	305	.49	8.68	9.23
20	.31	5.31	495	790	295	.48	9.10	9.24

Series E; 3% Gradient

Test No.	h1 ft.	u1 fps	q1 gpm	q2 gpm	$\Delta q = q_2 - q_1$	h2 ft.	V_o fps	V' fps
1	.08	4.14	100	610	510	.35	9.50	7.50
2	.08	4.14	100	615	505	.33	9.50	7.40
3	.11	6.04	200	705	505	.41	10.50	9.67
4	.11	6.04	200	710	510	.41	10.50	9.67
5	.07	4.73	100	800	700	.45	10.00	8.54
6	.07	4.73	100	795	695	.45	10.00	8.54
7	.08	4.14	100	430	330	.27	8.34	7.09
8	.07	4.73	100	430	330	.27	8.00	7.68
9	.12	5.53	200	525	325	.31	9.50	8.69
10	.13	5.11	200	525	325	.29	10.00	8.17

List of Symbols

h_1 = initial depth of flow, ft.
u_1 = initial average velocity, ft. per sec.
q_1 = initial discharge, gpm.
q_2 = final discharge, gpm
Δq = $q_2 - q_1$ = increment of discharge, gpm.
h_2 = final depth of flow, ft.
V_o = observed wave velocity, ft. per sec.
v' = $u_1 - \sqrt{gh_2}$ = wave velocity computed by Bazin's formula, ft. per sec.

2. The profile of the laboratory wave near its genesis rapidly becomes flatter as it moves down the flume, while Russell's wave changes little in a prismatic channel.

3. The observed velocity of the laboratory wave was the average velocity of the toe of the wave as it traversed a 40-foot reach in the flume, while the velocity of Russell's wave computed by the same formula

$$V = \sqrt{gh_2} + u_1 \quad (62)$$

is the velocity of the crest of a translatory wave.

4. The laboratory wave near its genesis is everywhere convex upwards and there is usually a small turbulent region at its toe, while Russell's wave has two points of inflection and no trace of turbulence anywhere.

5. Usually the heights of the laboratory waves, $h_2 - h_1$, exceeded the normal stage, h_1, sometimes in ratio greater than five to one. In Russell's theory and experiments the maximum height of wave considered was just equivalent to the normal stage, h_1.

Although the laboratory waves cannot be classified as Russell's wave of translation projected on flowing water, there is a striking agreement between the velocities of these two types of waves.

IV. Conclusions

1. The profile of the roll wave in equilibrium may be obtained in terms of elementary functions from the general hydrodynamic equations of a simple flood wave.

2. The profile of the monoclinal or rising flood wave in equilibrium may be obtained in terms of elementary functions from the general hydrodynamic equations of a simple flood wave.

3. The velocity of the monoclinal or rising flood wave in equilibrium, may be obtained from Kleitz's equation for the velocity of a constant flow.

4. The theoretical minimum velocity of a flood wave is obtained as the velocity of a wave of small height. It is apparently greater than the mean velocity of the channel in normal flow.

5. The maximum velocity of waves observed in the laboratory experiments agreed with the velocity of a wave of translation computed by Bazin's modification of Russell's formula.

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