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Price response in multiple item choice: spillover effects of reference price

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PRICE RESPONSE IN MULTIPLE ITEM CHOICE:

SPILLOVER EFFECTS OF REFERENCE PRICE

by

Kyuseop Kwak

An Abstract

Of a thesis submitted in partial fulfillment
of the requirements for the Doctor of
Philosophy degree in Business Administration
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Thesis Supervisor: Professor Gary J. Russell
Assistant Professor Sri Devi Duvvuri

ABSTRACT

In this thesis, we develop a SKU level market basket model and apply the model to investigate cross-category reference price effects. This research extends previous work on the category-level multivariate logit model (Russell and Petersen 2000). Our model is a generalization of the multivariate logit model which allows for both complementarity and substitution effects at the brand level.

The modeling effort in this thesis allows us to use conditional probability distributions of individual items to construct the final joint-distribution of all possible basket selections. The resulting model is very flexible and accommodates a large variety of market structure patterns. The model structure implies that the changes in brand-level marketing variables directly affect category incidence (by altering category attractiveness) and indirectly determine market basket composition. Because the model can be written in a closed form manner, we can easily study the pattern of brand price competition by computing a matrix of cross-price elasticities. We use scanner panel data for the yogurt category to demonstrate the structural flexibility of the model. The results from this application reveal asymmetric competition consistent with price-tier competition literature.

We use this model to investigate how consumers' responses to reference prices within a category spillover into their choices across multiple categories. The notion is that a consumer's subjective judgment of the fairness of the price levels in one category influences the choice decisions of related items in other categories. We begin with building within-category SKU-level model based on previous findings from single category reference price models (i.e., internal versus external reference prices, asymmetric response due to loss aversion, and heterogeneity in response across consumers). We then develop four alternative model specifications for cross-category spillover effects and test competing theories about those effects. Using scanner panel data for detergent and softener categories, we discover valuable implications for reference price effects. First, SKU-level reference price effects exist and improve forecasting ability. Second, those reference price effects influence category attractiveness, but do no spillover across categories. Finally, category-level reference dependent evaluation may exist but not be important in forecasting.

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CERTIFICATE OF APPROVAL

PH.D. THESIS

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To my parents, parents-in-law, Kyunghee, Dahyun, and Layoung

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CHAPTER 1
INTRODUCTION

Individual item decisions in a market basket choice setting are generally not independent of one another. Household choice decisions may be related across product categories because product categories serve as complements or substitutes. These demand relationships arise due to household consumption needs and limited shopping budgets. Multiple category models, defined as choice models which capture these types of demand effects, are of interest to both retailers and manufacturers because they assist in the development of cross-category marketing strategies.

Multi-product relationships play a central role in many recent articles in the marketing science literature (Russell et. al. 1999, Seetharaman et. al. 2005). Often, these studies develop models which allow the identification and quantification of the cross-category effects of marketing mix variables. An important stream of research is primarily focused on the category incidence decision (Manchanda et al 1999, Russell and Petersen 2000, Chib et al 2002, Duvvuri et al 2007), ignoring individual item choice. Within a category, items are considered perfect substitutes, and single category multinomial logit (e.g., Guadagni and Little 1983) or multinomial probit (e.g., Ainslie and Rossi 1998) models are used to predict item choice. Recently, researchers have studied interdependence among brands across multiple categories within a single modeling framework (Singh et al 2005, Song and Chintagunta 2006, Ma et al 2005). During a shopping trip, households are assumed to be in a pick-any choice situation in which any subset of items may be chosen. However, these models retain the idea that at most one item from a category can be purchased on any one shopping occasion.

Previous researches reveal that consumers' choice decisions are affected by observable marketing mix such as shelf price as well as unobservable effects such as reference price (Winer 1986, Kalyanaram and Winer 1995). Reference price effects exist when consumers use the price of a focal product as an anchor to judge the fairness of prices. From a managerial point of view, reference price effects are important because they affect how retailers implement pricing strategies. In cross category decisions, therefore, consumers' subjective judgment of the fairness of the price levels in one category may influence the choice decision of related items in another category.

Prices of items in one category do not always move in same direction at one point of time, so category level price typically measured as a weighted average of prices of items in the category (Manchanda et al 1999, Duvvuri et al 2007), may not be enough to capture the price variations of the category. Moreover, consumers may construct a category level reference point not solely based on a single item but based on total category attractiveness (Bell and Bucklin 1999). Therefore, we need a model to account for category level cross-effects by integrating the effects of items within a category.

In this dissertation, we first develop a SKU (stock keeping unit) level choice model which permits a variety of demand relationships across items in market basket selection. We then apply the model to study cross-category reference price spillover effects at SKU level.

The rest of dissertation is organized as follows. In Chapter 2, we review previous literature about multiple item (or category) choice and reference price. In Chapter 3, we develop a general SKU-level model which will be used for the rest of the dissertation. The model is considered a generalization of previous multiple category model (Russell and Petersen 2000) to SKU choice. Drawing on this theory, we construct a SKU-level model for the yogurt product category which links product characteristics (brand and flavor) to the pattern of SKU substitution and complementarity. Results and implications are discussed. In Chapter 4, we apply the model to investigate spillover effects of reference price across multiple categories using detergent and softener categories. We test four alternative theoretical frameworks including our proposed model and discuss the results and implication. In Chapter 5, we conclude the dissertation.

CHAPTER 2
LITERATURE REVIEW

2.1 Models of Cross-Category Analyses

A number of research articles started to incorporate cross-category relationships in their choice models (for review, see Russell et al. 1999 and Seetharaman et al 2005). Many of them try to identify and quantify cross-category choice effects of marketing-mix variables. We can classify those studies into a hierarchy depending on the nature of choice decision and the choice outcome the study investigates.

First, we can distinguish the model which actually deals with "pick-any" choices of items across categories in a basket selection from a multi-category analysis using traditional single category choice models such as Multinomial Logit (MNL). This distinction is due to the difference between *multinomial* and *multivariate* choice decision.

Multinomial choice means that a consumer chooses one item among substitutable alternatives in consideration at a specific time, whereas multivariate choice implies a choice of multiple items on the same shopping occasion. In order to develop a model for multivariate choices, one should incorporate the effects of other items within or across categories on target item choice decision directly in the model. Multivariate logit (e.g., Russell and Petersen 2000, Song and Chintagunta 2006) or multivariate probit (e.g., Manchanda et al. 1999, Chib et al 2002) is used for this purpose. On the contrary, many studies employ correlation among marketing mix sensitivity (e.g., price coefficient) across categories, with the assumption of single category utility maximization (e.g., Ainslie and Rossi 1998, Erdem 1998, Singh et al 2005). An alternative approach to deal with a multiple category purchase can be found from product bundling or assortment literature (e.g., McAlister 1979, Farquhar and Rao 1976, Bradlow and Rao 2000).

We can also classify the literature in terms of choice outcomes the research focuses on. Many earlier models focus on multiple category purchase incidences (e.g., Russell and Petersen 2000, Manchanda et al 1999). Recently, brand (or SKU) choices across categories are main interests of the research (e.g., Singh et al 2005). Some models use aggregate level data (Song and Chintagunta 2006), while most of studies use household level disaggregated data. The main advantage of household level data is that we observe the distribution of joint purchasing in categories across households. We can, in contrast, only observe the marginal distribution of item purchases at aggregate data (e.g., store-level).

According to above classification scheme, the literature can be classified as in Figure 2.1. We only show key articles representing each of the final nodes. Note that most category incidence models are multivariate choice models. Moreover, we have not found a good representative multivariate probit model for brand-level choice outcomes. We rather found that some models adopt a hybrid approach in which multivariate probit is used at category-level while multinomial probit with correlated marketing mix parameters is used at brand-level (Chib et al 2005). The model developed for this dissertation is a multivariate logit model at SKU level, which is comparable to a few other papers, i.e., Song and Chintagunta (2006, 2007) and Ma et al (2005). We explain how our model is different from those models later in detail. We also review main characteristics of the models for product bundle or assortment choice even though we do not include those models in this diagram.

2.1.1 Multiple Category Incidence Models

The models in this group focus on dependence in category incidence decision across categories. Some research employ the panel data multivariate probit (MVP) model to explain household level multiple category incidence decision (Manchanda et al 1999; Chib et al 2002; Duvvuri et al 2007). Russell and Petersen (2000), meanwhile, develop the multivariate logistic model (MVL).

Manchanda et al (1999) derive an MVP model by assuming that households' indirect utilities for various product categories follow a joint normal distribution. The model includes both own-category marketing mix and cross-category marketing mix in a deterministic utility and allows correlation of error terms of utilities across categories. The authors call the estimated coefficients for the cross-category price *complementarity* and the estimated correlation of utilities *coincidence*. The study reveals significant cross-category effects of prices and promotions with two pairs of complementary categories (i.e., cake mix and frosting; detergent and softener). Chib et al (2002) extend the model to estimate households' incidence outcomes in twelve product categories simultaneously. They show that multi-dimensional MVP fits better than a two-dimensional probit as adopted in Manchanda et al (1999). The authors also show that ignoring unobserved heterogeneity across households leads one to overestimate cross-category correlation and underestimate the effectiveness of the marketing mix. Duvvuri et al (2007) extend the model to allow marketing mix sensitivities to be correlated across categories and Duvvuri and Gruca (2007) apply factor analytic approach to better summarize the correlation patterns. Those studies reveal very significant cross-category correlations of price sensitivities and high second order factor structure for substitutable categories. Russell and Petersen (2000), meanwhile, study cross-category dependence of four paper good categories using a multivariate logit (MVL) model which was adopted from spatial statistics literature. They find that purchase probability of four paper goods are correlated even though the magnitudes are very small.

2.1.2 Brand Choice Models

In this section, we can sub-classify the research in terms of model specification. Some models employ traditional *multinomial* logit/probit model allowing marketing mix sensitivities or brand preferences to correlate across categories (Ainslie and Rossi 1998, Erdem 1998). Such models only can tell the extent of similarity of marketing sensitivities or brand preference across households. On the contrary, more recent papers employ *multivariate* logit model at individual shopping occasion to investigate the effects of a brand in one category on another brand in other categories (Ma et al 2005, Song and Chintagunta 2006, 2007). Chib et al (2005) develop a hybrid model to incorporate both multivariate probit for category incidence decision (i.e., correlation of errors for category-level utility) and multinomial probit for brand choice with correlated parameters.

Independent Category Model with Correlated Marketing Mix Effects

Ainslie and Rossi (1998) investigate the extent of similarities of households' marketing mix sensitivities (i.e., price, display and feature) across product categories. The authors decompose variance components of marketing mix coefficients into a household-specific component (common across categories) and a category-specific component (common across households). However, the authors assume independent normal distribution of the utility for each brand, which yields a multinomial probit model of brand choices. Seetharaman et al (1999) extend the Ainslie and Rossi (1998) model to investigate household state dependence behavior. Chib et al (2005) develop a multinomial probit model studying both category incidence and brand choice, allowing cross-category correlations at the category-level purchase incidence.

Alternatively, Erdem (1998) uses a multinomial logit brand choice model to investigate whether consumers' quality perceptions of brands are correlated across two product categories (i.e., toothbrush and toothpaste) on account of these brands sharing a common brand name (such as "Colgate"). Singh et al (2005) estimate brand choice models simultaneously across three closely related product categories (Potato Chips, Tortilla Chips and Pretzels), and two less closely related product categories (Sliced Cheese and Mayonnaise). The model allows both the marketing mix coefficients and the consumer's brand preferences to be correlated across product categories. However, the model is essentially developed from a household level MNL model of brand choice. The authors incorporated factor analytic decomposition to obtain a parsimonious representation of these cross-category correlations between MNL models of brand choices.

Multivariate Brand Choice Models

Recently, Song and Chintagunta (2006) adopt multivariate logit approach to investigate cross-category marketing mix effects with the aggregated store-level data. The authors start from individual consumer utility maximizing behavior and analytically aggregate it to model store level market share. They assume that aggregate sales or purchases are derived under the assumption that households are heterogeneous in their preferences and in their sensitivities to marketing activities. Moreover, Ma et al (2005) apply the multivariate logit model to household level data. They used two complementary categories, i.e., cake mix and frosting, to model households' category incidence and brand purchases. Using the estimated demand model as an input, the authors investigate the nature of retailer pricing behavior across the two complementary product categories under study.

Both Song and Chintagunta (2006) and Ma et al (2005) derive their models from individual utility maximization theory to fit a specific data (or market) structure. They assume a consumer purchases only one brand within a category while purchasing multiple brands from multiple categories. In other words, multinomial choice is assumed within a category but multivariate choice is assumed across categories even though cross-effects (equivalent to correlation of errors in utility in MVP model) are specified between brands. The papers do not provide a general framework to fit the model to various alternative market structures (e.g., substitutable categories while interdependent brand choice within a category). Moreover, Ma et al (2005) restrict a market basket to contain at most two items on a shopping trip, one item per each category.

2.2 Reference Price Literature

The effect of reference price has been very widely studied in marketing literature for many years (see Mazumdar et al 2005 for review). Reference price literature argues that consumers use the psychologically encoded prices when making choices as opposed to observed prices (Winer 1986, Kalyanaram and Winer 1995). We classify the research in terms of their focuses (Table 2.1). One stream of research focuses on the formation of the reference price construct (see Briesch et al 1997 for detail). In general, there are two alternative views about how reference prices may be formed. The first view is that consumers use the internal memory-based reference price (IRP or MBR) by retrieving the previous price of the brand from the past purchases (Kalwani et el 1990, Kalyanaram and Little 1994). The other view argues that consumers may not have a strong memory of the past prices and, therefore, use external stimulus-based reference price (ERP or SBR) from the distribution of current prices at the shopping trip (Hardie et al 1993). Some studies, meanwhile, have revealed that consumers consider both IRP and ERP at the same time (Mayhew and Winer 1992, Rajendran and Tellis 1994).

Along with this, a group of research has investigated consumer heterogeneity with respect to their emphasis on IRP versus ERP (e.g., Mazumdar and Papatla 2000, Moon et al 2006). Mazumdar and Papatla (2000) use the finite mixture model (Kamakura and Russell 1989) to find the segments based on the relative weight on IRP and ERP. With this procedure, they found that there are three to four segments depending on different emphasis on IRP and ERP in four different categories. Moon et al (2006) also show that consumers can be segmented depending on which reference price mechanism they use in choice process. The authors found that customers who use internal memory-based reference price (IRP) are more price sensitive than other customers.

Finally, a group of research studies the effects of reference price on brand choice. On the basis of the prospect theory (Kahneman and Tversky 1979), when an observed price is higher (lower) than the reference price, consumers encode it as a loss (gain).

Some studies developed the model based on the assumption that the effect of reference price is symmetric (Winer 1986, Mayhew and Winer 1992, Rajendran and Tellis 1994, Chang et al 1999). In other words, customers react equally to both gain and loss. On the contrary, most research assumes asymmetric effects of reference price (Kalwani et al 1990, Hardie et al 1993, Kalyanaram and Little 1994, Mazumdar and Papatla 2000). Loss aversion dictates that consumers react more strongly to price increases (i.e., loss) than to price decreases (i.e., gain). Nevertheless, Bell and Lattin (2000) and Chang et al (1999) claim that if we incorporate consumer heterogeneity in the model, the coefficient for loss aversion is significantly reduced and the effect disappeared for some category.

Despite an extensive amount of research on reference price effects, very little work has been done in the area of multiple-category brand choice. Many reference price studies simply compare reference price effects across multiple categories, once they estimate the model for single category at a time (e.g., Bell and Lattin 2000, Mazumdar and Papatla 2000). For the category level reference points effect, Bell and Bucklin (1999) show that category purchase decision is dependent on category reference value which is defined as the expected inclusive value (or category value) in the nested logit. To our best knowledge, only Erdem et al (2001) allow reference price sensitivities (along with other marketing mix sensitivities) to correlate across brands in different categories within a single modeling framework. They found significant correlation of price reference sensitivities across categories, implying price reference effects are not just category specific but also household dependent characteristic. Nevertheless, because of the multinomial structure of the model, we can only tell how similarly consumers react to price change with respect to the reference price across categories. It does not show how the reference price effects of one category influence the choice of another item in other category. For complementary categories (e.g., detergent and softener), decrease (increase) of actual price of the item relative to the reference price may positively (negatively) affect purchase probability of another brand in the other category on the same shopping trip. Substitutes (e.g., cola and water), on the contrary, may suggest the opposite effect of reference price in one brand on another brand in other category. Therefore, the model should have the property of translating the effects of reference price in one category to brand choices in another category.

2.3 Product Bundle and Reference Price

Studies in product bundle or assortment choice explain how a consumer decides whether or not to buy a bundle of products at a given shopping occasion on the basis of the product attributes (e.g., McAlister 1979, Farquhar and Rao 1976, Bradlow and Rao 2000). Studies in this literature group measure individual ratings for preference or utility on different offerings by directly building a utility over product characteristics, not specifying utility over the product offerings. Farquhar and Rao (1976), for example, developed a balance model with an assumption that subjects categorize attributes as falling into equibalancing, counterbalancing, desirable, and undesirable set. Bradlow and Rao (2000) extended this research using hierarchical Bayesian framework.

In this group of literature, the utility for the bundle is expressed commonly as a summation of utilities of attributes of the product within a bundle. In particular, Janiszewski and Cunha (2004) investigated how consumers construct the bundle utility with respect to price anchor. The authors found that when consumers evaluate a product bundle, e.g., pizza and cola, they first evaluate individual item separately based on gain and loss mechanism and then sum individual utilities to construct the utility for the bundle. The authors argue that anchoring and adjusting do not depend on which item is evaluated first. This theory is particularly interesting because this implies that when people construct a market basket model the order of choice does not influence much on the final basket construction.

2.4 Summary

There are many attempts to study cross-category choice effects. Some models, however, simply combine single category choice models with correlated parameters. Such approaches are not able to capture cross-category dependencies due to demand relationships such as complementarity and substitution. Moreover, some models are developed at the category level, which may not be sufficient for individual item management. Recently, new attempts have been made to employ multivariate logit structure to study cross-category effects at brand level. Such application is fruitful because it not only helps one understand true multi-category choice but also provides a basis to study impacts of other marketing phenomena (e.g., reference price) across categories. Due to interdependencies across categories, consumer's response to both observed (i.e., shelf price) and psychologically encoded price (i.e., reference price) in one category influences choice decision in another category. Moreover, due to individual differences among items in one category in terms of attributes and pricing policy, reference price effects must be defined at item (or SKU) level. We develop the cross-category model to take care of brand (or SKU level) reference price effects, which in turn translate to category level effects.

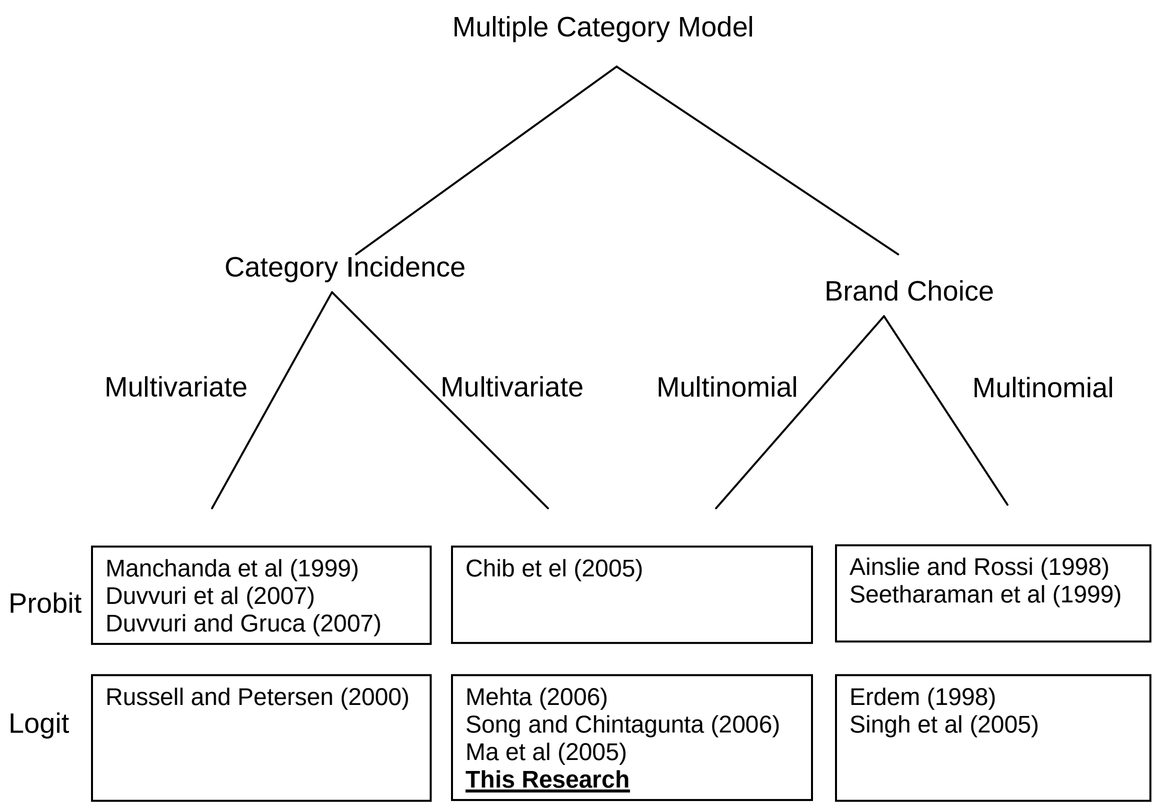
Table 2.1 Classification of Reference Price Effect Models

Reference Price Formation (for review, see Briesch et al 1997)		**Construction**
IRP	Kalwani et al (1990), Kalyanaram and Little (1994)	Retrieval of previous price of the past purchased brand
ERP	Mayhew and Winer (1992)	Retailer provided regular price of a brand
	Hardie, Johnson, and Fader (1993)	Current price of the past purchased brand
	Rajendran and Tellis (1994)	Lowest price in the category
	Mazumdar and Papatla (2000)	Weighted average of current prices of brands in the category
Heterogeneous use of IRP versus ERP		**Features**
IRP only	Kalwani et al (1990), Kalyanaram and Little (1994)	No heterogeneity
	Erdem, Mayhew, and Sun (2001)	Heterogeneity on marketing mix and reference price sensitivities
ERP only	Hardie, Johnson, and Fader (1993)	No heterogeneity
Both IRP & ERP	Mayhew and Winer (1992), Rajendran and Tellis (1994)	No heterogeneity. Different weights on IRP and ERP
	Bell and Lattin (2000)	Latent class. Either IRP or ERP model. IRP fits better
	Mazumdar and Papatla (2000)	Market segmentation based on relative emphasis on IRP and ERP
	Moon, Russell and Duvvuri (2006)	Market segmentation based on different mechanism of reference price usage

Table 2.1 Continued

Symmetric/Asymmetric effects of Gain-Loss on brand choice		Results
Symmetry	Winer (1986)	Significant effects
	Mayhew and Winer (1992)	Both IRP and ERP effects are significant
	Rajendran and Tellis (1994)	Significant effect when ERP is included
	Chang, Siddarth, and Weinberg (1999)	Not significant when heterogeneity is incorporated
Asymmetry	Kalwani et al (1990), Kalyanaram and Little (1994)	Loss aversion is supported
	Hardie, Johnson, and Fader (1993)	Loss aversion is supported for both price and quality
	Mazumdar and Papatla (2000)	Loss aversion is supported in only one segment
	Bell and Lattin (2000)	No evidence of loss aversion when heterogeneity is incorporated
	Erdem, Mayhew, and Sun (2001)	Reference price effects are correlated across categories

Figure 2.1 Multiple Category Models



CHAPTER 3
A SKU-LEVEL MODEL OF MULTIPLE ITEM CHOICE

In this chapter, we develop a SKU (stock keeping unit) level choice model which permits a variety of demand relationships across items in market basket selection. This model, a reinterpretation of the category level basket model of Russell and Petersen (2000), can be derived by assuming that the utility of one item in a market basket depends upon the set of items already selected. Starting from the conditional probability distributions of individual items, we can construct the final joint-distribution of all possible item selections. The resulting model implies that the probability of buying a basket of items on the same shopping trip depends upon the household's valuation of each item in the basket, adjusted for the demand relationships among the items. The key attribute of the model is flexibility in demand patterns: with suitable modification, the model can be applied to virtually any type of market structure. Because the cross-price elasticities of the model can be written in a closed form manner, we can easily study the pattern of brand price competition implied by the choice model.

The rest of the chapter is organized as follows. In the next section, we provide a discussion of the multivariate logit basket model, explaining how the model can be extended to build a general model of SKU choice. In particular, we show how prior assumptions about the substitutability and complementarity patterns among products can be used to generate new choice models appropriate for different types of market structure and various levels of choice decisions. Drawing on this theory, we then construct a SKU-level model for the yogurt product category which links product characteristics (brand and flavor) to the pattern of SKU substitution and complementarity. This choice model, which assumes perfect substitution across brand names and pick-any choice among flavors within a brand, has not previously appeared in the marketing science literature. Substantively, we provide evidence for price-tier competition in the yogurt category and demonstrate how product characteristics impact cross-price elasticities. We also discuss implications of the model for optimal pricing. We conclude this chapter with a discussion of model limitations and future research directions.

3.1 SKU-Level Model Development

In this section, we describe a SKU level market basket model which allows for a very flexible pattern of product competition. The model is built upon the conditional utility of a SKU choice, dependent on its own marketing mix and the other SKU choices in the chosen basket on a shopping trip. Following the general approach discussed in Russell and Petersen (2000), we demonstrate that the implied choice model for a market basket can be expressed as a variation of the multivariate logistic distribution (Cox 1972). Since the model is built at the SKU level, the model allows the researcher to investigate brand competition within a category as well as across categories throughout the system.

3.1.1 Model Description

Let us suppose that household *h* selects a basket *b* at time *t*. A basket *b* contains multiple items (zero, one or more items) from multiple categories. Let us denote j_c as an item $j\left(=1,2,\cdots,J_c\right)$ in category $c\left(=1,2,\cdots,C\right)$. The total number of alternatives is $N_J=\sum_{c=1}^{C}J_c$. Define a market basket as a $N_J\times 1$ vector of binary variables indicating the presence of the item in a basket

$$\mathbf{z}_{ht}^{b}=\left[z_{h1t}^{1},z_{h2t}^{1},\cdots,z_{hJ_1t}^{1},z_{h1t}^{2},z_{h2t}^{2},\cdots,z_{hJ_2t}^{2},\cdots,z_{hJ_Ct}^{C}\right]' \tag{3.1}$$

where z_{hjt}^{c} =1 if household *h* selects item *j* in category *c* at time *t* and equals zero otherwise. Because of choice characteristics (i.e., pick-any choice), there are 2^{N_J} possible baskets, including the null basket, that could be selected. Without any restriction, the model subsequently assigns a choice probability to each of 2^{N_J} baskets.

We now define the conditional choice probability for an item, given choices of all other items in a basket. The conditional utility consists of two parts: (a) a baseline utility dependent on SKU characteristics and (b) the demand interactions with other items in the basket. The utility of item *j* in category *c*, conditional on choices of other items under consideration, can be written as

(3.2) $$U\left(j_c \mid all\, else\right) = \pi_{hjt}^{c} + \sum_{c^*} \sum_k \theta_{hjk}^{c,c^*} z_{hkt}^{c^*} + \varepsilon_{hjt}$$

where π_{hjt}^{c} is a baseline utility for item *j* in category *c* of household *h* at time *t*, and ε_{hjt} is a Gumbel distributed random error. In general, the baseline utility depends upon both household preferences and marketing mix elements.

Note that there is no restriction on the number of items which may be chosen from a category, which implies that the summation of category *c** includes category *c*. For the purpose of model identification, we also assume that $\theta_{hjk}^{c,c^*} = 0$ for items $k = j$ in the same category ($c = c^*$). The parameter θ_{hjk}^{c,c^*} captures the influence of item *k* in category *c** on the utility of target item *j* in category *c*. Accordingly, $\theta_{hjk}^{c,c^*} > 0$ if item *k* has positive impact (complementarity effect) on item *j*'s utility, $\theta_{hjk}^{c,c^*} < 0$ if item *k* negatively influences item *j*'s utility (substitution effect), and $\theta_{hjk}^{c,c^*} = 0$ if there is no influence at all (independence).

Define the symmetric matrix, $\mathbf{\Theta}_h$, containing all the elements of θ_{hjk}^{c,c^*}, as

(3.3) $$\mathbf{\Theta}_h = \left[\mathbf{\Theta}_{h1}^{1}, \mathbf{\Theta}_{h2}^{1}, \cdots, \mathbf{\Theta}_{hJ_1}^{1}, \mathbf{\Theta}_{h1}^{2}, \cdots, \mathbf{\Theta}_{hJ_C}^{C}\right]'_{N_J \times 1} = \begin{bmatrix} 0 & \theta_{h12}^{1,1} & \cdots & \theta_{h1J_C}^{1,C} \\ \theta_{h21}^{1,1} & 0 & \cdots & \theta_{h2J_C}^{1,C} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{hJ_C1}^{C,1} & \theta_{hJ_C2}^{C,1} & \cdots & 0 \end{bmatrix}_{N_J \times N_J}$$

where $\mathbf{\Theta}_{hj}^{c}$ is the column vector for the corresponding item *j* in category *c* of the matrix $\mathbf{\Theta}_h$. Using matrix (3.3) and choice vector (3.1), we can rewrite equation (3.2) as follows:

(3.4) $$U\left(j_c \mid all\, else\right) = \pi_{hjt}^{c} + \mathbf{\Theta}_{hj}^{c}{}' \mathbf{z}_{ht}^{b} + \varepsilon_{hjt}$$

Assuming that the error in (3.4) follows a Gumbel distribution, the conditional choice probability of purchasing item *j* in category *c* becomes the logit model

$$\Pr\left(z_{hjt}^{c} \mid all\ other\ z's\right) = \frac{\left\{\exp\left(\pi_{hjt}^{c} + \mathbf{\Theta}_{hj}^{c}{}'\mathbf{z}_{ht}^{b}\right)\right\}^{z_{hjt}^{c}}}{1+\exp\left(\pi_{hjt}^{c} + \mathbf{\Theta}_{hj}^{c}{}'\mathbf{z}_{ht}^{b}\right)} \tag{3.5}$$

where, $z_{hjt}^{c} \in \{0,1\}$ is an indictor variable reporting whether or not item *j* is chosen.

Applying Brook's Lemma (Besag 1974), equations (3.4) and (3.5) now in turn imply a joint probability distribution of purchasing a basket of items. Assuming that the cross-effect matrix is symmetric (i.e., $\theta_{hjk}^{c,c*} = \theta_{hkj}^{c*,c}$), we show in Appendix A that the implied basket model has the form:

$$\Pr\left(\mathbf{z}_{ht} = \mathbf{z}_{ht}^{b}\right) = \frac{\exp\left(V_{hbt}\right)}{\sum_{b^*}\exp\left(V_{hb^*t}\right)} = \frac{\exp\left(\boldsymbol{\pi}_{ht}'\mathbf{z}_{ht}^{b} + \frac{1}{2}\left\{\mathbf{z}_{ht}^{b}{}'\mathbf{\Theta}_{h}\mathbf{z}_{ht}^{b}\right\}\right)}{\sum_{b^*}\exp\left(\boldsymbol{\pi}_{ht}'\mathbf{z}_{ht}^{b^*} + \frac{1}{2}\left\{\mathbf{z}_{ht}^{b^*}{}'\mathbf{\Theta}_{h}\mathbf{z}_{ht}^{b^*}\right\}\right)} \tag{3.6}$$

where $\boldsymbol{\pi}_{ht} = \left[\pi_{h1t}^{1}, \pi_{h2t}^{1}, \cdots, \pi_{hJ_1t}^{1}, \pi_{h1t}^{2}, \cdots, \pi_{hJ_Ct}^{C}\right]'$ is a $N_J \times 1$ column vector containing the individual (full) conditional baseline utilities for items in consideration. Here $\boldsymbol{\pi}_{ht}'\mathbf{z}_{ht}^{b}$ is a summation of baseline utilities for items in the chosen basket *b*.

3.1.2 Model Interpretation

It is interesting to note that the general form of equation (3.6) is similar to structural models developed from economic theory (e.g., Song and Chintagunta 2006, Ma et al. 2005). Accordingly, V_{hbt} can also be considered to be a second-order (quadratic) Taylor series approximation to the true (indirect) utility of the basket *b*. Note that the utility of a basket of items depends upon both the baseline utilities of all items in the basket $\boldsymbol{\pi}_{ht}'\mathbf{z}_{ht}^{b}$ and the demand relationships among the items in the basket $\mathbf{\Theta}_{h}$.

Intuitively, equation (3.6) states that the utility of a basket depends upon household valuations for each item in the basket, adjusted for interactions among the selected items. This theory exhibits clear relationships to the work in the consumer behavior literature dealing with product bundles. If all elements of $\mathbf{\Theta}_{h}$ are set to zero, then V_{hbt} is just the sum of the utilities of the items in the basket. This type of model was recently proposed by Janiszewski and Cunha (2004) in a study of consumer reactions to price anchors. More broadly, the idea that the valuation of a bundle depends both upon main effects and interactions of item attributes has strong support in the marketing literature (Farquhar and Rao, 1976; Chung and Rao 2003).

Using equation (3.6), we can easily derive the probability of choosing a category. Notice that the model assumes that choice is characterized by an IIA (independence of irrelevant alternatives) property relative to market baskets. Due to this property, we can write the category purchase incidence probability as

$$(3.7) \qquad \Pr\left(c=1\right)=\sum_{b\,\text{having any item in category}\,c}\frac{\exp\left(\boldsymbol{\pi}_{ht}'\mathbf{z}_{ht}^{b}+\frac{1}{2}\left\{\mathbf{z}_{ht}^{b}{}'\boldsymbol{\Theta}_{h}\mathbf{z}_{ht}^{b}\right\}\right)}{\sum_{b^*}\exp\left(\boldsymbol{\pi}_{ht}'\mathbf{z}_{ht}^{b^*}+\frac{1}{2}\left\{\mathbf{z}_{ht}^{b^*}{}'\boldsymbol{\Theta}_{h}\mathbf{z}_{ht}^{b^*}\right\}\right)}$$

where the summation in the numerator runs across all baskets containing at least one item in the particular category. The simplicity of this expression facilitates the use of the model in analyzing how marketing activity in one category impacts purchasing in other categories. More generally, the closed form expression in equation (3.6) enables the researcher to analytically derive the own and cross elasticities for marketing mix elements within and across categories. As we show subsequently, this derivation is possible even with very complicated market structures.

Formally, equation (3.6) is identical to the multivariate logit purchase incidence model of Russell and Petersen (2000). However, here we regard the specification as a representation of the probability of buying a bundle of SKU's on the same shopping occasion. This distinction is not trivial. As we show in the next section, interpreting the multivariate logit basket model as a SKU choice model allows the researcher to use *a priori* notions of market structure to derive new types of choice models. These models can accommodate complex patterns of pick-one and pick-any choice outcomes on the same shopping trip.

3.1.3 Market Structure and Choice

In principle, the model in equation (3.6) can be applied to any type of market structure with simple restrictions on the $\mathbf{\Theta}$ matrix. This allows the model to adapt to choice situations in which different types of constraints impact choice outcomes. Using this flexibility, we can test different possible market structures empirically by comparing model fits. These restrictions also facilitate model calibration by reducing the total number of possible baskets that may be chosen by the household.[1] In this section, we illustrate these points by considering two different structures. We begin with a structure which has previously appeared in the marketing science literature. We then discuss a highly unusual structure appropriate for the empirical work reported later in this chapter.

Multivariate Category-Multinomial Item Choice

A classical single category model assumes that items within a category are perfect substitutes: a household can only purchase at most one item on a shopping occasion. The vast majority of studies in the marketing science literature represent purchase probability using multinomial logit (e.g., Guadagni and Little 1983) or multinomial probit (e.g., Ainslie and Rossi 1998). Extending this structure to a set of complementary categories, we assume that households can purchase multiple categories (multivariate choice), but only one item within a category, given that the category is selected (multinomial choice). A graphical representation of this structure is shown in Figure 3.1.

Categories are complements so that a household chooses multiple categories at once, but items within categories are perfect substitutes, thus permitting at most one item

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[1] As number of items in consideration increases, summation over all possible baskets (denominator in equation (3.6)) quickly explodes without any restriction. Maximum Pseudo Likelihood Estimation (MPLE) (Besag 1974) and Markov Chain Monte Carlo Maximum Likelihood Estimation (MCMC MLE) (Wu and Huffer 1997) have been suggested to alleviate this problem. In this paper, due to the restrictions imposed on cross-effect matrix in equation (3.3), we can use traditional Maximum Likelihood Estimation (MLE) without facing severe dimensionality problems.

in each category to be chosen. This structure has been assumed in other studies (see, e.g., Song and Chintagunta 2006, Ma et al 2005). To accommodate this structure, we impose restrictions on the symmetric matrix $\boldsymbol{\Theta}$ in the following manner. Assuming three categories (A,B,C) with three items in each category (1,2,3), the $\boldsymbol{\Theta}$ matrix takes the form

(3.8) $$\boldsymbol{\Theta} = \left[\begin{array}{ccc:ccc:ccc} 0 & & & & & & & & \\ -\infty & 0 & & & & & & & \\ -\infty & -\infty & 0 & & & & & & \\ \hdashline \theta_{11}^{B,A} & \theta_{12}^{B,A} & \theta_{13}^{B,A} & 0 & & & & & \\ \theta_{21}^{B,A} & \theta_{22}^{B,A} & \theta_{23}^{B,A} & -\infty & 0 & & & & \\ \theta_{31}^{B,A} & \theta_{32}^{B,A} & \theta_{33}^{B,A} & -\infty & -\infty & 0 & & & \\ \hdashline \theta_{11}^{C,A} & \theta_{12}^{C,A} & \theta_{13}^{C,A} & \theta_{11}^{C,B} & \theta_{12}^{C,B} & \theta_{13}^{C,B} & 0 & & \\ \theta_{21}^{C,A} & \theta_{22}^{C,A} & \theta_{23}^{C,A} & \theta_{21}^{C,B} & \theta_{22}^{C,B} & \theta_{23}^{C,B} & -\infty & 0 & \\ \theta_{31}^{C,A} & \theta_{32}^{C,A} & \theta_{33}^{C,A} & \theta_{31}^{C,B} & \theta_{32}^{C,B} & \theta_{33}^{C,B} & -\infty & -\infty & 0 \end{array}\right]$$

where $\theta_{12}^{B,A}$ denotes the cross-effect parameter between item 1 in category B and item 2 in category A. The symbol $-\infty$ (negative infinity) implies perfect substitution, reflecting multinomial choice within a category.

Notice that baskets containing certain combinations of items (e.g., item 1 and 2 in the same category) are excluded from consideration because the choice probability of the basket equals zero (i.e., the numerator of equation (3.6) becomes $\exp(-\infty) = 0$). With this structure, the total number of possible baskets is reduced from 512 to 64. In model calibration, the parameters set to $-\infty$ are not actually estimated. Rather, the model in equation (3.6) is rewritten to exclude baskets with zero purchase probability.

It should be noted that we can further simplify the model by assuming that cross-category item effects are only dependent on category characteristics $\left(\theta_{12}^{B,A} = \theta^{B,A}\right)$. With this assumption, we only need to estimate three values of θ^{c,c^*} (category level cross-effects) in the $\boldsymbol{\Theta}$ matrix. This assumption is used for the study in Chapter 4. Hence, restrictions on $\boldsymbol{\Theta}$ matrix not only accommodate different assumptions about market structure but also assist the researcher in model calibration.

<u>Multinomial Category-Multivariate Item Choice</u>

An alternative market structure is one in which categories are considered substitutes while items within a category are dependent on each other. Although this structure may not appear intuitive, it can nevertheless be useful in some settings. For example, in the yogurt category, there are multiple flavors within each brand name. Households typically buy multiple flavors on a shopping trip, but restrict their purchases to SKU's having the same brand name. If we allow brand names to define categories and let SKU's be items, then the structure shown in Figure 3.2 is an appropriate description of the market structure.

The $\mathbf{\Theta}$ matrix corresponding to Figure 3.2 is a mirror image of the $\mathbf{\Theta}$ matrix corresponding to Figure 3.1. Assuming 3 categories and 3 items within each category as in previous example, a plausible structure of the $\mathbf{\Theta}$ matrix is

$$(3.9) \qquad \mathbf{\Theta} = \left[\begin{array}{ccc|ccc|ccc} 0 & & & & & & & & \\ \theta_{21}^{A,A} & 0 & & & & & & & \\ \theta_{31}^{A,A} & \theta_{32}^{A,A} & 0 & & & & & & \\ \hline -\infty & -\infty & -\infty & 0 & & & & & \\ -\infty & -\infty & -\infty & \theta_{21}^{B,B} & 0 & & & & \\ -\infty & -\infty & -\infty & \theta_{31}^{B,B} & \theta_{32}^{B,B} & 0 & & & \\ \hline -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & 0 & & \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & \theta_{21}^{C,C} & 0 & \\ -\infty & -\infty & -\infty & -\infty & -\infty & -\infty & \theta_{31}^{C,C} & \theta_{32}^{C,C} & 0 \end{array}\right]$$

Here, we have small basket models within each category while multinomial choice takes place at category level. Two items are perfect substitutes ($\theta_{jk}^{c,c*} = -\infty$) if they are in different categories ($c \neq c*$). Within a category, however, items may have any type of demand dependence (substitution, complementarity or independence). This demand structure reduces the number of possible baskets from 512 to 22.

3.1.4 Summary

At this point, the important properties of the basket model in equation (3.6) should be clear. The key feature of the model is its ability to represent a various types of demand structures. Aside from the structures discussed above, it is possible to show that models such as multinomial logit (e.g., Guadagni and Little 1983) and conditionally independent logit (e.g., Bawa et al. 1997) are also special cases of the general model.[2] Clearly, by altering the pattern of the $\mathbf{\Theta}$ matrix, we can restrict the number of possible baskets and thereby generate a wide variety of choice models. In the next section, we take advantage of this flexibility in modeling choice in the yogurt category.

3.2 Empirical Analyses

Although the proposed model has attractive theoretical properties, model calibration can be technically challenging. Here, we develop an application to household purchase histories from the yogurt product category using the structure of the $\mathbf{\Theta}$ matrix corresponding to Figure 3.2. Substantively, we show that households treat brand names as strong substitutes, and flavors (within a brand) as weak substitutes and complements. We also show that the general pattern of cross-category price elasticities is consistent with price tier competition.

3.2.1 Data Description

The data are taken from an ERIM scanner panel of yogurt purchase histories from the Sioux Falls area over a two and one-half year period. We select 59 households for the analysis. Although the yogurt has multiple brands, for model tractability we restrict attention to two brands (Dannon and Nordica) with the same product form (low fat, swirled). We select eight common flavors from both brands and collapse three flavors

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[2] The multinomial logit results from setting all off-diagonal elements of $\mathbf{\Theta}$ to minus infinity. The conditionally independent logit corresponds to setting all elements of $\mathbf{\Theta}$ to zeros.

into one composite flavor (Others). Taken together, we have total of twelve SKU's (six SKUs for each brand) in the analysis.[3] The six common flavors are Cherry, Mixed Berry, Peach, Raspberry, Strawberry and Others. By constructing the SKU set in this manner, we are able to study the role of brand and flavor in yogurt choice decisions.

We split the data into three consecutive periods. The first 8 months (240 days) of the data were used to create household-specific SKU loyalty variables. The remainder of the data was split into two sets: a model calibration period (1453 shopping trips over 545 days) and a holdout period (237 shopping trips over 120 days). Table 3.1 shows descriptive statistics for the two brands across these periods. Due to retailer marketing decisions, there is no variation in marketing mix activity across SKU's within a brand at the same time. There was no display activity at all for Dannon during the observed periods. Of the 59 households used for model calibration, 16 households did not make any purchases of these two brands during holdout period. For this reason, only 43 households enter the holdout data.

Table 3.2 presents a contingency table showing which items are purchased together. Diagonal elements are purchase frequencies for individual SKU's in the calibration data period. The table shows no cross purchases across brands, but many instances of joint purchasing of flavors within a brand.[4] For this reason, we assume the choice process for Dannon and Nordica follows the pattern shown in Figure 3.3: multinomial (pick-one) choice across brands, and multivariate (pick-any) choice within a brand. This type of demand pattern corresponds to the structure of the $\boldsymbol{\Theta}$ matrix discussed earlier in relation to equation (3.9) (multinomial category selection with multivariate choice within categories).

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[3] Multiple purchases of a single SKU are considered one SKU choice.

[4] In fact, 30 purchase events (out of 1,720 total observations) show simultaneous purchasing of items with different brand names. These records were excluded from the dataset prior to model calibration. Accordingly, there exist some households whose choice process is not entirely consistent with our model.

In calibrating the model (discussed below), we also make two other adjustments. Since the choice histories are restricted to only those instances in which either Dannon or Nordica is purchased, we never observe a null (empty) market basket. Consequently, we eliminate the null basket from the denominator of equation (3.6), thereby fixing the probability of observing the null basket to zero. (As explained in Appendix A, this procedure is justified by the IIA property of the choice model with respect to product bundles.) In addition, we also adjust the model specification to accommodate stock-outs in which some items are not available in the market.

3.2.2 Model Specification

We first define individual SKU's baseline utility π^{c}_{hjt} as

$$\pi^{p}_{hjt} = \alpha^{p}_{hj} + \beta^{p}_{1,hj} PRICE^{p}_{hjt} + \beta^{p}_{2,hj} DISP^{p}_{hjt} + \beta^{p}_{3,hj} FEAT^{p}_{hjt} + \beta^{p}_{4,hj} LOY^{p}_{hj} \tag{3.10}$$

where *p* indicates a brand, here taking on two values, 1 for Nordica and 2 for Dannon.

This specification allows for the influence of price (*PRICE*), in-store display (*DISP*) and feature advertising (*FEAT*). The notation LOY^{p}_{hj} denotes a loyalty variable that adjusts for the household's long-run propensity to buy the item *j* in category *c*. We define $LOY^{p}_{hj} = \log\left(\frac{n^{p}_{hj} + 0.5}{n_{h.} + 0.5N_J}\right)$ where n^{p}_{hj} is the number of purchases of the item *j* (in brand *p*) across the household's $n_{h.}$ shopping trips in the initial eight months of the dataset. Because marketing mix variables (such as price) are always the same for all flavors of a particular brand, we assume that marketing mix parameters vary only by brand. This assumption allows the baseline utility π_{hjt} for a specific SKU to be rewritten as follows:

$$\pi^{p}_{hjt} = \alpha^{p}_{hj} + \beta^{p}_{1,h} PRICE^{p}_{ht} + \beta^{p}_{2,h} DISP^{p}_{ht} + \beta^{p}_{3,h} FEAT^{p}_{ht} + \beta^{p}_{4,hj} LOY^{p}_{hj} \tag{3.11}$$

Note that we assume that all parameters in the model vary across households. Thus, the model calibration discussed below accommodates household parameter heterogeneity.

Next, we specify the structure of the cross-effect matrix $\mathbf{\Theta}_h$, following the structural restrictions shown in Figure 3.3. The two brands are treated as perfect substitutes, implying that any basket containing both Nordica and Dannon products will never be observed. That is, we restrict the cross-effects $\left(\theta_{hjk}^{N,D}\right)$ between any two items having different brand names to be $-\infty$, thus setting the choice probabilities of these baskets to zero. However, the cross-effects for flavors within a brand $\left(\theta_{hjk}^{N} \text{ or } \theta_{hjk}^{D}\right)$ are unrestricted (allowing for varying degrees of substitution, complementarity or independence).

These restrictions reduce the total number of possible baskets from 4096 ($=2^{12}$) to 128 ($=2\times 2^{6}$). The corresponding cross-effect matrix for all brands and flavors has the general form

$$\mathbf{\Theta}_h = \begin{bmatrix} \mathbf{\Theta}_h^N & -\infty \\ -\infty & \mathbf{\Theta}_h^D \end{bmatrix}_{12\times 12} \tag{3.12}$$

where $\mathbf{\Theta}_h^N$ and $\mathbf{\Theta}_h^D$ are 6×6 matrices for the flavors within each brand (Nordica and Dannon, respectively). These sub-matrices must be symmetrical, but need not necessarily be equal.

We complete the model specification by developing assumptions about the pattern of household response parameter heterogeneity. Define the vector containing all unique cross-effect parameters as $\boldsymbol{\theta}_h = vec\left(tril\left[\mathbf{\Theta}_h\right]\right)$, where ‘vec’ is a vectorization operator and ‘tril’ is a lower-triangular matrix operator. Using this notation, we assume that household response heterogeneity is governed by the relationships

$$\begin{aligned} \boldsymbol{\alpha}_h &\sim N\left(\bar{\boldsymbol{\alpha}}, \mathbf{\Sigma}_\alpha\right) \\ \boldsymbol{\beta}_h &\sim N\left(\bar{\boldsymbol{\beta}}, \mathbf{\Sigma}_\beta\right) \\ \boldsymbol{\theta}_h &\sim N\left(\bar{\boldsymbol{\theta}}, \mathbf{\Sigma}_\theta\right) \end{aligned} \tag{3.13}$$

where $\boldsymbol{\alpha}_h$ and $\boldsymbol{\beta}_h$ are vectors of item specific intercepts and marketing mix parameters.

In general, we can estimate any type of covariance structure for the parameters by properly specifying the variance-covariance matrices in equation (3.13). In this application, we assume for simplicity that all variance-covariance matrices are diagonal. Although this assumption requires that parameters independently vary across households, it does not imply that marketing actions of one item have no impact on other items. As we show below, such cross-item dependencies exist due to the fact that the cross-effect matrix in equation (3.12) is not equal to zero.

3.2.3 Model Calibration

We introduce an indicator variable Y_{ht}^{b}, which takes the value one if the basket *b* (i.e., $\mathbf{z}_{ht}^{b}$) is chosen and zero otherwise. Ignoring household parameter heterogeneity, the likelihood takes on the standard form

$$L = \prod_{h}\prod_{t}\prod_{b}\left\{\Pr\left(\mathbf{z}_{ht} = \mathbf{z}_{ht}^{b}\right)\right\}^{Y_{ht}^{b}} \tag{3.14}$$

where, as noted earlier, *h* denotes household and *t* denotes time. However, given our heterogeneity assumptions, it is necessary to construct the likelihood of each household by integrating out the random components of the parameters as

$$L_h = \int_{\xi}\prod_{t}\prod_{b}\left\{\Pr\left(\mathbf{z}_{ht} = \mathbf{z}_{ht}^{b} \mid \xi\right)\right\}^{Y_{ht}^{b}} f\left(\xi\right) d\xi \tag{3.15}$$

where ξ represents any random component in the model. This integral can be approximated using a certain number (*R*) of simulated values of individual parameters. In this way, we obtain

$$L_h \approx \widetilde{L_h} = \frac{1}{R}\sum_{r} L_h^{(r)} = \frac{1}{R}\sum_{r}\prod_{t}\prod_{b}\left\{\Pr\left(\mathbf{z}_{ht} = \mathbf{z}_{ht}^{b} \mid \xi^{(r)}\right)\right\}^{Y_{ht}^{b}} \tag{3.16}$$

which leads to $LL\left(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\theta}\right) \approx \sum_{h}\log\widetilde{L_h}\left(\boldsymbol{\alpha}_h,\boldsymbol{\beta}_h,\boldsymbol{\theta}_h\right)$, an approximation to the true log likelihood of all households and time points. Maximization of the approximate log likelihood with respect to all model parameters generates simulated maximum likelihood

(SML) estimates. Further details about the SML procedure and its applications to choice models can be found in Train (2003).

3.2.4 Model Comparison

Using the SML methodology, we test several variations of our proposed model on the yogurt purchase histories by altering the market structure implied by the $\mathbf{\Theta}_h$ matrix. In all model comparisons, we retain the assumption that brands are perfect substitutes. The first benchmark model, called *No Cross-Effects*, assumes independence among flavors within each brand, restricting all elements in the cross-effect matrix ($\mathbf{\Theta}_h$) to be zeros. In this instance, we only estimate marketing mix parameters including item specific intercepts. The second benchmark model, called *Same Cross-Effects*, allows cross-effects parameters to be estimated, imposing the restriction that cross-effects are based on flavor (not on brand). This makes the cross-effect submatrices identical across brands, i.e., $\mathbf{\Theta}_h^N = \mathbf{\Theta}_h^D$. The third benchmark model, called *Different Cross-Effects*, assumes that cross-effect parameters vary across brands, i.e., $\mathbf{\Theta}_h^N \neq \mathbf{\Theta}_h^D$. For each of these three variations, we estimate model parameters both *with* and *without* household parameter heterogeneity.

Table 3.3 displays the fit statistics for different models. We report values of the log-likelihood (for both calibration and holdout data), ρ^2, AIC (Akaike Information Criterion), and BIC (Bayesian Information Criterion). As expected, models incorporating household parameter heterogeneity fit substantially better than their counterparts without heterogeneity. Comparing fit statistics of the models incorporating heterogeneity, we can see that the heterogeneous *Different Cross-Effects* model (shown in the last row of Table 3) provides the best overall fit in calibration as well as holdout data. However, the level of improvement in fit of this model relative to the heterogeneous *No Cross-Effects* model suggests that the degree of dependence across flavors within a brand is modest. The elasticity analysis reported subsequently reinforces this observation.

3.2.5 Parameter Estimates

Table 3.4 presents parameter estimates for baseline utilities for the heterogeneous Different Cross-Effects model. Recall that coefficients for price, feature and display vary by brand, but not by flavor. Moreover, the display variable for Dannon is omitted from the model because Dannon (in our dataset) never uses display in its promotions. The parameter values suggest that Dannon is, in general, the preferred brand (larger brand intercepts). Moreover, households are relatively more sensitive to price changes in Dannon (-7.09) than Nordica (-2.68). All marketing mix coefficients have the correct signs (negative for price, positive for feature and display).

As explained earlier, all model coefficients are assumed to vary across households. For this reason, we also present the standard deviation of the parameters in Table 3.4. To develop some intuition for the variation in price sensitivity across households, we construct histograms of price coefficients by computing posterior means (see, e.g., Green 2003). Using the individual household's likelihood L_h, we compute posterior estimates of the parameters using the expression

(3.17)
$$\beta_{hj} \mid \beta_j, \Delta_j, \xi_{hj} = \frac{\sum_{r=1}^{R} \beta_{hj}^{(r)} L_h\left(\beta_{hj}^{(r)}\right)}{\sum_{r=1}^{R} L_h\left(\beta_{hj}^{(r)}\right)}$$

where $\boldsymbol{\beta}_h \sim N\left(\overline{\boldsymbol{\beta}}, \boldsymbol{\Sigma}_\beta\right)$. The histograms shown in Figures 3.4 and 3.5 show that price sensitivities vary by household and brand, but that the variability across households within a brand is relatively small.

Table 3.5 displays the cross-effect matrix $\boldsymbol{\Theta}_h$ for the heterogeneous Different Cross-Effects model. Although these coefficients also vary across households, for expositional purposes we only present estimates of the mean of the population. Some parameters, denoted by value $-\infty$ (negative infinity), are not estimated by the SML procedure. Rather, these parameters, implying perfect substitution across brand names, reflect restrictions imposed by the model specification. As expected from the model comparisons, many of the coefficients are small, suggesting independence of demand effects across flavors (within brands). However, a few show modest complementarity (e.g., strawberry and peach in Nordica) and substitutability (e.g., raspberry and cherry in Nordica). Overall, the market structure is characterized by substitution across brands and weak dependence (both substitution and complementarity) across flavors within brands.

3.2.6 Elasticity Analysis

Although the model restricts cross-effect matrix $\mathbf{\Theta}_h$ to be symmetrical, this does not necessarily mean that two items affect each other symmetrically. Additional insights can be obtained by investigating own and cross price elasticities. Cross price elasticity is defined as the percentage change in market share of item *j* due to one percent change in price of item *k*, namely $\eta_{jk}^{c,c*} = \%\Delta MS\left(j_c\right)/\%\Delta PRICE\left(k_{c*}\right)$. If item *j* and *k* have different brand names ($c \neq c*$), the cross elasticity has the same form as cross price elasticities in a multinomial logit model. This arises from the assumption that brands are perfect substitutes: no basket can contain items with different brand names. However, if item *j* and *k* have the same brand name ($c = c*$), cross-elasticities are influenced by the cross-effect terms $\left(\theta_{jk}\right)$. Due to the structure of the model, price changes of one item (*k*) influence choice of the other item (*j*) not only directly through θ_{jk} but also indirectly through other terms having subscript of either *j* or *k* (θ_{jl} or θ_{kl}).

We derive individual household level price elasticities and analytically aggregate them to develop aggregate own and cross price market share elasticities (cf. Russell and Kamakura 1994, Russell and Petersen 2000). Details are discussed in Appendix B. Using the model estimates from the heterogeneous Different Cross-Effects model, aggregate cross-price elasticities are presented in Table 3.6. Two general patterns are evident. First, cross elasticity patterns are not symmetric even though the cross effect matrix (Table 3.5) is symmetric. Across brands, the perfect substitution assumption leads to positive cross-price elasticities. Within each brand (across flavors), cross-elasticity patterns are mixed in terms of signs. Second, in general, within-brand cross-elasticities are very small in magnitude (close to zero), whereas between-brand cross-elasticities are relatively large (due to strong substitution effects). This reinforces the general finding that flavors (within brands) have modest demand interactions.

The elasticity matrix also provides insights into price competition among the two yogurt brands. First, the own-price elasticities (on the diagonal of Table 3.6) indicate that Dannon flavors are more sensitive to their own price changes than are Nordica flavors. Second, Dannon price changes have a larger impact on Nordica choice share than vice versa. For example, a one percent price decrease of Dannon raspberry will lead to about 0.4 percent decrease in market share of Nordica products. In contrast, a one percent price decrease in Nordica leads to less than 0.1 percent of Dannon's market share. Figure 3.6 provides a summary of this pattern. Each bar of the histogram represents the average cross-price elasticity reporting the impact of price changes of a given brand and flavor combination on the flavors of the other brand. The figure clearly shows that Dannon dominates Nordica with respect to inter-brand price competition.

This finding is broadly consistent with price-tier competition (Blattberg and Wisniewski 1989, Sivakumar and Raj 1997, Sivakumar 2000). This theory argues that brands which are perceived to be of higher quality (and typically, are higher priced) have more impact on lower quality (lower priced) brands, than vice versa. We note that Dannon is priced higher than Nordica, and relies less on promotional techniques such as display and feature (Table 3.1). Both factors argue that Dannon is perceived by consumers to be of higher quality than Nordica. In fact, in the United States, Dannon and Yoplait (not included in this study) largely define the premium national brand segment of the yogurt market (Orgish 2002). Accordingly, the pattern of demand asymmetry in Figure 3.6, in which Dannon dominates Nordica, is quite reasonable.

3.2.7 Analysis of Optimal Prices

To further understand the elasticity structure uncovered by our analysis, we conduct a simple optimization study. Define $\Pr\left(j_c \mid \boldsymbol{\beta}, \boldsymbol{\Theta}, \mathbf{mix}\right)$ as the purchase probability of item *j* in brand *c* given marketing mix parameters $\boldsymbol{\beta}$, cross-effects matrix $\boldsymbol{\Theta}$, and marketing mix of all items. In our model, this probability takes the form

(3.18) $$\Pr\left(j_c \mid \boldsymbol{\beta}, \boldsymbol{\Theta}, \mathbf{mix}\right) = \sum_{\text{Basket } b \text{ having item } j \text{ in brand } c} \Pr\left(\mathbf{z} = \mathbf{z}^b\right)$$

where the summation runs over all baskets containing the specified item. Therefore, the expected profit (ψ) for the basket is equal to

(3.19) $$\psi = \sum_c \sum_{j_c} \left[\left\{ PRICE\left(j_c\right) - COST\left(j_c\right) \right\} \Pr\left(j_c \mid \boldsymbol{\beta}, \boldsymbol{\Theta}, \mathbf{mix}\right) \right]$$

where $COST\left(j_c\right)$ is the retailer's variable cost. Averaging equation (3.19) over households (with different response parameters) yields the average profit per basket for the retailer.

To obtain optimal prices, we maximize average profit per basket with respect to the prices of the yogurt SKU's. We found that direct optimization of equation (3.19) did not yield useful results because the function is not globally concave. For this reason, we modified equation (3.19) to

(3.20) $$\psi = \sum_c \sum_{j_c} \left[\left\{ PRICE\left(j_c\right) - COST\left(j_c\right) \right\} \exp(\tau - \lambda\mu) \Pr\left(j_c \mid \boldsymbol{\beta}, \boldsymbol{\Theta}, \mathbf{mix}\right) \right]$$

where τ is the observed average yogurt price in our dataset, μ is the average of the yogurt SKU prices inserted into the choice model, and λ is a subjectively determined tuning parameter. The $\exp(\tau - \lambda\mu)$ term acts as a penalty function to prevent the average optimal prices from deviating too much from the observed average price. Our optimization results are based upon $\lambda = 5$. In managerial terms, the penalty function is a reflection of the fact that prices which are too high will obviously drive demand for yogurt to zero.

Table 3.7 displays the results of this procedure for our data. In conducting the optimization, we set non-price variables to their long-run averages in the dataset and assumed that variable costs were 80% of current prices. The cost assumption implies that the variable cost of Nordica is lower than that of Dannon (because Nordica’s current price is lower than Dannon’s current price). This cost differential is fully consistent with the idea that when prices are equal, the margin of a national brand (Dannon) is typically lower than the margin of a regional brand (Nordica). We ran the optimization procedure twice: once allowing all twelve SKU prices to be freely determined, and once constraining prices of all flavors within a brand to be the same.

Three broad conclusions emerge from the optimization exercise. First, despite the penalty function, the recommended optimal prices are somewhat larger than the current prices. This boosts profitability, but decreases the average number of items in each basket. Second, the overall pattern of prices is supportive of current practice: the mean price of the Dannon flavors is higher than the mean price of the Nordica flavors. This is true both in the constrained and unconstrained optimization conditions. Third, allowing the prices of flavors within each brand to vary leads to a substantial increase in profitability. Put another way, forcing prices to vary only by brand harms profitability.

In Figure 3.7, we display the relationship between the unconstrained optimal prices and the SKU’s price sensitivity. We define price sensitivity to be the absolute value of the SKU’s own price elasticity found in Table 3.6. The letter *N* designates a Nordica flavor, while the letter *D* designates a Dannon flavor. The lines are the points of a regression of optimal price on price sensitivity for each brand. The pattern in Figure 3.7 reflects the notion that optimal pricing is a compromise between a SKU’s price sensitivity (lower sensitivity allows higher price) and a SKU’s cost structure (higher costs yield higher prices). The interplay of these two factors leads to the recommendation that Dannon be priced higher, despite the fact that Dannon faces a more elastic demand than Nordica.

The optimization exercise conducted here is only suggestive of the way that managers could use the model to set prices. First, retailers obviously cannot set prices in the yogurt category using only a subset of brands. A more realistic model would consider a larger number of SKU's in the category and would replace the penalty function in equation (3.20) with a function relating overall category sales to a category price index. Second, retailers may find that allowing prices of individual flavors to vary is too expensive to implement because of shelf stocking costs. Moreover, allowing the price range of Nordica SKU's to overlap with the price range of Dannon SKU's (as in unconstrained optimal prices in Table 3.7) may be unacceptable to both retailers and manufacturers. Third, increasing prices of all brands is clearly not desirable if the price level of the yogurt category plays a key role in the retailer's positioning relative to competitors. Nevertheless, the results of this analysis lead to the interesting conclusion that a price differential between Nordica and Dannon is justified. This conclusion suggests that the model developed here is a reasonable way of understanding competition in the yogurt category.

3.3 Discussion

This chapter develops a SKU level model of market basket choice which allows for great degree of flexibility in demand interactions across items. The approach, a generalization of the Russell and Petersen (2000) category incidence basket model to SKU basket choice, offers the researcher the ability to constrain the probability distribution of baskets to correspond to known aspects of market structure. We applied the model in an analysis of SKU competition in the yogurt product category.

3.3.1 Theoretical Contributions

The model developed here has a number of advantages. First, the model assumes that households evaluate baskets of products by summing valuations of individual items and then adjusting the joint value for demand interactions (perceived substitutability or complementarity). This theory is attractive because it is consistent both with structural economics models (e.g., Song and Chintagunta 2006; Ma et al. 2005) and with consumer behavior research on bundling (e.g., Janiszewski and Cunha 2004). Second, the model is very flexible. Because the cross-effect parameters $\boldsymbol{\Theta}$ can take on any values (positive, negative or zero), the model can be applied to any type of market structure (complements, substitutes or independents). For example, in the current study, we study a pick-any process within two product groupings (brands) that are assumed to be perfect substitutes. Third, due to the closed form of the model, we can analytically derive cross-elasticity to understand brand competition. Such flexibility enables retailers to interpret complex demand patterns and to develop pricing policies consistent with market structure.

3.3.2 Substantive Contributions

We use the model structure to fit a non-standard model of SKU choice to the yogurt product category. Substantively, we show that households treat brand names as strong substitutes, and flavors (within a brand) as weak substitutes and complements. We also show that the general pattern of SKU price competition is consistent with the Blattberg and Wisniewski (1989) theory of price tier competition. The estimated cross-price elasticities clearly show that price changes of the premium brand (Dannon) have more impact on the lower-tier brand (Nordica) than vice versa. We also demonstrate how optimal prices might be set using the model output. These results suggest that a pricing rule based upon brand name is consistent with the pattern of brand competition in the yogurt category.

3.3.3 Limitations and Extensions

The key limitation of the model proposed is one of tractability as the number of SKU's increases. In part, this issue arises because the denominator of the market basket choice function (equation (3.6)) contains as many terms as possible baskets. In this research, we avoid the dimensionality problem by using structural assumptions to limit the number of possible baskets. Although this approach will prove workable in some settings, the ideal methodology would simulate the denominator of the choice model, but still allow for household parameter heterogeneity. In addition, the number of cross-effect parameters increases geometrically with the number of SKU's in the study. The obvious solution here is to constrain the pattern of the cross-effect matrix to reduce the number of parameters in the model. Successfully addressing these problems would allow retailers to use the model to study brand price competition for large numbers of SKU's, both within and across categories.

Table 3.1 Descriptive Statistics of Yogurt

		Calibration Data	Holdout Data
Number of Observations		1453	237
Number of Households		59	43
NORDICA	**Price**	0.5431 (0.098)	0.5436 (0.091)
	Display	0.0378 (0.189)	0.0171 (0.129)
	Feature	0.1709 (0.372)	0.2429 (0.423)
DANNON	**Price**	0.6494 (0.042)	0.6906 (0.001)
	Display	0.0 (NA)	0.0 (NA)
	Feature	0.0514 (0.219)	0.0127 (0.112)

Note: Price is the mean dollar per eight ounce container during the period. Display and Feature are the mean of indicator (0-1) variables during the period. Standard deviations are shown in parentheses. Statistics for the Dannon Display variable are not available (NA) because Dannon does not employ displays in this dataset. Individual flavors have the same means and standard deviations on marketing mix elements as the corresponding brand name.

Table 3.2 SKU Contingency Table for Yogurt

	NORDICA FLAVORS						DANNON FLAVORS						
	Cherry	Mixed Berry	Peach	Rasp berry	Straw berry	Others	Cherry	Mixed Berry	Peach	Rasp berry	Straw berry	Others	Total
NORDICA FLAVORS													
Cherry	153	29	33	31	44	74	0	0	0	0	0	0	364
Mixed Berry	29	146	22	45	28	76	0	0	0	0	0	0	346
Peach	33	22	127	41	54	55	0	0	0	0	0	0	332
Raspberry	31	45	41	173	48	74	0	0	0	0	0	0	412
Strawberry	44	28	54	48	154	73	0	0	0	0	0	0	401
Others	74	76	55	74	73	299	0	0	0	0	0	0	651
DANNON FLAVORS													
Cherry	0	0	0	0	0	0	136	34	18	33	24	57	302
Mixed Berry	0	0	0	0	0	0	34	257	27	47	36	133	534
Peach	0	0	0	0	0	0	18	27	110	33	21	45	254
Raspberry	0	0	0	0	0	0	33	47	33	310	34	95	552
Strawberry	0	0	0	0	0	0	24	36	21	34	185	90	390
Others	0	0	0	0	0	0	57	133	45	95	90	495	915
Total	364	346	332	412	401	651	302	534	254	552	390	915	5453

Table 3.3 Model Comparison

Model Description	k	LL_Cal	ρ^2 Cal	AIC	BIC	LL_Hold	ρ^2 Hold
Multivariate Logistic without Heterogeneity							
No Cross-Effects	29	-4904.89	0.301	9867.78	10020.94	-868.63	0.241
Same Cross-Effects	44	-4883.08	0.304	9854.17	10086.55	-873.39	0.237
Different Cross-Effects	59	-4863.31	0.307	9844.62	10156.22	-872.18	0.238
Multivariate Logistic with Heterogeneity							
No Cross-Effects	58	-4027.23	0.426	8112.47	8265.63†	-782.51	0.316
Same Cross-Effects	88	-4002.82	0.429	8093.64	8326.02	-813.03	0.289
Different Cross-Effects	118	-3951.82†	0.437†	8021.64†	8333.24	-779.27†	0.319†

Note: Cal denotes calibration data and Hold denotes holdout data. Fit measures are defined as follows: AIC = -2LL + 2k, BIC = -2LL + 2*log(N)*k, and ρ^2 =1-(LL/LL_null) where k is number of parameters, N is sample size, LL is log likelihood of the model and LL_null is the log likelihood with all parameters set to zero. The dagger symbol † denotes the model with the best performance on each measure.

Table 3.4 Model Coefficents

	Intercept		Loyalty		Price		Display		Feature	
	Mean	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**
NORDICA										
Cherry	0.686 (0.734)	0.911** (0.154)	0.620** (0.226)	0.403** (0.069)	-2.678** (0.539)	1.332** (0.114)	0.095 (0.157)	-0.149 (0.106)	0.621** (0.099)	0.140 (0.082)
Mixed Berry	0.004 (0.708)	0.322* (0.135)	0.480* (0.216)	0.401** (0.049)	-2.678** (0.539)	1.332** (0.114)	0.095 (0.157)	-0.149 (0.106)	0.621** (0.099)	0.140 (0.082)
Peach	1.500* (0.686)	1.091** (0.156)	0.941** (0.203)	0.598** (0.058)	-2.678** (0.539)	1.332** (0.114)	0.095 (0.157)	-0.149 (0.106)	0.621** (0.099)	0.140 (0.082)
Raspberry	1.637* (0.672)	-0.160 (0.112)	0.766** (0.195)	0.512** (0.054)	-2.678** (0.539)	1.332** (0.114)	0.095 (0.157)	-0.149 (0.106)	0.621** (0.099)	0.140 (0.082)
Strawberry	-0.721 (0.577)	0.912** (0.143)	0.135 (0.156)	0.093* (0.037)	-2.678** (0.539)	1.332** (0.114)	0.095 (0.157)	-0.149 (0.106)	0.621** (0.099)	0.140 (0.082)
Others	2.682** (0.565)	-0.840** (0.126)	0.816** (0.147)	0.233** (0.037)	-2.678** (0.539)	1.332** (0.114)	0.095 (0.157)	-0.149 (0.106)	0.621** (0.099)	0.140 (0.082)
DANNON										
Cherry	4.733** (0.802)	-0.411** (0.150)	1.151** (0.172)	0.252** (0.053)	-7.088** (1.058)	1.853 (0.108)	NA	NA	0.161 (0.182)	0.171 (0.192)
Mixed Berry	6.091** (0.794)	0.025 (0.114)	1.733** (0.180)	0.623** (0.086)	-7.088** (1.058)	1.853 (0.108)	NA	NA	0.161 (0.182)	0.171 (0.192)
Peach	5.989** (0.886)	0.441* (0.207)	2.294** (0.289)	0.693** (0.084)	-7.088** (1.058)	1.853 (0.108)	NA	NA	0.161 (0.182)	0.171 (0.192)
Raspberry	5.899** (0.763)	0.121 (0.118)	1.403** (0.146)	0.132* (0.061)	-7.088** (1.058)	1.853 (0.108)	NA	NA	0.161 (0.182)	0.171 (0.192)
Strawberry	6.549** (0.820)	-0.890** (0.145)	1.678** (0.171)	0.287** (0.044)	-7.088** (1.058)	1.853 (0.108)	NA	NA	0.161 (0.182)	0.171 (0.192)
Others	6.461** (0.748)	-0.057 (0.078)	1.678** (0.141)	0.458** (0.052)	-7.088** (1.058)	1.853 (0.108)	NA	NA	0.161 (0.182)	0.171 (0.192)

Note: The notation S.D. indicates standard deviation of the household heterogeneity distribution. Significance levels are denoted as $p < 0.05$ (*) and $p < 0.01$ (**). Values in parentheses are standard errors. Marketing mix coefficients are constrained to be the same across all flavors within the same brand. Because Dannon has no display activity, coefficients denoted NA (not available) are not estimated.

Table 3.5 Cross-Effect Matrix for Yogurt

		NORDICA						DANNON					
		CH	**MX BY**	**PCH**	**RBY**	**ST**	**OT**	**CH**	**MX BY**	**PCH**	**RBY**	**ST**	**OT**
NORDICA	**CH**	0.0	-0.371	-0.016	-0.812**	0.050	-0.404	−∞	−∞	−∞	−∞	−∞	−∞
	MX BY	-0.371	0.0	0.272	-0.264	-0.507*	0.266	−∞	−∞	−∞	−∞	−∞	−∞
	PCH	-0.016	0.272	0.0	-0.616	0.934**	-0.085	−∞	−∞	−∞	−∞	−∞	−∞
	RBY	-0.812**	-0.264	-0.615	0.0	0.063	-0.470	−∞	−∞	−∞	−∞	−∞	−∞
	ST	0.050	-0.507*	0.934**	0.063	0.0	-0.077	−∞	−∞	−∞	−∞	−∞	−∞
	OTHER	-0.404	0.266	-0.085	-0.470	-0.077	0.0	−∞	−∞	−∞	−∞	−∞	−∞
DANNON	**CH**	−∞	−∞	−∞	−∞	−∞	−∞	0.0	-0.468*	0.047	-0.027	-0.622*	-0.627**
	MX BY	−∞	−∞	−∞	−∞	−∞	−∞	-0.468*	0.0	-0.053	-0.462*	-0.533*	-0.011
	PCH	−∞	−∞	−∞	−∞	−∞	−∞	0.047	-0.053	0.0	0.137	-0.679	-0.364
	RBY	−∞	−∞	−∞	−∞	−∞	−∞	-0.027	-0.462*	0.137	0.0	-0.550*	-0.376
	ST	−∞	−∞	−∞	−∞	−∞	−∞	-0.622*	-0.533*	-0.679	-0.550*	0.0	-0.219
	OTHER	−∞	−∞	−∞	−∞	−∞	−∞	-0.627**	-0.011	-0.364	-0.380	-0.219	0.0

Note: Significance levels are denoted as $p < 0.05$ (*) and $p < 0.01$ (**). All parameters (not equal to negative infinity or zero) exhibit heterogeneity across households. For clarity, the standard deviations of these parameters are not shown. The zeros on the diagonal of the table and the $-\infty$ values are structural restrictions of the model. These coefficients are not estimated from the data. Codes for flavors are as follows: CH = cherry, MX BY = mixed berry, PCH = peach, RBY = raspberry, ST = strawberry and OT = other.

Table 3.6 SKU Price Competition for Yogurt

		NORDICA						DANNON					
		CH	**MX BY**	**PCH**	**RBY**	**ST**	**OT**	**CH**	**MX BY**	**PCH**	**RBY**	**ST**	**OT**
NORDICA	**CH**	-2.303	0.001	-0.032	0.036	-0.042	-0.004	0.177	0.222	0.068	0.382	0.320	0.487
	MX BY	0.001	-2.311	-0.054	-0.007	0.008	-0.171	0.178	0.233	0.067	0.383	0.313	0.477
	PCH	-0.028	-0.044	-2.233	0.027	-0.160	-0.076	0.166	0.203	0.058	0.343	0.276	0.387
	RBY	0.023	-0.004	0.020	-2.207	-0.037	0.004	0.175	0.217	0.067	0.375	0.315	0.452
	ST	-0.032	0.006	-0.143	-0.045	-2.290	-0.079	0.190	0.253	0.073	0.415	0.341	0.555
	OTHER	-0.001	-0.042	-0.023	0.001	-0.027	-1.823	0.176	0.206	0.067	0.352	0.290	0.449
DANNON	**CH**	0.047	0.045	0.051	0.074	0.065	0.180	-4.866	0.024	-0.040	-0.152	0.075	0.152
	MX BY	0.037	0.036	0.038	0.056	0.053	0.130	0.015	-3.778	-0.017	0.046	0.046	-0.169
	PCH	0.045	0.042	0.044	0.070	0.062	0.169	-0.100	-0.067	-4.937	-0.172	0.096	0.009
	RBY	0.041	0.038	0.042	0.063	0.057	0.144	-0.061	0.030	-0.028	-3.423	0.035	0.007
	ST	0.044	0.040	0.043	0.068	0.060	0.151	0.038	0.038	0.020	0.045	-3.873	-0.082
	OTHER	0.033	0.031	0.030	0.049	0.049	0.118	0.039	-0.070	0.001	0.004	-0.041	-2.542

Note: Table displays the percentage change in the aggregate choice share of the row SKU with respect to a one percent increase in the price of the column SKU. Diagonal elements are own-price elasticities. Codes for flavors are as follows: CH = cherry, MX BY = mixed berry, PCH = peach, RBY = raspberry, ST = strawberry and OT = other.

Table 3.7 Optimal Pricing

		Current		Unconstrained Optimal		Constrained Optimal	
		Price	**Sales Index**	**Price**	**Sales Index**	**Price**	**Sales Index**
NORDICA	**Cherry**	0.543	0.056	0.518	0.076	0.604	0.059
	Mixed Berry	0.543	0.052	0.478	0.074	0.604	0.054
	Peach	0.543	0.064	0.524	0.082	0.604	0.065
	Raspberry	0.543	0.088	0.612	0.094	0.604	0.093
	Strawberry	0.543	0.070	0.554	0.087	0.604	0.073
	Others	0.543	0.210	0.778	0.159	0.604	0.224
DANNON	**Cherry**	0.649	0.065	0.652	0.079	0.697	0.056
	Mixed Berry	0.649	0.101	0.682	0.100	0.697	0.089
	Peach	0.649	0.026	0.556	0.055	0.697	0.022
	Raspberry	0.649	0.157	0.727	0.125	0.697	0.140
	Strawberry	0.649	0.123	0.701	0.109	0.697	0.109
	Others	0.649	0.238	0.756	0.166	0.697	0.216
	Average Basket Size		**1.250**		**1.206**		**1.199**
	Profit Per Basket		**0.014**		**0.017**		**0.015**

Note: Sales Index is defined as the overall probability of the item falling in any basket at the given prices. Average Basket Size is the expected number of items in the basket at the given price point. Profit is the value of the profit function evaluated at the given prices. The Constrained Optimal solution forces all prices within each brand to be identical. The Unconstrained Optimal solution allows all twelve prices to be freely determined. In the Unconstrained Optimal solution, the mean price across flavors is .577 for Nordica and .679 for Dannon.

Figure 3.1 Multivariate Category Multinomial Item Choice

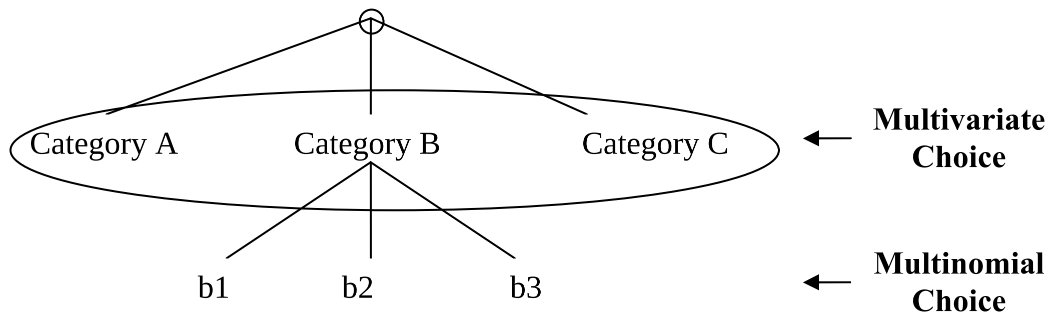


Figure 3.2 Multinomial Category Multivariate Item Choice

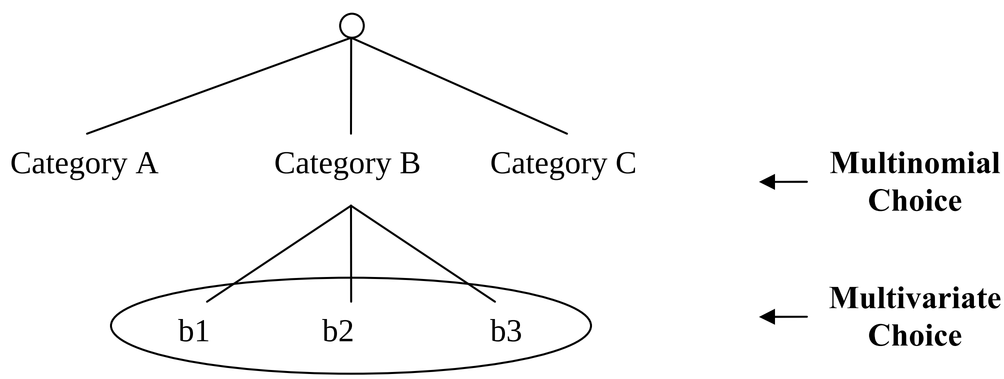


Figure 3.3 Market Structure of Yogurt Category

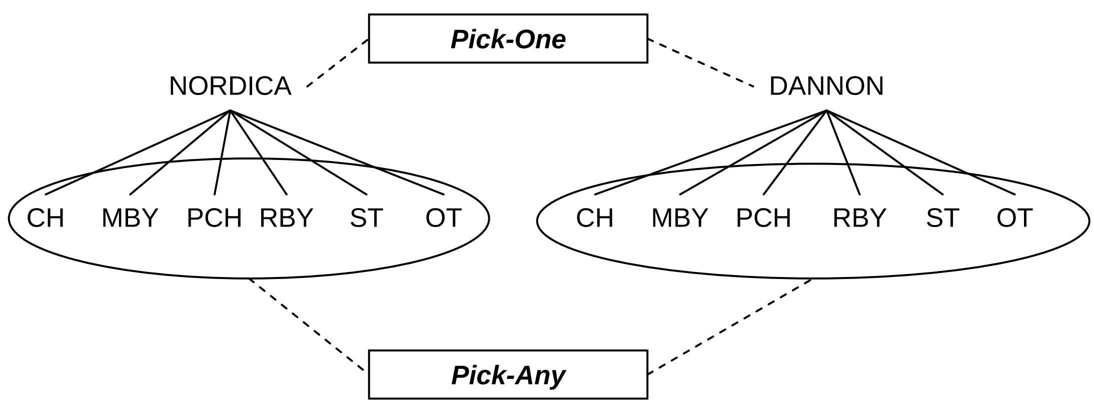


Figure 3.4 Individual Posterior Price Coefficients for NORDICA (mean = -2.7)

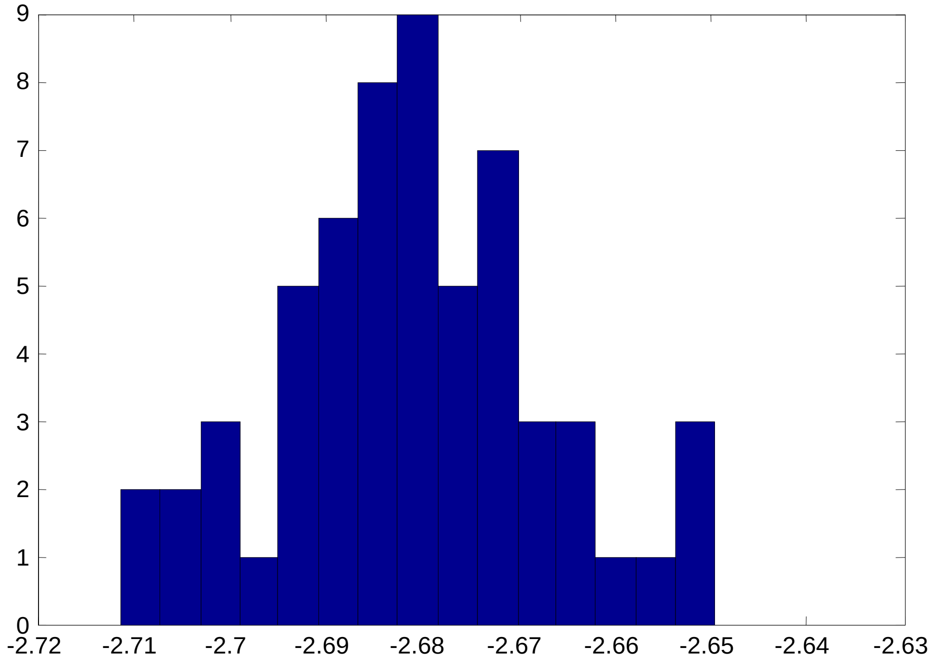


Figure 3.5 Individual Posterior Price Coefficients for DANNON (mean = -7.1)

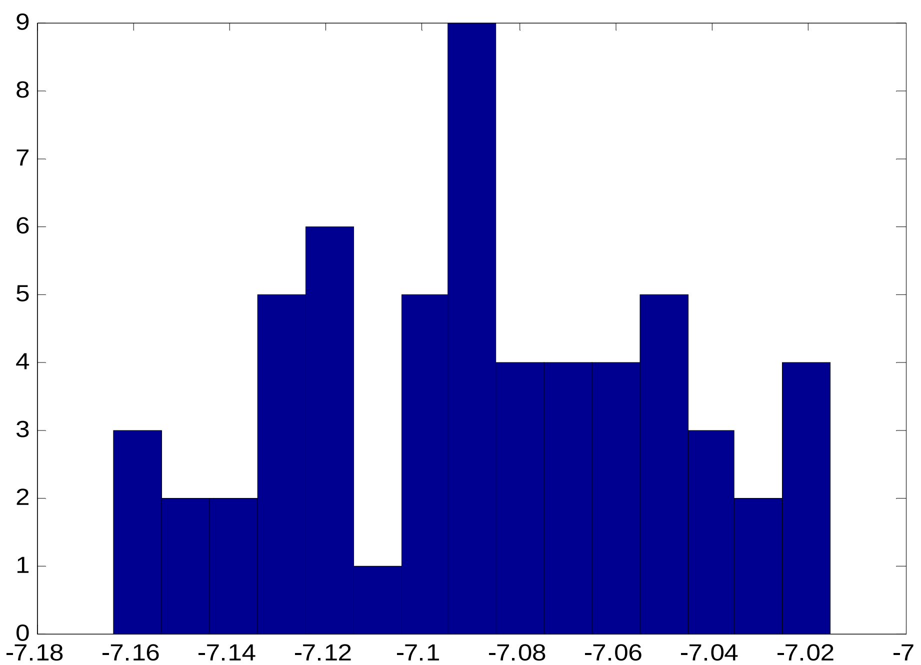


Figure 3.6 Asymmetric Price-Tier Competition

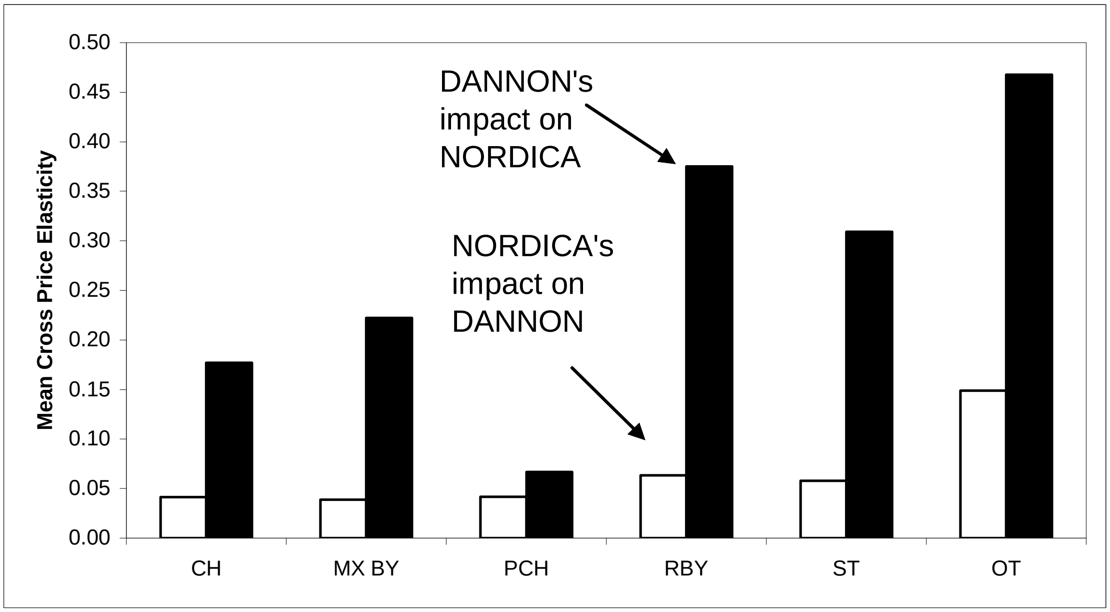
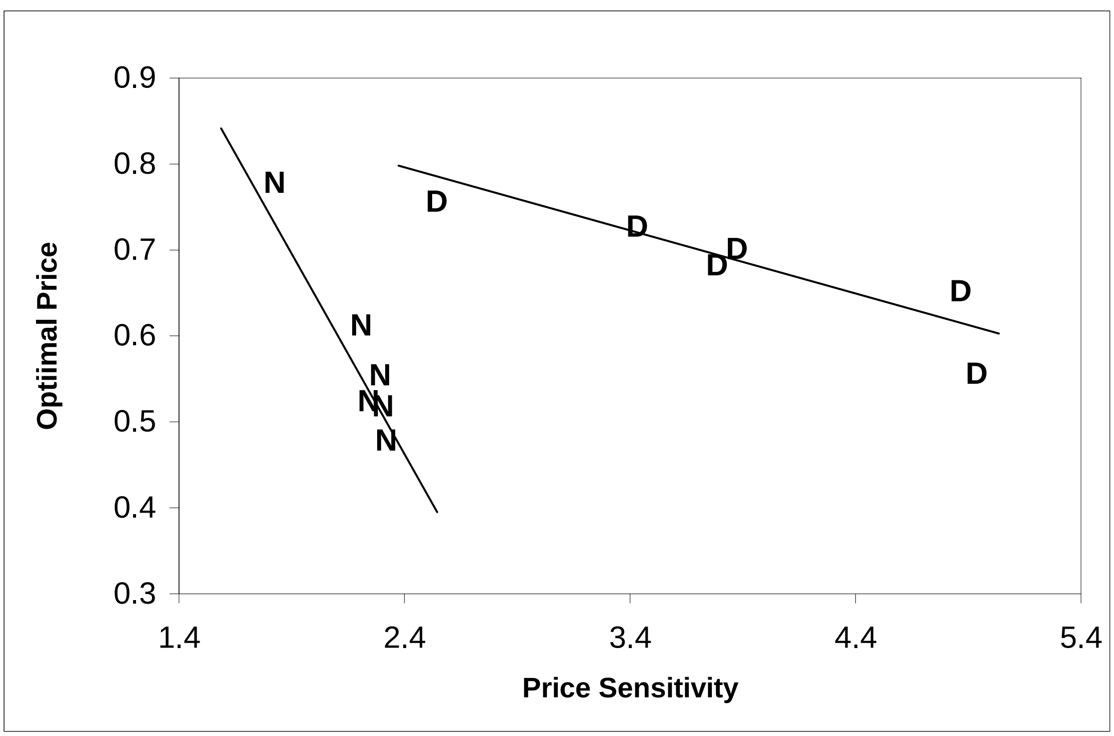


Figure 3.7 Analysis of Optimal Prices



CHAPTER 4
SPILLOVER EFFECTS OF
REFERENCE PRICE ON CROSS-CATEGORY CHOICE

In this chapter, we generalize the multi-item choice model developed in Chapter 3 to understand how reference price mechanisms work in a cross-category setting at SKU level. The basket model we developed and tested in Chapter 3 has the property that any marketing mix effect of one item flows through the system to influence another item in the same basket which may contain items from multiple categories. We build a reference price spillover mechanism to study this phenomenon. Upon this mechanism, we develop four alternative model specifications implying different mechanisms for reference price spillover effects. While those models agree on how they model within-category reference price effects, they differ in how the cross-category spillover effects are specified.

The rest of the chapter is organized as follows. First, we redefine basket composition and construct SKU-level reference price model. The model is built upon previous literature about reference price discussed in Chapter 2. Next, we extend the model to investigate spillover effect of reference price across categories. We build four models with comparable theoretical frameworks. We test these models with household laundry product categories, i.e., liquid detergent, powder detergent, liquid softener, and dry softener. We discuss results and implications of the model based upon the best fitting model. We conclude this chapter with a discussion of model limitations and future research directions.

4.1 Basket Definition and Market Structure

In this Chapter, we use a typical market structure assumption used in most cross-category choice studies. Those studies are described in detail in Chapter 3.1.3, i.e., Multivariate Category-Multinomial Item Choice. The choice structure is in fact same as Figure 3.1 illustrated in Chapter 3. We assume that items are considered perfect substitutes (i.e., multinomial choice) within each category. Categories could be either substitutes or complements and thus multivariate incidence is assumed across categories.

For illustration, let us assume there are three categories in consideration and each category has two items. Categories are denoted by capital letters A, B, and C. SKUs are denoted by lower letters with subscripted numbers. Thus, all six items in consideration are $\{a_1, a_2, b_1, b_2, c_1, c_2\}$. The choice structure can be represented as in Figure 4.1.

The multivariate choice at category level implies that there are eight (= 2^3) possible category-level baskets including the null basket (when none of the categories are chosen). Meanwhile, total number of possible baskets at SKU-level is 27 $(= 3 \times 3 \times 3)$ because each category has three alternatives including no-choice option[5].

Let us suppose that household *h* selects a SKU-level basket *b* at time *t*. A basket *b* contains multiple items (zero, one or more items) from multiple categories. Let us denote $j(c)$ as an item $j(=1,2,\cdots,J_C)$ in category $c(=1,2,\cdots,C)$. The total number of alternatives is $N = \sum_{c=1}^{C} J(c)$. Define a market basket as a $N \times 1$ vector of binary variables indicating the presence of the item in a basket *b*:

(4.1) $$\mathbf{y}_{ht}^{b} = \left[y_{h1(1)t}^{b}, y_{h2(1)t}^{b}, \cdots, y_{hJ(1)t}^{b}, y_{h1(2)t}^{b}, y_{h2(2)t}^{b}, \cdots, y_{hJ(2)t}^{b}, \cdots, y_{hJ(C)t}^{b} \right]'$$

where $y_{hj(c)t}^{b}$ =1 if household *h* selects item *j* in category *c* at time *t* and equals zero otherwise. Without any restriction, the model subsequently assigns a choice probability to each of 2^N baskets.

We define another basket $\tilde{b}$ which only consists of indicators for categories. Indicator variable $z_{hct}^{\tilde{b}}$ =1 if household *h* selects category *c* at time *t* and equals zero otherwise. Then, the following vector represents category-level basket composition:

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5 In general, assuming *C* categories, {1, 2, …, C}, and each category *c* has J_c SKUs (or items), one final basket contains at most *C* items on a shopping trip, one item per category. Therefore, total number of possible baskets at SKU level is $\#b = \{(J_1+1)\times(J_2+1)\times\cdots\times(J_C+1)\}$ including the null basket.

(4.2) $$\mathbf{z}_{ht}^{\tilde{b}} = \left[z_{h1t}^{\tilde{b}}, z_{h2t}^{\tilde{b}}, \cdots, z_{hCt}^{\tilde{b}} \right]'$$

where $\tilde{b}$ stands for the basket at category level in contrast with *b* for the basket at item (or SKU) level. Let us define the notation $b(\tilde{b})$ or $\tilde{b}(b)$ to represent a unique basket which has category-level basket $\tilde{b}$ and SKU-level basket *b*. Note that, without any restriction, there are 2^C baskets available at category level. Hence, there are multiple item level basket combinations $(\mathbf{y}^b)$ having same $\mathbf{z}^{\tilde{b}}$ at the category level[6].

Correspondingly, cross-effects matrix $\mathbf{\Theta}$ is set as in equation (3.8) presented in Chapter 3. The cross-effects within same category will be set as $-\infty$ (negative infinity) implying perfect substitutes whereas cross-effects between items from different categories will be free to be estimated. For the study of spillover effects across categories, however, we set cross-effects parameters within same category block have same values regardless of the item. This implies that demand relationships (i.e., substitutability, complementarity, and independence) are defined at category level. Assuming four categories in consideration (i.e., A, B, C and D), this restriction allows us to specify cross-effects matrix at the category level as follows:

(4.3) $$\mathbf{\Theta}_h = \begin{bmatrix} 0 & \theta_h^{AB} & \theta_h^{AC} & \theta_h^{AD} \\ & 0 & \theta_h^{BC} & \theta_h^{BD} \\ & & 0 & \theta_h^{CD} \\ & & & 0 \end{bmatrix}$$

For the expositional purpose, we provide our general model in equation (3.6) again using notation in equations (4.1), (4.2) and (4.3).

[6] For two categories {A,B} and two items {1,2} within each category, $\mathbf{z}^{\tilde{b}}$ for {a1,0,0,b2} and {0,a2,b1,0} is same even though these two are represented with different $\mathbf{y}_{ht}^{b}$ $(\{1,0,0,1\}$ and $\{0,1,1,0\}$ respectively$)$. Common $\mathbf{z}^{\tilde{b}}$ for these two $\mathbf{y}_{ht}^{b}$'s is {1,1}.

$$(4.4)\qquad \Pr\left\{b\left(\tilde{b}\right)\right\} = \frac{\exp\left(V_{hb(\tilde{b})t}\right)}{\sum_{b^*(\tilde{b}^*)} \exp\left(V_{hb^*(\tilde{b}^*)t}\right)} = \frac{\exp\left(\boldsymbol{\pi}'_{ht}\mathbf{y}^{b}_{ht} + \frac{1}{2}\left\{\mathbf{z}^{\tilde{b}}_{ht}{}'\boldsymbol{\Theta}_h\mathbf{z}^{\tilde{b}}_{ht}\right\}\right)}{\sum_{b^*(\tilde{b}^*)} \exp\left(\boldsymbol{\pi}'_{ht}\mathbf{y}^{b^*}_{ht} + \frac{1}{2}\left\{\mathbf{z}^{\tilde{b}^*}_{ht}{}'\boldsymbol{\Theta}_h\mathbf{z}^{\tilde{b}^*}_{ht}\right\}\right)}$$

Note that cross effect matrix only operates with category level basket indicators $\mathbf{z}^{\tilde{b}}_{ht}$ whereas baseline utility $\boldsymbol{\pi}_{ht}$ works with item (or SKU) level indicators $\mathbf{y}^{b}_{ht}$.

4.2 Within Category Reference Price Model

In order to study spillover effects of reference price in our basket model framework, we must define the baseline utility properly. Incorporating prospect theory (Kahneman and Tversky 1979), we define the baseline utility as follows:

$$(4.5)\qquad \begin{aligned} \pi^{c}_{hjt} = \psi^{c}_{h} + \alpha^{c}_{hj} + \beta^{c}_{hG}GAIN^{c}_{hjt} + \beta^{c}_{hL}LOSS^{c}_{hjt} + \beta^{c}_{hI}INV^{c}_{ht} + \beta^{c}_{1,h}LOY^{c}_{hj} \\ + \beta^{c}_{2,h}PRICE^{c}_{hjt} + \beta^{c}_{3,h}DISP^{c}_{hjt} + \beta^{c}_{4,h}FEAT^{c}_{hjt} \end{aligned}$$

where ψ^{c}_{h} is a category specific intercept and α^{c}_{hj} is another intercept for item *j* in category *c*. $GAIN^{c}_{hjt}$ ($LOSS^{c}_{hjt}$) is defined when the reference price is larger (smaller) than actual shelf price of the item *j* on a shopping trip *t*. The notation LOY^{c}_{hj} denotes a loyalty variable that adjusts for the household's long-run propensity to buy the item *j* in category *c*. We define $LOY^{c}_{hj} = \log\left(\frac{n^{c}_{hj} + 0.5}{n_{h.} + 0.5J_c}\right)$ where n^{c}_{hj} is the number of purchases of the item *j* (in category *c*) across the household's $n_{h.}$ shopping trips in the initialization period. The inventory term, INV^{c}_{ht}, is an estimate of the relative category inventory for household *h* at time *t*. The relative inventory variable is mean-centered to zero for each household and directly comparable across households (details in Appendix C). A negative value for inventory therefore indicates that a given household's estimated supply of product on hand is below its average. This inventory captures household's idiosyncratic needs to purchase a product within a category so it is fully dependent on category (same across items within a category).

Due to the assumption of perfect substitutes within a category, simple algebra gives a conditional item choice probability given a category from a general model in equation (4.4) [7]. The inventory variable (*INV*) and category-level parameters do not influence the item choice probability. Thus, the conditional item choice will be a multinomial logit with item specific variables and parameters:

$$(4.6)\quad \Pr\left(j_c \mid c\right)_{ht} = \frac{\exp\left(\pi^c_{hjt}\right)}{\sum_{k\in c}\exp\left(\pi^c_{hkt}\right)} = \frac{\exp\left(\psi^c_h + \beta^c_{hI} INV^c_{ht}\right)\exp\left(\tilde{\pi}^c_{hjt}\right)}{\exp\left(\psi^c_h + \beta^c_{hI} INV^c_{ht}\right)\sum_{k\in c}\exp\left(\tilde{\pi}^c_{hkt}\right)} = \frac{\exp\left(\tilde{\pi}^c_{hjt}\right)}{\sum_{k\in c}\exp\left(\tilde{\pi}^c_{hkt}\right)}$$

where
$$\tilde{\pi}^c_{hjt} = \alpha^c_{hj} + \beta^c_{hG} GAIN^c_{hjt} + \beta^c_{hL} LOSS^c_{hjt} + \beta^c_{1,h} LOY^c_{hj} + \beta^c_{2,h} PRICE^c_{hjt} + \beta^c_{3,h} DISP^c_{hjt} + \beta^c_{4,h} FEAT^c_{hjt}$$

As reviewed in Chapter 2, researchers have used different approaches to operationalize the effect of reference price in the model. Some only use internal memory-based reference price (e.g., Hardie et al 1993) while most studies include external stimulus-based reference price as well (e.g., Mayhew and Winer 1992, Rajendran and Tellis 1994). In this dissertation, we follow a hybrid approach used by Mazumdar and Papatla (2000) including both Internal Reference Price (IRP) and External Reference Price (ERP) in the same framework. Some may argue that consumers only use one of these reference price points in judgment of fairness of the price (Kalwani et al 1990, Kalyanaram and Little 1994). However, many studies find that customers use both reference price mechanisms (Mayhew and Winer 1992, Rajendran and Tellis 1994). In particular, Rajendran and Tellis (1994) find that loss aversion effect becomes significant when ERP is included in the model.

We operationalize the GAIN (LOSS) variables as a weighted average of the deviations of IRP and ERP from the current shelf price of the item. The weight λ_h is the probability that household *h* uses IRP and $\left(1-\lambda_h\right)$ is the probability that *h* uses ERP. Therefore, if λ_h is close to one, it implies that a consumer use only internal memory to compute GAIN (or LOSS):

---

7 $\Pr\left(j_c \mid c\right) = \sum_{b(\tilde{b})\text{ having item } j \text{ in category } c} \Pr\left\{b\left(\tilde{b}\right)\right\}$

$$GAIN_{hjt}^{c} = \lambda_h^c \left(IRP_{hjt}^{c} - PRICE_{hjt}^{c}\right)_+ + \left(1-\lambda_h^c\right)\left(ERP_{ht}^{c} - PRICE_{hjt}^{c}\right)_+ \tag{4.7}$$
$$LOSS_{hjt}^{c} = \lambda_h^c \left(PRICE_{hjt}^{c} - IRP_{hjt}^{c}\right)_+ + \left(1-\lambda_h^c\right)\left(PRICE_{hjt}^{c} - ERP_{ht}^{c}\right)_+$$

where the notation $\left(x\right)_+$ denotes a variable which equals zero if *x* is negative and equals *x* if *x* is positive. Therefore, values of both $GAIN_{hjt}^{c}$ and $LOSS_{hjt}^{c}$ variables are always positive so that the parameter for $GAIN_{hjt}^{c}$, i.e., β_{hG}^{c} is positive and the parameter for $LOSS_{hjt}^{c}$, i.e., β_{hL}^{c} must be negative. This implies that utility of the item decreases (increases) when actual price is higher (lower) than reference point.

4.2.1 Operationalization of IRP and ERP

There is little guidance for the operationalization of IRP and ERP. In terms of IRP, a majority of research modeled IRP as a weighted average of past prices with varying carry-over weights (Mazumdar and Papatla 2000, Kalyanaram and Little 1994). On the contrary, some studies (e.g., Mayhew and Winer 1992) use the respective brand's price on last purchase occasion. Some other researchers (Kalwani et al 1990, Winer 1986) use more complicated function including household and brand characteristics, such as frequency of promotion, price trend, deal-proneness of household to measure reference price. We use a weighted average of past prices with a carry-over weight γ as

$$IRP_{hjt}^{c} = \gamma^c IRP_{hj[t-1]}^{c} + \left(1-\gamma^c\right) PRICE_{hj[t-1]}^{c} \tag{4.8}$$

where $j[t\text{-}1]$ indicates the item purchased at the last shopping occasion.

This operationalization is more reasonable because current psychological research suggests that consumers do not *remember* previous purchase price correctly (Dickson and Sawyer 1990), but they *know* their reference price (Monroe and Lee 1999). Consumers just know whether the respective brand is expensive or not although they do not remember exactly how much they spent in the last purchase. Equation (4.8) implies that if a consumer does not remember previous price but has an implicit memory then $\gamma\left(0 \le \gamma \le 1\right)$ will be close to one. If it is close to zero, then consumers remember previous price accurately. For example, Greenleaf (1995) finds the best model fit, using weekly peanut butter data, when γ is 0.925. Using panel data on refrigerated orange juice, Hardie et al (1993) obtain the best fit when γ is 0.83.

Unlike IRP, ERP is defined at the category level. Hence, a common ERP is compared to prices of all items in a category. There are also many different approaches to define ERP in the model. For example, Rajendran and Tellis (1994) use three different measures such as the highest, lowest, and mean of current price of a brand. They find the lowest price has the largest impact. Hardie et al. (1993) find that, among different operationalizations, the current price of the brand chosen on last purchase occasion fits the data best. Meanwhile, Mazumdar and Papatla (2000) model ERP as current prices of brands weighted by loyalties of the respective brands, where loyalty variables are defined as in Guadagni and Little (1983). Consistent with earlier work by Mazumdar and Papatla (2000), we define the external reference price point to be a weighted averaged price of items within a category, where the weight is defined by our loyalty variable. Carefully looking at loyalty equation above, we can see that taking exponential makes the values sum up to one across items, i.e., $\sum_k \exp\left(LOY_{hk}^c\right) = 1$. Thus, the ERP equation is defined as

(4.9)
$$ERP_{ht}^c = \sum_{j \in c} \left[\frac{\exp\left(LOY_{hj}^c\right)}{\sum_k \exp\left(LOY_{hk}^c\right)} \cdot PRICE_{hjt}^c \right]$$

There are two reasons of using this operationalization. First, since we are dealing with cross-category choice, there are a large number of no-purchase occasions in a category when a consumer buys other categories only. In many cases, we have no information about the previously chosen brand until a consumer makes his/her first choice. Thus, ERP definition depending on past purchase history (e.g., Hardie et al 1993) becomes difficult. Second, we actually test other operationalizations, e.g., lowest, highest, and average of prices in a category, in our empirical work and find that our definition provides the best fit. This implies that consumers pay attention to current prices of other items in the category and construct a reference point at the store.

4.3 Model for Cross Category Spillover Effects

Using the general model in equation (4.4) and item level model in equation (4.6), we can derive the category-level basket choice model. Enumerating SKU-level basket $\mathbf{y}^b$ choice probabilities having same category cluster $\mathbf{z}^{\tilde{b}}$, we have the following equation:

$$\text{(4.10)} \qquad \Pr\left\{B = \mathbf{z}_{ht}^{\tilde{b}}\right\} = \frac{\exp\left\{\sum_{c} W_{ht}^{c} z_{hct}^{\tilde{b}} + \sum_{c<c^*} \theta_h^{cc^*} z_{hct}^{\tilde{b}} z_{hc^*t}^{\tilde{b}}\right\}}{\sum_{\tilde{b}^*} \exp\left\{\sum_{c} W_{ht}^{c} z_{hct}^{\tilde{b}^*} + \sum_{c<c^*} \theta_h^{cc^*} z_{hct}^{\tilde{b}^*} z_{hc^*t}^{\tilde{b}^*}\right\}}$$

where $W_{ht}^{c} = \log\left\{\sum_{j\in c} \exp\left(\pi_{hjt}^{c}\right)\right\}$ and $\theta_h^{cc^*}$ is an element in cross-category matrix in equation (4.3). Therefore, if $\theta_h^{cc^*} > 0$ then category c and c* are considered complements and if $\theta_h^{cc^*} < 0$ then these categories are considered substitutes. In addition, $\theta_h^{cc^*} = 0$ implies that these two categories are independent.

The implication of this decomposition is that reference price effects in each category spillover to other categories via category attractiveness W_{ht}^{c} and cross-effect parameters θ^{cc^*}. Reference price effects influence category attractiveness (and therefore category incidence) represented by W_{ht}^{c} and thereby the effects influence other category's utility via cross-effects parameters θ^{cc^*}. Even though reference price variables and effects are defined at SKU-level in baseline utility, the effects can flow through categories in the same basket. This relationship is illustrated in Figure 4.2.

This model specification is particularly consistent with previous consumer behavior literature in product bundle choice. Janiszewski and Cunha (2004) find that when people make a bundle choice, e.g., pizza and cola, they evaluate individual items separately according to gain and loss mechanism and sum those individual evaluations to construct the utility for the bundle. In our framework, we specify reference price mechanism in SKU-level baseline utility (equation 4.5). Then, category-level basket utility is determined by sum of attractiveness, measured as W_{ht}^{c}, of individual categories in the basket selection. In addition, we adjust demand relationships using the cross-effect θ and category incidence variables $z_{hjt}^{\tilde{b}}$. This adjustment is essential because consumers construct their own baskets based on their perception of demand relationships, i.e., substitutability, complementarity and independence.

4.4 Alternative Model Specifications

To investigate reference price spillover effects, we construct three alternative models in addition to our proposed model (*Basket Model* or BM). These models have same specification for within-category reference price effects as in equation (4.6). They only differ in terms of the spillover mechanism. We call them (1) *Independence Model* (IM), (2) *Generalized Nested Logit* (GNL), (3) *Multilevel Reference Effect Model* (MREM) as opposed to the proposed *Basket Model* (BM) in equation (4.10).

4.4.1 Independence Model

For this model, we assume that the cross-effects matrix $\boldsymbol{\Theta}$ is a null matrix, i.e., all cross-effects are zero. This allows us to estimate a model where choices across categories are independent. We can write this model as follows:

$$\text{(4.11)} \qquad \Pr\left\{B = \mathbf{z}_{ht}^{\tilde{b}}\right\} = \frac{\exp\left\{\sum_{c} W_{ht}^{c} z_{ht}^{\tilde{b}}\right\}}{\sum_{\tilde{b}^*} \exp\left\{\sum_{c} W_{ht}^{c} z_{ht}^{\tilde{b}^*}\right\}}, \text{ where } W_{ht}^{c} = \log\left\{\sum_{j\in c} \exp\left(\pi_{hjt}^{c}\right)\right\}$$

Interestingly, this specification is consistent with that proposed by Erdem et al (2001). They combine individual category models by allowing marketing mix and reference price parameters to correlate across categories.

4.4.2 Generalized Nested Logit

As is evident from our framework, the choice process we propose consists of two levels: (1) a basket construction at category level, and (2) SKU choices given categories.

Figure 4.1 illustrates two-stage choice process for multinomial nested logit (Ben-Akiva and Lerman 1985) which could be an alternative for the proposed theory. The difference between our theory and nested logit framework is, however, that nested logit does not allow multiple item choice (multivariate choice) at any level of choice decision. On the contrary, our theory allows any possible combination of multiple items (or categories). Thus, we need a new derivation to accommodate multivariate nature of our study.

We start from the construction of new choice tree based upon our conceptual framework. First, let us assume that household *h* visits the store and decides a cluster of categories she wants to buy. Then, the first stage of choice decision must be a basket of categories. Next, we assume that the same household actually constructs a basket by selecting individual items from chosen categories. This collection of items from multiple categories should be the element of the second stage of the choice. Based on example in Figure 4.1, there are eight possible baskets including the null basket in first stage of choice decision at category level. Those are $\{0,0,0\}$, $\{A,0,0\}$, $\{0,B,0\}$, $\{0,0,C\}$, $\{A,B,0\}$, $\{A,0,C\}$, $\{0,B,C\}$, and $\{A,B,C\}$. Once one of eight alternatives is chosen, there exist multiple possible combinations of SKUs. The number of final objects is different depending on basket selection. For example, under $\{A,0,0\}$, consumers can only choose either $\{a_1\}$ or $\{a_2\}$. However, there are four possible combinations under $\{A,B,0\}$. Those are $\{a_1,b_1\}$, $\{a_1,b_2\}$, $\{a_2,b_1\}$, and $\{a_2,b_2\}$. By the same token, the cluster $\{A,B,C\}$ has eight different combinations of SKUs. Therefore, the new choice structure or decision tree becomes Figure 4.3.

Correspondingly, a basket for SKU choices can be written in vector forms. For example, $\{a_1\}$ implies $\mathbf{z}^{\tilde{b}}=[1,0,0]'$ at category level and $\mathbf{y}_{ht}^{b}=[1,0,0,0,0,0]'$ at item level. Similarly, $\{b_1,c_2\}$ implies $\mathbf{z}^{\tilde{b}}=[0,1,1]'$ at category level and $\mathbf{y}_{ht}^{b}=[0,0,1,0,0,1]'$ at item level.

If we define utility of the basket as the linear combination of category-level basket utility and item-level basket utility with appropriate assumptions (equation D1 in

Appendix D), we can derive a multivariate version of nested logit (i.e., *Generalized Nested Logit*). The model allows random errors in basket utilities to be correlated. Full derivation of the model is in Appendix D. The final model at the category level is written in the following equation:

$$\Pr\left\{B = \mathbf{z}_{ht}^{\tilde{b}}\right\} = \frac{\exp\left\{\sum_{c}\left(\mu W_{ht}^{c}\right) z_{hct}^{\tilde{b}} + \sum_{c<c^*}\theta_h^{cc^*} z_{hct}^{\tilde{b}} z_{hc^*t}^{\tilde{b}}\right\}}{\sum_{\tilde{b}^*}\exp\left\{\sum_{c}\left(\mu W_{ht}^{c}\right) z_{hct}^{\tilde{b}^*} + \sum_{c<c^*}\theta_h^{cc^*} z_{hct}^{\tilde{b}^*} z_{hc^*t}^{\tilde{b}^*}\right\}} \tag{4.12}$$

where $W_{ht}^{c} = \log\left\{\sum_{j\in c}\exp\left(\pi_{hjt}^{c}\right)\right\}$

The parameter μ captures correlations among indirect utilities of alternative baskets under the common category cluster[8]. Based upon Ben-Akiva and Lerman (1985), $1-(\mu)^2$ equals the correlation of the indirect utilities for any collection of SKUs sharing common category composition. The closer the correlation is to unity, the closer μ is to zero. Conversely, if the correlation is zero, μ becomes one, which implies that alternatives share no common utility elements. Thus, our proposed *Basket Model* is a special case of *Generalized Nested Logit*, when people regard all distinct baskets of SKUs as perfect substitutes.

4.4.3 Multilevel Reference Effect Model

The final alternative is based upon Bell and Bucklin (1999). Their original work investigates the effect of reference dependent evaluation both at SKU-level and at category level. They conceptualize that category incidence decision, i.e., whether or not to choose the category, depends on a reference point for category attractiveness. Using multinomial nested logit framework, they define GAIN and LOSS at category level as difference between category value (or inclusive value) of current period and future expectation of the value. Since inclusive value in nested logit framework captures

8 $\mu = \mu_g / \mu_d$ in appendix D. We set $\mu_d = 1$ for identification.

category attractiveness, they argue that a consumer experiences gain (loss) if she observes higher (lower) category attractiveness compared with expected category attractiveness. Even though their original work is confined to single category choice, we extend the model to multi-category choice context adopting GNL approach. Therefore, the model incorporates gain and loss mechanisms at category as well as SKU level. The model can be written as

$$\text{(4.13)} \quad \Pr\left\{B = \mathbf{z}_{ht}^{\tilde{b}}\right\} = \frac{\exp\left\{\sum_{c}\left(\mu_1 W_{ht}^{c} + \mu_2 WG_{ht}^{c} + \mu_3 WL_{ht}^{c}\right) z_{hct}^{\tilde{b}} + \sum_{c<c^*} \theta_h^{cc^*} z_{hct}^{\tilde{b}} z_{hct}^{\tilde{b}}\right\}}{\sum_{\tilde{b}^*} \exp\left\{\sum_{c}\left(\mu_1 W_{ht}^{c} + \mu_2 WG_{ht}^{c} + \mu_3 WL_{ht}^{c}\right) z_{hct}^{\tilde{b}^*} + \sum_{c<c^*} \theta_h^{cc^*} z_{hct}^{\tilde{b}^*} z_{hct}^{\tilde{b}^*}\right\}}$$

where $W_{ht}^{c} = \log\left\{\sum_{j\in c} \exp\left(\pi_{hjt}^{c}\right)\right\}$

$WG_{ht}^{c} = W_{ht}^{c} - E\left(W_{ht^*}^{c}\right)$ when $W_{ht}^{c} > E\left(W_{ht^*}^{c}\right)$, where $E\left(W_{ht^*}^{c}\right) = W_{h[t-1]}^{c}$

$WL_{ht}^{c} = E\left(W_{ht^*}^{c}\right) - W_{ht}^{c}$ when $W_{ht}^{c} < E\left(W_{ht^*}^{c}\right)$

The term $E\left(W_{ht^*}^{c}\right)$ is defined as expected category value (or inclusive value). Bell and Bucklin (1999) argue that household compares current category attractiveness W_{ht}^{c} with household's future belief about the category attractiveness $E\left(W_{ht^*}^{c}\right)$. They posit that household constructs this belief or expectation by looking back to the values that they themselves have realized in the recent past. This argument is consistent with Loewenstein (1988), who points out that past experience serves as the basis for reference point in intertemporal choice. Therefore, $E\left(W_{ht^*}^{c}\right)$ is defined as category value that consumer realized at the last purchase occasion, i.e., $W_{h[t-1]}^{c}$.

4.4.4 Summary of Model Development

In sum, there are four distinct but theoretically comparable frameworks to study cross-category spillover of reference price effects. All four models have same specification for SKU-level reference price effects, built upon previous reference price literature. Differences among four specifications arise from (1) the ability of capturing spillover effects, (2) the accountability of correlation among utilities of alternative baskets, and (3) the presence of multilevel reference dependent evaluation. *Independence model* (IM) consistent with Erdem et al (2001) does not model spillover effects or multilevel reference dependent evaluation. We also do not incorporate correlations among basket utilities for this model. The proposed *Basket Model* (BM) specifies spillover effects but has no category-level reference dependent evaluation. *Generalized Nested Logit* (GNL) generalizes BM framework, allowing correlation among utilities of baskets. Finally, *Multilevel Reference Effect Model* (MREM) extended from Bell and Bucklin (1999) has all components in the model. Four approaches are compared in Table 4.1.

4.5 Empirical Analyses

We use scanner panel data for household laundry product categories (i.e., liquid detergent, powder detergent, liquid softener, and dry softener) to test the four specifications (Table 4.1) to study the reference price spillover mechanism. We can see that IM is the most restricted version and MREM is the most general model. For each specification, we also estimate the model without reference price effects within category as a comparison.

4.5.1 Likelihood and Model Estimation Strategy

Combining equation (4.6) and (4.10), we can write the full likelihood function for the final basket choice. Three other alternative models have same form of likelihood but have different category-level basket choice probability, $\Pr\left\{B=\mathbf{z}_{ht}^{\tilde{b}(b)}\right\}$, described in previous section. Without considering household heterogeneity, the full likelihood becomes:

$$L=\prod_{h}\prod_{t}\prod_{b}\left[\Pr\left\{B=\mathbf{z}_{ht}^{\tilde{b}(b)}\right\}\left\{\prod_{c}\left(\prod_{j_c\in c}\left[\Pr\left(j_c\mid c\right)_{ht}\right]^{y_{hjt}^{c}}\right)^{z_{ht}^{c|b}}\right\}\right]^{r_{ht}^{b}} \tag{4.14}$$

where r_{ht}^{b} = 1 if household *h* select basket *b* at time *t*, zero otherwise. To account for household heterogeneity, we make distributional assumption for all parameters in the model except for carry-over parameter γ for IRP construction. Let $\boldsymbol{\xi}_h$ represent all household level random coefficients in the model distributed normal with $\overline{\boldsymbol{\xi}}$ and $\boldsymbol{\Omega}$ as the mean vector and covariance matrix respectively. The distributional assumption is

$$\text{(4.15)} \qquad \boldsymbol{\xi}_h = \begin{bmatrix} \boldsymbol{\alpha}_h \\ \boldsymbol{\beta}_h \\ \boldsymbol{\lambda}_h \\ \boldsymbol{\psi}_h \\ \boldsymbol{\theta}_h \end{bmatrix} \sim N\left(\overbrace{\begin{bmatrix} \overline{\boldsymbol{\alpha}} \\ \overline{\boldsymbol{\beta}} \\ \overline{\boldsymbol{\lambda}} \\ \overline{\boldsymbol{\psi}} \\ \overline{\boldsymbol{\theta}} \end{bmatrix}}^{\overline{\boldsymbol{\xi}}}, \overbrace{\begin{bmatrix} \boldsymbol{\Omega}_\alpha & & & & \\ 0 & \boldsymbol{\Omega}_\beta & & & \\ 0 & 0 & \boldsymbol{\Omega}_\lambda & & \\ 0 & 0 & 0 & \boldsymbol{\Omega}_\psi & \\ 0 & 0 & 0 & 0 & \boldsymbol{\Omega}_\theta \end{bmatrix}}^{\boldsymbol{\Omega}} \right)$$

Note that we assume diagonal matrix for each $\boldsymbol{\Omega}$ for the ease of estimation. The vector containing all unique cross-effect parameters as $\boldsymbol{\theta}_h = vec\left(tril\left[\boldsymbol{\Theta}_h\right]\right)$, where 'vec' is a vectorization operator and 'tril' is a lower-triangular matrix operator. Incorporating this distributional assumption, the final likelihood becomes:

$$\text{(4.16)}\ L(\boldsymbol{\xi}) = \prod_h \int_{\xi} \prod_t \prod_b \left[\Pr\left\{ B = \mathbf{z}_{ht}^{\tilde{b}(b)} \mid \boldsymbol{\xi} \right\} \left\{ \prod_c \left(\prod_{j_c \in c} \left[\Pr\left(j_c \mid c; \boldsymbol{\xi} \right)_{ht} \right]^{y_{hjt}^{c}} \right)^{z_{ht}^{c|b}} \right\} \right]^{r_{ht}^{b}} f(\boldsymbol{\xi})\, d\boldsymbol{\xi}$$

To estimate this model, we use simulated maximum likelihood (SML; Train 2003) using Monte-Carlo simulation. We replace multidimensional integration with randomly generated values. For this dissertation, we adopt two-stage estimation approach because the likelihood can be decomposed into two parts: conditional item choice and category-level basket choice. In the first stage, we estimate within-category SKU level model (equation 4.6) for each category, applying household heterogeneity. In this stage, we have parameters, i.e., mean and variance, which specify distribution of parameters for within-category model. Then, we recover individual parameters for every household including households who have never purchased a certain category. In the second stage, we estimate category-level basket model (equation 4.10), imputing within-category household-level individual parameters into the model. This two-stage approach eases the computational burden substantially without hurting consistency of parameter estimates (Ben-Akiva and Lerman 1985). Standard errors of the parameters can be estimated using the procedure (i.e., BHHH estimator) used in a typical maximum likelihood estimation (Greene 2003). The detail of two-stage simulated maximum likelihood approach is fully described in Appendix E.

4.5.2 Data Description

We use A.C.Nilesen scanner panel data for categories of detergent (liquid and powder) and fabric softener (liquid and dry/sheet). Total number of weeks of data set is 124 weeks (from January 1993 to March 1995). We first select households who made at least five purchases from any of four categories. The reason for using this criterion is: (1) we need enough information to construct IRP and (2) it increases the possibility of having households who purchase multiple categories at the same shopping trip. Next, we pick relatively large brand groups defined as composite of similar SKUs. This reduces number of items and thus helps us manage the complicated estimation procedure. Taken together, we have total 21 items from four categories in consideration. There are eight items in liquid detergent category, six items in powder detergent category, three items in liquid softener and four items in dry (or sheet) softener category. Some brand names are common across categories. Total number of households selected is 595 and total number of observation is 7,424. We then randomly select 440 households for calibration and 155 households for holdout validation. The first 40 weeks are used to initialize some variables such as loyalty, inventory and IRP in the analyses. We also delete data points which define the null basket, in which a household visits the store but makes purchases in categories other than four included in the study. In empirical estimation, we adjust the model to account for not having null basket. This renormalization is discussed in Appendix F.

We begin with classifying household category purchase occasions depending on the number of categories purchased together. In Table 4.2, we can see that most purchases are made with only one category. Purchases of more than three categories are very rare. The table also shows clusters of categories in chosen baskets at category level. We can also observe that the pair-wise purchases of detergent and softener items are more frequent than that of pairs of detergent or pairs of softener items.

Table 4.3 displays descriptive statistics of marketing mix variables for all 21 items in the study. To account for size differences of items within a category, price variables are computed as price per unit for each category. Thus, cents per ounce is used for liquid detergent, powder detergent, and liquid softener. Cents per sheet is used for dry/sheet softener.

4.5.3 Model Performance

Table 4.4 exhibits model performances across the four different models. We provide log-likelihoods, AIC, BIC, and ρ^2 for the models without and with reference price effects. We can clearly see that the performance of the models without reference price effects is worse (e.g., lower ρ^2 and higher AIC or BIC) than those with reference price effects. This indicates that reference price effects exist within category and accounting for such effects is very important to understand households' choice decisions. Next, comparing models across columns, we can see that GNL (third column) and MREM (fourth column) perform much better than the other two models. Further, GNL performs as well as MREM. Considering parsimony, we can say that GNL is the best model based on fit statistics and holdout validation. To make our results more conclusive, we need to analyze model performance by comparing each model in turn.

First, we surprisingly find that allowing for cross effects to capture cross-category demand relationship (from IM to BM) does not improve the model fit much (e.g., 0.5138 → 0.5141 for calibration ρ^2 and 0.4433 → 0.4452 for holdout ρ^2). This may be due to the following two reasons: (1) The categories used for the study may have strong consumption complementarity, but not purchase complementarity. Because the model only captures cross-category dependency at purchase occasions, we do not know the strength of consumption complementarity. (2) Given all category specific information such as household inventory level and household brand preference (e.g., loyalty), we may be able to recover basket choices without incorporating cross-category dependency.

Second, allowing for correlations among errors of basket utilities improves the model performance substantially (from BM to GNL). Calibration ρ^2, for example, has increased from 0.5141 to 0.6199 and holdout ρ^2 has increased from 0.4452 to 0.5669. This implies that the random components contain a lot of variation in common across baskets under common category cluster.

Finally, category-level reference dependent evaluation is not supported because the fit statistics do not improve from GNL to MREM, e.g., 0.6199 → 0.6202 for calibration ρ^2 and 0.5669 → 0.5635 for holdout ρ^2. It implies that accounting for category-level reference effects is not very important as long as we incorporate item specific reference price effects. This result conforms to the argument by Janiszewski and Cunha (2004) in which product bundle evaluation is sum of individual item evaluation based on gain and loss mechanisms at item level.

4.5.4 Parameter Estimates

<u>Within-Category SKU level model</u>

Let us look at Table 4.5 which presents parameter estimates for within-category reference price effects model. Most of the results are consistent with previous findings from the literature. We summarize some interesting findings. First, we find that all parameters have expected signs and they are significant. For example, all price coefficients are negative and coefficients for other marketing mix, i.e., feature and display, have positive signs. In addition, coefficients for GAIN are positive and those for LOSS are negative.

Second, we find the evidence of loss aversion except for liquid detergent category. For liquid detergent, GAIN coefficient is slightly larger than LOSS coefficient in absolute value (2.03 versus -1.32). Nevertheless, all other three categories show larger coefficient for LOSS than for GAIN. This result is consistent with prospect theory (Kahneman and Tversky 1979).

Third, IRP effects dominate reference price effect. Relative weight parameter λ^c is larger than 0.5 implying GAIN and LOSS depend more on IRP than on ERP. In particular, liquid softener category supports IRP only model having $\lambda^c = 1$. This strong IRP dependency has been reported in previous literature (Mazumdar and Papatla 2000).

Fourth, people do not remember exact price from previous shopping occasions, but they have a kind of implicit memory (or knowledge) about the price level (Monroe and Lee 1999). Carry-over parameter γ is about 0.8 for all four categories implying that IRP depends on long history of past prices. This result also conforms to findings from previous studies such as Greenleaf (1995) and Hardie et al (1993).

Cross-Category Basket Model

Parameter estimates for category-level basket model is presented in Table 4.6. Investigating this table, we can compare different specifications about reference price spillover effects across categories in detail.

First, like within-category model, parameters have correct signs implied by theory. For example, parameters for household inventory show negative values for all categories. This implies that purchase likelihood of the category decreases as household inventory level is high.

Second, we find very strong positive correlation among random components of SKU level basket utilities under same cluster of categories. Parameter μ attached to inclusive value in GNL is 0.27 which implies correlation among baskets under same category cluster is about 0.92 $\left(=1-0.27^2\right)$. This result confirms the finding from model performance comparison (Table 4.4) which shows big leap in model fit from BM to GNL.

Finally, we find weak support of category level reference dependent evaluation proposed by MREM. We do not find an improvement of the model fit from GNL to MREM (e.g., calibration ρ^2 : 0.6199 → 0.6202 and holdout ρ^2 : 0.5699 → 0.5635). Parameters for category-level reference dependent evaluation mechanism, nevertheless, are significant and have expected signs. We find positive effect of *WG* (gain effect of category reference point) and negative effect of *WL* (loss effect of category reference point). This tells us that category-level basket utility increases when household experiences larger category attractiveness in current period than expected category attractiveness. The result, however, do not support loss aversion at the category level as opposed to the effects found in SKU level. Coefficient for gain effect (*WG*) is 0.17 whereas that for loss effect (*WL*) is -0.05.

Cross-Effect Matrix

We present cross-effect matrix $\mathbf{\Theta}$ from GNL in Table 4.7 because this model provides the best model fit and forecasting (Table 4.4). On the basis of the pattern and sizes of this matrix, we can infer the demand relationships among categories which are translated to cross-category spillover effects.

According to the table, we first find statistically significant substitutability between liquid detergent and powder detergent (θ = -1.51). This implies that these two categories are competing with each other in consumer's mind and thus favorable marketing mix effects including reference price effects for a category (i.e., gain) may decrease choice probability of items in the other category.

Meanwhile, all other pairs of categories show mild complementarity, but all of them are not statistically significant. Thus, we may conclude that those categories are selected mostly independently. Such independency provides very weak support to our theory about spillover effects of reference price effects for categories studied in this dissertation. According to the spillover mechanism illustrated in Figure 4.2, reference price effects along with other marketing mix effects transfer to other categories due to cross category dependences captured by cross-effect matrix $\boldsymbol{\Theta}$. Therefore, small values of θs imply no strong spillover effects.

4.5.5 Summary

On the basis of the model comparison and parameter estimates, we can summarize the following results. First, there is strong evidence for within-category reference price effects. Within-category models show significant gain/loss effects and also support loss aversion in most categories. Second, correlations in utility across baskets under same category cluster are high. The model fit improves substantially when we allow correlation in random components among utilities of baskets (i.e., BM → GNL). In addition, implied correlation is very large, i.e., 0.92. Third, we find weak evidence for category-level reference evaluation mechanism. The model fit statistics for MREM does not improve compared to GNL. However, parameter estimates for gain and loss effects at category-level are statistically significant. Finally, we find substitutability between liquid detergent and powder detergent category. We, nevertheless, find independencies among other pairs of categories (i.e., weak complementarity but insignificant). This may imply weak evidence for cross-category spillover effects because our theory suggests that demand relationship stimulates the spillover effects. To investigate this effect more closely, we need further investigation such as scenario analyses described in the following section.

4.6 Scenario Analyses

If spillover effects occur as we predict in the theory (Figure 4.2), choice probabilities of the items in other categories will change if price (or reference price) of one item in a category has shifted. If price of the item goes up, for instance, all items in substitutable categories (including the same category) gain market share due to the spillover effects of smaller gain and larger loss for the item. On the contrary, all items in complementary categories lose market share because of the spillover effects. We conduct two scenario analyses in which we compute cross-price elasticities depending on scenario. In Scenario 1, we change current price of an item assuming all others remain the same and then we compute choice probability of SKUs. The new choice probabilities based on this scenario are influenced by changes of price and ERP. Because IRP is based on previous price history, price change at specific time point does not influence IRP in that period. In Scenario 2, meanwhile, we directly change IRP, implicitly assuming that actual shelf price of previous period has changed. This scenario purely measures the impact of IRP on choice probabilities. The scenarios are summarized in Table 4.8.

The likelihood provides two probabilities, one for conditional SKU choice, $\Pr\left(j_c \mid c\right)$ and the other for category level basket choice, $\Pr\left(B = \mathbf{z}_{ht}^{\tilde{b}}\right)$. To compute cross-price elasticities, we must compute unconditional item choice probabilities, $\Pr\left(j_c\right)$ for each item using above two probabilities. We first reconstruct item level basket choice probabilities for all possible baskets $\Pr\left(b = \mathbf{y}^b\right)$ using the following equation:

$$\Pr\left(b = \mathbf{y}^b\right)_{ht} = \Pr\left(B = \mathbf{z}^{\tilde{b}}\right) \prod_c \Pr\left(j_c \mid c\right)^{r_{hc(\tilde{b})t}} \tag{4.17}$$

Using equation (4.17), we can construct unconditional choice probability $\Pr\left(j_c\right)$ by summing up all item level baskets containing item *j* in category *c*. Then, cross price elasticities (cross IRP elasticities for Scenario 2) are computed in the following way:

$$\eta_{jk} = \frac{\%\Delta \Pr\left(j_c\right)}{\%\Delta PRICE_k} = \left[\frac{\Pr\left(j_c\right)_{after} - \Pr\left(j_c\right)_{before}}{\Delta PRICE_k}\right] \cdot \left[\frac{PRICE_k}{\Pr\left(j_c\right)_{before}}\right] \tag{4.18}$$

The elasticity computation results based on Scenario 1 is displayed in Table 4.9. The output shows that all own elasticities are negative while cross-price elasticities within the category are all positive due to perfect substitutability. Cross-elasticities across categories have mixed signs but all signs are same within same block. This happens because we restrict that cross-effect parameters only operate at the category level. If there is any change in any item in one category, the effects influence all items in other category in the same direction. As we know from cross-effects patterns in Table 4.7, items in liquid detergent and items in powder detergent show positive cross elasticities implying substitutability between these two categories. Nevertheless, we find very small cross price elasticities across different categories, which is implied by insignificant and small values of cross-effects terms (Table 4.7). One possible reason of having small cross-elasticities even for the block between liquid detergent and powder detergent could be trade off between substitutability and complementarity among categories. As we discussed in Chapter 3, cross-elasticities not only depend on direct relationship between two categories θ^{AB} for category A and B, but also depend on indirect effect through other categories such as θ^{AC} and θ^{CB}. Thus, even though cross-effects between liquid detergent and powder detergent show strong substitutability, the final effects reflected in cross-elasticities might be attenuated by complementarity between liquid detergent and softeners as well as complementarity between softeners and powder detergent.

We presented the results from Scenario 2 in Table 4.10. For Scenario 2, we increased IRP by 10% resulting in larger gain (smaller loss) for its own item. This gain effect increases its own choice probability but decreases the choice probabilities of other items in same category because of substitutability. This implies that if the retail price has been set high in the past resulting in high IRP then changing back to normal price increases choice probability of the item. Nevertheless, because the effect only depends on IRP change, the effect sizes are much smaller than those from Scenario 3 (Table 4.9). Cross elasticities across categories show expected pattern, but all values are very close to zero confirming no spillover effects.

4.7 Discussion

This chapter customizes a SKU level market basket model proposed in Chapter 3, investigating reference price spillover effects across categories. We first specify within category reference price model built upon previous literature for reference price. We then construct category level basket model representing cross-category spillover effects of reference price. Therefore, reference price effects within a category can flow through other categories in our system. We develop four alternative models based upon different conceptual frameworks. We apply those competing models to household laundry detergent to understand reference price spillover effects.

4.7.1 Theoretical Contributions

The models developed in this chapter have a number of advantages. First, we construct a SKU level baseline models in which both IRP and ERP operate with household heterogeneity. Previous research has tried to identify these two reference price mechanisms and segment households based on the emphasis of one mechanism over the other (e.g., Mazumdar and Papatla 2000, Moon et al 2006). We use a continuous distributional assumption and find that IRP effect is the dominant mechanism for the households in our data. In particular, liquid softener category only supports IRP model. We also find the models with reference price outperform those without reference price effects. Second, we build four different models explaining reference price spillover effects across categories. Those are *Independence Model* assuming no spillover effects, *Basket Model* allowing for spillover effects, *Generalized Nested Logit* allowing for correlation among utilities of baskets, and *Multilevel Reference Effect Model* assuming category level reference dependent evaluation. Interestingly, although GNL and BM are developed from different theoretical framework, BM can be viewed as a special case of GNL when we set a proper restriction. If we set parameter attached to inclusive value

(W), μ to be one, the GNL is exactly the same model as BM in which all baskets in the consideration are assumed to be uncorrelated and considered perfect substitutes.

4.7.2 Substantive Contributions

We test four different models to understand reference price spillover effects. We find that reference price effects exist at SKU level within a category. The effects are so strong that retailers should not disregard dynamics of pricing when they set pricing policy. Large carry-over parameter, i.e., $\gamma \approx 0.8$, implies that people use history of past prices to construct IRP. However, we do not find strong evidence for spillover effects across categories. The models with cross-effect parameters show that spillover effects are not very apparent in the categories studied here. Cross price elasticities based on scenarios also show that values in off-diagonal blocks are close to zeros meaning no spillover effects. We also find that category level reference dependent evaluations, originally suggested by Bell and Bucklin (1999), do not operate across categories. Multiple category generalization of Bell and Bucklin (1999), named as MREM performs as good as GNL even though the parameters for multilevel effects are significant. This result is consistent with theory in consumer behavior literature. For product bundle evaluation, Janiszewski and Cunha (2004) suggest that consumers evaluate individual items separately based on price anchoring and adjusting and sum those individual utilities to form bundle utility.

Taken together, reference price effects strongly influence SKU choices within the category and these effects influence category attractiveness, thereby affecting category incidence decisions. However, reference price effects mainly stay within the category and do not spillover to other categories. Finally, consumers do not go through category level reference dependent evaluation in their market basket selection.

4.7.3 Limitations and Extensions

This study shares a key limitation identified in previous chapter. That is, as the number of SKU's increase, the model estimation becomes intractable. Nevertheless, according to our assumption about market structure (i.e., multivariate choice at category level and multinomial choice within category), we only need to evaluate baskets at category level. This significantly reduces the total number of baskets involved in the analyses. Therefore, as long as we keep the number of categories manageable, we can easily estimate the model.

Another limitation is that we do not fully investigate dynamic aspects of reference price effects across categories. We did scenario analyses, selecting one point of time. Such analyses provide valuable insights into the short-term effect of price change (and therefore reference price). Understanding long-term effect of price effect will be beneficial to the retailers to set category pricing policy. Greenleaf (1995) uses dynamic programming approach to figure out optimal pricing policy for items within a category. Due to multiple category nature in our research, such approach is very difficult to implement. Future research should conduct an analysis to understand long term effect of pricing policy across categories.

Finally, further studies are needed for better understanding about the distinction between purchase complementarity and consumption complementarity. One research avenue should involve the replication of this study with other categories which may possess stronger purchase complementarity than those used in this study. We can not generalize weak spillover effects of reference price to other categories until we fully understand the distinction between purchase complementarity and consumption complementarity. The other investigation must be the extension of the model enabling researchers to detect consumption complementarity as well as purchase complementarity.

Table 4.1 Models for Reference Price Spillover Effect

	SKU-Level Reference Price	**Spillover Effect**	**Correlation among Baskets**	**Category-Level Reference Point**
Independence Model (IM)	**Yes**	No	No	No
Basket Model (BM)	**Yes**	**Yes**	No	No
Generalized Nested Logit (GNL)	**Yes**	**Yes**	**Yes**	No
Multilevel Reference Effect Model (MREM)	**Yes**	**Yes**	**Yes**	**Yes**

Note: Correlations are allowed for baskets under same construction of categories (i.e., item level baskets under same upper category level basket tree).

Table 4.2 Multiple Category Purchase Incidences

# of Categories Purchased	Category Basket Composition $\left(\mathbf{z}^{\tilde{b}}\right)$	Frequency
One Category only	{1,0,0,0}	1470
	{0,1,0,0}	2442
	{0,0,1,0}	1008
	{0,0,0,1}	1171
	Sum	**6091**
Two Categories	{1,1,0,0}	58
	{1,0,1,0}	163
	{1,0,0,1}	255
	{0,1,1,0}	259
	{0,1,0,1}	418
	{0,0,1,1}	90
	Sum	**1243**
Three Categories	{1,1,1,0}	24
	{1,1,0,1}	16
	{1,0,1,1}	22
	{0,1,1,1}	25
	Sum	**87**
Four Categories	{1,1,1,1}	3
	Sum	**3**
TOTAL		**7424**

Note: Each elements represents liquid detergent, powder detergent, liquid softener and dry softener respectively. For example, {1,0,1,0} stands for the basket having liquid detergent and liquid softener only.

Table 4.3 Descriptive Statistics of Detergents and Softeners

Category	Item	# Choice	Price	Feature	Display
Liquid Detergent	AH	268	6.722	0.004	0.019
	ALL	191	8.263	0.033	0.016
	CHEERS	313	7.496	0.016	0.003
	ERA	312	5.646	0.000	0.000
	SURF	157	10.464	0.108	0.076
	TIDE	572	7.245	0.032	0.019
	WISK	375	10.964	0.122	0.144
	OTHERS	570	5.679	0.085	0.145
Powder Detergent	ALL	269	5.532	0.123	0.123
	CHEERS	642	8.113	0.067	0.067
	SURF	738	6.926	0.110	0.153
	TIDE	1264	7.425	0.167	0.235
	WISK	620	7.147	0.102	0.162
	OTHERS	1338	6.056	0.043	0.051
Liquid Softener	DOWNY	987	12.696	0.061	0.054
	SNUGGLE	772	10.979	0.090	0.227
	OTHERS	368	6.899	0.025	0.088
Dry Softener	BOUNCE	851	6.030	0.061	0.065
	SNUGGLE	636	5.919	0.069	0.094
	OTHERS	737	5.867	0.030	0.036
	CONTROL	449	4.509	0.040	0.022

Note: Others in Dry Softener category only contain national brands. Prices are price per unit of each category. Cents per ounce is used for Liquid Detergent, Powder Detergent, and Liquid Softener. Cents per sheet is used for Dry (or Sheet) Softener.

Table 4.4 Fit Statistics for Model Comparison

			IM	BM	GNL	MREM
Calibration	Data	k	96	108	110	114
		N	4745	4745	4745	4745
		HHs	440	440	440	440
	No Reference Price	Null L-L	-21604.0	-21604.0	-21604.0	-21604.0
		Model L-L	-13885.2	-13621.2	-11098.7	-11006.0
		AIC	27962.4	27458.3	22417.4	22240.0
		BIC	28583.0	28156.5	23128.5	22977.0
		ρ^2	0.3573	0.3695	0.4863	0.4906
	With Reference Price	Null L-L	-21604.0	-21604.0	-21604.0	-21604.0
		Model L-L	-10503.0	-10497.0	-8212.5	-8204.3
		AIC	21222	21234	16669	16660.6
		BIC	21920.2	22009.8	17457.7	17475.2
		ρ^2	0.5138	0.5141	0.6199	0.6202

			IM	BM	GNL	MREM
Holdout	Data	N	1380	1380	1380	1380
		HHs	155	155	155	155
	No Reference Price	Null L-L	-5127.6	-5127.6	-5127.6	-5127.6
		Model L-L	-3710.5	-3695.7	-3027.0	-3024.8
		ρ^2	0.2764	0.2793	0.4097	0.4101
	With Reference Price	Null L-L	-5127.6	-5127.6	-5127.6	-5127.6
		Model L-L	-2854.6	-2845.0	-2220.7	-2238.0
		ρ^2	0.4433	0.4452	0.5669	0.5635

Note: IM - Independence Model, BM - Basket Model, GNL - Generalized Nested Logit, MREM - Multilevel Reference Effect Model. k stands for number of parameters, N for number of observations, and HHs for number of households

Table 4.5 Parameter Estimates for Within-Category SKU-Level Models

	Liq-Det		Pow-Det		Liq-Soft		Dry-Soft	
	Mean	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**
Price	-0.43*	0.35**	-0.19	0.84**	-0.69**	0.60**	-2.20**	0.70**
	(0.20)	(0.04)	(0.12)	(0.10)	(0.17)	(0.08)	(0.50)	(0.14)
Feature	1.63**	3.39**	0.75**	0.27	0.76**	1.53	0.32	2.50**
	(0.73)	(0.69)	(0.23)	(0.53)	(0.84)	(1.17)	(0.77)	(1.14)
Display	0.17	2.36**	0.40*	0.54	1.02*	2.40**	1.65**	0.52
	(0.56)	(0.57)	(0.20)	(0.35)	(0.69)	(0.76)	(0.62)	(0.94)
Loyalty	2.28**	1.34**	1.32**	0.90**	2.09**	1.39**	2.22**	1.68**
	(0.17)	(0.16)	(0.12)	(0.23)	(0.26)	(0.24)	(0.27)	(0.24)
Gain	2.03**	1.48**	1.80**	0.22	0.81**	0.46	2.78**	2.43**
	(0.27)	(0.16)	(0.15)	(0.37)	(0.19)	(0.20)	(0.53)	(0.34)
Loss	-1.32**	0.58*	-2.34**	1.07**	-1.86**	1.06**	-5.24**	2.82**
	(0.25)	(0.16)	(0.19)	(0.17)	(0.26)	(0.20)	(0.69)	(0.45)
λ_h^c	0.63**	0.06	0.84*	0.01	1.00		0.65*	1.85**
	(0.15)	(0.13)	(0.30)	(0.39)	(NA)		(0.28)	(0.55)
γ^c	0.82**		0.76**		0.82**		0.82**	
	(0.16)		(0.09)		(0.15)		(0.09)	
item1	-	-	-	-	-	-	-	-
item2	0.66*	1.07**	0.76*	1.64**	-1.89**	2.62**	-0.63*	2.03**
	(0.33)	(0.19)	(0.37)	(0.22)	(0.39)	(0.32)	(0.29)	(0.29)
item3	-1.15**	2.51**	0.68*	1.14**	-5.10**	1.28**	0.19	1.45**
	(0.49)	(0.47)	(0.24)	(0.15)	(0.99)	(0.47)	(0.25)	(0.25)
item4	-1.54**	1.35**	1.52**	1.92**			-4.40**	4.64**
	(0.36)	(0.30)	(0.29)	(0.26)			(0.75)	(0.51)
item5	-0.71	1.51**	0.76**	0.49				
	(0.62)	(0.22)	(0.25)	(0.30)				
item6	0.42	1.98**	0.87**	2.24**				
	(0.31)	(0.31)	(0.24)	(0.29)				
item7	0.22	2.14**						
	(0.64)	(0.28)						
item8	-1.52**	2.34**						
	(0.35)	(0.26)						

Note: The notation S.D. indicates standard deviation of the household heterogeneity distribution. Significance levels are denoted as $p < 0.05$ (*) and $p < 0.01$ (**). Values in parentheses are standard errors. λ is a relative weights on IRP versus ERP in construction of reference price. γ is exponential smoothing parameter for IRP.

Table 4.6 Parameter Estimates for Category Level Model

		IM		BM		GNL		MREM	
		Mean	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**
Category Intercepts (ψ^c)	**LiqDet**	0.17	3.07**	1.01**	2.80**	-1.09**	1.18**	-1.07**	1.05**
		(0.13)	(0.10)	(0.25)	(0.09)	(0.22)	(0.08)	(0.22)	(0.1)
	PowDet	-4.12**	3.54**	-3.63**	3.69**	-2.02**	1.17**	-1.72**	1.03**
		(0.12)	(0.08)	(0.25)	(0.08)	(0.22)	(0.08)	(0.22)	(0.09)
	LiqSft	5.49**	4.15**	5.58**	4.45**	0.31	1.30**	0.05	0.91**
		(0.13)	(0.10)	(0.28)	(0.12)	(0.25)	(0.09)	(0.25)	(0.09)
	DrySft	8.03**	2.34**	8.14**	1.71**	0.41	0.64**	-0.03	0.67**
		(0.11)	(0.10)	(0.24)	(0.09)	(0.25)	(0.08)	(0.26)	(0.12)
Household Inventory (β^c_{hI})	**LiqDet**	-0.82**	0.94**	-0.99**	1.10**	-0.77**	0.49**	-0.74**	0.50**
		(0.09)	(0.10)	(0.11)	(0.11)	(0.09)	(0.08)	(0.09)	(0.11)
	PowDet	-0.89**	1.28**	-0.90**	0.84**	-0.79**	0.77**	-0.70**	0.55**
		(0.06)	(0.08)	(0.06)	(0.08)	(0.07)	(0.09)	(0.06)	(0.06)
	LiqSft	-1.24**	0.60*	-1.12**	0.39	-1.02**	0.05	-1.16**	0.59**
		(0.14)	(0.19)	(0.13)	(0.16)	(0.11)	(0.14)	(0.13)	(0.14)
	DrySft	-3.43**	1.58**	-3.23**	1.29**	-2.69**	1.34**	-2.42**	1.22**
		(0.23)	(0.19)	(0.19)	(0.12)	(0.19)	(0.16)	(0.19)	(0.17)
Inclusive Values (μ)	**W**	-		-		0.27**	0.02	0.21**	0.03*
						(0.01)	(0.01)	(0.01)	(0.01)
	WG	-		-		-		0.17**	0.25**
								(0.03)	(0.04)
	WL	-		-		-		-0.05*	0.02
								(0.02)	(0.04)

Note: LiqDet - Liquid Detergent, PowDet - Powder Detergent, LiqSft - Liquid Softener, DrySft: Dry (or Shee) Softener. WG = W-E(W) when W > E(W). WL = E(W)-W when W < E(W). The notation S.D. indicates standard deviation of the household heterogeneity distribution. Significance levels are denoted as $p < 0.05$ (*) and $p < 0.01$ (**). Values in parentheses are standard errors.

Table 4.7 Cross-Effects Matrix based on Generalized Nested Logit

	Liq-Det	Pow-Det	Liq-Soft	Dry-Soft
Liq-Det	0.0	-1.51** (0.31)	0.23 (0.22)	0.33 (0.22)
Pow-Det	-1.51** (0.31)	0.0	0.07 (0.21)	0.33 (0.21)
Liq-Soft	0.23 (0.22)	0.07 (0.21)	0.0	0.07 (0.22)
Dry-Soft	0.33 (0.22)	0.33 (0.21)	0.07 (0.22)	0.0

Note: LiqDet - Liquid Detergent, PowDet - Powder Detergent, LiqSft - Liquid Softener, DrySft: Dry (or Shee) Softener. Significance levels are denoted as $p < 0.05$ (*) and $p < 0.01$ (**). Values in parentheses are standard errors.

Table 4.8 Scenario Analyses Description

	Scenario	Reference Price Effect	Spillover Effect
Scenario 1 (ERP effect)	PRICE(j) 10% increase	Larger LOSS (j) Smaller GAIN (j) No IRP(j) change	Increased MS(k) in Sub-Cat Decreased MS(k) in Comp-Cat
Scenario 2 (IRP effect)	IRP(j) 10% increase	Larger GAIN (j) Smaller LOSS (j) No ERP change (j)	Decreased MS(k) in Sub-Cat Increased MS(k) in Comp-Cat

Note: Scenario 1, in fact, has both own price and ERP effects. MS(k) stands for market share for item k. In empirical analysis, Market share is replaced by unconditional choice probability, Pr(k). Sub-Cat stands for substitutable categories including own category. Comp-Cat stands for complementary categories.

Table 4.9 Results from Scenario 1 (Price and ERP effect) – Decrease Own Price by 10%

	SKU1	SKU2	SKU3	SKU4	SKU5	SKU6	SKU7	SKU8	SKU9	SKU10	SKU11	SKU12	SKU13	SKU14	SKU15	SKU16	SKU17	SKU18	SKU19	SKU20	SKU21
MS1	**-8.15**	0.85	0.63	0.63	0.54	1.00	0.53	0.44	0.01	0.01	0.03	0.04	0.01	0.03	-0.02	-0.01	0.00	-0.01	-0.04	-0.02	-0.02
MS2	0.54	**-11.96**	0.78	0.66	0.59	1.04	1.13	0.33	0.01	0.01	0.04	0.03	0.04	0.02	-0.02	-0.01	0.00	-0.04	-0.05	-0.02	0.00
MS3	0.59	0.59	**-6.95**	0.45	0.45	0.83	0.77	0.43	0.00	0.01	0.02	0.02	0.01	0.01	-0.02	0.00	0.00	-0.04	-0.03	-0.04	0.00
MS4	0.55	0.77	0.65	**-3.98**	0.50	0.94	0.76	0.36	0.01	0.01	0.03	0.02	0.01	0.02	-0.04	-0.02	0.00	0.00	-0.01	-0.01	0.00
MS5	0.34	1.11	0.78	0.65	**-16.33**	1.08	1.03	0.70	0.00	0.00	0.02	0.02	0.00	0.01	0.00	-0.01	0.00	0.00	-0.01	0.00	0.01
MS6	0.38	0.55	0.51	0.38	0.33	**-4.27**	0.47	0.35	0.00	0.02	0.03	0.02	0.02	0.02	-0.04	-0.02	0.00	-0.01	0.00	-0.01	0.00
MS7	0.38	1.03	0.54	0.40	0.74	0.90	**-15.87**	0.58	0.00	0.01	0.02	0.01	0.01	0.02	-0.01	0.00	0.00	-0.03	-0.02	-0.02	-0.01
MS8	0.32	0.66	0.48	0.47	0.74	0.75	0.70	**-2.66**	0.00	0.01	0.01	0.02	0.01	0.01	-0.02	-0.01	0.00	0.00	0.00	-0.02	0.00
MS9	0.01	0.04	0.02	0.02	0.01	0.03	0.03	0.02	**-5.99**	0.80	0.97	0.99	0.75	0.71	-0.03	-0.05	0.00	-0.04	-0.04	-0.07	-0.01
MS10	0.00	0.01	0.01	0.00	0.00	0.02	0.01	0.00	0.24	**-8.54**	0.58	0.84	0.43	0.43	0.00	0.00	0.00	-0.04	-0.02	-0.03	0.00
MS11	0.01	0.02	0.02	0.01	0.01	0.03	0.02	0.01	0.29	0.78	**-3.96**	0.93	0.67	0.76	0.00	0.00	0.00	-0.03	-0.03	-0.03	-0.01
MS12	0.01	0.02	0.01	0.01	0.01	0.03	0.01	0.01	0.23	0.68	0.64	**-3.34**	0.44	0.41	-0.01	0.00	0.00	-0.02	-0.02	-0.03	-0.01
MS13	0.01	0.03	0.02	0.01	0.00	0.02	0.02	0.01	0.29	0.59	0.85	0.86	**-6.33**	0.58	0.00	0.00	0.00	-0.04	-0.03	-0.05	-0.01
MS14	0.01	0.01	0.01	0.01	0.01	0.03	0.01	0.01	0.22	0.51	0.67	0.65	0.48	**-2.96**	0.00	0.00	0.00	-0.02	-0.01	-0.03	-0.01
MS15	0.00	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**-3.55**	0.96	0.14	0.01	0.01	0.00	0.00
MS16	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	1.65	**-6.00**	0.15	0.00	0.00	0.00	0.00
MS17	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.25	1.24	**-3.86**	-0.01	0.00	0.00	0.00
MS18	0.00	-0.01	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	**-6.03**	0.59	0.66	0.28
MS19	0.00	-0.01	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.51	**-4.72**	0.57	0.21
MS20	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.58	0.53	**-3.70**	0.22
MS21	0.00	0.00	-0.01	0.00	0.00	-0.01	-0.01	0.00	0.00	0.00	-0.01	-0.01	0.00	-0.01	0.00	0.00	0.00	0.48	0.47	0.55	**-1.38**

Note: Table displays the percentage change in the aggregate choice share of the row SKU with respect to a one percent increase in the price of the column SKU.

Table 4.10 Results from Scenario 2 (IRP only effect) – Increase IRP by 10%

	SKU1	SKU2	SKU3	SKU4	SKU5	SKU6	SKU7	SKU8	SKU9	SKU10	SKU11	SKU12	SKU13	SKU14	SKU15	SKU16	SKU17	SKU18	SKU19	SKU20	SKU21
MS1	**1.11**	-0.12	-0.43	-0.26	-0.08	-0.41	-0.18	-0.09	0.00	0.00	-0.02	-0.02	0.00	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
MS2	-0.05	**2.07**	-0.13	-0.17	-0.19	-0.25	-0.35	-0.07	0.00	0.00	-0.04	-0.02	-0.04	-0.01	0.01	0.01	0.00	0.01	0.00	0.00	0.00
MS3	-0.11	-0.08	**2.21**	-0.16	-0.04	-0.38	-0.19	-0.07	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00
MS4	-0.10	-0.14	-0.21	**1.28**	-0.14	-0.37	-0.16	-0.10	0.00	0.00	-0.02	-0.01	0.00	-0.01	0.02	0.01	0.00	0.00	0.00	0.00	0.00
MS5	-0.02	-0.27	-0.08	-0.19	**6.20**	-0.28	-0.14	-0.51	0.00	0.00	-0.01	-0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
MS6	-0.05	-0.07	-0.17	-0.13	-0.06	**1.46**	-0.09	-0.08	0.00	-0.01	-0.02	-0.01	-0.01	-0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00
MS7	-0.02	-0.27	-0.22	-0.12	-0.10	-0.24	**4.61**	-0.12	0.00	0.00	-0.01	0.00	0.00	-0.01	0.01	0.01	0.00	0.01	0.00	0.01	0.00
MS8	-0.03	-0.07	-0.11	-0.14	-0.55	-0.26	-0.13	**0.68**	0.00	0.00	-0.01	-0.01	-0.01	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
MS9	0.00	-0.01	0.00	-0.01	0.00	-0.01	-0.01	0.00	**2.00**	-0.22	-0.78	-0.64	-0.32	-0.35	0.00	0.02	0.00	0.00	0.00	0.00	0.00
MS10	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	-0.05	**3.41**	-0.53	-0.62	-0.18	-0.31	0.00	0.00	0.00	0.01	0.00	0.01	0.00
MS11	0.00	-0.01	-0.01	0.00	0.00	-0.01	-0.01	0.00	-0.11	-0.31	**2.91**	-0.61	-0.33	-0.43	0.00	0.00	0.00	0.00	0.01	0.01	0.00
MS12	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	-0.07	-0.30	-0.48	**1.94**	-0.22	-0.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MS13	0.00	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	-0.09	-0.22	-0.74	-0.57	**2.87**	-0.31	0.00	0.00	0.00	0.00	0.00	0.01	0.00
MS14	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	-0.07	-0.21	-0.55	-0.38	-0.18	**1.61**	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MS15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**1.81**	-0.52	-0.05	0.00	0.00	0.00	0.00
MS16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.17	**2.55**	-0.04	0.00	0.00	0.00	0.00
MS17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.02	-0.53	**1.09**	0.01	0.00	0.00	0.00
MS18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.76**	-0.05	-0.13	-0.05
MS19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	-0.03	**0.48**	-0.07	-0.02
MS20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	-0.11	-0.05	**0.61**	-0.04
MS21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05	-0.02	-0.07	**0.28**

Note: Table displays the percentage change in the aggregate choice share of the row SKU with respect to a one percent increase in the IRP of the column SKU.

Figure 4.1 Tree Structure for Multinomial Nested Logit

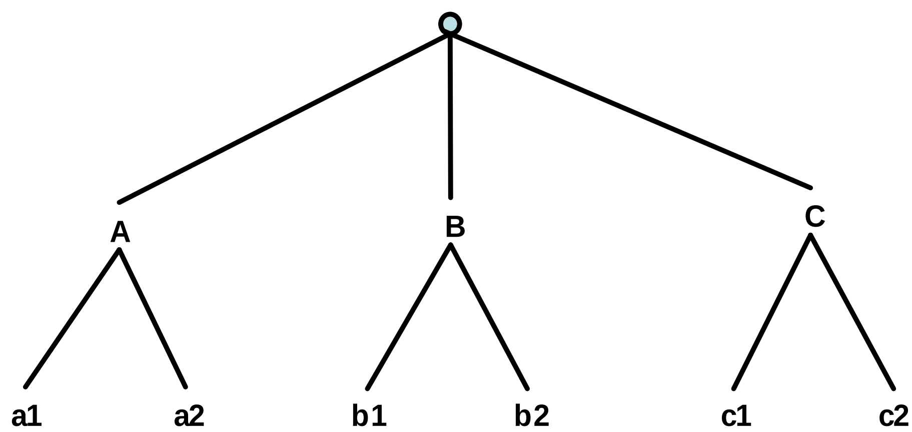


Figure 4.2 Cross-Category Reference Price Mechanism

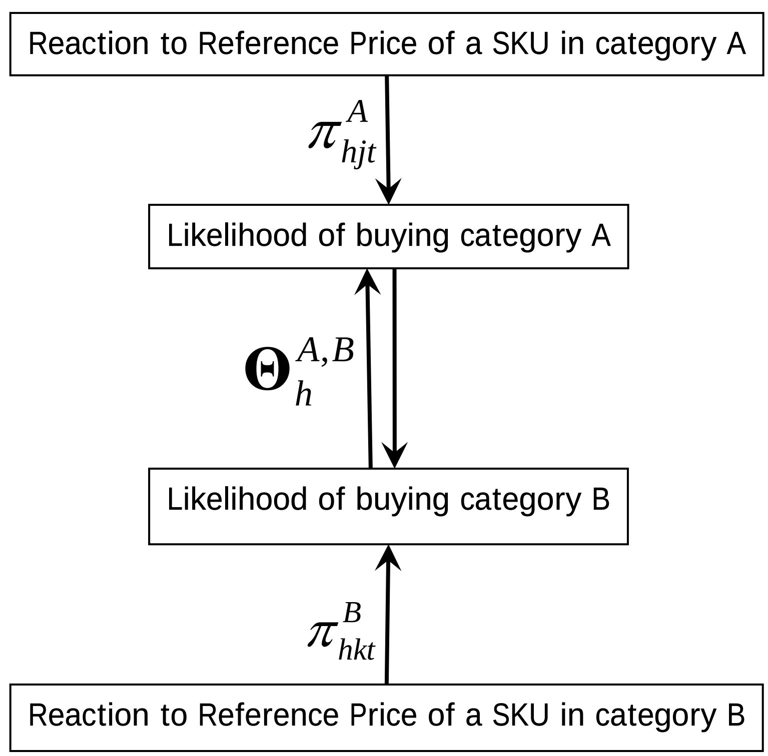
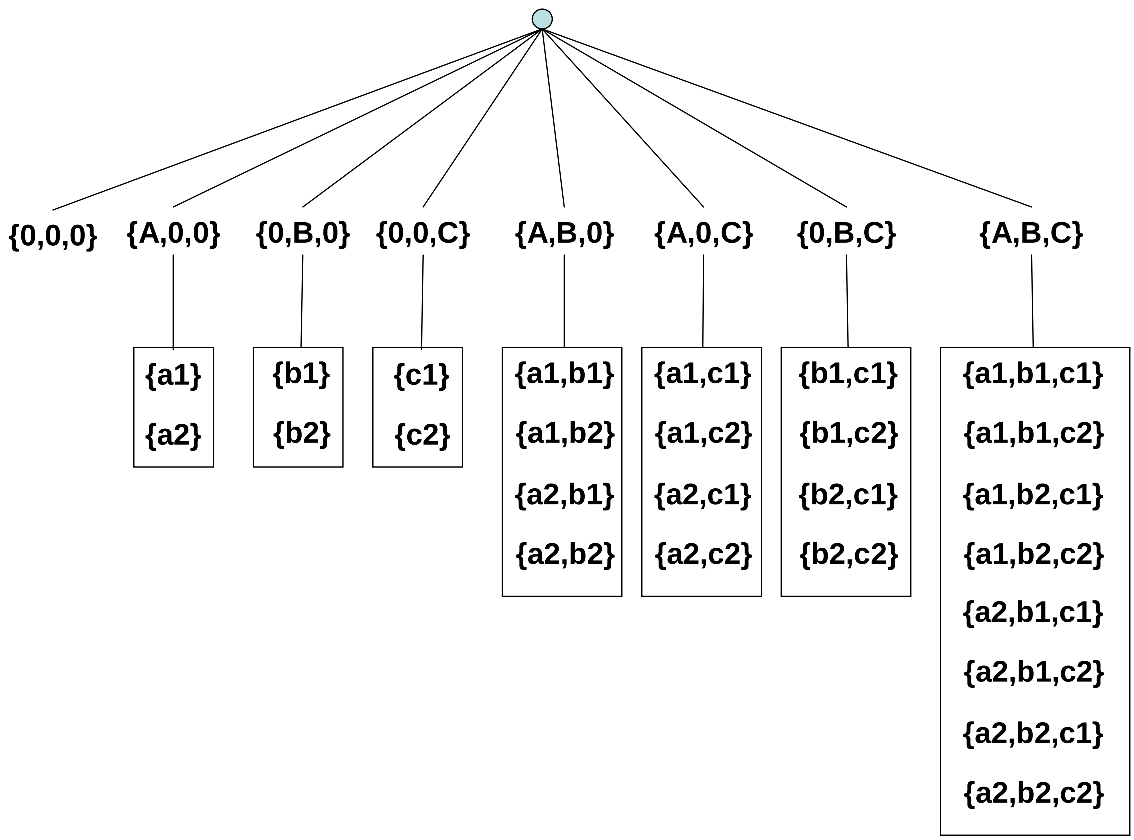


Figure 4.3 Tree Structure for Generalized Nested Logit



CHAPTER 5
CONCLUSION

This dissertation develops a SKU level model of market basket choice which allows for great degree of flexibility in demand interactions across items. The approach, a generalization of the Russell and Petersen (2000) category incidence basket model to SKU basket choice, offers the researcher the ability to constrain the probability distribution of baskets to correspond to known aspects of market structure. In Chapter 3, we apply the model in an analysis of SKU competition in the yogurt product category. In Chapter 4, we customize the model to understand how reference price mechanisms work in cross-category setting, so called cross-category spillover effects. We first specify within category reference price model built upon previous literature for reference price. We then construct four alternative model specifications at the category level, in which every model has different theoretical implication about cross-category spillover effects of reference price. We apply those competing models to household laundry product categories.

5.1 Summary of the Dissertation

In Chapter 3, we show that households treat brand names as strong substitutes, and flavors (within a brand) as weak substitutes and complements. We also show that the general pattern of SKU price competition is consistent with the Blattberg and Wisniewski (1989) theory of price tier competition. The estimated cross-price elasticities clearly show that price changes of the premium brand (Dannon) have more impact on the lower-tier brand (Nordica) than vice versa. We also demonstrate how optimal prices might be set using the model output. These results suggest that a pricing rule based upon brand name is consistent with the pattern of brand competition in the yogurt category.

In Chapter 4, we find that reference price effects exist at SKU level within a category. The effects are so strong that retailers should take this dynamics into account for their pricing policy. In particular, consumers construct implicit knowledge (Monroe and Lee 1999) about price level based on history of the pricing patterns of the product. We, however, find that within-category reference price effects may not spillover across categories at least for the categories studied in this dissertation. The models with cross-effect parameters do not improve the model fit compared to that with independence assumption. Moreover, cross price elasticities based on scenarios also show that spillover effects are not very strong. Finally, the model for category level reference dependent evaluation (MREM), a multiple category extension of Bell and Bucklin (1999) reveals significant gain and loss effects, but incorporating those effects does not improve the forecasting ability.

The failure of observing spillover effects can be attributed to several factors. First, category demand relationships might not be strong enough to observe spillover effects. In our model specification, reference price effects influence category attractiveness, thereby affecting category incidence, and then demand relationships among categories dictate the strength of the spillover effects. At category incidence decision, previous models have inconsistent implications about cross-category dependencies. Manchanda (1999) find that price change of detergent has influence on choice of softener category (i.e., complementarity), whereas Russell and Petersen (2000) do not report strong category dependencies among four paper good categories. Song and Chintagunta (2007) incorporate both category incidence decision and brand choice decision and find no strong influence of price change on a brand on other brands in another category, observing very small cross-brand elasticities across categories.

Second, purchase complementarity (purchase of categories at the same time) might not be as strong as consumption complementarity (use of categories at the same time). The categories we have chosen, i.e., detergents and softeners, may possess strong consumption complementarity but have very weak purchase complementarity because of different package size and usage pattern. The models developed in this thesis are only able to capture purchase complementarity.

Third, co-incidence or unobserved random components might be the major source for cross-category elasticities. Co-incidence is suggested by Manchanda (1999) and typically captured by correlations among random components in utilities of categories. Duvvuri and Gruca (2007) and Duvvuri et al (2007) confirmed that cross-category correlations in error structure are significant. In addition, Song and Chintagunta (2007) claimed that co-incidence due to unobserved category promotion effects might be the major source for cross-category elasticities as the cross-brand elasticities due to the conditional purchase quantity are much smaller. In our data, only 18% (1333 out of 7424 purchase occasions) are considered multiple category choices.

Fourth, within each household, category choices might be independently made for the basket construction. Previous models using store-level data have found significant cross-category effects for the categories used in this dissertation, whereas some models using household panel data have not found strong category dependencies. Song and Chintagunta (2006), for example, have found significant complementarity between liquid softener and both liquid and powder detergents. On the contrary, Russell and Peterson (2000), which has same specification at the category level, suggest that no specification for store traffic effect in the model might result in small estimated cross-category dependencies. They argue that the major impact of category pricing might be on store choice. Therefore, researchers may be able to observe category dependencies in store-level aggregated data even though category choice decisions are independently made within a household (i.e., no spillover effects). In addition, knowing household inventory level and brand preference, we may be able to forecast consumer's basket construction within each household.

5.2 Contribution of the Dissertation

This dissertation contributes to the literature in two ways. First, in terms of modeling effort, we provide a very flexible multivariate choice model to be used not only for multi-category analysis but also for any kind of pick-any choice situation (e.g., multiple channel selection or cross-selling across channels). Even though a few recent papers adopt similar framework, our model is so general that other models are special cases of our modeling framework. In addition, we derive a generalized (or multivariate) nest logit (GNL) which nests our basket model as a special case. We apply those models to scanner panel data to understand price competition and marketing mix spillover effects across categories. We also incorporate household heterogeneity.

This dissertation also enriches reference price effects literature, which is the substantive issue of this dissertation and what distinguishes this dissertation to other cross-category research. We investigate reference price effects across categories at SKU level. We test four different theoretical frameworks based upon previous literature as an attempt to understand reference price spillover mechanisms. This is very important because it enables retailers to coordinate cross-category pricing policy at SKU level. We found that reference price effects mainly confine at SKU level and do not have a large influence on item choices in other category. In addition, we do not find evidence of category-level reference dependent evaluation occurs at category choice in market basket selection.

5.3 Future Research Directions

In terms of modeling effort, the proposed basket model (with other variations developed in Chapter 4) is very flexible in modeling any kind of multivariate pick-any choice situations. Researchers may find the proposed approaches are suitable for a variety of research areas. Some examples might include product assortment decisions, cross-selling, product recommendations (Moon and Russell 2006), and multi-channel coordination. The proposed approach, nevertheless, has limitations as well. As the number of alternatives increases, thereby increasing number of baskets, the model estimation becomes very difficult. We try to avoid this problem by reducing number of baskets (Chapter 3) or by decomposing choice decisions (Chapter 4). Further investigation is needed to develop new estimation methodologies utilizing simulation techniques, e.g., MCMC MLE or Bayesian statistics. In addition, we restrict our attention to within-category dynamics and cross-category spillover effect at fixed time point. The methodology which allows full investigation of inter-temporal cross-category spillover effects, e.g.., dynamic programming, might be a potential research direction.

In terms of theoretical perspectives, we may also need additional research to expand our understanding of reference price effects and of its cross-category implications. First, combining household level data and store level data for better insights might be a beneficial (Russell and Kamakura 1994), because previous literature has suggested that different level of aggregation of data provide different insights. Second, we may replicate the study using categories which potentially show strong purchase complementarity, e.g., cake mix and frosting. Third, we may extend our research to understand cross-category implications of reference price in different types of goods or purchasing environment. Reference price effects might operate differently for bundling durable goods, e.g., computer, monitor, and printer. Those effects may also differ in online purchasing environment where consumers can construct reference point very easily by searching other online stores as opposed to grocery shopping which requires physical presence in different stores.

APPENDIX A
DERIVATION OF BASKET MODEL

Let $\mathbf{z} = (z_1, z_2, \cdots, z_J)$ be any basket of item choices. Brook's Lemma, cited in Besag (1974), states that the joint distribution of random variable z_j, namely $\Pr(z_1, z_2, \cdots, z_J)$, is proportional to a series of ratios through a certain constant $\Pr(z_{10}, z_{20}, \cdots, z_{J0})$ where z_{j0} is an arbitrary reference value of the random variable z_j. Specifically, Brook's Lemma states that

$$\text{(A1)}\ \frac{\Pr(z_1, z_2, \cdots, z_J)}{\Pr(z_{10}, z_{20}, \cdots, z_{J0})} = \frac{\Pr(z_1 \mid z_2, z_3, \cdots, z_J)}{\Pr(z_{10} \mid z_2, z_3, \cdots, z_J)} \frac{\Pr(z_2 \mid z_{10}, z_3, \cdots, z_J)}{\Pr(z_{20} \mid z_{10}, z_3, \cdots, z_J)} \cdots \frac{\Pr(z_J \mid z_{10}, z_{20}, \cdots, z_{J-1,0})}{\Pr(z_{J0} \mid z_{10}, z_{20}, \cdots, z_{J-1,0})}$$

In this research, we define the joint distribution $\Pr(z_1, z_2, \cdots, z_J)$ to be the probability of choosing a basket of products.

We derive a basket choice model in equation (3.6) in the following way. First, we assume (without loss of generality) that the reference values $z_{j0} = 0$ so that the proportionality constant $\Pr(z_{10}, z_{20}, \cdots, z_{J0})$ is equal to $\Pr(0, 0, \cdots, 0)$. Second, we assume that all the conditional probabilities on the right hand side of (A1) have the form of the binary logit expressions found in equation (3.5). Third, we assume that all elements of the $\mathbf{\Theta}_h$ in equation (3.3) are symmetric. This last assumption essentially means that the probability of observing a basket depends only on the contents of the basket – not upon the order in which the various products in the basket are purchased.

The basket choice probability given in equation (3.6) is obtained by inserting these assumptions into (A1) and using the fact that the sum of the probabilities over all baskets must add to one. The symmetry of $\mathbf{\Theta}_h$ ensures that the basket model generated by (A1) always has the same form, regardless of the order in which the SKU's are placed in the $\mathbf{z} = (z_1, z_2, \cdots, z_J)$ vector. The model implicitly assumes that any basket – including the null basket with probability equal to $\Pr(0, 0, \cdots, 0)$ – may be observed. However, the analyst can restrict the model to a subset of baskets by restricting the possible values of $\mathbf{z}$ and then renormalizing the resulting model.

APPENDIX B
DERIVATION OF PRICE ELASTICITIES

We first derive household-level price elasticities. Let *j* and *k* denote two different SKU's. Item *j* and *k* could either have same brand or different brands.

At specific time point *t* for household *h*, elasticities are computed as

$$E\left(j,j\right)_{ht}=\frac{\partial\left[\ln\Pr\left(j\right)_{ht}\right]}{\partial\left[\ln PRICE\left(j\right)_{ht}\right]}=\frac{\partial\Pr\left(j\right)_{ht}}{\partial PRICE\left(j\right)_{ht}}\frac{PRICE\left(j\right)_{ht}}{\Pr\left(j\right)_{ht}} \quad \text{(B1)}$$

$$E\left(j_c,k_{c*}\right)_{ht}=\frac{\partial\left[\ln\Pr\left(j_c\right)_{ht}\right]}{\partial\left[\ln PRICE\left(k_{c*}\right)_{ht}\right]}=\frac{\partial\Pr\left(j_c\right)_{ht}}{\partial PRICE\left(k_{c*}\right)_{ht}}\frac{PRICE\left(k_{c*}\right)_{ht}}{\Pr\left(j_c\right)_{ht}} \quad \text{(B2)}$$

Using equation (3.6), we can write probability of buying item *j* by enumerating all probabilities of baskets containing item *j*. We can also define the probability of choosing item *j* and *k* in one basket in the same way. These can be written as

$$\Pr\left(j\right)_{ht}=\sum_{b\,\text{containg item } j}\frac{\exp\left(\boldsymbol{\pi}'_{ht}\mathbf{z}^{b}_{ht}+\frac{1}{2}\left\{\mathbf{z}^{b}_{ht}{}'\boldsymbol{\Theta}_h\mathbf{z}^{b}_{ht}\right\}\right)}{\sum_{b^*}\exp\left(\boldsymbol{\pi}'_{ht}\mathbf{z}^{b^*}_{ht}+\frac{1}{2}\left\{\mathbf{z}^{b^*}_{ht}{}'\boldsymbol{\Theta}_h\mathbf{z}^{b^*}_{ht}\right\}\right)} \quad \text{(B3)}$$

$$\Pr\left(j,k\right)_{ht}=\sum_{b\,\text{containg item } j\,\text{and}\,k}\frac{\exp\left(\boldsymbol{\pi}'_{ht}\mathbf{z}^{b}_{ht}+\frac{1}{2}\left\{\mathbf{z}^{b}_{ht}{}'\boldsymbol{\Theta}_h\mathbf{z}^{b}_{ht}\right\}\right)}{\sum_{b^*}\exp\left(\boldsymbol{\pi}'_{ht}\mathbf{z}^{b^*}_{ht}+\frac{1}{2}\left\{\mathbf{z}^{b^*}_{ht}{}'\boldsymbol{\Theta}_h\mathbf{z}^{b^*}_{ht}\right\}\right)} \quad \text{(B4)}$$

where $\pi^p_{hjt}=\alpha^p_{hj}+\beta^p_{1,h}PRICE_{hpt}+\beta^p_{2,h}DISP_{hpt}+\beta^p_{3,h}FEAT_{hpt}+\beta^p_{4,h}LOY_{hj}$ and marketing mix parameters are brand specific, not item specific. Simple algebra gives us

$$\frac{\partial\Pr\left(j_p\right)_{ht}}{\partial PRICE\left(j_p\right)_{ht}}=\beta^p_{1,h}\Pr\left(j_p\right)_{ht}\left\{1-\Pr\left(j_p\right)_{ht}\right\} \qquad \text{if } j \text{ has brand } p$$

$$\frac{\partial\Pr\left(j_p\right)_{ht}}{\partial PRICE\left(k_q\right)_{ht}}=-\beta^q_{1,h}\Pr\left(j_p\right)_{ht}\Pr\left(k_q\right)_{ht} \qquad \text{if brand } p\neq q \quad \text{(B5)}$$

$$\frac{\partial\Pr\left(j_p\right)_{ht}}{\partial PRICE\left(k_p\right)_{ht}}=\beta^p_{1,h}\left\{\Pr\left(j_p,k_p\right)_{ht}-\Pr\left(j_p\right)_{ht}\Pr\left(k_p\right)_{ht}\right\} \text{if } j\neq k \text{ for same brands}$$

Applying elasticity definitions in (B1) and (B2), the corresponding household-level elasticities are computed as

$$
\text{(B6)}\quad
\begin{aligned}
&E\left(j_p, j_p\right)_{ht} = \beta^p_{price}\left\{1-\Pr\left(j_p\right)_{ht}\right\} PRICE^p_{hjt} \\
&E\left(j_p, k_q\right)_{ht} = -\beta^q_{price}\Pr\left(k_q\right)_{ht} PRICE^q_{hkt} \qquad \text{if } p \neq q \\
&E\left(j_p, k_p\right)_{ht} = \beta^p_{price}\Pr\left(k_p\right)_{ht}\left\{S\left(j_p, k_p\right)_{ht}-1\right\} PRICE^p_{hkt} \ \text{ if } j \neq k \text{ for same brand}
\end{aligned}
$$

where $S\left(j,k\right)_{ht} = \Pr\left(j,k\right)_{ht} / \left\{\Pr\left(j\right)_{ht}\Pr\left(k\right)_{ht}\right\}$. When there are more than two items in a category, cross-elasticity between two items in same category is not solely determined by cross-effect terms but by $S\left(j,k\right)$. Notice that $S\left(j,k\right) > 0$ for complements, $S\left(j,k\right) < 0$ for substitutes, and $S\left(j,k\right) = 0$ for independence.

To compute aggregate elasticities, we first define the overall choice shares as $MS_j = \sum_h \sum_t \Pr\left(j\right)_{ht} / NT$ where NT (the number households (N) times average number of shopping trips (T) per household) is the number of total observations in a particular data set. Following procedures outlined in Russell and Kamakura (1994), we differentiate this expression and use the household level derivatives in (B5) to infer market share elasticities. This process yields the expressions:

$$
\text{(B7)}\quad
\begin{aligned}
&\eta_{jj} = \frac{\%\Delta MS\left(j_p\right)}{\%PRICE^p_j} = \left[\frac{\left\{\sum_h \beta^p_h \Pr\left(j_p\right)_h\left[1-\Pr\left(j_p\right)_h\right]\right\}/N}{MS\left(j_p\right)}\right]\widehat{PRICE}^p_j \\
&\eta^{p,q}_{jk} = \frac{\%\Delta MS\left(j_p\right)}{\%PRICE^q_k} = -\left[\frac{\left\{\sum_h \beta^q_h \Pr\left(j_p\right)_h \Pr\left(k_q\right)_h\right\}/N}{MS\left(j_p\right)}\right]\widehat{PRICE}^q_k, \qquad \text{for brands } p \neq q \\
&\eta^p_{jk} = \frac{\%\Delta MS\left(j_p\right)}{\%PRICE^p_k} = \left[\frac{\left\{\sum_h \beta^p_h \left[\Pr\left(j_p, k_p\right)_h - \Pr\left(j_p\right)_h \Pr\left(k_p\right)_h\right]\right\}}{MS\left(j_p\right)}\right]\widehat{PRICE}^p_k, \ j \neq k
\end{aligned}
$$

APPENDIX C
DEFINITION OF INVENTORY VARIABLE

We adopt the approach used by Bucklin and Gupta (1992) and Duvvuri et al (2007) to construct category-level inventory variable for a household. The inventory for household *h*, in category *c*, and at time *t*, $\widetilde{INV}_{ht}^{c}$ is defined as:

(C1) $$\widetilde{INV}_{ht}^{c} = \widetilde{INV}_{h[t-1]}^{c} + Q_{h[t-1]}^{c} - S_{h}^{c} \cdot T_{ht[t-1]}$$

where $Q_{h[t-1]}^{c}$ is the quantity purchased (in ounces) by household *h* in category *c* at at time *t*-1; $T_{ht[t-1]}$ is the time interval (in weeks) between trips at *t*-1 and *t*, and S_{h}^{c} is the average weekly consumption of household *h* in category *j*.

The average weekly consumption of a household in a category is computed as total quantity of a product (in ounces) purchased by the household in the estimation period divided by the total number of weeks. The inventory in the first time period for each household is set to zero. Finally, we take a mean-centering approach to derive the final category-level inventory variable for each household.

(C2) $INV_{ht}^{c} = \widetilde{INV}_{ht}^{c} - \overline{INV}_{h}^{c}$ where $\overline{INV}_{h}^{c}$ is household specific average inventory

APPENDIX D
MULTIVARIATE NESTED LOGIT MODEL

D.1 Basket Utility

We first write a final basket utility as a linear combination of category basket utility (upper level of the choice tree in Figure 4.3) and SKU basket utility (lowest level of the choice tree in Figure 4.3). Utility for household *h* of choosing basket *b* at time *t* is then defined as

(D1) $$U_{hbt} = \tilde{V}_{hgt,b} + \tilde{V}_{hs(g)t,b} + \varepsilon_{hbt}$$

$\tilde{V}_{hgt,b}$ = the systematic utility associated to a combination of categories in basket *b*.

$\tilde{V}_{hs(g)t,b}$ = the systematic utility associated to SKUs in categories *g* chosen in the basket *b*

ε_{hbt} = the random utility component, following iid Gumbel distribution with the scale parameter μ_b normalized to one across baskets.

Now, we redefine $\tilde{V}_g$ (drop subscript *h* and *t* for simplicity) using utility of each category selected in basket *b*. Because $\tilde{V}_g$ is defined at category level, all components are also determined at category level. Therefore, $\tilde{V}_g$ can be written as

(D2) $$\tilde{V}_g = \sum_c V_c z_c^{\tilde{b}} + \sum_c \sum_{c^*>c} \theta_{cc^*} z_c^{\tilde{b}} z_{c^*}^{\tilde{b}} + \varepsilon_g$$

where V_c is utility associated to a category *c* and θ^{cc^*} captures interaction with category *c* and *c**. Indicator variable $z_c^{\tilde{b}}$ is corresponding category level basket to $y_{j(c)}^b$ at item level and it is defined as in first section in Chapter 4. Error term ε_g is iid Gumbel distributed error for a basket of categories with parameter μ_g. This error is not category specific, but a basket specific. The category specific utility V_c is typically defined as $V_c = \psi_c + \lambda_c INV_c$, where ψ_c is category specific intercept and INV is inventory variable.

We also redefine $\tilde{V}_s$ using utility of each SKU as follows:

(D3) $$\tilde{V}_s = \sum_c \left(\sum_{j(c)} \pi_{j(c)} y_{j(c)}^b \right) z_c^{\tilde{b}} + \varepsilon_s$$

where $\pi_{j(c)}$ is a deterministic utility of item *j* in category *c*. Indicator variable $y^{b}_{j(c)} = 1$ if item *j* of category *c* is in basket *b*, otherwise zero. Error term ε_s is iid Gumbel error with parameter μ_s. Item specific utility $\pi_{j(c)} = \alpha_{j(c)} + \beta_{1,c} PRICE_{j(c)} + \beta_{2,c} PROM_{j(c)}$, where $\alpha_{j(c)}$ is a item specific intercept in category *c*. Variable *PRICE* and *PROM* are price and promotion (e.g., feature and display) variables.

Then, equation (D1) can be rewritten using (D2) and (D3) as follows:

(D4)
$$\begin{aligned} U_{b(\tilde{b})} &= \tilde{V}_g + \tilde{V}_s + \varepsilon_b \\ &= \sum_c V_c z_c^{\tilde{b}} + \sum_c \sum_{c^*>c} \theta_{cc^*} z_c^{\tilde{b}} z_{c^*}^{\tilde{b}} + \sum_c \left(\sum_{j(c)} \pi_{j(c)} y^{b}_{j(c)} \right) z_c^{\tilde{b}} + \varepsilon_g + \varepsilon_s + \varepsilon_b \end{aligned}$$

where $b(\tilde{b})$ stands for item level basket *b* with category level basket $\tilde{b}$. Error terms ε_g, ε_s, and ε_b are independent.

D.2 Joint Choice Model

First, we derive the basket choice model assuming errors associated with each level, i.e., ε_g and ε_s, have zero variances. Then, we have one global error term, ε_b, for final basket which is iid Gumbel error with parametr μ_b normalized to one across baskets. Then, following standard utility maximization argument, we will have the following logit choice model for the basket:

(D5)
$$\begin{aligned} \Pr(b) &= \frac{\exp\left(\tilde{V}_g + \tilde{V}_s\right)}{\sum_{\{g^*\}\in \tilde{b}^*, \{s^*\}\in b^*} \exp\left(\tilde{V}_{g^*} + \tilde{V}_{s^*}\right)} \\ &= \frac{\exp\left(\sum_c V_c z_c^{\tilde{b}} + \sum_c \sum_{c^*>c} \theta_{cc^*} z_c^{\tilde{b}} z_{c^*}^{\tilde{b}} + \sum_c \left(\sum_{j(c)} \pi_{j(c)} y^{b}_{j(c)} \right) z_c^{\tilde{b}} \right)}{\sum_{b^*(\tilde{b}^*)} \exp\left(\sum_c V_c z_c^{\tilde{b}^*} + \sum_c \sum_{c^*>c} \theta_{cc^*} z_c^{\tilde{b}^*} z_{c^*}^{\tilde{b}^*} + \sum_c \left(\sum_{j(c)} \pi_{j(c)} y^{b^*}_{j(c)} \right) z_c^{\tilde{b}^*} \right)} \end{aligned}$$

Note that denominator is sum of all possible basket utilities, where each utility is defined by the final choice object in Figure 4.3.

D.2.1 Marginal Choice Probability

Now, we derive a marginal probability of choosing a basket of categories (in upper level choice). For marginal probabilities, we first define Inclusive Value (W_c) as defined in a typical nested logit framework:

$$W_c = \ln\left\{\sum_{j\in c}\exp\left(\pi_{j(c)}\right)\right\} \tag{D6}$$

Using this definition for inclusive value and indicator variable $z_c^{\tilde{b}}$, we can rewrite the denominator of choice probability in equation (D5) at category level. Define a vector $\mathbf{W} = [W_1, W_2, \cdots, W_C]'$ and $\mathbf{V} = [V_1, V_2, \cdots, V_C]'$. Simple algebra proves that the denominator in equation (D5) can be rewritten as

$$\begin{aligned}
\sum_{\{g\}\in\tilde{b},\{s\}\in b}\exp\left(\tilde{V}_g + \tilde{V}_s\right) &= \sum_{b(\tilde{b})}\exp\left(\sum_c V_c z_c^{\tilde{b}} + \sum_c\left(\sum_{j(c)}\pi_{j(c)} y_{j(c)}^b\right) z_c^{\tilde{b}} + \sum_c\sum_{c^*>c}\theta_{cc^*} z_c^{\tilde{b}} z_{c^*}^{\tilde{b}}\right) \\
&= \sum_{b(\tilde{b})}\exp\left(\sum_c V_c z_c^{\tilde{b}} + \sum_c\sum_{c^*>c}\theta_{cc^*} z_c^{\tilde{b}} z_{c^*}^{\tilde{b}}\right)\exp\left\{\sum_c\left(\sum_{j(c)}\pi_{j(c)} y_{j(c)}^b\right) z_c^{\tilde{b}}\right\} \\
&= \sum_{\tilde{b}}\exp\left(\sum_c V_c z_c^{\tilde{b}} + \sum_c\sum_{c^*>c}\theta_{cc^*} z_c^{\tilde{b}} z_{c^*}^{\tilde{b}}\right)\exp\left\{\sum_c W_c z_c^{\tilde{b}}\right\} \\
&= \sum_{\tilde{b}}\exp\left(\mathbf{V}\mathbf{z}^{\tilde{b}} + \mathbf{W}\mathbf{z}^{\tilde{b}} + \frac{1}{2}\mathbf{z}^{\tilde{b}\prime}\boldsymbol{\Theta}\mathbf{z}^{\tilde{b}}\right)
\end{aligned} \tag{D7}$$

Note that same category level basket ($\tilde{b}$) has $J(c)$ number of item level baskets (*b*), which has only one item within same category. Thus, sum of $\exp\left\{\sum_c\left(\sum_{j(c)}\pi_{j(c)} y_{j(c)}^b\right) z_c^{\tilde{b}}\right\}$ across *b*'s under same $\tilde{b}$ will be $\prod_c\left\{\sum_{j(c)}\exp\left(\pi_{j(c)}\right)\right\}^{z_c^{\tilde{b}}}$ when we rearrange each component carefully. This is, then, equivalent to $\exp\left\{\sum_c W_c z_c^{\tilde{b}}\right\}$.

Next, we start to derive marginal probability of a basket having only one category. For example, choice probability of a basket {A,0,0} is:

(D8)

$$\begin{aligned}\Pr\left(\{A,0,0\}\right) &= \sum_{A\in g,\{a_1,a_2\}\in s} \frac{\exp\left(\tilde{V}_g+\tilde{V}_s\right)}{\sum_{\{g^*\}\in\tilde{b}^*,\{s^*\}\in b^*}\exp\left(\tilde{V}_{g^*}+\tilde{V}_{s^*}\right)} \\ &= \frac{\exp\left(V_A\right)\left\{\exp\left(\pi_{a_1}\right)+\exp\left(\pi_{a_2}\right)\right\}}{\sum_{\{g^*\}\in\tilde{b}^*,\{s^*\}\in b^*}\exp\left(\tilde{V}_{g^*}+\tilde{V}_{s^*}\right)} \\ &= \frac{\exp\left(V_A+W_A\right)}{\sum_{\tilde{b}}\exp\left(\mathbf{V}\mathbf{z}^{\tilde{b}^*}+\mathbf{W}\mathbf{z}^{\tilde{b}^*}+\frac{1}{2}\mathbf{z}^{\tilde{b}^*\prime}\mathbf{\Theta}\mathbf{z}^{\tilde{b}^*}\right)}\end{aligned}$$

By the same token, marginal probability of a basket having two categories is defined as

(D9)

$$\begin{aligned}\Pr\left(\{A,B,0\}\right) &= \sum_{\{A,B\}\in g,\{a_1,a_2,b_1,b_2\}\in s} \frac{\exp\left(\tilde{V}_g+\tilde{V}_s\right)}{\sum_{\{g^*\}\in B^*,\{s^*\}\in b^*}\exp\left(\tilde{V}_{g^*}+\tilde{V}_{s^*}\right)} \\ &= \frac{\exp\left(V_A+V_B+\theta_{AB}\right)\left\{\exp\left(\pi_{a_1}\right)+\exp\left(\pi_{a_2}\right)\right\}\left\{\exp\left(\pi_{b_1}\right)+\exp\left(\pi_{b_2}\right)\right\}}{\sum_{\tilde{b}^*}\exp\left(\mathbf{V}\mathbf{z}^{\tilde{b}^*}+\mathbf{W}\mathbf{z}^{\tilde{b}^*}+\frac{1}{2}\mathbf{z}^{\tilde{b}^*\prime}\mathbf{\Theta}\mathbf{z}^{\tilde{b}^*}\right)} \\ &= \frac{\exp\left(V_A+V_B+W_A+W_B+\theta_{AB}\right)}{\sum_{\tilde{b}^*}\exp\left(\mathbf{V}\mathbf{z}^{\tilde{b}^*}+\mathbf{W}\mathbf{z}^{\tilde{b}^*}+\frac{1}{2}\mathbf{z}^{\tilde{b}^*\prime}\mathbf{\Theta}\mathbf{z}^{\tilde{b}^*}\right)}\end{aligned}$$

Finally, marginal probability of a basket having three categories is

(D10)

$$\begin{aligned}\Pr\left(\{A,B,C\}\right) &= \sum_{\{A,B,C\}\in g,\{a_1,a_2,b_1,b_2,c_1,c_2\}\in s} \frac{\exp\left(\tilde{V}_g+\tilde{V}_s\right)}{\sum_{g^*\in\tilde{b}^*,s^*\in b^*}\exp\left(\tilde{V}_{g^*}+\tilde{V}_{s^*}\right)} \\ &= \frac{\exp\left(V_A+V_B+V_C+W_A+W_B+W_C+\theta_{AB}+\theta_{AC}+\theta_{BC}\right)}{\sum_{\tilde{b}^*}\exp\left(\mathbf{V}\mathbf{z}^{\tilde{b}^*}+\mathbf{W}\mathbf{z}^{\tilde{b}^*}+\frac{1}{2}\mathbf{z}^{\tilde{b}^*\prime}\mathbf{\Theta}\mathbf{z}^{\tilde{b}^*}\right)}\end{aligned}$$

Now, we can generalize numerator of each marginal choice probability using matrices $\mathbf{z}^b$ and $\mathbf{\Theta}$. The marginal choice probability is now written as

(D11)

$$\Pr\left(\mathbf{z}^{\tilde{b}}\right) = \frac{\exp\left(\mathbf{V}\mathbf{z}^{\tilde{b}}+\mathbf{W}\mathbf{z}^{\tilde{b}}+\frac{1}{2}\mathbf{z}^{\tilde{b}\prime}\mathbf{\Theta}\mathbf{z}^{\tilde{b}}\right)}{\sum_{\tilde{b}^*}\exp\left(\mathbf{V}\mathbf{z}^{\tilde{b}^*}+\mathbf{W}\mathbf{z}^{\tilde{b}^*}+\frac{1}{2}\mathbf{z}^{\tilde{b}^*\prime}\mathbf{\Theta}\mathbf{z}^{\tilde{b}^*}\right)}$$

where for equation (D8) $\mathbf{z}^{\tilde{b}} = \{1,0,0\}$, for equation (D9) $\mathbf{z}^{\tilde{b}} = \{1,1,0\}$, and for equation (D10) $\mathbf{z}^{\tilde{b}} = \{1,1,1\}$.

D.2.2 Conditional Choice Probability

Now, we derive conditional choice probabilities given the basket of categories. For one category node, conditional choice probability of SKUs is same as multinomial logit for SKU given a category:

(D12) $$\Pr\left(a_1 \mid \{A,0,0\}\right) = \frac{\exp\left(\pi_{a_1}\right)}{\exp\left(\pi_{a_1}\right)+\exp\left(\pi_{a_2}\right)}$$

For two category node, conditional choice probability of SKUs is defined as

(D13)
$$\begin{aligned}
\Pr\left(\{a_1,b_1\} \mid \{A,B,0\}\right) &= \frac{\exp\left(\pi_{a_1}+\pi_{b_1}\right)}{\exp\left(\pi_{a_1}+\pi_{b_1}\right)+\exp\left(\pi_{a_1}+\pi_{b_2}\right)+\exp\left(\pi_{a_2}+\pi_{b_1}\right)+\exp\left(\pi_{a_2}+\pi_{b_2}\right)} \\
&= \frac{\exp\left(\pi_{a_1}\right)\exp\left(\pi_{b_1}\right)}{\left\{\exp\left(\pi_{a_1}\right)+\exp\left(\pi_{a_2}\right)\right\}\left\{\exp\left(\pi_{b_1}\right)+\exp\left(\pi_{b_2}\right)\right\}} \\
&= \frac{\exp\left(\pi_{a_1}\right)}{\exp\left(\pi_{a_1}\right)+\exp\left(\pi_{a_2}\right)} \cdot \frac{\exp\left(\pi_{b_1}\right)}{\exp\left(\pi_{b_1}\right)+\exp\left(\pi_{b_2}\right)} \\
&= \Pr\left(a_1 \mid \{A,0,0\}\right)\Pr\left(b_1 \mid \{0,B,0\}\right)
\end{aligned}$$

By the same token, conditional choice probability of three category node is defined as

(D14) $$\Pr\left(\{a_1,b_1,c_1\} \mid \{A,B,C\}\right) = \Pr\left(a_1 \mid \{A,0,0\}\right)\Pr\left(b_1 \mid \{0,B,0\}\right)\Pr\left(c_1 \mid \{0,0,C\}\right)$$

Therefore, conditional SKU choice probability can be written in general form as

(D15) $$\Pr\left(\{j(A),k(B),\cdots,m(C)\} \mid \{z_A,z_B,\cdots,z_C\}\right) = \prod_c \left[\prod_{j\in c}\left(\frac{\exp\left(\pi_{j(c)}\right)}{\sum_{j^*(c^*)}\exp\left(\pi_{j^*(c^*)}\right)}\right)^{y_{j(c)}}\right]^{z_c}$$

Finally, unconditional basket choice probability is given by

(D16)

$$\Pr\left\{b\left(\tilde{b}\right)\right\} = \Pr\left(\mathbf{z}^{\tilde{b}}\right)\Pr\left(\mathbf{y}^{b} \mid \mathbf{z}^{\tilde{b}}\right)$$
$$= \left[\frac{\exp\left(\mathbf{V}'\mathbf{z}^{\tilde{b}} + \mathbf{W}'\mathbf{z}^{\tilde{b}} + \frac{1}{2}\mathbf{z}^{\tilde{b}\prime}\boldsymbol{\Theta}\mathbf{z}^{\tilde{b}}\right)}{\sum_{\tilde{b}*}\exp\left(\mathbf{V}'\mathbf{z}^{\tilde{b}*} + \mathbf{W}'\mathbf{z}^{\tilde{b}*} + \frac{1}{2}\mathbf{z}^{\tilde{b}*\prime}\boldsymbol{\Theta}\mathbf{z}^{\tilde{b}*}\right)}\right]$$
$$\times\left[\prod_{c}\left\{\prod_{j\in c}\left(\frac{\exp\left(\pi_{j(c)}\right)}{\sum_{s*(c*)}\exp\left(\pi_{j(c)}\right)}\right)^{y_{j(c)}^{b}}\right\}^{z_c^{\tilde{b}}}\right]$$

D.3 Multivariate Nested Logit Model

Now, we generalize joint choice model allowing one of error term has non-zero variance, which allows basket utilities are correlated due to same components. We assume ε_g follows a Gumbel distribution with parameter μ_g. This means that basket utilities are correlated because same categories appear in different baskets. Then, according to nested logit framework (Ben-Akiva and Lerman 1985), the following model is derived:

(D17)

$$\Pr\left\{b\left(\tilde{b}\right)\right\} = \left[\frac{\exp\left\{\frac{\mu_g}{\mu_b}\left(\mathbf{V}'\mathbf{z}^{\tilde{b}} + \mathbf{W}'\mathbf{z}^{\tilde{b}} + \frac{1}{2}\mathbf{z}^{\tilde{b}\prime}\boldsymbol{\Theta}\mathbf{z}^{\tilde{b}}\right)\right\}}{\sum_{\tilde{b}*}\exp\left\{\frac{\mu_g}{\mu_b}\left(\mathbf{V}'\mathbf{z}^{\tilde{b}*} + \mathbf{W}'\mathbf{z}^{\tilde{b}*} + \frac{1}{2}\mathbf{z}^{\tilde{b}*\prime}\boldsymbol{\Theta}\mathbf{z}^{\tilde{b}*}\right)\right\}}\right]$$
$$\times\left[\prod_{c}\left\{\prod_{j\in c}\left(\frac{\exp\left(\pi_{j(c)}\right)}{\sum_{s*(c*)}\exp\left(\pi_{j(c)}\right)}\right)^{y_{j(c)}^{b}}\right\}^{z_c^{\tilde{b}}}\right]$$

Note that scale parameter μ_b of ε_b is typically normalized to unity for the identification purpose.

D.4 Implication of IV parameter

The scale parameter μ_g / μ_b (unless we set $\mu_b = 1$) rescales all parameters associated with category-level basket choice. According to Ben-Akiva and Lerman (1985), $1-\left(\mu_g / \mu_b\right)^2$ equals the correlation of the indirect utilities for any pair of collections of SKUs sharing common category basket selection. The closer the correlation is to unity, the closer μ_g / μ_b is to zero. Conversely, if the correlation is zero, μ_g / μ_b becomes one, which implies that alternatives share no common utility elements. Therefore, the joint choice model developed in equation (D16) assumes that indirect utilities of all basket choices are uncorrelated. Meanwhile, multivariate nested logit in equation (D17) allows the correlation among indirect utilities of basket choices if they have common categories. The proof is presented in (D18), following Ben-Akiva and Lerman (1985):

(D18)
$$\begin{aligned}
\frac{\mu_g}{\mu_d} &= \left[\frac{\operatorname{var}\left(\varepsilon_d\right)}{\operatorname{var}\left(\varepsilon_d\right)+\operatorname{var}\left(\varepsilon_g\right)}\right]^{1/2} \\
&= \left[1-\frac{\operatorname{var}\left(\varepsilon_g\right)}{\operatorname{var}\left(\varepsilon_g\right)+\operatorname{var}\left(\varepsilon_d\right)}\right]^{1/2} \\
&= \left[1-\frac{\operatorname{cov}\left(U_s, U_{s*}\right)}{\left[\operatorname{var}\left(U_s\right)\operatorname{var}\left(U_{s*}\right)\right]^{1/2}}\right]^{1/2} \\
&= \sqrt{1-corr\left(U_s, U_{s*}\right)}
\end{aligned}$$

Based on Figure 4.3, for example, indirect utility of $\{a_1, b_2\}$ and that of $\{a_1, b_2, c_1\}$ is uncorrelated because their category level baskets are different. Meanwhile, indirect utility of $\{a_1, b_2, c_1\}$ and that of $\{a_2, b_1, c_2\}$ are correlated because they share same categories.

APPENDIX E
TWO-STAGE SIMULATED MAXIMUM LIKELIHOOD

We can estimate the model (4.14), using two-stage estimation procedure because the model consists of two parts: category-level basket model and conditional brand choice model. We first estimate conditional brand choice model for each category separately, incorporating household parameter heterogeneity. The first stage estimation produces individual household level parameters for within-category model specification in equation (4.6). Those individual parameter estimates for within-category model, then, are imputed in cross-category basket model at category level in the second stage estimation. Therefore, we only need to estimate category-level household parameters specified in cross-category basket model in equation (4.10). This two stage estimation approach is not very statistically efficient, but provides fully consistent parameter estimates (Ben-Akiva and Lerman 1985).

We begin with taking logarithm on equation (4.14) and get Log-Likelihood as

$$\text{(E1)} \qquad LL = \sum_h \sum_t \sum_{b(\tilde{b})} \left[\gamma^b \left\{ \log \Pr\left\{ B = \mathbf{z}_{ht}^{b(\tilde{b})} \right\} + \sum_c z^{c|b} \sum_{j \in c} y_{hjt}^c \log \Pr\left(j_c \mid b \right)_{ht} \right\} \right]$$

This can be re-written as

$$\text{(E2)} \quad LL = \sum_h \sum_t \sum_{\tilde{b}} \gamma^{\tilde{b}} \log \Pr\left\{ B = \mathbf{z}_{ht}^{b(\tilde{b})} \right\} + \sum_h \sum_t \sum_b \gamma^b \left\{ \sum_c z^{c|b} \sum_{j \in c} y_{hjt}^c \log \Pr\left(j_c \mid b \right)_{ht} \right\}$$

Note that final log-likelihood has two components: one for category-basket choice and the other for conditional item choice. In estimation, we incorporate simulated maximum likelihood (SML, Train 2002) approach to account for household heterogeneity. In addition, carefully investigating the second part of equation (i.e., conditional choice), it is obvious that we can even break it down for each category. If a category is not in the basket (i.e., $z^{c|\tilde{b}} = 0$), item choice probability becomes zero for the category. Therefore, final likelihood contains only choice probabilities for each category when there is a choice incidence. This implies that item choices can be independently determined for each category, given basket choice (which lets us know which categories are selected). Finally, null basket has no impact on item choice probability because the null basket implies all $z^{c|\tilde{b}} = 0$ which makes entire second part of equation disappear for the null basket.

E.1 Within-Category Model Estimation

Due to the characteristic described above, we estimate each category separately. For each category, we first select households who purchase at least three purchases. Some households appear in the data for one category but disappear in the data for another category. The likelihood for within-category SKU level becomes

(E3) $$L = \prod_h L_h = \prod_h \prod_t \prod_b \left\{ \prod_c \left(\prod_{j_c \in c} \left[\frac{\exp\left(\tilde{\pi}^c_{hjt}\right)}{\sum_{k \in c} \exp\left(\tilde{\pi}^c_{hkt}\right)} \right]^{y^c_{hjt}} \right)^{z^{c|b}_{ht}} \right\}^{r^b_{ht}}$$

where $$\tilde{\pi}^c_{hjt} = \alpha^c_{hj} + \beta^c_{hG} GAIN^c_{hjt} + \beta^c_{hL} LOSS^c_{hjt} + \beta^c_{1,h} LOY^c_{hj} + \beta^c_{2,h} PRICE^c_{hjt} + \beta^c_{3,h} DISP^c_{hjt} + \beta^c_{4,h} FEAT^c_{hjt}$$

We make the following distributional assumption for parameters in within-category model for each category:

(E4) $$\boldsymbol{\alpha}^c \sim N\left(\overline{\boldsymbol{\alpha}}^c, \boldsymbol{\Omega}^c_\alpha\right)$$
$$\boldsymbol{\beta}^c \sim N\left(\overline{\boldsymbol{\beta}}^c, \boldsymbol{\Omega}^c_\beta\right)$$
$$\boldsymbol{\lambda}^c \sim N\left(\overline{\boldsymbol{\lambda}}^c, \boldsymbol{\Omega}^c_\lambda\right)$$

Note that λ^c_h is a parameter for relative weight between IRP and ERP in construction of GAIN (or LOSS). Carry-over parameter γ is not household specific. In addition, $\boldsymbol{\beta}^c$ do not contain β^c_{hI} for inventory variable because it disappears in within-category model. For the standard errors of the parameters, we estimate information matrix using BHHH estimator (Greene 2003) or the outer product of gradient estimator.

The estimator is just the reciprocal of the sum of squares of the first derivatives of the likelihood function.

Next, we estimate individual SKU-level parameters using multinomial logit framework (with SML approach). We generate *R* random values for each parameter[9]. From the estimation, we have mean and variance for each parameter due to distributional assumption and SML procedure. Using these statistics (i.e., prior) and likelihood of individual household, we can recover household level posterior estimates, for example $\hat{\beta}_{hj}$ as

(E5) $$\hat{\beta}_{hj} \mid \overline{\beta}_j, \Delta_j = \frac{\sum_{r=1}^{R} \beta_{hj}^{(r)} L_h\left(\beta_{hj}^{(r)}\right)}{\sum_{r=1}^{R} L_h\left(\beta_{hj}^{(r)}\right)}$$

where $\beta_{hj} \sim N\left(\overline{\beta}_j, \Delta_j\right)$. Even though our parameter estimates are only based on those who are in the calibration data set, we can recover individual parameter for those who are not in the data set. Assuming parameter estimates for the distribution is for population, we can use the same procedure in equation (E5) for households not in the data set.

E.2 Category-Level Basket Choice Model

To calibrate category-level basket model, the first part of full likelihood in equation (E2) is used. Therefore, category-level likelihood becomes

(E6) $$L = \prod_h L_h = \prod_h \prod_t \prod_b \left(\frac{\exp\left\{ \sum_c W_{ht}^c z_{ht}^{c|b} + \sum_{c<c^*} \theta_h^{cc^*} z_{ht}^{c|b} z_{ht}^{c^*|b} \right\}}{\sum_{b^*} \exp\left\{ \sum_c W_{ht}^c z_{ht}^{c|b^*} + \sum_{c<c^*} \theta_h^{cc^*} z_{ht}^{c|b^*} z_{ht}^{c^*|b^*} \right\}} \right)^{r_{ht}^b}$$

---

[9] Following Train (2003), we use Halton sequence to draw normal random variables. We draw total 125 random variables for the estimation

The category attractiveness or inclusive value, $W_{ht}^{c}=\log\left\{\sum_{j\in c}\exp\left(\pi_{hjt}^{c}\right)\right\}$ contains category-level variables and parameters: category specific intercept ψ_{h}^{c} and inventory $\beta_{hI}^{c}INV_{ht}^{c}$. Therefore, we can rewrite W_{ht}^{c} as follows:

$$
\begin{aligned}
W_{ht}^{c} &= \log\left\{\sum_{j\in c}\exp\left(\pi_{hjt}^{c}\right)\right\} \\
&= \log\left\{\sum_{j\in c}\exp\left(\begin{matrix}\psi_{h}^{c}+\alpha_{hj}^{c}+\beta_{hG}^{c}GAIN_{hjt}^{c}+\beta_{hL}^{c}LOSS_{hjt}^{c}+\beta_{hI}^{c}INV_{ht}^{c}+\beta_{1,h}^{c}LOY_{hj}^{c} \\ +\beta_{2,h}^{c}PRICE_{hjt}^{c}+\beta_{3,h}^{c}DISP_{hjt}^{c}+\beta_{4,h}^{c}FEAT_{hjt}^{c}\end{matrix}\right)\right\} \\
&= \log\left\{\exp\left(\psi_{h}^{c}+\beta_{hI}^{c}INV_{ht}^{c}\right)\sum_{j\in c}\exp\left(\begin{matrix}\alpha_{hj}^{c}+\beta_{hG}^{c}GAIN_{hjt}^{c}+\beta_{hL}^{c}LOSS_{hjt}^{c}+\beta_{1,h}^{c}LOY_{hj}^{c} \\ +\beta_{2,h}^{c}PRICE_{hjt}^{c}+\beta_{3,h}^{c}DISP_{hjt}^{c}+\beta_{4,h}^{c}FEAT_{hjt}^{c}\end{matrix}\right)\right\} \\
&= \psi_{h}^{c}+\beta_{hI}^{c}INV_{ht}^{c}+\tilde{W}_{ht}^{c}
\end{aligned}
\quad \text{(E7)}
$$

where $\tilde{W}_{ht}^{c}=\log\left\{\sum_{j\in c}\exp\left(\tilde{\pi}_{hjt}^{c}\right)\right\}$. At this stage, we impute individual household level parameters from within-category model estimation in first stage. Thus, we only have unobserved parameters at the category-level in this stage. Those are category intercepts ψ_{h}^{c}, inventory parameter β_{hI}^{c}, and cross-effects parameters θ_{h}^{cc*}. For the estimation, the following assumption is made for these parameters:

$$
\begin{aligned}
\boldsymbol{\psi}^{c} &\sim N\left(\bar{\boldsymbol{\psi}}^{c},\boldsymbol{\Omega}_{\lambda}^{c}\right) \\
\boldsymbol{\beta}_{I}^{c} &\sim N\left(\bar{\boldsymbol{\beta}}_{I}^{c},\boldsymbol{\Omega}_{I}^{c}\right) \\
\boldsymbol{\theta}^{cc*} &\sim N\left(\bar{\boldsymbol{\theta}}^{cc*},\boldsymbol{\Omega}_{\theta}^{cc*}\right)
\end{aligned}
\quad \text{(E8)}
$$

As we noted, all other household level parameters from within-category model are imputed in the likelihood. Finally, household level likelihood in equation (E6) will be approximated by

$$
L_{h}\approx\tilde{L}_{h}=\frac{1}{R}\sum_{r}\prod_{t}\prod_{b}\left(\frac{\exp\left\{\sum_{c}W_{ht}^{c(r)}z_{ht}^{c|b}+\sum_{c<c*}\theta_{h}^{cc*(r)}z_{ht}^{c|b}z_{ht}^{c*|b}\right\}}{\sum_{b*}\exp\left\{\sum_{c}W_{ht}^{c(r)}z_{ht}^{c|b*}+\sum_{c<c*}\theta_{h}^{cc*(r)}z_{ht}^{c|b*}z_{ht}^{c*|b*}\right\}}\right)^{\gamma_{ht}^{b}}
\quad \text{(E9)}
$$

where $W_{ht}^{c(r)}$ is defined by (1) posterior household-level parameters and (2) randomly generated category level parameters (mean and variance is estimated) for all households.

(E10)

$$W_{ht}^{c(r)} = \psi_h^{c(r)} + \beta_{hI}^{c(r)} INV_{ht}^c + \tilde{W}_{ht}^c$$
$$= \psi_h^{c(r)} + \beta_{hI}^{c(r)} INV_{ht}^c + \log\left\{\sum_{j\in c}\exp\begin{pmatrix}\hat{\alpha}_{hj}^c + \hat{\beta}_{hG}^c GAIN_{hjt}^c + \hat{\beta}_{hL}^c LOSS_{hjt}^c + \hat{\beta}_{1,h}^c LOY_{hj}^c \\ + \hat{\beta}_{2,h}^c PRICE_{hjt}^c + \hat{\beta}_{3,h}^c DISP_{hjt}^c + \hat{\beta}_{4,h}^c FEAT_{hjt}^c\end{pmatrix}\right\}$$

where $\psi_h^{c(r)}$ and $\beta_{hI}^{c(r)}$ are *r*-th randomly generated values for all households. For other parameters such as $\hat{\beta}_h^c$, individual household level parameters from the first stage estimation are imputed.

We then maximize the following log-likelihood for the estimation:

$$\text{(E11)}\; LL \approx \sum_h \log \tilde{L}_h = \sum_h \log\left[\frac{1}{R}\sum_r\prod_t\prod_b\left(\frac{\exp\left\{\sum_c W_{ht}^{c(r)} z_{ht}^{c|b} + \sum_{c<c^*}\theta_h^{cc^*(r)} z_{ht}^{c|b} z_{ht}^{c^*|b}\right\}}{\sum_{b^*}\exp\left\{\sum_c W_{ht}^{c(r)} z_{ht}^{c|b^*} + \sum_{c<c^*}\theta_h^{cc^*(r)} z_{ht}^{c|b^*} z_{ht}^{c^*|b^*}\right\}}\right)^{\gamma_{ht}^b}\right]$$

Because some households never purchase certain categories, individual basket choice probability, i.e., L_h should be redefined by how many categories are purchased by each household. Therefore, we use renormalization approach described in Appendix F. For standard errors of the parameters, same procedure (i.e., BHHH estimator) is used as for within-category parameters.

APPENDIX F
RENORMALIZING BASKET PROBABILITY

Assume that there are *g* numbers of categories in consideration. The probability of selection a basket *b*, given that category *g* has never been chosen, is rewritten by removing baskets which contain category *g*. This leads to the renormalization of $\Pr\left\{B=\mathbf{z}_{ht}^{\tilde{b}}\right\}$. First, define category-level basket as

$$\mathbf{z}_{ht}^{\tilde{b}}=\left\{z_{h1t}^{\tilde{b}},z_{h1t}^{\tilde{b}},\cdots z_{hft}^{\tilde{b}},z_{hgt}^{\tilde{b}}\right\} \tag{F1}$$

where $z_{hgt}^{\tilde{b}}=1$ if category *g* is in the basket $\tilde{b}$ and zero otherwise.

Assume that household *h* has no record of buying category the last category *g*. Then, basket expression can be written as

$$\mathbf{z}\left(\tilde{b}\mid g=0\right)_{ht}=\left\{z_{h1t}^{\tilde{b}},z_{h2t}^{\tilde{b}},\cdots z_{hft}^{\tilde{b}},0\right\} \tag{F2}$$

Due to the basket composition in (F2), $\Pr\left\{B=\mathbf{z}\left(b\mid g=0\right)\right\}_{ht}$ can be expressed as

$$\Pr\left\{B=\mathbf{z}\left(\tilde{b}\mid g=0\right)\right\}_{ht}=\frac{\exp\left\{\sum_{g\notin c}W_{ht}^{c}z_{hct}^{\tilde{b}}+\sum_{g\notin c<c^{*}}\theta_{h}^{cc^{*}}z_{hct}^{\tilde{b}}z_{hc^{*}t}^{\tilde{b}}\right\}}{\sum_{\tilde{b}^{*}}\exp\left\{\sum_{g\notin c}W_{ht}^{c}z_{hct}^{\tilde{b}^{*}}+\sum_{g\notin c<c^{*}}\theta_{h}^{cc^{*}}z_{hct}^{\tilde{b}^{*}}z_{hc^{*}t}^{\tilde{b}^{*}}\right\}} \tag{F3}$$

We assign zero probability to observe any basket which contains category *g*, and denominator also does not contain such baskets. Equation (F3) then can be rewritten by redefining basket expression using only observable categories. Therefore, define $\tilde{b}$ removing category *g* in its composition as

$$\mathbf{z}_{ht}^{\tilde{b}^{*}}=\left\{z_{h1t}^{\tilde{b}^{*}},z_{h1t}^{\tilde{b}^{*}},\cdots z_{hft}^{\tilde{b}^{*}}\right\} \tag{F4}$$

Then, equation (F3) can be rewritten as

$$\Pr\left\{B=\mathbf{z}\left(\tilde{b}^{*}\right)\right\}_{ht}=\frac{\exp\left\{\sum_{c}W_{ht}^{c}z_{hct}^{\tilde{b}^{*}}+\sum_{c<c^{\dagger}}\theta_{h}^{cc^{\dagger}}z_{hct}^{\tilde{b}^{*}}z_{hc^{*}t}^{\tilde{b}^{*}}\right\}}{\sum_{\tilde{b}^{\dagger}}\exp\left\{\sum_{c}W_{ht}^{c}z_{hct}^{\tilde{b}^{\dagger}}+\sum_{c<c^{\dagger}}\theta_{h}^{cc^{\dagger}}z_{hct}^{\tilde{b}^{\dagger}}z_{hc^{*}t}^{\tilde{b}^{\dagger}}\right\}} \tag{F5}$$

where c stands for categories but $\tilde{b}*$ does not contain category g.

This is equivalent to saying that for households who have purchased only some of total categories in consideration in history, we can redefine their baskets only based on categories they purchased. This can be generalized to any number of baskets. Therefore, if we assume that every household purchases at least one basket during the entire period of data set, then we redefine our basket model removing null basket, i.e., $\mathbf{z}\left(\tilde{b}\right)_{ht} = \left\{0, 0, \cdots, 0\right\}$. This approach was used in Russell and Petersen (2000):

(F6) $$\Pr\left\{B = \mathbf{z}\left(\tilde{b} \mid \tilde{b} \neq \mathbf{0}\right)\right\}_{ht} = \frac{\exp\left\{\sum_{c} W_{ht}^{c} z_{hct}^{\tilde{b}} + \sum_{c<c^*} \theta_{h}^{cc^*} z_{hct}^{\tilde{b}} z_{hc^*t}^{\tilde{b}}\right\}}{\sum_{b^* \neq \mathbf{0}} \exp\left\{\sum_{c} W_{ht}^{c} z_{hct}^{\tilde{b}^*} + \sum_{c<c^*} \theta_{h}^{cc^*} z_{hct}^{\tilde{b}^*} z_{hc^*t}^{\tilde{b}^*}\right\}}$$

This reduces total number of baskets from 2^g to $2^g - 1$. Therefore, depending on how we select data set, we can decide whether or not include the null basket in our estimation. Mathematically, the inclusion of the null basket does not make a difference. However, due to number of observations affecting estimation procedure, it makes a practical difference.

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