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A distributed control approach to optimal economic dispatch of power generators

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A DISTRIBUTED CONTROL APPROACH TO OPTIMAL ECONOMIC DISPATCH OF POWER GENERATORS

by

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CERTIFICATE OF APPROVAL

MASTER'S THESIS

This is to certify that the Master's thesis of

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Soura Dasgupta

To my parents and brother.

ABSTRACT

In this dissertation, we propose a novel distributed approach to the control of generators in the electric grid. Specifically, we consider the problem of the optimal economic dispatch of generator; we present a simple, distributed algorithm, which adjusts the power-frequency set-points of generators to correct for power imbalances arising from generation and load fluctuations. In this algorithm each generator independently adjusts its real-power output based on its estimate of the aggregate power imbalance in the network; such as an estimate can be independently obtained by each generator through local measurements of the frequency deviation on the grid. Eventually, over the course of network operation, the distributed algorithm achieves the equal-marginal-cost power allocation among generators while driving the power imbalance exponentially to zero. In the absence of power losses, we prove the eventual optimality of the distributed algorithm under mild assumptions (strict convexity and positivity of cost functions) and present simulation results to compare its performance with traditional (centralized) dispatch algorithms. Furthermore, we present numerical simulation results that show that the distributed algorithm performs well even in the presence of power losses and other constraints. We argue that distributed control methods are especially attractive for electric grids with smart meters and other advanced capabilities at the end node and grids with high penetration of alternative energy generators and we identify interesting open problems for future work in this area.

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CHAPTER 1

INTRODUCTION

This thesis presents a new distributed approach to optimal economic dispatch of power generators. The basic idea behind this approach is as follows. Each generator independently makes incremental adjustments to its power frequency set-point in response to a power imbalance as indicated by frequency deviations on the grid. We show that it is possible to design a distributed adjustment procedure that achieves the minimum cost allocation of the generation, with each generator using only local knowledge of its own marginal cost of generation.

The motivation for this work is the ongoing trend towards increasing penetration of alternative energy sources especially wind-turbine generators in the electric grid [1]. Alternative energy sources such as wind are inherently unpredictable and intermittent and therefore can benefit from a more flexible real-time control. Our approach also lends itself naturally to Distributed Generation (DG) where a large number of small-scale power sources such as roof-top wind turbines, solar panels, and bio-fuels can provide power to the electric grid [2].

In this introductory chapter, we provide background information to motivate the problem considered in this thesis. We start with some basic information on the historical development of the power grid in the US; we highlight the recent trends towards a more decentralized and intelligent power grid driven by alternative energy technologies and distributed generation, and Smart Grids (SG). We then outline some of the challenges for the electric grid presented by the above trends and argue that a distributed control approach, exemplified by the dispatch algorithm presented in this thesis present a promising way to address these challenges.

1.1 Historical Background

The origins of the modern power system can be traced to Thomas Edison's famous invention [3] of a practical light bulb in 1879. This invention created a demand for electric power in residences and businesses. Responding to this demand, Edison formally unveiled Pearl Street Station on 4 September 1882, and this can be considered as the starting point of the modern electric grid.

Edison's early power system had some important features – underground cables, electric meters, wiring, fuses, switches, and sockets [4]. However, most important of all, he used Direct Current (DC), and this became the early standard. Unfortunately, the DC power system operated on low voltages; the resulting large voltage drops and transmission losses restricted the feasible range of the power system to an area not much larger than a few city blocks.

The Alternating Current (AC) system advocated by Nikola Tesla and Westinghouse was capable of long distance transmission as proved in a successful demonstration by Tesla and Westinghouse of sending electric power from Niagara Falls to Buffalo, in November, 1896. The ability of AC power systems to transmit power efficiently over large distances eventually led to a centralized power system that covers large areas with central power generators. However, since the power delivered to households and consumers was still at low voltages, the transformer became the inevitable gateway to set the power grid, and the electric grid came to consist to two identifiably distinct parts – a centralized generation and transmission system, and a distribution network. Standard voltage levels were established worldwide, for example, 120 / 240 VAC at 60 Hertz in North America and 230 / 400 V at 50 Hertz in Europe. Over the course of the 20'th century the electric grid grew and became almost universal in most parts of the world. It also became an indispensable part of modern industrial life, and because of its importance, the electric system was usually regulated or controlled as public utilities by governments everywhere.

The later decades of the 20th century brought a series of important changes to the electric grid in the US (and other parts of the world; however we focus on the US system here). First, some steps were taken towards deregulation of the power market starting with the 1978 Public Utility Regulation Policy Act (PURPA) [5]. The main stated goal of deregulation was efficiency through price competition among providers. The power utility no longer had a monopoly over generation, and independent power producers (IPP) [6] were given the ability to supply power to the grid. While traditionally most of power supplied to the electric grid came from large-scale centralized sources, the ability of independent producers being able to supply power to the grid raises the possibility of using distributed generation (DG). Usually, DG is defined as small decentralized generators, and a capacity of generator in DG is usually less than tens of MW [7].

Simultaneously, environmental and political issues over fossil fuels have created a high level of interest in using alternative energy sources. DG is often well-suited to such alternative energy technologies; while it is economically infeasible to operate a small nuclear or coal-fired power plant, wind, solar and bio-fuels lend themselves naturally to a distributed generation model.

Wind energy in particular has been historically very important especially in Europe and has been growing at very fast rate in recent years [8]. However, the intermittency of alternative energy is not well suited to traditional power control. For example, wind turbines only generate electricity when the wind blows. Wind may blow when electricity is needed, or it may not. This presents some serious challenges to the power system and these challenges will grow as the proportion of power supplied by alternative energy sources increases. We explore some of these challenges in more detail in the rest of this chapter.

1.2 Alternative Energy in the Power Grid

The first modern energy source that was used on an industrial scale was coal. Coal was a main source of power for the steam engine that helped bring about the industrial revolution, in the 18th century. Since then, the generation and consumption of energy resources have increased enormously. However, the energy supply today still relies largely on one class of sources – fossil fuels. In 2008, fossil fuels including oil, coal, and natural gas, account for 69% of the global electricity production [10]. The fact that fossil fuels are non-renewable and that they are rapidly being depleted has led to an increased interest in alternative energy.

According to the Oxford Dictionary, alternative energy is defined as “energy generated in ways that do not deplete natural resources or harm the environment, especially by avoiding the use of fossil fuels and nuclear power” [11]. The most common alternative energy sources are solar, wind, tide, and hydro which are renewable forever for free. Although they represent only a small portion of total energy generation – 18% in 2008 [10] – their energy capacity in world market grew very rapidly in recent years, for example at rates of 10-60 percent annually for wind power, solar photovoltaic, and bio-fuels respectively, from 2004 to 2009 [10]. Also investment in this area has been increasing; in 2008 and 2009, about $130 billion and $150 billion were spent, respectively [10]. So we can expect that alternative energy sources will play an increasingly important role in power grid.

1.3 20% Wind Energy by 2030

Among all the alternative energy sources, the fastest growing source is wind energy which also receives by far the largest levels of investment. For example, in the year 2009, 38 GW of wind capacity was installed worldwide which represents a 41% increasing over 2008 [10]. China added 13.8 gigawatts (GW) wind power capacity in 2009, which was more than any other country in 2009 [10]. In economic terms, wind power received more than 60% of utility-scale alternative energy sources investment in 2009 [10]. The worldwide investments in wind energy were $55.5 billion and $62.7 billion in 2008 and 2009, respectively [10].

The U.S. Department of Energy (DoE) aims to increase wind energy capacity to 20% of U.S. electricity needs by 2030 [1]. Statistically, to reach 20% of U.S. electricity demand, wind power capacity would have to reach more than 300 GW [1]. This number appears too big to achieve, especially when compared to the modest current wind power capacity (25 GW in 2009 [10]), however prospects for wind power development are quite bright. In 2009, 10 GW of wind generators were added [10]. The U.S. government also provides federal tax credits to accelerate development of the wind power industry [1].

Why does the world invest largest amounts of money to wind relative to the other alternative energy technologies? A recent comparative study [12] of alternative energy technologies suggests some reasons. First, wind power technology is more mature; it is faster to build and operate than other alternative energy sources. Usually, manufacturing and lifetime of wind turbine are 2-5 years and 30 years, respectively [12]. Also, it is very environmentally friendly – it doesn't emit carbon dioxide, water is not needed to generate electricity, and it doesn't take much area compared to same amount of electricity generation of other alternative energy sources.

1.4 Challenges for Alternative Power Generation

Generally, the amount of electricity generated at any time is determined by the power consumption at that time. The grid operator maintains a reserve of idle generators to keep following fluctuations of power consumption. Wind turbines, however, cannot be used in this role as a reserve to follow load fluctuations. Regardless of the load, the power generated by a wind turbine only depends on the wind speed. The problem that is caused by instability of wind is not an insufficient quantity of supply but the difficulty of integration into the electric grid. Adding uncontrollable wind turbines to power grid means there are fluctuations in not only consumption but also generation. The other generators in the grid are forced to decrease their generation when wind turbines generate more power. Thus in a grid with a large number of wind generators, there will be fluctuations in both consumption and generation.

1.5 Distributed Generation and Smart Grid

The Institute of Electrical and Electronics Engineers (IEEE) defines Distributed Generation (DG) as the generation of electricity by facilities sufficiently smaller than central generating plants as to allow interconnection at nearly any point in a power system [7]. Another general definition of DG is: "an electric power source connected directly to the distribution network or on the customer site" [13]. In other words, DG is composed of a set of small size generators that connects into the low-voltage distribution network and supplies power locally. Alternative energy sources are inherently distributed and thus are important motivators for DG.

The concept of DG is also closely related to the idea of a Smart Grid (SG). The term Smart Grid has been in use since at least 2005, when the article "Toward A Smart Grid", authored by S. Massoud Amin and Bruce F. Wollenberg appeared in the September/October issue of *IEEE P&E Magazine* (Vol. 3, No.5, pgs 34-41) [9]. The two defining features of the Smart Grid are the ability to self-balance and customer's participation. One enabling technology for the smart grid is the Smart Meter (SM) which enables two-way communication between customer and supplier. The Smart Meter technology offers the possibility that consumers can adjust their energy consumption by making use of real-time information about power supply/demand conditions in the grid. This is especially important in a grid with a large penetration of intermittent alternative energy sources. A study on smart grids [14] lists key characteristics of SG – self ability to correct generation shortage or excessiveness, consumer friendly generation system, and economic efficiency.

1.6 Our Contribution

So far in this chapter we have highlighted the trends towards a decentralized electric grid with alternative energy sources playing an increasingly important role and with enabling technologies such as Distributed Generation and Smart Grids driving this trend in the future. In this thesis, we present a new technique for distributed control that is well-suited to these trends. Specifically we present a distributed algorithm for frequency control and optimal economic dispatch that minimizes cost of information in an iterative and online fashion. While our work is motivated by the trends towards a more distributed grid, even for a centralized power grid, our distributed algorithm offers certain advantages such as simplicity, ease of implementation and robustness to model uncertainties as we describe in subsequent chapters. Specifically the traditional approach to the dispatch problem involves numerically solving an optimization problem; this optimization problem can quickly become intractable when power losses and practical constraints on the power of generators are taken into account, and this necessitates the use of complex computational techniques such as particle swarm algorithms, neural networks and Monte-Carlo methods [23], [25]. Our distributed approach automatically takes power losses into account and is able to handle certain constraints in a straightforward way.

1.7 Outline of the Rest of the Thesis

The rest of the thesis is organized as follows. Chapter 2 introduces the problem of economic dispatch and its implementation. Our new distributed technique for economic dispatch is presented in Chapter 3 and some of its properties are analyzed, as well as numerical simulations to demonstrate the performance of the proposed technique. Finally, Chapter 4 concludes with a brief discussion of open issues for future work.

CHAPTER 2

CONTROL OF POWER GENERATION

In this chapter, we describe the process of controlling the power produced by generators in an electric grid. This control is usually implemented in a complex process of Automatic Generation Control (AGC) that consists of multiple sub-processes operating over a range of different time-scales. The primary purpose of AGC is to meet load requirements while maintaining the voltage and frequency of the grid within the specified limits. A secondary goal is to allocate the generation in an efficient way that minimizes cost of generation and maximizes the lifetime of the generators.

The main contribution of this thesis is a new distributed algorithm for optimum economic dispatch of generators, and this chapter we provide the necessary background to motivate the advantages of the distributed approach and contrast it with the traditional centralized methods of generation control.

2.1 The Three Levels of Generation Control

Three distinct generation control processes are used in the modern electric grid [15]. First one is primary control. Its objective is to respond to short-term imbalances between load and generation. When there is a load fluctuation (e.g. a consumer switching an electric appliance on or off), the primary controller adjusts power output of generator quickly to restore balance between load and generation. This process results in a frequency deviation [21].

Continuous fluctuation forces generator to keep adjusting generation; over time the adjustments may make the cumulative frequency deviations too large. Secondary control stabilizes generators and makes sure they do not deviate excessively from the rated frequency. While primary control operates on individual generators separately and restores balance quickly (on the order of 30 seconds or less), secondary control operates periodically on a longer time-scale (on the order of 5-15 minutes) over a set of generators that form part of a Load Serving Entity (LSE) to maintain system frequency at its rated value; in the process, the secondary controller may reallocate power among the different generators within its area. Both primary and secondary generation control processes focus on stable power generation and supply, but do not consider economic aspects, i.e. costs of generation.

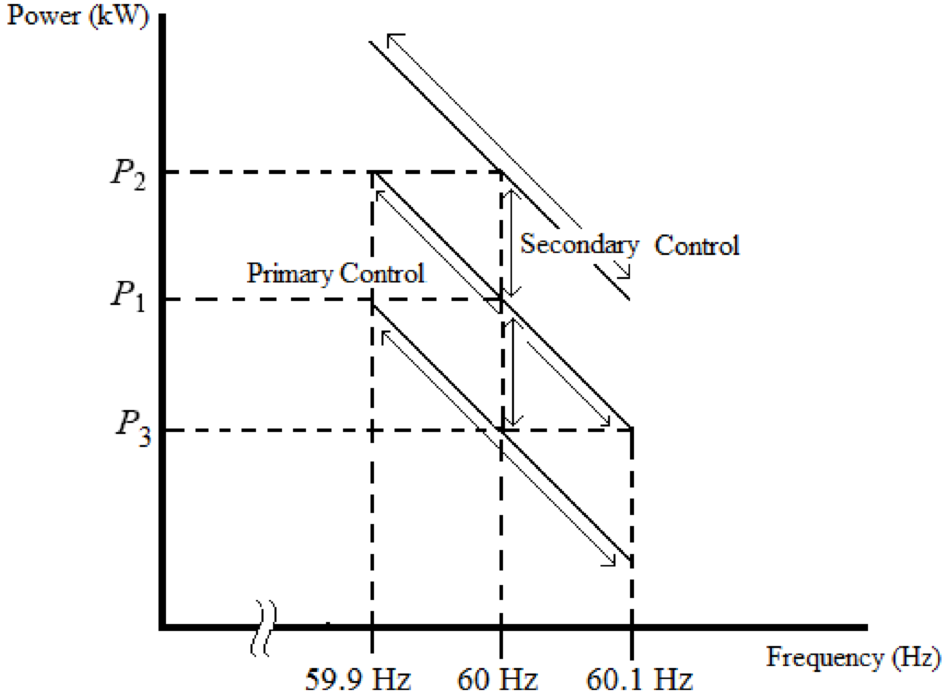

Figure 2-1 Power-frequency curve

Figure 2-1 graphically explains primary and secondary control. As the figure shows, each generator has a power-frequency (P-f) curve. Usually this curve has a negative slope (or "droop") i.e. if the generator supplies power larger than its rated level, this results in a frequency drop and so on. Thus the primary controller changes the operating point of the generator to a new point on the P-f curve. In contrast, the secondary control modifies the P-f curve so that the generator supplies the required amount of power at the rated frequency thus removing any frequency deviation.

Tertiary control, also called economic dispatch (ED), adjusts power allocation of each generator to minimize the total cost of generation across all generators. Thus the tertiary control is an optimization process. In the present electric grid, this process is typically carried out at a centralized control center [16]; this requires the control center to have accurate knowledge of the cost of each generator and this becomes more difficult if the number of generators becomes large (as in DG) or if alternative energy generators are widely deployed (because of their intermittency).

2.2 Cost Functions

The cost of power is determined by many factors such as fuel costs, idling costs, maintenance and heating costs; in addition the power allocation must satisfy constraints such as limits on maximum and minimum generation power of each generator. Furthermore, there may be transmission losses in the network, and the amount of loss will depend on, among other factors, the transmission distance from each generator to the load. Finally the above costs will in general depend not only on the active power produced by each generator but also potentially on its power factor. In practice, the cost of generation is modeled heuristically as a fixed function of its (active) power output, and it is this modeled cost that is minimized by the ED process.

The cost function describes this modeled relationship between generation and cost numerically. Physical intuition leads us to expect that they are in a monotonic relationship – when generation rises, cost also rises. We further assume convexity for all cost functions i.e. each additional unit of generation becomes incrementally more expensive. This assumption will be discussed in more detail in chapter 3, and corresponds to the physically reasonable idea that it is more expensive to generate an additional unit of power as the generator is operated closer to its capacity. Cost functions are frequently modeled [18] as quadratic or, more generally, polynomial functions. For example, a quadratic cost function has the following form:

$$J_i(P_i)=a_iP_i^2+b_iP_i+c_i \tag{2.1}$$

where J_i is cost of generation (\$/hr), P_i is power generation of *i*-th plant (KW) and a_i, b_i, and c_i are cost coefficients of *i*-th plant. As mentioned before, there may be also constraints on the allowed range of values for P_i [18]:

$$P_i^{min}\leq P_i\leq P_i^{max} \tag{2.2}$$

Such constraints make the ED optimization process significantly harder to solve. Frequently, the controller first solves the unconstrained optimization problem and when the resulting optimal power generation lies in prohibited zone (i.e. violates the constraints), the controller uses heuristic methods to reformulate the optimization problem to enforce the constraints. We show in chapter 3 that a distributed approach is able to handle maximum/minimum constraints in a simple way.

Power losses are usually modeled, for the purposes of economic dispatch, as a second order function of the P_i:

$$P_{loss}=\sum_{i=1}^{N}\sum_{j=1}^{N}P_iB_{ij}P_j \tag{2.3}$$

A generalized version of the above including linear and constant terms is also sometimes used, and is called Kron's loss formula [18]:

$$P_{loss}=\sum_{i=1}^{N}\sum_{j=1}^{N}P_i\,B_{ij}\,P_j+\sum_{i=1}^{N}B_{0i}\,P_i+B_{00} \tag{2.4}$$

The coefficients B_{ij} are called loss coefficients or *B*-coefficients and their units are 1/KVA, constant, and KVA for B_{ij}, B_{0i}, and B_{00}, respectively [18].

Physically power loss is attributable to resistance in transmission lines. The loss coefficients, above, can therefore be obtained from impedances and currents in the network. Thus, the loss coefficients in general change when the state of power system changes. However, in this thesis, we assume that the state of the power system always stays within a small range of deviations. Therefore we assume that the loss coefficients can be modeled as approximately constant, and the power losses depend only on the P_i as stated in (2.4).

2.3 Economic Dispatch

Thus the economic dispatch problem can be formulated as follows:

Minimize the total cost, which is sum of J_i, with respect to P_i

$$J = \sum_{i=1}^{N} J_i(P_i) \tag{2.5}$$

under the constraint

$$\sum_{i=1}^{N} P_i = P_{load} + P_{loss} \tag{2.6}$$

Equation (2.6) is represented graphically in Figure 2-2, which shows a set of generators supplying power to loads and incurring power losses on the power buses.

The most direct way of solving the dispatch problem is by using Lagrangian techniques, which are based on the fact that the marginal cost – derivative of the cost function (2.1) – of each generator is equal in the optimal allocation. Usually an analytical solution is not feasible, however the optimal solution can be obtained using a numerical method such as lambda iteration; the lambda iteration method works as follows. Initially an arbitrary value is assumed for the marginal cost which is denoted by λ; If the corresponding power allocations lead to the total generation power exceeding the loads and power losses, then this indicates that our assumed value of marginal cost was too large.

$$\sum_{i=1}^{N} P_i(\lambda) > P_{load} + P_{loss} \tag{2.7}$$

Therefore the value of λ is reduced and the process is repeated [22].

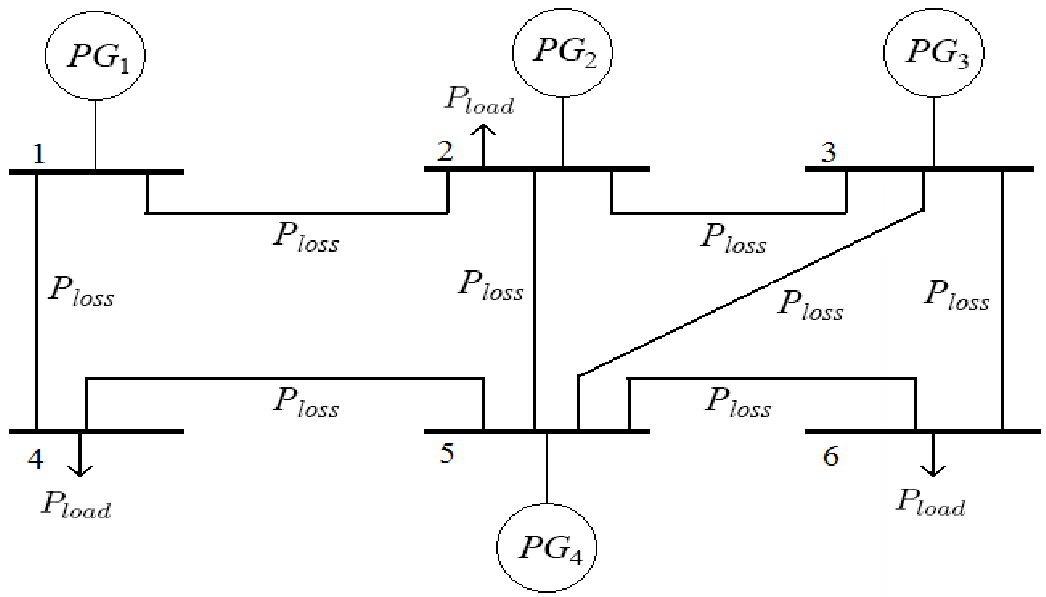

Figure 2-2 Power system bus

Lagrangian methods, however, do not work well when constraints and power losses are included. Many methods have been proposed in the literature to solve the optimization problem under these conditions, based on genetic algorithms, neural networks, particle swarm optimization and such other numerical techniques [23]. These methods, however, are based on complicated computational techniques whose behavior is not well-understood.

The distributed approach presented in this thesis offers a simple alternative to these methods and we consider this approach in detail next.

CHAPTER 3

DISTRIBUTED CONTROL

In this chapter, we present a new distributed control technique for the economic dispatch of power generators. This control process adjusts power generation iteratively and independently for each generator. It restores power balance exponentially fast and guarantees minimum cost allocation over time under some assumptions. The algorithm is also able to accommodate constraints on the allowed power allocations and also power losses in the network.

3.1 Model and Assumptions

In the sequel, the following model will be used. First, if there are N generators in power bus system, power imbalance at time-step k is denoted by:

$$\Delta P[k] = P_{load}[k] + P_{loss}[k] - \sum_{i=1}^{N} P_i\,[k] \tag{3.1}$$

where P_{load}, P_{loss} are the total power consumed in the loads and transmission losses in the grid, respectively, and P_i is the active power generation corresponding to the rated frequency on the P-f curve of generator *i*. We neglect the effects of reactive power flows, voltage deviations and transients as is standard for economic dispatch problems.

As explained in Chapter 2, whenever a power imbalance occurs in the grid, the primary controller on each generator acts to reduce this imbalance to zero. However, this process introduces a frequency deviation and this frequency deviation is assumed to be proportional to the imbalance between the total rated power that all generators are set to supply at the rated frequency and the total power consumed (including loads and losses) in the grid:

$$\Delta f[k] = -\beta \Delta P[k] \tag{3.2}$$

where β is constant. We assume that each generator can indirectly measure the power imbalance in the grid by observing the frequency deviation on the grid. This measurement does not require any coordination or centralized action, and can be independently performed by each generator. This is analogous to the Area Control Error (ACE) [27] observed by the secondary controller in a traditional Load Frequency Control (LFC) implementation. We assume that β remains constant for all values of $P_i[k]$ and $\Delta P[k]$. This is a reasonable assumption for small frequency deviations.

We describe next a distributed algorithm that uses this measured power imbalance to adjust the rated generation power at each generator in such a way as to achieve lower total cost of generation and to reduce the frequency deviation to zero. Thus the distributed algorithm combines the functions traditionally performed by the secondary and tertiary controllers.

The algorithm is basically divided into two processes: pull-up and pull-down processes, one of which is activated depending on the sign of power imbalance. Positive imbalance in (3.1) means that power generation is lower than consumption, so the control pulls up power generation. Likewise, with a negative imbalance in (3.1), power generation at each generator is reduced. The amount by which each generator increases or reduces its power depends on the cost function of the generators.

The cost function of generator i is denoted by $J_i(P_i)$, and the marginal cost by:

$$J_i'(P_i) = \frac{dJ_i(P_i)}{dP_i} \tag{3.3}$$

The cost function $J_i(P_i)$ is assumed to be twice differentiable and a convex. For example, in the case of a quadratic cost functions, this implies that the coefficient a_i in (2.1) is positive; in fact we assume the slightly stronger condition for all $i \in \{1,\cdots,N\}$:

$$J_i''(P_i) > \eta_1 \tag{3.4}$$

where $\eta_1>0$.

The marginal cost of all generators is assumed to be always positive. Furthermore, we assume there exists η_2>0 and $J_i^{'}(P_i)$>η_2 for all $i \in \{1,\cdots,N\}$. This assumption is physically reasonable: as explained in Chapter 2, the power generation and cost are expected to have a monotonic relationship because generating more power, in general, requires more fuel and other resources, i.e., higher cost, and idling costs when a generator output is zero.

Finally, for simplicity, we assume in our mathematical analysis that there are no other constraints on the $P_i[k]$, i.e., we neglect the commonly imposed maximum and minimum limits on the power of the active generators. This assumption is not necessary for the working of the algorithm, however it makes the mathematical presentation considerably simpler. We also neglect power losses in our mathematical analysis for simplicity. A more general analysis taking power losses into account is presented in [26].

3.2 Distributed Control Algorithm

The distributed algorithm works as follows. At time step k, i-th generator updates its generation, iteratively, as follows:

$$P_i[k+1] = \begin{cases} P_i[k] + \alpha_1 \Delta P[k] \left(\dfrac{1}{J_i^{'}(P_i) J_i^{''}(P_i)} \right), & \text{if } \Delta P[k] \geq 0 \\ P_i[k] + \alpha_2 \Delta P[k] \left(\dfrac{J_i^{'}(P_i)}{J_i^{''}(P_i)} \right), & \text{otherwise} \end{cases} \tag{3.5}$$

where α_1 and $\alpha_2$are parameters controlling the rate of adaptation.

The intuition behind the algorithm can be explained as follows. When $\Delta P[k]$ is larger than zero, power generation should be pulled up in order to satisfy the load. Intuitively, to get lower cost, we would like to have generators with low marginal costs increase their allocation more rapidly than high marginal cost generators: the cost of low marginal cost generator increases by a smaller amount with the same amount of additional generation. Therefore, when $\Delta P[k]$ is positive, marginal cost is in the denominator of (3.5) to increase generation in inverse proportion to the marginal cost.

Likewise, when $\Delta P[k]$ is negative, generation should be lowered, and, therefore, generation of generator which has high marginal cost should be reduced more than low marginal cost generator, to achieve lower cost allocations. Therefore, marginal cost appears in the numerator in (3.5) to decrease generation in proportion to marginal cost. The inclusion of the second derivative reflects the fact that a large second derivative causes larger changes to the marginal costs. Thus the inclusion of the second derivative in denominator prevents abrupt changes in cost of generation.

3.3 Properties of Distributed Algorithm

To analyze the properties of the distributed control algorithm, it is more convenient to consider the continuous time version.

$$\frac{dP_i(t)}{dt} = \begin{cases} \alpha_1 \Delta P(t) \left(\dfrac{1}{J_i'(P_i) J_i''(P_i)} \right), & \text{if } \Delta P(t) \geq 0 \\ \alpha_2 \Delta P(t) \left(\dfrac{J_i'(P_i)}{J_i''(P_i)} \right), & \text{otherwise} \end{cases} \tag{3.6}$$

In (3.6) we use the same notation as for the discrete-time version above. For example, $\Delta P(t)$ is power imbalance at time step t, which can be expressed as

$$\Delta P(t) = P_{load}(t) + P_{loss}(t) - \sum_{i=1}^{N} P_i(t) \tag{3.7}$$

and so on. As mentioned earlier, we neglect power losses in our analysis for simplicity.

We now prove the existence of a solution to (3.6). Suppose $P_{load}(t)$ is constant, $P_i(t) > 0$ for all $i \in \{1,2,\cdots,N\}$, and $\Delta P(0)$ is finite. Then the solution to (3.6) exists and is unique. Further, for all $i \in \{1,2,\cdots,N\}$, and all $t \geq 0$, $J_i'(P_i) > 0$.

We first prove this for the case of $\Delta P(0) > 0$. In the interval $t \in [0,T)$, (3.6) becomes:

$$\frac{d}{dt}\{J_i'(P_i(t))\}^2 = 2\alpha_1 \Delta P(t) \tag{3.8}$$

From (3.7) we get

$$\Delta\dot{P}(t) = -\sum_{i=1}^{N} \dot{P}_i(t) \tag{3.9}$$

With a positive $\Delta P(t)$, the $\dot{P}_i(t)$ are positive because power will be increased to restore the balance, as noted in (3.6). Therefore, in (3.9), $\Delta\dot{P}(t) < 0$ is guaranteed for all t, given the initial condition $\Delta P(0) > 0$. Thus, $\Delta P(t)$ is decreasing and is hence bounded by $\Delta P(0)$. Extending this argument for t = ϵ and beyond, a unique continuous solution exists for all $t \in [0,T)$. Then the existence and uniqueness of the solution is guaranteed as by definition $\Delta P(T) = 0$ and $\Delta P = 0$ is a stationary point of the algorithm.

Now suppose $\Delta P(0) < 0$. Then, in the interval $t \in [0,T)$, (3.6) becomes

$$\frac{d}{dt}\{J_i'\big(P_i(t)\big)\} = \alpha_2 J_i'\big(P_i(t)\big)\Delta P(t) \tag{3.10}$$

Observe because of convex feature of the marginal cost, $J_i'\big(P_i(t)\big)$ uniquely specifies $P_i(t)$ and hence $\Delta P(t)$. Thus at least for some $\epsilon > 0$, and $t \in [0,\epsilon)$, the $P_i(t)$ are strictly decreasing as are the $J_i'\big(P_i(t)\big)$. Further, with a negative $\Delta P(t)$, the $\dot{P}_i(t)$ are negative because power will be reduced to restore the balance, as noted in (3.6). Therefore, in (3.9), $\Delta\dot{P}(t) > 0$ is guaranteed in $\Delta P(0) < 0$ initial condition. Hence, $\Delta P(t)$ is increasing on this interval and has magnitude bounded by $|\Delta P(0)|$. Thus arguing as above a unique continuous solution exists for all $t \in [0,T)$. Then the existence and uniqueness of the solution is again guaranteed as $\Delta P = 0$ is a stationary point of the algorithm.

The case of $\Delta P(0) = 0$ is trivial, because it represents a stationary point of the algorithm.

Finally, if $\Delta P(0) > 0$, the $P_i(t)$ are non-decreasing. Consequently, under assumption there exists $\eta_2 > 0$ and $J_i'(P_i) > \eta_2$ for all $i \in \{1,\cdots,N\}$, as stated above, $J_i'\big(P_i(t)\big) > 0$ for all t. On the other hand if $\Delta P(0) < 0$, (3.10) holds, ΔP is non-decreasing and as $|\Delta P(t)| < |\Delta P(0)|$, again $J_i'\big(P_i(t)\big) > 0$ for all t.

Next, we show that under the distributed algorithm, $\Delta P(t)$ converges exponentially to zero. Once again the case of $\Delta P(0) = 0$ is trivial: in this case power imbalance stays at 0. Now, consider the case of $\Delta P(0) > 0$. From (3.6), $\dot{P}_i(0) > 0$ for all $i \in \{1,2,\cdots,N\}$. Further, $\Delta P(t)$ remains in positive for some $t \in [0,t_1]$, and, therefore, $\dot{P}_i(t) > 0$ and $P_i(t) > P_i(0)$ for $t \in [0,t_1]$. Since J_i is strictly convex, $J_i^{'}(P_i(t)) > J_i^{'}(P_i(0))$ for all $t \in [0,t_1]$.

Also,

$$\frac{d\Delta P(t)}{dt} = \frac{d}{dt}(P_{load}(t) + P_{loss}(t) - \sum_{i=1}^{N} P_i\,(t))$$

$$= -\alpha_1 \Delta P(t) \sum_{i=1}^{N} \left(\frac{1}{J_i^{'}(P_i) J_i^{''}(P_i)} \right) \tag{3.11}$$

We mentioned earlier the assumption that marginal cost is positive; in fact we make the slightly stronger assumption that there is a γ_1, such that

$$\alpha_1 \sum_{i=1}^{N} \left(\frac{1}{J_i^{'}(P_i) J_i^{''}(P_i)} \right) > \gamma_1 > 0 \tag{3.12}$$

Now, we can prove $\Delta P(t)$ exponentially converges to zero. From (3.11) and (3.12)

$$\Delta \dot{P}(t) < -\gamma_1 \Delta P(t)$$

$$\int_0^t \frac{\Delta \dot{P}(t)}{\Delta P(t)} d\tau < -\int_0^t \gamma_1 d\tau$$

$$ln\Delta P(t) < ln\Delta P(0) - \gamma_1 t$$

$$\therefore \Delta P(t) < \Delta P(0) e^{-\gamma_1 t} \tag{3.13}$$

Next, suppose that $\Delta P(0) < 0$. In this case,

$$\frac{d\Delta P(t)}{dt} = \frac{d}{dt}(P_{load}(t) + P_{loss}(t) - \sum_{i=1}^{N} P_i\,(t))$$

$$= -\alpha_2 \Delta P(t) \sum_{i=1}^{N} \left(\frac{J_i^{'}(P_i)}{J_i^{''}(P_i)} \right) \tag{3.14}$$

By (3.6), $\dot{P}_i(t) < 0$ and $P_i(t) < P_i(0)$, while $\Delta P(t)$<0. We started with $\Delta P(0) < 0$, and, since $\Delta P = 0$ is a stationary trajectory, $\Delta P(t)$ remains negative until it converges to zero. Also, there is a γ_2 which satisfies

$$\gamma_2 > \alpha_2 \sum_{i=1}^{N} \left(\frac{J_i'(P_i)}{J_i''(P_i)} \right) > 0 \tag{3.15}$$

By following same steps as (3.13), it can be easily proved that $\Delta P(t)$ is also exponentially converges to zero when $\Delta P(0) < 0$.

$$\Delta \dot{P}(t) < -\gamma_2 \Delta P(t)$$

$$\Delta \dot{P}(t) < \gamma_2 |\Delta P(t)|$$

$$\int_0^t \frac{\Delta \dot{P}(t)}{|\Delta P(t)|} d\tau < \int_0^t \gamma_2 d\tau$$

$$ln|\Delta P(t)| < ln|\Delta P(0)| + \gamma_2 t$$

$$\therefore |\Delta P(t)| < |\Delta P(0)| e^{-\gamma_2 t} \tag{3.16}$$

Next we show that the distributed algorithm attains the minimum cost allocation of power among the generators under certain conditions. Suppose $\Delta P(t) \neq 0$ and $J_i'(P_i(t)) \neq J_j'\left(P_j(t)\right)$ for whole procedure. Then following is guaranteed:

$$\left(\frac{d(J_i'(P_i(t)) - J_j'\left(P_j(t)\right))}{dt} \right) \left(J_i'(P_i(t)) - J_j'\left(P_j(t)\right) \right) < 0$$

$$(J_i''(P_i) \frac{dP_i(t)}{dt} - J_j''\left(P_j\right) \frac{dP_j(t)}{dt}) \left(J_i'(P_i(t)) - J_j'\left(P_j(t)\right) \right) < 0 \tag{3.17}$$

First, suppose $\Delta P(t) > 0$. Then, (3.17) can be shown as follows:

$$\alpha_1 \Delta P(t) (J_i''(P_i) \frac{1}{J_i'(P_i) J_i''(P_i)} - J_j''\left(P_j\right) \frac{1}{J_j'\left(P_j\right) J_j''\left(P_j\right)}) \left(J_i'(P_i(t)) - J_j'\left(P_j(t)\right) \right)$$

$$= \alpha_1 \Delta P(t) (\frac{1}{J_i'(P_i)} - \frac{1}{J_j'\left(P_j\right)}) \left(J_i'(P_i(t)) - J_j'\left(P_j(t)\right) \right)$$

$$= -\frac{\alpha_1 \Delta P(t)}{J_i'(P_i) J_j'\left(P_j\right)} \left(J_i'(P_i(t)) - J_j'\left(P_j(t)\right) \right)^2 < 0 \tag{3.18}$$

Likewise, in $\Delta P(t) < 0$ case, (3.17) can be proved as follows:

$$\alpha_2 \Delta P(t)(J_i''(P_i)\frac{J_i'(P_i)}{J_i''(P_i)} - J_j''(P_j)\frac{J_j'(P_j)}{J_j''(P_j)})\left(J_i'(P_i(t)) - J_j'(P_j(t))\right)$$

$$= -\alpha_2|\Delta P(t)|\left(J_i'(P_i(t)) - J_j'(P_j(t))\right)^2 < 0 \qquad (3.19)$$

Equation (3.17) can be interpreted as follows. While $\Delta P(t) \neq 0$, regardless of the sign of $\Delta P(t)$, marginal costs of different generators will tend to converge. The signs of marginal costs and their derivatives are opposite by (3.17). It means, the smaller marginal cost, increases more than the larger one, in a direction to force $\Delta P(t) = 0$.

If $\Delta P(t) \neq 0$, i.e., a non-zero power imbalance is maintained by power load fluctuations (or by deliberately introducing a small dither in the generation power), the marginal costs move continuously closer to each other. Eventually, all the marginal costs become equal, which corresponds to the lowest cost allocation assuming negligible power losses.

$$J_k'(P_k)=\lambda, \forall\, k \in 1,\cdots,N \qquad (3.20)$$

And the λ is guaranteed as the optimal point by Lagrangian optimization method.

The last property we consider is the following: consider the stationary point where $\Delta P(t) = 0$, but for some $\{i, j\} \subset \{1, \cdots, \mathrm{N}\}$, $J_i'(P_i(t)) \neq J_j'(P_j(t))$. Then this stationary point is unstable.

Suppose $P_i(t)^*$ is the stationary point in question, for all $i \in \{1,\cdots,N\}$. Because its solution existence is guaranteed, with every perturbation $\delta P_i(t)$, and $P_i(0) = P_i(t)^* + \delta P_i(t)$,

$$\lim_{t\to\infty} P_i(t) \qquad (3.21)$$

exists.

By definition of an unstable stationary point, [24] it then suffices to show that for every $\epsilon > 0$ there is a perturbation $\delta P_i(t)$, obeying $\|\delta P_i(t)\| < \epsilon$, such that with $P_i(0) = P_i(t)^* + \delta P_i(t)$,

$$\sum_{i=1}^{N} P_i(t) \neq P_{load}(t) \tag{3.22}$$

and

$$\lim_{t\to\infty} P_i(t) \neq P_i(t)^* \tag{3.23}$$

Indeed with i, j as in the hypothesis, one can always find a perturbation $\delta P_i(t)$, obeying $\|\delta P_i(t)\|<\epsilon$, such that with $P_i(0) = P_i(t)^* + \delta P_i(t)$, the i-th and j-th marginals corresponding to $P_i(0)$ are closer to each other than is the case with the i-th and j-th marginals corresponding to $P_i(t)^*$. Then from (3.17), (3.23) must hold.

The significance of this instability property is as follows. While $\Delta P(t) \neq 0$, the algorithm will tend to drive the marginals closer. If $\Delta P(t)$, becomes zero before the marginals are equalized, then the slightest noise in the $P_i(t)$ or load fluctuations that enforce the condition $\Delta P(t) \neq 0$, will again tend to drive the marginals closer to each other. Over time the practical effect of this is to equalize the marginals.

3.4 The Effect of Constraints and Transmission Losses

As discussed in Chapter 2, power constraint makes the optimization problem of dispatch much harder to solve. Under the distributed algorithm, maximum and minimum constraints can be handled in a very simple way: when the adjusted power for generator i from (3.6) exceeds the maximum or minimum constraint, the power for generator is simply left unchanged. The other generators make power adjustments as before and eventually converge to an allocation that satisfies the constraints. This is possible in the distributed control because it solves optimization problem iteratively.

Likewise, the distributed algorithm works well with power losses as long as the losses are not too large. While in the presence of losses, the “equal marginal costs” condition no longer represents the lowest cost allocation, the level of sub-optimality is found to be small, and the algorithm still drives frequency deviations to zero exponentially fast, while satisfying all constraints.

3.5 Simulation without Power Constraint and Loss

We now present some numerical simulation results to illustrate the performance of the distributed algorithm. In these simulations, one time step for *k* is taken to be ten seconds. This assumption is physically reasonable for the typical response times of power generators. Also, in order to prevent the frequency deviation from going to zero before the generation allocation gets optimal condition, we apply a small random dither to the generation powers; specifically when the absolute value of the power imbalance becomes smaller than a threshold, a random dither is applied independently by each generator chosen uniformly from an interval:

$$-\varepsilon \le d \le \varepsilon \tag{3.24}$$

where d is random dither value and ε specifies the size of the dither.

Our first simulation is of a three generator power bus system without any power constraints or losses. The cost functions and detail system description is obtained from [18]. Table 3-1 shows the coefficients of three generators. The total power load is constant at 800kW in this system.

Table 3-1. Generator Data for Simulation 1

Unit	a_i	b_i	c_i	ε
P_1	0.004	5.3	500	
P_2	0.006	5.5	400	8
P_3	0.009	5.8	200	

Below Fig. 3-1 shows how the distributed control iteratively allocates optimal generation from the starting non-optimal value. Comparing the marginal cost plot with cost plot of Fig 3-1, we note that even before it reaches optimal value, i.e., before the marginal costs converge, the algorithm achieves good performance with low cost. We also note that the lowest cost from [18] is reached at approximately at 2000 sec and maintained thereafter. Detailed numerical results are given in Table 3-2.

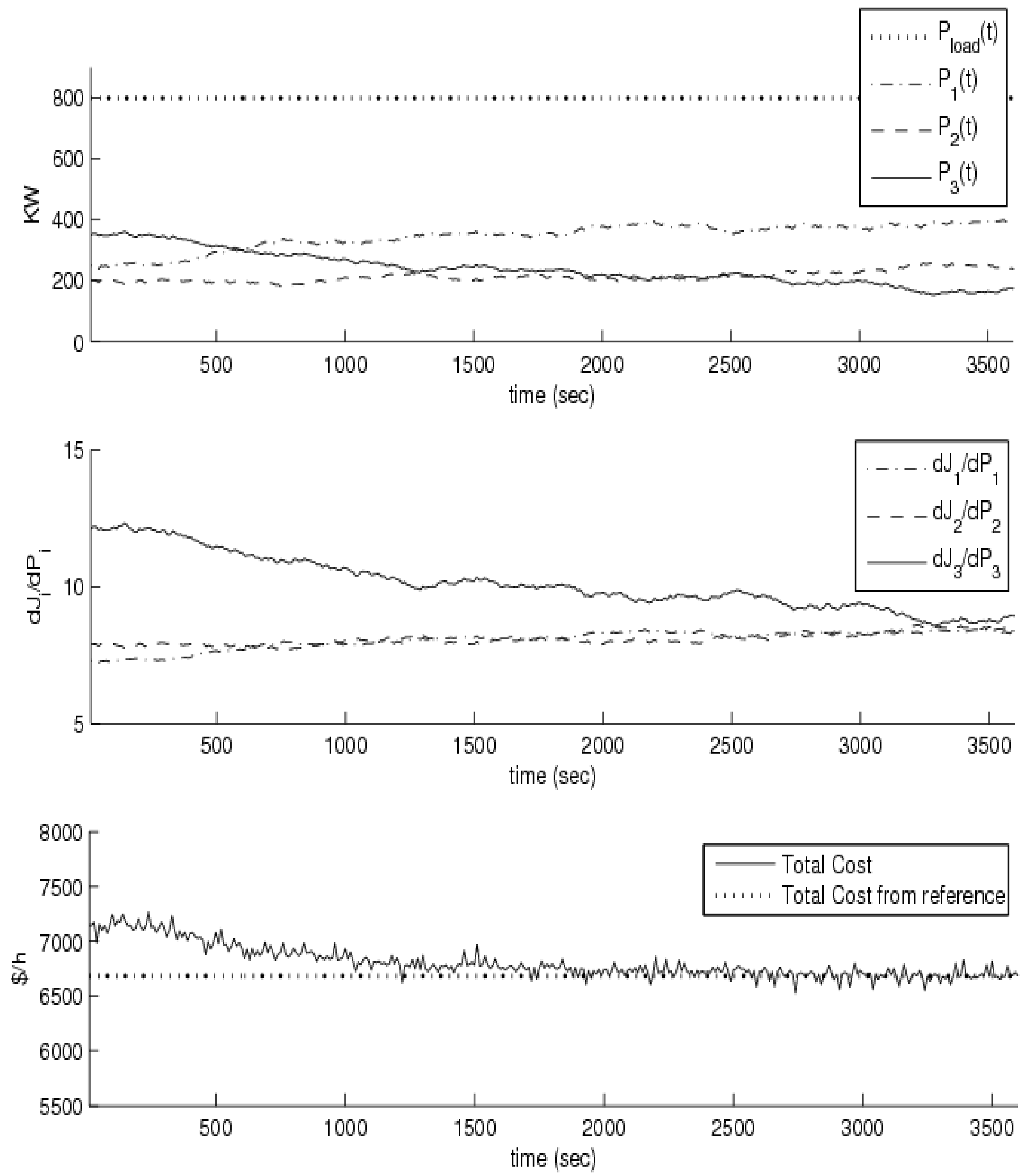

Figure 3-1. Dynamics of the distributed control without power constraint and loss

Table 3-2. Numerical results of Simulation 1

Unit	The distributed Control (at 2140 sec)	The distributed Control (at 3550 sec)	Ref. [18]
P_1 (kW)	382.6733	395.1714	400
P_2 (kW)	214.7282	243.2989	250
P_3 (kW)	202.2399	161.3855	150
P_{Total} (kW)	799.6514	799.8558	800
Total Cost ($/h)	6712.759	6682.804	6682.5

Next we consider Fig. 3-2 which shows that the algorithm forces $\Delta P = 0$ very fast when there is no dither. Of course if the load power is constant and no dither is applied, then while the power imbalance is reduced to zero rapidly, the minimum cost allocation is not achieved.

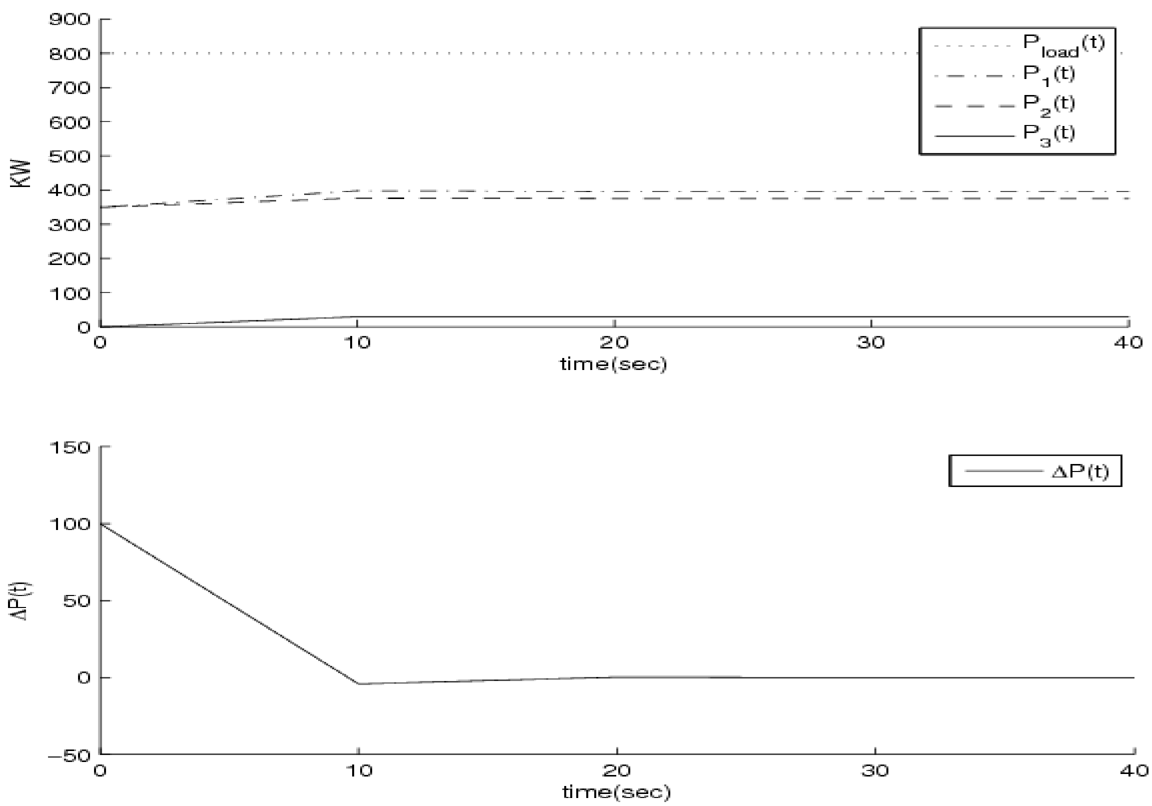

Figure 3-2. Change of ΔP for first simulation without consideration of economic status

3.6 Simulation with Power Constraint and Loss

Next we present results from the simulation of a more complicated system that includes constraints and losses. This example is taken from [19].

Table 3-3. Generator Data for Simulation 2

Unit	a_i	b_i	c_i	$P_{i(min)}$	$P_{i(max)}$	ε
P_1	0.00533	11.69	213.1	50	200	
P_2	0.00889	10.333	200	37.5	150	1.5
P_3	0.00741	10.833	240	45	180	

Below are loss coefficients.

$$B = \begin{bmatrix} 0.06760 & 0.00953 & -0.00507 \\ 0.00953 & 0.05210 & 0.00901 \\ -0.00507 & 0.00901 & 0.02940 \end{bmatrix} \times 10^{-2}$$

$$B_0 = [-0.07660 \quad -0.00342 \quad 0.01890]$$

$$B_{00} = 0.040357$$

Below Fig. 3-3 and Table 3-4 show the results. They show the distributed control works well even with power constraint and loss. Finally, Fig. 3-4 shows the control forces $\Delta P = 0$ in a few iterations, even with power loss.

Table 3-4. Numerical results of Simulation 2

Unit	The distributed Control (at 2050 sec)	The distributed Control (at 3600 sec)	Ref. [19]
P_1 (kW)	61.688	62.106	71.685
P_2 (kW)	98.629	91.507	69.239
P_3 (kW)	55.600	61.771	73.679
P_{loss} (kW)	6.378	5.942	4.603
P_{Total} (kW)	215.917	215.384	214.602
Total Cost ($/h)	3125.342	3117.101	3114.943

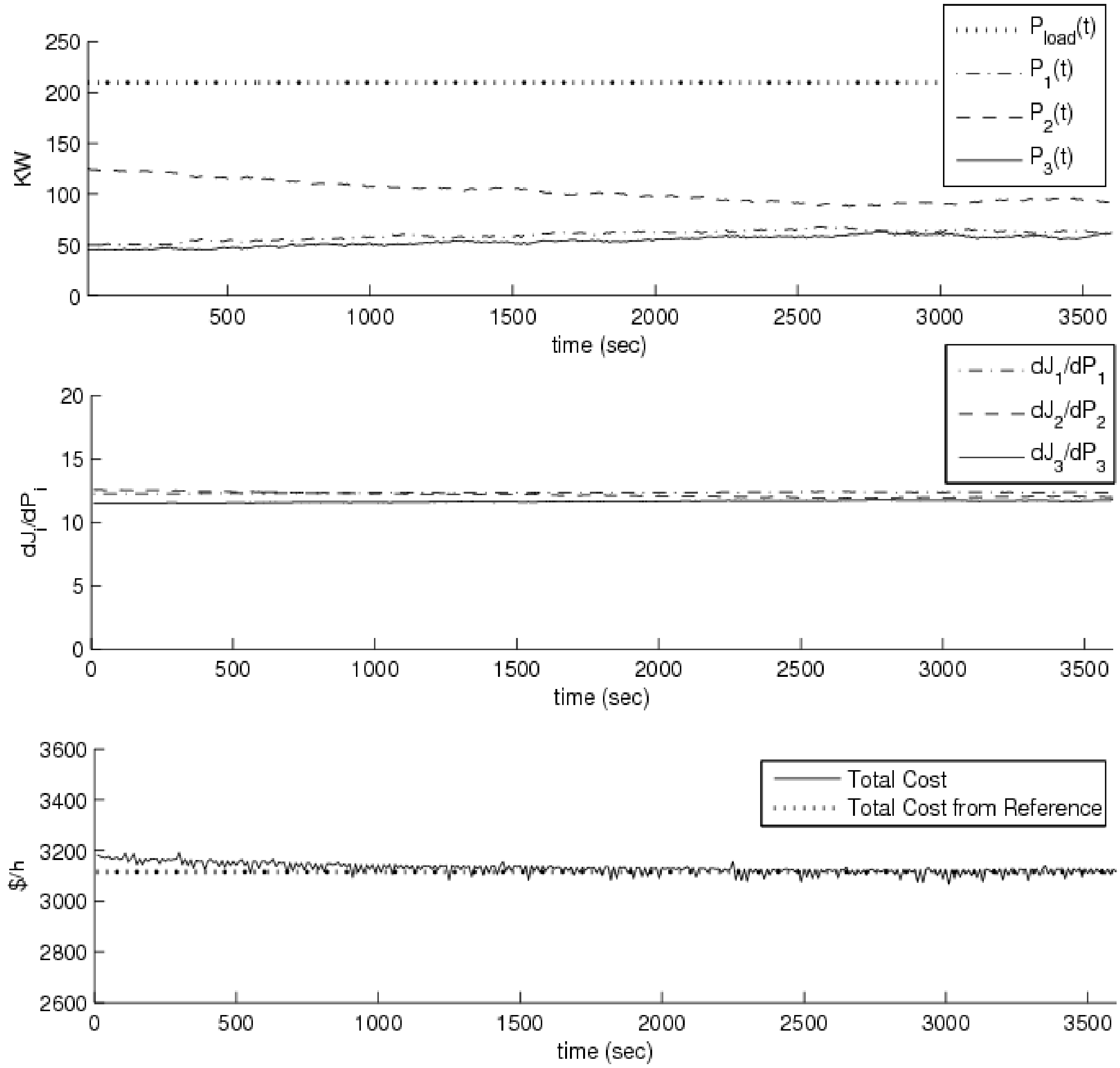

Figure 3-3. Dynamics of the distributed control with power constraint and loss

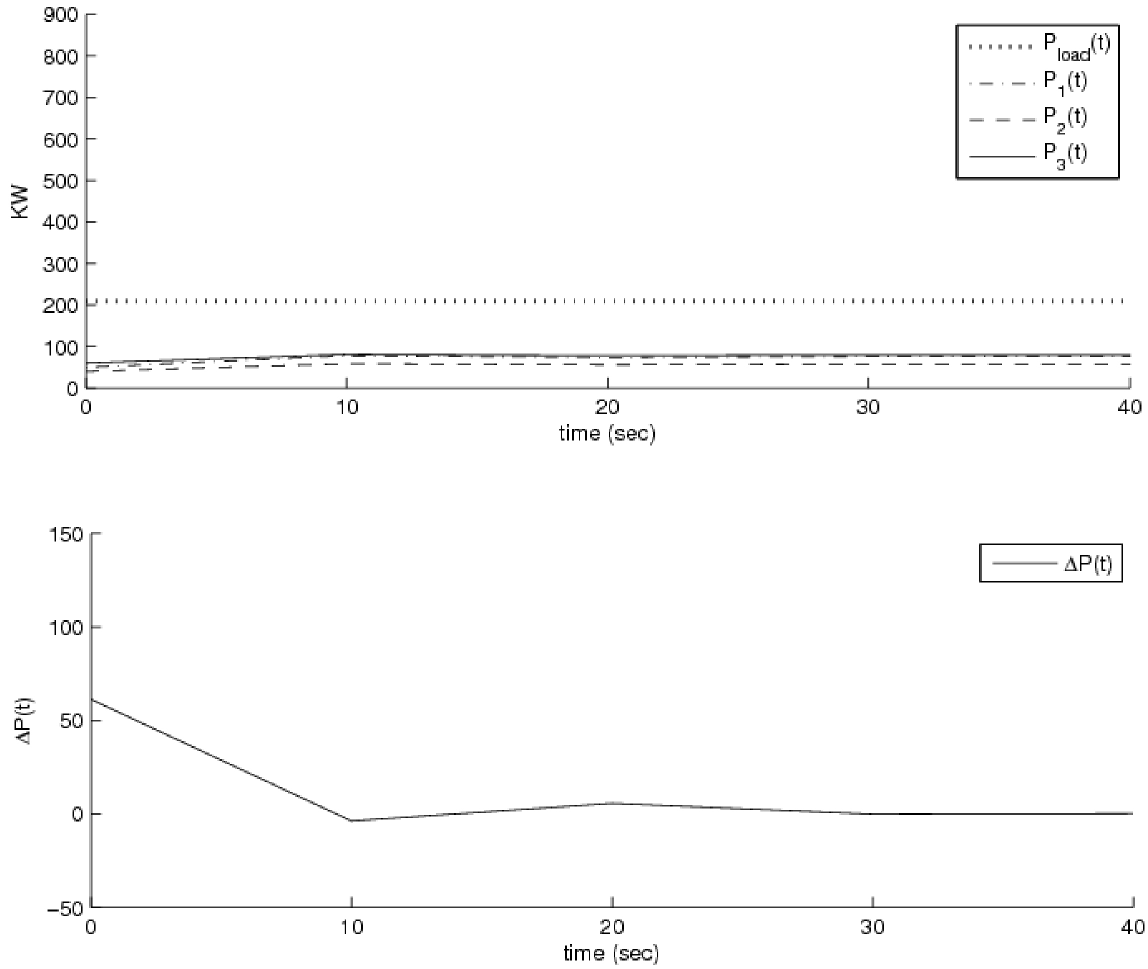

Figure 3-4. Change of Δ*P* for second simulation without consideration of economic status

CHAPTER 4

CONCLUSION

We presented a simple distributed algorithm to control the power allocation, which requires each generator only to have knowledge of its own generation cost function and the overall power imbalance. We presented analytical arguments to show that the algorithm achieves minimum cost allocation over time under some assumptions on the cost functions. The preliminary work reported in this paper shows that the distributed approach has many advantages especially for electric grids with a high penetration of alternative energy generators.

This work brings up many interesting open issues.

4.1 Algorithm Optimization and Extensions

The basic algorithm for optimal dispatch that we have described in this thesis can be refined and extended in many ways. While we have shown that the algorithm has good asymptotic convergence properties, this requires a persistent perturbation of the system for e.g. in the form of load fluctuations. An alternative is to apply a “dither” to the power output of each generator. The optimal design of such a dithering scheme depends on the statistics of load fluctuations seen on the electric grid and a detailed analysis of these statistics is an interesting open problem. Also the parameters α_1 and α_2 of the distributed algorithm were chosen in a heuristic manner for our numerical simulations; these parameters determine the convergence rate of the algorithm i.e. higher values of α_1 and α_2 leads to faster convergence. However we cannot make these parameters too large because in a discrete-time implementation, this may lead to overshoot and even possible instability. A systematic investigation of these tradeoffs is another interesting area for future work. Finally, while we have shown that our algorithm works well even in the presence of small power losses, we have not directly addressed the possibility of modifying the basic algorithm to take losses into account directly and this presents another challenging open problem.

4.2 Application to Smart Grids

As mentioned in Chapter 1, our work in this thesis was motivated primarily by the anticipated needs of the future electric grid where loads and generators respond intelligently to the real-time state of the grid to maximize efficiency of energy use. While we have focused on minimizing cost of generation in this thesis, an important open problem is extending this distributed approach to also control loads.

4.3 Distributed for the Electric Grid

In this thesis, we have focused on the problem of optimal economic dispatch. Our work illustrates the possibilities for distributed control not only for the dispatch problem, but more generally for control of the electric grid. Some examples of other possible applications include voltage regulation, reactive power control, unit commitment and load forecasting.

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