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Black-hole/near-horizon-CFT duality and 4 dimensional classical spacetimes

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BLACK-HOLE/NEAR-HORIZON-CFT DUALITY AND 4 DIMENSIONAL CLASSICAL SPACETIMES

by

Leo L. Rodriguez

An Abstract

Of a thesis submitted in partial fulfillment of the
requirements for the Doctor of Philosophy
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Thesis Supervisor: Professor Vincent G.J. Rodgers

ABSTRACT

In this thesis we accomplish two goals: We construct a two dimensional conformal field theory (CFT), in the form of a Liouville theory, in the near horizon limit for three and four dimensions black holes. The near horizon CFT assumes the two dimensional black hole solutions that were first introduced by Christensen and Fulling (1977 Phys. Rev. D 15 2088104) and later expanded to a greater class of black holes via Robinson and Wilczek (2005 Phys. Rev. Lett. 95 011303). The two dimensions black holes admit a $Diff(S^1)$ or Witt subalgebra, which upon quantization in the horizon limit becomes Virasoro with calculable central charge. These charges and lowest Virasoro eigen-modes reproduce the correct Bekenstein-Hawking entropy of the four and three dimensions black holes via the Cardy formula (Blöte et al 1986 Phys. Rev. Lett. 56 742; Cardy 1986 Nucl. Phys. B 270 186). Furthermore, the two dimensions CFT's energy momentum tensor is anomalous, i.e. its trace is nonzero. However, In the horizon limit the energy momentum tensor becomes holomorphic equaling the Hawking flux of the four and three dimensions black holes. This encoding of both entropy and temperature provides a uniformity in the calculation of black hole thermodynamics and statistical quantities for the non local effective action approach.

We also show that the near horizon regime of a Kerr-Newman-AdS ($KNAdS$) black hole, given by its two dimensional analogue a la Robinson and Wilczek, is asymptotically AdS_2 and dual to a one dimensional quantum conformal field theory (CFT). The s-wave contribution of the resulting CFT's energy-momentum-tensor together with the asymptotic symmetries, generate a centrally extended Virasoro algebra, whose central charge reproduces the Bekenstein-Hawking entropy via Cardy's Formula. Our derived central charge also agrees with the near extremal Kerr/CFT Correspondence in the appropriate limits. We also compute the Hawking temperature of the $KNAdS$ black hole by coupling its Robinson and Wilczek two dimensional analogue (RW2DA) to conformal matter.

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CERTIFICATE OF APPROVAL

PH.D. THESIS

This is to certify that the Ph.D. thesis of

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CHAPTER 1
INTRODUCTION

The thermodynamic and statistical properties of black holes have provided a unique insight and test for theories of quantum gravity. Though a fully formulated quantum field theory of gravity is lacking, a multitude of candidates exists, with string theory and loop quantum gravity leading in popularity. Despite a variety of theories, it is the hope that any serious candidate reproduce a variant of the Bekenstein-Hawking entropy [1]

$$S_{BH} = \frac{A}{4\hbar G} \tag{1.1}$$

and Hawking temperature [2, 3]

$$T_H = \frac{\hbar\kappa}{2\pi}, \tag{1.2}$$

where A is the horizon area and κ the surface gravity of the black hole. The fact that these quantities depend on both $\hbar$ and G is evident of their quantum gravitational origin.

Effective quantum gravity theories have had much success in reproducing (1.1) and (1.2) via analysis of anomalous energy momentum tensors,

$$\begin{cases} g^{\mu\nu}\left\langle T_{\mu\nu}\right\rangle \equiv \left\langle {T_\mu}^\mu \right\rangle \neq 0 & \text{Trace Anomaly (CFT)} \\ \left\langle \nabla_\nu {T_\mu}^\nu \right\rangle - \nabla_\nu \left\langle {T_\mu}^\nu \right\rangle \neq 0 & \text{Gravitational Anomaly} \end{cases}, \tag{1.3}$$

of non-local effective actions [4] and holographic one and two dimensional conformal field theories [5], respectively. We will discuss this in more detail next and sketch where the work of this thesis fits into the current plethora of approaches to quantum gravitational affects in the near horizon regime of black holes.

1.1 Quantum Gravity

The main difficulty with constructing a quantum field theory of gravity lies in its renormalizabilty. To see this let us exam the vacuum theory with zero cosmological constant given by the Einstein-Hilbert functional:

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R. \tag{1.4}$$

Next, we will consider small perturbations to second order in $h_{\mu\nu}$ where we assumed a spacetime:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{1.5}$$

To second order the theory (1.4) is linear in h and behaves as a gauge theory, i.e.

$$\frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}g_{\alpha\beta} = \eta_{\mu\nu} + h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu + \mathcal{O}\left(h^2\right). \tag{1.6}$$

Imposing the Hilbert gauge

$$\partial_\mu h^\mu{}_\nu - \partial_\nu h^\mu{}_\mu = 0 \tag{1.7}$$

the Lagrangian density, to lowest order reads:

$$\mathcal{L}_0 = \frac{1}{2}h_{\alpha\beta}\partial_\gamma V^{\alpha\beta\mu\nu}\partial^\gamma h_{\mu\nu}, \tag{1.8}$$

where $V^{\alpha\beta\mu\nu} = \frac{1}{2}\delta^{\alpha\mu}\delta^{\beta\nu} - \frac{1}{4}\delta^{\alpha\beta}\delta^{\mu\nu}$. From the lowest order Lagrangian we obtain the graviton propagator in Hilbert gauge

$$G_{\alpha\beta\mu\nu} = \frac{\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha} - \delta_{\mu\nu}\delta_{\alpha\beta}}{k^2 + i\epsilon}, \tag{1.9}$$

which allows the study of the Feynman rules for the tree level diagrams in Figure 1.1, from which we obtain the simple formula for the naive degree of momentum diver-

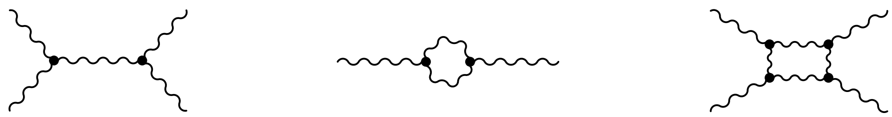

Figure 1.1: First few graviton graviton diagrams.

gences:

$$D = 4 - E + n, \tag{1.10}$$

where E is the number of external legs and n the number of vertexes. This simple formula indicates non-renormalizabilty due to the fact that it increases with the number of vertexes. In other words, an infinite sum within the path integral to all orders in $\hbar$ will be plagued by infinite ever-diverging diagrams. Thus the theory is non-renormalizable. There are several ideas and theories to attack this problem and for a comprehensive review of approaches to quantum gravity see [6, 5, 7].

1.2 Effective Action and Hawking Temperature

Analysis of an anomalous energy momentum tensor to compute (1.2) was first carried out by Christensen and Fulling in [8]. Considering the most general solution to the conservation equation

$$\nabla_\mu T^\mu{}_\nu = 0, \tag{1.11}$$

they found that by restricting to the $r-t$ plane of a free scalar field in Schwarzschild geometry the energy momentum tensor exhibits a trace anomaly leading to the result:

$$\langle T^r{}_t \rangle = \frac{1}{768\pi G^2 M^2} = \frac{\pi}{12} {T_H}^2, \tag{1.12}$$

which is exactly the luminosity (Hawking flux, Hawking radiation) of the 4-dimensional black hole in units $\hbar = 1$ (similarly ${T_{U(1)}}^{r}{}_{t}$ carries the electromagnetic Pointing flux).

A similar approach was studied in [9] where the authors determined the s-wave contribution of a scalar field to the 4-dimensional effective action for an arbitrary spherically symmetric gravitational field. Applying their results to a Schwarzschild black hole, the authors showed the energy momentum tensor of the non-local effective action to contain the Hawking Flux. Other closely related approaches for scalar fields and two dimensional theories include [10, 11, 12, 13, 14]. The general idea follows from studying the effective action of the functional:

$$Z(\varphi, g) = \int \mathcal{D}\varphi e^{-iS^D[\varphi,g]}, \tag{1.13}$$

where $S^D[\varphi, g]$ is the action of a free scalar field in D dimensions on the background spacetime ${g^{(D)}}_{\mu\nu}$. For the case where $D = 2$ the effective action is given by the non-local Polyakov Action [15, 16]

$$\Gamma_{Polyakov} = \frac{1}{96\pi}\int d^2x\sqrt{-g^{(2)}}R^{(2)}\frac{1}{\Box_{g^{(2)}}}R^{(2)}, \tag{1.14}$$

which has shown to play an important role for computing quantum gravitational quantities near black hole horizons [17, 18, 19, 20] and exhibits a unique relationship to conformal algebras [21, 22].

Another method for computing Hawking radiation, first introduced by Robinson and Wilczek [23], considers a quantum chiral-scalar field theory of 2-dimensions in the near horizon limit of a static 4-dimensional black hole. A two dimensional chiral field theory is known to exhibit a gravitational anomaly of the form:

$$\nabla_\mu \langle {T^\mu}_\nu \rangle = \frac{1}{96\pi\sqrt{-g^{(2)}}}\epsilon^{\gamma\rho}\partial_\rho\partial_\lambda {\Gamma^{(2)\lambda}}_{\nu\gamma}, \tag{1.15}$$

where ${g^{(2)}}_{\mu\nu}$ contains the leftover components of the 4-dimensional metric which are not redshifted away in the near horizon limit of the functional

$$S_{\text{free scalar}} = \frac{1}{2}\int d^4x\sqrt{-g}\nabla_\mu\varphi\nabla^\mu\varphi. \tag{1.16}$$

Robinson and Wilczek showed in the near horizon regime of a Schwarzschild black hole, that to ensure a unitary quantum field theory, the black hole should radiate as a thermal bath of temperature equaling T_H. In other words, quantum gravitational effects in the near horizon regime cancel the chiral/gravitational anomaly [24]. This method has been expanded to include gauge/gravitational anomalies and covariant anomalies [25, 26, 27, 28] and has successfully reproduced the black hole temperature for charged-rotating black holes [18, 19], dS/AdS black holes [29, 30], rotating dS/AdS black holes [31, 32], black rings and black string [33, 34], 3-dimensional black holes [35, 36] and black holes of non spherical topologies [37]. This method provides a fundamental reason for black hole thermodynamics based on symmetry principles of a near horizon quantum field theory. It also provides a Robinson and Wilczek two dimensional analogue (RW2DA), for higher dimensional black holes besides the Schwarzschild case. This is a rather useful fact since the Ricci tensor in 2-dimensions is always Einstein, i.e.[1]

$${R^{(2)}}_{\mu\nu} - \frac{1}{2}{g^{(2)}}_{\mu\nu}R^{(2)} \equiv 0. \tag{1.17}$$

Thus classically, in 2-dimensions, there are no general relativistic dynamics and any gravitational effects that are present must have risen from some (effective) quantum gravity of metric ${g^{(2)}}_{\mu\nu}$.

[1] In 2-dimensions the curvature of any Riemannian manifold is completely characterized by its scalar variant. This is because any 2-form has only one independent component. Thus for any Riemannian-Levi-Cevitia connection 2-form $\omega_{\alpha\beta}$, $d\omega_{12} = K\mathrm{vol}^2$,where $K = \frac{1}{(2)((2)-1)}R^{(2)}$ is the Gauss curvature

1.3 Holographic CFT and Entropy

Combining two dimensional near horizon physics with holography has provided a unique scenario for studying black hole entropy by asserting that quantum gravity in two dimensions is dual to a conformal field theory of equal or lesser dimension. This duality is richly exemplified in the well known AdS/CFT correspondence of string theory [38]. One simplistic viewpoint of this correspondence is to analyze the asymptotic symmetry group of an AdS space. Then choosing a particular set of boundary conditions the asymptotic symmetry group my includ the generators of conformal symmetry. In fact in their seminal work [39], Brown and Henneaux showed the algebra of the asymptotic symmetry group of three dimensional gravity with a negative cosmological constant is a Virasoro algebra (conformal algebra) with calculable central charge. This is widely considered to be the first example of an AdS_3/CFT_2 correspondence. Applying this to the three dimensional BTZ-black hole [40], Strominger [41] reproduced the Hawking-Bekenstein Entropy [1]

$$S_{BH} = \frac{A}{4\hbar G} \tag{1.18}$$

via Cardy's Formula [42, 43]. This idea has been generalized and applied to various black holes in near horizon regimes and at asymptotic infinity by Carlip and others [17, 44, 20, 45, 46, 47, 48, 49, 50, 51], where the general idea is summarized as follows. Given a set of consistent metric boundary or fall-off conditions, there exists an associated asymptotic symmetry group (ASG). This ASG is generated by a finite set of diffeomorphisms which have a mode decomposition into a set of discrete ξ_n for all $n \in \mathbb{Z}$ satisfying a $Diff(S^1)$ subalgebra:

$$i\left\{\xi_m, \xi_n\right\} = (m-n)\xi_{m+n}. \tag{1.19}$$

Consistency necessitates that these generators, ξ_n, be finite and well behaved at the respective boundary. Upon quantization, $\xi_n \to \mathcal{Q}_n$ via Hamiltonian or Covariant Lagrangian techniques, Brown and Henneaux showed [39]

$$[\mathcal{Q}_m, \mathcal{Q}_n] = (m-n)\mathcal{Q}_{m+n} + \frac{c}{12}m\left(m^2-1\right)\delta_{m+n,0} \tag{1.20}$$

where c is a calculable central extension. We should note that (1.20) assumes a fixed normalization of the lowest Virasoro mode due to the linear term in the center. This ambiguity was first addressed by string theory in [52, 53, 41, 54], where it was shown that the massive BTZ black hole is a solution to low energy superstring theory lying in the Neveu-Schwarz sector (antiperiodic BC). This implies a mass shift $\mathcal{Q}_0 = \frac{c}{24}$ and thus fixes the normalization such that:

$$\left(\mathcal{Q}_0 - \frac{c}{24}\right)|0\rangle = 0. \tag{1.21}$$

In the case for non-supersymmetric theories the requirement for the generators of the ASG to include a proper $SL(2,\mathbb{R})$ subgroup, i.e. $\{\mathcal{Q}_{-1}, \mathcal{Q}_0, \mathcal{Q}_1\}$ form a proper $\mathfrak{sl}(2,\mathbb{R})$ subalgebra, is synonymous to the requirement that the vacuum be annihilated according to (1.21). The Bekenstein-Hawking entropy is then obtained from Cardy's Formula [42, 43] in terms of c and the proper normalized lowest eigenmode via:

$$S = 2\pi\sqrt{\frac{c\cdot\mathcal{Q}_0}{6}}. \tag{1.22}$$

Applying the above outline to the two dimensional dilaton black hole

$$\begin{aligned} ds^2 &= g^D{}_{\mu\nu}dx^\mu dx^\nu \\ &= -\left[(\lambda x)^2 - \frac{2M}{\lambda}(\lambda x)^3\right]dt^2 + \left[(\lambda x)^2 - \frac{2M}{\lambda}(\lambda x)^3\right]^{-1}dx^2, \end{aligned} \tag{1.23}$$

Cadoni computed the following central extension and zero mode [46]:

$$c = 48\frac{M^2}{\lambda^2} \text{ and } \xi_0 = \frac{M^2}{2\lambda^2}. \tag{1.24}$$

Cadoni further showed by conformally mapping (1.23) to the s-wave sector of the Schwarzschild metric:

$$g^{(2)}{}_{\mu\nu} = 2\phi g^{D}{}_{\mu\nu} \tag{1.25}$$

with

$$\lambda^2 = \frac{1}{G} \tag{1.26}$$

and

$$x = \frac{G}{r} \tag{1.27}$$

(1.24) and (1.22) reproduced the Bekenstein-Hawking Entropy for the respective 4-dimensional black hole. In other words, together with Robinson and Wilczek's results [23] both entropy and temperature of the Schwarzschild black hole induces some effective two dimensional semiclassical spacetime

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2. \tag{1.28}$$

Our goal will be to expound and extend this idea to include a greater class of rotating, charged and other types of black holes.

1.4 Kerr/CFT Correspondence

A recent study by Guica, Hartman, Song and Strominger [55] proposed that the near horizon geometry of an extremal Kerr black hole is holographically dual to a two dimensional chiral CFT[2] with non vanishing left central extension c_L. The

[2]Chiral in this part means that the generators of conformal symmetry split into two holomorphic sectors each with its own central extension.

Kerr geometry is given by the line element:

$$\begin{aligned} ds^2 &= g^{(4)}{}_{\mu\nu} dx^\mu dx^\nu \\ &= -\frac{\Delta(r)}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right)^2 + \frac{\rho^2}{\Delta(r)}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 \\ &+ \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left(adt - \frac{r^2+a^2}{\Xi}d\phi\right)^2, \end{aligned} \tag{1.29}$$

where

$$\begin{aligned} \Delta(r) &= \left(r^2+a^2\right) - 2GMr, \\ \Delta_\theta &= 1, \\ \rho^2 &= r^2 + a^2\cos^2\theta \text{ and} \\ \Xi &= 1 \end{aligned} \tag{1.30}$$

and the horizons are located at $r^{\pm} = GM \pm \sqrt{G^2M^2 - a^2}$. An extremal Kerr black hole is one for which $GM = a$, leaving one horizon located at $r^+ = GM$. By constructing the Frolov-Thorne vacuum for generic Kerr geometry [56], which reduces to the Hartle-Hawking vacuum [57] for $a \to 0$, GHSS obtain a non vanishing left Frolov-Thorne temperature T_L in the near horizon, extremal limit. The Hartle-Hawking vacuum is obtained by quantizing a scalar field in the Schwarzschild spacetime. This spacetime has a well defined time-like killing vector which allows for the definition of well behaved positive frequency base modes. A given quantum scalar will then be given by a standard Fourier decomposition of these base modes and their Hermitian conjugates, who's Fourier coefficients, $\hat{a}^\dagger$ and $\hat{a}$ act as creation and annihilation operators. The vacuum annihilated by $\hat{a}$ is called the Hartle-Hawking vacuum. The extension of this construction to a Kerr black hole is not obvious and is only well behaved near the horizon, which is all that is necessary for Kerr/CFT.

The temperature together with c_L inside the thermal Cardy formula [58] reproduces the Bekenstein-Hawking entropy for the extremal Kerr black hole:

$$S_{BH} = \frac{\pi^2}{3} c_L T_L. \tag{1.31}$$

This correspondence has been extended to various exotic black holes in string theory, higher dimensional theories and gauged supergravities to name a few [59, 60, 61, 62, 63, 64, 65, 66, 58, 67, 68].

One of the main arguments of the Kerr/CFT correspondence is to apply the rich ideas of holographic duality to more astrophysical objects/black-holes, such as the nearly extremal GRS 1915+105, a binary black hole system $11000pc$ away in Aquila [69]. In [55] the authors show that GRS 1915+105 is holographically dual to a two dimensional chiral CFT with $c_L = (2 \pm 1) \times 10^{79}$ and in the extremal limit the inner most stable circular orbit corresponds to the horizon. Thus, the authors conclude, any radiation emanating from the inner most circular orbit should be well described by the two dimensional chiral CFT, making the Kerr/CFT correspondence an essential theoretical tool in an astrophysical observation.

Despite the various models observationalists employ they all incorporate four main quantities: black hole mass M and spin J, poloidal magnetic field at the horizon B_0 and (Eddington) luminosity L for both supermassive [70] and stellar [71] black holes. This provides a new testable playing field for holography, i.e. to use some induced two dimensional CFT in the near horizon regime of extremal and non-extremal black holes to model the four main quantities in accordance with observation. In particular, the origin or mechanism of B_0 is unclear from a theoretical standpoint, since it must be due to an accreting disk for non gauged black holes. Yet, it might find its origin in some black-hole/CFT duality.

1.5 Goals and Outline of Our Work

The aim of this thesis is to apply the rich ideas of holography and effective action approache to the two dimensional near horizon theory of four dimensional black holes. The near horizon theories are derived via Robinson and Wilczek's dimensional reduction procedure and are referred to as two dimensional analogues (RW2DA). The salient new features of this this thesis are:

- The study of the conformal equivalence between solution spaces of two dimensional near horizon CFTs and their centers in order to compute the Bekenstein-Hawking entropies of more general black holes based on the well known dilaton gravity theory.
- The introduction of a new method for computing Hawking flux by studying the holomorphic behavior of the energy momentum tensor of conformal matter at a predefined metric boundary.
- The demonstration that the near horizon, a la Robinson and Wilczek, of a Kerr-Newman-AdS ($KNAdS$) metric is asymptotically AdS for a suitable choice of metric fall off conditions. Then using a covariant Lagrangian technique and a Liouville type action resulting from the s-channel of a minimally coupled scalar, the asymptotic quantum generator (charge) algebra is computed, which is a centrally extended Virasoro algebra. This central extension together with the lowest normalized eigen-mode reproduce the $KNAdS$ Bekenstein-Hawking entropy inside Cardy's formula.

The thesis is outlined as follows: Chapter 2 is devoted to the relevant back ground and motivational material necessary to understand the methodology used to obtain the main results. Readers with a working knowledge in general relativity, black hole physics, quantum field theory in curved spacetime and conformal field theory may skip this chapter, or refer to it when prompted. Chapter 3 contains the first part of the original research. The RW2DAs of a large class of black holes are analyzed and used to compute entropy and temperature. The conformal equivalence between two dimensional spacetimes are exploited in what is called the "Cadoni map". Chapter 4 contains the second part of the main results of this thesis, namely the asymptotic symmetry generators, both classical and quantum, of the RW2DA field theory for the general $KNAdS$ black hole and its coupling to conformal matter in order to obtain Hawking temperature. In Chapter 5 we discuss our results and possible future directions.

CHAPTER 2
BLACK HOLES AND QUANTUM FIELDS REDUX

In this chapter we will follow the conventions outlined in [72, 73, 74, 75, 76, 77] where the Einstein-Hilbert action with cosmological constant and matter sector, is given by the functional:

$$S_{EH} = \int d^4x\sqrt{-g}\left\{\frac{1}{16\pi G}\left(R - 2\Lambda\right) + \mathcal{L}_{matter}\right\}. \tag{2.1}$$

The Einstein field equation for the above functional, resulting from variation with respect to the inverse metric tensor is

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = 8\pi G T_{\mu\nu}, \tag{2.2}$$

where $T_{\mu\nu}$ is the energy momentum tensor of the matter sector in (2.1). Solutions to the Einstein field equation are given by the spacetime pair $(\mathcal{M}, g_{\mu\nu})$, where $\mathcal{M}$ is an n-dimensional Riemannian Manifold and $g_{\mu\nu}$ its metric tensor.

2.1 Black Holes

In this thesis, we will consider a black hole any region in spacetime, $(\mathcal{M}, g_{\mu\nu})$, that cannot be mapped to conformal infinity. This includes for the most part a curvature singularity hiding behind a killing horizon or coordinate singularity. As an illustration of the above definition let us examine the Schwarzschild metric in Kruskal coordinates

$$ds^2 = -(1 - 2GM/r)dudv + r^2 d\Omega^2 \tag{2.3}$$

where we employed the transformations $u = t - r*$,$v = t + r*$ and $r* = \int dr \frac{1}{1-\frac{2GM}{r}}$. We can conformally map (2.3) to Minkowski spacetime via the transformations

$$U = \tan \frac{u}{\sqrt{2GM}} \ \& \ V = \tan \frac{v}{\sqrt{2GM}} \tag{2.4}$$

$$T = V + U \ \& \ R = V - U \tag{2.5}$$

modulo a conformal factor $2 \cos U \cos V$. The transformations (2.4) rescale $\pm\infty$ to $\pm\frac{\pi}{2}$ and thus the entire Schwarzschild spacetime can be contained on a finite region of the $U - V$ plane as depicted in the conformal diagram of Figure 2.1 with labels defined in Table 2.1. In Figure 2.1 the region bounded by the lines $r = 2GM$ and

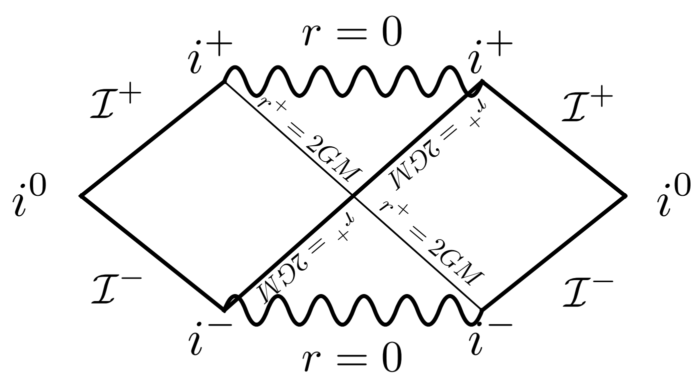

Figure 2.1: Conformal map of the Schwarzschild metric. The surface $r = 2GM$ is located at $U = V = 0$, the singularity at $r = 0$ is the line $U + V = \pm\frac{\pi}{2}$, and the points $(U, V) = (\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$ are the limits of $r \to \pm\infty$.

$r = 0$ is the black hole and time like curves emanating from here terminate at the horizon and never reach $\mathcal{I}^+$, thus a black hole is said to be present.

Label	Name	Definition
$\mathcal{I}^+$	Future Null ∞	$v = \infty$, $u = finite$
$\mathcal{I}^-$	Past Null ∞	$u = -\infty$, $v = finite$
i^o	Spatial ∞	$r = \infty$, $t = finite$
i^+	Future Time-like ∞	$t = \infty$, $r = finite$
i^-	Past Time-like ∞	$t = -\infty$, $r = finite$

Table 2.1: Labels for Fig. 2.1

2.2 Killing Vectors and Horizons

Given a Riemannian spacetime metric $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ a vector ξ is called Killing if

$$\mathcal{L}_\xi g_{\mu\nu} = 0, \tag{2.6}$$

where $\mathcal{L}_\xi$ denotes the Lie derivative along ξ. Writing (2.6) out in components yields the Killing equation:

$$\begin{aligned} \xi^\alpha \nabla_\alpha g_{\mu\nu} - g_{\alpha\nu}\nabla_\mu \xi^\alpha - g_{\mu\alpha}\nabla_\nu \xi^\alpha = 0 \\ \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0 \end{aligned}. \tag{2.7}$$

In most spacetimes considered there are two such vectors, a time-like and an angular-like, $\xi^\alpha_{(t)} \equiv t^\alpha = \frac{\partial x^\alpha}{\partial t}$ and $\xi^\alpha_{(\varphi)} \equiv \Omega_H \varphi^\alpha = \frac{\partial x^\alpha}{\partial \varphi}$. Combining these two gives a general Killing vector:

$$\xi^\alpha = a\xi^\alpha_{(t)} + b\xi^\alpha_{(\varphi)} \tag{2.8}$$

$$= t^\alpha + \Omega_H \varphi^\alpha, \tag{2.9}$$

where $\Omega_H = -\frac{g_{t\phi}}{g_{\phi\phi}}|_{r^+}$ is the angular velocity of the black hole at the horizon $r = r^+$. A Killing vector ξ also satisfies a non-homogeneous wave equation, which is obtained by its action on the Rieman tensor;

$$-R^{\mu}{}_{\nu\alpha\beta}\xi^{\nu} = (\nabla_\beta\nabla_\alpha - \nabla_\alpha\nabla_\beta)\xi^{\mu}, \tag{2.10}$$

which can be rearranged via the Jacobi Identity to yield

$$\nabla_\mu\nabla_\alpha\xi_\beta = -R_{\mu\nu\alpha\beta}\xi^{\nu} \tag{2.11}$$

and contracting over α and μ gives

$$\Box\xi_\beta = -R_{\beta\nu}\xi^{\nu}. \tag{2.12}$$

The isometries are manifested in the inner product between the four velocity and the Killing vectors, i.e.

$$u_\alpha\xi^\alpha = u_\alpha\xi^\alpha_{(t)} + u_\alpha\xi^\alpha_{(\varphi)} \tag{2.13}$$

$$= \tilde{E} + \tilde{L}, \tag{2.14}$$

where $\tilde{E}$ and $\tilde{L}$ are the conserved quantities energy and angular momentum. (2.6) also implies that ξ satisfies a geodesic and geodetic equation, i.e.

$$\begin{cases} (\nabla_\beta\xi^\alpha)\xi^\beta = 0 & r > r^+ \\ (\nabla_\beta\xi^\alpha)\xi^\beta = \kappa\xi^\alpha & r = r^+ \end{cases}. \tag{2.15}$$

Next, we will define the notion of a Killing horizon and its relevance in black hole physics. Let ξ be a Killing vector, then a Killing horizon is the surface defined by

$$\xi_\alpha\xi^\alpha = 0, \tag{2.16}$$

i.e. the surface on which the Killing vectors become null generators. A trivial example is the Killing vector $\xi = x\partial_t + t\partial_x$ in Minkowski space. In this case, ξ is null on the surface $x = \pm t$, thus defining a Killing horizon. Another example is the event horizon of a black hole, which we will denote by $\xi_\mu \xi^\mu|_{r^+} = 0$. It is important to note that every event horizon defines a Killing horizon, but not ever Killing horizon defines an event horizon. This is easily verified by the example in Minkowski space, which exhibits no coordinate or curvature singularities. To every Killing horizon we can associate a quantity κ called the surface gravity. It is the force required, by an observer at infinity, to hold a particle (of unit mass) stationary at the horizon[1]. Yet, before calculating κ explicitly we need to first make some general statements about the geometry of the horizon. Combining (2.15), and (2.16) we conclude that the Killing vectors define a congruence of null geodesics at the horizon, which is necessarily hypersurface orthogonal. Thus we may employ Frobenius' theorem [73], which states that for null generators ξ^α on a horizon the congruence is hypersurface orthogonal if and only if

$$(\nabla_{[\beta}\xi_\alpha)\xi_{\gamma]} = 0. \tag{2.17}$$

Expanding (2.17) and acting on it with $\nabla^\beta \xi^\alpha$ and making use of (2.7) and (2.15) yields

$$\begin{aligned}(\nabla^\beta \xi^\alpha)(\nabla_\beta \xi_\alpha)\xi_\gamma &= -2\kappa^2 \xi_\gamma \\ \Longrightarrow \kappa^2 &= -\frac{1}{2}(\nabla^\beta \xi^\alpha)(\nabla_\beta \xi_\alpha),\end{aligned} \tag{2.18}$$

which allows us to determine the surface gravity of any given spacetime ds^2 by extracting the Killing vectors thereof.

2.3 Surface Element for Null Generators

Let Σ be a null surface then we may write the directed surface element as

$$d\Sigma_\mu = -\xi_\mu \sqrt{h} d^3 y, \tag{2.19}$$

[1]We will only be concerned with the surface gravity of black holes.

where h_{ab} is the metric of the hypersurface (horizon) and $y^a = (\lambda, \theta^1, \theta^2)$ are its coordinates and ξ_μ acts as the normal vector. We are implying an embedding of the hypersurface such that for some auxiliary null vector N^{α}[2]. The metric is given by

$$g^{\alpha\beta} = -2\xi^{[\alpha}N^{\beta]} + h^{ab}e_a^\alpha e_b^\beta, \text{ completeness relation} \tag{2.20}$$

where

$$e_a^\alpha = \frac{\partial x^\alpha}{\partial y^a} \tag{2.21}$$

are the pull backs from the original spacetime to the embedding. Given (2.21) we can rewrite (2.19) as

$$d\Sigma_\mu = \xi^\nu dS_{\mu\nu} d\lambda, \tag{2.22}$$

where

$$dS_{\mu\nu} = 2\xi_{[\mu}N_{\nu]}\sqrt{h}d^2\theta. \tag{2.23}$$

We can draw a direct relation between (2.19) and (2.23) via Stokes theorem, which states that for any p-1 form ω

$$\int_{\partial\mathcal{M}} \omega = \int_{\mathcal{M}} d\omega. \tag{2.24}$$

A direct consequence of (2.24) for any antisymmetric contravariant two tensor is that

$$\int_\Sigma \left(\nabla_\beta B^{\alpha\beta}\right) d\Sigma_\alpha = \frac{1}{2}\oint_{\partial\Sigma} B^{\alpha\beta} dS_{\alpha\beta}. \tag{2.25}$$

(2.25) has the form of a Gauss theorem and it will come in handy when calculating mass and angular momentum transfer across a horizon.

[2] $N^2 = 0$, $N_\alpha\xi^\alpha = -1$

2.4 The Laws of Black Hole Mechanics

The laws of black hole mechanics where formulated by Bardeen, Carter and Hawking during the period from 1971-1973 [3], though Israel presented the first rigorous proof of the third law in 1986 [78]. In this section we work in units of $G = c = \hbar = 1$ and the laws are as follows.

Law 2.1. [Zerothe Law of Black Hole Mechanics] The zeroth law states that the surface gravity of a stationary black hole is constant cross the event horizon, i.e. we need to show that

$$\boxed{\partial_\mu \kappa = 0 \text{ and } (\partial_\mu \kappa)\, e^\mu{}_a = 0}, \tag{2.26}$$

where $e^\mu{}_a$ is the pull back from the 4-dimensional spacetime to the horizon. This my be established by differentiating (2.18) and employing (2.11) to yield:

$$\xi^\mu \partial_\mu \kappa = 0, \tag{2.27}$$

which shows the first equation in (2.26). The second equation can be determined by assuming geodesic completeness on the horizon and thus, since $(\partial_\mu \kappa)\, e^\mu{}_a = 0$ in the bifurcation two-sphere[3] of the spacetime, it holds across the entire horizon.

Law 2.2. [First Law of Black Hole Mechanics] Our starting point will be the Komar formulae [73]

$$\begin{aligned} M_{tot} &= \frac{-1}{8\pi} \oint_S (\nabla^\alpha \xi^\beta_{(t)}) dS_{\alpha\beta} \\ J_{tot} &= \frac{1}{16\pi} \oint_S (\nabla^\alpha \xi^\beta_{(\varphi)}) dS_{\alpha\beta} \end{aligned} \tag{2.28}$$

from which we will derive the Smarr formula and expressions for the mass and angular momentum transfer across the horizon. Taking the Komar formulae and

[3] $(u, v) = (0, 0)$ in the conformal diagram of Figure 2.1

applying them across the hrizon of a black hole we get

$$\begin{aligned} M_{BH} &= \frac{-1}{8\pi}\oint_{\mathcal{H}}(\nabla^{\alpha}t^{\beta})dS_{\alpha\beta} \\ J_{BH} &= \frac{1}{16\pi}\oint_{\mathcal{H}}(\nabla^{\alpha}\varphi^{\beta})dS_{\alpha\beta}, \end{aligned} \tag{2.29}$$

where BH stands for black hole and $\mathcal{H}$ denotes its horizon. Next, consider

$$\begin{aligned} M_{BH} - 2\Omega_H J_{BH} &= \frac{-1}{8\pi}\oint_{\mathcal{H}}(\nabla^{\alpha}t^{\beta})dS_{\alpha\beta} - \frac{1}{8\pi}\oint_{\mathcal{H}}(\nabla^{\alpha}\Omega_H)\varphi^{\beta}dS_{\alpha\beta} \\ &= \frac{-1}{8\pi}\oint_{\mathcal{H}}(\nabla^{\alpha}\xi^{\beta})dS_{\alpha\beta} \\ &= \frac{-1}{4\pi}\oint_{\mathcal{H}}(\nabla^{\alpha}\xi^{\beta})\xi_{\alpha}N_{\beta}dS \\ &= \frac{-1}{4\pi}\oint_{\mathcal{H}}\kappa\xi^{\beta}N_{\beta}dS \\ &= \frac{\kappa}{4\pi}A_{BH} \end{aligned}$$

thus arriving at the general Smarr formula

$$M_{BH} = \frac{\kappa}{4\pi}A_{BH} + 2\Omega_H J_{BH}. \tag{2.30}$$

next, considering the integral $\oint_S(\nabla^{\alpha}\xi^{\beta})dS_{\alpha\beta}$ and using (2.25) we have

$$\begin{aligned} \oint_S(\nabla^{\alpha}\xi^{\beta})dS_{\alpha\beta} &= 2\int_{\Sigma}\nabla_{\alpha}\nabla^{\alpha}\xi^{\beta}d\Sigma_{\beta} \\ &= 2\int_{\Sigma}R^{\alpha}{}_{\beta}\xi^{\beta}d\Sigma_{\alpha} \end{aligned} \tag{2.31}$$

and invoking the Einstein field equation[4] we have

$$\oint_S \nabla^{\alpha}\xi^{\beta}dS_{\alpha\beta} = -16\pi\int_{\Sigma}\left(T_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}T\right)n^{\alpha}\xi^{\beta}\sqrt{h}d^3y, \tag{2.32}$$

where n^{α} is the outward normal to the surface. Now we can now rewrite the Komar formulae as

$$M = 2\int_{\Sigma}\left(T_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}T\right)n^{\alpha}\xi^{\beta}_{(t)}\sqrt{h}d^3y \tag{2.33}$$

[4] $R_{\mu\nu} = 8\pi(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$

and

$$J = -\int_{\Sigma}\left(T_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}T\right)n^{\alpha}\xi^{\beta}_{(\varphi)}\sqrt{h}d^3y. \tag{2.34}$$

Next, we introduce the vector currents

$$\epsilon^{\alpha} = -T^{\alpha}_{\beta}\xi^{\beta}_{(t)} \tag{2.35}$$

and

$$l^{\alpha} = T^{\alpha}_{\beta}\xi^{\beta}_{(\varphi)}, \tag{2.36}$$

which interpret to the energy and angular momentum flux. In fact for a perfect fluid $T^{\alpha\beta} = \rho u^{\alpha}u^{\beta}$ they take the form $\epsilon^{\alpha} = \tilde{E}j^{\alpha}$ and $l^{\alpha} = \tilde{L}j^{\alpha}$, where j^{α} is the current density ρu^{α} and $\tilde{E}$ and $\tilde{L}$ are the conserved quantities (2.13). From energy conservation we know that j^{α} is divergenceless and thus we conclude that

$$\oint_{\partial V}\epsilon^{\alpha}d\Sigma_{\alpha} = 0 \text{ and } \oint_{\partial V}l^{\alpha}d\Sigma_{\alpha} = 0 \tag{2.37}$$

by Stokes theorem. (2.37) is the statement that the total energy transfer across a closed surface ∂V is conserved. If we take a partition H of this closed surface we have that the energy transfer through it is

$$\delta M = -\int_{\mathcal{H}} T^{\alpha}{}_{\beta}\,\xi^{\beta}_{(t)}d\Sigma_{\alpha} \text{ and } \delta J = \int_{\mathcal{H}} T^{\alpha}{}_{\beta}\,\xi^{\beta}_{(\varphi)}d\Sigma_{\alpha}. \tag{2.38}$$

Using (2.38) we can now derive the First Law of Black Hole Mechanics as follows: Given the linear combination $\delta M - \Omega_H\delta J$, we have

$$\begin{aligned}\delta M - \Omega_H\delta J &= -\int_{\mathcal{H}} T^{\alpha}{}_{\beta}\left(t^{\beta} + \Omega_H\varphi^{\beta}\right)d\Sigma_{\alpha}\\ &= \int_{\mathcal{H}} T_{\alpha\beta}\xi^{\alpha}\xi^{\beta}dSd\lambda.\end{aligned} \tag{2.39}$$

Next, we need to relate the integrand to the geometric evolution of the horizon to complete the integration over λ. This may be done via Raychaudhuri's equation $\frac{d\theta}{d\lambda} = \kappa\theta - 8\pi T_{\alpha\beta}\xi^{\alpha}\xi^{\beta}$, which is an evolution equation for the expansion parameter

θ of a two dimensional medium, i.e. θ is the fractional rate of change of the congruence's cross-sectional area ($\theta = \frac{1}{dS}\frac{ddS}{d\lambda}$ and $\theta|_{\lambda=\pm\infty} = 0$). Thus, substituting for $T_{\alpha\beta}\xi^\alpha\xi^\beta$ yields

$$\begin{aligned} \delta M - \Omega_H \delta J &= -\frac{1}{8\pi}\int d\lambda \oint_{\mathcal{H}} \left(\frac{d\theta}{d\lambda} - \kappa\theta\right) dS \\ &= \frac{\kappa}{8\pi}\int\oint_{\mathcal{H}} \theta dS d\lambda \\ &= \frac{\kappa}{8\pi}\int\oint_{\mathcal{H}} \frac{1}{dS}\frac{ddS}{d\lambda} dS d\lambda \\ &= \frac{\kappa}{8\pi}\oint_{\mathcal{H}} dS|_{\pm\infty} \\ &= \frac{\kappa}{8\pi}\delta A \end{aligned} \tag{2.40}$$

thus arriving at the First Law of Black Hole Mechanics:

$$\boxed{\delta M = \frac{\kappa}{8\pi}\delta A + \Omega_H \delta J}. \tag{2.41}$$

Law 2.3. [Second Law of Black Hole Mechanics] The second law was established by Hawking in 1971 [79], which states that the area of a black hole can never decrease:

$$\boxed{\delta A \geq 0}. \tag{2.42}$$

This follows directly from the focusing theorem and from the observation that the null generators' geodesics have no future endpoints in the given spacetime. The focusing theorem states that assuming the strong energy condition $R_{ab}k^a k^b \geq 0$, for null vector k^a, an initially negative expansion θ implies that the generators will converge in a caustic at $\theta = -\infty$. This is a contradiction to the initial observation and we conclude that $\theta \geq 0$.

Law 2.4. [Third Law of Black Hole Mechanics] The third law follows from the weak energy condition

$$T_{\mu\nu}u^\mu u^\nu > 0 \tag{2.43}$$

for an observer moving with four velocity u^α in a bounded energy momentum tensor $T_{\mu\nu}$. It states that the surface gravity of a black hole cannot be reduced to zero in a finite advanced time $v = t + r^*$. The third law may be illustrated by help of the general Vaidya spacetime [80] given by the line element:

$$ds^2 = -f dv^2 + 2dvdr + r^2 d\Omega^2, \tag{2.44}$$

where $f = 1 - \frac{2m(v)}{r} + \frac{q^2(v)}{r^2}$. This metric describes a spacetime in which the mass and charge vary with time due to some fictitious irradiating charged null dust with

$$T^{\mu\nu} = T^{\mu\nu}_{dust} + T^{\mu\nu}_{U(1)}, \tag{2.45}$$

where

$$\begin{cases} T^{\mu\nu}_{dust} = \rho l^\mu l^\nu & \rho = \frac{1}{4\pi r^2}\frac{\partial}{\partial v}\left(m - \frac{q^2}{2r}\right) \\ T^{\mu\nu}_{U(1)} = P\mathrm{diag}(-1,-1,1,1) & P = \frac{q^2}{8\pi r^4} \end{cases}. \tag{2.46}$$

For a charged spacetime the surface gravity vanishes in the case of extremality, i.e. $m(v_0) = q(v_0)$ for some advanced time $v_0 < \infty$. Assume an observer is restricted to moving along the radial direction then $T^{\mu\nu}u^\mu u^\nu = \rho\left(\frac{dv}{d\tau}\right)^2 + P$. Now since we require (2.43) this implies $\rho > 0$. In particular we have the relation on the horizon $r^+ = m\sqrt{m^2 - q^2}$:

$$4\pi\left(r^+\right)^3 \rho\left(r^+\right) = m\dot{m} - q\dot{q} + \sqrt{m^2 - q^2}\dot{m} > 0, \tag{2.47}$$

where dot means differentiation with respect to v. The above equation implies that

$$m(v_0)\dot{\Delta}(v_0) > 0, \tag{2.48}$$

where $\Delta = m - q$. This means that if we assume the black hole becomes extremal in some advanced time v_0 then Δ must be decreasing, i.e. there exists v_0 for which

$$\dot{\Delta}(v_0) < 0 \tag{2.49}$$

and Δ will become zero in a finite time, but this is a contradiction to (2.48). Thus the weak energy condition prevents the black hole from becoming extremal in a finite advanced time.

2.5 Black Hole Thermodynaimcs

The laws of black hole mechanics, Law 2.1, Law 2.2 and Law 2.3, bear a striking resemblance to the laws of thermodynamics[5] with $\kappa \sim$ temperature, $A \sim$ entropy and $M \sim$ internal energy. This duality was first noticed by Bekenstein [1] and solidified by Hawking [2] by examining quantum processes near black holes. The exact duality is outlined in Table 2.2. Classically, black holes where considered a

Thermodynamics	Black Hole
Temperature T	$\frac{\hbar}{2\pi}\kappa$
Entropy S	$\frac{A}{4\hbar G}$
Energy E	M
Thermo. Equilib.	$\kappa = constant$
1^{st} Law	1^{st} Law
$\delta S > 0$	$\delta A > 0$

Table 2.2: Black Hole Thermodynamic Analogy.

region in spacetime from which escape was impossible. Yet, by combining a general relativistic and quantum field theoretic description of the region just outside the event horizon, Hawking and his contemporaries demonstrated that black holes are not so black after all. Instead they behave as thermodynamic objects, where the

[5] In the special case of an isolated system in thermal equilibrium.

horizon area encodes information about the quantum spacetime, thus their relevance to the study of quantum gravity. Another interesting fact is that black hole entropy scales with the area instead of volume as compared to a traditional thermodynamic system. These interesting facts about black hole thermodynamics are summarized in the principle of holographic scaling:

Principle 2.1 (Holographic Scaling)**.**

- The entropy depends on area and not volume
- The horizon area encodes information at the quantum level
- Any viable quantum gravity candidate should reproduce the duality of Table 2.2

2.6 Canonical Quantum Fields in Curved Space

In this section we will restrict our analysis to a scalar field theory. We begin by first reviewing quantization in flat spacetime and then extend to a curved background. The Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{2}\left(\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + m^2\varphi^2\right) \tag{2.50}$$

with line element

$$\eta_{\mu\nu}dx^\mu dx^\nu = -dt^2 + d\vec{x}^2. \tag{2.51}$$

Varying the functional $\int d^4x\mathcal{L}$ with respect to φ yields the equation of motion:

$$\Box\varphi - m^2\varphi = 0. \tag{2.52}$$

A solution to this equation is given by the complex exponential

$$\varphi = \varphi_0 e^{ik_\mu x^\mu}, \tag{2.53}$$

where $k^{\mu} = (\omega, \vec{k})$. Substituting the solution (2.53) into (2.52) implies the relativistic dispersion:

$$\omega^2 = k^2 + m^2. \tag{2.54}$$

An orthonormal inner product is defined by the constant time integral

$$(\varphi_1, \varphi_2) = -i \int d^3x \left(\varphi_1 \partial_t \varphi_2^* - \varphi_2^* \partial_t \varphi_1\right), \tag{2.55}$$

which implies an orthonormal set of modes

$$f_k = \frac{e^{ik_\mu x^\mu}}{\sqrt{(2\pi)^3 2\omega}} \tag{2.56}$$

since

$$(f_{k_1}, f_{k_2}) = \delta^3 (k_1 - k_2). \tag{2.57}$$

Given the dispersion relation (2.54) we will take ω to always be a positive number and thus complement the set (2.56) with its complex conjugate, f_k^*. This strategy allows for the definition of both positive and negative frequency modes:

$$\begin{cases} \partial_t f_k = -i\omega f_k & \text{positive} \\ \partial_t f_k^* = i\omega f_k^* & \text{negative} \end{cases}, \tag{2.58}$$

with the following orthogonal relationships

$$(f_{k_1}, f_{k_2}) = \delta^3 (k_1 - k_2), \quad \left(f_{k_1}^*, f_{k_2}^*\right) = -\delta^3 (k_1 - k_2) \text{ and } \left(f_{k_1}, f_{k_2}^*\right) = 0. \tag{2.59}$$

$\{f_k, f_k^*\}$ form a complete set and any solution to (2.52) is $\in$ Span $(\{f_k, f_k^*\})$, i.e.

$$\varphi(t, x) = \int d^3k \left[a_k f_k(t, x) + a_k^\dagger f_k^*(t, x)\right]. \tag{2.60}$$

To canonically quantize this theory we promote the fields φ and $\pi = \frac{\partial \mathcal{L}}{\partial(\partial_0 \varphi)}$ to operators satisfying the equal time commutation relations:

$$\begin{aligned}[\varphi(t,x),\varphi(t,x')] =& 0\\ [\pi(t,x),\pi(t,x')] =& 0\\ [\varphi(t,x),\pi(t,x')] =& i\delta^3(x-x')\end{aligned} \tag{2.61}$$

these relations imply the standard creation and annihilation operator algebra

$$\begin{aligned}[\hat{a}_k,\hat{a}_{k'}] =& 0\\ [\hat{a}_k^\dagger,\hat{a}_{k'}^\dagger] =& 0\\ [\hat{a}_k,\hat{a}_{k'}^\dagger] =& \delta^3(k-k')\end{aligned} \tag{2.62}$$

with Fock state

$$|n_k\rangle = \frac{1}{\sqrt{n_k!}}\left(\hat{a}_k^\dagger\right)^{n_k}|0\rangle \tag{2.63}$$

and number operator

$$\hat{n}_k = \hat{a}_k^\dagger \hat{a}_k. \tag{2.64}$$

The above analysis may be repeated for scalar field in a curved background with Lagrangian density:

$$\mathcal{L} = -\frac{1}{2}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + m^2\varphi^2 + \xi R\varphi^2\right), \tag{2.65}$$

where ξ may have the following numerical couplings

$$\xi = \begin{cases} 0 & \text{minimal} \\ \frac{d-2}{4(d-1)} & \text{conformal} \end{cases}. \tag{2.66}$$

The field equation now takes the form

$$\Box\varphi - m^2\varphi - \xi R\varphi = 0 \tag{2.67}$$

and adhere to the modified orthogonal inner product:

$$(\varphi_1,\varphi_2) = -i\int_\Sigma \sqrt{\gamma}d^3x\left(\varphi_1\nabla_\mu\varphi_2^* - \varphi_2^*\nabla_\mu\varphi_1\right)n^\mu, \tag{2.68}$$

where ∇_μ is the covariant derivative compatible with $g_{\mu\nu}$, γ_{ij} is the metric of the hyper surface Σ obtained from $g_{\mu\nu}|_{t=0}$ and n^μ is its unit normal. Defining the canonical momentum as $\pi = \frac{\partial \mathcal{L}}{\partial(\nabla_0 \varphi)}$ we impose the canonical commutation relations

$$\begin{aligned}
[\varphi(t,x), \varphi(t,x')] =& 0 \\
[\pi(t,x), \pi(t,x')] =& 0 \\
[\varphi(t,x), \pi(t,x')] =& \frac{i}{\sqrt{-g}} \delta^3(x - x')
\end{aligned} \tag{2.69}$$

and assuming a complete orthonormal decomposition

$$\varphi = \sum_i \left(\hat{a}_i f_i + \hat{a}_i^\dagger f_i^* \right) \tag{2.70}$$

such that

$$(f_i, f_i) = \delta_{ij}, \;\; (f_i^*, f_i^*) = -\delta_{ij} \text{ and } \; (f_i, f_i^*) = 0 \tag{2.71}$$

arrive at the creation annihilation commutator algebra:

$$\begin{aligned}
[\hat{a}_i, \hat{a}_j] =& 0 \\
[\hat{a}_i^\dagger, \hat{a}_j^\dagger] =& 0 \\
[\hat{a}_i, \hat{a}_{j'}^\dagger] =& \delta_{ij}
\end{aligned} \tag{2.72}$$

with Fock state

$$|n_i\rangle = \frac{1}{\sqrt{n_i!}} \left(\hat{a}_i^\dagger \right)^{n_i} |0_f\rangle \tag{2.73}$$

and number operator

$$\hat{n}_{fi} = \hat{a}_i^\dagger \hat{a}_i. \tag{2.74}$$

We should note that the choice of basis $\{f_i, f_i^*\}$ is not unique and we could have instead chosen an alternative basis $\{g_i, g_i^*\}$ such that

$$\varphi = \sum_i \left(\hat{b}_i g_i + \hat{b}_i^\dagger g_i^* \right), \tag{2.75}$$

where $\hat{b}^\dagger$ and $\hat{b}$ are a new set of creation and annihilation operators satisfying:

$$\begin{aligned}[\hat{b}_i, \hat{b}_j] =&0\\ [\hat{b}_i^\dagger, \hat{b}_j^\dagger] =&0\\ [\hat{b}_i, \hat{b}_{j'}^\dagger] =&\delta_{ij}\end{aligned} \tag{2.76}$$

and new Fock state $|n_i\rangle = \frac{1}{\sqrt{n_i!}}\left(\hat{b}_i^\dagger\right)^{n_i}|0_g\rangle$. This fact begs the question what is the difference between the vacuum states $|0_f\rangle$ and $|0_g\rangle$? Or analogues, how do the excitations above $|0_f\rangle$ and $|0_g\rangle$ differ? This question may be addressed via the Bogolyubov transformations:

$$\begin{aligned}g_i =&\sum_j \left(\alpha_{ij} f_j + \beta_{ij} f_j^*\right)\\ f_i =&\sum_j \left(\alpha_{ij}^* g_j - \beta_{ij} g_j^*\right),\end{aligned} \tag{2.77}$$

where

$$\begin{aligned}\alpha_{ij} =&(g_i, f_i)\\ \beta_{ij} =&-(g_i, f_i^*)\end{aligned} \tag{2.78}$$

are called the Bogolyubov coefficients and satisfy

$$\sum_k \left(\alpha_{ik}\alpha_{jk}^* - \beta_{ik}\beta_{jk}^*\right) =\delta_{ij} \tag{2.79}$$

$$\sum_k \left(\alpha_{ik}\beta_{jk} - \beta_{ik}\alpha_{jk}\right) =0. \tag{2.80}$$

Given the above, the Bogolyubov coefficients may also be used to transform between the operators:

$$\begin{aligned}\hat{a}_i =&\sum_j \left(\alpha_{ij}\hat{b}_j + \beta_{ij}^*\hat{b}_j^\dagger\right)\\ \hat{b}_i =&\sum_j \left(\alpha_{ij}^*\hat{a}_j - \beta_{ij}^*\hat{a}_j^\dagger\right).\end{aligned} \tag{2.81}$$

Next, let us imagine a system with vacuum state $|0_f\rangle$, in which there are no excitations above the ground state, i.e. no particles are observed. We may now calculate the particle excitations as observed using the g-modes, in other words we evaluate the expectation value of $\hat{n}_{gi}$ with respect to the f-vacuum:

$$\begin{aligned}
\langle 0_f \left|\hat{n}_{gi}\right| 0_f\rangle &= \langle 0_f \left|\hat{b}_i^\dagger \hat{b}_i\right| 0_f\rangle \\
&= \left\langle 0_f \left|\sum_{jk} \left(\alpha_{ij}\hat{a}_j^\dagger - \beta_{ij}\hat{a}_j\right)\left(\alpha_{ik}^*\hat{a}_k - \beta_{ik}^*\hat{a}_k^\dagger\right)\right| 0_f \right\rangle \\
&= \sum_{jk} \beta_{ij}\beta_{ik}^* \left\langle 0_f \left|\hat{a}_j\hat{a}_k^\dagger\right| 0_f \right\rangle \\
&= \sum_{jk} \beta_{ij}\beta_{ik}^* \left\langle 0_f \left|\left(\hat{a}_k^\dagger\hat{a}_j + \delta_{jk}\right)\right| 0_f \right\rangle \\
&= \sum_{j} \left|\beta_{ij}\right|^2
\end{aligned} \quad . \tag{2.82}$$

This shows that the number of g-particles in the f-vacuum is nothing but the square of the beta sector of the Bogolyubov coefficients.

2.7 Effective Action in Curved Space

Given a generic theory S of fields φ we can always write a quantum theory as given by the path integral

$$Z = \int D\varphi e^{iS}. \tag{2.83}$$

We will restrict our analysis to the case when S is a two dimensional free scalar field. We define the effective field theory as the logarithm of the partition function, i.e.

$$\Gamma_{effective} = \ln Z. \tag{2.84}$$

There are many ways to determine the functional Γ with the heat kernel approach leading in popularity. In this approach the strategy is to first Wick rotate (2.83) from Lorentzian to Euclidean time, $t \to i\tau$ and expand the fields as in an orthogonal set of functions satisfying the eigen value problem:

$$\Box\varphi_n = \lambda_n\varphi_n, \tag{2.85}$$

where $\varphi = \sum_n c_n \varphi_n$. For the free two dimensional scalar this strategy has the advantage of reducing the functional of (2.83) to:

$$\int \prod_{n=0}^{\infty} \frac{dc_n}{\sqrt{2\pi}} e^{-\frac{1}{2}\lambda_n c_n^2} = \sqrt{\prod_{n=0}^{\infty} \lambda_n}. \tag{2.86}$$

This equates the computation of the effective action with computing the functional determinant of the differential operator $\Box_g$, i.e.

$$\Gamma = \frac{1}{2} \ln \det \Box_g. \tag{2.87}$$

At first glance the expressions in (2.86) and (2.87) are infinitely divergent due to the infinite spectrum of $\Box_g$ and a standard tool to regulate these divergencies is via analytic continuation of the zeta function. The zeta function for $\Box_g$ is defined as

$$\zeta_{\Box_g}(s) = \sum_{n=0}^{\infty} \left(\frac{1}{\lambda_n}\right)^s = \mathrm{Tr}\left(\Box_g^{-s}\right) \tag{2.88}$$

and satisfies the relation:

$$\frac{d\zeta_{\Box_g}(s)}{ds} = \frac{d}{ds} \sum_{n=0}^{\infty} e^{-s \ln \lambda_n} = -\sum_{n=0}^{\infty} e^{-s \ln \lambda_n} \ln \lambda_n. \tag{2.89}$$

Thus, the effective action is recast in terms of the zeta function as:

$$\Gamma = -\left.\frac{1}{2} \frac{d\zeta}{ds}\right|_{s=0}. \tag{2.90}$$

The heat kernel is defined as

$$\hat{K}(\tau) \equiv e^{-\Box_g \tau} \tag{2.91}$$

and relates to the the zeta function via the integral

$$\zeta(s) = \frac{1}{\Gamma(s)} \int d\tau \tau^{s-1} \mathrm{Tr} \hat{K}(\tau). \tag{2.92}$$

The operator $\hat{K}$ derives its name from the diffusion equation:

$$\frac{d}{d\tau} \left\langle x \left| \hat{K}(\tau) \right| x' \right\rangle = \frac{d^2}{dx^2} \left\langle x \left| \hat{K}(\tau) \right| x' \right\rangle. \tag{2.93}$$

The task of computing the effective action is now reduced to solving the PDE (2.93), then using its solution in (2.92) to determine the zeta function in terms of the parameter s and finally evaluating its derivative in (2.90). Applying this outline to the theory of (2.65) with $d = 2$ and $m = 0$ yields the Polyakov two dimensional gravitational action:

$$\Gamma_{grav} = \frac{1}{96\pi} \int d^2x \sqrt{-g} R \Box_g^{-1} R. \tag{2.94}$$

In the case of a massless charged scalar field, $\partial_\mu \to \partial_\mu - i\mathcal{A}_\mu$ the effective action will have obtain a gauge field sector:

$$\Gamma = \Gamma_{grav} + \Gamma_{U(1)}, \tag{2.95}$$

where

$$\Gamma_{grav} = \frac{1}{96\pi} \int d^2x \sqrt{-g^{(2)}} R^{(2)} \frac{1}{\Box_{g^{(2)}}} R^{(2)} \tag{2.96}$$

$$\Gamma_{U(1)} = \frac{e^2}{2\pi} \int F \frac{1}{\Box_{g^{(2)}}} F. \tag{2.97}$$

2.8 The Unruh Effect

The Unruh effect is basically the fact that an observer in Minkowski space at rest will disagree with the thermal spectrum around it as compared to an observer accelerating with uniform acceleration a. In other words an observer accelerating at constant acceleration a will observe a thermal bath of particles with temperature

$$T_U = \frac{a}{2\pi} \tag{2.98}$$

called the Unruh temperature [81]. To arrive at this conclusion we will start with the metric written in inertial coordinates

$$ds^2 = -dt^2 + dx^2 \tag{2.99}$$

and define coordinate adapted for uniform acceleration given by the transformations

$$t = \frac{1}{a} e^{a\xi} \sinh(a\eta) \tag{2.100}$$

$$x = \frac{1}{a} e^{a\xi} \cosh(a\eta), \tag{2.101}$$

in which the metric reads:

$$ds^2 = e^{2a\xi}\left(-d\eta^2 + d\xi^2\right). \tag{2.102}$$

The region where $-\infty < \eta$ and $\xi < \infty$ covering the wedge $x > |t|$ is called Rindler space, as depicted in by region I in Figure 2.2. As mentioned earlier the presence of

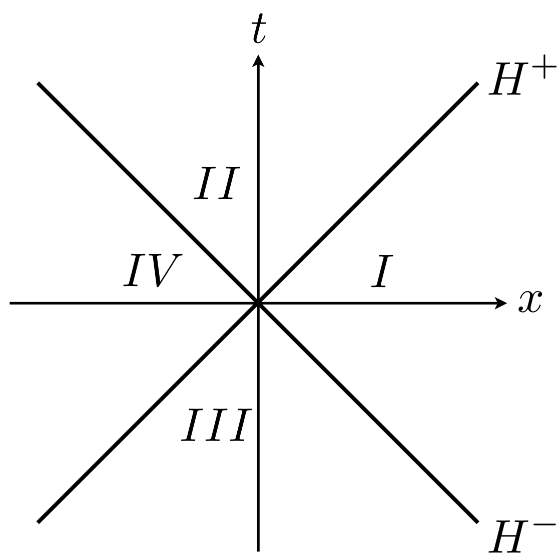

Figure 2.2: Rindler coordinates. The Region I is accessible to a positive constant accelerating observer. The coordinates (η, ξ) may be used in region IV as well with opposite orientation. $H^{\pm}$ corresponds to a Killing horizon of the symmetries generated by ∂_η, which are Lorentz boosts.

a Killing horizon does not necessarily imply a black hole, yet we may still compute the surface gravity following from (2.15), which gives

$$\kappa = a. \tag{2.103}$$

We will proceed to quantize the free scalar field in Rindler coordinates. The equation of motion (2.52) reads

$$e^{-2a\xi}\left(-\partial_\eta^2 + \partial_\xi^2\right)\varphi = 0 \tag{2.104}$$

with normalized eigen mode solution

$$\begin{aligned} g_k^{(1)} &= \begin{cases} \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega\eta + ik\xi} & I \\ 0 & IV \end{cases} \\ g_k^{(2)} &= \begin{cases} 0 & I \\ \frac{1}{\sqrt{4\pi\omega}} e^{i\omega\eta + ik\xi} & IV \end{cases}. \end{aligned} \tag{2.105}$$

The distinction between the two modes stems from the fact that the future time-like killing vectors of region I and IV differ by on overall minus sign and both modes will be positive frequency with respect to these isometries:

$$\begin{aligned} \partial_\eta g_k^{(1)} &= -\, i\omega g_k^{(1)} \\ \partial_{-\eta} g_k^{(2)} &= -\, i\omega g_k^{(2)} \end{aligned}. \tag{2.106}$$

Following the canonical quantization prescription outlined in Section 2.6 we obtain the quantum field

$$\varphi = \int dk \left(\hat{b}_k^{(1)} g_k^{(1)} + \hat{b}_k^{(1)\dagger} g_k^{(1)*} + \hat{b}_k^{(2)} g_k^{(2)} + \hat{b}_k^{(2)\dagger} g_k^{(2)*}\right) \tag{2.107}$$

analogous to (2.60). Using the modes (2.105) we define a set of alternate Rindler modes which are analytic and well defined along the entire surface $t = 0$ and thus share the same vacuum as Minkowski. These modes properly normalized are:

$$\begin{aligned} h_k^{(1)} =& \frac{1}{\sqrt{2\sinh\left(\frac{\pi\omega}{a}\right)}}\left(e^{\pi\omega/2a}g_k^{(1)} + e^{-\pi\omega/2a}g_{-k}^{(2)*}\right) \\ h_k^{(2)} =& \frac{1}{\sqrt{2\sinh\left(\frac{\pi\omega}{a}\right)}}\left(e^{\pi\omega/2a}g_k^{(2)} + e^{-\pi\omega/2a}g_{-k}^{(1)*}\right) \end{aligned} \tag{2.108}$$

and the quantum scalar field may now be expanded in terms of these new modes as

$$\varphi = \varphi = \int dk\left(\hat{c}_k^{(1)}h_k^{(1)} + \hat{c}_k^{(1)\dagger}h_k^{(1)*} + \hat{c}_k^{(2)}h_k^{(2)} + \hat{c}_k^{(2)\dagger}h_k^{(2)*}\right). \tag{2.109}$$

Equation (2.108) should be recognized as a Bogolyubov transformation and from Section 2.6 we have:

$$\begin{aligned} b_k^{(1)} =& \frac{1}{\sqrt{2\sinh\left(\frac{\pi\omega}{a}\right)}}\left(e^{\pi\omega/2a}\hat{c}_k^{(1)} + e^{-\pi\omega/2a}\hat{c}_{-k}^{(2)\dagger}\right) \\ b_k^{(2)} =& \frac{1}{\sqrt{2\sinh\left(\frac{\pi\omega}{a}\right)}}\left(e^{\pi\omega/2a}\hat{c}_k^{(2)} + e^{-\pi\omega/2a}\hat{c}_{-k}^{(1)\dagger}\right) \end{aligned}. \tag{2.110}$$

We know ask the question about particles observed in region I, i.e. we need to compute

$$\begin{aligned} \left\langle 0_M \left| \hat{n}_R^{(1)} \right| 0_M \right\rangle =& \left\langle 0_m \left| \hat{b}_k^{(1)\dagger}\hat{b}_k^{(1)} \right| 0_M \right\rangle \\ =& \frac{1}{2\sinh\frac{\pi\omega}{a}}\left\langle 0_M \left| e^{-\pi\omega/a}\hat{c}_{-k}^{(1)}\hat{c}_{-k}^{(1)\dagger} \right| 0_M \right\rangle \\ =& \frac{e^{-\pi\omega/a}}{2\sinh\frac{\pi\omega}{a}}\delta(0) \\ =& \frac{1}{e^{2\pi\omega/a}-1}\delta(0), \end{aligned} \tag{2.111}$$

where the $\delta(0)$ is due to the fact that our basis consists of plane waves and we used the fact that $c_{-k}^{(1)\dagger}\left| 0_M \right\rangle$ is a one particle normalized state. The result of (2.111) is a thermal Planck distribution with temperature:

$$T_U = \frac{a}{2\pi}, \tag{2.112}$$

thus arriving at the Unruh effect.

2.9 The Hawking Effect

In this section we will draw a direct analogy to the Unruh effect, by extending the above analysis to quantum fields in a black hole background. We have already seen how to quantize a scalar field in a curved background in Section 2.6 and Section 2.7. We will now apply these results directly to the $s-$wave sector of the Schwarzschild black hole of Section 2.1, given by the two dimensional metric:

$$g^{(2)}_{\mu\nu} = \begin{pmatrix} -\left(1-\frac{2GM}{r}\right) & 0 \\ 0 & \left(1-\frac{2GM}{r}\right)^{-1} \end{pmatrix}. \tag{2.113}$$

We will be considering the massless theory of (2.65) in this background. When restricting to two dimensions (2.65) is naturally conformally coupled and it will be to our interest to exploit this convenience via light cone and Kruskal coordinates. The light cone is defined via the transformations

$$u = t - r^* \text{ and } v = t + r^*, \tag{2.114}$$

where $\frac{dr^*}{dr} = \left(1-\frac{2GM}{r}\right)^{-1}$[6]. In these coordinates the metric takes the form:

$$ds^2 = \left(1-\frac{2GM}{r}\right) dudv. \tag{2.115}$$

The light cone has the advantage that it is naturally conformally flat and coincides with Minkowski at asymptotic infinity. Yet we will need another coordinate system to cover the region on the black hole horizon and its interior. The desired coordinate system is the Kruskal one given by the transformations:

$$U = -4Me^{-u/4M} \text{ and } V = -4Me^{-v/4M} \tag{2.116}$$

[6]We use (u, v) instead of $x^{\pm}$ in this section to be consistent with current literature on this subject.

and define the Kruscal light cone via

$$U = T - R \text{ and } V = T + R. \tag{2.117}$$

The metric in Kruskal reads

$$ds^2 = \frac{2GM}{r} e^{1-\frac{2GM}{r}} dU dV \tag{2.118}$$

and we see the Kruskal choice eliminates the coordinate singularity for the curvature one at $r = 0$ and thus we now have covered the entire spacetime.

A quantized light cone mode expansion is given by:

$$\varphi = \int \frac{d\Omega}{\sqrt{4\pi\Omega}} \left(e^{-i\Omega u} \hat{b}_\Omega + e^{i\Omega u} \hat{b}^\dagger_\Omega + e^{-i\Omega v} \hat{b}_{-\Omega} + e^{i\Omega v} \hat{b}^\dagger_{-\Omega} \right), \tag{2.119}$$

where the operators $\hat{b}_{\pm\Omega}$ correspond to excitations observed by a stationary observer a uniform distance from the horizon. This is directly analogous to a uniformly accelerating observer in Rindler space. Similarly the mode expansion in Kruskal coordinates reads:

$$\varphi = \int \frac{d\omega}{\sqrt{4\pi\omega}} \left(e^{-i\omega U} \hat{a}_\omega + e^{i\omega U} \hat{a}^\dagger_\omega + e^{-i\Omega V} \hat{a}_{-\omega} + e^{i\omega V} \hat{a}^\dagger_{-\omega} \right), \tag{2.120}$$

where $\hat{a}_{\pm\omega}$ correspond to excitations measured by an observer falling past the horizon into the black hole.

We see now that we have two different sets of creation and annihilation both annihilating the vacuums:

$$\hat{a}_{\pm\omega} \left|0_K\right\rangle = 0 \text{ and } \hat{b}_{\pm\Omega} \left|0_{lc}\right\rangle = 0 \tag{2.121}$$

and just as in Section 2.8 we can now compute the Bogolyubov coefficients and thus the particle spectrum of the black hole at some distance from the black hole. Yet, a direct analogy to the Unruh system, studied in Section 2.8, may be drawn due to the fact that the massless two dimensional scalar in Rindler space is conformally equivalent to its variant in both light cone and Krusakal coordinates. This analogy is detailed in Table 2.3. From this analogy we deduce that a calculation of the

Rindler	Schwarzschild
Stationary $\|0_M\rangle$	Free Fall $\|0_K\rangle$
Accelerated $\|0_R\rangle$	$r = const\ \|0_{lc}\rangle$
$\kappa = a$	$\kappa = \frac{1}{4GM}$
$U = -\frac{1}{a}e^{-au}$	$U = -4GMe^{-u/GM}$
$V = -\frac{1}{a}e^{-av}$	$V = -4GMe^{-v/GM}$

Table 2.3: Analogy between the Unruh and Hawking effect for conformally coupled massless two dimensional scalar field and Rindler space the light cone coordinates $u = \eta - \xi$ and $v = \eta + \xi$.

density of state will yield a thermal spectrum

$$n_\Omega = \frac{1}{e^{\frac{2\pi\Omega}{\kappa}} - 1} \tag{2.122}$$

thus arriving at the Hawking temperature of a Schwarzschild black hole:

$$T_H = \frac{1}{8\pi GM} \tag{2.123}$$

2.10 Hawking Effect and Energy Momentum

It was first shown by Christensen and Fulling [8] and expounded upon by others [9, 17, 82] that the quantum energy momentum tensor of conformal matter near a black hole, encodes quantum information about the respective spacetime. We will exploit and modify this fact in latter chapters of this thesis, but for now we will show how the Hawking temperature may be alternatively derived by analyzing the energy momentum tensor of the effective action (2.94). The energy momentum tensor is defined as

$$\langle T_{\mu\nu}\rangle = \frac{2}{\sqrt{-g}}\frac{\delta\Gamma}{\delta g^{\mu\nu}} \tag{2.124}$$

and in the case of the functional (2.94) we obtain:

$$\begin{aligned}\langle T_{\mu\nu}\rangle =& \frac{1}{48\pi}\left\{-2\nabla_\mu\nabla_\nu\left(\Box_g^{-1}R\right) + \nabla_\mu\left(\Box_g^{-1}R\right)\nabla_\nu\left(\Box_g^{-1}R\right)\right. && (2.125)\\ &\left.+g_{\mu\nu}\left[2R - \frac{1}{2}\nabla_\gamma\left(\Box_g^{-1}R\right)\nabla^\gamma\left(\Box_g^{-1}R\right)\right]\right\}. && (2.126)\end{aligned}$$

Choosing (2.113) as our spacetime we obtain

$$\langle T_{\mu\nu}\rangle = \begin{pmatrix} \frac{r^4\left(A^2+\alpha^2\right)-4G^2M^2}{96\pi r^2(r-2GM)^2} & \frac{Ar\alpha}{48\pi r-96GM\pi} \\ \frac{Ar\alpha}{48\pi r-96GM\pi} & \frac{\left(A^2+\alpha^2\right)r^4-16GMr+28G^2M^2}{96\pi r^4} \end{pmatrix}, \tag{2.127}$$

where A and α are integration constants left over from evaluating $\left(\Box_g^{-1}R\right)$. The integration constants may be determined by choosing Unruh vacuum boundary conditions (UBC) [81]:

$$\begin{cases}\langle T_{++}\rangle = 0 & r\to\infty \\ \langle T_{--}\rangle = 0 & r\to r_+\end{cases}, \tag{2.128}$$

where we have introduced light cone coordinates $x^{\pm} = t \pm r^*$. These conditions basically ensure that there is no ingoing flux at $r = \infty$ and a free falling observer should observe regular energy momentum at $r = r_+$. Applying the UBC yields:

$$\begin{aligned}A =& -\alpha\\ \alpha =& -\frac{1}{4GM}\end{aligned}. \tag{2.129}$$

Now we may analyze the $\langle T^r{}_t\rangle$ component after substitution of the integration constants. We find

$$\langle T^r{}_t\rangle = \frac{1}{768\pi G^2M^2}, \tag{2.130}$$

which is the energy flux of the energy momentum tensor and in this case known as the Hawking flux

$$HF = \frac{\pi}{12} T_H^2 \tag{2.131}$$

$$T_H = \frac{1}{8\pi GM}. \tag{2.132}$$

We now have an alternative method for computing black hole temperature via the path integral quantization and analyzing the quantum energy momentum tensor of the resulting effective action.

2.11 Generators of Conformal Symmetries

In this section we will focus only on certain features of a conformal field theory that will be used and recalled in some of the main calculations of the latter chapters. We will restrict ourselves to two dimensions and analyze the generator algebra and energy momentum tensor in this setting. A conformal field theory is one, who's action functional $S = \int d^2x \sqrt{-g}\mathcal{L}$ is invariant under a conformal transformation:

$$g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta} = \Omega g_{\mu\nu}. \tag{2.133}$$

Let us consider an infinitesimal coordinate transformation where $x'^\mu = x^\mu + \xi^\mu$, for some ξ such that $\xi^2 \ll 1$. We know from Section 2.2 how the metric varies infinitesimally and thus introduce the conformal Killing equation

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = \Lambda g_{\mu\nu}, \tag{2.134}$$

where $\Omega = 1 + \Lambda + \mathcal{O}(\xi^2)$. Tracing (2.134) yields the convient relation:

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = \nabla_\mu \xi^\mu g_{\mu\nu}. \tag{2.135}$$

For convenience, but without loss of generality, let us consider a flat spcetime[7] and the complex variables:

$$z = x^0 + ix^1 \qquad \xi = \xi^0 + i\xi^1$$

$$\bar{z} = x^0 - ix^1 \qquad \bar{\xi} = \xi^0 - i\xi^1.$$

Taking $\xi(z)$ to be holomorphic we see that $z \to f(z)$ is an infinitesimal conformal transformation, with the trivial example $f(z) = z + \xi(z)$.

Next, let us determine the generators of infinitesimal conformal transformations. Consider the holomorphic functions

$$z' = z + \sum_{n\in\mathbb{Z}} \xi_n \left(-z^{n+1}\right) \tag{2.136}$$

$$\bar{z}' = \bar{z} + \sum_{n\in\mathbb{Z}} \xi_n \left(-\bar{z}^{n+1}\right), \tag{2.137}$$

where we have performed a Laurent expansion about zero of $\xi(z)$. The generators corresponding to infinitesimal conformal transformations are

$$l_n = -z^{n+1}\partial_z \text{ and } \bar{l}_n = -\bar{z}^{n+1}\partial_{\bar{z}}. \tag{2.138}$$

Since n takes its values form the integers it becomes apparent that the generators form an infinite set, i.e. the algebra of conformal transformations in two dimensions is infinite dimensional. The generators satisfy the commutation relations:

$$\begin{aligned} [l_m, l_n] &= (m-n)l_{m+n} \\ [\bar{l}_m, \bar{l}_n] &= (m-n)\bar{l}_{m+n} \\ [l_m, \bar{l}_n] &= 0 \end{aligned} \tag{2.139}$$

known as one copy of a Witt algebra.

An interesting subset of the Witt algebra is the conformal group $SL(2,\mathbb{C})/\mathbb{Z}_2$ generated by $\{l_{-1}, l_0, l_1\}$. To see this we perform the change of variables $z = re^{i\phi}$

[7] In two dimensions any two Riemannian metrics are conformally equivalent and thus for a conformal field theory statements in Minkowski space will hold in general for any curved space.

and obtain the generators:

$$l_{-1} = - \partial_z \qquad \textit{translation} \tag{2.140}$$

$$l_0 + \bar{l}_0 = - r\partial_r \qquad \textit{dilatation} \tag{2.141}$$

$$i\left(l_0 - \bar{l}_0\right) = - \partial_\phi \qquad \textit{rotation} \tag{2.142}$$

$$l_1 = - z^2\partial_z \qquad \textit{modular.} \tag{2.143}$$

In other words $\{l_{-1}, l_0, l_1\}$ generate transformations of the form

$$z \to \frac{az+b}{cz+d} \tag{2.144}$$

this transformation is clearly invariant under a sign change and invertible if $(ad - bc) = 1$, thus arriving at $SL(2,\mathbb{C})/\mathbb{Z}_2$.

The Witt algebra does not take into account the possibility of a center. Mathematically any Lie algebra may be centrally extended as long as it vanishes with respect to the Jacobi identity:

$$[L_m, [L_n, L_r]] + [L_r, [L_m, L_n]] + [L_n, [L_r, L_m]] = 0. \tag{2.145}$$

In physical theories it is common to violate symmetry upon quantization, thus giving rise to anomalies. Mathematically this is the case when transitioning from a center less Witt algebra to a centrally extended one and the center is commonly referred to as the conformal anomaly. A center satisfying the condition (2.145) is given by

$$C_{mn} = \frac{c}{12} m\left(m^2 - 1\right)\delta_{m+n,0} \tag{2.146}$$

where c is called the central charge and gives rise to the Virasoro algebra:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12} m\left(m^2 - 1\right)\delta_{m+n,0}. \tag{2.147}$$

2.12 The Energy Momentum Tensor of a Conformal Field

Let us recall Noether's theorem, which states for every continuous symmetry in in a field theory there exists an associated conserver current such that $\nabla_\mu J^\mu = 0$. In the case of conformal symmetries we have the obvious current

$$J_\mu = T_{\mu\nu}\xi^\nu, \tag{2.148}$$

where $T_{\mu\nu}$ is defined as in (2.124). Taking the divergence of (2.148) and employing (2.135) gives the equation:

$$\Lambda T^\mu{}_\mu = 0, \tag{2.149}$$

which is the statement that the energy momentum tensor of a conformal field is always traceless. As mentioned in the previouse section upon quantization anomalies may apear. This was in fact the case when we quantized the free two dimensional scalar field and computed its quantum energy momentum tensor in Section 2.10. A quick calculation would show that the energy momentum tensor of the Polyakov action is not traceless, but satisfies:

$$\left\langle T^\mu{}_\mu \right\rangle = \frac{1}{24\pi} R. \tag{2.150}$$

This is known as the trace anomaly of a quantum conformal field theory and has the general form in two dimensions

$$\left\langle T^\mu{}_\mu \right\rangle = \frac{c}{24\pi} R, \tag{2.151}$$

which includes the central charge of the conformal anomaly.

Next, we may associate to each conserved current a conserved charge

$$Q = \int J_\mu dx^\mu \tag{2.152}$$

which is the generator of symmetry transformations of any field, i.e.

$$\delta_\xi \varphi = \{Q, \varphi\}\,. \tag{2.153}$$

This allows us to define the quantum symmetry generators of a quantum conformal field theory as:

$$\mathcal{Q}_n = \frac{1}{2\pi i}\int dz T(z) l_n(z), \tag{2.154}$$

where we have expressed the energy momentum tensor in complex coordinates and defined $T = T_{zz}$. $\mathcal{Q}_n$ is now the quantum generator of conformal symmetry and from our discussion at the end of Section 2.11 we should expect $\mathcal{Q}_n$ to satisfy a Virasoro algebra. This is in fact the case which may be verified by computing the blacket:

$$\delta_{l_n}\mathcal{Q}_m = [\mathcal{Q}_m, \mathcal{Q}_n]. \tag{2.155}$$

One way to compute this bracket is to introduce the operator product expansion (OPE)

$$T(z)T(w) = \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial_w T(w)}{(z-w)} + \cdots \tag{2.156}$$

and the contour $C(w)$ as depicted in Figure 2.3 which stems form Wilson's hy-

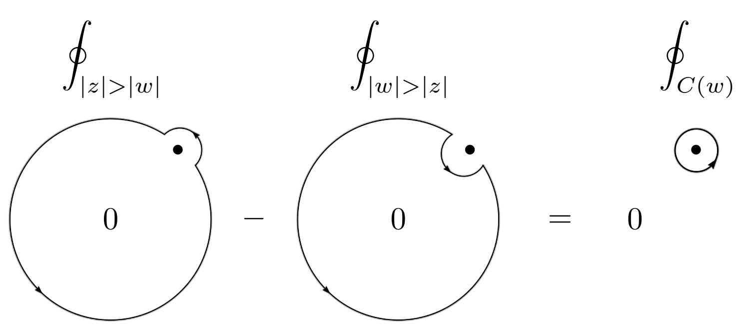

Figure 2.3: Appropriate contour for computing commutators of radial ordered operators.

pothesis on short distance expansions and the fact that any correlation function of a given quantum field theory only makes sense when defined as a time ordered product. Analogous, the product (2.156) only makes sense for $z > w$, eliminating the ambiguity wether w is inside or outside the contour, which is referred to as radial ordering and the radial ordering of $[,]$ is ensured by the contour relation of Figure 2.3, i.e.:

$$\oint dz[A(z), B(w)] = \oint_{|z|>|w|} A(z)B(w) - \oint_{|w|>|z|} B(w)A(z). \tag{2.157}$$

Upon application of these tools yields the Virasoro algebra:

$$[\mathcal{Q}_m, \mathcal{Q}_n] = (m-n)\mathcal{Q}_{m+n} + \frac{c}{12}m\left(m^2-1\right)\delta_{m+n,0}. \tag{2.158}$$

The OPE and contour $C(w)$ provide another benefit to analyze how the energy momentum tensor responds to an infinitesimal conformal transformation:

$$\begin{aligned} \delta_\xi T(z) =& \frac{1}{2\pi i}\oint_C (z)\xi(w)T(w)T(z) \\ =& \xi T' + 2T\xi' + \frac{c}{12}\xi''' \end{aligned}. \tag{2.159}$$

We see that in the case of a quantum conformal field theory the energy momentum tensor transforms as a quadratic differential form, as should, with the addition of an anomalous term $\frac{c}{12}\xi'''$[8].

2.13 The Eentropy of a Conformal Field

In this section we will present a heuristic derivation of the Cardy formula. For a micro canonical ensemble the relation ship between the entropy and density of states is given by

$$S = ln(\rho), \tag{2.160}$$

[8]The factor of $\frac{1}{12}$ is dependent upon the choice of conformal coordinates and normalization of $\mathcal{Q}$. For conformal light cone coordinates and unit normalization, the the factor is $\frac{1}{24\pi}$

where S is the entropy and ρ the density of states. This will be our starting point, and we will need to manipulate the partition function in order to yield ρ. This procedure applied to conformal field theories gives rise to Cardy's formula. Given a CFT who's symmetry is generated by a centrally extended Virasoro algebra, Cardy's basic result relies on the partition function

$$Z_0 = Tr e^{2\pi i (L_0 - \frac{c}{24})\tau} e^{-2\pi i (\bar{L}_0 - \frac{c}{24})\bar{\tau}} \tag{2.161}$$

beeing modular invariant, i.e. invariant under the transformation $\tau \to \frac{-1}{\tau}$. Applyed to the torus gives:

$$Z = Tr e^{2\pi i (L_0)\tau} e^{-2\pi i (\bar{L}_0)\bar{\tau}} \tag{2.162}$$

$$= \sum \rho(\Delta, \bar{\Delta}) e^{2\pi i (\Delta)\tau} e^{-2\pi i (\bar{\Delta})\bar{\tau}} \tag{2.163}$$

The above functional has a direct analogy to a unitary theory for Δ and $\bar{\Delta}$ the eigen values of L_0 and $\bar{L}_0$. This analogy is observed by inserting a complete set of states into the trace, allowing the extraction of ρ via contour integration and Fourier's trick. Assume τ is a complex variable, then

$$\rho(\Delta, \bar{\Delta}) = \frac{1}{(2\pi i)^2} \int \frac{dq}{q^{\Delta+1}} \frac{d\bar{q}}{\bar{q}^{\bar{\Delta}+1}} Z(q, \bar{q}) \tag{2.164}$$

where $q = e^{2\pi i \tau}$ and $\bar{q} = e^{-2\pi i \bar{\tau}}$. We will focus on the Δ integral and add the other contributions later. It is easily observed that $Z(\tau) = e^{\frac{2\pi i c \tau}{24}} Z_0(\tau)$ and employing the modular invariance yilds:

$$Z(\tau) = e^{\frac{2\pi i c \tau}{24}} Z_0\left(\frac{-1}{\tau}\right) = e^{\frac{2\pi i c \tau}{24}} e^{\frac{2\pi i c}{24\tau}} Z\left(\frac{-1}{\tau}\right) \tag{2.165}$$

and thus

$$\rho(\Delta) = \int d\tau e^{-2\pi i \Delta \tau} e^{\frac{2\pi i c \tau}{24}} e^{\frac{2\pi i c}{24\tau}} Z\left(\frac{-1}{\tau}\right). \tag{2.166}$$

This integral may be evaluated by means of a saddle point approximation. By assuming that $Z\left(\frac{-1}{\tau}\right)$ varies slowly near the extremum $\tau = i\sqrt{\frac{c}{24\Delta}}$, so that:

$$\begin{aligned}
\rho(\Delta) &\cong e^{2\pi\Delta\sqrt{\frac{c}{24\Delta}}} e^{\frac{-2\pi c}{24}\sqrt{\frac{c}{24\Delta}}} e^{\frac{2\pi c}{24}\left(\sqrt{\frac{c}{24\Delta}}\right)^{-1}} Z(i\infty) \\
&= e^{2\pi\sqrt{\frac{c\Delta}{6}}} Z(i\infty) \qquad (2.167) \\
\Rightarrow \rho(\Delta, \bar{\Delta}) &= e^{2\pi\left(\sqrt{\frac{c\Delta}{6}}+\sqrt{\frac{\bar{c}\bar{\Delta}}{6}}\right)}.
\end{aligned}$$

Nex, the entropy is given by the logarithm of the density of state, such that

$$S = 2\pi\left(\sqrt{\frac{c\Delta}{6}} + \sqrt{\frac{\bar{c}\bar{\Delta}}{6}}\right) \qquad \text{Cardy's Formula} \qquad (2.168)$$

arriving at the desired result.

2.14 *AdS*/CFT

In this section we will motivate a duality between a quantum gravity theory on a $d+1$-dimensional AdS bulk space and a d-dimensional CFT at the asymptotic boundary. This duality, known as the AdS/CFT correspondence and depicted in Figure 2.4, was first rigorously conjectured and formulated by Maldacena who showed that a type II B string theory on $AdS_5 \times S^5$ is dual to a $\mathcal{N} = 4$, $d = 4$ super-Yang-Mills theory [38]. To begin we will need to briefly review some aspects of quantum conformal field theory and AdS not perviously addressed.

As in any quantum field theory we may classify a field operator $\mathbb{O}_\Delta(x^\mu)$ via its transforms under the respective symmetry group. In the case of a conformal field theory the symmetry group is the conformal group in $d+1$ dimensions, $SO(d+1,1)$ and the field operator transforms under scaling as

$$\mathbb{O}_\Delta(x^\mu) \rightarrow \mathbb{O}_\Delta(\lambda x^\mu) = \lambda^{-\Delta}\mathbb{O}_\Delta(x^\mu), \qquad (2.169)$$

where Δ is called the scaling dimension. The Partition function is defined in the

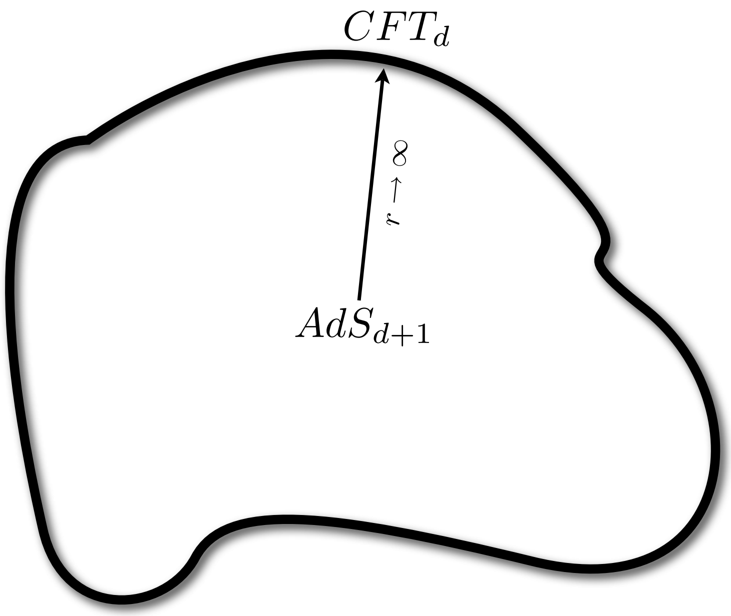

Figure 2.4: Cartoon depiction of the AdS/CFT correspondence.

usual sense

$$Z_{CFT}\left[\varphi_\Delta\right] = \left\langle e^{\int d^dx \varphi_\Delta(x)\mathbb{O}_\Delta(x)} \right\rangle, \tag{2.170}$$

where $\langle\rangle$ denotes path integration and $\varphi_\Delta(x)$ is a conformal field. Treating $\varphi_\Delta(x)$ as a source, we may employ functional methods to study the correlator:

$$\langle \mathbb{O}_{\Delta_1}\mathbb{O}_{\Delta_2}\ldots\rangle = \frac{\delta Z_{CFT}}{\delta\varphi_{\Delta_1}(x_1)\delta\varphi_{\Delta_2}(x_2)\ldots}, \tag{2.171}$$

which is conformally invariant if and only if the partition function is invariant under the scaling:

$$\varphi_\Delta(x) \to \lambda^{d-\Delta}\varphi_\Delta(\lambda x) \tag{2.172}$$

since

$$\begin{aligned}\int d^d x \varphi_\Delta(x) \mathbb{O}_\Delta(x) =& \int d^d(\lambda x) \varphi_\Delta(\lambda x) \mathbb{O}_\Delta(\lambda x) \\ =& \lambda^{d-\Delta} \int d^d x \varphi_\Delta(x) \mathbb{O}_\Delta(x).\end{aligned} \tag{2.173}$$

The duality between a quantum gravity and a quantum conformal field theory relies heavily on the fact that the isometry group of the respective spacetime is isomorphic to the conformal group. This makes AdS a natural choice since its isometries generate conformal transformations of the respective field content on the space. The metric for AdS with radius R is given by the line element:

$$ds^2 = \frac{R^2}{r^2} dr^2 + \frac{r^2}{R^2} \left(-dt^2 + dx^i dx_i\right), \tag{2.174}$$

where r is denoted as the radial coordinate and $r \to \infty$ is the asymptotic boundary and $r = 0$ may be considered a horizon.

The boundary $r \to \infty$ is particularly interesting, since the asymptotic conformal isometries generate conformal transformations of the (conformal) d-dimensional fields $\bar{\varphi} = \varphi(\infty, x)$. In other words the asymptotic AdS isometries generate a d-dimensional conformal group for boundary fields $\bar{\varphi}$. Such invariant boundary fields may be interpreted as sourcing an operator $\mathbb{O}_\Delta(x)$ of scaling dimension Δ:

$$Z\left[\bar{\varphi}\right] = \int_{\varphi = \bar{\varphi}} \mathcal{D}\varphi e^{-S[\varphi]} = \left\langle e^{\int d^d x \bar{\varphi}_\Delta(x) \mathbb{O}_\Delta(x)} \right\rangle, \tag{2.175}$$

where Z is conformally invariant provided φ behaves near the boundary as

$$\varphi(r, x) \sim \left(\frac{1}{r}\right)^{d-\Delta} \bar{\varphi} + \mathcal{O}\left(\frac{1}{r}\right)^{d-\Delta+1}, \tag{2.176}$$

which implies

$$\bar{\varphi}(x) \to \lambda^{d-\Delta} \bar{\varphi}(\lambda x) \tag{2.177}$$

and following from (2.172) we conclude Z must be a quantum conformal field theory. The above conclusion will hold in general irrespective of the field species and in the case of a free massless two dimensional scalar it is not to difficult to show that the scaling dimension satisfies:

$$\begin{aligned}\Delta =& \frac{d}{2} + \sqrt{\frac{d}{2} + m^2 R^2} \\ =& 2\end{aligned} \tag{2.178}$$

i.e. the conformal dimension must be equal to the spacetime dimension.

In Section 2.12 we have already seen that for a given CFT we may derive the local field $T^{\mu\nu}$ by variation of the CFT action functional via the massless symmetric spin two field $g_{\mu\nu}$. This means that the AdS theory must include a graviton in its field content, giving rise the the AdS/CFT correspondence: The partition function of a quantum gravity theory on an asymptotically $d+1$-dimensional AdS, as a function of the boundary values of its field content, is equivalent to the partition function of a d-dimensional CFT where the boundary fields source an operator $\mathbb{O}_\Delta(x)$ of scaling dimension Δ

$$Z_{grav}\left[\bar{\varphi}\right] = Z_{CFT}\left[\bar{\varphi}\right] = \left\langle e^{\int d^d x \bar{\varphi}_\Delta(x) \mathbb{O}_\Delta(x)} \right\rangle . \tag{2.179}$$

2.15 Gravity and 2-Dimensions

For a given two dimensional Riemannian-Levi-Cevita connection 2-form $\omega_{\alpha\beta}$, the Gauss curvature is simply:

$$d\omega_{12} = K \mathrm{vol}^2 \tag{2.180}$$

and relates to the Ricci scalar curvature as

$$K = \frac{1}{(2)((2)-1)} R^{(2)} . \tag{2.181}$$

This implies that the curvature of any Riemann-Surface is completely determined by its scalar variant and the Einstein equation is always trivially satisfied. Following a Cartan application to (2.180) and (2.181) it is not difficult to show in two dimensions that:

$$R^{(2)}_{\mu\nu} = \frac{1}{2} g_{\mu\nu} R^{(2)}. \tag{2.182}$$

The above equation may also be realized from a gravitational view point by tracing the Einstein equation, which implies that in two dimensions there exists no classical energy momentum configurations which can gravitate. Thus classically, in two dimensions, there are no general relativistic dynamics and any gravitational effects that are present must have quantum gravitational implication/origin. This is an interesting conclusion since the existence of two dimensional black holes has been well established, see [83] for a comprehensive review, and their associated theories usually contain additional field content beyond the metric such as dilatons or auxiliary scalar fields. In other words, the classical Einstein equations in two dimensions do not admit black hole solutions. Thus any two dimensional black hole must originate from some two dimensional effective quantum gravity theory such as string theory.

One example, dilaton gravity, is found by studying the s-wave sector of classical four dimensional gravity. Almost all salient features of black holes are encoded in their respective s-wave sectors, irrespective of the symmetries of the spacetime. To arrive at two dimensional dilaton gravity, we begin with the four dimensional metric ansatz

$$ds^2 = {g^{(2)}}_{\mu\nu} dx^\mu dx^\nu + \frac{1}{\lambda^2} e^{-2\varphi} d\Omega^2_{(2)} \tag{2.183}$$

and substituting into the Einstein-Hilbert action (2.1) and integrating out the angular degrees of freedom we are left with the theory:

$$S_{DG} = \frac{1}{2\pi} \int d^2x \sqrt{-g^{(2)}} e^{-2\varphi(r)} \left\{ R^{(2)} + 2 \left(\nabla\varphi\right)^2 + \lambda^2 e^{2\varphi} \right\}, \tag{2.184}$$

with a dimensionless coupling of $\frac{e^{-2\varphi(r_+)}}{2\pi} = \frac{4\pi r_+^2}{16\pi G}$ and $\lambda^2 = \frac{\pi}{2G}$. A black hole solution to this theory was discussed briefly in Section 1.3 and its relationship to conformal field theory and quantum gravity. Though the functional (2.184) is in general not a conformal field theory, we know from the c-Theorem [44, 84] that (2.184) must flow, under the renormalization group, to a quantum conformal field theory. The c-Theorem establishes the following relationships of the renormalization group of a two dimensional field theory with beta function β, coupling g and invariant under a one parameter group of transformations:

- There exists a function $c(g)$ such that $\left[\dot{c}(g) = \beta(g)\frac{\partial c}{\partial g}\right]_{g^*}$, where g^* is a fixed point of β.
- At a fixed point the two dimensional field theory has an infinite conformal symmetry.
- The value of c at the fixed point is the central extension of the generator algebra which is Virasoro.

and is proved in [84]. There exists strong evidence [17, 20, 44, 85, 86, 87], that in the near horizon this CFT takes the form of a Liouville Theory [88]

$$S_{Liouville} = \frac{c}{96\pi} \int d^2x \sqrt{-g^{(2)}} \left\{ -\Phi \Box_{g^{(2)}} \Phi + 2\Phi R^{(2)} \right\} \tag{2.185}$$

with effective dimensionless coupling proportional to:

$$\frac{A}{16\pi G}, \tag{2.186}$$

where the numerator originates from the dimensional reduction procedure and the denominator is a remnant of the parent classical gravitational theory (general relativity).

To see this, let us study the quantum theory of (2.184) in the conformal gauge

$$\begin{aligned} g_{+-} &= -\frac{1}{2}e^{-2\rho} \\ g_{++} &= g_{--} = 0 \end{aligned} \tag{2.187}$$

and from our discussion in Section 1.3 and Section 2.14 we know this gauge leaves unfixed a set of diffeomorphisms

$$\begin{aligned} \delta_{\xi^+} g_{++} &= 0 \\ \delta_{\xi^-} g_{--} &= 0, \end{aligned} \tag{2.188}$$

which generate a conformal group at asymptotic infinity and the corresponding quantum charges of $T_{\pm\pm}$ generate a Virasoro algebra with calculable central charge. This is a familiar setting and is well understood from the study of bosonic string theory with conformally invariant sigma model

$$S = -\frac{1}{2\pi}\int d^2x\sqrt{-\gamma}\left\{g_{\mu\nu}\nabla X^\mu \nabla X^\nu + \frac{1}{2}\Phi R_\gamma + T\right\}, \tag{2.189}$$

where γ_{mn} is a fiducial metric, $X^\mu = (\rho, \varphi)$ and the couplings $g_{\mu\nu}$, Φ and T[9] are functions of X^μ with known gravitational beta-functions to lowest order:

$$\beta^g_{\mu\nu} = 2\nabla_\mu \nabla_\nu \Phi + R_{\mu\nu} + \cdots. \tag{2.190}$$

From the above we see that conformal symmetry ($\beta^g_{\mu\nu} = 0$) severally restricts the possible quantum gravity theories in two dimensions. One obvious choice is the Liouville functional (2.185) for field redefinition $\Phi \to 2\Phi$ in (2.190).

A similar analysis for a large class of non-extremal weakly isolated horizons, including cosmological and of non-spherical spacetimes, by Chung [89, 90] showed

[9] We have restricted to the case $B_{\mu\nu} = 0$ since we are only interested in gravity at this stage.

by considering near-horizon Gauss Null Coordinates, given by the line element

$$ds^2 = \tilde{r}F(\tilde{r})du^2 + 2dud\tilde{r} + 2\tilde{r}h_i dudx^i + g_{ij}dx^i dx^j \tag{2.191}$$

which takes the form

$$ds^2 = 2g_{+-}\left(dx^+dx^- + h_{+i}dx^+dx^i\right) + g_{ij}dx^i dx^j \tag{2.192}$$

on the light cone,[10] that in the near horizon regime general relativity reduces to a two dimensional Liouville type conformal field theory to $\mathcal{O}(\tilde{r})$. This was done by considering diffeomorphisms $\xi^{\pm}$ preserving specific metric boundary conditions on the isolated horizon, then evaluating the Einstein-Hilbert Action for $g'_{\mu\nu} = g_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu}$ and integrating out the angular degrees of freedom. This near horizon theory again exhibited the same pre-factor $\frac{A}{16\pi G}$ and a center proportional to this coupling.

It may seem unclear why we would consider studying quantum gravity in two dimensions, especially since in four dimensions the problem is yet to be solved. Yet when restricting our analysis to the near horizon regime of black holes the tools from two dimensions become very useful, since in this isotropic region the only relevant degrees of freedom are contained in the $r-t$ plane. In other words we may extract four dimensional quantum black hole quantities from two dimensional quantum gravity in the near horizon regime. This will be the premise of the original research presented in this thesis.

[10]For the line element (2.191) the horizon is located at $\tilde{r} = 0$ and $x^{\pm}$ are defined in terms of $(u, \tilde{r})$.

CHAPTER 3
BLACK-HOLE/NEAR-HORIZON-CFT DUALITY AND THE CADONI MAP

In this chapter we present a method for computing black hole temperature and entropy from a near horizon quantum conformal field theory. The goal is to make contact with methods from both effective action approaches (Section 1.2 and Section 2.10), holographic duality (Section 2.11 and Section 2.15) and the conformal equivalence between spacetimes in two dimensions. We have addressed a large class of four dimensional black holes and the three dimensional BTZ as well. Yet it is not clear that the methods of this chapter apply to most general four dimensional static black hole of Kerr-Newman-AdS. This particular spacetime exhibits four complex horizon radii for which only one does not diverge in the Schwarzschild limit. This problem is addressed in Chapter 4 with an elegant solution. The work in this chapter is published in [17].

Motivated by Section 2.15 we will model the near horizon regime with a two dimensional Liouville type quantum field theory

$$S_{Liouville} = \frac{1}{96\pi}\int d^2x\sqrt{-g^{(2)}}\left\{-\Phi\Box_{g^{(2)}}\Phi + 2\Phi R^{(2)}\right\}. \tag{3.1}$$

We make this choice based on the fact that in this regime all mass and angular terms of (1.16) fall of exponentially fast upon transformation from $r \to r_*$, were $\frac{\partial r}{\partial r_*} = f(r)$ [23]. This leaves us with an infinite collection of two dimensional free scalars in spherically symmetric spacetime ${g^{(2)}}_{\mu\nu}$[1]. The effective action of each

[1] ${g^{(2)}}_{\mu\nu}$ may always be assumed spherically symmetric since any Riemannian Space in 2-dimensions is conformally flat.

partial wave is given by the Polyakov action of Section 2.7:

$$\Gamma_{Polyakov} = \frac{1}{96\pi}\int d^2x\sqrt{-g^{(2)}}R^{(2)}\frac{1}{\Box_{g^{(2)}}}R^{(2)} \tag{3.2}$$

and integrating out Φ in $S_{Liouville}$ yields $\Gamma_{Polyakov}$. In the case where the original four dimensional metric is not spherically symmetric [18, 19], a $U(1)$ gauge sector appears in addition to (3.2), which adds a gauge anomaly to Robinson and Wilczek's method for computing Hawking Radiation. In this chapter we will ignore this contribution since we are mainly focused on Hawking effects and address the gauge field sector in Chapter 4.

The energy momentum tensor for (3.1) was defined in Section 2.10:

$$\begin{aligned}\langle T_{\mu\nu}\rangle &= -\frac{2}{\sqrt{-g^{(2)}}}\frac{\delta S_{Liouville}}{\delta g^{(2)\mu\nu}} \\ &= -\frac{1}{48\pi}\left\{\partial_\mu\Phi\partial_\nu\Phi - \nabla_\mu\partial_\nu\Phi + {g^{(2)}}_{\mu\nu}\left[2R^{(2)} - \frac{1}{2}\nabla_\alpha\Phi\nabla^\alpha\Phi\right]\right\}\end{aligned} \tag{3.3}$$

and the equation of motion for the auxiliary scalar Φ is:

$$\Box_{g^{(2)}}\Phi = R^{(2)} \tag{3.4}$$

As an ansatz for the two dimensional metric ${g^{(2)}}_{\mu\nu}$, we choose the RW2DA a la Robinson and Wilczek of Section 1.2. Thus given a ${g^{(2)}}_{\mu\nu}$ we are free to solve (3.4) and (3.3) up to integration constants. The integration constants are addressed by adopting Unruh Vacuum boundary conditions [81] with a slight modification [17]

$$\begin{cases} T_{++} = 0 & r \to \infty,\ l \to \infty \\ T_{--} = 0 & r \to r_+ \end{cases} \tag{3.5}$$

where $x^{\pm} = t \pm r_*$ are light-cone coordinates, r_+ is the horizon radius defined as the largest real root of $f(r) = 0$ and l is the de Sitter radius. At the horizon and asymptotic infinity for $\left(\Lambda = \pm\frac{1}{l^2}\right) = 0$ and at the horizon only for $\Lambda \neq 0$, (3.3) will be dominated by one holomorphic component. This component equals the Hawking flux of the four and three dimensional black holes, which determines the Hawking temperature. In other words, at the boundary[2] of the RW2DA there exists a one dimensional quantum conformal field theory who's holomorphic energy momentum tensor contains the higher dimensional black hole's Hawking temperature.

The entropy will be determined by counting the horizon microstates of ${g^{(2)}}_{\mu\nu}$ via the Cardy formula (1.22). Following the outline proposed in [45] we construct a near horizon $Diff(S^1)$ or Witt subalgebra satisfying (1.19) based on the isometries of ${g^{(2)}}_{\mu\nu}$. In the horizon limit ($\mathcal{I}^+$ boundary) the Diff(S^1) subalgebra takes the form:

$$i2\left\{\xi^+_m, \xi^+_n\right\} = (m-n)\xi^+_{m+n}, \tag{3.6}$$

where the factor 2 comes from neglecting the asymptotic infinity limit ($\mathcal{I}^-$ boundary). On the $\mathcal{I}^+$ boundary the energy momentum tensor is holomorphic given by the $\langle T_{++}\rangle$ component. Next, we define the charge on the $\mathcal{I}^+$ boundary[3]

$$Q_n = \frac{3A}{\pi G}\int dx^+ \left\langle T_{++}\right\rangle \xi^+_n, \tag{3.7}$$

where A is the horizon area of the higher dimensional black hole. The coefficient on the integral of Q_n is chosen such that in the case when the higher dimensional black hole is Schwarzschild $Q_n = \frac{1}{16\pi G}\int dx^+ \frac{12}{\pi\left(T_H^2\right)^2}\left\langle T_{++}\right\rangle \xi^+_n$, where we have normalized the units of the energy momentum tensor. For a 1-dimensional CFT with holomorphic energy momentum tensor $T(z)$ we have from Section 2.12:

$$\delta_{\xi(z)}T(z) = \xi T' + 2T\xi' + \frac{k}{24\pi}\xi''', \tag{3.8}$$

where k is the central extension associated with the CFT and not with the microstates of ${g^{(2)}}_{\mu\nu}$. In the case for two dimensional quantum scalar $k = 1$ [91]. Thus, given the transformation (3.8) and compactifying the $\mathcal{I}^+$ boundary to a circle with period $(1/2 \cdot 1/T_H)$, where the 1/2 takes the $\mathcal{I}^-$ boundary into account, we

[2] Boundary is used loosely and refers to either $r = r_+$ or $r \to \infty$.
[3] Q_n is only conserved on the $\mathcal{I}^+$ boundary.

obtain the following charge algebra:

$$[Q_m, Q_n] = (m-n)Q_n + \frac{c}{12}m\left(m^2-1\right)\delta_{m+n,0}, \tag{3.9}$$

where c is the central extension associated with the microstates of ${g^{(2)}}_{\mu\nu}$. The Bekenstein-Hawking entropy of the four and three dimensional black holes is then given by Q_0 and c via (1.22).

Finally we compare our results to [46] by conformally mapping ${g^D}_{\mu\nu}$ to ${g^{(2)}}_{\mu\nu}$

$$ {g^{(2)}}_{\mu\nu} = 2\phi {g^D}_{\mu\nu} \tag{3.10}$$

for some conformal factor $2\phi = (\lambda x)^2$, where (1.24) is invariant under conformal transformations [13, 92]. This is what we will refer to as the Cadoni map.

We will now apply the method, outlined above, to various four and three dimensional black holes with zero and non zero cosmological constant and construct their Hawking flux, associated entropy and temperature.

3.1 Spherically Symmetric Solutions

In this class we will consider the Scwarzschild (SS) and Reissner-Nordström (RNS) black holes. Their two dimensional analogues have the form [93, 23]

$$ {g^{(2)}}_{\mu\nu} = \begin{pmatrix} -f(r) & 0 \\ 0 & \frac{1}{f(r)} \end{pmatrix}, \tag{3.11}$$

where

$$ f_{SS}(r) = 1 - \frac{2GM}{r} \tag{3.12}$$

and

$$ f_{RNS}(r) = 1 - \frac{2GM}{r} + \frac{Q^2G}{r^2} \tag{3.13}$$

Next, using the above ansatz and solving (3.4) we get:

$$\Phi_{SS} = C_2 t + C_1 r + \ln r - (1 - 2GMC_1)\ln(r - 2GM) + C_3 \tag{3.14}$$

and

$$\begin{aligned}\Phi_{RNS} =& C_2 t + C_1 r + \frac{C_1\sqrt{G}\,(2GM^2 - Q^2)}{\sqrt{GM^2 - Q^2}} \arctan\left(\frac{GM - r}{\sqrt{G^2M^2 - GQ^2}}\right) \\ &+ 2\ln r - (1 - GMC_1)\ln\left(r^2 - 2GMr + GQ^2\right) + C_3\end{aligned} \tag{3.15}$$

Using these auxiliary fields in (3.3) and transforming to light cone coordinates we obtain:

$$\begin{aligned}\left\langle T^{SS}_{++}\right\rangle &= \frac{-r^4(C_1 + C_2)^2 + 8rGM - 12G^2M^2}{192\pi r^4} \\ \left\langle T^{SS}_{--}\right\rangle &= \frac{-r^4(C_1 - C_2)^2 + 8rGM - 12G^2M^2}{192\pi r^4} \\ \left\langle T^{SS}_{+-}\right\rangle = \left\langle T^{SS}_{-+}\right\rangle &= \frac{GM(r - 2GM)}{24\pi r^4}\end{aligned} \tag{3.16}$$

and

$$\begin{aligned}\left\langle T^{RNS}_{++}\right\rangle =& \left[-r^6(C_1 + C_2)^2 + 8r^3GM - 12r^2G\left(GM^2 + Q^2\right)\right. \\ &\left.+24rG^2MQ^2 - 8G^2Q^4\right] / \left[192\pi a^6\right] \\ \left\langle T^{RNS}_{--}\right\rangle =& \left[-r^6(C_1 - C_2)^2 + 8r^3GM - 12r^2G\left(GM^2 + Q^2\right)\right. \\ &\left.+24rG^2MQ^2 - 8G^2Q^4\right] / \left[192\pi a^6\right] \\ \left\langle T^{RNS}_{+-}\right\rangle =& \left\langle T^{RNS}_{-+}\right\rangle = \frac{G\,(2rM - 3Q^2)\,(r^2 - 2rGM + GQ^2)}{48\pi r^6}\end{aligned} \tag{3.17}$$

The fact that both energy momentum tensors are not holomorphic/anti-homolomorphic signals the existence of a conformal anomaly taking the form

$$\left\langle T_\mu{}^\mu\right\rangle = -\frac{1}{24\pi}R^{(2)} \tag{3.18}$$

due to the trace anomaly [91]. Imposing (3.5) to eliminate C_1 and C_2 our final steps are to analyze $\left\langle T_{\mu\nu}\right\rangle$ at the horizon and construct the conformal map (3.10). At the horizon the energy momentum tensors are dominated by one holomorphic component $\langle T_{++}\rangle$ given by

$$\left\langle T_{++}^{SS}\right\rangle = \frac{1}{768\pi G^2 M^2} = \frac{\pi}{12}\left(T_H\right)^2 \tag{3.19}$$

and

$$\begin{aligned}\left\langle T_{++}^{RNS}\right\rangle =& \frac{G^2\left(GM^2-Q^2\right)\left(2M\sqrt{G\left(GM^2-Q^2\right)}+2GM^2-Q^2\right)}{48\pi\left(\sqrt{G\left(GM^2-Q^2\right)}+GM\right)^6} \\ =& \frac{\pi}{12}\left(T_H\right)^2,\end{aligned} \tag{3.20}$$

which are in agreement with Hawking's original results [2, 3].

Following [45], we compute the near horizon diffeomorphisms satisfying (1.19). We get:

$$\xi_n^{SS} = 4GMe^{in\kappa t}\partial_t + \frac{in(r-2GM)^2 e^{in\kappa t}}{GM}\partial_r \tag{3.21}$$

and

$$\begin{aligned}\xi_n^{RNS} =& \frac{\left(\sqrt{G\left(GM^2-Q^2\right)}+GM\right)^3 e^{i\kappa nt}}{G\left(M\sqrt{G\left(GM^2-Q^2\right)}+GM^2-Q^2\right)}\partial_t + \\ & \frac{ine^{i\kappa nt}\left(r^2-2rGM+GQ^2\right)^2}{rG\left(rM-Q^2\right)}\partial_r,\end{aligned} \tag{3.22}$$

where κ is the horizon surface gravity. Transforming to $x^{\pm}$ coordinates and taking the horizon limit, we obtain the $\mathcal{I}^+$ boundary charge algebras:

$$\left[Q_m, Q_n\right]_{SS} = (m-n)Q_n + 2GM^2 m\left(m^2-1\right)\delta_{m+n,0}, \tag{3.23}$$

and

$$\begin{aligned}\left[Q_m, Q_n\right]_{RNS} =& (m-n)Q_n + \\ & \left(M\sqrt{G\left(GM^2-Q^2\right)}+GM^2-\frac{Q^2}{2}\right) m\left(m^2-1\right)\delta_{m+n,0},\end{aligned} \tag{3.24}$$

which imply

$$Q_0^{SS} = GM^2 \tag{3.25}$$

$$c^{SS} = 24GM^2 \tag{3.26}$$

and

$$Q_0^{RNS} = \frac{\left(\sqrt{G\left(GM^2 - Q^2\right)} + GM\right)^2}{4G} \tag{3.27}$$

$$c^{RNS} = \frac{6\left(\sqrt{G^2M^2 - GQ^2} + GM\right)^2}{G} \tag{3.28}$$

and using these values in (1.22) we get:

$$S_{SS} = 4\pi GM^2 = \frac{A}{4G} \tag{3.29}$$

and

$$S_{RNS} = \frac{\pi\left(\sqrt{G\left(GM^2 - Q^2\right)} + GM\right)^2}{G} = \frac{A}{4G}. \tag{3.30}$$

Finally, conformally mapping ${g^{(2)}}_{\mu\nu}$ to ${g^{(D)}}_{\mu\nu}$ implies:

$$\lambda_{SS}^2 = \frac{1}{G} \tag{3.31}$$

$$x_{SS} = \frac{G}{r} \tag{3.32}$$

$$c_{SS} = 48GM^2 \tag{3.33}$$

$$\xi_0^{SS} = \frac{GM^2}{2} \tag{3.34}$$

as in [46] and

$$\lambda_{RNS}^2 = \frac{4GM^2}{\left(\sqrt{G^2M^2 - GQ^2} + GM\right)^2} \tag{3.35}$$

$$x_{RNS} = \frac{G\left(2rM - Q^2\right)}{2r^2M} \tag{3.36}$$

$$c_{RNS} = \frac{12\left(\sqrt{G^2M^2 - GQ^2} + GM\right)^2}{G} \tag{3.37}$$

$$\xi_0^{RNS} = \frac{\left(\sqrt{G^2M^2 - GQ^2} + GM\right)^2}{8G} \tag{3.38}$$

where λ_{RNS} and x_{RNS} are such that

$$\lim_{Q\to 0} \lambda_{RNS} = \lambda_{SS} \text{ and } \lim_{Q\to 0} x_{RNS} = x_{SS} \tag{3.39}$$

Using (1.22) we find the respective entropies:

$$S_{SS} = 4\pi GM^2 = \frac{A}{4G} \tag{3.40}$$

and

$$S_{RNS} = \frac{\pi\left(\sqrt{G\left(GM^2 - Q^2\right)} + GM\right)^2}{G} = \frac{A}{4G}. \tag{3.41}$$

We see that our central extension and zero-mode relate to Cadoni's via

$$c = \frac{c_c}{2} \tag{3.42}$$

and

$$Q_0 = 2\xi_0. \tag{3.43}$$

Yet, their respective products are equal and invariant under two dimensional conformal transformations and produce entropies in agreement with the Bekenstein-Hawking area law [1] for $\hbar = 1$. Thus, we may choose to conformally map into Cadoni's solution for all ${g^{(2)}}_{\mu\nu}$ for calculational simplicity.

We will proceed to solidify our main argument by applying the methods of

Section 1.5 to several more black hole solutions of various types.

3.2 Axisymmetric Solutions

For this class we analyze the Kerr (K) and Kerr-Newman (KN) Black holes with two dimensional analogues [18, 93]

$$g^{(2)}{}_{\mu\nu} = \begin{pmatrix} -f(r) & 0 \\ 0 & \frac{1}{f(r)} \end{pmatrix}, \tag{3.44}$$

where

$$f(r) = \frac{\Delta}{r^2 + J^2} \tag{3.45}$$

and

$$\Delta = \begin{cases} r^2 - 2rGM + J^2 & K \\ r^2 - 2rGM + GQ^2 + J^2 & KN \end{cases}. \tag{3.46}$$

The auxiliary scalars read:

$$\begin{aligned} \Phi_K =& rC_1 + tC_2 + (C_1GM - 1)\log\left(r^2 - 2rGM + J^2\right) + \log\left(r^2 + J^2\right) + \\ & \frac{2C_1G^2M^2\arctan\left(\frac{r-GM}{\sqrt{J^2-G^2M^2}}\right)}{\sqrt{J^2 - G^2M^2}} + C_3 \end{aligned} \tag{3.47}$$

and

$$\begin{aligned} \Phi_{KN} = rA + tC_2 + (C_1GM - 1)\log\left(r^2 - 2rGM + GQ^2 + J^2\right) + \\ \log\left(r^2 + J^2\right) + \frac{C_1G\left(2GM^2 - Q^2\right)\arctan\left(\frac{r-GM}{\sqrt{G(Q^2-GM^2)+J^2}}\right)}{\sqrt{G\left(Q^2 - GM^2\right) + J^2}} + C_3 \end{aligned} \tag{3.48}$$

from which we obtain the energy momentum tensors:

$$
\begin{aligned}
\left\langle T^K_{++}\right\rangle =&- \Big[r^8(C_1+C_2)^2+4r^6J^2(C_1+C_2)^2-8r^5GM+6r^4\left(C_1^2J^4+\right.\\
&\left.2C_1J^4C_2+2G^2M^2+J^4C_2^2\right)+16r^3GJ^2M+4r^2\left(C_1^2J^6+\right.\\
&\left.2C_1J^6C_2-10G^2J^2M^2+J^6C_2^2\right)+24rGJ^4M+C_2^2J^8+\\
&2C_1J^8C_2-4G^2J^4M^2+J^8C_2^2\Big]\Big/\left[192\pi\left(r^2+J^2\right)^4\right]\\
\left\langle T^K_{--}\right\rangle =&- \Big[r^8(C_1-C_2)^2+4r^6J^2(C_1-C_2)^2-8r^5GM+6r^4\left(C_1^2J^4-\right.\\
&\left.2C_1J^4\alpha+2G^2M^2+J^4\alpha^2\right)+16r^3GJ^2M+4r^2\left(C_1^2J^6-\right.\\
&\left.2C_1J^6C_2-10G^2J^2M^2+J^6C_2^2\right)+24rGJ^4M+C_1^2J^8-\\
&2C_1J^8C_2-4G^2J^4M^2+J^8C_2^2\Big]\Big/\left[192\pi\left(r^2+J^2\right)^4\right]\\
\left\langle T^K_{+-}\right\rangle =&\left\langle T^K_{-+}\right\rangle=\frac{rGM\left(r^2-3J^2\right)\left(r^2-2rGM+J^2\right)}{24\pi\left(r^2+J^2\right)^4}
\end{aligned}
\tag{3.49}
$$

and

$$\begin{aligned}
\langle T^{KN}_{++}\rangle =& -\left[r^8(C_1+C_2)^2+4r^6J^2(C_1+C_2)^2-8r^5GM+6r^4\left(C_1^2J^4+\right.\right.\\
&2C_1J^4C_2+2G^2M^2+2GQ^2+J^4C_2^2\Big)-8r^3GM\left(3GQ^2-\right.\\
&\left.2J^2\right)+4r^2\left(C_1^2J^6+2C_1J^6C_2+2G^2\left(Q^4-5J^2M^2\right)+\right.\\
&\left.2GJ^2Q^2+J^6C_2^2\right)+24rGJ^2M\left(GQ^2+J^2\right)+C_1^2J^8+\\
&\left.2C_1J^8C_2-4G^2J^4M^2-4G^2J^2Q^4-4GJ^4Q^2+J^8C_2^2\right]/\\
&\left[192\pi\left(r^2+J^2\right)^4\right]\\
\langle T^{KN}_{--}\rangle =& -\left[r^8(C_1-C_2)^2+4r^6J^2(C_1-C_2)^2-8r^5GM+6r^4\left(C_1^2J^4-\right.\right.\\
&2C_1J^4C_2+2G^2M^2+2GQ^2+J^4C_2^2\Big)-8r^3GM\left(3GQ^2-\right.\\
&\left.2J^2\right)+4r^2\left(C_1^2J^6-2C_1J^6C_2+2G^2\left(Q^4-5J^2M^2\right)+\right.\\
&\left.2GJ^2Q^2+J^6C_2^2\right)+24rGJ^2M\left(GQ^2+J^2\right)+C_1^2J^8-\\
&\left.2C_1J^8C_2-4G^2J^4M^2-4G^2J^2Q^4-4GJ^4Q^2+J^8C_2^2\right]/\\
&\left[192\pi\left(r^2+J^2\right)^4\right]\\
\langle T^{KN}_{+-}\rangle =& \langle T^{KN}_{-+}\rangle=\left[G\left(2r^3M-3r^2Q^2-6rJ^2M+J^2Q^2\right)\left(r^2-\right.\right.\\
&\left.\left.2rGM+GQ^2+J^2\right)\right]/\left[48\pi\left(r^2+J^2\right)^4\right]
\end{aligned} \tag{3.50}$$

which exhibits conformal anomaly (3.18). Applying (3.5) and taking the horizon limit we obtain the holomorphic pieces

$$\langle T^{K}_{++}\rangle = \frac{\pi\left(G^2M^2-J^2\right)}{12\left(4\pi GM\sqrt{G^2M^2-J^2}+4\pi G^2M^2\right)^2}=\frac{\pi}{12}\left(T_H\right)^2 \tag{3.51}$$

and

$$\begin{aligned}
\langle T^{KN}_{++}\rangle =& -\frac{G\left(Q^2-GM^2\right)+J^2}{48\pi\left(\left(\sqrt{G^2M^2-GQ^2-J^2}+GM\right)^2+J^2\right)^2}\\
=& \frac{\pi}{12}\left(T_H\right)^2
\end{aligned} \tag{3.52}$$

agreeing with Hawking's result [2, 3]. Next, from (3.10) and applying similar boundary conditions as in (3.39) we obtain

$$\lambda_K^2 = \frac{4GM^2}{\left(\sqrt{G^2M^2 - J^2} + GM\right)^2 + J^2} \tag{3.53}$$

$$x_K = \frac{aG}{a^2 + J^2} \tag{3.54}$$

$$c_K = \frac{12\left(\left(\sqrt{G^2M^2 - J^2} + GM\right)^2 + J^2\right)}{G} \tag{3.55}$$

$$\xi_0^K = \frac{\left(\sqrt{G^2M^2 - J^2} + GM\right)^2 + J^2}{8G} \tag{3.56}$$

and

$$\lambda_{KN}^2 = \frac{4GM^2}{\left(\sqrt{G^2M^2 - GQ^2 - J^2} + GM\right)^2 + J^2} \tag{3.57}$$

$$x_{KN} = \frac{2aGM - GQ^2}{2a^2M + 2J^2M} \tag{3.58}$$

$$c_{KN} = \frac{12\left(\left(\sqrt{G^2M^2 - GQ^2 - J^2} + GM\right)^2 + J^2\right)}{G} \tag{3.59}$$

$$\xi_0^{KN} = \frac{\left(\sqrt{G^2M^2 - GQ^2 - J^2} + GM\right)^2 + J^2}{8G} \tag{3.60}$$

which give the respective entropies:

$$S_K = 2\pi M\left(\sqrt{G^2M^2 - J^2} + GM\right) = \frac{A}{4G} \tag{3.61}$$

and

$$S_{KN} = \pi\left(2M\left(\sqrt{G\left(GM^2 - Q^2\right) - J^2} + GM\right) - Q^2\right) = \frac{A}{4G} \tag{3.62}$$

reproducing the Bekenstein-Hawking area law [1] and continuing the trend of Section 3.1.

3.3 Spherically Symmetric $SSdS$ and Rotating BTZ

Now, we turn our attention to black holes with non zero cosmological constant:

$$\Lambda = \begin{cases} \frac{1}{l^2} & dS \\ -\frac{1}{l^2} & AdS \end{cases}, \tag{3.63}$$

where l is the de Sitter radius. In this black hole class we consider the spherically symmetric dS ($SSdS$) with line element

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3}\right) dt^2 + \left(1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3}\right)^{-1} dr^2 \\ &+ r^2 d\Omega \end{aligned} \tag{3.64}$$

and the three dimensional BTZ black hole with line element

$$\begin{aligned} ds^2 = &-\left(-8GM + \frac{r^2}{l^2} + \frac{16GJ^2}{r^2}\right) dt^2 + \left(-8GM + \frac{r^2}{l^2} + \frac{16GJ^2}{r^2}\right)^{-1} dr^2 \\ &+ r^2\left(d\phi - \frac{4GJ}{r^2}dt\right)^2 . \end{aligned} \tag{3.65}$$

Their two dimensional analogues [36, 29] are as in (3.44) where

$$f(r) = \begin{cases} 1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3} & SSdS \\ -8GM + \frac{r^2}{l^2} + \frac{16GJ^2}{r^2} & BTZ \end{cases} \tag{3.66}$$

Following the steps outlined in Section 1.5 we obtain the energy momentum tensors:

$$\begin{aligned} \left\langle T^{SSdS}_{++}\right\rangle = &-\left[r^4\left(3C_1^2 + 6C_1C_2 + 3C_2^2 - 4\Lambda\right) + 24r^3GM\Lambda\right. \\ &\left.-24rGM + 36G^2M^2\right] / \left[576\pi r^4\right] \\ \left\langle T^{SSdS}_{--}\right\rangle = &\left[r^4\left(-3C_1^2 + 6C_1C_2 - 3C_2^2 + 4\Lambda\right) - 24r^3GM\Lambda\right. \\ &\left.+24rGM - 36G^2M^2\right] / \left[576\pi r^4\right] \\ \left\langle T^{SSdS}_{+-}\right\rangle = &\left\langle T^{SSdS}_{-+}\right\rangle = -\left[r^6\Lambda^2 - 3r^4\Lambda + 12r^3GM\Lambda - 18rGM\right. \\ &\left.+36G^2M^2\right] / \left[432\pi a^4\right] \end{aligned} \tag{3.67}$$

and

$$\begin{aligned}
\left\langle T_{++}^{BTZ}\right\rangle =& -\left[r^6\left(C_1^2l^2+2C_1l^2C_2-32GM+l^2C_2^2\right)+384r^4G^2J^2\right.\\
&\left.-1536r^2G^3J^2l^2M+2048G^4J^4l^2\right]/\left[192\pi r^6l^2\right]\\
\left\langle T_{--}^{BTZ}\right\rangle =& -\left[r^6\left(C_1^2l^2-2C_1l^2C_2-32GM+l^2C_2^2\right)+384r^4G^2J^2\right.\\
&\left.-1536r^2G^3J^2l^2M+2048G^4J^4l^2\right]/\left[192\pi r^6l^2\right]\\
\left\langle T_{+-}^{BTZ}\right\rangle =& \left\langle T_{-+}^{BTZ}\right\rangle = -\left[\left(r^4+48G^2J^2l^2\right)\left(r^4-8r^2Gl^2M\right.\right.\\
&\left.\left.+16G^2J^2l^2\right)\right]/\left[48\pi r^6l^4\right]
\end{aligned} \tag{3.68}$$

with conformal anomaly (3.18). Applying (3.5) we obtain the holomorphic piece

$$\langle T_{++}\rangle = \frac{\pi}{12}\left(T_H\right)^2 \tag{3.69}$$

for both spacetimes in their respective horizon limits and agreeing as before with Hawking's results [2, 3]. Next, their respective entropies are computed via (1.22), (3.10),

$$\begin{aligned}
\lambda^2_{SSdS} =& - \left[16GM^2\Lambda^2\left(\sqrt{\Lambda^3\left(9G^2M^2\Lambda-1\right)}-3GM\Lambda^2\right)^{2/3}\right] / \\
& \left[\left(\left(\sqrt{3}-i\right)\left(\sqrt{\Lambda^3\left(9G^2M^2\Lambda-1\right)}-3GM\Lambda^2\right)^{2/3}\right.\right. \\
& \left.\left. + \left(-\sqrt{3}-i\right)\Lambda\right)^2\right] && (3.70) \\
x_{SSdS} =& \frac{r^3\Lambda+6GM}{6rM} && (3.71) \\
c_{SSdS} =& - \left[3\left(\left(\sqrt{3}-i\right)\left(\sqrt{\Lambda^3\left(9G^2M^2\Lambda-1\right)}-3GM\Lambda^2\right)^{2/3}\right.\right. \\
& \left.\left. + \left(-\sqrt{3}-i\right)\Lambda\right)^2\right] / \left[G\Lambda^2\left(\sqrt{\Lambda^3\left(9G^2M^2\Lambda-1\right)}\right.\right. \\
& \left.\left. -3GM\Lambda^2\right)^{2/3}\right] && (3.72) \\
\xi_0^{SSdS} =& - \left[\left(\left(\sqrt{3}-i\right)\left(\sqrt{\Lambda^3\left(9G^2M^2\Lambda-1\right)}-3GM\Lambda^2\right)^{2/3}\right.\right. \\
& \left.\left. + \left(-\sqrt{3}-i\right)\Lambda\right)^2\right] / \left[32G\Lambda^2\left(\sqrt{\Lambda^3\left(9G^2M^2\Lambda-1\right)}\right.\right. \\
& \left.\left. -3GM\Lambda^2\right)^{2/3}\right] && (3.73)
\end{aligned}$$

and

$$\begin{aligned}
\lambda^2_{BTZ} =& \frac{4GM^2}{\sqrt{\sqrt{G^2l^2\left(l^2M^2-J^2\right)}+Gl^2M}} && (3.74) \\
x_{BTZ} =& \frac{\left(-r^4+r^2l^2-16G^2J^2l^2+8r^2Gl^2M\right)}{\left(2r^2l^2M\right)} && (3.75) \\
c_{BTZ} =& \frac{12\sqrt{\sqrt{G^2l^2\left(l^2M^2-J^2\right)}+Gl^2M}}{G} && (3.76) \\
\xi_0^{BTZ} =& \frac{\sqrt{\sqrt{G^2l^2\left(l^2M^2-J^2\right)}+Gl^2M}}{8G} && (3.77)
\end{aligned}$$

reproducing the Bekenstein-Hawking area law [1]

$$S = \frac{A}{4G} \tag{3.78}$$

in both cases via (1.22). Thus, we have shown that both entropy and temperature induce an effective two dimensional quantum gravity in the near horizon regime of four dimensional Schwarzschild, Reissner-Nordström, Kerr, Kerr-Newman and Spherically Symmetric dS and three dimensional BTZ black hole.

CHAPTER 4
BLACK-HOLE/NEAR-HORIZON-CFT DUALITY FROM AdS_2/CFT_1 CORRESPONDENCE AND KERR-NEWMAN-AdS

In this chapter we address the general classical Kerr-Newman-AdS black hole and its RW2DA analogue theory. Our goal here will be to show that the near horizon is asymptotically AdS_2 and then apply the AdS/CFT correspondence to compute its entropy. The Hawking temperature will be computed via the techniques of the previous chapter by computing the holomorphic energy momentum tensor of a quantum conformal field on the horizon. The work of this chapter is published in [82].

4.1 Near Horizon Geometry

The Kerr-Newman-AdS metric is a solution to the Einstein-Hilbert Action with negative cosmological constant coupled to a Maxwell field given by the line element [31, 94]:

$$\begin{aligned} ds^2 &= {g^{(4)}}_{\mu\nu} dx^\mu dx^\nu \\ &= -\frac{\Delta(r)}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right)^2 + \frac{\rho^2}{\Delta(r)}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 \\ &+ \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left(adt - \frac{r^2+a^2}{\Xi}d\phi\right)^2, \end{aligned} \tag{4.1}$$

where

$$\begin{aligned} \Delta(r) &= \left(r^2+a^2\right)\left(1+\frac{r^2}{l^2}\right) - 2GMr + GQ^2, \\ \Delta_\theta &= 1 - \frac{a^2}{l^2}\cos^2\theta, \\ \rho^2 &= r^2 + a^2\cos^2\theta \text{ and} \\ \Xi &= 1 - \frac{a^2}{l^2} \end{aligned} \tag{4.2}$$

and M is the mass, a is the angular momentum per unit mass, Q is the charge, G is Newton's constant and l the de Sitter radius. In general there are four horizon radii for which $\Delta(r)$ vanishes, but only two are physical. Of these two, only one, r_+, reduces to Kerr-Newman, Kerr, Reissner-Nördstrom and Schwarzschild black hole horizons in the appropriate limits. Thus, given we choose this respective horizon radius, any closed forms for entropy and temperature will hold in general for all sub-leading black holes in their respective limits.

The RW2DA is found by examining the functional

$$S^{(4)}[\varphi, g] = -\frac{1}{2}\int dx^4 \sqrt{-g}\nabla_\mu\varphi\nabla^\mu\varphi \tag{4.3}$$

in the regime where r is close to r_+. Expanding φ in terms of spherical harmonics, transforming to tortoise coordinates and integrating out the angular degrees of freedom, we obtain the near horizon theory [31, 95]:

$$S^{(4)}[\varphi, g] \xrightarrow{r\sim r_+} S^{(2)}[\varphi_{lm}, g^{(2)}] = \frac{(r_+^2 + a^2)}{2\Xi}\int dtdr\varphi^*_{lm}\left[\frac{1}{f(r)}\left(\partial t - i\mathcal{A}_t\right)^2 - \partial_r f(r)\partial_r\right]\varphi_{lm} \tag{4.4}$$

where

$$f(r) = \frac{\Delta(r)}{r^2 + a^2} \tag{4.5}$$

with RW2DA

$$g^{(2)}{}_{\mu\nu} = \begin{pmatrix} -f(r) & 0 \\ 0 & \frac{1}{f(r)} \end{pmatrix} \tag{4.6}$$

and a gauge field containing the contributions of the $U(1)$ charge and angular momentum

$$\mathcal{A}_t = -\frac{eQr}{r^2 + a^2} - \frac{\Xi am}{r^2 + a^2}. \tag{4.7}$$

The above dimensional reduction suggests that in the near horizon regime the

$KNAdS$ metric has the form:

$$ds^2 = g^{(2)}{}_{\mu\nu} dx^\mu dx^\nu + B(\varphi)\left[d\phi - \mathcal{A}_t dt\right]^2 + C(\varphi)\, d\theta^2 \tag{4.8}$$

assuming we consider φ as a component of the gravitational field. Motivated by the approaches in Section 2.15, we will consider a slight modification and elevating φ_{lm} to a gravitational field via the field redefinition

$$\varphi_{lm} = \sqrt{\frac{6}{G}}\psi_{lm}, \tag{4.9}$$

where ψ_{lm} is now unit less and the $\sqrt{6}$ was chosen to recover the Einstein coupling $\frac{1}{16\pi G}$ in the quantum gravitational effective action of (4.4) within the s-wave approximation.

4.2 Effective Action and Asymptotic Symmetries

Applying the field redefinition (4.9) to (4.4) yields:

$$S^{(2)}[\psi, g] = \frac{3(r_+^2 + a^2)}{G\Xi} \int d^2x \sqrt{-g^{(2)}} \psi^*_{lm} \left[D_\mu \left(\sqrt{-g} g_{(2)}{}^{\mu\nu} D_\nu\right)\right] \psi_{lm}, \tag{4.10}$$

where D_μ is the gauge covariant derivative. The effective action of each partial wave is given by the sum of two functionals [18, 96],

$$\Gamma_{(lm)} = \Gamma_{grav} + \Gamma_{U(1)}, \tag{4.11}$$

where

$$\begin{aligned} \Gamma_{grav} =& \frac{(r_+^2 + a^2)}{16\pi G\Xi} \int d^2x \sqrt{-g^{(2)}} R^{(2)} \frac{1}{\Box_{g^{(2)}}} R^{(2)} \text{ and} \\ \Gamma_{U(1)} =& \frac{3e^2(r_+^2 + a^2)}{\pi G\Xi} \int F \frac{1}{\Box_{g^{(2)}}} F. \end{aligned} \tag{4.12}$$

We will discuss the s-wave contribution of (4.11) shortly and instead turn our attention to computing the ASG of (4.10). The asymptotic or large r behavior of

(4.6) and (4.7) are given by

$$g^{(0)}{}_{\mu\nu} = \begin{pmatrix} -\frac{r^2}{l^2} - 1 + \frac{2GM}{r} - \frac{GQ^2}{r^2} + \mathcal{O}\left(\left(\frac{1}{r}\right)^3\right) & 0 \\ 0 & \frac{l^2}{r^2} + \mathcal{O}\left(\left(\frac{1}{r}\right)^3\right) \end{pmatrix}, \tag{4.13}$$

$$\mathcal{A}^{(0)}{}_t = \frac{eQ^2}{r^2} + \mathcal{O}\left(\left(\frac{1}{r}\right)^3\right) \tag{4.14}$$

and define an asymptotically AdS_2 configuration with Ricci Scalar, $R = -\frac{2}{l^2} + O\left(\left(\frac{1}{r}\right)^1\right)$. We also impose the following metric and gauge field boundary or fall-off conditions:

$$\delta g_{\mu\nu} = \begin{pmatrix} \mathcal{O}(r) & \mathcal{O}\left(\left(\frac{1}{r}\right)^0\right) \\ \mathcal{O}\left(\left(\frac{1}{r}\right)^0\right) & \mathcal{O}\left(\left(\frac{1}{r}\right)^3\right) \end{pmatrix} \text{ and } \delta\mathcal{A} = \mathcal{O}\left(\left(\frac{1}{r}\right)^3\right) \tag{4.15}$$

A set of diffeomorphisms preserving the asymptotic metric structure is given by

$$\xi_n = \xi_1(r)\frac{e^{in\kappa(t\pm r^*)}}{\kappa}\partial_t + \xi_2(r)\frac{e^{in\kappa(t\pm r^*)}}{\kappa}\partial_r, \tag{4.16}$$

where r^* is the tortoise coordinate defined by $dr^* = \frac{1}{f(r)}dr$,

$$\xi_1 = \frac{iAr^4 e^{in\kappa r^*}}{n\kappa\left(-2Gl^2Mr + Gl^2Q^2 + l^2r^2 + r^4\right)},\ \xi_2 = Are^{in\kappa r^*}, \tag{4.17}$$

A is an arbitrary normalization constant and κ is the surface gravity of the KNAdS black hole. Applying this set of diffeomorphisms to the gauge field we find

$$\delta_\xi \mathcal{A}_\mu = \left(-\frac{3\left(eQ^2ne^{int\kappa}\right)}{r^2} + \mathcal{O}\left(\left(\frac{1}{r}\right)^3\right), \mathcal{O}\left(\left(\frac{1}{r}\right)^4\right)\right). \tag{4.18}$$

Thus to satisfy all the imposed fall of conditions we must consider total symmetries of the action, which implies

$$\delta_\xi \to \delta_{\xi+\Lambda}, \tag{4.19}$$

where

$$\Lambda = -\frac{3ieQ^2e^{int\kappa}}{r^2\kappa}. \tag{4.20}$$

Evaluating the gauge field under this total symmetry we find

$$\delta_{\xi+\Lambda}\mathcal{A} = \mathcal{O}\left(\left(\frac{1}{r}\right)^3\right) \tag{4.21}$$

in accordance with (4.15). Finally switching to light cone coordinates $x^{\pm} = t \pm r^*$,[1] the set $\xi_n^{\pm}$ is well behaved on the $r \to \infty$ boundary and obey the centerless Virasoro or $Diff(S^1)$ subalgebra

$$i\left\{\xi_m^{\pm}, \xi_n^{\pm}\right\} = (m-n)\xi_{m+n}^{\pm}. \tag{4.22}$$

Evaluating the wave equation

$$D_\mu\left(\sqrt{-g}{g_{(2)}}^{\mu\nu}D_\nu\right)\psi_{lm} = 0 \tag{4.23}$$

in this asymptotic behavior we find a product solution for ψ_{lm}, which is complex hypergeometrical in r, but decays exponentially fast in t for higher orders in m. Thus we only consider the s-wave contribution to (4.11), $\Gamma_{00} = \Gamma$, leaving us with a near horizon effective action:

$$\begin{aligned}\Gamma =& \frac{(r_+^2+a^2)}{16\pi G\Xi}\int d^2x\sqrt{-g^{(2)}}R^{(2)}\frac{1}{\Box_{g^{(2)}}}R^{(2)} \\ &+ \frac{3e^2(r_+^2+a^2)}{\pi G\Xi}\int F\frac{1}{\Box_{g^{(2)}}}F.\end{aligned} \tag{4.24}$$

The above functional may be recast in the familiar form of a Liouville type CFT by introducing auxiliary scalars Φ and B satisfying

$$\Box_{g^{(2)}}\Phi = R \text{ and } \Box_{g^{(2)}}B = \epsilon^{\mu\nu}\partial_\mu A_\nu. \tag{4.25}$$

[1]Large r behavior will be synonymous with large x^+ behavior.

In terms of these new fields our near horizon CFT takes its final form:

$$\begin{aligned} S_{NHCFT} =& \frac{(r_+^2+a^2)}{16\pi G\Xi}\int d^2x\sqrt{-g^{(2)}}\left\{-\Phi\Box_{g^{(2)}}\Phi + 2\Phi R^{(2)}\right\} \\ &+ \frac{3e^2(r_+^2+a^2)}{\pi G\Xi}\int d^2x\sqrt{-g^{(2)}}\left\{-B\Box_{g^{(2)}}B \right. \\ &\left. + 2B\left(\frac{\epsilon^{\mu\nu}}{\sqrt{-g^{(2)}}}\right)\partial_\mu A_\nu\right\} \end{aligned} \tag{4.26}$$

4.3 Energy Momentum and The Virasoro algebra

The energy momentum tensor and $U(1)$ current of (4.26) are defined as:

$$\begin{aligned} \langle T_{\mu\nu}\rangle =& \frac{2}{\sqrt{-g^{(2)}}}\frac{\delta S_{NHCFT}}{\delta g^{(2)\mu\nu}} \\ =& \frac{r_+^2+a^2}{8\pi G\Xi}\left\{\partial_\mu\Phi\partial_\nu\Phi - 2\nabla_\mu\partial_\nu\Phi + {g^{(2)}}_{\mu\nu}\left[2R^{(2)} - \frac{1}{2}\nabla_\alpha\Phi\nabla^\alpha\Phi\right]\right\} \\ &+ \frac{6e^2(r_+^2+a^2)}{\pi G\Xi}\left\{\partial_\mu B\partial_\nu B - \frac{1}{2}g_{\mu\nu}\partial_\alpha B\partial^\alpha B\right\} \text{ and} \\ \langle J^\mu\rangle =& \frac{1}{\sqrt{-g^{(2)}}}\frac{\delta S_{NHCFT}}{\delta \mathcal{A}_\mu} = \frac{6e^2(r_+^2+a^2)}{\pi G\Xi}\frac{1}{\sqrt{-g^{(2)}}}\epsilon^{\mu\nu}\partial_\nu B \end{aligned} \tag{4.27}$$

and the equation of motions for the auxiliary fields are:

$$\begin{aligned} \Box_{g^{(2)}}\Phi =& R^{(2)} \\ \Box_{g^{(2)}}B =& \epsilon^{\mu\nu}\partial_\mu\mathcal{A}_\nu \end{aligned} \tag{4.28}$$

Thus, given the metric (4.6) and gauge field (4.7) and adopting modified Unruh Vacuum boundary conditions (MUBC)

$$\begin{cases} \langle T_{++}\rangle = \langle J_+\rangle = 0 & r\to\infty,\ l\to\infty \\ \langle T_{--}\rangle = \langle J_-\rangle = 0 & r\to r_+ \end{cases}, \tag{4.29}$$

where the modification takes the AdS radius into account, all relevant integration constants of (4.27) and (4.28) are determined. for large r and to $\mathcal{O}(\frac{1}{l})^2$, the resulting energy momentum tensor is dominated by one holomorphic component, $\langle T_{--}\rangle$. Expanding this component and the $U(1)$ current in terms of the boundary fields

(4.13) and (4.14), we compute their responses to the total symmetry $\delta_{\xi_n^-+\Lambda}$, yielding:

$$\begin{cases} \delta_{\xi_n^-+\Lambda} \langle T_{--} \rangle = \xi_n^- \langle T_{--} \rangle' + 2 \langle T_{--} \rangle (\xi_n^-)' + \frac{r_+^2+a^2}{4\pi G \Xi} (\xi_n^-)''' + \mathcal{O}\left(\left(\frac{1}{r}\right)^3\right) \\ \delta_{\xi_n^-+\Lambda} \langle J_- \rangle = \mathcal{O}\left(\left(\frac{1}{r}\right)^3\right) \end{cases} \tag{4.30}$$

This shows that $\langle T_{--} \rangle$ transforms asymptotically as the energy momentum tensor of a one dimensional CFT with center:

$$\frac{c}{24\pi} = \frac{r_+^2 + a^2}{4\pi G \Xi} \Rightarrow c = \frac{3A}{2\pi G}, \tag{4.31}$$

where $A = \frac{4\pi\left(r_+^2+a^2\right)}{\Xi}$ is the horizon area of the KNAdS black hole. It is well known that a two dimensional CFT exhibits a conformal/trace anomaly of the form [91]

$$\left\langle T_\mu{}^\mu \right\rangle = -\frac{c}{24\pi} R^{(2)} \tag{4.32}$$

and evaluating the trace of (4.27) agrees with the above equation yielding the same center as in (4.31).

The entropy of our near horizon CFT will be determined by counting the microstates of the total quantum asymptotic symmetry generators on the $r \to \infty$ boundary via the Cardy formula (1.22). The quantum generators are defined via the charge:

$$\mathcal{Q}_n = \lim_{r\to\infty} \int dx^- \langle T_{--} \rangle \xi_n^-, \tag{4.33}$$

Computing its response to a total symmetry and compactifying the x^- coordinate to a circle from $0 \to 2\pi/\kappa$ yields the charge algebra:

$$\delta_{\xi_m^-+\Lambda} \mathcal{Q}_n = [\mathcal{Q}_m, \mathcal{Q}_n] = (m-n)\mathcal{Q}_n + \frac{c}{12} m \left(m^2 - 1\right) \delta_{m+n,0}, \tag{4.34}$$

which takes the familiar form of a centrally extended Virasoro algebra.

4.4 Entropy

Summarizing our results from (4.31) through (4.34) we have:

$$
\begin{aligned}
c &= \frac{3A}{2\pi G} \\
\mathcal{Q}_0 &= \frac{A}{16\pi G}
\end{aligned}
\tag{4.35}
$$

Substituting this into the Cardy Formula (1.22) we obtain:

$$S = 2\pi\sqrt{\frac{c\mathcal{Q}_0}{6}} = \frac{A}{4G}, \tag{4.36}$$

which is in agreement with the Bekenstein-Hawking entropy of the four dimensional KNAdS black hole. Taking the limit of (4.31) to Kerr and to extremality yields

$$\lim_{l\to\infty,\ Q\to 0,\ M\to a} c = 12J, \tag{4.37}$$

which is the same value of the left central charge obtained in the Kerr/CFT correspondence [55], further strengthening the proposal of GHSS.

4.5 Temperature

To compute the black hole temperature, we will couple the metric (4.6) to a single quantum conformal field Φ with Liouville functional

$$S_{Liouville} = \frac{1}{96\pi}\int d^2x\sqrt{-g^{(2)}}\left\{-\Phi\Box_{g^{(2)}}\Phi + 2\Phi R^{(2)}\right\} \tag{4.38}$$

and following the steps (4.27) through (4.29), we obtain an energy momentum tensor which is dominated by one holomorphic component in the limit $r = r^+$ given by:

$$\langle T_{++}\rangle = -\frac{f'\left(r^+\right)^2}{192\pi}. \tag{4.39}$$

This is the value of the Hawking Flux of the KNAdS black hole, from which we obtain the known Hawking temperature[31, 94]:

$$HF = -\frac{\pi}{12}\left(T_H\right)^2 \Rightarrow T_H = \frac{f'\left(r^+\right)}{4\pi}. \tag{4.40}$$

CHAPTER 5
CONCLUSION

To conclude, we have analyzed quantum black hole properties in the near horizon regime via CFT techniques of quantum effective actions and extended the analysis of Chapter 3 to the more general KNAdS spacetime in Chapter 4. The main premiss is that the near horizon of four dimensional black holes is dual to a two dimensional Liouville type quantum CFT whose conformal symmetry is generated by the centrally extended Virasoro algebra. The central charge and lowest Virasoro eigen-mode (4.35) together reproduce the correct form of the Bekenstein-Hawking entropy and analysis of the RW2DA (4.6) coupled to a single quantum conformal field reproduce the known form of the Hawking temperature.

It is interesting to note that the lowest Virasoro eigen-mode satisfies

$$\mathcal{Q}_0 = GM_{irr}^2 \tag{5.1}$$

where M_{irr}^2 is the irreducible black hole mass, i.e. the final mass state after radiating away its angular momentum via a Penrose type process. This suggests that the eigen value of a CFT's Hamiltonian is proportional to the irreducible mass of its black hole dual.

In (4.9) we elevated the scalar field to a gravitational one. This was first suggested and outlined by Solodukhin in [87] and extended to compute Hawking radiation by RW in their seminal work [23]. Yet, in this approach the scalar field is still treated mathematically as a matter field. It is also unclear the exact details of the four dimensional gravitational theory, perhaps an ultraviolet complete extension of general relativity that dimensionally reduces to (4.26) except that it has the same coupling as standard Einstein gravity.

It still remains an open question to generalize the methods of this note to more exotic, higher dimensional black holes. In [33, 34, 37] the authors showed that the RW method for computing Hawking radiation via gauge and gravitational anomalies holds for their respective exotic black holes in arbitrary topologies and thus we believe our construction for a near horizon CFT dual should extend to these cases as well.

REFERENCES

[1] J. D. Bekenstein, "Black holes and entropy," *Phys. Rev.* **D7** (1973) 2333–2346.

[2] S. W. Hawking, "Particle Creation by Black Holes," *Commun. Math. Phys.* **43** (1975) 199–220.

[3] J. M. Bardeen, B. Carter, and S. W. Hawking, "The Four laws of black hole mechanics," *Commun. Math. Phys.* **31** (1973) 161–170.

[4] A. Wipf, "Quantum fields near black holes," `arXiv:hep-th/9801025`.

[5] S. Carlip, "Horizon constraints and black hole entropy," `arXiv:gr-qc/0508071`.

[6] G. K. Au, "The Quest for quantum gravity," `arXiv:gr-qc/9506001`.

[7] C. Rovelli, "Strings, loops and others: A critical survey of the present approaches to quantum gravity," `arXiv:gr-qc/9803024`.

[8] S. M. Christensen and S. A. Fulling, "Trace Anomalies and the Hawking Effect," *Phys. Rev.* **D15** (1977) 2088–2104.

[9] V. F. Mukhanov, A. Wipf, and A. Zelnikov, "On 4-D Hawking radiation from effective action," *Phys. Lett.* **B332** (1994) 283–291, `arXiv:hep-th/9403018`.

[10] R. Balbinot and A. Fabbri, "4D quantum black hole physics from 2D models?," *Phys. Lett.* **B459** (1999) 112–118, `arXiv:gr-qc/9904034`.

[11] R. Balbinot and A. Fabbri, "Hawking radiation by effective two-dimensional theories," *Phys. Rev.* **D59** (1999) 044031, `arXiv:hep-th/9807123`.

[12] M. Cadoni, "Trace anomaly and Hawking effect in generic 2D dilaton gravity theories," *Phys. Rev.* **D53** (1996) 4413–4420, `arXiv:gr-qc/9510012`.

[13] M. Cadoni, "Trace anomaly and Hawking effect in 2D dilaton gravity theories." 1997.

[14] S.-Q. Wu, J.-J. Peng, and Z.-Y. Zhao, "Anomalies, effective action and Hawking temperatures of a Schwarzschild black hole in the isotropic coordinates," *Class. Quant. Grav.* **25** (2008) 135001, `arXiv:0803.1338 [hep-th]`.

[15] V. Mukhanov and S. Winitzki, *Intro. To Quantum Effects In Gravity.* Cambridge, 2007.

[16] A. M. Polyakov, "Quantum geometry of bosonic strings," *Phys. Lett.* **B103** (1981) 207–210.

[17] L. Rodriguez and T. Yildirim, "Entropy and Temperature From Black-Hole/Near-Horizon-CFT Duality," *Class. Quant. Grav.* **27** (2010) 155003, `arXiv:1003.0026 [hep-th]`.

[18] S. Iso, H. Umetsu, and F. Wilczek, "Anomalies, Hawking radiations and regularity in rotating black holes," *Phys. Rev.* **D74** (2006) 044017, `arXiv:hep-th/0606018`.

[19] K. Murata and J. Soda, "Hawking radiation from rotating black holes and gravitational anomalies," *Phys. Rev.* **D74** (2006) 044018, `arXiv:hep-th/0606069`.

[20] S. Carlip, "Conformal field theory, (2+1)-dimensional gravity, and the BTZ black hole," *Class. Quant. Grav.* **22** (2005) R85–R124, `arXiv:gr-qc/0503022`.

[21] B. Rai and V. G. J. Rodgers, "From Coadjoint Orbits to Scale Invariant WZNW Type Actions and 2-D Quantum Gravity Action," *Nucl. Phys.* **B341** (1990) 119–133.

[22] G. W. Delius, P. van Nieuwenhuizen, and V. G. J. Rodgers, "The Method of Coadjoint Orbits: an Algorithm for the Construction of Invariant Actions," *Int. J. Mod. Phys.* **A5** (1990) 3943–3984.

[23] S. P. Robinson and F. Wilczek, "A relationship between Hawking radiation and gravitational anomalies," *Phys. Rev. Lett.* **95** (2005) 011303, `arXiv:gr-qc/0502074`.

[24] S. Das, S. P. Robinson, and E. C. Vagenas, "Gravitational anomalies: a recipe for Hawking radiation," *Int. J. Mod. Phys.* **D17** (2008) 533–539, `arXiv:0705.2233 [hep-th]`.

[25] R. Banerjee, "Covariant Anomalies, Horizons and Hawking Radiation," *Int. J. Mod. Phys.* **D17** (2009) 2539–2542, `arXiv:0807.4637 [hep-th]`.

[26] R. Banerjee and S. Kulkarni, "Hawking Radiation and Covariant Anomalies," *Phys. Rev.* **D77** (2008) 024018, `arXiv:0707.2449 [hep-th]`.

[27] R. Banerjee and S. Kulkarni, "Hawking Radiation, Effective Actions and Covariant Boundary Conditions," *Phys. Lett.* **B659** (2008) 827–831, `arXiv:0709.3916 [hep-th]`.

[28] R. Banerjee and S. Kulkarni, "Hawking Radiation, Covariant Boundary Conditions and Vacuum States," *Phys. Rev.* **D79** (2009) 084035, `arXiv:0810.5683 [hep-th]`.

[29] S. Gangopadhyay, "Hawking radiation from black holes in de Sitter spaces via covariant anomalies," *Gen. Rel. Grav.* **42** (2010) 1183–1187, `arXiv:0910.2079 [hep-th]`.

[30] Q.-Q. Jiang, "Hawking radiation from black holes in de Sitter spaces," *Class. Quant. Grav.* **24** (2007) 4391–4406, `arXiv:0705.2068 [hep-th]`.

[31] Q.-Q. Jiang and S.-Q. Wu, "Hawking radiation from rotating black holes in anti-de Sitter spaces via gauge and gravitational anomalies," *Phys. Lett.* **B647** (2007) 200–206, `arXiv:hep-th/0701002`.

[32] Z. Xu and B. Chen, "Hawking radiation from general Kerr-(anti)de Sitter black holes," *Phys. Rev.* **D75** (2007) 024041, `arXiv:hep-th/0612261`.

[33] B. Chen and W. He, "Hawking Radiation of Black Rings from Anomalies," *Class. Quant. Grav.* **25** (2008) 135011, `arXiv:0705.2984 [gr-qc]`.

[34] J.-J. Peng and S.-Q. Wu, "Covariant anomalies and Hawking radiation from charged rotating black strings in anti-de Sitter spacetimes," *Phys. Lett.* **B661** (2008) 300–306, `arXiv:0801.0185 [hep-th]`.

[35] S. Nam and J.-D. Park, "Hawking radiation from covariant anomalies in 2+1 dimensional black holes," *Class. Quant. Grav.* **26** (2009) 145015, `arXiv:0902.0982 [hep-th]`.

[36] M. R. Setare, "Gauge and gravitational anomalies and Hawking radiation of rotating BTZ black holes," *Eur. Phys. J.* **C49** (2007) 865–868, `arXiv:hep-th/0608080`.

[37] E. Papantonopoulos and P. Skamagoulis, "Hawking Radiation via Gravitational Anomalies in Non- spherical Topologies," *Phys. Rev.* **D79** (2009) 084022, `arXiv:0812.1759 [hep-th]`.

[38] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," *Adv. Theor. Math. Phys.* **2** (1998) 231–252, `arXiv:hep-th/9711200`.

[39] J. D. Brown and M. Henneaux, "Central Charges in the Canonical Realization of Asymptotic Symmetries: An Example from Three-Dimensional Gravity," *Commun. Math. Phys.* **104** (1986) 207–226.

[40] M. Banados, C. Teitelboim, and J. Zanelli, "The Black hole in three-dimensional space-time," *Phys. Rev. Lett.* **69** (1992) 1849–1851, `arXiv:hep-th/9204099`.

[41] A. Strominger, "Black hole entropy from near-horizon microstates," *JHEP* **02** (1998) 009, `arXiv:hep-th/9712251`.

[42] H. W. J. Blöte, J. A. Cardy, and M. P. Nightingale *Phys. Rev. Lett.* **56** no. 742, (1986) .

[43] J. A. Cardy *Nucl. Phys.* **B270** no. 186, (1986) .

[44] S. Carlip, "Black hole entropy from conformal field theory in any dimension," *Phys. Rev. Lett.* **82** (1999) 2828–2831, `arXiv:hep-th/9812013`.

[45] S. Carlip, "Entropy from conformal field theory at Killing horizons," *Class. Quant. Grav.* **16** (1999) 3327–3348, `arXiv:gr-qc/9906126`.

[46] M. Cadoni, "Statistical entropy of the Schwarzschild black hole," *Mod. Phys. Lett.* **A21** (2006) 1879–1888, `arXiv:hep-th/0511103`.

[47] G. Kang, J.-i. Koga, and M.-I. Park, "Near-horizon conformal symmetry and black hole entropy in any dimension," *Phys. Rev.* **D70** (2004) 024005, `arXiv:hep-th/0402113`.

[48] S. Silva, "Black hole entropy and thermodynamics from symmetries," *Class. Quant. Grav.* **19** (2002) 3947–3962, `arXiv:hep-th/0204179`.

[49] R. Banerjee, S. Gangopadhyay, and S. Kulkarni, "Hawking radiation and near horizon universality of chiral Virasoro algebra," *Gen. Rel. Grav.* **42** (2010)

2865–2871.

[50] E. Barnes, D. Vaman, and C. Wu, "All 4-dimensional static, spherically symmetric, 2-charge abelian Kaluza-Klein black holes and their CFT duals." 2010.

[51] D. Astefanesei and Y. K. Srivastava, "CFT Duals for Attractor Horizons," *Nucl. Phys.* **B822** (2009) 283–300, `arXiv:0902.4033 [hep-th]`.

[52] O. Coussaert and M. Henneaux, "Supersymmetry of the (2+1) black holes," *Phys. Rev. Lett.* **72** (1994) 183–186, `arXiv:hep-th/9310194`.

[53] O. Coussaert and M. Henneaux, "Self-dual solutions of 2+1 Einstein gravity with a negative cosmological constant," `arXiv:hep-th/9407181`.

[54] S. Carlip, "What we don't know about BTZ black hole entropy," *Class. Quant. Grav.* **15** (1998) 3609–3625, `arXiv:hep-th/9806026`.

[55] M. Guica, T. Hartman, W. Song, and A. Strominger, "The Kerr/CFT Correspondence," *Phys. Rev.* **D80** (2009) 124008, `arXiv:0809.4266 [hep-th]`.

[56] V. P. Frolov and K. S. Thorne, "RENORMALIZED STRESS - ENERGY TENSOR NEAR THE HORIZON OF A SLOWLY EVOLVING, ROTATING BLACK HOLE," *Phys. Rev.* **D39** (1989) 2125–2154.

[57] J. B. Hartle and S. W. Hawking, "Path Integral Derivation of Black Hole Radiance," *Phys. Rev.* **D13** (1976) 2188–2203.

[58] D. D. K. Chow, M. Cvetic, H. Lu, and C. N. Pope, "Extremal Black Hole/CFT Correspondence in (Gauged) Supergravities," *Phys. Rev.* **D79** (2009) 084018, `arXiv:0812.2918 [hep-th]`.

[59] J. Rasmussen, "On the CFT duals for near-extremal black holes," `arXiv:1005.2255 [hep-th]`.

[60] J. Rasmussen, "A near-NHEK/CFT correspondence," *Int. J. Mod. Phys.* **A25** (2010) 5517–5527, `arXiv:1004.4773 [hep-th]`.

[61] C.-M. Chen, Y.-M. Huang, J.-R. Sun, M.-F. Wu, and S.-J. Zou, "On Holographic Dual of the Dyonic Reissner-Nordstróm Black Hole," `arXiv:1006.4092 [hep-th]`.

[62] B. Chen and J. Long, "On Holographic description of the Kerr-Newman-AdS-dS black holes," `arXiv:1006.0157 [hep-th]`.

[63] R. Li, M.-F. Li, and J.-R. Ren, "Entropy of Kaluza-Klein Black Hole from Kerr/CFT Correspondence," `arXiv:1004.5335 [hep-th]`.

[64] A. Castro, A. Maloney, and A. Strominger, "Hidden Conformal Symmetry of the Kerr Black Hole," *Phys. Rev.* **D82** (2010) 024008, `arXiv:1004.0996 [hep-th]`.

[65] C. Krishnan, "Hidden Conformal Symmetries of Five-Dimensional Black Holes," *JHEP* **07** (2010) 039, `arXiv:1004.3537 [hep-th]`.

[66] T. Azeyanagi, N. Ogawa, and S. Terashima, "The Kerr/CFT Correspondence and String Theory," *Phys. Rev.* **D79** (2009) 106009, `arXiv:0812.4883 [hep-th]`.

[67] H. Lu, J. Mei, and C. N. Pope, "Kerr/CFT Correspondence in Diverse Dimensions," *JHEP* **04** (2009) 054, `arXiv:0811.2225 [hep-th]`.

[68] B. C. da Cunha and A. R. de Queiroz, "Kerr-CFT From Black-Hole Thermodynamics," *JHEP* **08** (2010) 076, `arXiv:1006.0510 [hep-th]`.

[69] J. E. McClintock *et al.*, "The Spin of the Near-Extreme Kerr Black Hole GRS 1915+105," *Astrophys. J.* **652** (2006) 518–539, `arXiv:astro-ph/0606076`.

[70] R. A. Daly, "Bunds on Black Hole Spins," *The Astrophysical Journal* **696** no. 1, (2009) L32–L36.

[71] R. A. Remillard and J. E. McClintock, "X-ray Properties of Black-Hole Binaries," *Ann. Rev. Astron. Astrophys.* **44** (2006) 49–92, `arXiv:astro-ph/0606352`.

[72] S. Carroll, *Spacetime and Geometry.* Addison Wesley, 2004.

[73] E. Poisson, *A Relativist's Toolkit.* Cambridge, 2004.

[74] V. Mukhanov and S. Winitzki, "Introduction to quantum effects in gravity,". Cambridge, UK: Cambridge Univ. Pr. (2007) 273 p.

[75] R. Blumenhagen and E. Plauschinn, "Introduction to conformal field theory," *Lect. Notes Phys.* **779** (2009) 1–256.

[76] P. C. Argyres, "Introduction to the AdS/CFT Correspondence," *Lect. Notes Phys.* **828** (2011) 57–69.

[77] M. B. Green, J. H. Schwarz, and E. Witten, *Supperstring Theory*, vol. 1. Cambridge, 1987.

[78] W. Israel, "Third Law of Black-Hole Dynamics: A Formulation and Proof," *Phys. Rev. Lett.* **57** no. 4, (Jul, 1986) 397–399.

[79] S. W. Hawking, "Gravitational Radiation from Colliding Black Holes," *Phys. Rev. Lett.* **26** no. 21, (May, 1971) 1344–1346.

[80] P. C. Vaidya, "Newtonian Time in General Relativity," *Nature* **171** (1953) 260–261.

[81] W. G. Unruh, "Notes on black hole evaporation," *Phys. Rev.* **D14** (1976) 870.

[82] B. K. Button, L. Rodriguez, C. A. Whiting, and T. Yildirim, "A Near Horizon CFT Dual for Kerr-Newman-AdS," `arXiv:1009.1661 [hep-th]`. Accepted in: Int. J. Mod. Phys. A, May 2011.

[83] A. Strominger, "Les Houches lectures on black holes," 1994.

[84] A. B. Zamolodchikov *JETP Lett.* **43** (1986) 730.

[85] S. Carlip, "Liouville lost, Liouville regained: Central charge in a dynamical background," *Phys. Lett.* **B508** (2001) 168–172, `arXiv:gr-qc/0103100`.

[86] S. P. de Alwis, "Quantum black holes in two-dimensions," *Phys. Rev.* **D46** (1992) 5429–5438, `arXiv:hep-th/9207095`.

[87] S. N. Solodukhin, "Conformal description of horizon's states," *Phys. Lett.* **B454** (1999) 213–222, `arXiv:hep-th/9812056`.

[88] N. Seiberg, "Notes on quantum Liouville theory and quantum gravity," *Prog. Theor. Phys. Suppl.* **102** (1990) 319–349.

[89] H. Chung, "Dynamics of Diffeomorphism Degrees of Freedom at a Horizon," *Phys. Rev.* **D83** (2011) 084017, `arXiv:1011.0623 [gr-qc]`.

[90] H. Chung, "Hawking Radiation and Entropy from Horizon Degrees of Freedom," `arXiv:1011.0624 [gr-qc]`.

[91] P. D. Francesco, P. Mathieu, and D. Sénéchal, *Conformal Field Theory*. Springer, 1997.

[92] M. Cadoni and S. Mignemi, "On the conformal equivalence between 2-d black holes and Rindler space-time," *Phys. Lett.* **B358** (1995) 217–222, `arXiv:gr-qc/9505032`.

[93] S. Iso, H. Umetsu, and F. Wilczek, "Hawking radiation from charged black holes via gauge and gravitational anomalies," *Phys. Rev. Lett.* **96** (2006) 151302, `arXiv:hep-th/0602146`.

[94] M. M. Caldarelli, G. Cognola, and D. Klemm, "Thermodynamics of Kerr-Newman-AdS black holes and conformal field theories," *Class. Quant. Grav.* **17** (2000) 399–420, `arXiv:hep-th/9908022`.

[95] S. P. Robinson, *Two quantum effects in the theory of gravitation*. Ph.D., Massachusetts Institute of Technology, 2005.

[96] H. Leutwyler, "Gravitational Anomalies: A Soluble Two-Dimensional Model," *Phys. Lett.* **B153** (1985) 65.