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Ability, education choice and life cycle earnings

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ABILITY, EDUCATION CHOICE AND LIFE CYCLE EARNINGS

by

Yu-Chien Kong

An Abstract

Of a thesis submitted in partial fulfillment of the
requirements for the Doctor of Philosophy
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Thesis Supervisors: Professor B. Ravikumar
Assistant Professor Guillaume Vandenbroucke

ABSTRACT

This dissertation consists of two chapters. In the first chapter, I explain changes in the life-cycle earnings profile for different birth cohorts. The second chapter assesses the quantitative importance of federal aid for college education in explaining college premium.

In the first chapter, I document the life-cycle earnings profile for the 25-year-old college- and high school-educated white men in 1940, 1950, 1960 and 1970. I find that later cohorts have flatter average life-cycle earnings profile. Using a version of the Ben-Porath model, I propose an explanation based on the composition effect. In my model, all individuals have a high school diploma and are differentiated by their ability. They must decide whether to work or go to a four-year college. There is a threshold ability above which individuals choose to attend college and below which they work. All cohorts face the same ability distribution and an exogenous sequence of wage rate per unit of human capital that grows at a constant rate. A higher initial level of wage rate increases college attainment implying that the average ability is lower for both college- and high school-educated individuals. From the Ben-Porath model, lower ability individuals have less steep increment in their earnings. This implies that the average college (and high school) life-cycle earnings profile for the 1970 cohort will be flatter than that of the 1940 cohort. My model is able to quantitatively explain 67 and 35 percent of the flattening in the average life-cycle earnings profile for college and high school-educated individuals, respectively.

Since the late 1970s, there has been a strong increase in the college premium.

While most papers focus on skill-biased technical change, the second chapter explores the role of federal aid as a possible source of inequality. I build a model where all individuals have a high-school diploma but are heterogeneous with respect to their innate abilities and initial human capital. They decide whether to attend college to accumulate more human capital before working, or to start working right away. The production function for human capital in college requires two inputs: human capital and goods. In this context, two mechanisms are key for the behavior of the college premium. First, federal aid makes it easier to afford the goods input in the human capital technology. This induces college students to accumulate more human capital and consequently, they have higher earnings. Second, as more individuals attend college due to rising income, the composition of college graduates changes: more low-ability individuals attend college, implying a decrease in average college earnings. A calibrated version of the model accounts fully for the rise in the college premium. Federal aid alone accounts for about 70 percent of the rise.

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CERTIFICATE OF APPROVAL

PH.D. THESIS

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To my mother and my aunt: Shu-Chen and Shu-Ying Huang. I thank them for their continued support

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In the first chapter, I document the life-cycle earnings profile for the 25-year-old college- and high school-educated white men in 1940, 1950, 1960 and 1970. I find that later cohorts have flatter average life-cycle earnings profile. Using a version of the Ben-Porath model, I propose an explanation based on the composition effect. In my model, all individuals have a high school diploma and are differentiated by their ability. They must decide whether to work or go to a four-year college. There is a threshold ability above which individuals choose to attend college and below which they work. All cohorts face the same ability distribution and an exogenous sequence of wage rate per unit of human capital that grows at a constant rate. A higher initial level of wage rate increases college attainment implying that the average ability is lower for both college- and high school-educated individuals. From the Ben-Porath model, lower ability individuals have less steep increment in their earnings. This implies that the average college (and high school) life-cycle earnings profile for the 1970 cohort will be flatter than that of the 1940 cohort. My model is able to quantitatively explain 67 and 35 percent of the flattening in the average life-cycle earnings profile for college and high school-educated individuals, respectively.

Since the late 1970s, there has been a strong increase in the college premium. While most papers focus on skill-biased technical change, the second chapter explores the role of federal aid as a possible source of inequality. I build a model where all individuals have a high-school diploma but are heterogeneous with respect to their innate abilities and initial human capital. They decide whether to attend college to accumulate more human capital before working, or to start working right away. The production function for human capital in college requires two inputs: human capital and goods. In this context, two mechanisms are key for the behavior of the college premium. First, federal aid makes it easier to afford the goods input in the human capital technology. This induces college students to accumulate more human capital and consequently, they have higher earnings. Second, as more individuals attend college due to rising income, the composition of college graduates changes: more low-ability individuals attend college, implying a decrease in average college earnings. A calibrated version of the model accounts fully for the rise in the college premium. Federal aid alone accounts for about 70 percent of the rise.

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CHAPTER 1
COLLEGE ATTAINMENT AND THE CHANGING LIFE CYCLE PROFILE OF EARNINGS

1.1 Introduction

Among the notable changes that affected the labor market since the 1940s, the behavior of the life-cycle earnings profiles of successive cohorts has received little attention, with the notable exception of Kambourov and Manovskii (2009). In this paper, I document that these profiles are noticeably flatter for recent cohorts than for older cohorts, and I propose a quantitative theory to account for this movement.

Figures 1.1 and 1.2 plot the average life-cycle earnings profile for the 1940, 1950, 1960, and 1970 cohorts for college- and high school-educated white men, respectively.[1] These two earnings profiles are also referred to as the conditional earnings profiles in the paper.[2] One observation from these two figures is that for each successive birth cohort, the earnings profiles are getting flatter and flatter. Take for instance, from Figure 1.1, the average annual real earnings of 25-year old college-educated individuals in 1940 increases 3.96 times by the time they are 55 years old. However, those for 25-year old college-educated individuals in 1970 increases only 2.19 times in the same length of time. A similar trend is observed for high school-educated individuals: 3.44 times for those in the 1940 cohort and 1.28 times for those in the

[1]Refer to Appendix A.1 for the construction of life-cycle earnings profile.

[2]Figure 1.3 shows the life-cycle earnings profile for the average earnings of both college- and high school-educated individuals. This is also referred to as the unconditional life-cycle earnings profile in the paper.

1970 cohort (see Figure 1.2 and Table 1.1). This paper aims to provide a quantitative explanation for this observed phenomenon.

I construct a model of education choice and human capital accumulation building upon the work of Ben-Porath (1967), Heckman et al. (1998), and Huggett et al. (2006). All individuals have a high school diploma and decide whether or not to go to college. Individuals are heterogeneous with respect to their ability to accumulate human capital across their life cycle.[3] There are two technologies for human capital accumulation: on-the-job and college. Individuals who devote more time and resources (hereafter referred to as goods) and those with higher abilities will accumulate more human capital. The optimal choice of schooling implies that there is a threshold ability above which individuals choose to attend college and below which they choose to work. As with the Ben-Porath model, my model delivers that all individuals have a hump-shaped life-cycle earnings profile and that individuals with higher ability accumulate human capital faster and, hence, have higher earnings. All cohorts face the same ability distribution and an exogenous wage rate per unit of human capital that grows at a constant rate. This means that all cohorts face the same growth in wage rate but recent cohorts start with a higher initial level. As the wage rate increases, more individuals attend college, i.e. the threshold ability is lower for recent cohorts. This implies that in recent cohorts, new college-educated individuals have lower ability than the college-educated individuals of older cohorts.

[3] Ability has become a standard feature of human capital models and original work by Mincer (1958), Becker (1964) and Ben-Porath (1967) link human capital investment to life cycle earnings.

This selection mechanism reduces the overall earnings growth for college-educated individuals in recent cohorts. A similar mechanism operates for high school-educated individuals: the remaining high school-educated individuals in recent cohorts are less able than in older cohorts on average. Since growth in earnings is smaller for individuals with lower abilities, this implies that the average life-cycle earnings profile for college-educated individuals from 1970 cohort is flatter than that for the 1940 cohort. This is the same for high school-educated individuals.

I calibrate the model parameters to moments characterizing the earnings profile of the 1940 cohort not conditioning on education. Using the calibrated parameters, I conduct the following experiment. I compute optimal decisions for a sequence of cohorts, starting with 1940 and ending with the 1970 cohort. Each cohort differs from its predecessor in only one dimension: the level of wage rate per unit of human capital that it faces at the beginning of its life. I report the earnings profiles of these cohorts and their education choices. I find that the earnings profiles flatten, between the 1940 and 1970 cohorts, in the model as in the data and that educational attainment rises in line with the data as well. Quantitatively, my model is able to explain approximately 67 percent of the flattening for college-educated individuals, 35 percent of that for high school-educated individuals and 50 percent of that for all individuals.

The paper contributes to the literature in the following ways. First, using Census data, I am able to build the earnings profiles for earlier cohorts than previously examined in the literature.[4] As a result, I was able complement some of the previous

[4]Bernhardt et al. (1999) identifies two cohorts of young white men using National Longi-

explanations by proposing a theory that is consistent with the flattening observed before the baby boom generation enters the labor market. Early evidence on the flattening are given by Welch (1979) and Berger (1985). They document flatter earnings profile at the time when the baby boom generation is entering the labor market and suggest that earnings profiles are flatter for cohorts that have a larger size. In my analysis, the 1970 cohort would correspond to the baby boom generation. However, as documented above, the flattening of life-cycle earnings profile is observed for successive cohorts starting from the 1940 cohort, where fertility is in fact decreasing in the birth years of the 1940, 1950, and 1960 cohorts. This suggest that the flattening has little connection with the cohort's size.[5]

Second, there is a large literature on earnings heterogeneity in macreconomics. One dimension is in life cycle earnings. As documented above, I observe a systematic flattening in the life-cycle earnings profiles of successive cohorts starting from the 1940 cohort. The question is: What has changed in the economy to account for the observed phenomenon. For Kambourov and Manovskii (2009), this is due to recent cohorts losing occupation-specific human capital as they change jobs more often than their predecessors. In my paper, the explanation of this complex phenomenon is parsimonious: It is the changing composition of the labor force, as more individuals

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tudinal Survey: the *original* cohort followed from 1966-1981 and the *recent* cohort, followed from 1979-1994. They find evidence "deterioration in wage gains for recent cohorts." Kambourov and Manovskii (2009) document the same finding using data on male heads of households from the Panel Study of Income Dynamics (PSID) over the 1968-1997 period and Current Population Survey (CPS) over the period 1963-2004. Beaudry and Green (2000) document similar pattern in the Canadian data.

[5] Kambourov and Manovskii (2009)'s results also arrive at the same conclusion.

choose to attend college, that is causing the flattening. Why do I think composition effect is important? The twentieth century United States is a period of mass education in two levels of education: high school and college. The high school movement officially ended in 1940 and from 1940 onwards, it is a period of massive expansion in tertiary education. As a result, college attainment increases markedly. Consequently, this is a significant factor that can result in serious changes in the characteristics of this labor force.

Based on the proposed explanation, the difference in policy implication can be profound. One possible reason why earnings is growing slower for recent cohorts can be due to a slower accumulation of individual human capital across the life cycle. The impact of this explanation can be far-reaching because the works of Erosa et al. (2010) and Manuelli and Seshadri (2005) implies that human capital is an important determinant of economic growth. Consequently, slower accumulation of individual human capital can impede economic growth. My explanation of the flattening of the life-cycle earnings profile through composition effect however hints that there may be nothing to get agitated about: The average earnings profile looks flatter because the profile of the 'average' individual is changing across cohorts. The pattern of individual human capital accumulation across the life cycle is however not changing across cohorts.

Third, this paper shows that there is important differences between cohort-based and cross-sectional profiles. Thus, it is not always useful to make comparisons based on cross-sectional data when the cohort-specific element is nontrivial. Consider the following illustration: cross-sectional comparison of earning of a 55-year old college-educated individual to a 25-year old college-educated individual is what is traditionally calculated as the experience premium. However, since recent cohorts have a lower life-cycle earnings profile than older cohorts, the calculated experience premium contains a cohort-specific element that we do not want as well as the accumulated experience element that we wanted. In the situation where the cohort-specific characteristic is not inconsequential, using cross-sectional data can be at best misleading.

The rest of the paper proceeds as follows. In section 1.2, I introduce the model. I calibrate the model in section 1.3 and presents results in section 1.4. In section 1.5, I discuss two exercises. I conclude in section 1.6.

1.2 Model

1.2.1 Environment

The economy is populated by overlapping generations of individuals. Each individual lives for T periods and is ex-ante heterogeneous in terms of ability, $a \in R_+$. Ability denotes his capacity to accumulate human capital over his life cycle and is distributed according to a time-invariant cumulative distribution function, A. I assume that ability is observable for each individual before schooling and consumption decisions are made. Ability is immutable with education.

There are two levels of education: high school and four-year college. Time is discrete and indexed by $t = 0, 1, ..., \infty$. Each individual is endowed with one unit of time per period. He enters with a high school education and chooses whether or not to go to college. If he chooses not to go to college, he enters the labor market and divides his time between work and human capital accumulation. If he decides to go to college, he will first spend s periods studying full time and will then go to work.

There is a perfect credit market in which each individual can borrow and save at a constant exogenous rate, $r - 1$. There is also no uncertainty and preferences are defined on consumption sequences only. The wage rate per unit of human capital is given exogenously and is assumed to admit a constant growth rate, g. Since there are no borrowing constraint, uncertainty and leisure, the problem of maximizing lifetime utility is equivalent to maximizing lifetime earnings.

1.2.2 Technologies

Each individual with ability a enters the model with H^{hs} level of human capital. The ability distribution completely determines the distribution of initial human capital through the following equation:

$$H^{hs}(a) = z_h a,$$

where z_h is a productivity parameter common to all individuals.

There are two technologies for accumulating human capital: in college and on the job. The college human capital accumulation function for each individual with ability a is given by

$$H^{col}(a, e, H^{hs}) = (ze)^{\eta}(aH^{hs})^{1-\eta},$$

where e represents present value expenditure towards the services affecting the quality of college education and z represents the productivity parameter that is common to all individuals.

Each individual with ability a has the following on-the-job human capital accumulation function.

$$h^{'} = (1-\delta)h + H(a,n,h)$$

$$H(a,n,h) = a(nh)^{\phi},$$

where $\phi \in (0,1)$, δ is the depreciation rate, $n \in (0,1)$ is the fraction of time committed towards accumulation of human capital on the job, h is the accumulated human capital inherited from the last period and $h^{'}$ is the accumulated human capital in this period.

In this model, human capital accumulated from education is an input to the production of on-the-job human capital, which is indirectly productive in the labor market. Consequently, there exists a tight link between ability and the level of human capital accumulated and ultimately the lifetime earnings of an individual. Thus, the ability of an individual is a representation of the capacity to both learn and earn.

Lastly, the wage rate per unit of human capital (w) is exogenous and assumes to grow at a constant rate g:

$$w_{t+1} = gw_t.$$

Similar to the Ben-Porath model, earnings inequality between and within education levels can be generated only be differences in the level of human capital and investment behavior (where both are functions of heterogenous ability). This is because both high- and low-skilled individuals command the same wage rate per unit of human capital.

1.2.3 Individual's problem

Each individual enters the model with a high school diploma and chooses to have college education or not. He will choose the schooling level that gives him the highest net lifetime earnings. Once he enters the labor market, he cannot return to school. After a schooling decision is made, he will solve for the optimal sequence of time investment on-the-job, $\{n_{\tau,j}(h)\}_{j=1}^{T}$, to maximize lifetime earnings.

I solve the problem backwards and in two steps. Since all individuals regardless of education level will ultimately enter into employment, in step one, given an arbitrary level of human capital, I can solve the on-the-job human capital accumulation problem. Using the optimal solution from the on-the-job problem, I proceed to step two to solve the schooling choice problem by choosing the level of education that maximizes an individual's net lifetime earnings.

1.2.3.1 Human capital accumulation on the job

I formulate this part in the spirit of Ben-Porath (1967) and in the language of dynamic programming. For a given arbitrary level of human capital, each individual maximizes lifetime earning by choosing, in each period, the time he wants to spend accumulating human capital on the job and thereby determining the decision rules for $h_{\tau,j}(h)$ and $n_{\tau,j}(h)$ and value function, $V_{\tau,j}(h)$. The value function, $V_{\tau,j}(h)$, gives the maximum present value of lifetime earnings of an individual from cohort τ at age

j.

The problem is

$$V_{\tau,j}(h) = w_{\tau+j-1}h(1-n) + \left(\frac{1}{r}\right) V_{\tau,j+1}(h')$$

subject to

$$\begin{aligned} h' &= (1-\delta)h + H(a,n,h) \\ n &\in (0,1) \\ h & \quad \text{given,} \end{aligned}$$

where $w_{\tau+j-1}$ is the exogenous wage rate per unit of human capital an individual from cohort τ receives at age j .

The on-the-job human accumulation problem for a college- and high school-educated individual differ in two aspects. The first is the age that the individual enters the labor market. Each high school-educated individual enters at age $j = 1$ and each college-educated individual enters at age $j = s+1$. For the college-educated individual, there are earnings forgone for the s periods they spend in college education. Each high school-educated individual from cohort τ will enter labor market at age $j = 1$ with w_τ. Recalling that there is no differentiation of skills through prices, consequently, s periods later, both the college- and high school-educated individuals at the age $j = s + 1$ will face the same $w_\tau g^s$. Second, for the earnings forgone because of college education, it is compensated through a college-educated individual's higher initial level of on-the-job human capital. A high school-educated individual enters with level H^{hs} whereas a college-educated individual enters with a higher level $H^{col}\left(a, e, H^{hs}\right)$.

Solving the dynamic programming problem by value function iteration using terminal condition $V_{\tau,T+1} = 0$, I obtain the sequence of value function at each age j, $\{V_{\tau,j}(h)\}_{j=1}^{T}$. It takes the following form:

$$V_{\tau,j}(h) = \alpha_{\tau,j} + \beta_{\tau,j} h$$

where

$$\begin{aligned}
\alpha_{\tau,j} &= -w_{\tau+j-1}\left(\frac{a\phi\beta_{\tau,j+1}}{rw_{\tau+j-1}}\right)^{\frac{1}{1-\phi}} + \left(\frac{1}{r}\right)\alpha_{\tau,j+1} + \left(\frac{1}{r}\right)\beta_{\tau,j+1} a\left(\frac{a\phi\beta_{\tau,j+1}}{rw_{\tau+j-1}}\right)^{\frac{\phi}{1-\phi}} \\
\beta_{\tau,j} &= w_{\tau+j-1} + \left(\frac{1}{r}\right)(1-\delta)\beta_{\tau,j+1}.
\end{aligned}$$

Taking the first-order condition with respect to n, the optimal rule for n is

$$n_{\tau,j}(h) = \left[\frac{a\phi\beta_{\tau,j+1}}{rw_{\tau+j-1}}\right]^{\frac{1}{1-\phi}} h^{-1}. \tag{1.1}$$

For interior solution, $n_{\tau,j} \in (0,1)$,

$$0 < \left[\tfrac{a\phi\beta_{\tau,j+1}}{rw_{\tau+j-1}}\right]^{\frac{1}{1-\phi}} h^{-1} < 1.$$

The first inequality is automatically satisfied. The second inequality tells us that the condition for interior solution is

$$h > A_{\tau,j}(a) = \left[\frac{a\phi\beta_{\tau,j+1}}{rw_{\tau+j-1}}\right]^{\frac{1}{1-\phi}}.$$

The optimal decision rules are as follows:

$$h_{\tau,j}(h) = \begin{cases} aA_{\tau,j}(a)^{\phi} + (1-\delta)h, & \text{for } h > A_{\tau,j}(a) \\ ah^{\phi} + (1-\delta)h, & \text{for } h < A_{\tau,j}(a) \end{cases}$$

$$n_{\tau,j}(h) = \begin{cases} A_{\tau,j}(a)h^{-1}, & \text{for } h > A_{\tau,j}(a) \\ 1, & \text{for } h < A_{\tau,j}(a) \end{cases}$$

1.2.3.2 Net lifetime earnings (High school)

Since each individual from cohort τ enters the model with high school education at age $j = 1$, I evaluate the value function at $V_{\tau,1}(h)$. The first wage rate that each individual from cohort τ faces as he enter the labor market is w_τ with $H^{hs}(a)$ level of human capital. The maximized value of net lifetime earning is then

$$\begin{aligned} \widetilde{V}_{\tau}^{hs}(a) &= V_{\tau,1}\left(H^{hs}(a)\right) \\ &= \alpha_{\tau,1} + \beta_{\tau,1}H^{hs}. \end{aligned}$$

1.2.3.3 Net lifetime earnings (College)

Since each college-educated individual from cohort τ with ability a will start work at age $j = s + 1$, I evaluate the value function is at $V_{\tau,s+1}(h)$. He will enter the labor market with human capital level $(ze)^{\eta}\left(aH^{hs}\right)^{1-\eta}$, where e is endogenously determined.

The maximization problem is

$$\widetilde{V}_{\tau}^{col}(a) = \max_{e}\left\{\left(\frac{1}{r}\right)^{s}\alpha_{\tau,s+1} + \left(\frac{1}{r}\right)^{s}\beta_{\tau,s+1}\,(ze)^{\eta}\left(aH^{hs}\right)^{1-\eta} - e\right\}.$$

Deriving the first-order condition with respect to e gives the optimal expenditure for a college education and the level of human capital that a college-educated individual enters the labor market with:

$$\begin{aligned} e_{\tau}^{col} &= \left[\left(\frac{1}{r}\right)^{s} \beta_{\tau,s+1} z^{\eta}\eta\right]^{\frac{1}{(1-\eta)}} aH^{hs} \\ h_{\tau}^{col} &= \left[\left(\frac{1}{r}\right)^{s} \beta_{\tau,s+1} z\eta\right]^{\frac{\eta}{(1-\eta)}} aH^{hs}. \end{aligned}$$

The optimal net lifetime earnings of each college-educated individual is

$$\widetilde{V}_{\tau}^{col}(a) = \left(\frac{1}{r}\right)^{s} \alpha_{\tau,j+1} + \left[\left(\frac{1}{r}\right)^{s} z^{\eta}\beta_{\tau,s+1}\right]^{\frac{1}{1-\eta}} \kappa a H^{hs},$$

where

$$\kappa = \eta^{\frac{\eta}{1-\eta}} - \eta^{\frac{1}{1-\eta}}.$$

1.2.4 Schooling decision

Each individual compares net lifetime earnings between a college education and a high school education and decides whether or not to attend college with the following decision rules:

$$\begin{aligned} \widetilde{V}_{\tau}^{hs}(a) &< \widetilde{V}_{\tau}^{col}(a) \text{ – choose college} \\ \widetilde{V}_{\tau}^{hs}(a) &> \widetilde{V}_{\tau}^{col}(a) \text{ – no college.} \end{aligned}$$

In particular, the unique cohort-specific threshold a_{τ}^{*} is given by

$$\begin{aligned} \widetilde{V}_{\tau}^{hs}(a) &= \widetilde{V}_{\tau}^{col}(a) \\ \alpha_{\tau,1} + \beta_{\tau,1}H^{hs} &= \left(\frac{1}{r}\right)^{s} \alpha_{\tau,j+1} + \left[\left(\frac{1}{r}\right)^{s} z^{\eta}\beta_{\tau,s+1}\right]^{\frac{1}{1-\eta}} \kappa a H^{hs}. \end{aligned}$$

1.2.5 Dynamics

The model is driven by an exogenous wage rate per unit of human capital. This section discusses the changes in net lifetime earnings when wage rate per unit of human capital changes. $\alpha_{\tau,j}$ and $\beta_{\tau,j}$ can be re-expressed as

$$\begin{aligned}\alpha_{\tau,j} &= aw_\tau \sum_{t=j}^{T} g^{t-1} C \left(\frac{\zeta^{T-t}-1}{\zeta-1}\right)^{\frac{1}{1-\phi}} \left(\frac{1}{r}\right)^{t-j} \\ \beta_{\tau,j} &= w_\tau g^{j-1} \left(\frac{\zeta^{T-j+1}-1}{\zeta-1}\right),\end{aligned}$$

where

$$\begin{aligned}C &= \left(\frac{g}{r}\right)^{\frac{1}{1-\phi}} \left(\phi^{\frac{\phi}{1-\phi}} - \phi^{\frac{1}{1-\phi}}\right) \\ \zeta &= \left(\frac{1}{r}\right)(1-\delta)\, g.\end{aligned}$$

Taking partial derivative with respect to w_τ, it is fairly clear that both $\frac{\partial \widetilde{V}_\tau^{hs}(a)}{\partial w_\tau} > 0$ and $\frac{\partial \widetilde{V}_\tau^{col}(a)}{\partial w_\tau} > 0$.

1.3 Calibration

1.3.1 Strategy

In this section, I discuss my calibration strategy, which includes two steps. First, I assign some parameters values using prior information. Then, the remaining parameters are obtained by calibrating the model to earnings of the 1940 cohort. Recall that the aim of this paper is to explain the flattening of the two conditional life cycle earnings profiles.

In this baseline calibration exercise, I calibrate the model parameters to the unconditional earnings of the 1940 cohort.[6] In particular, the targets are (i) inverse of the coefficient of variation and (ii) the fraction of earnings at selected ages to the sum of mean earnings at ages 25, 35 45, and 55.

Using the model, the two conditional earnings profiles for the 1940 cohort are generated endogenously. So, the first objective is to test whether the model is able to generate the split between high school- and college-educated individuals correctly without using any information specific to each education group. Getting the conditional earnings profiles correct for the 1940 cohort will provide confidence to this calibration exercise. The model is then assessed based on how much flattening can be explained by the mechanism.

1.3.2 Details

One model period represents one calendar year. Each individual lives 42 model periods, ($T = 42$), and enters the model at age 18 and exits at age 59. Retirement decision is not modeled in this paper. Upon entering the model, each individual chooses whether to enter college or not. Those who do will spend four years in college ($s = 4$) whereas each high school-educated individual will start employment immediately. The gross interest rate is $r = 1.05$. The depreciation rate, (δ), taken from Huggett et al. (2006), is 0.0114.

The list of the remaining eight parameters is

$$\theta = \{z, z_h, w_1, g, \phi, \eta, \hat{\mu}, \hat{\sigma}\},$$

[6] Appendix A.3 discusses how the model generates average unconditional earnings.

which consists of college human capital accumulation function productivity parameter (z), productivity parameter from high school human capital accumulation function (z_h), the initial wage rate per unit of human capital for the first model cohort (w_τ, where $\tau = 1$), growth rate (g) in w_τ, on-the-job human capital accumulation function parameter (ϕ), college human capital production function parameter (η), mean ($\hat{\mu}$) and standard deviation ($\hat{\sigma}$) of lognormal ability distribution. These eight parameters are going to be calibrated through the solving of nonlinear equations to minimize the distance between the selected data moments and their corresponding model-generated moments. I will discuss explicitly how this is done in later paragraphs. Refer to Table 1.2 for a quick summary.

In the model, the wage rate per unit of human capital is growing over time. The economic intuition for this is that skills gets paid more in the labor market as economies grow. Allowing the wage rate per unit of human capital to grow at a constant rate is somewhat of an extreme way of modeling. The way I will think about constant growth g is that individuals know on average how much wage rate per unit of human capital is going to grow, but they do not know every point of the sequence.[7] The observed wage rate in the data is equal to the wage rate per unit of human capital (w) multiplied by the level of human capital; however, neither of is observed empirically. I let w grow at a constant rate g to impose discipline on the sequence of w, where g is in turn disciplined by the data through calibration.

[7]One example is that individuals know that the U.S. is going to grow on average 2% per annum. However, they are unable to predict correctly every peak and trough of the business cycle.

This will allow the story to be illustrated by the changes in the human capital. This approach is not uncommon in human capital literature. Depending on the question of interest, authors either use this approach or an alternative approach, where the level of human capital is kept constant to allow changes in w to capture the story. The latter approach is usually used for skill-biased technological change when the price of human capital, w, is the focus.

Production function parameters such as ϕ and η govern the shape and the increase in the earnings profile. Proper parameterization ensures that the earnings profile behave regularly. The parameters ϕ and η in theory are between zero and one. However, the Ben-Porath model generates earnings to infinity very easily, meaning that the model is extremely sensitive to changes in these two parameters. Consequently, the combination of values these two parameters can take in reality is much smaller. For example, the parameter ϕ is surveyed by Browning et al. (1999) to take values between 0.5 to almost 1. Under this range of values, the parameter η cannot exceed 0.55 or earnings can go to infinity. The productivity parameter z has almost the same effect as ϕ and η except that it affects the college earnings profile only. It, however, has a nontrivial role in making sure that when plotted across different levels of ability, lifetime earnings of college-educated individuals cuts high school lifetime earnings from below. This guarantees that lifetime earnings of high-school individuals are higher than that of college-educated individuals when ability is low and vice versa when ability is high. Productivity parameter z_h affects the initial level of human capital coming from high school education. Therefore it has a level effect on earnings.

A high z_h limits the growth of earnings simply because earnings start from a higher level. This parameter is useful in controlling the extent of growth in the earnings. The parameter w_τ has the same function. The parameter z_h also has a direct significance in ensuring that there are no corner solutions in the choice of $n_{\tau,j}(h)$. I want to avoid corner solutions because when $n_{\tau,j} = 1$, the individual is accumulating human capital full time. This corresponds to a semblance of schooling that is not quite defined in the context of the model. The growth rate, g, determines the extent of a leftward shift in lifetime earnings when plotted across different levels of ability. A higher g induces a greater shift. Together η, ϕ, z, z_h, w_1 and g determine the sequence of the cohort-specific ability thresholds.

The parameters $\hat{\mu}$ and $\hat{\sigma}$ characterize the lognormal ability distribution. These parameters govern the range and frequency of ability levels that enter the earning functions but do not alter ability thresholds. The separate determination of thresholds and distribution creates the need to properly specify the location parameter to guarantee a sensible fit between the two.[8] This separation makes matching targets singularly difficult. In particular, I cannot be sure of the direction of change in the simulated ratios when I change these parameters. Below is an illustration of this problem.

For example, a higher z indicates a higher lifetime earnings for any college-

[8]It is worth noting that even though parameters η and z are college specific, they can indirectly affect the earnings of high school-educated individuals through the support of the ability distribution. This is because as the threshold ability changes, the parameters $\hat{\mu}$ and $\hat{\sigma}$ will need to accommodate this change.

educated individual. However, a higher z can result in lower average lifetime earnings for college-educated individuals for this reason: From the determination of threshold ability, a higher z results in a faster increase in lifetime earnings when plotted against levels of ability. This means that, given no changes in the lifetime earnings of high-school individuals, the threshold is lower for a higher z compared to a lower one. So, although individually, higher z results in higher earnings, collectively, average earnings can be lower through lower conditional average ability. Since ability enters the model in a nontrivial manner, the effect of an increase in z on the average earnings profile is unknown.

Using prior information, I calibrate the eight remaining parameters. The chosen targets are as follows:

1. Inverse of the coefficient of variation at age 35, $\left(\frac{\mu_{35}}{\sigma_{35}}\right)$[9]

2. Inverse of the coefficient of variation at age 45, $\left(\frac{\mu_{45}}{\sigma_{45}}\right)$

3. Inverse of the coefficient of variation at age 55, $\left(\frac{\mu_{55}}{\sigma_{55}}\right)$

4. Fraction of average earnings at age 25 to sum of mean earnings at ages 25, 35, and 45, $\left(\Omega^{25}_{25,35,45}\right)$[10]

5. Fraction of average earnings at age 35 to sum of average earnings at ages 35,

[9] μ_{35} is the normalized mean earnings at age 35.
This is mathematically equal to $\frac{E_{35}}{E_{25}}$, where E_j is the mean earnings of all individuals at age j.

[10] Mathematically this is $\frac{E_{25}}{E_{25}+E_{35}+E_{45}}$, where E_j is the average earnings of all individuals at age j.

45, and 55, $\left(\Omega^{35}_{35,45,55}\right)$

6. Fraction of average earnings at age 45 to sum of average earnings at ages 25, 45, and 55, $\left(\Omega^{45}_{25,45,55}\right)$

7. Fraction of average earnings at age 55 to sum of average earnings at ages 25, 35, and 55, $\left(\Omega^{25}_{25,35,55}\right)$

8. The 90/10 ratio at age 25

Since model units are different from data units, the chosen targets are all unit free. The aim is to minimize the distance between ratios produced by the Census data and the model-simulated data. Therefore, a measure of distance is built using both the simulated data and the Census data. Below is a system of nonlinear equations in eight unknowns. For a given wage rate per unit of human capital sequence, the parameters are calibrated such that the ratio between observed data and their model counterpart is close to 1: $F(\theta) = 1$.

$$F(\theta) = \left\{ \begin{array}{lll} 2.914 & / & \frac{\mu_{35}}{\sigma_{35}} \\ 2.927 & / & \frac{\mu_{45}}{\sigma_{45}} \\ 2.824 & / & \frac{\mu_{55}}{\sigma_{55}} \\ 0.166 & / & \Omega^{25}_{25,35,45} \\ 0.231 & / & \Omega^{35}_{35,45,55} \\ 0.401 & / & \Omega^{45}_{25,45,55} \\ 0.544 & / & \Omega^{55}_{25,35,55} \\ 5.357 & / & \frac{E^{90}_{25}}{E^{10}_{25}} \end{array} \right\}$$

The values of calibrated parameters and the fit of calibration are presented in Tables 1.2 and 1.3, respectively. Once calibrated parameters are obtained, they are substituted into the model to simulate earnings data.

1.4 Results

This section discusses the results from the baseline calibration. Recall that the main objective is to explain the flattening of the conditional life-cycle earnings profile. The model is calibrated to unconditional earnings and conditional earnings profiles are generated endogenously. In the following paragraphs, I discuss the main results.

1.4.1 Flattening of the unconditional earnings profile

Figure 1.4 compares the life-cycle earnings profile of all individuals from the model-simulated data (hereafter, the model) and the Census data (hereafter, the data).[11] The dotted lines represent the data and the solid lines represent the model. The magenta and blue lines represent the 1940 and 1970 cohorts, respectively. The 1940 cohort lines shows the model fit. The first observation is that the model tracks data very well. (The precise magnitudes are in Table 1.4 in the "ALL" column.) This is especially true towards the end of the life cycle where the model is able to produce the dip in the growth of earnings from ages 45 to 55. The implication is that the

[11]Note that from Census data, I calculate the average annual real earnings for all individuals and not annual real earnings of the average individual. This two items are different in the model as ability enters in a non-linear manner and cannot be factored out. Therefore, even though, in the model, the average individual stays the same across cohorts, average annual real earnings is changing due to the increase in college attainment over cohorts.

depreciation rate of 0.0114 also has a role in disciplining the ability distribution. This is because ability enters earnings in a non-trivial way. Thus, by fixing the depreciation rate to 0.0114, the model is forcing the ability distribution to be right so that the model-simulated earnings can produce the dip that is consistent with data.

The model is able to explain about half of the flattening in the unconditional life-cycle earnings profile. At first glance, the model performs very well. The model's unconditional earnings profile for the 1940 cohort is very close to the data and the mechanism generates a decent amount of flattening. However, what is the significance of the flattening in the unconditional earnings profile generated by the model?

The significance of the flattening in the unconditional earnings profile generated by the model is as follows. It is difficult to verify a decrease in the average abilities of college- and high school-educated individuals since data on innate ability is not available. However, we can indirectly verify the decrease in ability using theoretical implication of the model. If the model is generating flatter earnings profiles for both college- and high school-educated individuals in the recent cohorts, the composition effect implies that the unconditional earnings profiles for recent cohorts should also be flatter. In this case, the model predicts flatter unconditional earning profile across cohorts. This is consistent with the behavior of the unconditional earnings profile from data.

1.4.2 Flattening of the conditional earnings profiles

Figures 1.5 and 1.6 show the earnings profile for college- and high school-educated individuals, respectively. The solid lines represent the model and the dotted lines represent the data. The magenta and blue represent the 1940 and 1970 cohorts, respectively. Remember that part of the exercise is to test if the model is able to correctly generate the split between college and high school-educated individuals. At first glance, the model is doing reasonably well with one exception.

For the data, life-cycle earnings profile for college-educated individuals for the 1940 cohort has a strong increase from ages 35 to 45; giving the profile a somewhat convex shape before dipping at age 55. The model is not able to reproduce this result because of model limitation. The model is exogenously driven by a sequence of wage rate per unit of human capital that grows at a constant rate. Therefore, the shape of the earnings profile is crucially dictated by changes in the human capital investment decision: Individuals choose to accumulate more human capital when young as they have the rest of their life to recoup the investment. Following the same logic, as they age, they choose to accumulate less and less human capital. Therefore, the corresponding earnings profile is such that earnings growth is the fastest when an individual is young; as the individual ages, his earnings increases at a decreasing rate. Thus, the model is not able to generate an earnings profile where the increase in earnings between ages 35-45 is faster than that between ages 25-35.

Both simulated life-cycle earnings profiles are consistently concave. This is one of the artifacts of the Ben-Porath model. This shows that the depreciation rate (δ)

of 0.0114 is able to produce a decrease in average earnings at the end of the working life cycle as documented in Figure 1 from Huggett et al. (2006).

1.4.2.1 Performance of the mechanism

In this paragraph, I discuss the performance of the mechanism. First, the model is able to produce flattening of the life-cycle earnings profiles for both college- and high school-educated individuals but of different magnitudes. The model is able to account for a large proportion of flattening in the life-cycle earnings profile for college-educated individuals (see Figure 1.5), but only one-third of that for high school-educated individuals (see Figure 1.6). The first two rows of Table 1.4 report for the data and the model life-cycle earnings profile at ages 25 and 55 for high school-educated, college-educated and all individuals. The first line provides the results for the 1940 cohort and the second for the 1970 cohort.

The third line provides a measure of the extent of the flattening, which is useful for determining which education group has the most and least flattening. Based on the data, the average annual earnings of high school-educated individuals in the 1940 cohort increases 3.439 times by the time he is 55 years old; that for the 1970 cohort is 1.284 over the same length of time. The number 2.679 is obtained by dividing 3.439 by 1.284. I construct this for the rest of the columns. For the data (see columns three to five), the greatest flattening occurs in the life-cycle earnings profile for high school-educated individuals, the result for all individuals falls between the other two. For the model, however (see columns six to eight), the greatest flattening occurs in the the life-cycle earnings profile for college-educated individuals, with, the result for all individuals again falling between the other two. Accordingly, the model cannot reproduce the fact that the greatest flattening is observed for high school-educated individuals; however through composition effect, flattening for all individuals is consistently between that for the high school- and college-educated individuals. In section 1.4.4, I discuss why the model generates more flattening for college-educated individuals than high school-educated individuals.

Table 1.5 notes the proportion of the data explained by the model. Columns two and three calculate for the data and the model, respectively, the change in the lifetime increment of 1970 cohort from 1940 cohort. The change is consistently smaller for the model than the data, which means that the model is not able to explain fully the flattening observed in the data. Taking the change produced by the model (0.219) divided by the change observed in data (0.626) for high school-educated individuals, I arrive at 0.349. Therefore, the model is able to explain 35 percent of the flattening observed in the data for high school-educated individuals; 67 percent of that for college-educated individuals and 50 percent for all individuals.

1.4.3 College attainment

Figure 1.7 shows the college attainment generated by the model (red line) and that in the data for 45 year olds (dotted black line).[12] I choose 45 year olds because at this age, educational attainment would be stabilized. Recall that the model has

[12] I use the 40 to 49-year-old group from the Census data to allow for more data points.

not been calibrated to match college attainment for 1940 cohort, and yet, since this is a story of changing composition, it is critical that the model gets the right number of people into college for exercise to have validity.

Based on Figure 1.7, it appears that the model is doing well. The model is able to generate increasing college attainment over time. It slightly under-predicts the data for the 1970 cohort and over-predicts for the 1960 cohort; however, on average, the model is able to correctly predict college attainment. In Section 1.3, I mentioned that the model generates infinite earnings easily, which means the model is extremely sensitive to small changes in parameters. Because of this sensitivity, the model easily overselects the number of people who go to college; making it difficult to correctly predict college attainment.

1.4.4 More on the flattening of earnings profile

The model is not able to replicate that there is greater flattening in the life-cycle earnings profile for high school-educated individuals than college-educated individuals. In the following paragraphs, I discuss why this is so.

From the model, the two cohorts are differ in two aspects: (i) their initial wage rate per unit of human capital (w_τ), which is higher for recent cohort, and (ii) their conditional average ability, which is lower for the recent cohort, where (ii) is caused by the changing composition of the labor force induced by increasing college attainment. Here, I remove the composition effect by keeping ability levels constant across cohorts.

In Figures 1.8, I plot the life-cycle earnings profile for an individual with ability level, a, such that he will have only high school education in both 1940 and 1970 cohort. I do the same for a college-educated individual in Figure 1.9. The ability level chosen for the high-school and college-educated individual is $a = 0.02$ and $a = 0.04$, respectively. The magenta color represent the 1940 cohort and blue represents the 1970 cohort.

From Figure 1.8, the life-cycle earnings profile does not change whether he is from the 1940 or the 1970 cohort. Examination of Equation 1.1 shows why this is so. Equation 1.1 reduces to Equation 1.2 after some algebra. For the high school-educated individual's optimal time spent in human capital accumulation on the job, n, to be cohort specific, it can only come from the level of initial human capital he has when he enters the labor market. However, since $H^{hs} = z_h a$, the sequence of optimal time spent in human capital accumulation on the job for a high school-educated individual is only age dependent but not cohort specific. Since n is only age dependent, the optimal sequence of accumulated human capital on the job will also not be cohort specific. The only cohort specific element in Equation 1.3 $w_{\tau+j-1}$ factors out after normalization. Consequently, the life-cycle earnings profile for a high school-educated individual with ability level $a = 0.02$ does not vary across cohort:

$$\begin{aligned} n_{\tau,j}(h) &= \left[\frac{a\phi\beta_{\tau,j+1}}{rw_{\tau+j-1}}\right]^{\frac{1}{1-\phi}} h^{-1} \\ &= \left(\frac{a\phi g}{r}\right)^{\frac{1}{1-\phi}} \left(\frac{\zeta^{T-j}-1}{\zeta-1}\right)^{\frac{1}{1-\phi}} h^{-1}, \end{aligned} \tag{1.2}$$

where

$$\zeta = \left(\frac{1}{r}\right)(1-\delta)\,g.$$

$$Earnings^{hs}_{\tau,j} = w_{\tau+j-1} * h_j * (1 - n_j) \tag{1.3}$$

In contrast, the college-educated individual enters the labor market with a human capital level of $H^{col}_{\tau}\left(a, e^{col}_{\tau}, H^{hs}\right)$ where the optimum expenditure on college education, e^{col}_{τ}, is cohort specific. Therefore, applying the same argument as before (see Equation 1.4), life-cycle earnings profile for a college-educated individual with ability level $a = 0.04$ flattens across cohorts. (Figure 1.9)

$$Earnings^{col}_{\tau,j} = w_{\tau+j-1} * h_{\tau,j} * (1 - n_{\tau,j}) \tag{1.4}$$

Hence, the model not only has a composition effect, it also has a level effect from w_{τ}. The level effect is explained as follows. Individuals will be willing to spend more on college education, (e^{col}_{τ}), because they know they will face a higher earnings in the future. The connection comes from the fact that human capital accumulated in college is increasing in e^{col}_{τ}. As the flattening of high school life-cycle earnings profile in the model is only affect by composition effect and college life-cycle earnings profile is affected by both the composition and level effects. Because of this difference, the model generates a more flattening in the life-cycle earnings profile of college-educated individuals than that of high school-educated individuals.

1.5 Discussion

In this section, I discuss two other cases apart from the baseline case. In Case 1, I calibrate the model to the *conditional* life-cycle earnings profiles of the 1940 cohort. In Case 2, I allow the growth rate of the wage rate per unit of human capital (g) to be differentiated by education, i.e., g^{hs} and g^{col}.

1.5.1 Case 1

In this exercise, I calibrate the model to the conditional life-cycle earnings profiles for the 1940 cohort and examine the amount of flattening produced. I am also interested in how well the model tracks data in the unconditional life-cycle earnings profile of the 1940 cohort and how much flattening is produced. This exercise reverses the experiment discussed in the baseline calibration. In a way, this is a simpler exercise because instead of relying on the model to endogenously produce the split between education groups, this exercise directly fixes the model to the conditional earnings profiles of the 1940 cohort and evaluates extend of the flattening produced by the mechanism. This exercise also serves to check the baseline calibration. The question I am interested in answering here is whether having a different calibration strategy for the model produces significantly different results.

Tables 1.6 and 1.7 report the values of the calibrated parameters the fit of the calibration, respectively. Since the targets in Case 1 are different from those in the baseline calibration, it is unsurprising that values of the parameters are different. However, the values are not vastly dissimilar to the ones in Table 1.2.

1.5.1.1 Flattening of the conditional earnings profiles

Figure 1.10 shows for college-educated individuals and Figure 1.11 for the high school-educated individuals the life-cycle earnings profile produced by model (solid lines) and data (dotted lines). Table 1.8 reports the values from these figures. The magenta and blue lines represent the 1940 and 1970 cohorts, respectively. Remember that this exercise calibrates the model to the conditional earnings profiles of the 1940 cohort then lets the mechanism run to produce the average earnings profile for the subsequent cohorts. Therefore, it is not surprising that the simulated life-cycle earnings profiles for the 1940 cohort closely match those produced by the data. Again, however, the simulated profile is not able to match the increase in average college earnings at age 35 very well. Otherwise, the model is pretty good at matching all of the other points on the life-cycle earnings profile.

Table 1.9 reports the proportion of the data explained by the model. It explains 36 percent of the flattening observed in the data for high school-educated individuals and 67 percent of that for college-educated individuals. The numbers are 35 percent and 67 percent, respectively in the baseline.

1.5.1.2 Flattening of the unconditional earnings profile

Figure 1.12 shows the results for the unconditional life-cycle earnings profile. For the 1940 cohort, the model tracks the data very well. (See the "ALL" column in Table 1.8.) Recall that I did not target any unconditional moments in this calibration. Given this fact, the model performs extremely well and gives confidence in the validity of this calibration. The model is able to explain 47 percent (50 percent in the baseline) of the flattening in the unconditional life-cycle earnings profile. This number again lies between those for college- and high school-educated individuals.

1.5.1.3 College attainment

Figure 1.13 shows college attainment generated by the model (solid red line) compared with the data (dotted black line). Recall that the model is not calibrated to the fraction of individuals with college degree for the 1940 cohort. Thus, the model predicts the right proportion of college-educated individuals in the 1940 cohort extremely well. The mechanism also reasonably predicts to the proportion of college-educated individuals for the 1970 cohort: 0.32 in the data and 0.361 (0.364 in baseline) in the model.

Overall, the results for the baseline and Case 1 is not significantly different. This is good because even though the choice of experiment taxes the model to different extents, the model does not produce totally different results which, would raise question about the validity of the baseline calibration if it did.

1.5.2 Case 2

In Case 2, I explore the possibility that human capital produced by high school- and college-educated individuals are non-substitutable. To achieve this, I let the growth rate of wage rate per unit of human capital for high school- and college-educated individuals to grow at g^{hs} and g^{col}, respectively.

Since Case 2 is really baseline case with one additional parameter, I employ the calibration strategy of the baseline in this exercise. In addition, I use the proportion of college-educated individuals at age 35 in the 1940 cohort to pin down the prices of human capital.

The results of this exercise are shown in Figures 1.14, 1.15, 1.16, and 1.17, and reported in Tables 1.10, 1.11 1.12 and 1.13.

The results of Case 2 do not warrant extensive discussion here because allowing for different growth rates introduces little change in the values of the parameters. Also, the flattening observed in both the conditional and the unconditional life-cycle earnings profiles change only slightly. However, the take away point is this: Calibrating to earnings of the 1940 cohort tells us that calibrated value for g^{hs} is lower than g^{col}. I am interested to know if the difference is economically significant. I took an average of g^{hs} and g^{col} and rerun the model. Figures 1.18, 1.19, 1.20, and 1.21 show the earnings profiles plotted with the average growth rate. The earnings profiles do not change drastically. The proportion of data that is explained by the model is 45 percent, 31 percent and 62 percent for all, high school- and college-educated individuals, respectively. (For Case 2, the numbers are 47 percent, 35 percent and 64 percent.) The only difference is that college attainment is consistently underpredicted for almost all cohorts. Overall, I think this does not give support to the empirical observation by Goldin and Katz (2008) that skill biased technical change (SBTC) starts as early as 1940.

1.6 Conclusion

I begin the paper by documenting the flattening of the life-cycle earnings profile across cohorts. Using empirical observations: increase in average real annual earnings over time and increasing education attainment, I build a mechanism to explain the observed flattening of earnings profile. The model is able to replicate the trend observed in the data as well as produce the correct movements for conditional average abilities. In terms of the flattening of the life-cycle earnings profile, the model also performs well. In the baseline exercise, it explains 67 percent of the flattening for college-educated individuals, around 35 percent of that for high school-educated individuals and half of the flattening for all individuals. The model is also able to deliver the concavity in the life-cycle earnings profile for both education groups and consistently predicts the correct proportion of college-educated individuals. The baseline experiment is: I calibrate the model to the unconditional life-cycle earnings profile for the 1940 and leave the model to determine endogenously the conditional life-cycle earnings profile. I find that the model-simulated conditional life-cycle earnings profile is very close to that of the data for the 1940 cohort. This result gives confidence in the validity of the baseline calibration exercise. It is, however, a model artifact that more flattening is observed in the simulate profile for college-educated individuals than high school-educated individuals.

In Case 1, I take a the direct route and calibrate the model to the conditional life-cycle earnings profiles. In Case 2, I differentiate the growth rate per unit of human capital based on education. Both calibration exercises produce a decent about of flattening, although, not significantly different from the baseline case. The take away point in Case 2 is: Although calibration results give a higher price of human capital for college-educated individuals, this difference is not significant enough to cause major changes in the earnings profiles. Therefore, this does not give support to the empirical observation by Goldin and Katz (2008) that skill biased technical change starts as early as 1940s.

Overall, I would argue that this Ben-Porath type model has performed well in many aspects, albeit some its parsimonious outlook.

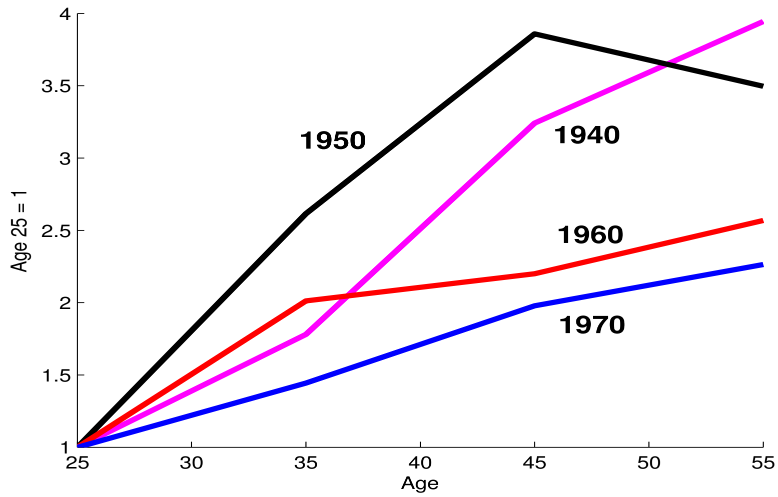

Figure 1.1: Life-cycle earnings profiles of college-educated workers by cohort and normalized to 1 at age 25

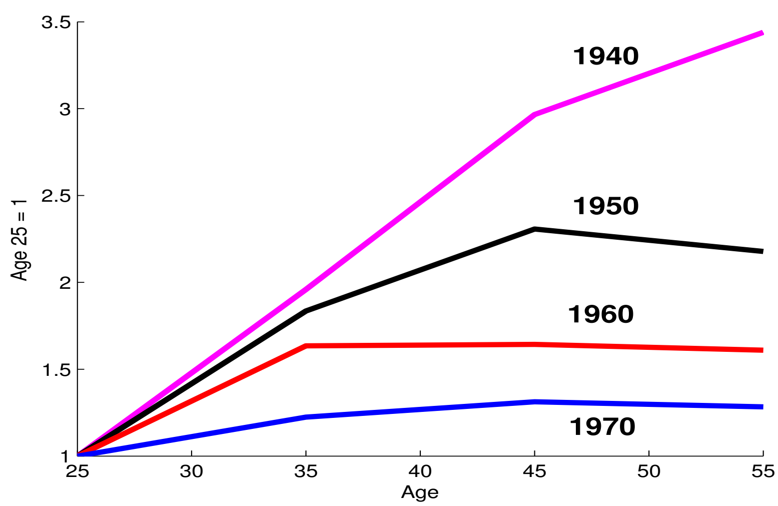

Figure 1.2: Life-cycle earnings profiles of high school-educated workers by cohort and normalized to 1 at age 25

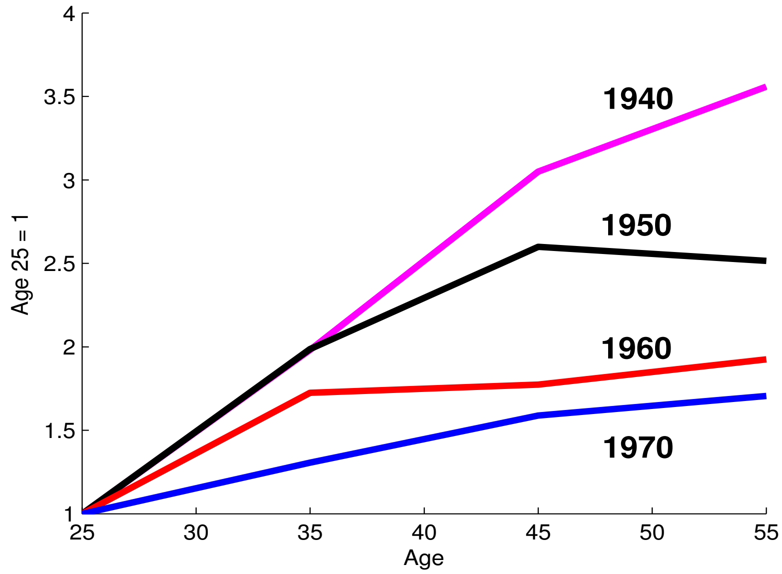

Figure 1.3: Life-cycle earnings profiles of high-school and college-educated workers by cohort and normalized to 1 at age 25

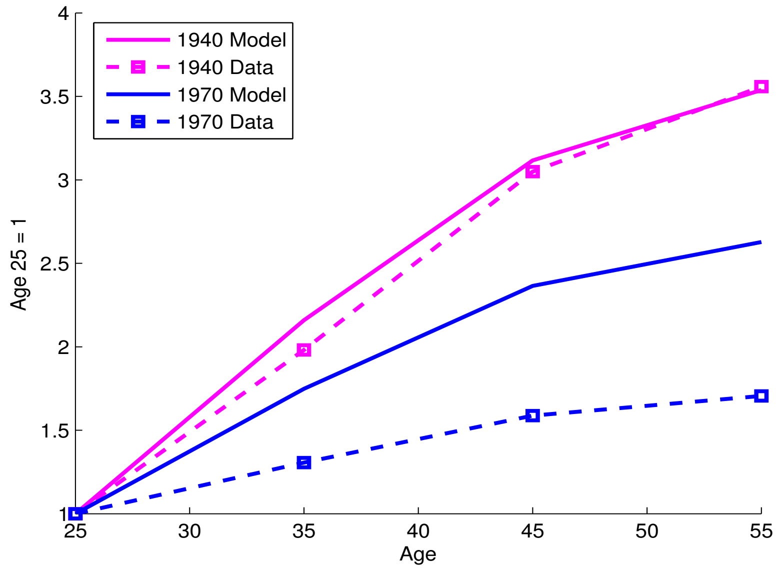

Figure 1.4: Life-cycle earnings profiles of high school- and college-educated workers by cohort – Model vs. Data (Baseline)

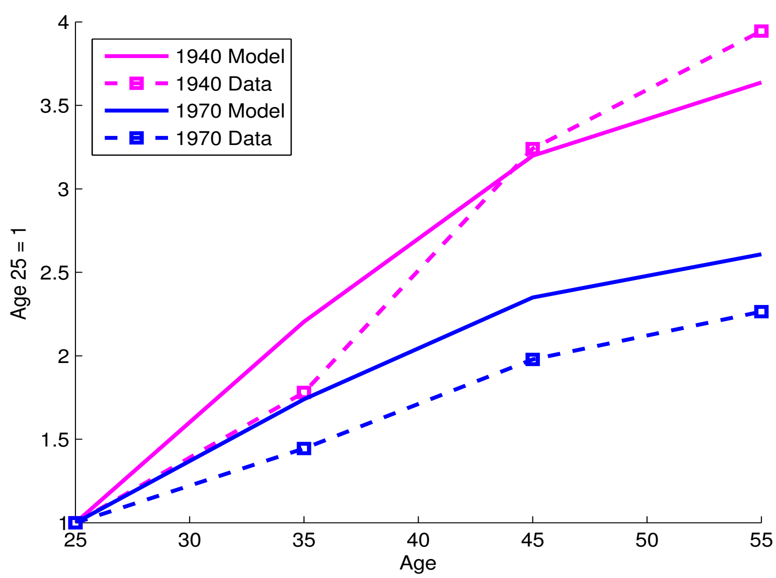

Figure 1.5: Life-cycle earnings profiles of college-educated workers by cohort – Model vs. Data (Baseline)

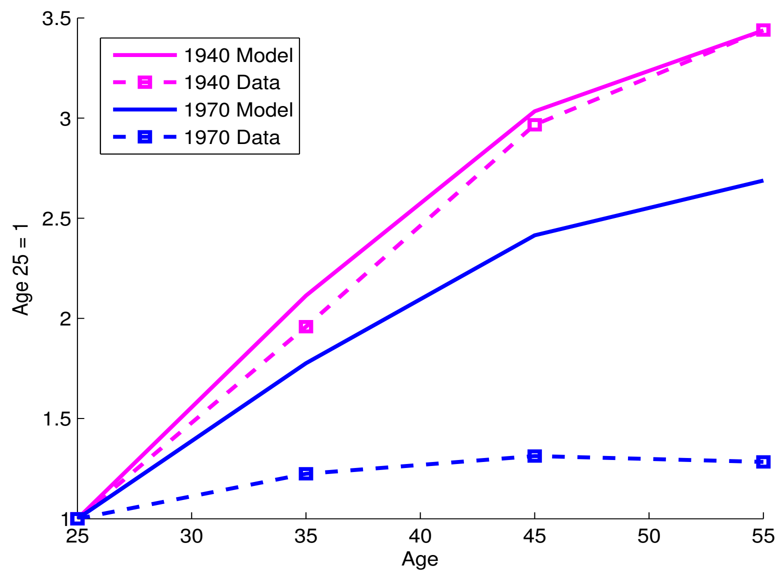

Figure 1.6: Life-cycle earnings profiles of high school-educated workers by cohort – Model vs. Data (Baseline)

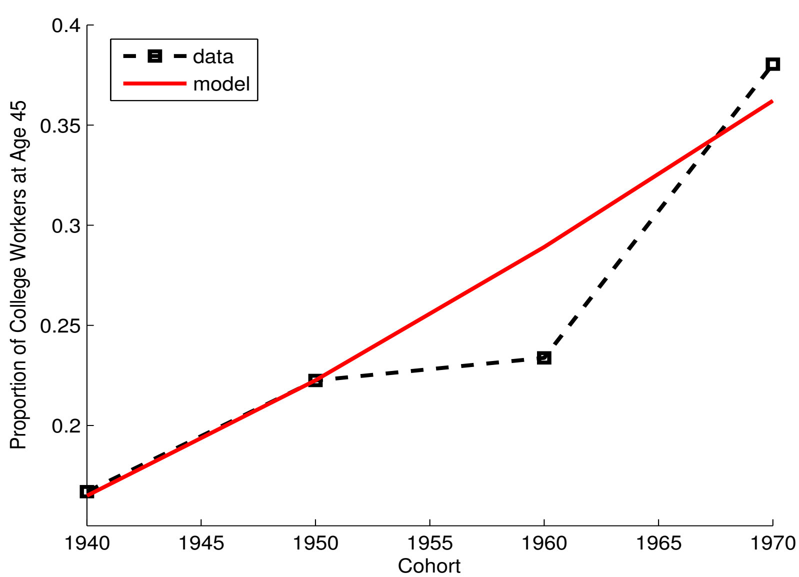

Figure 1.7: Proportion of college-educated workers by cohort at age 45 – Model vs. Data (Baseline)

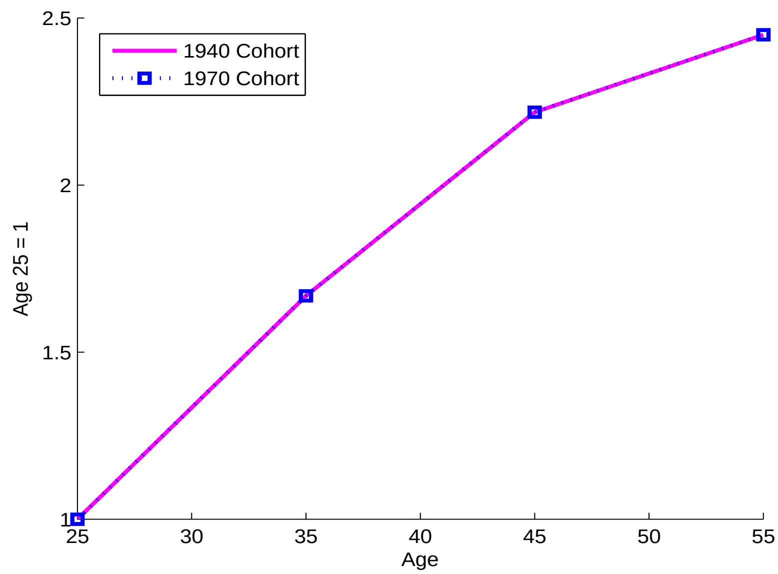

Figure 1.8: Life-cycle earnings profiles of a high school-educated worker with ability=0.02 in the 1940 and 1970 cohorts (Baseline)

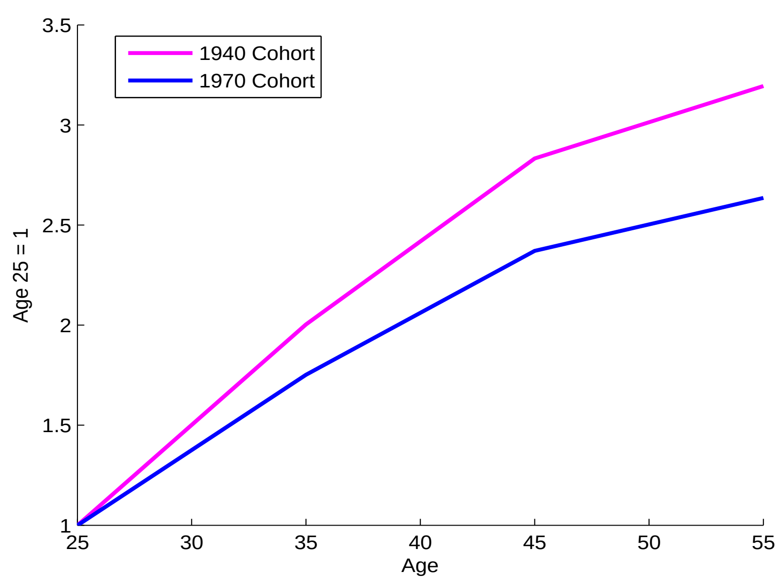

Figure 1.9: Life-cycle earnings profiles of a college-educated worker with ability=0.04 in the 1940 and 1970 cohorts (Baseline)

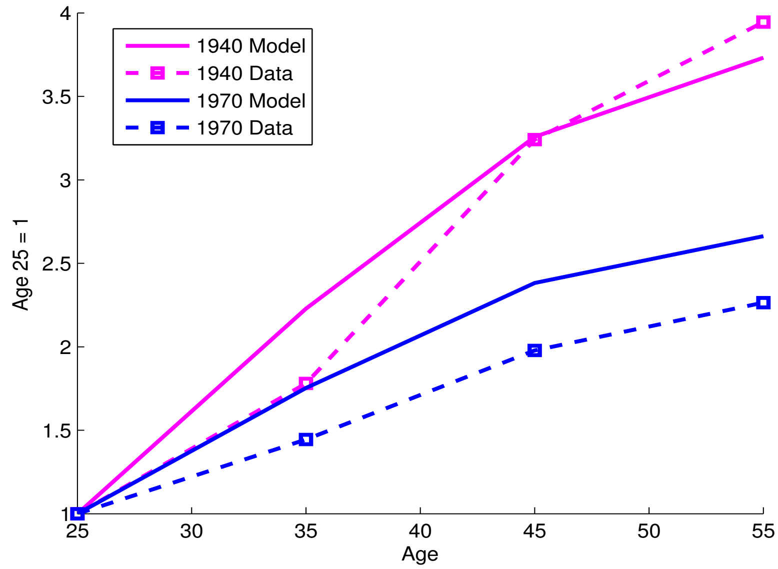

Figure 1.10: Life-cycle earnings profiles of college-educated workers by cohort – Model vs. Data (Case 1)

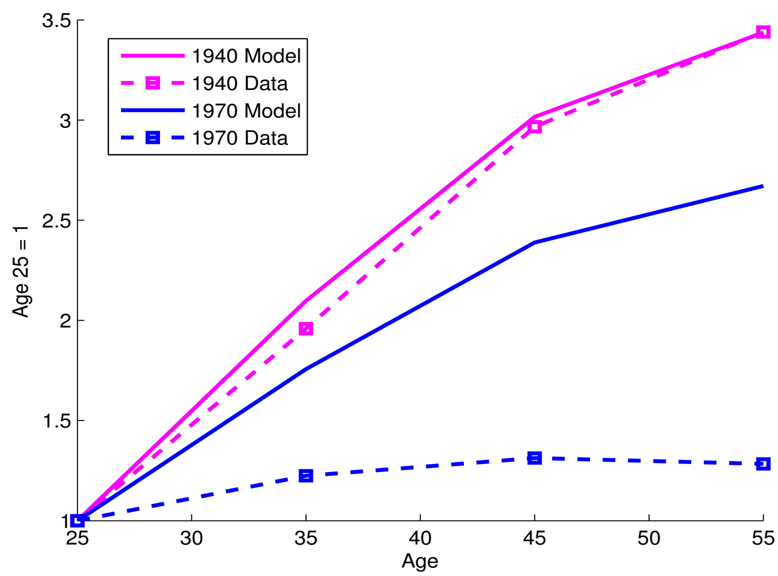

Figure 1.11: Life-cycle earnings profiles of high school-educated workers by cohort – Model vs. Data (Case 1)

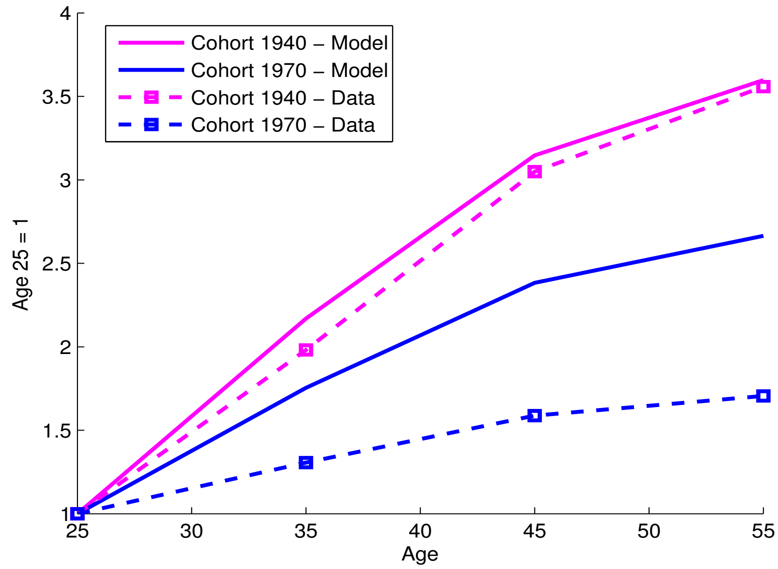

Figure 1.12: Life-cycle earnings profiles of high-school and college-educated workers by cohort – Model vs. Data (Case 1)

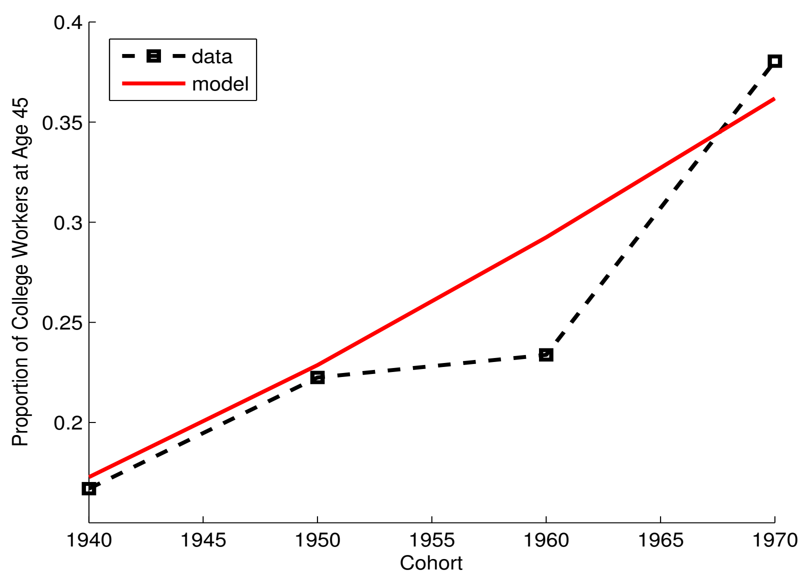

Figure 1.13: Proportion of college-educated workers by cohort at age 45 – Model vs. Data (Case 1)

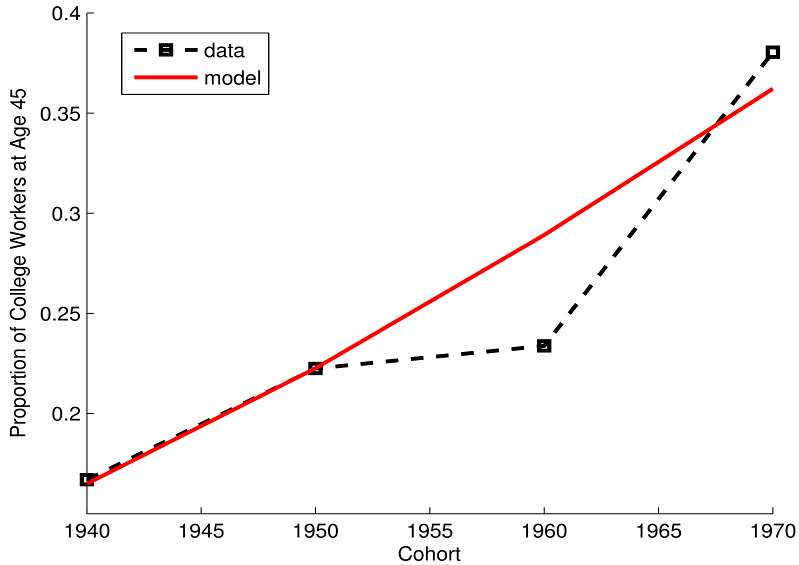

Figure 1.14: Proportion of college-educated workers by cohort at age 45 – Model vs. Data (Case 2)

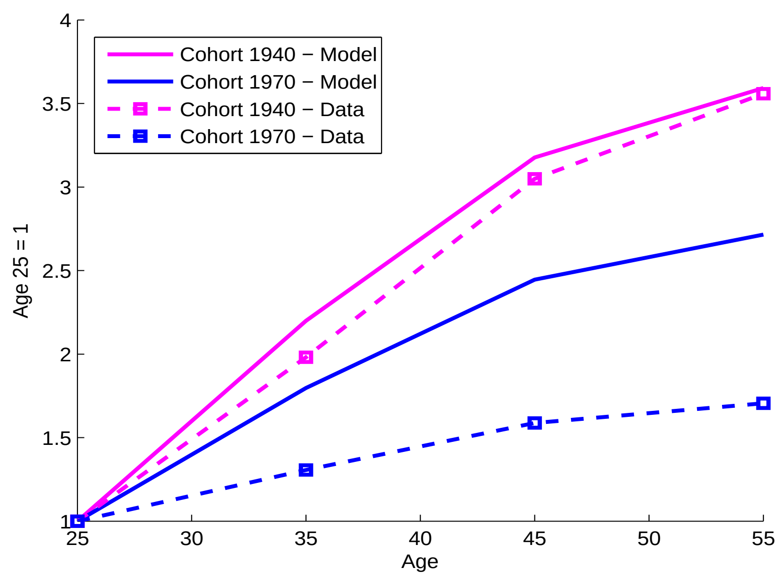

Figure 1.15: Life-cycle earnings profiles of high-school and college-educated workers by cohort – Model vs. Data (Case 2)

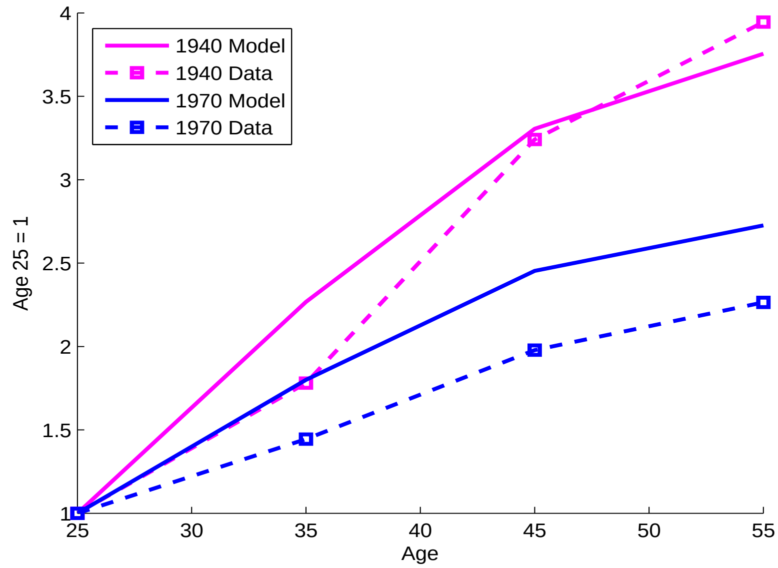

Figure 1.16: Life-cycle earnings profiles of college-educated workers by cohort – Model vs. Data (Case 2)

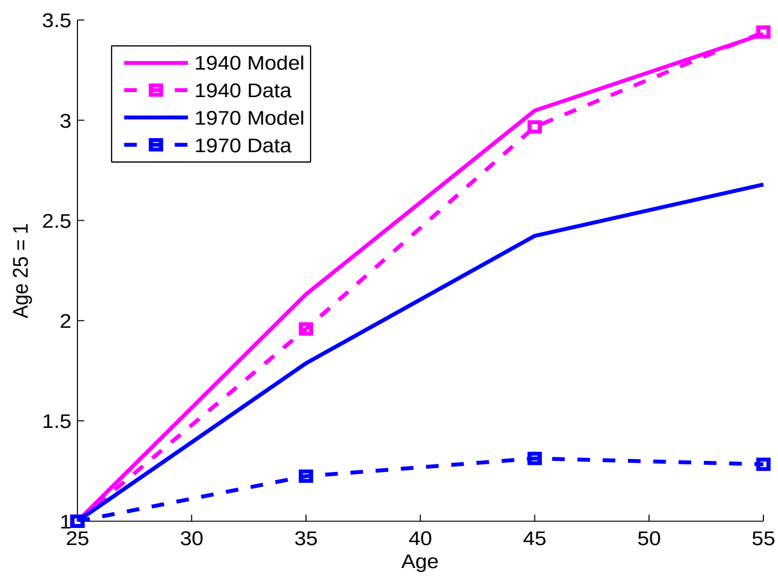

Figure 1.17: Life-cycle earnings profiles of high school-educated workers by cohort – Model vs. Data (Case 2)

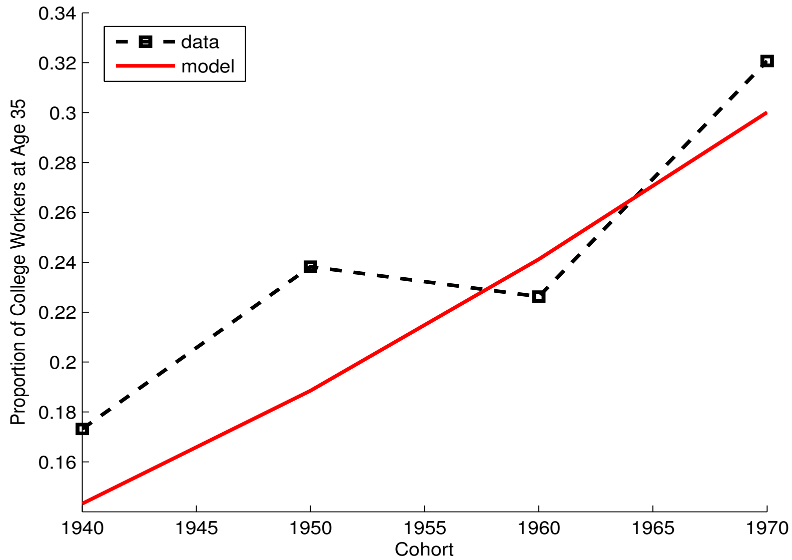

Figure 1.18: Proportion of college-educated workers by cohort at age 35 – Model vs. Data (Case 2, average ‘g’)

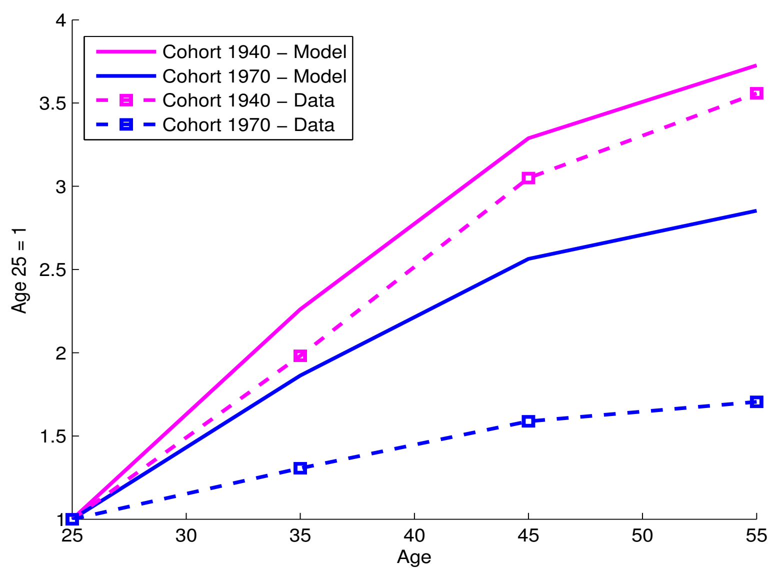

Figure 1.19: Life-cycle earnings profiles of high-school and college-educated workers by cohort – Model vs. Data (Case 2, average ‘g’)

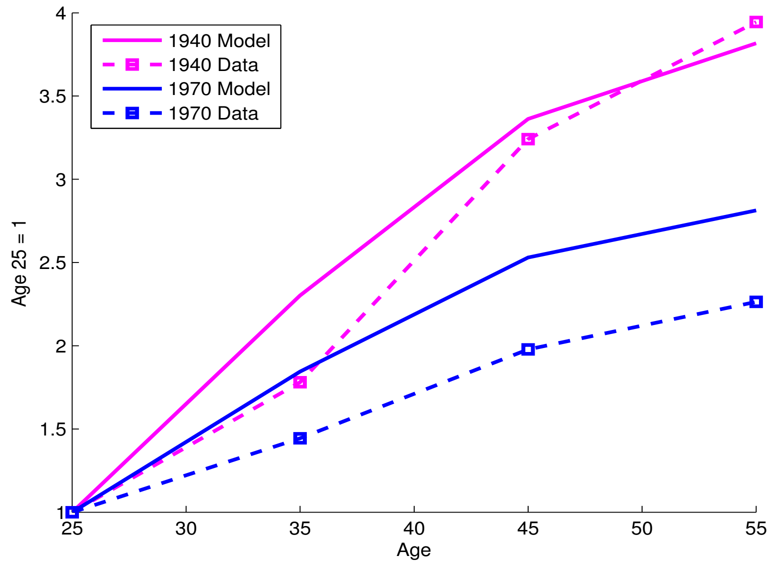

Figure 1.20: Life-cycle earnings profiles of college-educated workers by cohort – Model vs. Data (Case 2, average 'g')

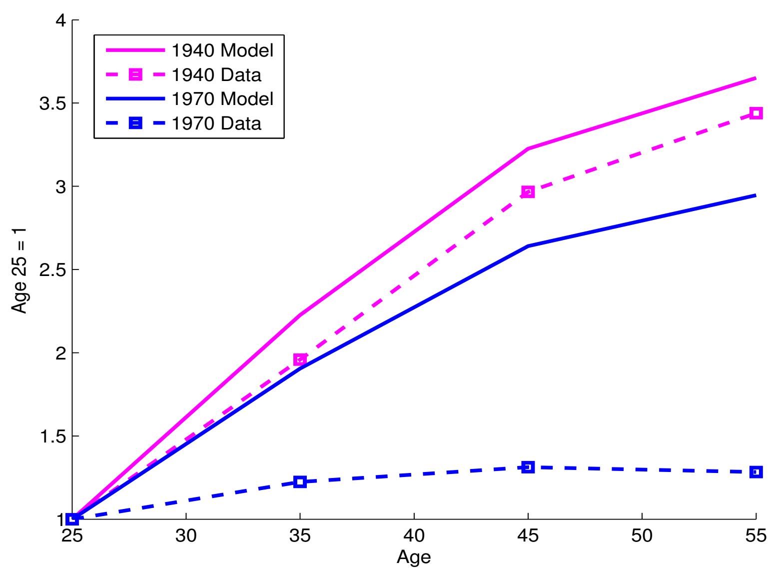

Figure 1.21: Life-cycle earnings profiles of high school-educated workers by cohort – Model vs. Data (Case 2, average 'g')

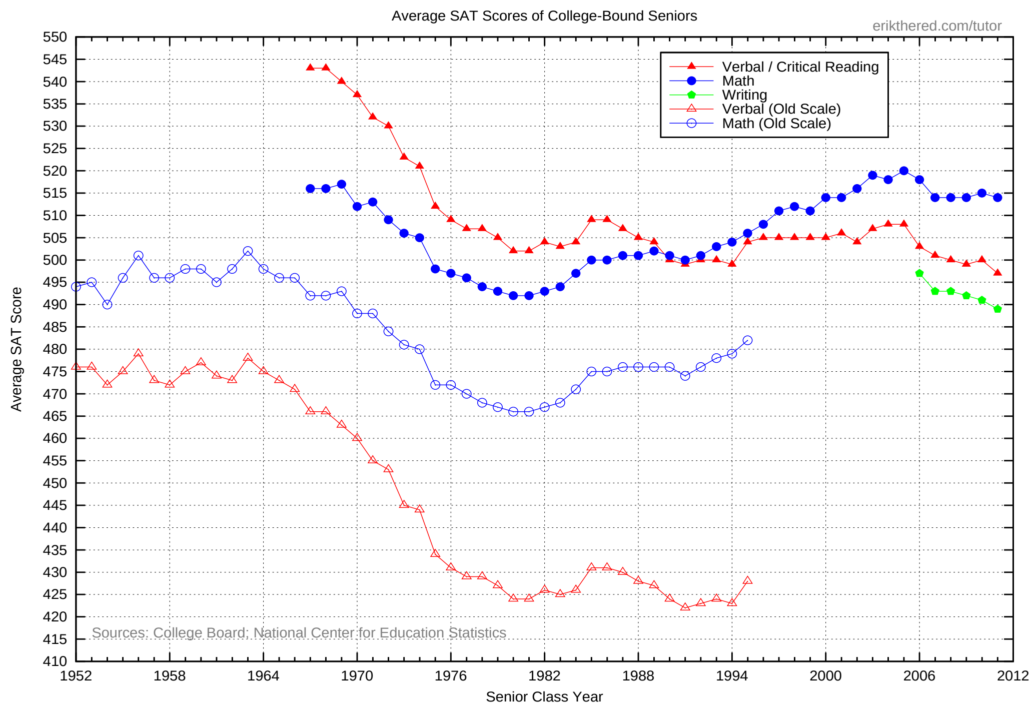

Figure 1.22: Historical average of SAT scores

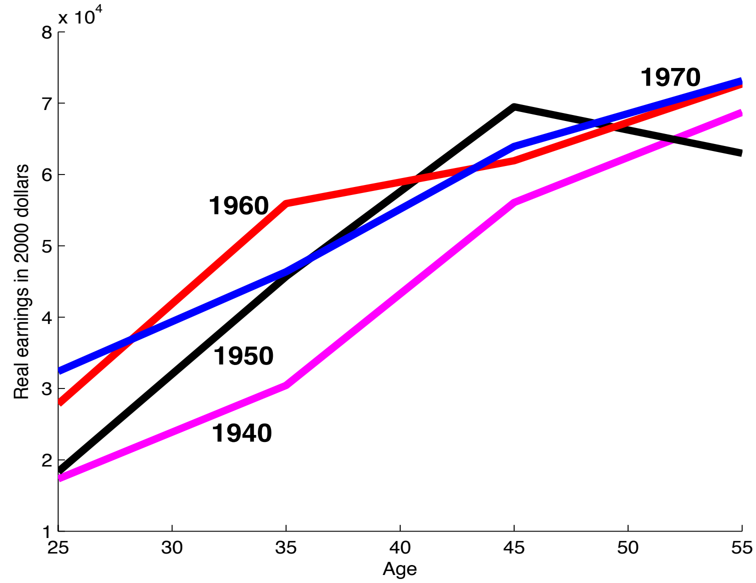

Figure 1.23: Average annual real earnings for college-educated workers by cohort

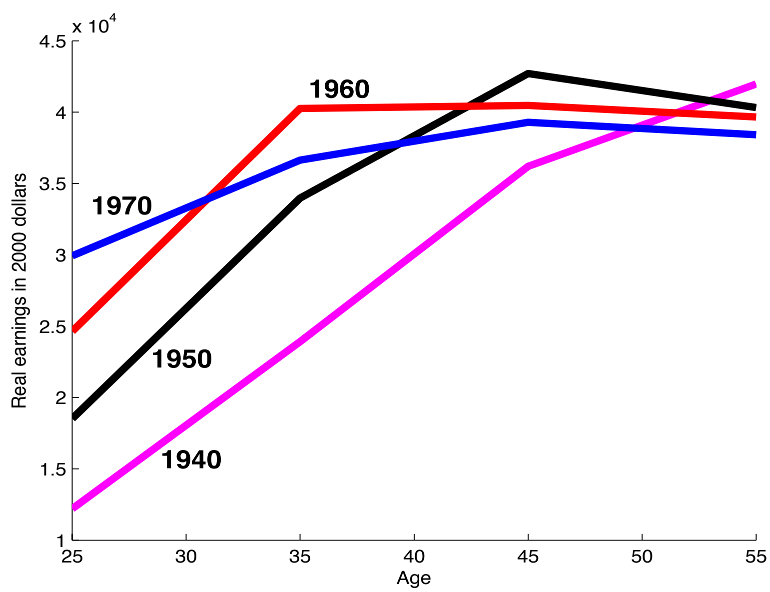

Figure 1.24: Average annual real earnings for high school-educated workers by cohort

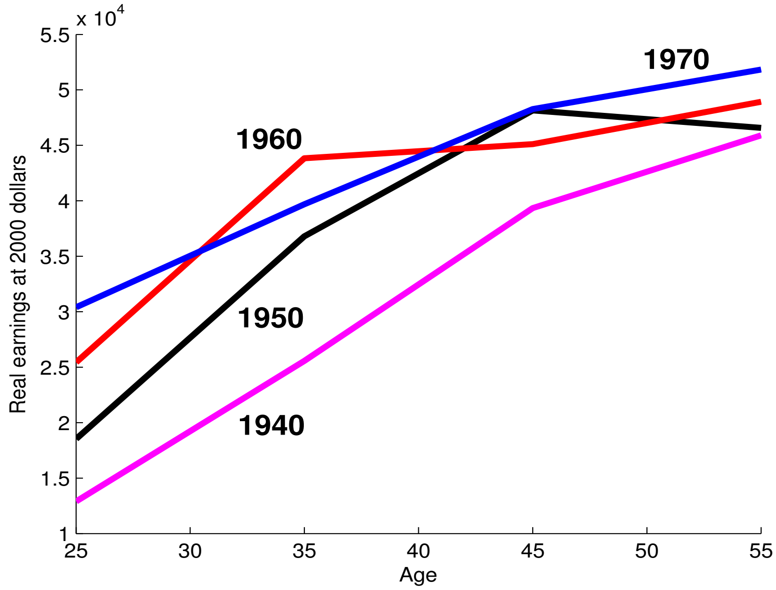

Figure 1.25: Average annual real earnings for high-school and college-educated workers by cohort

Table 1.1: Earnings profiles according to education

$Cohorts$		HS	COL	$HS+COL$
	Age 25	Age 55		
1940	1	3.439	3.945	3.559
1970	1	1.284	2.264	1.705
$\frac{1940}{1970}$	1	2.679	1.742	2.087

Table 1.2: Calibrated parameters (Baseline)

Parameter	Description	Value
T	number of periods	42
r	interest rate	1.05
s	time spent in college	4
δ	depreciation rate	0.0114
ϕ	on-the-job HC accumulation function parameter	0.6500
η	college HC accumulation function parameter	0.4900
z	college HC accumulation function productivity parameter	3.3082
z_h	productivity parameter common to all workers	1.0111
w_1	initial wage rate per unit of human capital	4.2429
g	growth rate of w_1	1.0061
$\hat{\mu}$	mean of lognormal ability distribution	-3.7293
$\hat{\sigma}$	standard deviation of lognormal ability distribution	0.3611

Note: Highlighted parameters are calibrated

Table 1.3: Calibration (Baseline)

Target	Model	Data	$\frac{Data}{Model}$
$\frac{\mu_{35}}{\sigma_{35}}$	3.049	2.914	0.956
$\frac{\mu_{45}}{\sigma_{45}}$	2.681	2.927	1.094
$\frac{\mu_{55}}{\sigma_{55}}$	2.583	2.824	1.094
$\Omega^{25}_{25,35,45}$	0.159	0.166	1.041
$\Omega^{35}_{35,45,55}$	0.245	0.231	0.942
$\Omega^{45}_{25,45,55}$	0.407	0.401	0.985
$\Omega^{55}_{25,35,55}$	0.528	0.544	1.030
$\frac{E^{90}_{25}}{E^{10}_{25}}$	5.698	5.357	0.940

Table 1.4: Earnings profiles – Model vs. Data (Baseline)

	Age 25	*Age* 55					
		Data			Model		
Cohorts		*HS*	*COL*	*ALL*	*HS*	*COL*	*ALL*
1940	1	3.439	3.945	3.559	3.439	3.637	3.500
1970	1	1.284	2.264	1.705	2.687	2.606	2.593
$\frac{1940}{1970}$	1	2.679	1.742	2.087	1.280	1.395	1.348

Table 1.5: Proportion of data explained (Baseline)

	$\frac{X_{1970}-X_{1940}}{X_{1940}}$		$\frac{\frac{X^m_{1970}-X^m_{1940}}{X^m_{1940}}}{\frac{X^d_{1970}-X^d_{1940}}{X^d_{1940}}}$
	Data	Model	
High school	0.626	0.219	0.349
College	0.426	0.283	0.665
All	0.521	0.258	0.496

Note: Columns 2 and 3 show the change in lifetime increment of 1970 cohort from 1940 cohort. Column 4 shows the proportion of data explained by the model.

Table 1.6: Calibrated parameters (Case 1)

Parameter	Description	Value
T	number of periods	42
r	interest rate	1.05
s	time spent in college	4
δ	depreciation rate	0.0114
ϕ	on-the-job HC accumulation function parameter	0.6424
η	college HC accumulation function parameter	0.4944
z	college HC accumulation function productivity parameter	3.3665
z_h	productivity parameter common to all workers	1.0175
w_1	initial wage rate per unit of human capital	3.9184
g	growth rate of w_1	1.0063
$\hat{\mu}$	mean of lognormal ability distribution	-3.7718
$\hat{\sigma}$	standard deviation of lognormal ability distribution	0.3992

Note: Highlighted parameters are calibrated

Table 1.7: Calibration (Case 1)

Target	Model	Data	$\frac{Data}{Model}$
$\Omega^{h25}_{25,35,45}$	0.164	0.169	1.032
$\Omega^{h35}_{35,45,55}$	0.245	0.234	0.955
$\Omega^{h45}_{25,45,55}$	0.405	0.401	0.990
$\Omega^{h55}_{25,35,55}$	0.526	0.538	1.022
$\Omega^{c25}_{25,35,45}$	0.154	0.166	1.077
$\Omega^{c35}_{35,45,55}$	0.242	0.199	0.821
$\Omega^{c45}_{25,45,55}$	0.408	0.396	0.971
$\Omega^{c55}_{25,35,55}$	0.536	0.587	1.094

Table 1.8: Earnings profiles – Model vs. Data (Case 1)

	Age 25	*Age* 55					
		Data			Model		
Cohorts		*HS*	*COL*	*ALL*	*HS*	*COL*	*ALL*
1940	1	3.439	3.945	3.559	3.437	3.731	3.597
1970	1	1.284	2.264	1.705	2.671	2.663	2.636
$\frac{1940}{1970}$	1	2.679	1.742	2.087	1.287	1.401	1.350

Table 1.9: Proportion of data explained (Case 1)

	$\frac{X_{1970}-X_{1940}}{X_{1940}}$		$\frac{\frac{X^m_{1970}-X^m_{1940}}{X^m_{1940}}}{\frac{X^d_{1970}-X^d_{1940}}{X^d_{1940}}}$
	Data	Model	
High school	0.626	0.223	0.355
College	0.426	0.286	0.672
All	0.521	0.245	0.471

Note: Columns 2 and 3 show the change in lifetime increment of 1970 cohort from 1940 cohort. Column 4 shows the proportion of data explained by the model.

Table 1.10: Calibrated parameters (Case 2)

Parameter	Description	Value
T	number of periods	42
r	interest rate	1.05
s	time spent in college	4
δ	depreciation rate	0.0114
ϕ	on-the-job HC accumulation function parameter	0.6475
η	college HC accumulation function parameter	0.4895
z	college HC accumulation function productivity parameter	3.4450
z_h	productivity parameter common to all workers	1.0020
w_1	initial wage rate per unit of human capital	4.0290
g^{col}	growth rate of w_1 for college	1.0054
$\hat{\mu}$	mean of lognormal ability distribution	-3.6950
$\hat{\sigma}$	standard deviation of lognormal ability distribution	0.3520
g^{hs}	growth rate of w_1 for high school	1.0048

Note: Highlighted parameters are calibrated

Table 1.11: Calibration (Case 2)

Target	Model	Data	$\frac{Data}{Model}$
$\frac{\mu_{35}}{\sigma_{35}}$	3.127	2.914	0.932
$\frac{\mu_{45}}{\sigma_{45}}$	2.678	2.927	1.093
$\frac{\mu_{55}}{\sigma_{55}}$	2.560	2.824	1.103
$\Omega^{25}_{25,35,45}$	0.157	0.166	1.057
$\Omega^{35}_{35,45,55}$	0.245	0.231	0.941
$\Omega^{45}_{25,45,55}$	0.409	0.401	0.980
$\Omega^{55}_{25,35,55}$	0.529	0.544	1.029
$\frac{E^{90}_{25}}{E^{10}_{25}}$	5.418	5.357	0.989
Λ	0.170	0.173	1.020

Table 1.12: Earnings profiles – Model vs. Data (Case 2)

	Age 25	*Age* 55					
		Data			Model		
Cohorts		*HS*	*COL*	*ALL*	*HS*	*COL*	*ALL*
1940	1	3.439	3.945	3.559	3.430	3.755	3.595
1970	1	1.284	2.264	1.705	2.679	2.726	2.715
$\frac{1940}{1970}$	1	2.679	1.742	2.087	1.280	1.377	1.323

Table 1.13: Proportion of data explained (Case 2)

	$\frac{X_{1970}-X_{1940}}{X_{1940}}$		$\frac{\frac{X^m_{1970}-X^m_{1940}}{X^m_{1940}}}{\frac{X^d_{1970}-X^d_{1940}}{X^d_{1940}}}$
	Data	Model	
High school	0.626	0.219	0.349
College	0.426	0.274	0.643
All	0.521	0.244	0.469

Note: Columns 2 and 3 show the change in lifetime increment of 1970 cohort from 1940 cohort. Column 4 shows the proportion of data explained by the model.

CHAPTER 2
FEDERAL AID AND U.S. EARNINGS INEQUALITY

2.1 Introduction

There have been notable changes in the U.S. wage structure since the 1970s. Figures 2.1 and 2.2 show the strong increase in the college premium over time. While most papers in the literature focus on skill-biased technical change, this paper explores the role of federal aid as a possible source of inequality. My proposed theory is consistent with the following observed facts: (i) the increase in college attainment, (ii) the establishment and the subsequent increase in federal aid for college education, and (iii) the increase in the college premium.

Figure 2.3 shows the college attainment of 25- to 34-year-old individuals over time. From this picture, it is clear that education attainment, particularly college attainment, has increased. In this paper, I introduce another fact: increasing federal aid toward college education over time. The Higher Education Act of 1965 established a system of grants and loans to help students finance their college education. Federal aid makes up as much as 68 percent of all the direct financial aid available to students, which makes the federal government the single biggest player in aid provision.[1] Figure 2.4 shows the breakdown of federal aid over time. As the figure shows, grants and loans make up the biggest component in federal aid. Figures 2.5, 2.6, and 2.7 show the total amount of aid, the number of recipients and the average amount received

[1]Trends in Student Aid 2003.

per recipient for both Pell grants and GSL. The total volume and the number of recipients received have grown over time. The increase in recipients for Guaranteed Student Loans (GSL) far outgrew the increase in college enrollment (Figure 2.8). Figure 2.9 shows the percentage of students receiving Pell grants and GSL since 1965. By 1977, about 35 percent of the students received Pell grants and the same percentage is achieved for GSL in 1981. By 2003, as many as 60 percent of full-time students had one or more sources of federal aid to help finance their college education.[2] Figure 2.10 shows the comparison between the average federal aid received and the cost of obtaining various types of post-high-school education; the average amount per recipient indicates that the role of federal aid in college financing cannot be neglected.

Inequality can be defined in many ways. In the context of education differentials, it is known as the college premium. In this paper, I think of inequality generated through education selection in terms of the ratio between the present value of average lifetime earnings for college-educated individuals to that of a high school-educated individuals. I choose this ratio metrics because when an individual makes an education choice, he is essentially making a comparison between the stream of future earnings from each education level. Taking the costs and benefits into account, he will choose the level that gives the highest future earnings. This is the view taken in this paper.

Earnings can be understood as follows: $earnings = w * h(a, q, H^{hs})$, where w is the wage rate per unit of human capital, h is human capital, a is ability, q is federal aid, and H^{hs} is the level of initial human capital. Human capital, h, is increasing in

[2]Ibid.

both a and q. There are three potential ways to increase earnings. The first is though the wage rate (w) and the second and third are through human capital (h). In this paper, the mechanism focuses on the effect on the college premium through federal aid (q) and ability (a). The effect working through q is called the federal aid effect and through a is known as selection effect.

I construct a model of education choice and human capital accumulation to explain the relationship between inequality and federal aid. The production of human capital requires goods input (i.e., the expenditures of individuals for a college education). Federal aid is modeled as a subsidy: For every unit of goods invested, some proportion is given for free, which makes the accumulation of human capital cheaper than otherwise. This will induces individuals to accumulate more human capital. A larger accumulated stock of human capital will result in higher earnings. Since federal aid is available only to college students, this can disproportionately increase average college earnings relative to average high-school earnings.

In the model, individuals are heterogeneous with respect to their ability to accumulate human capital and their initial human capital. Individuals with higher ability are able to accumulate human capital at a faster rate and thereby have more earnings than individuals with lower ability. Since college education entails some cost, only people with the ability to reap high enough returns choose to go to college. Consequently, for a level of initial human capital, individuals are sorted into education groups according to their ability: Only an individual with high ability will go to college. However, as earnings of college-educated individuals increase, it may increasingly become worthwhile for more individuals to pay for a college education. This increase in attending college changes the composition of individuals in both high-school and college groups in terms of average ability. The selection effect on college premium is thus ambiguous. The quantitative questions still remain: In which direction does the selection effect has on the college premium, and which effect – the selection or federal effect – dominates? My model can almost fully explain the rise in the college premium. Federal aid alone explains around 70 percent of the increase in college premium.

The rest of the paper proceeds as follows. Section 2.2 introduces the model. Section 2.3 discusses the calibration strategy and presents parameter values. Section 2.4 discusses the results and Section 2.5 concludes.

2.2 Model

2.2.1 Environment

The economy is populated by overlapping generations of individuals. There are two levels of education: high school and college. Each individual enters the model with a high school diploma and chooses whether to go to college. If he decides to attend college, he will go for s periods and then enter the labor market. If he chooses not to attend college, he will enter the labor market immediately. Once an individual chooses to enter the labor market, he cannot return to school. In this model, human capital is accumulated only in school and not in the labor market. Each individual enrolled in college is a full-time student. Each working individual has one unit of time endowment per period that he supplies to the market inelastically.

Time is discrete and indexed by $t = 0, 1, ..., \infty$. Each individual lives for T periods and is ex-ante heterogeneous in terms of ability, $a \in R_+$, and initial human capital, $H^{hs} \in R_+$. The ability of each individual includes both cognitive and non cognitive skills and describes his capacity to learn and to accumulate human capital. Initial human capital is the level of human capital of an individual when he enters the model. Since each individual enters the model with a high school diploma, initial human capital is the level of human capital an individual has when he finishes high school. Human capital accumulates with more education but ability does not. Each individual knows his level of ability and initial human capital before schooling and consumption decisions are made. Ability and initial human capital are distributed according to the cumulative distributions, A and H, respectively. Both distributions are time invariant.

There is a perfect credit market in which each individual can borrow and save at a constant exogenous rate $r - 1$. There is no uncertainty, and preferences are defined on consumption sequences only. The wage rate per unit of human capital is given exogenously and is assumed to admit a constant growth rate g. Since there are no borrowing constraints, uncertainty, and leisure, the problem of maximizing lifetime utility is equivalent to maximizing lifetime earnings.

2.2.2 Technologies

There is no accumulation of human capital on the job. Once an individual decides to enter the labor market, his level of human capital is fixed for the rest of his model life. An individual that chooses to go to college will accumulate human capital based on the following accumulation function, and his level of human capital will remain constant after he leaves college:

$$h_{\tau,t+1} = a^{\phi^c} \left(h_{\tau,t}^{\theta} [(1+q_\tau)x_t]^{1-\theta} \right)^{\zeta} \tag{2.1}$$
$$h_{\tau,1} = H^{hs},$$

where ϕ^c, θ, and $\zeta \in (0,1)$ and x_t is the amount of goods committed toward accumulation of human capital in college at time t, q_τ is the subsidy from the government for college education for cohort τ, h_t is the accumulated human capital inherited from the last period, and h_{t+1} is the accumulated human capital in this period. Human capital does not depreciate in this model.

The wage rate per unit of human capital (w) is exogenous and is assumed to grow at a constant rate g:

$$w_{t+1} = g w_t.$$

Similar to the Ben-Porath model, earning inequality between and within education levels can only be generated by the difference in the level of human capital and investment behavior. This is because both college- and high school-educated individuals command the same wage rate.

2.2.3 Individual's problem

An individual enters the model with a high school diploma and chooses whether to have college education. He will choose the schooling level that affords him the highest net lifetime earnings. Once he enters the labor market, he cannot return to school. There is no decision to be made in the labor market.

2.2.3.1 Net lifetime earnings (High school)

An high school-educated individual from cohort τ with ability a and initial human capital H^{hs} will start work at age $j = 1$. The first wage rate that workers from cohort τ face as they enter the labor market is w_τ. There is no decision for the high school-educated individual. The maximized value of net lifetime earnings is then

$$U_\tau^{hs}(H^{hs}) \;\; = \;\; \left\{ w_\tau H^{hs} \sum_{t=1}^{T} \left(\frac{g}{r}\right)^{t-1} \right\}.$$

2.2.3.2 Net lifetime earnings (College)

A college-educated individual from cohort τ with ability a and initial human capital H^{hs} starts work at age $j = s + 1$. He accumulates human capital in college. After college, his stock of human capital remains constant. For every period in college, the individual chooses the amount of goods (x_t) he wants to invest in the accumulation of human capital.

The maximization problem is

$$U_\tau^{col}(a, H^{hs}) = \max_{\{x_t\}_{t=1}^{s}} \left\{ w_\tau h_{\tau,s+1} \sum_{t=s+1}^{T} \left(\frac{g}{r}\right)^{t-1} - \sum_{t=1}^{s} \left(\frac{1}{r}\right)^{t-1} x_t \right\}$$

subject to

$$
\begin{aligned}
h_{\tau,t+1} &= a^{\phi^c}\left(h_{\tau,t}^{\theta}[(1+q_\tau)x_t]^{1-\theta}\right)^{\zeta} \\
h_{\tau,1} &= H^{hs},
\end{aligned}
$$

where $U_\tau^{col}\left(a, H^{hs}\right)$ is the maximum net lifetime earnings of a college-educated individual from cohort τ with ability level a and initial level of human capital $H^h s$.

After some algebraic manipulation, this problem can be expressed as

$$
U_\tau^{col}(a, H^{hs}) = \max_x \left\{a(s)A(s)(1+q_\tau)^{B(s)}x^{B(s)}\left(H^{hs}\right)^{D(s)}W_\tau(s) - xC(s)\right\}
$$

where

$$
\begin{aligned}
A(s) &= \left(\frac{r}{\theta\zeta}\right)^{\sum_{t=1}^{s}(t-1)(1-\theta)\zeta(\theta\zeta)^{s-t}} \\
a(s) &= \left(a^{\phi^c}\right)^{\sum_{t=1}^{s}(\theta\zeta)^{s-t}} \\
B(s) &= \sum_{t=1}^{s}(1-\theta)\zeta(\theta\zeta)^{s-t} \\
C(s) &= \sum_{t=1}^{s}\left(\frac{1}{\theta\zeta}\right)^{t-1} \\
D(s) &= (\theta\zeta)^s \\
W_\tau(s) &= w_\tau\sum_{t=s+1}^{T}\left(\frac{g}{r}\right)^{t-1}.
\end{aligned}
$$

The first-order condition with respect to x is

$$
x = \left[\frac{a(s)A(s)(1+q_\tau)^{B(s)}W_\tau(s)\left(H^{hs}\right)^{D(s)}B(s)}{C(s)}\right]^{\frac{1}{1-B(s)}}
$$

$$
x_{\tau,t} = \left(\frac{r}{\theta\zeta}\right)^{t-1}x_\tau, \text{ where } t = 1, ..., s.
$$

The optimal net lifetime earnings for a college worker is expressed as

$$U_\tau^{col}(a, H^{hs}) = \kappa(s)\left[\left(\frac{a(s)A(s)(1+q_\tau)^{B(s)}W_\tau(s)\left(H^{hs}\right)^{D(s)}}{C(s)^{B(s)}}\right)^{\frac{1}{1-B(s)}}\right],$$

where

$$\kappa(s) = B(s)^{\frac{B(s)}{1-B(s)}} - B(s)^{\frac{1}{1-B(s)}}.$$

2.2.4 Schooling decision

An individual compares net lifetime earnings between a college and high-school education and decides whether to attend college according to the following decision rules:

$$\begin{aligned} U_\tau^{hs}(H^{hs}) &< U_\tau^{col}(a, H^{hs}) - \text{choose college} \\ U_\tau^{hs}(H^{hs}) &> U_\tau^{col}(a, H^{hs}) - \text{no college} . \end{aligned}$$

In particular, the unique cohort specific threshold a_τ^* is given by

$$U_\tau^{hs}(H^{hs}) = U_\tau^{col}(a, H^{hs}).$$

2.2.5 Mechanism of the model

It is worthwhile to spend some time discussing the mechanism of the model. In my model where labor is supplied inelastically to the market so earnings consist of a wage rate per unit of human capital, w, and the level of human capital, $h(a, q, H^{hs})$. The contemporaneous college premium is the ratio between average earnings of college-educated individuals to that of high school-educated individuals at a point in time for a single cohort:

$$\text{College premium} = \frac{Earnings_{\tau,j}^{col}}{Earnings_{\tau,j}^{hs}},$$

where

$$\text{Earnings} \;=\; wh(a, q, H^{hs}).$$

There are two potential ways to change college premium. The first is through wage rate per unit of human capital (w). The way is to distinguish between the wage rate for college (w^{col}) and high school (w^{hs}). When we allow for $w^c > w^h$ at every point in time, we have skill-biased technical change (SBTC).

The second way to change the college premium is through the mechanism of the model. The mechanism relies on the changes in the level of human capital across cohorts. There are at least two ways that human capital can change. The first is through the ability threshold. Equation 2.2 characterizes the ability threshold for a particular cohort τ:

$$a^*_\tau = \left\{ \frac{\left(H^{hs}\right)^{1-\frac{D(s)}{1-B(s)}} \sum_{t=1}^{T} \left(\frac{g}{r}\right)^{t-1}}{w_\tau^{\frac{B(s)}{1-B(s)}} \left(1+q_\tau\right)^{\frac{B(s)}{1-B(s)}} \kappa \left[\left(\frac{A(s)}{c(s)^{B(s)}}\right) g^3 \sum_{t=s+1}^{T} \left(\frac{g}{r}\right)^{t-1} \right]^{\frac{1}{1-B(s)}}} \right\}^{\frac{1-B(s)}{\phi^c \sum_{t=1}^{s} (\theta\zeta)^{s-t}}} \tag{2.2}$$

where we know

$$\begin{aligned} 1 - B(s) &> 0 \\ \frac{1-B(s)}{\phi^c \sum_{t=1}^{s} (\theta\zeta)^{s-t}} &> 0 \\ \frac{B(s)}{1-B(s)} &> 0. \end{aligned}$$

For a given level of initial human capital, the ability threshold decreases as the initial wage rate per unit of human capital w_τ increases. So, more individuals with lower ability are induced to go to college. This implied that the average abilities of both high school- and college-educated individuals are decreasing over cohorts.

In the context of the college premium, when the average earnings of both college- and high school-educated individuals drop, college premium can potentially increase or decrease. I call this the selection effect: As individuals respond to the income effect from the increase in w_τ, their schooling decision changes the composition of both the pool of college- and high school-educated individuals over cohorts. It is a quantitative question whether the selection effect works for or against increasing the college premium.

The second is through accumulating more human capital for every unit of goods (x) invested in college education; I call this the federal aid effect. From Equation 2.1, we can see that for every unit of x spent on college education, the government subsidizes a fraction q such that each individual aid recipient gets a little more human capital than otherwise. Federal aid induces individuals to accumulate more human capital during college. Since federal aid is pertains only to college individuals, it increases the stock of accumulated human capital, resulting in higher college earning and therefore the college premium *ceteris paribus*. However, the presence of federal aid induces additional selection effects as a side effect. This is also evident from Equation 2.2: Individuals see the potential of "cheaper" human capital and higher future earnings. Consequently, it may be worthwhile for the "marginal" individual to attend college at the expense of incurring some cost. Technically, this decreases the cohort-specific threshold such that the average individual of both the college and high school education groups are less able to accumulate human capital over cohorts.

It is not clear from analytical solutions whether the college premium will increase. To determine the effects quantitatively, this requires proper parameterization of the model. In the next subsection, I discuss the parameterization under which individuals accumulate more human capital as the government increases federal aid $\left(\text{i.e., } \frac{\partial h_{s+1}}{\partial q} > 0\right)$.

2.2.6 Federal aid effect

In this section, I examine some properties of the accumulated human capital from college education and its effect when the subsidy, q, increases. The closed-form solution of the level of human capital at the end of an individual's college education is given by

$$h_{s+1}(a) = [A(s)a(s)]^{\frac{1}{1-B(s)}} \left[\frac{B(s)W(s)}{C(s)}(1+q_\tau)\right]^{\frac{B(s)}{1-B(s)}} \left(H^{hs}\right)^{\frac{D(s)}{1-B(s)}}.$$

Partial derivation with respect to q gives

$$\frac{\partial h_{s+1}}{\partial q} = [A(s)a(s)]^{\frac{1}{1-B(s)}} \left[\frac{B(s)W(s)}{C(s)}\right]^{\frac{B(s)}{1-B(s)}} \left(H^{hs}\right)^{\frac{D(s)}{1-B(s)}} (1+q)^{\frac{B(s)}{1-B(s)}-1}.$$

For q to have a positive effect on subsidy,

$$\frac{B(s)}{1-B(s)} - 1 > 0 \rightarrow B(s) > \frac{1}{2},$$

where the constant $B(s)$ is increasing in ζ and decreasing in θ.

As discussed above, it is unclear how human capital will ultimately change. However, for the federal aid effect to have some impact, proper parameterization should first allow for $\frac{\partial h_{s+1}}{\partial q} > 0$, then $\frac{\partial h_{s+1}}{\partial q \partial a} > 0$ should also be met so that the federal aid effect dominates the selection effect. And for the federal aid effect to be quantitatively significant, $\left[\frac{\partial h_{s+1}}{\partial q \partial a}\right]_{q>0} >> \left[\frac{\partial h_{s+1}}{\partial a}\right]_{q=0}$.

Section 2.3 discusses how calibration strategy, and Section 2.4 discusses the model simulated-results compared with observed data.

2.3 Calibration

2.3.1 Calibration strategy

This section discusses the calibration strategy. There are two steps to calibration. First, some parameters are assigned values using prior information. Then the remaining parameters are obtained by calibrating the model to key statistics of the 1960 cohort. In particular, I am interested in determining whether federal aid is quantitatively significant in explaining the rising earnings inequality in the late 1970s.

2.3.1.1 Parameters determined with prior information

One model period represents one calendar year. The model has 42 model periods ($T = 42$). This means that each individual enters the model at age 18 and exits at age 59. Retirement decision is not modeled in this paper. Upon entering the model, an individual chooses whether to obtain a college education. A college student will spend 4 years in college ($s = 4$), whereas a high school-educated individual starts employment immediately. The gross interest rate is set to $r = 1.048$.

The federal aid sequence (q_τ) is calculated from data. The sequence of q_τ is the proportion of the subsidy received for every dollar invested in college education. To find its data counterpart, I first find the total yearly cost for college education adjusted for inflation for every academic year starting from the academic year 1963-64 in a four-year college. The total cost comprises of average tuition, fees, and room and board.[3] As discussed previously, federal aid can come in several forms; Pell grants and loans are the largest two aid components. A Pell grant is conceptually close to the idea that q is free. A subsidized loan also has this similar property. Consequently, considering only Pell grants can potentially understate the increase in the impact of federal aid, especially in the 1980s when loans overtook grants as the dominant component in federal aid (see Figure 2.4); this trend still persists today. I take the average amount of Pell grants and average subsidized GSL per full-time student in the year as the dollar amount of federal aid. The dollar amount of GSL subsidized by the federal government needs to be calculated.

A student loan is subsidized in two aspects.[4] First, the interest incurred while in college is paid for by the federal government. Second, the interest paid on that loan is lower than on an unsubsidized loan. I ignore the second aspect in the calculation as the repayment plans for college loans are complex. Individuals can choose various types of repayment plans and to decide whether to lock in a fixed interest rate. Also, the additional option of consolidating college debts while still in school or after graduation can yield different interest rates paths for the same borrower. This makes the calculation of an "average" loan virtually impossible.

[3]The total cost is for a four-year college is made up of public universities and four-year public colleges. It does not include private institutions. So total cost is a weighted average (by enrollment) of the cost from these two types of institutions in the year 1967 as this is the earliest data found. Alternatively, I could use an average. However, enrollment patterns shows that more students attend public four-year colleges than public universities. Thus the total cost using average will overstate the total cost faced by an average student and understate the level proportion of federal aid, q in the 1960s.
Data source: Trends in Student Aid: 1963 to 1983.

[4]The name GSL is subsequently changed to Stafford loans.

The yearly interest payment for a subsidized loan is calculated based on the first aspect and as with a Pell grant, it is applicable only to students while they are in college. This is consistent with the model where human capital accumulation and the subsidy from the federal government, q, take place while an individual is in college. I use the unsubsidized interest rate because this is the total amount that is paid by the federal government for a college individual in a given year. The unsubsidized interest rate for GSL follows the 3-month Treasury bill interest rate plus 2.3%. I consider the interest rate as an average of the 3-month Treasury bill interest rate over the years 1963 to 2003 plus 2.3%. The average amount of interest payment for a student loan taken by a full-time student is calculated in this way.

As q is the proportion of out-of-pocket expenditures that is free. Conceptually, it should be $\frac{Subsidy}{Cost-Subsidy}$ and not $\frac{Subsidy}{Cost}$ because x in the model corresponds to out-of-pocket expenditures where the out-of-pocket expenditures is average cost for an academic year less the about of subsidy received from the federal government i.e., $(Cost - Subsidy)$. I calculate this number for every year starting from academic year 1963-64 and ending with the academic year 2002-03. In this exercise, I calculate an average growth rate g_q and let allow the subsidy q to grow at a constant rate. Figure 2.11 plots the sequence of subsidies (q) plugged into the model.

2.3.1.2 Calibrated parameters

The list of the remaining ten parameters is

$$\Psi = \left\{\theta, \zeta, \phi^c, w_1, g, \mu_a, \sigma_a, \mu_{H^{hs}}, \sigma_{H^{hs}}, \rho_{a,H^{hs}}\right\},$$

which consists of college human capital accumulation function productivity parameters (ϕ^c,θ, and ζ); the level of the wage rate per unit of human capital for the first cohort of the model (w_1); the growth rate (g) in w_1; the mean (μ_a) and standard deviation (σ_a) of the lognormal ability distribution; the mean ($\mu_{H^{hs}}$) and standard deviation ($\sigma_{H^{hs}}$) of the lognormal initial human capital distribution; and lastly, the correlation between ability and initial human capital ($\rho_{a,H^{hs}}$). These ten parameters are calibrated by solving at least ten nonlinear equations to minimize the distance between the selected data moments and their corresponding model conterparts. This is discussed explicitly in later paragraphs. Refer to Table 2.1 for a quick summary.

The human capital production function parameters such as θ, ζ, and ϕ^c govern the level of accumulated human capital. The parameter θ determines how important today's level of human capital is tomorrow. A lower value of θ places greater emphasis on today's expenditure x_t and the federal subsidy, q, and vice versa. Remember the discussion in Section 2.2.6; for the federal aid effect to have an impact, θ is ideally small. A smaller value of θ increases the average earnings of college-educated individuals relative to that for the high school-educated individuals and will induce more individuals to go to college. The parameter ζ imposes concavity to the human capital accumulation function. This, in turn, helps to limit the level of accumulated human capital for the college individual. This parameter helps to keep the college premium from exploding. Parameter ϕ^c also has the same function as ζ.

The level of the wage rate per unit of human capital for the first model cohort (w_1) is critical in pinning down the ratio of average earnings between college- and high school-educated individuals. The growth rate g is one of the parameters that will determine the extent of increase in lifetime earnings. Together θ, ζ, ϕ^c, w_1, and g determine the sequence of cohort-specific ability thresholds.

The parameters μ_a ,σ_a, $\mu_{H^{hs}}$, $\sigma_{H^{hs}}$, and $\rho_{a,H^{hs}}$ characterize the joint lognormal ability and initial human capital distribution. This governs the range and frequency of ability and initial human capital levels that enter the earning functions, but it does not alter ability thresholds. The separate determination of thresholds and distribution creates the need to properly specify the location parameter to guarantee a sensible fit between the two. This separation also produces a situation that makes matching targets singularly difficult. In particular, the direction of change in the simulated ratios is uncertain when these parameters are changes, as illustrated by the following examples.

As an example consider that for a given level of initial human capital (H^{hs}), a higher ζ means higher lifetime earnings of a college-educated individual for any cohort. However, a higher ζ results in steeper lifetime earnings when plotted against levels of ability. This means that, given no changes in the lifetime earnings of the high school-educated individual, the threshold of any cohort can be lower for a higher ζ compared with a lower one. Consequently, the mean of the ability distribution will have to adjusted lower to ensure the proportion of college attainment is matched. The resulting range of ability that enters the lifetime-earnings profile is lower than for a lower ζ. Thus the effect of an increase in ζ on earnings is unknown since ability enters the model in an non trivial manner.

2.3.2 Targets

Using prior information, I proceed to calibrate the ten remaining parameters to the 1960 cohort. The chosen targets are listed below:

1. Proportion of college-educated individuals (R1), 1 moment

2. Mean of normalized real earnings for ages 20-29, 30-39, 40-49, 50-59 (R9, R12, and R15), 3 moments

3. Standard error of normalized real earnings for ages 20-29, 30-39, 40-49, 50-59 (R19-R22), 4 moments

4. Skewness ($\frac{mean}{median}$) of real earnings for ages 20-29, 30-39, 40-49, 50-59 (R8, R11, R14, and R17), 4 moments

5. College premium of the present value (PV) of lifetime earnings (R27), 1 moment

This calibration exercise uses earnings data to pin down the parameters of the model that do not have explicit data counterpart. The main idea is that the parameters should behave in a way that is consistent with the observable outcome of education– that is, earnings observed in the labor market. Therefore, information about the parameters should be embedded in the earnings data. The mean, standard error, and skewness targets are calculated using real earnings of both college- and high school-educated individuals. Real earnings are then normalized by the mean of real earnings for the 20- to 29-year-old group. Therefore, there are only three mean moments since the mean of normalized real earnings for this group is matched by default.

All parameters jointly determine earnings, but some parameters have a greater influence some targets than others. The mean, standard error, and skewness moments help to pin down the joint distribution. Human capital parameters are also determined by the mean and skewness moments. As lifetime earnings affect college attainment, the human capital parameters are played a role in making sure that the correct proportion of people attend college. Lastly, recall that the exercise is to fix the model at 1960 cohort and see how the model performs, the wage rate per unit of human capital for the first model cohort and the growth rate is determined largely by the college premium.

Since model units are different from data units, the chosen targets are all units free. The aim of the calibration exercise is to minimize the distance between ratios calculated from Census data and their model counterparts. Therefore a measure of distance is built using both simulated and Census data. Below is a system of thirteen nonlinear equations solving for ten unknowns. For given wage rate and subsidy sequences, parameters are calibrated such that each element in Ψ simultaneously minimizes this function. This means that the ratio between observed data and their model counterparts is close to 1: $F(\Psi) = 1$.

$$F(\Psi) = \left\{ \begin{array}{lcl} 0.23 & \div & R1 \\ 1.52 & \div & R9 \\ 1.64 & \div & R12 \\ 1.70 & \div & R15 \\ 0.48 & \div & R19 \\ 0.66 & \div & R20 \\ 0.88 & \div & R21 \\ 1.03 & \div & R22 \\ 0.98 & \div & R8 \\ 1.07 & \div & R11 \\ 1.15 & \div & R14 \\ 1.19 & \div & R17 \\ 1.39 & \div & R27 \end{array} \right\}$$

The values of calibrated parameters and the fit of calibration are presented in Tables 2.1 and 2.2, respectively. Once calibrated parameters are obtained, they are substituted into the model to simulate earnings data.

2.3.3 Simulation of model data

I take a random draw from the joint lognormal ability and initial human capital distribution. This draw of ability is kept unchanged for every cohort. I could have used a new draw for each cohort but my intent is to keep each cohort strictly identical except for w_τ and q_τ.[5]

The number of random draws of ability levels and initial human capital determines the number of individuals for each cohort. Since the number of draws does not change from one cohort to another, my model is one of constant population. This means that any result from this model is not caused by the effect of increase cohort size. The model is simulated for 70,000 individuals for one age group. Since there are ten age groups for each cohort, 700,000 individuals are simulated for each birth cohort. The model is able to generate his sequence of lifetime earnings for each individual.

To make the conditions for comparison between simulated and Census data as close as possible, some considerations are made. First, we know that all college-educated individual have zero earnings for the first four periods of their model life when they are in still college. These entries must be removed before averages are calculated as average earnings are calculated only for employed workers. Second, since the top and bottom 1 percent of earnings are removed when calculating average earnings in Census data, this is also done for the simulated data. Third, the average earnings for high school-educated individuals calculated from Census data do not include earnings of individuals with less than high school or one to three years of college education. Similarly, the average earnings for college-educated individuals calculated from Census data do not include individuals with more than four years of college education. This is to ensure that observed data conform as closely to model

[5]It is observed that any draw below 40 can significantly change the results.

specification as possible.

2.4 Results

2.4.1 Baseline case

In this section, I discuss the results from the model and compare them with observed data. Recall that the exercise is to fix the model to the 1960 cohort and evaluate the effectiveness of the mechanism.

As argued previously, inequality generated from schooling decisions is perhaps best thought of in terms of lifetime inequality, because when an individual makes a decision on schooling, he considers which option will provide the highest future earnings. Using synthetic cohorts, I calculated the sequence of average earnings for college- and high school-educated individuals at ages 25, 35, 45, and 55. Applying discounting to the sequences, I construct a measure of the present value of average lifetime earnings for both college- and high school-educated individuals. The college premium is then calculated as a ratio of average lifetime earnings of college-educated individuals to that of high school-educated individuals (hereafter, this ratio is denoted as PVCP).

Figure 2.12 shows the comparison between data and model. The dashed blue line and the solid magenta line represent data and model measures of PVCP, respectively. The PVCP for the 1960 cohort matches perfectly with data as this point is a target in the calibration exercise. The mechanism is then allowed to run to evaluate the effectiveness of the mechanism. The increase in the magenta line shows that the model is able to match the increase in PVCP well. Table 2.3 calculates the numbers to Figure 2.12. Using this measure of the college premium, the model can explain the rise in the present value of lifetime college premium fully.

The measure of the college premium is customarily calculated in the literature by comparing average earnings of specific age groups of college- and high school-educated individuals. I call this the contemporaneous college premium. To see how well the model perform with respect to this measure, I calculated the sequence of the contemporaneous college premium and compare it with its model counterpart. In this measure, I take the ratio of average earnings of college-educated individuals 22 to 28 years of age to that for the high school-educated individuals 18 to 24 years of age ($\frac{E^{col}_{22-28}}{E^{hs}_{18-24}}$). The age groups are chosen so that the number of years of experience in the labor market for both education groups is the same.

Figure 2.13 shows the comparison with data. Once again, the dashed blue line represents data and the solid magenta line represents model. Contrary to PVCP, the contemporaneous college premium for the year 1960 is not calibrated to: The contemporaneous measure of college premium for 1960 does not match perfectly with data but it is reasonably close. I normalize the contemporaneous college premium from the model to that of the data (See Figure 2.14). This allows me to evaluate the extent of the increase in the contemporaneous college premium explained by the model. Table 2.4 provides these numbers. The model is able to explain 42 percent of the rise in the contemporaneous college premium. Considering the parsimonious nature of this model, I would argue that the model is performing reasonably well.

Another item of interest is the college attainment since this is a education selection story. Figure 2.15 shows the comparison of the model with data. The blue dashed line represents data and the magenta solid line is from the model. Recall that only the first observation, the college attainment for the 1960 cohort, is calibrated to. On the whole, the model is performing extremely well compared to the data with the exception of the 1970 cohort where the model produces a proportion of 0.27 and its data counterpart is 0.32.

2.4.2 Counterfactual

The aim of the paper is to quantitatively evaluate the importance of federal aid on inequality. A counterfactual exercise is simulated for cases with no federal aid (i.e., $q_\tau = 0$). The solid black lines in Figures 2.12, 2.13, and 2.14 shows the results under this scenario. Table 2.5 shows that federal aid explains around 67 percent of the rise in PVCP. A counterfactual performed on the contemporaneous college premium shows that federal aid explains approximately 25 percent of the rise in the contemporaneous college premium (See Table 2.6). Lastly, college attainment is lower when there is no federal aid (See the solid black line in Figure 2.15).

There is one potential problem with this counterfactual exercise. This stems from the fact that this is a partial equilibrium model. It is observed from the previous paragraph, when there is no federal aid, college attainment declines. However, when college attainment declines, the marginal product of the high-level skills provided by college-educated individuals may rise and there may be a possible feedback on college premium. But at the end of the day, this is an issue of the position that is taken on the elasticity of substitution between the two levels of skills: High-leveled skills provided by the college-educated individuals and low-leveled skills provided by high school-educated individuals. If the technology on the production side is linear, then the two level of skills are perfect substitutes and they command the same price. This is the case in this paper. Any specification other than linear technology will have the feedback effect and this effect is not captured by the partial equilibrium model. Lastly, it is worth mentioning that this issue does not matter for baseline results as the baseline case is consistent with the behavior of prices.

2.5 Conclusion

I begin this paper by introducing a possible federal aid effect to explain earnings inequality between college- and high school-educated individuals. I build a model of schooling choice to address the role of federal aid on the rise of earnings inequality in the late 1970s. The model features discrete schooling choice and individual heterogeneity so that people are sorted into education groups. Federal aid is modeled as a subsidy that is a proportion of what each individual chooses to spend on college. The mechanism of the model relies on both the selection and the federal aid effects. Skill-biased technological change is not build into the model: Regardless of education attainment, all individuals receive the same wage rate per unit of human capital. While it is clear that the federal aid effect increases the college premium through the channel of increased accumulation of human capital while the individuals are in college, analytically the effect of selection is not certain. It is thus a quantitative issue whether federal aid has any impact on earnings inequality.

I find that federal aid is quantitatively important in explaining the rise inequality in the late 1970s. My main results are that my mechanism can explain fully of the rise in PVCP and around 42 percent of the rise in the contemporaneous college premium. The federal aid effect alone is able to account for 67 percent and 25 percent of the rise in earnings inequality as measured by PVCP and the contemporaneous college premium, respectively. Given the parsimonious nature of the model, I would consider that the model has performed well.

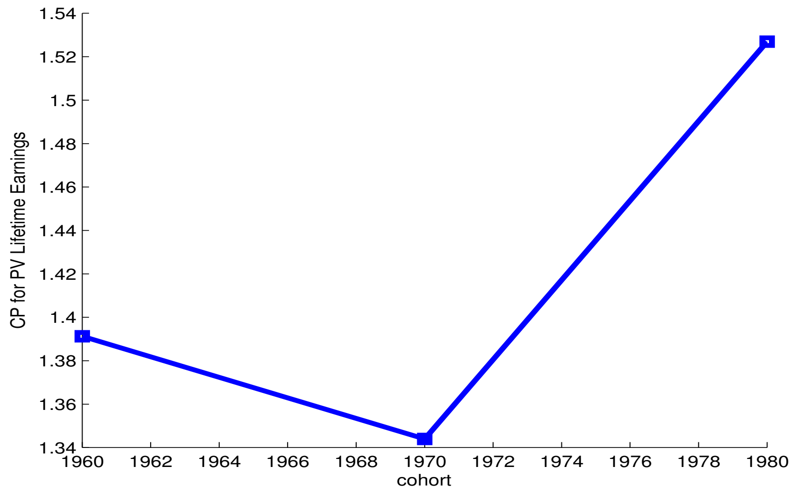

Figure 2.1: Present value of lifetime college premium

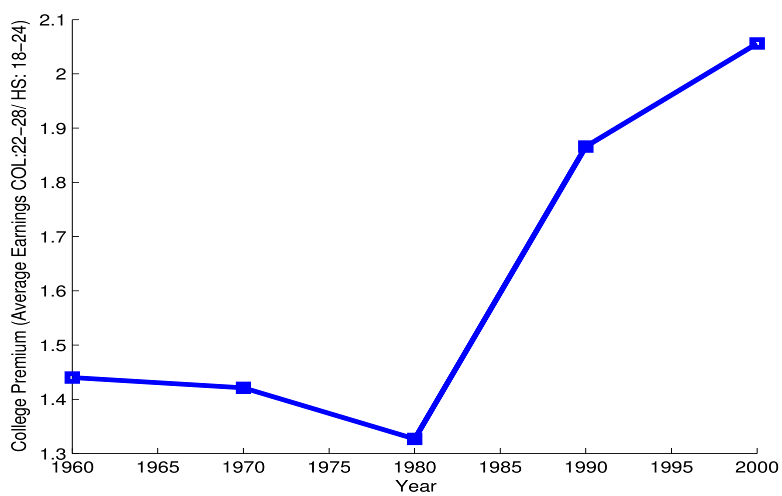

Figure 2.2: Contemporaneous college premium

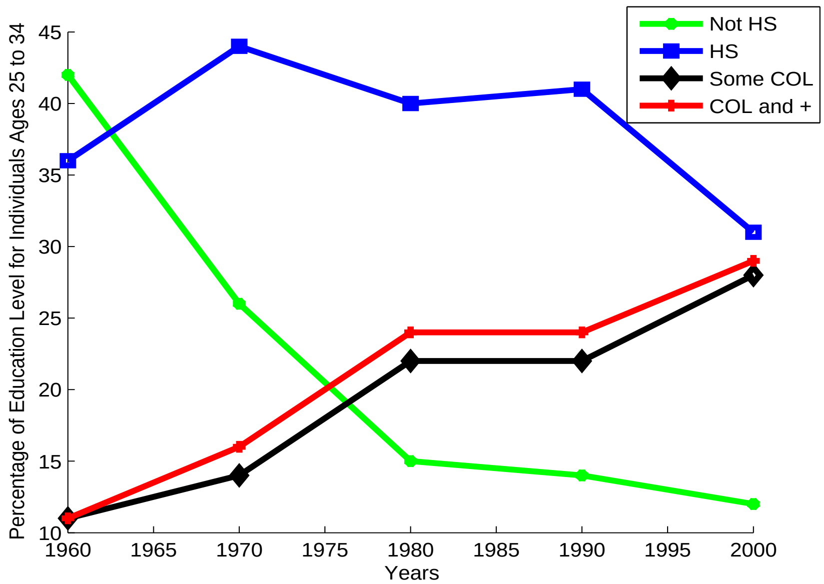

Figure 2.3: Education attainment of ages 25-34 over time

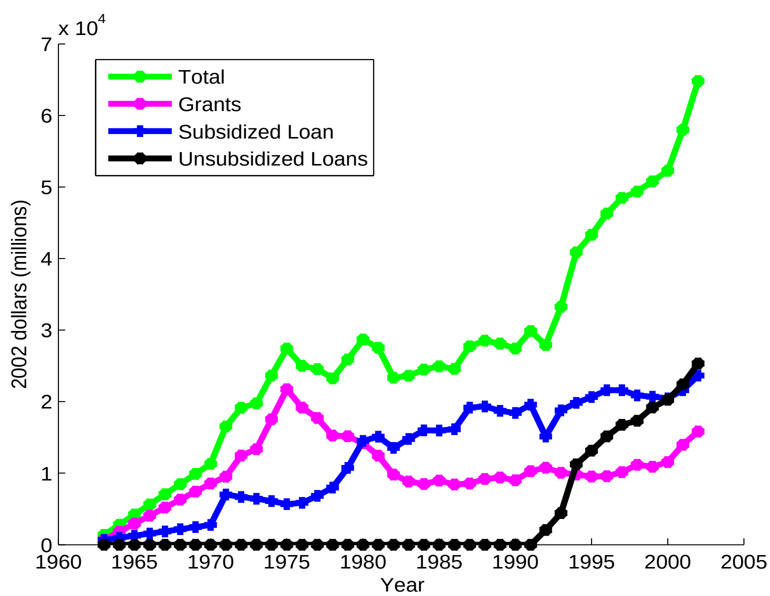

Figure 2.4: Amount of federal aid over time

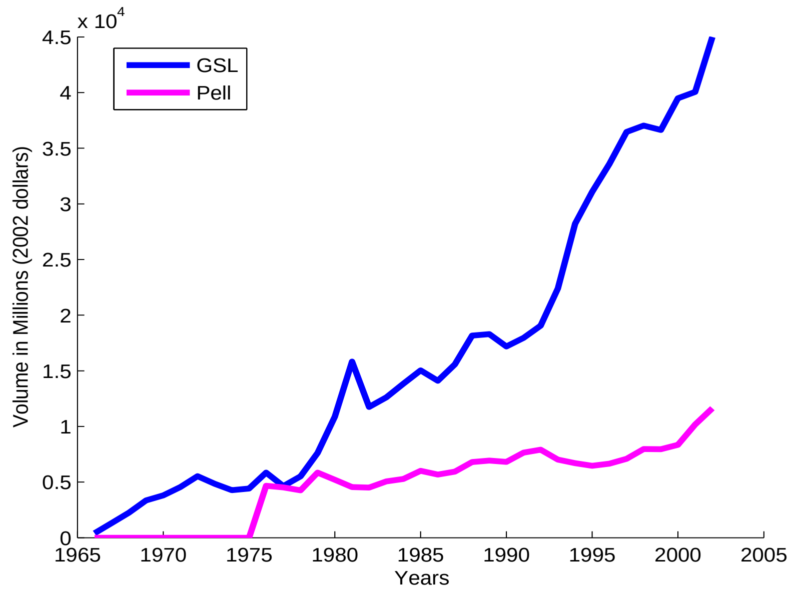

Figure 2.5: Volume of Pell grants and GSL

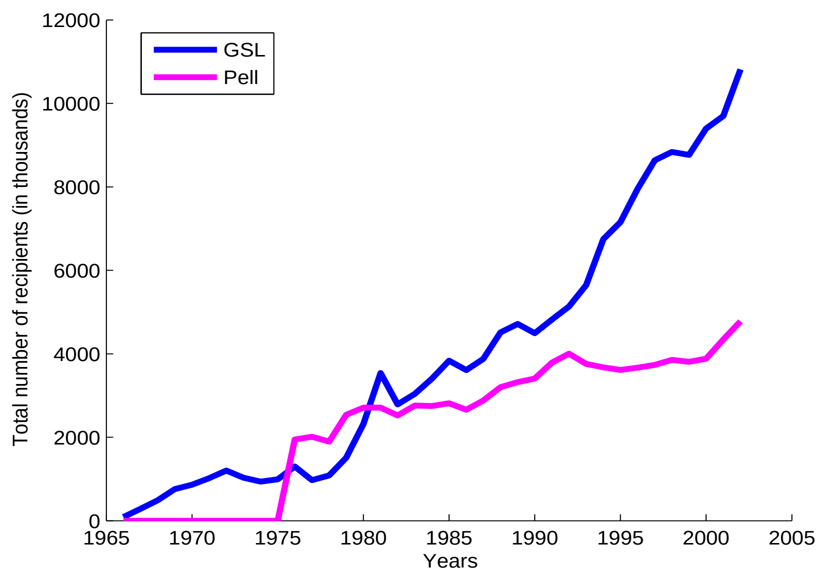

Figure 2.6: Number of Recipients for Pell grants and GSL

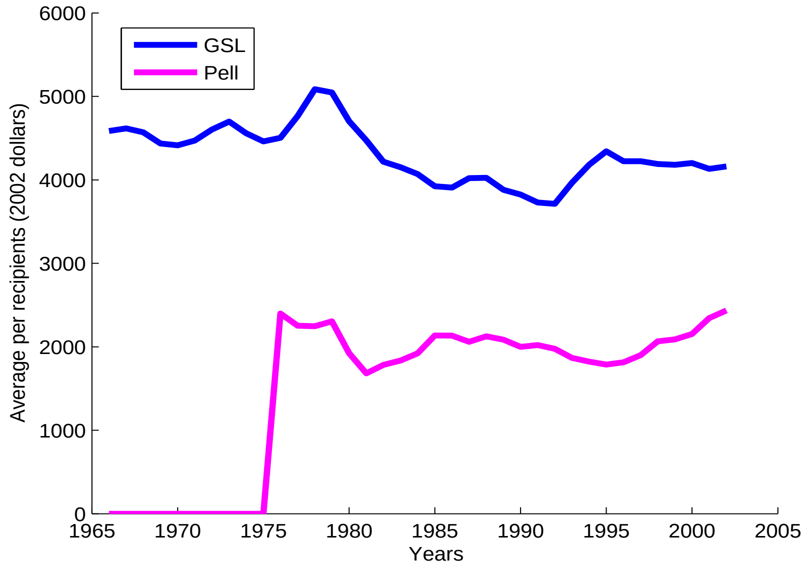

Figure 2.7: Average amount per recipient in 2002 dollars

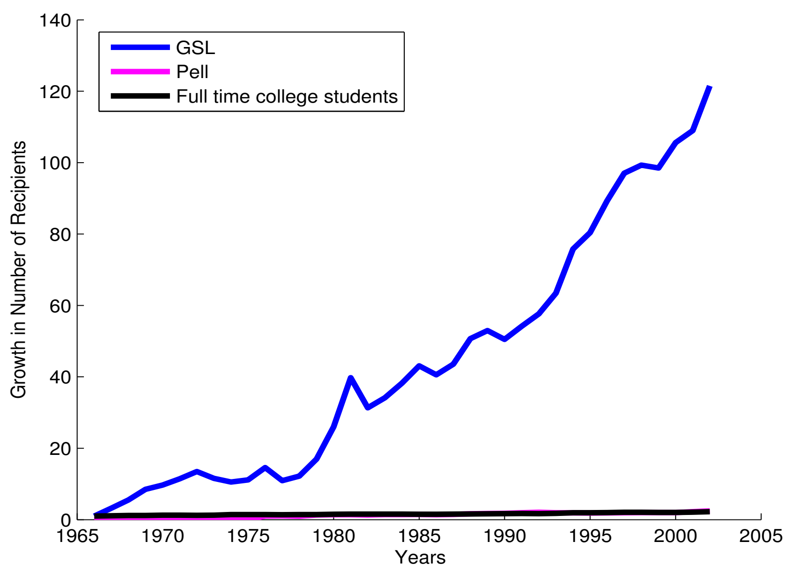

Figure 2.8: Number of recipients of federal aid and college enrollment

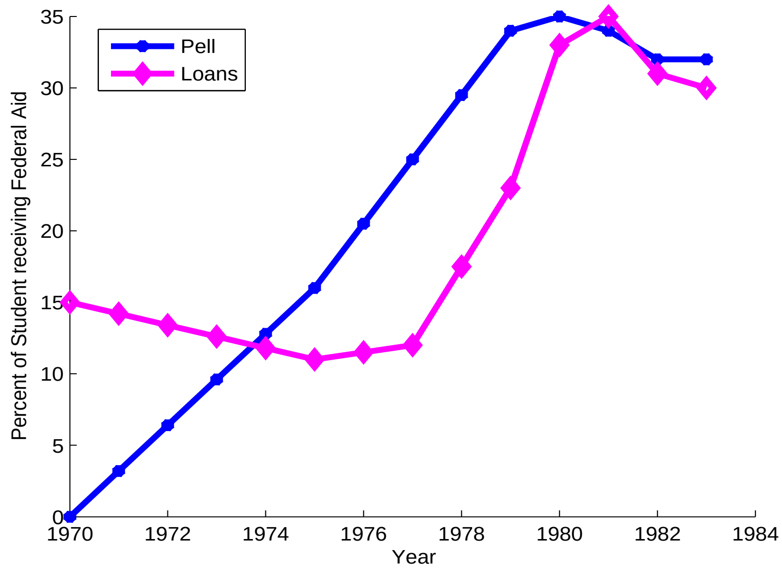

Figure 2.9: Percent of students receiving federal aid

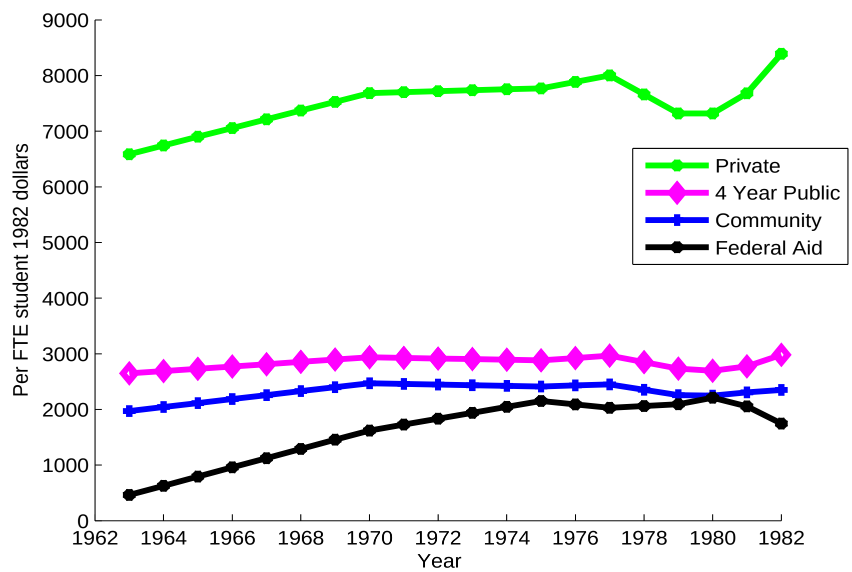

Figure 2.10: Comparing federal aid and cost of attending college

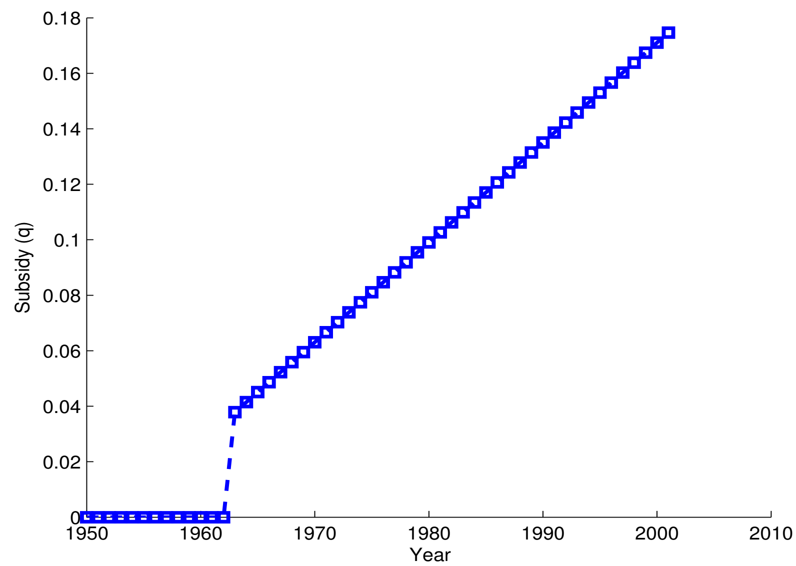

Figure 2.11: The sequence of subsidy, q

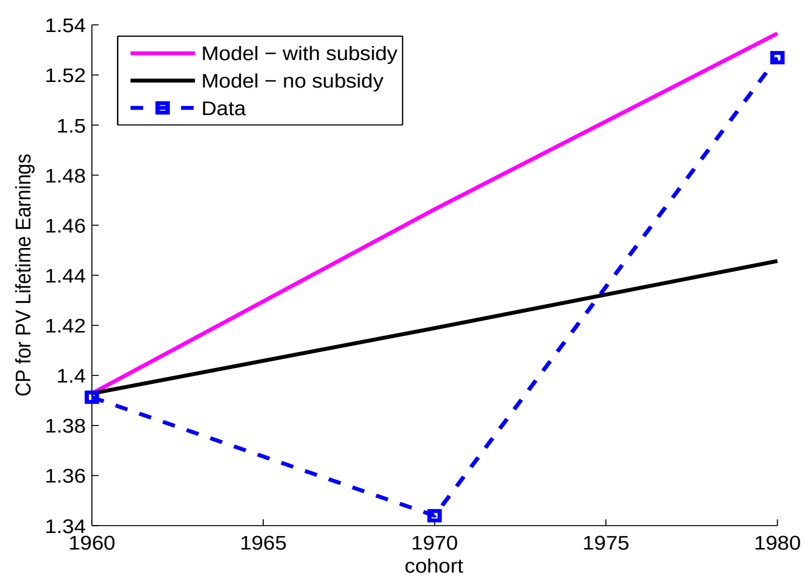

Figure 2.12: College premium – Model vs. Data

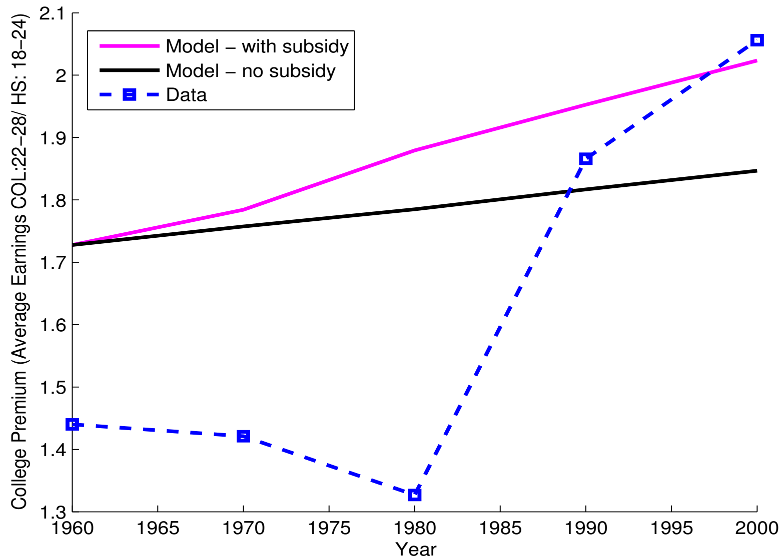

Figure 2.13: Contemporaneous college premium – Model vs. Data

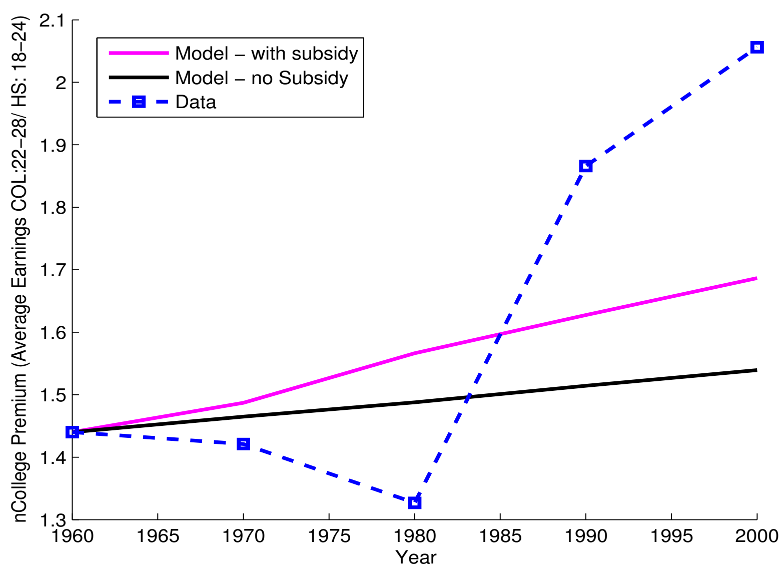

Figure 2.14: Normalized contemporaneous college premium – Model vs. Data

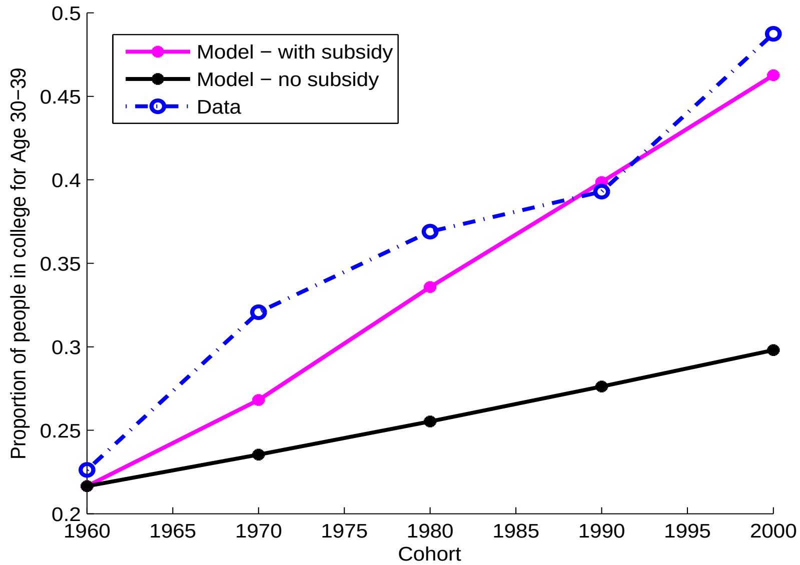

Figure 2.15: Proportion of college-educated individuals – Model vs. Data

Table 2.1: Calibrated parameters

Parameter	Description	Value
T	number of periods	42
r	interest rate	1.048
s	time spent in college	4
θ	human capital production function parameter	0.318
ζ	human capital production function parameter	0.697
ϕ^c	parameter for college workers	0.235
w_1	initial wage rate per unit of human capital	41.2
g	growth rate of w_τ	1.0021
μ_a	mean of lognormal ability distribution	12.8
σ_a	s.d. of lognormal ability distribution	0.5435
$\mu_{H^{hs}}$	mean of lognormal initial human capital distribution	17.7
$\sigma_{H^{hs}}$	s.d. of lognormal initial human capital distribution	0.605
$\rho_{a,H^{hs}}$	correlation between ability and initial human capital	0.56

Note: Highlighted numbers are calibrated.

Table 2.2: Calibration

Target	Model	Data	$\frac{Data}{Model}$
Proportion of college-educated individuals	0.22	0.23	1.03
Mean of normalized real earnings at age 30-39	1.05	1.52	1.45
Mean of normalized real earnings at age 40-49	1.07	1.64	1.54
Mean of normalized real earnings at age 50-59	1.09	1.70	1.56
Standard error of normalized real earnings at age 20-29	0.61	0.48	0.80
Standard error of normalized real earnings at age 30-39	0.63	0.66	1.05
Standard error of normalized real earnings at age 40-49	0.65	0.88	1.36
Standard error of normalized real earnings at age 50-59	0.66	1.03	1.56
Skewness of real earnings at age 20-29	1.1827	0.98	0.83
Skewness of real earnings at age 30-39	1.1820	1.07	0.90
Skewness of real earnings at age 40-49	1.1820	1.15	0.98
Skewness of real earnings at age 50-59	1.1820	1.19	1.008
Present value of lifetime college premium	1.393	1.391	0.999

Note: Earnings are normalized by average earnings at age 20-29.

Table 2.3: Proportion of increase in the present value of lifetime college premium explained

	Year	Lifetime College Premium	Change
Data			
	1960	1.391	
	1980	1.527	0.0978
Model			
	1960	1.393	
	1980	1.535	0.102
Explains			1.042

Table 2.4: Proportion of increase in the contemporaneous college premium explained

	Year	Contemporaneous College Premium	Change
Data			
	1960	1.440	
	2000	2.056	0.428
Model			
	1960	1.718	
	2000	2.023	0.178
Explains			0.415

Table 2.5: Proportion of the present value of lifetime college premium explained by the federal aid effect

	Year	Lifetime College Premium	Change	Explains
Model: $q > 0$				
	1960	1.390		
	2000	1.535	0.102	1.042
Model: $q = 0$				
	1960	1.390		
	1980	1.444	0.0366	0.374
Federal Aid Explains				0.668

Table 2.6: Proportion of the contemporaneous college premium explained by the federal aid effect

	Year	Contemporaneous College Premium	Change	Explains
Model: $q > 0$				
	1960	1.718		
	2000	2.023	0.178	0.415
Model: $q = 0$				
	1960	1.730		
	2000	1.839	0.070	0.164
Federal Aid Explains				0.250

APPENDIX A
MATERIAL FOR CHAPTER 1

A.1 Construction of the earnings profile from data

The data comes from the IPUMS-USA. In particular, I use Census data from 1940 to 2000. I look at only employed white males with the following education levels: (i) high school diploma, (ii) four-year college degree, and (iii) all individuals i.e. individuals with high school diploma and individuals with four-year college degree. The earnings profiles from (i) and (ii) are also referred to as the conditional earnings profiles, while that from (iii) is known as the unconditional earnings profile.[1]

I construct life-cycle earnings profiles for synthetic cohorts according to education level. I calculate the average annual real earning, in 2000 dollars, of college-educated individuals who are 25 years old in 1940, 1950, 1960 and 1970. I denote them as the 1940, 1950, 1960, and 1970 cohorts, respectively.[2] I recalculate the average earnings of each cohort every 10 years over 30 years to create a sequence of average earnings for college-educated individuals across the life cycle.[3] I repeat the same exercise for high school-educated individuals and all individuals.[4] Figures 1.1,

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[1]I select only employed white males because labor hours for white male individuals do not change considerably on the intensive margin, focusing on them reduces unnecessary complications in the data. In particular, I do not want the analysis to be convoluted by factors such as increase in labor hours on the extensive margin due to increases in the female labor-force-participation rate.

[2]From the Census data, I group 20-29, 30-39 year old etc. I use these groups to allow for more data points, but refer to the groups as 25 year olds, 35 year olds, etc.

[3]See Figure 1.23.

[4]See Figures 1.24 and 1.25.

1.2, and 1.3 are obtained by normalizing each sequence by its first observation. A slower rise in average earnings is shown by a flatter slope and vice versa. That is, the flattening of life cycle earnings over cohorts indicates that earlier cohorts have greater increments in their life-cycle earnings profile than later cohorts.

A.2 Data

The main source of data is US census data from the IPUMS-USA. I used 1 percent sample for all years 1940 to 2000 except for 1970, for which I used 1970 Form 1 State sample.

The income variable is INCWAGE. It reports each respondent's total pre-tax annual wage and salary income. INCWAGE includes wages, salaries, commissions, cash, bonuses, tips and other monetary income from an employer. Payments-in-kind or reimbursements for business expenses are not included. Since INCWAGE is expressed in nominal terms, it needs to be adjusted for comparisons over time.

The education variable is EDUCD (detailed version). EDUCD denotes a respondent's highest educational attainment. This is denoted either as the highest year of school completed or highest degree earned. Classifications have evolved over time. For comparability, for 1940-1980, all respondents are classified according to the highest year of school completed. From 1990 onwards, respondents who have completed high school are classified according to highest degree earned and high school drop outs are classified according to highest year of school completed. A college degree is differentiated by assigning each degree the number of years it typically takes to achieve the degree: 2 years of college for associate's degrees; 4 years of college for bachelor's degrees; and 5+ years of college for graduate and professional degrees.[5]

Since I am looking at earnings of employed white males, I use RACED (=100), SEX (= 1), EMPSTATD (= 10) and CLASSWRKD ($\geq$ 20, $<$ 29) to filter out the subsample needed. I use AGE to create the age intervals 20- 29, 30-39, 40-49 and 50-59 to obtain the four data points for each cohort.

For occupation, I use OCC1950. Classification is according to the 1950 Census Bureau occupational classification system for occupational data, which uses a three-digit code to sort respondents. 1940 and pre-1940 data are reclassified to the 1950 classification system for comparability. I group respondents into occupation groups with broad categories 000, 100, 200, 300, 400, 500, 600, 700, 810 and 910.

A.3 The unconditional earnings profile

The unconditional average earnings from the model is calculated from

$$E_{\tau,j} = p_\tau E^{col}_{\tau,j} + (1 - p_\tau)E^{hs}_{\tau,j}.$$

where $E_{\tau,j}$, $E^{col}_{\tau,j}$ and $E^{hs}_{\tau,j}$ is the average annual earnings for all , college- and high school-educated individuals for cohort τ and age j, respectively. And p is the proportion of individuals with college degree from cohort τ. The earnings profile for each cohort is then calculated by normalizing by earnings at age 25 ($E_{\tau,25}$).

[5] HS[60,64], COL[100,110), COL+[100,116]

APPENDIX B
MATERIAL FOR CHAPTER 2

B.1 Data

Earnings comes from various years of Census date. Refer to Appendix A.2 for more details.

Data for federal aid comes from College Board publication "Trends in Student Aid" for years 1963 to 1968 and 2003. Another source comes from "ACE Historical Fact Sheet on Higher Education" from the U.S. department of Education, Office of Postsecondary Education. Education attainment data comes from College Board publication "Education Pays". Expenditure for college education is from yet another College Board publication "Trends in College Pricing" Table 5b.

REFERENCES

Acemoglu, Daron, "Technical Change, Inequality and the Labor Market," *Journal of Economic Literature*, March 2002, *XL*, 7–72.

Angrist, Joshus, Victor Chernozhukov, and Ivan Fernandez-Val, "Quantile Regression under Misspecification, with an Application to the US Wage Structure," *Econometrica*, 2006, *74* (2), 539–563.

Autor, David, Frank Levy, and Richard J. Murnane, "The Skill Content of Recent Technological Change: An Empirical Exploration," *The Quarterly Journal of Economics*, November 2003, *118* (4), 1279–1333.

_ , Lawrence Katz, and Melissa Kearney, "Rising Wage Inequality: The Role of Composition and Prices," NBER Working Papers W11628, NBER 2005.

Beaudry, P. and D. A. Green, "Cohort Patterns in Canadian Earnings: Assessing the Role of Skill Premia in Inequality Trends," *Canadian Journal of Economics*, 2000, *34* (4), 907–936.

Becker, G. S., *Human Capital*, NBER, 1964.

Ben-Porath, Yoram, "The Production of Human Capital and the Life-Cycle of Earnings," *Journal of Political Economy*, 1967, *75* (4), 352–65.

Berger, M. C., "The Effect of Cohort Size on Earnings Growth: A Reexamination of the Evidence," *Journal of Political Economy*, 1985, *93* (3), 561–573.

Bernhardt, A., M. Handcock, and M. Scott, "Inequality and Mobility: Trends in Wage Growth for Young Adults," Working Paper 99-03, The Pennsylvania State University 1999.

Browning, M., L. Hansen, and J. Heckman, *Handbook of Macroeconomics*, Elsevier Science B.V, Amsterdam, 1999.

Card, David and John E. DiNardo, "Skill Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles," *Journal of Labor Economics*, October 2002, *20* (4), 733–783.

_ and Thomas Lemieux, "Can Falling Supply Explain the Rising Return to College for Younger Men? A Cohort Based Analysis," *The Quarterly Journal of Economics*, May 2001, *116* (2), 705–746.

Carneiro, Pedro and James Heckman, "The Evidence on Credit Constraints in Post-Secondary Schooling," *The Economic Journal*, October 2002, *112*, 705–734.

Deaton, Angus and Christina Paxson, "Growth and Saving Among Individuals and Households," Working Paper, Princeton University 1999.

Erosa, Andrés, Tatyana Koreshkova, and Diego Restuccia, "How Important Is Human Capital? A Quantitative Theory Assessment of World Income Inequality," *Review of Economic Studies*, October 2010, *77* (4), 1421–1449.

Glomm, Gerhard and B. Ravikumar, "Human Capital Accumulation and Endogenous Public Expenditures," *Canadian Journal of Economics*, 2001, *34(3)*, 807–826.

Gokhale, Jagadeesh, Laurance J. Kotlikoff, John Sabelhaus, Barry Bosworth, and Robert Haveman, "Understanding the Postwar Decline in U.S. Savings: A Cohort Analysis," *Brookings Institute Press*, 1996, *1996* (1), 315–407.

Goldin, Claudia and Lawrence Katz Katz, *The Race Between Education and Technology*, Harvard University Press, 2008.

Heathcote, Johathan, Kjetil Storeletten, and Giovanni L. Violante, "The Cross-Sectional Implications of Rising Wage Inequality in the United States," Mimeo, New York University 2004.

Heckman, James, Lance Lochner, and Christopher Taber, "Explaining Rising Wage Inequality: Explorations with A Dynamic General Equilibrium Model of Labor Earnings with Heterogenous Agents," *Review of Economic Dynamics*, 1998, *1*, 1–58.

_ , _ , and Petra Todd, "Fifty Years of Mincer Earnings Regressions," IZA Discussion Paper No 775, IZA 2003.

_ , _ , and Ricardo Cossa, "Learning-by-Doing VS On-the-Job Training: Using Variation Induced by the EITC to Distinguish between Models of Skill Formation," NBER Working Paper W9083, NBER 2002.

Hendricks, Lutz and Todd Schoellman, "Student Abilities During the Expansion of US Education," Manuscript 2012.

Huggett, Mark, Gustavo Ventura, and Amir Yaron, "Human Capital and Earnings Distribution Dynamics," *Journal of Monetary Economics*, 2006, *53* (265 - 290).

Juhn, Chinhui, Kevin Murphy, and Brooks Pierce, "Wage Inquality and the Rise in Returns to Skill," *Journal of Political Economy*, June 1993, *101* (3), 410–42.

Kambourov, Gueorgui and Iourii Manovskii, "Accounting for the Change in Life-Cycle Profile of Earnings," Manuscript 2009.

Katz, Lawrence and David Autor, "Changes in the Wage Structure and Earnings Inequality," in Orley Ashenfelter and David Card, eds., *Handbook of Labor Economics*, 1 ed., Vol. 3, Elsevier, 1999, chapter 26, pp. 1463–1555.

Lebow, David, Louis Sheiner, Larry Slifman, and Martha Starr-McCluer, "Recent Trends in Compensation Practices," Finance and Economics Discussion Series 1999-32, Board of Governors of the Federal Reserve System 1999.

Lemieux, Thomas, "Decomposing Changes in Wage Distributions: A Unified Approach," *Canadian Journal of Economics*, November 2002, *35* (4), 646–688.

_ , "Increasing Residual Wage Inequality: Composition Effects, Noisy Data or Rising Demand for Skill?," *American Economics Review*, June 2006, *96* (3), 461–498.

_ , "Post-Secondary Education and Increasing Wage Inequality," Technical Report W12077, NBER 2006.

Macurdy, Thomas and Thomas Mroz, "Measuring Macroeconomic Shifts in Wage from Cohort Specification," Mimeo, Stanford University 1995.

Manuelli, Rodolfo E. and Ananth Seshadri, "Human Capital and the Wealth of Nations," Manuscript 2005.

Mincer, J., "Investment in Human Capital and Personal Income Distribution," *Journal of Political Economy*, 1958, *66* (4), 281–302.

Restuccia, Diego and Guillaume Vandenbroucke, "The Evolution of Education: A Macroeconomics Analysis," *International Economic Review*, forthcoming.

Ruggles, Steven, J. Trent Alexander, Katie Genadek, Ronald Goeken, Matthew B. Schroeder, and Matthew Sobek, "Integrated Public Use Microdata Series: Version 5.0 [Machine-readable database]. Minneapolis: University of Minnesota, 2010.," Technical Report.

Welch, F., "Effects of Cohort Size on Earnings: The Baby Boom Babies Financial Bust," *Journal of Political Economy*, 1979, *87* (5), S65–S97.