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A Distributed Algorithm for Optimal Dispatch in Smart Power Grids with Piecewise Linear Cost Functions

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A DISTRIBUTED ALGORITHM FOR OPTIMAL DISPATCH IN SMART POWER GRIDS WITH PIECEWISE LINEAR COST FUNCTIONS

by

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CERTIFICATE OF APPROVAL

MASTER'S THESIS

This is to certify that the Master's thesis of

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has been approved by the Examining Committee for the thesis requirement for the Master of Science degree in Electrical and Computer Engineering at the August 2013 graduation.

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To my parents and my husband

ABSTRACT

We consider the optimal economic dispatch of power generators in a smart electric grid for allocating power between generators to meet load requirements at minimum total cost. We assume that each generator has a piecewise linear cost function. We first present a polynomial time algorithm that achieves optimal dispatch. We then present a decentralized algorithm where, each generator independently adjusts its power output using only the aggregate power imbalance in the network, which can be observed by each generator through local measurements of the frequency deviation on the grid. The algorithm we propose exponentially erases the power imbalance, while eventually minimizing the generation cost.

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CHAPTER 1
INTRODUCTION

In this thesis, we present a simple, distributed algorithm for optimal economic dispatch, [1] of power generators in a smart electric grid. The specific goal is to generate the required power at minimum total cost. We assume that generators have access to the power imbalance in the grid. We note that such imbalances produce a proportional frequency deviation. Consequently, local measurement of frequency deviation provides each generator a quantity proportional to the overall imbalance. Using the information about the instantaneous power imbalance in the grid, the generators adjust their generations. The algorithm we propose exponentially erases the power imbalance, while eventually achieving the total minimum cost. Consequently, this distributed algorithm achieves the optimal economic dispatch. The algorithm has been presented in [2], [3] and [4]. The key difference between this work and the earlier work is that our earlier work assumes that the production cost of each generator is a twice differentiable function. By contrast, for reason to be explained in the sequel, in this thesis these costs are assumed to be piecewise linear.

1.1 Traditional Electric Grids

Electric power grids have evolved as a centralized and unidirectional system of transmission and distribution of electricity. They constitute a large, mature and highly interconnected system in which thousands of electricity generation stations transmit the power to the grids through high capacity power lines which are then divided to deliver the power to small industrial, commercial and domestic users. The availability of resources highly influences the structure and location of generation stations i.e. nuclear power plants are located close to cooling water resources and hydroelectric dams are sited in mountain areas. To keep the environment clean and free from pollution, oil and coal plants are constructed as removed from populated areas as is economically possible. This results in possibly huge power losses in transmission lines [5]. Being centralized in nature, these power grids are also vulnerable to the domino effect of failures. The ever increasing demand of electricity over time is leading to more power stations and in the design of new electricity demand patterns that depend on the daily peak hours due to domestic heating and air conditioning. Peaking power generators are used during peak hours for a short period each day and result in high cost to electricity companies and utilities. Moreover, these traditional grids rely primarily on fossil fuels that do not have an infinite reservoir. In early 21st century, renewable energy has emerged as a suitable energy source for electricity generation that might play a much larger role in the future with its infinite availability and its benign interactions with the environment.

Most renewable energy resources are, however, intermittent and non-dispatchable at precisely the times when underlying resources are not available; they cannot be turned on at will. In such situations, utility can be well served by having a backup network of dispatchable traditional power plants. Therefore, the use of renewable energy requires more advanced controlled techniques for grid stations to deal with this intermittency [6]. Furthermore, growing concern of reducing power line losses and

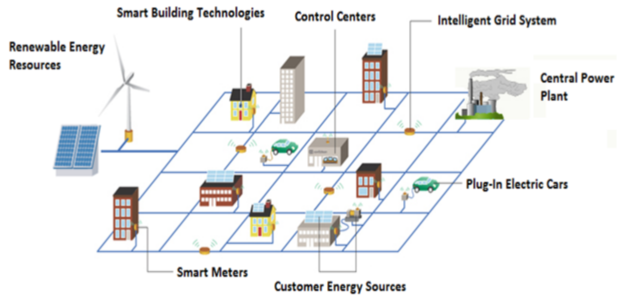

Figure 1.1: A model of smart grid.

power failures has led to calls for a more robust energy grid that is less dependent on centralized power stations, and more reactive to exigencies [7, 8].

1.2 Smart Power Grids

The next generation power grids known as Smart power grids are supposed to deal with most of the shortcomings of those existing power grids [7]. IEEE defines smart grid as an automated, widely distributed energy delivery network characterized by a two-way flow of electricity and information, capable of monitoring and responding to changes in everything from power plants to customer preferences to individual appliances. The smart grid helps utilities meet customers power demands and prevent customers from wasting power [8]. It is an updated grid system that has the ability to incorporate and deal with unpredictable energy resources i.e. wind energy, solar energy etc. For example, in the night hours, when sunlight is not available for generating the electricity from solar panels, the traditional and central power plants can be utilized. Thus, the optimization of electrical grid may ultimately distribute power more effectively, with less environmental impact.

1.3 Distribution Generation in Smart Power Grids

Our work is motivated by conceived next generation power grids that are expected to be more flexible and will permit a greater penetration of higly variable alternative energy resources such as wind and solar energy. But the intermittent nature of the availability of renewable energy supply, and the growing variability of demand profiles call for a smart grid infrastructure that is inherently less centralized and can react more quickly to short term conditions, [7, 8]. Instead of using large centralized facilities, generation of electricity from many small energy sources also improves security of supply and the use of renewable energy resources has lower environmental impacts.

1.4 Three Levels of Control in Power system

Electric power systems must deal with the problem of maintaining a real time balance between load and generation. Minute to minute load fluctuations result in disturbance of millions of individual loads. Generation may also suffer instantaneous variations in its regular schedule because of unexpected trips or power failures. The trend in system frequency is a measure of imbalance between load and generation. A certain amount of active power called frequency control reserve is available to correct the frequency of the system. Generally, three levels of controls are used to remove the power imbalance between load and generation and they are accomplished in multiple time scales [9].

Primary control is implemented on generating units individually that adjusts the power of each unit to compensate the imbalance of power between generation and consumption side and correct the frequency variation. Secondary control restores the frequency to its nominal value and maintains the power interchange between several control areas. It is a centralized automatic control scheme for reducing the Area Control Error (ACE) to zero. ACE is the measure of imbalance between rated generation capacity and power consumption within the control area.

$$ACE = \Delta(Net\ interchange) + (1/\beta)\Delta(f) \tag{1.1}$$

Here, Δ Net interchange is the instantaneous difference between actual and scheduled interchange and $1/\beta$ is the frequency bias. Only the generating units that are located in that area, where the imbalance is originated, participate in this control. This control is known as Load Frequency Control or sometimes Automatic Generation Control. Load Frequency Control operates on generating units of a control area every 5-15 minutes. Tertiary control restores the primary and secondary control as economically as possible. In existing grids, this control is implemented in a centralized fashion, and requires the central system to have the knowledge of all generators, making the control problem complex as the number of generators increases [9]. In tertiary control, a dispatch algorithm adjusts the power to the network generators to minimize the cost of generation but these allocations may change from time to time due to the change in load and actions of secondary controllers.

1.5 Economic Dispatch

In the last decades of the 20th century, many states and provinces in the US have deregulated electricity markets and introduced competetion in the electricity industry [10, 11]. Deregulation divides the electricity services into three main segments (generation, transmission and distribution) and allows the energy suppliers to compete on prices. In this deregulated model, private entities own the generation side and their privately owned generators produce electricity and then sell it on electricity markets. Energy suppliers purchase this electricityfor distribution to consumers. A competetive market offers more options to energy suppliers in order to provide more benefits to consumers. Power producers that act as deregulated companies are assigned generation schedules according to the price competetion of wholesale markets. Good dispatch algorithms can be very useful for generation companies to maximize their profits in the wholesale market [12]. The dispatch algorithm discussed in this thesis is very attractive to market-based pricing process and being distibuted in nature, is also very suitable for small grids with alternative energy generators.

1.6 Related work

Traditionally the dispatch problem is treated as a multivariable optimization problem to be numerically solved at a centralized controller. The old style Economic dispatch assumes that a centralized entity has command and control authority, and full, physical operating cost information authority to set MW power output levels of all generators. It seeks to minimize instantaneous operating costs while meeting the load demand. It does so by formulating and a multivariable constrained optimization problem [1] that is then solved using such Lagrangian techniques such as lambda iteration [13]. Complex numerical *centralized* optimization methods such as genetic algorithms, particle swarm optimization or Monte-Carlo Methods [14, 15] are often employed to determine the minimum cost allocation of power across generators. The idea of using frequency deviation to control power imbalance between load and generation has already been discussed in [2–4]. The authors in [2–4] proposed the idea of frequency adaptive power energy rescheduler that operates by the changes in the frequency of electric power systems above and below a standard frequency. It adjusts the power of system by using the locally available measurement without considering any utility-consumer communication and achieves zero power imbalance and minimum cost allocation of generation across all generators of grid. The key idea behind this work is that power imbalance in the grid can be observed independently by each generator through local measurement of the frequency deviation on grid, and therefore no centralized controller and no explicit coordination between the generators is needed. Each of [2–4] assumes that the cost of production of each generator is twice differentiable. Their proposed algorithm changes the powers of generators according to the power imbalance that is proportional to the frequency deviation. In many cases, however, cost function are not twice differentiable. See e.g. [16, 17] where the costs are obtained by linearly interpolating tabulated operating points. In such a case the algorithms of [2–4] are ill-posed and cannot be used.

1.7 Our Contribution

The first contribution of this thesis is to provide a centralized polynomial time algorithm that achieves optimal dispatch when generator cost functions are piecewise linear and convex. This convexity assumption is standard in all proposed algorithm in literature, and conforms to experimental data whose piecewise linear interpolation provides the cost functions available in literature. It recognizes that marginal costs increase with increased production. The second contribution of this thesis is to formulate a distributed algorithm in which each generator adjusts its power only on the basis of its own cost function, current power generation and the locally measured frequency deviation and without even recourse to the information of number of generators on the grid, let alone their characteristics. The algorithm eventually achieves optimal economic dispatch despite the lack of twice differentiability.

The adjective "eventually" bears elaboration. We first show that this algorithm exponentially erases load imbalances. Yet the fact that only these load imbalances inform each generator of the state of global grid implies that once the deficit is erased, the algorithm must cease its operation. We show, however, that generic nonoptimal stationary points cannot be sustained under small load fluctuations. Thus in face of inevitable numerical errors and load fluctuations. Under such fluctuations at these stationary points, cause the cost function to strictly decrease. Consequently this decentralized algorithm eventually achieves optimality. In a sense this work can be viewed as a novel excursion from the traditional consensus theory, [18–26].

1.8 Outline of Rest of Thesis

The rest of thesis is organized as follows. We first introduce the dispatch problem considered in this thesis and our centralized dispatch algorithm in chapter 2. The goal of this centralized approach is primarily to set up the conditions of optimality that informs the analysis of the distributed algorithm presented and analyzed in chapter 3. We also illustrate the performance of the algorithm using numerical simulations in this chapter. Then we conclude in chapter 4.

CHAPTER 2
PROBLEM DESCRIPTION AND A CENTRALIZED ALGORITHM

2.1 Model and Assumptions

We model the economic dispatch problem as follows. We assume that there are N generators supplying power to the network. We denote the total power consumed by $P_L > 0$ which is assumed to be constant and the active power set point for generator i at the rated system frequency by $P_i,\ i \in 1\{\dots N\}$. As a result, the power imbalance in the system is given by

$$P_L = \sum_{i=1}^{N} P_i \text{ and for all } i \in \{1, \cdots, N\}, P_i \geq 0. \tag{2.1}$$

We neglect the effects of reactive power flows, voltage deviations and transients as is standard for economic dispatch problems. We also neglect power losses here for simplicity.

Note that P_i represents the active power *set-point*; the actual active power produced by each generator is determined by its primary controller which uses P_i as a reference. More precisely, the primary controller on each generator responds to a power deficit (or surplus) Δ by increasing (or decreasing) its generated power above (or below) its generation set-point P_i until the total generated power matches the total load. This action by the controller, however, has the side-effect of introducing a small frequency deviation that is proportional to the original imbalance Δ.

In other words, the total imbalance between the rated generation power and the load, after the controllers have reached steady-state, results in a proportional frequency deviation $\Delta f = \beta\Delta$ on the grid. This frequency deviation can be monitored continuously by each generator which thus can directly monitor the power imbalance Δ. This is analogous to the Area Control Error (ACE) signal observed by the secondary controller in a traditional Load Frequency Control (LFC) implementation [27]. We assume that β remains constant for all values of P_i and Δ. This is a reasonable assumption for small frequency deviations.

Let $J_i(P)$ be the cost function for generator i. The goal of the dispatch algorithm is to choose the P_i to minimize the total *steady state cost* $\sum_{i=1}^{N} J_i(P_i)$, where $J_i(P_i)$ is the cost incurred when the i-th generator, and force the power imbalance $\Delta(k)$ to zero. As noted in the introduction, we will assume that the cost of power generation by each generator is modeled as a continuous piecewise linear cost function. This is quite common in a number of settings, where the cost functions are obtained by linearly interpolating generation cost at a discrete set of operating points.

2.2 Properties of Piecewise Linear Cost Functions

As noted in the introduction, we consider continuous piece wise linear cost functions that are shown in Figure 2.1. In particular, for some finite integer M_i, positive numbers S_{ij} and nonnegative numbers C_{ij}, $C_{i0} \geq 0$ and $C_{i,M_i} = \infty$, there holds for all $j \in \{1, \cdots, M_i\}$

$$\begin{aligned} J_i(P_i) &= S_{ij}\left(P_i - C_{i,j-1}\right) + J_i\left(C_{i,j-1}\right) \\ &\quad \forall\ C_{i,j-1} \leq P_i \leq C_{i,j}. \end{aligned} \tag{2.2}$$

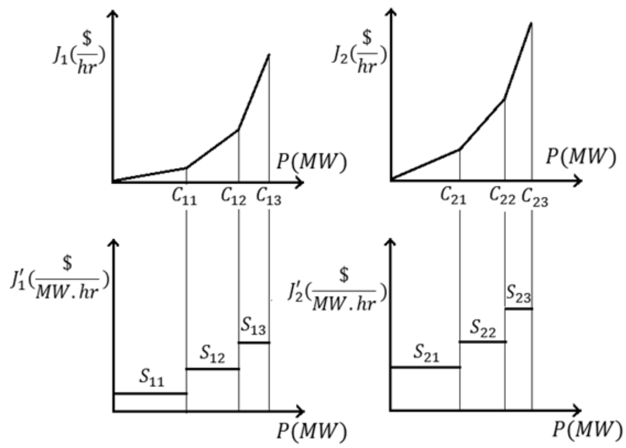

Figure 2.1: Cost function and marginal cost curves for generators 1 and 2.

We assume that the slopes S_{ij} are strictly increasing in j. In particular, for all $j \in \{1, \cdots, M_i - 1\}$ and $i \in \{1, \cdots, N\}$:

$$0 < S_{i,j} < S_{i,j+1}. \tag{2.3}$$

We note that (2.3) is tantamount to a convexity condition. This is standard among cost functions considered in the literature, [17]. The convexity condition of course implicitly reflects the appealing reality that the marginal cost increases with production. We also assume that unless $i = k$ and $j = l$,

$$S_{ij} \neq S_{kl}. \tag{2.4}$$

This assumption holds generically. Its violation makes the optimum nonunique. However, the algorithms we consider in this thesis will still find one of these nonunique optima.

Observe, for all $i \in \{1, \cdots, N\}$:

$$J_i(0) = 0, \tag{2.5}$$

i.e. each generator has a zero idling cost.

Define $P = [P_1, \cdots, P_N]$. Then for some $P_L > 0$, the optimization problem is:

Problem 2.1. Find P to minimize:

$$J(P) = \sum_{i=1}^{N} J_i(P_i), \tag{2.6}$$

subject to

$$P_L = \sum_{i=1}^{N} P_i \text{ and for all } i \in \{1, \cdots, N\}, P_i \geq 0 \tag{2.7}$$

2.3 Optimal Solution for Dispatch Problem

Our global goal is to formulate an algorithm that achieves eventual optimality in a completely decentralized fashion, where each generator uses only local measurements of the frequency deviation. In earlier work [2–4] lagrangian technique is used to describe the optimality for minimum cost allocation of power. This technique cannot work here, as we are using piecewiselinear cost function.Therefore, to characterize conditions of optimality, we present and analyze first a centralized algorithm that to our knowledge is new, and finds the optimal in polynomial time, with respect to the total number of slopes.

We first define a set of partially ordered triples $x_i \in \mathbb{R}_+ \times \mathbb{Z}^2$:

$$\mathcal{X} = \left\{ x_1, x_2, \cdots, x_{\sum_{i=1}^{N} M_i} \right\} \tag{2.8}$$

In particular there hold:

(A) $x_i(1) < x_{i+1}(1)$.

(B) For every i there exists $k \in \{1, \cdots, N\}$ and $l \in \{1, \cdots, M_i\}$, such that: $x_i(1) = S_{kl}$.

(C) With k, l as in (B), $x_i(2) = k$ and $x_i(3) = l$.

Thus $\mathcal{X}$ sorts the slopes S_{kl} in ascending order. These are in the first element of each x_i. The remaining two elements contain the generator and the slope index of that generator, respectively. For example in the cost functions depicted in Figure 2.1, the first three x_i are the triples:

$$x_1 = [S_{11}, 1, 1], x_2 = [S_{21}, 2, 1] \text{ and } x_3 = [S_{12}, 1, 2].$$

2.4 Centralized Algorithm

Before formally presenting the let us explain it using the example of Figure 2.1. Suppose, $P_L \le C_{11}$. Then clearly the optimal allocation is $P_1 = C_{11}$ and $P_2 = 0$, as the slope $S_{11} < S_{21}$. If on the other hand $C_{11} < P_L \le C_{11} + C_{21}$, then the next largest slope is C_{21}, one must select $P_1 = C_{11}$ and $P_2 = P_L - C_{11}$. Likewise, as S_{12} has the third smallest slope, should $C_{11} + C_{21} < P_L \le C_{21} + C_{12}$ then one has, $P_2 = C_{21}$ and $P_1 = P_L - C_{21}$.

It is also clear why the violation of (2.4) renders the optimum nonunique. Indeed suppose

$$C_{11} + C_{21} < P_L < \min\{C_{21} + C_{12}, C_{11} + C_{22}\}$$

and $S_{12} = S_{22}$. Then both the allocation pairs

$$P_1 = P_L - C_{21}, P_2 = C_{21}$$

and

$$P_1 = C_{11}, P_2 = P_L - C_{11}$$

achieve precisely the same costs.

The algorithm below formalizes these ideas.

With the quantities defined above the optimum power allocations $P^* = [P_1^*, \cdots, P_N^*]$ are selected as below:

Algorithm 2.1 Centralized Algorithm

Set $P_i = 0$ for all $i \in \{1, \cdots, N\}$.

Set $j = 1$.

While $\sum_{i=1}^{N} P_i < P_L$ do

$k = x_j(2)$

$l = x_j(3)$

$P_k = C_{kl}$

$j \to j + 1$

end

For all $i \in \{\{1, \cdots, N\} - \{k\}\}$, $P_i^* = P_i$.

$P_k^* = P_L - \sum_{i \neq k} P_i^*$.

It is clear that, under (2.3) and (2.4), the allocations made by Algorithm 2.1 is unique. We will show in the sequel that under (2.3) and (2.4) this is the unique solution to Problem 2.1. If η is the total number of slopes:

$$\eta = \sum_{i=1}^{N} M_i.$$

Then sorting of slopes completes in $\eta \log \eta$ iterations and while loop takes η iterations. So, overall complexity of the algorithm is $O(\eta \log \eta)$.

In the rest of this section we show that the Algorithm 2.1 does indeed solve Problem 2.1. First we show that the constraints are satisfied.

Lemma 2.1. *With the various quantities as defined in the foregoing, and $P_L > 0$, the P_i^* selected by Algorithm 2.1 satisfy (2.1).*

Proof. The last line of Algorithm 2.1 ensures the satisfaction of the first constraint in (2.1). As $P_L > 0$, the "while loop" executed atleast once. Without loss of generality, suppose that after the last update in the "while loop" $k = 1$. Clearly, as $C_{ij} > 0, P_i^* \geq 0$ for all $i \in \{2, \cdots, N\}$. It remains to show that $P_1^* \geq 0$. It is clear that prior to termination of the while loop, for some m, $P_1 = C_{1m} \geq 0$. By the nature of while loop for this m,

$$C_{1m} + \sum_{i=2}^{N} P_i < P_L$$

Thus there holds:

$$\begin{aligned} P_1^* &= P_L - \sum_{i=2}^{N} P_i^* \\ &> C_{1m} + \sum_{i=2}^{N} P_i - \sum_{i=2}^{N} P_i^* \end{aligned}$$

$$= C_{1m}$$

$$\geq 0$$

■

In the sequel we will say that P_i *is at a corner* if for some $j \in \{1, \cdots, M_i\}$, $P_i = C_{ij}$. Observe that the power allocations made by Algorithm 2.1 require that there is at most one allocated power that is not at a corner. We show that this is necessary for optimality.

Lemma 2.2. *Suppose $P_L > 0$, and under (2.4) and (2.3), some P, solves Problem 2.1. Consider the index set $\mathcal{I}(P) \subset \{1, \cdots, N\}$ with the following property: For every $i \in \mathcal{I}(P)$, and all $j \in \{1, \cdots, M_i\}$, $P_i \neq C_{ij}$. Then $|\mathcal{I}(P)| \leq 1$.*

Proof. To establish a contradiction, suppose for some

$$1 < |\mathcal{I}(P)| = m \leq N$$

Without loss of generatlity assume that $|\mathcal{I}(P)| = \{1, \cdots, m\}$. By definition of $|\mathcal{I}(P)|$, for all $i \in \mathcal{I}(P)$, there exist a $j_i \in \{1, \cdots, M_i - 1\}$, such that:

$$C_{i,j_i-1} < P_i < C_{i,j_i}$$

In view of (2.3), again without loss of generality assume that:

$$S_{1,j_1} > S_{2,j_2}$$

$$J(P) = J_1(P_1) + J_2(P_2) + \cdots + J_N(P_N)$$

$$J(P) = S_{1,j_1}(P_1 - C_{1,j_1-1}) + J_1(C_{1,j_1-1}) + S_{2,j_2}(P_2 - C_{1,j_2-1}) + J_2(C_{1,j_2-1}) + \cdots + J_N(P_N)$$

Define:

$$0 < \delta < \min\{P_1 - C_{1,j_1-1}, P_2 - C_{1,j_2-1}\}.$$

Define $\bar{P} = [\bar{P}_1, \cdots, \bar{P}_N]^T$, with

$$\bar{P}_1 = P_1 - \delta$$

$$\bar{P}_2 = P_2 - \delta$$

And for all $i \in \{3, \cdots, N\}$, $\bar{P}_i = P_i$. By definition of δ at least one among $\bar{P}_1$ and $\bar{P}_2$ is at a corner. Then,

$$J(P) = S_{1,j_1}(\bar{P}_1 + \delta - C_{1,j_1-1}) + J_1(C_{1,j_1-1}) + S_{2,j_2}(\bar{P}_2 - \delta - C_{1,j_2-1}) + J_2(C_{1,j_2-1}) + \cdots + J_N(P_N)$$

$$J(P) = -\delta(-S_{1,j_1} + S_{2,j_2}) + J_1(\bar{P}_1) + J_2(\bar{P}_2) + \cdots + J_N(\bar{P}_N)$$

$$J(P) = -\delta(-S_{1,j_1} + S_{2,j_2}) + J(\bar{P})$$

then there holds:

$$J(\bar{P}) - J(P) = \delta(S_{2,j_2} - S_{1,j_1}) < 0$$

Thus, P does not solve Problem 2.1. The contradiction proves the result. ■

We next argue that at an optimum, if there is a generator not operating at a corner then it must have the highest marginal cost.

Lemma 2.3. *Under (2.3), (2.4), $P_L > 0$, and $N > 1$, suppose there is a P that solves Problem 2.1. Further suppose $\mathcal{I}(P)$, defined in Lemma 2.2 has the solitary* *element i. Then for all $k \neq i$*

$$J_i'(P_i) > J_k'(P_k)$$

Proof. To establish a contradiction suppose for some l,

$$J_i'(P_i) < J_l'(P_l)$$

This strict inequality is necessary because of (2.4). From Lemma 2.2, P_l is at corner and for some m,

$$S_{im} < J_i'(P_i) < S_{i,m+1}$$

Consider $\bar{P} = [\bar{P}_1, \cdots, \bar{P}_N]^T$, with

$$\bar{P}_i = P_i + \delta$$

$$\bar{P}_l = P_l - \delta$$

And for all $k \notin \{i, l\}$, $\bar{P}_k = P_k$. For sufficienty small $\delta > 0$, $J_i'(P_i) = J_i'(\bar{P}_i)$ and $J_l'(P_l) = J_l'(\bar{P}_l)$. Then it is readily checked that $J(P) > J(\bar{P})$ establishing that P cannot be the minimum and a contradiction. ■

We next show that no $P \neq P^*$ can solve problem 2.1.

Lemma 2.4. *Under (2.3), (2.4), $P_L > 0$, and $N > 1$, suppose there is a P that solves Problem 2.1. Then $P = P^* = [P_1^*, \cdots, P_N^*]$ with P_i^* defined in Algorithm 2.1.*

Proof. There is no loss of generality in assuming that for all i, $P_i^* > 0$, as otherwise we can laways reduce the number of generators by

Without the loss of generality, suppose for all $i \in \{1, \cdots, N-1\}$, and some $k_i \in \{1, \cdots, M_i\}$,

$$P_i^* = C_{i,k_i} \tag{2.9}$$

By definition for some $k_N \in \{1, \cdots, N_N\}$,

$$C_{N,k_N+1} \geq P_N^* \geq C_{N,k_N} \tag{2.10}$$

From the statement of Algorithm 2.1 for all $i \in \{1, \cdots, N-1\}$, and (2.3), there holds;

$$S_{N,k_N} > S_{i,k_i} \tag{2.11}$$

In fact without loss of generality, also from (2.3), assume that for all $i \in \{1, \cdots, N-1\}$,

$$S_{i,k_i} < S_{i+1,k_{i+1}} \tag{2.12}$$

We observe from the details of Algorithm 2.1 for all $i \in \{1, \cdots, N\}$

$$S_{i,k_{i+1}} > S_{N,k_N} \tag{2.13}$$

To establish a contradiction, suppose there is a $P \neq P^*$ that solves Problem 2.1. then from Lemma 2.2, $|\mathcal{I}(P)| \leq 1$. Since $P \neq P^*$, there exists, $i, j \in \{1, \cdots, N\}$, such that

$$P_i > P_i^* \text{ and } P_j < P_j^* \tag{2.14}$$

Further at least one among these two are not in $|\mathcal{I}(P)|$. Consider the following two cases that exaust all possibilities.

Case I $i \notin |\mathcal{I}(P)|$**:** In this case for some k,

$$J_i'(P_i) = S_{i,k} > S_{N,k_N} \tag{2.15}$$

Further from Lemma 2.3, P_j is also at a corner, and as $P_j < P_j^*$,

$$J_j'(P_j) < S_{N,k_N} \tag{2.16}$$

Consider $\bar{P} = [\bar{P}_1, \cdots, \bar{P}_N]^T$, with

$$\bar{P}_i = P_i - \delta$$

$$\bar{P}_j = P_j + \delta$$

And for all $k \notin \{i, j\}$, $\bar{P}_k = P_k$. For sufficiently small δ, $J(\bar{P}) < J(P)$, contradicting the hypothesis that P is a solution to Problem 2.1.

Case II $i \in |\mathcal{I}(P)|$**:** Then from Lemma 2.2, P_j is at a corner. In this case from Lemma 2.3,

$$J_j'(P_j) < J_i'(P_i)$$

Then with $\bar{P}$ as above, for sufficiently small δ, $J(\bar{P}) < J(P)$, again establishing a contradiction. ■

Armed with these lemmas, we now show that P^* generated by Algorithm 2.1, does indeed provide the unique solution to the optimization Problem 2.1.

Theorem 2.1. *Under (2.3), (2.4), $P_L > 0$, and $N > 1$, P^* defined in Algorithm 2.1 is the unique solution to Problem 2.1.*

Proof. For $P_L > 0$, the set of P satisfying (2.1) is compact and non empty. Thus there is at least one P that solves Problem 2.1, From Lemma 2.4 such a P can only be a P^* generated by Algorithm 2.1. Under (2.4) such a P^* is unique. ■

2.5 Nonunique Solutions

We now remark on the violation of (2.4). In this case there are potentially multiple outputs of Algorithm 2.1, depending on how one resolves conflicts when two slopes are equal. However, the techniques used in the foregoing proofs can be expanded to show that:

- There are multiple solutions to Problem 2.1.
- The set of these solutions are only those that are potential outputs of Algorithm 2.1.

We assume a very simple case here to explain this scenario. For some $P_L > 0$ and $N > 1$,

$$J(P) = J_1(P_1) + J_2(P_2) + \cdots + J_N(P_N)$$

Without loss of generality, we make the following assumptions:

$$P_i = C_{i,j_i-1}$$

$$S_{1,j_1} = S_{2,j_2}$$

For all $k \neq \{1,2\}$,

$$S_{k,j_k} > S_{1,j_1} = S_{2,j_2}$$

Then,

$$J(P) = S_{1,j_1}(C_{1,j_1-1}-C_{1,j_1-2})+J_1(C_{1,j_1-2})+S_{2,j_2}(C_{2,j_2-1}-C_{2,j_2-2})+J_2(C_{2,j_2-2})+\cdots+J_N(P_N)$$

Consider a new value of load power $\bar{P}_L = P_L+\Delta P$ and $\Delta P < \min\{C_{1,j_1}-C_{1,j_1-1}, C_{2,j_2}-C_{2,j_2-1}\}$. Then the cost becomes,

$$J(\bar{P}) = J_1(\bar{P}_1) + J_2(\bar{P}_2) + \cdots + J_N(\bar{P}_N)$$

with $\bar{P}_k = P_k$ for all $k \neq \{1, 2\}$.

Now we have two possible choices for $\bar{P}_1$ and $\bar{P}_2$. Let us first assume that $\bar{P}_1 = P_1 + \Delta P$ and $\bar{P}_2 = P_2$ Then,

$$\begin{aligned}
J_1(\bar{P}_1) + J_2(\bar{P}_2) &= J_1(P_1 + \Delta P) + J_2(P_2) \\
&= S_{1,j_1}(C_{1,j_1-1} + \Delta P - C_{1,j_1-2}) + J_1(C_{1,j_1-2}) + J_2(P_2) \\
&= S_{1,j_1}(C_{1,j_1-1} - C_{1,j_1-2}) + J_1(C_{1,j_1-2}) + S_{1,j_1}\Delta P + J_2(P_2) \\
&= J_1(P_1) + S_{2,j_2}(C_{2,j_2-1} + \Delta P - C_{2,j_2-2}) + J_2(C_{2,j_2-2}) \\
&= J(P_1) + J(P_2 + \Delta P).
\end{aligned}$$

Indeed the last represents a second possible choice for optimality: $\bar{P}_1 = P_1$ and $\bar{P}_2 = P_2 + \Delta P$. It is clear that total cost is identical for both cases. There can be many other solutions for above problem. One of them is $\bar{P}_1 = P_1 + 0.5\Delta P$ and $\bar{P}_2 = P_2 + 0.5\Delta P$. But it is obvious that all of these solution are the optimal ones.

2.6 Conclusion

In this chapter, we have considered the problem of economic dispatch with piecewise linear cost functions. We have introduced a centralized algorithm to set a benchmark for optimality. We have also shown that the centralized algorithm provides the optimal solution for Problme 2.1. Using this optimality criteria, we will discuss our distributed algorithm in next chapter.

CHAPTER 3
A DISTRIBUTED ALGORITHM FOR OPTIMAL DISPATCH

3.1 Introduction

The previous chapter presented a polynomial time algorithm for solving Problem 2.1. Its implementation, however, requires that a centralized authority oversee its impementation. In this chapter, we build on some earlier work to propose a decentralized solution. Specifically, as noted in Chapter 2 every generator can obtain a quantity proportional to the load deficit through a local measurement of the frequency deviation in the power grid.

We assume that each generator only has access to the following quantities:

- The frequency deviation.
- Its own generated power.
- Its cost function.

Except this information, it knows nothing; not even the number of generators in the grid.

3.2 Our Distributed Algorithm

Under above premise, we consider the algorithm below, that was first proposed in [2], and analyzed when the cost functions were twice differentiable. In the sequel let k denote the diecrete time instant, $P_i[k]$ the power output by the i-th generator at time k, and for a fixed load P_L, the load deficit,

$$\Delta[k] = P_L - \sum_{i=1}^{N} P_i[k]. \tag{3.1}$$

Then the decentralized algorithm we propose proceeds as follows. For $a_i > 0$, there hold:

$$P_i[k+1] = \begin{cases} P_i[k] + \frac{a_1 \Delta[k]}{J_i'(P_i[k])} & \Delta[k] \geq 0 \\ P_i[k] + a_2 \Delta[k] J_i'\left(P_i[k]\right) & \text{else} \end{cases}. \tag{3.2}$$

Clearly, the algorithm works with the information described above, and a_i subsume β, the constant of proportionality relating the load deficit to the frequency deviation. We note that under (2.2) there is an ambiguity in the value of the marginal costs at the transition points C_{ij}. We resolve this with the following convention:

$$J_i'(P_i) = S_{ij} \quad \forall C_{i,j-1} < P_i \leq C_{i,j}. \tag{3.3}$$

The motivation of the algorithm is as follows. When the load deficit is positive, the generators must increase their production. Intuition suggest that for optimality, generators with higher marginal costs must increase their production at a proportionately lower rate. On the flip side, if the load deficit is negative, the generators must decrease their production. In this case, for optimality, generators with higher marginal costs must decrease their production at a proportionately higher rate. This is precisely what (3.2) seeks to achieve.

Technically, an algorithm such as (3.2) may cause the $P_i[k]$ to dip below zero. To keep things precise, we assume that for all $i \in \{1, \cdots, N\}$, there holds:

$$J_i(P_i) = S_{i1} P_i + C_{i0} \text{ if } P_i < 0. \tag{3.4}$$

Under these conditions, we analyze the properties of (3.2) when the cost functions are piecewise linear. The first theorem below shows that under mild conditions (3.2) exponentially erases the load deficit.

Theorem 3.1. *Consider (3.2) under (2.2), (3.1), (3.3), with C_{ij}, S_{ij} obeying the properties described in Chapter 2. Then there exist $a_i^* > 0$, such that for all $0 < a_i \leq a_i^*$ and arbitrary $P_i[0]$, the following holds exponentially:*

$$\lim_{k\to\infty} \Delta[k] = 0,$$

Further, with a_i as above, there exist $M > 0$ and $\delta_i(M)$, for all $i \in \{1, \cdots, N\}$, such that for $P_L > M$ and $P_i[0] \geq \delta_i(M)$, for all $k \geq 0$ there holds:

$$P_i[k] \geq 0. \tag{3.5}$$

Proof. Whenever $\Delta[k] < 0$. From (3.2), there holds:

$$P_i[k+1] = P_i[k] + a_2\Delta[k]J_i'(P_i[k]) \tag{3.6}$$

Then from (3.1) there holds:

$$\Delta[k+1] = P_L + \sum_{i=1}^{N} P_i[k+1]$$

$$= P_L + \sum_{i=1}^{N} P_i[k] + a_2\Delta[k]J_i'(P_i[k])$$

$$\Delta[k+1] = \Delta[k](1 - a_2\sum_{i=1}^{N} J_i'(P_i[k])) \tag{3.7}$$

Under the assumptions on $J_i'(P_i[k])$ there is an a_2^*, such that for all $0 < a_2 \leq a_2^*$,

$$0 < a_2\sum_{i=1}^{N} J_i'(P_i[k]) < 1$$

Then exponential convergence of $\Delta[k]$ follows for $\Delta[0] < 0$ for such a set of a_2. Likewise when $\Delta[k] > 0$. From (3.2), there holds:

$$P_i[k+1] = P_i[k] + \frac{a_1 \Delta[k]}{J_i'(P_i[k])}$$

Then from (3.1) there holds:

$$\Delta[k+1] = P_L + \sum_{i=1}^{N} P_i[k+1]$$

$$= P_L + \sum_{i=1}^{N} P_i[k] + \frac{a_1 \Delta[k]}{J_i'(P_i[k])}$$

$$\Delta[k+1] = \Delta[k](1 - a_1 \sum_{i=1}^{N} \frac{1}{J_i'(P_i[k])}) \tag{3.8}$$

Under the assumptions on $J_i'(P_i[k])$ there is an a_1^*, such that for all $0 < a_1 \leq a_1^*$,

$$0 < a_1 \sum_{i=1}^{N} \frac{1}{J_i'(P_i[k])} < 1$$

Then exponential convergence of $\Delta[k]$ follows for $\Delta[0] > 0$ for such a set of a_1. ■

Evidently, the decentralized algorithm ensures the exponential erasure of the load deficit. What about achieving optimality with respect to Problem 2.1? Clearly $\Delta = 0$ is a stationary point of this algorithm. This accords with intuition as no generator is aware of any information about the global scenario beyond that supplied by the frequency deviation and *ipso facto* the load deficit. Thus when the frequency deviation decays to zero, regardless of whether optimality is achieved, the algorithm terminates. Yet simulations presented later, show that under load fluctuations, the algorithm eventually does converge to an optimum. Evidently a stationary point cannot be sustained under load fluctuations, unless it corresponds to a minimum. We show this below for a generic stationary point that is not an optimum.

To show this, we consider a setting at which the algorithm is initialized from a stationary point

$$P_i[-1] = \bar{P}_i \neq P_i^*, \tag{3.9}$$

such the load deficit is zero:

$$P_L = \sum_{i=1}^{N} \bar{P}_i. \tag{3.10}$$

Generically, it is highly unlikely that any generator will converge to a corner of its cost function. Thus, we assume that for all i, $\bar{P}_i$, are sufficiently removed from a corner of $J_i(\cdot)$. Now suppose at time $k = -1$, there is an impulsive load fluctuation of δ, and at $k = 0$ the load returns to its value. In other words,

$$P_L[-1] = P_L + \delta \text{ and } P_L[0] = P_L. \tag{3.11}$$

Now, these two changes will cause the algorithm to be activated. For sufficiently small δ the resulting changes will not be of a magnitude large enough for any generator's operating point to cross a corner. Of course the algorithm will still drive the load imbalance to zero. Then we show that this new steady state corresponds to a cost function that is strictly smaller than what it was prior to the impulsive fluctuation. Thus, indeed this generic stationary point cannot be sustained. To be specific we prove the following theorem.

Theorem 3.2. *Consider a set of powwr values, $\bar{P}_i \neq P_i^*$ that obeys (3.10). Suppose for all i, $\bar{P}_i$, is not at a corner of $J_i(\cdot)$, (3.9) holds and for some $\delta \neq 0$, (3.11) holds.*

Then there exists a δ^**, such that for all* $0 < \delta \leq \delta^*$ *there holds:*

$$\lim_{k\to\infty} \sum_{i=1}^{N} J_i(P_i[k]) < \sum_{i=1}^{N} J_i(\bar{P}_i).$$

Proof. Define

$$\bar{J} = \sum_{i=1}^{N} J_i(\bar{P}_i)$$

Clearly, for small enough δ^*, no $P_i[k]$ ever crosses a corner of $J_i(\cdot)$. Then for distinct $s_i > 0$, some c_i, for all $k \geq -1$, there holds:

$$J_i(P_i[k]) = s_i P_i[k] + c_i$$

Consider the following cases.

Case I $\delta > 0$**:** then

$$P_i[0] = \bar{P}_i + a_1 \frac{\delta}{s_i}$$

Then,

$$\Delta[0] = -a_1 \delta \sum_{i=1}^{N} \frac{1}{s_i} < 0$$

Further:

$$J(P[0]) = \bar{J} + a_1 N \delta$$

From the algorithm,

$$\Delta[k+1] = \Delta[k](1 - a_2 \sum_{i=1}^{N} s_i)$$

We know that,

$$J_i(P_i[k+1]) = s_i P_i[k+1] + c_i$$

$$= s_i P_i[k] + c_i + a_2 s_i^2 \Delta[k] = J(P_i[k]) + a_2 s_i^2 \Delta[k]$$

From here we get,

$$J(P[k+1]) = J(P[k]) + a_2\Delta[k]\sum_{i=1}^{N} s_i^2$$

Now,

$$\lim_{k\to\infty} J(P[k]) = J(P[0]) - a_1\delta\frac{\sum_{i=1}^{N} s_i^2}{\sum_{i=1}^{N} s_i}\sum_{i=1}^{N}\frac{1}{s_i}$$

$$= \bar{J} + a_1N\delta - a_1\delta\frac{\sum_{i=1}^{N} s_i^2}{\sum_{i=1}^{N} s_i}\sum_{i=1}^{N}\frac{1}{s_i}$$

$$\lim_{k\to\infty} J(P[k]) = \bar{J} + a_1\delta(N - \frac{\sum_{i=1}^{N} s_i^2}{\sum_{i=1}^{N} s_i}\sum_{i=1}^{N}\frac{1}{s_i}) \tag{3.12}$$

We will now show that

$$A = \frac{\sum_{i=1}^{N} s_i^2}{\sum_{i=1}^{N} s_i}\sum_{i=1}^{N}\frac{1}{s_i} \geq N$$

Further equality holds iff all s_i are equal. To see this, observe that A is sclae invarient.

Thus without loss of generality we choose,

$$\sum_{i=1}^{N} s_i = 1 \tag{3.13}$$

Then,

$$A = (\sum_{i=1}^{N} s_i^2)\sum_{i=1}^{N}\frac{1}{s_i} \geq N$$

Now under (3.13), there holds,

$$A = \sum_{j=1}^{N}\sum_{i=1}^{N} s_i^2\frac{1}{s_j}$$

$$= \sum_{i=1}^{N} s_i + \sum_{\substack{j\neq i\\ j=0}}^{N}\sum_{i=1}^{N} s_i^2\frac{1}{s_j}$$

$$= 1 + \sum_{\substack{j\neq i\\ j=0}}^{N}\sum_{i=1}^{N} s_i^2\frac{1}{s_j}$$

Without loss of generality assume that $s_i \leq s_{i+1}$. Then for all $N \geq 2$:

$$A = 1 + \sum_{i=1}^{N} \sum_{j=1}^{i-1} \frac{s_i^2}{s_j} + \sum_{i=1}^{N} \sum_{j=i+1}^{N} \frac{s_i^2}{s_j}$$

$$\geq 1 + \sum_{i=1}^{N} \sum_{j=1}^{i-1} 1 = 1 + \sum_{i=1}^{N} (i-1)$$

$$= 1 + \frac{(N-1)N}{2} \geq N$$

In above, equality holds only if the s_i are all equal. Thus A achieves its minimum when the s_i are identical and in fact $\frac{1}{N}$. In this case, A clearly equals N. Thus as long as the s_i are distinct from (3.13) there holds:

$$\lim_{k \to \infty} J(P[k]) < \bar{J} \tag{3.14}$$

Case II $\delta < 0$: Then

$$P_i[0] = \bar{P}_i + a_2 |\delta| s_i$$

Then,

$$\Delta[0] = a_2 |\delta| \sum_{i=1}^{N} s_i > 0$$

Further:

$$J(P[0]) = \bar{J} - a_2 |\delta| \sum_{i=1}^{N} s_i^2$$

From the algorithm,

$$\Delta[k+1] = \Delta[k](1 - a_1 \sum_{i=1}^{N} \frac{1}{s_i})$$

We know that,

$$J_i(P_i[k+1]) = s_i P_i[k+1] + c_i$$

$$= s_i P_i[k] + c_i + a_1 \Delta[k] = J(P_i[k]) + a_1 \Delta[k]$$

From here we get,

$$J(P[k+1]) = J(P[k]) + Na_1\Delta[k]$$

Now,

$$\begin{aligned}
\lim_{k\to\infty} J(P[k]) &= J(P[0]) + a_2|\delta|\frac{\sum_{i=1}^{N} s_i}{\sum_{i=1}^{N}\frac{1}{s_i}} \\
&= \bar{J} - a_2|\delta|\sum_{i=1}^{N} s_i^2 + a_2|\delta|\frac{\sum_{i=1}^{N} s_i}{\sum_{i=1}^{N}\frac{1}{s_i}} \\
\lim_{k\to\infty} J(P[k]) &= \bar{J} - a_2|\delta|(\sum_{i=1}^{N} s_i^2 - N\frac{\sum_{i=1}^{N} s_i}{\sum_{i=1}^{N}\frac{1}{s_i}})
\end{aligned}$$

Then, (3.14) holds as:

$$A = (\sum_{i=1}^{N} s_i^2)\sum_{i=1}^{N}\frac{1}{s_i} \geq N$$

■

Thus fluctuations from generic stationary points that do not correspond to an optimum lead to strict decline in the cost function.

3.3 Numerical Results

We now perform some simulations to see the performance of our Distributed Algorithm. We also simulate our Centralized Algorithm to set a benchmark for observing distributed algorithm results. We now run (3.2) with a five generator system with the cost functions defined below. These piecewise linear cost functions have been obtained from [16]. The units are in Megawatts.

$$J_1(P_1) = \begin{cases} 8.4620P_1 & \text{if } 0 \leq P_1 \leq 50 \\ 8.839P_1 - 18.85 & \text{if } 50 \leq P_1 \leq 68 \\ 9.24P_1 - 46.018 & \text{if } 68 \leq P_1 \leq 87 \\ 9.808P_1 - 95.434 & \text{if } 87 \leq P_1 \leq 107 \\ 10.379P_1 - 156.531 & \text{if } 107 \leq P_1 \leq 126 \end{cases} \tag{3.15}$$

$$J_2(P_2) = \begin{cases} 9.2291P_2 & \text{if } 0 \leq P_2 \leq 45 \\ 9.374P_2 - 6.5205 & \text{if } 45 \leq P_2 \leq 59 \\ 9.677P_2 - 24.3975 & \text{if } 59 \leq P_2 \leq 78 \\ 10.109P_2 - 58.0935 & \text{if } 78 \leq P_2 \leq 98 \\ 10.66P_2 - 112.0915 & \text{if } 98 \leq P_2 \leq 118 \end{cases} \tag{3.16}$$

$$J_3(P_3) = \begin{cases} 9.8575P_3 & \text{if } 0 \leq P_3 \leq 60 \\ 9.9P_3 - 2.55 & \text{if } 60 \leq P_3 \leq 7 \\ 9.94P_3 - 5.35 & \text{if } 70 \leq P_3 \leq 109 \\ 9.99P_3 - 10.8 & \text{if } 109 \leq P_3 \leq 128 \\ 10.13P_3 - 28.72 & \text{if } 128 \leq P_3 \leq 147 \end{cases} \tag{3.17}$$

$$J_4(P_4) = \begin{cases} 8.9348P_4 & \text{if } 0 \leq P_4 \leq 55 \\ 8.994P_4 - 3.256 & \text{if } 55 \leq P_4 \leq 87 \\ 9.315P_4 - 31.183 & \text{if } 87 \leq P_4 \leq 125 \\ 9.562P_4 - 62.058 & \text{if } 125 \leq P_4 \leq 164 \\ 9.763P_4 - 95.022 & \text{if } 164 \leq P_4 \leq 192 \end{cases} \tag{3.18}$$

$$J_5(P_5) = \begin{cases} 9.4232P_5 & \text{if } 0 \leq P_5 \leq 40 \\ 9.559P_5 - 5.432 & \text{if } 40 \leq P_5 \leq 47 \\ 9.779P_5 - 16.712 & \text{if } 47 \leq P_5 \leq 56 \\ 9.987P_5 - 27.42 & \text{if } 56 \leq P_5 \leq 76 \\ 10.421P_5 - 60.404 & \text{if } 76 \leq P_5 \leq 86 \end{cases} \tag{3.19}$$

We depict the results in the figures below with differing values of P_L to capture various scenarios. In each case the the steady state cost attained is the optimum. The a_i are depicted in the plots. In these plots, we have initialized the power of generators with some random values and then distributed algorithm forces them to adjust their powers with minimum total cost.

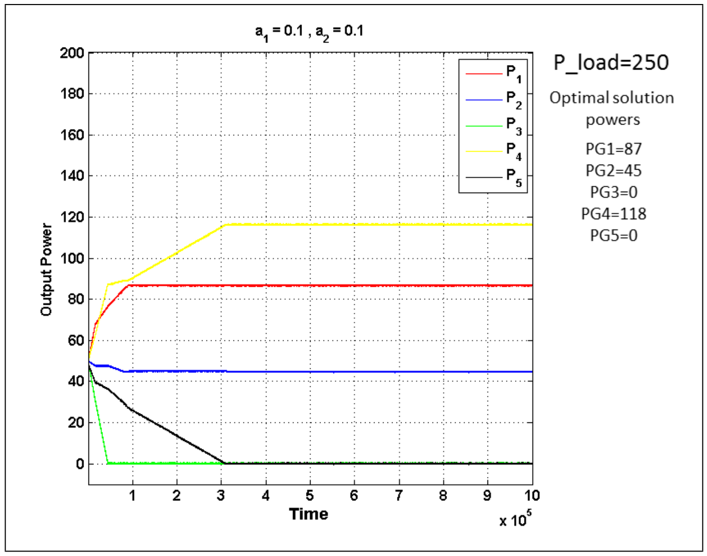

Figure 3.1: Convergence of power imbalance and total cost under the distributed dispatch algorithm with $P_L = 250$MW.

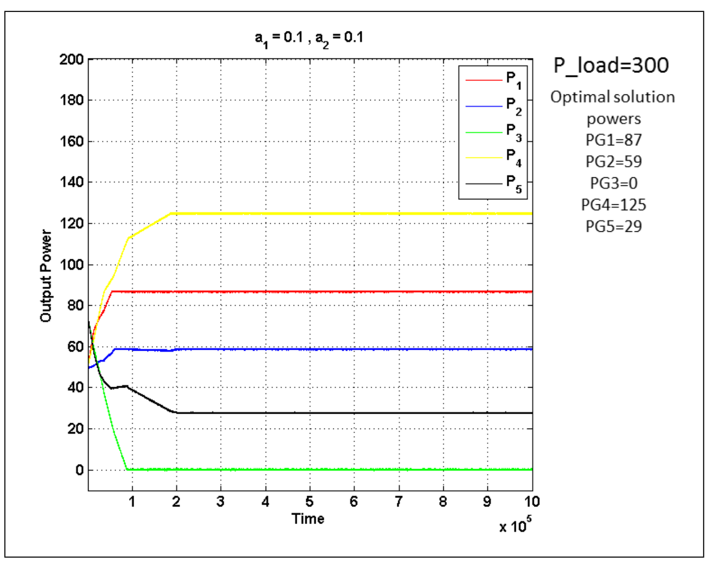

Figure 3.2: Convergence of power imbalance and total cost under the distributed dispatch algorithm with $P_L = 300$MW.

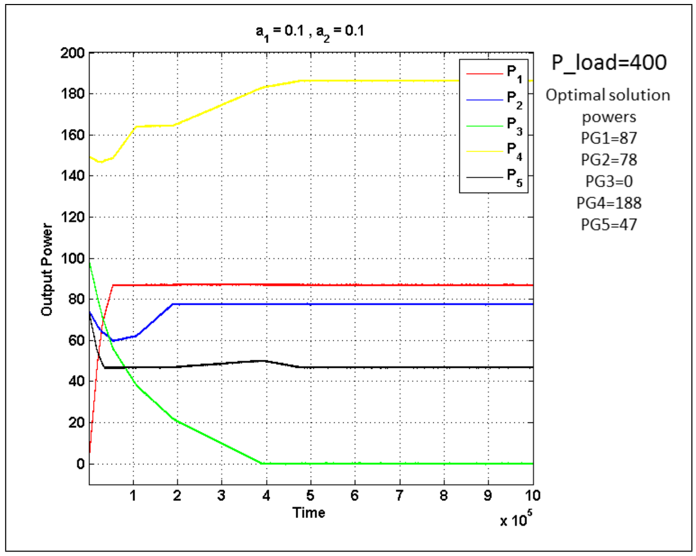

Figure 3.3: Convergence of power imbalance and total cost under the distributed dispatch algorithm with $P_L = 400$MW.

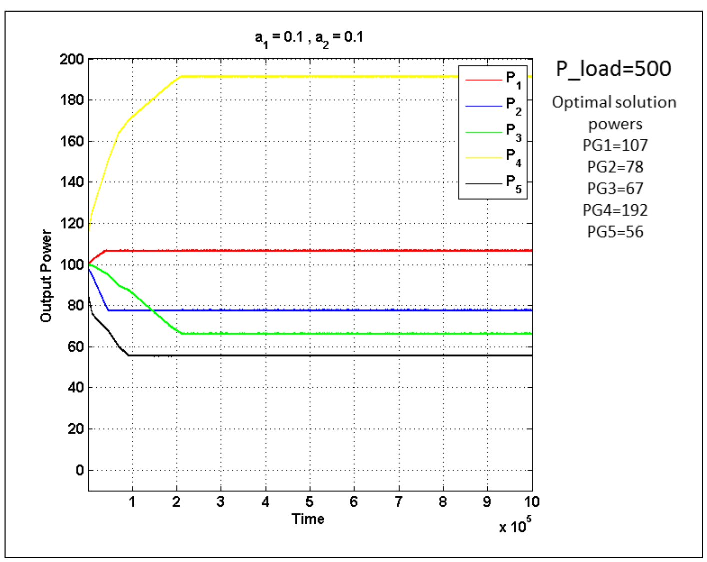

Figure 3.4: Convergence of power imbalance and total cost under the distributed dispatch algorithm with $P_L = 500$MW.

Here, it is very important to mention that in above simulations, we have introduced a random and small value of power imbalance to imitate the frequency deviation. Without introducing this power deficit, our algorithm would stop without allocating the powers of generators optimally. It is worth mentioning that after achieving the optimal allocations, generators do not change their powers even in the presence of small power imbalance. It is exactly what Theorem 3.2 says that only stationary point that is locally stable is minimum global. Before being steady, there could be many stationary points but distributed algorithm continues until it reaches to the minimum global that is exactly equal to the benchmark.

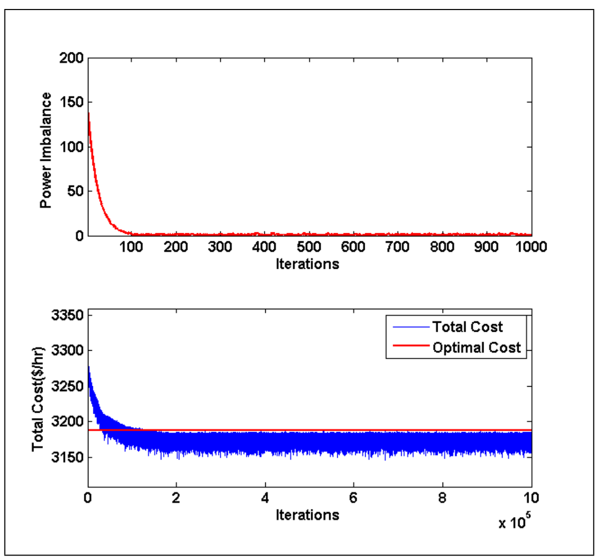

Figure 3.5: Convergence of power imbalance and total cost of power generators with $P_L = 350$MW.

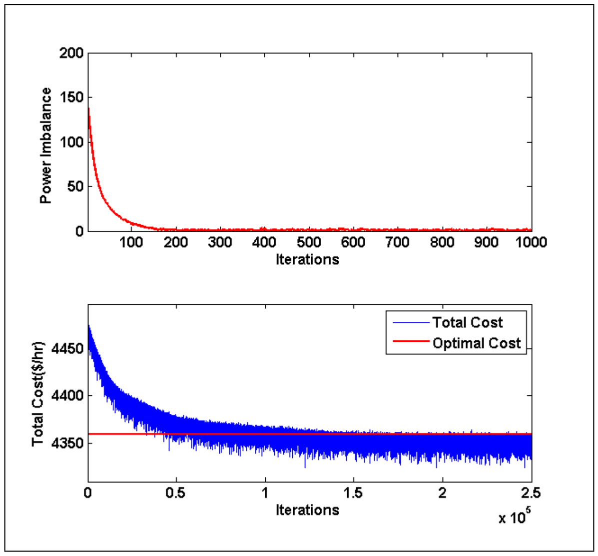

Figure 3.6: Convergence of power imbalance and total cost of power generators with $P_L = 470\text{MW}$.

We consider Figure 3.5 and Figure 3.6 to show the convergence of power imbalance to zero and convergence of total cost of generators to the optimal point. In above figures, initial and final values of total cost are not far apart. The reason is that slopes of piecewise linear cost functions that we are using in our simulations are very close to each other and for the purpose of simplicity, we have used small number of generators and small number of segments in each generators. By using a large data, we can observe a large difference between initial and optimal costs.

CHAPTER 4
CONCLUSION AND FUTURE WORK

4.1 Conclusion

We have presented a decentralized consensus based algorithm for achieving optimal dispatch, with piecewise linear cost functions that eventually achieves optimal power allocation while meeting load requirements. Our decentralized algorithm relies on local frequency deviation measurements. Additionally we have provided a simple polynomial time algorithm for centralized optimization that avoids the complexities in existing algorithms and helps characterize the optimal solution.

Our distributed algorithm is well suited in smart power grid with alternative energy generators. In case of renewable energy resources, when underlying energy resources are not available, this distributed algorithm can be used to turn on the backup network of dispatch able power generators economically.

4.2 Future Work

An important future area of research is to tune this algorithm to grid dynamics to avoid instabilities, though it is safe to conjecture that sufficiently small a_i and large enough sampling intervals in (3.2) should prevent grid instabilities. Convergence rate of our algorithm increases by using the large values of parameters .On the other hand, fluctuations and instabilities grow up with the increase in these parameters. Some further work can also be performed to investigate this problem systematically to deal with its pros and cons.

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