Spring 2016

Essays in industrial organization

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ESSAYS IN INDUSTRIAL ORGANIZATION

by

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A thesis submitted in partial fulfillment of the
requirements for the Doctor of Philosophy
degree in Economics
in the Graduate College of
The University of Iowa

May 2016

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CERTIFICATE OF APPROVAL

PH.D. THESIS

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To my Julie, who for the last six years has matched my blood, sweat and tears drop for drop.

And truth is knowledge of things as they are, and as they were, and as they are to come.

Doctrine and Covenants 93:24

ACKNOWLEDGEMENTS

First, for the entirety of my thesis, I am indebted to my advisors, Rabah Amir and Antonio Galvao, for seeing an entirely overly-optimistic PhD student with interests only partially correlated with their own and saying, "Yes, we'll work with you." I am better for the encouragement and guidance from N. Eugene Savin from the beginning of my academic career through the end of my thesis and for helping me better understand what it is to be an educator and an academic. I am also grateful to Anne Villamil and Suyong Song whose excitement for my project and helpful suggestions have been a great blessing.

The first chapter would likely not have see the light of day if not for encouragement from Nicholas Yannelis, who saw potential in the project and encouraged me to pursue something new. I am grateful for insight and wisdom concerning the lawyer market from Eric Anderson, Stanley Neeleman and Reese Hansen. I also owe a great debt to Deb Tiemens for graciously allowing me access to the school-level data used in this paper and to Jerry Organ for leading me to new data sources. This chapter benefited from helpful comments from Amil Petrin, Gautam Gowisankaran, Ayça Kaya, Kyungmin Kim, Nicolas Ziebarth, David Frisvold, Martin Gervais, Sam Burer and Nick Street as well as comments from seminar participants at the University of Iowa, Brigham Young University, the Midwest Econometrics Group Meetings, and the SAET Conference on Current Trends in Economics.

The second chapter is joint work with Rabah Amir and Jim Jin and is under revision for the *Journal of Economic Theory*. The paper benefited from helpful discussions with Wayne Barrett concerning some of the matrix analysis in the paper, discussions between my coauthors and Heracles Polemarchakis, and comments from the editor and two anonymous referees.

I have been inspired and motivated by my academic heritage, beginning with my grandfather, Horace Lundberg, who not only was the first person in his family to go to college, but went on to be the founding Dean of the School of Social Welfare Administration at Arizona State University, and continuing with my loving mother, "Lady" Constance (Lundberg) Erickson, who raised me in a university as she served as the Associate Dean of the Law School at Brigham Young University and first Dean of the Howard W. Hunter Law Library. This is as much their work as it is my own. And I am forever grateful for an amazing father, Boyd Erickson, who taught me, among most things I know in life, that I can both love math and love people.

I am grateful for the constant optimism and support from Renea Jay who has not only given invaluable administrative support, but who has been a sounding board and source of encouragement through the entirety of my graduate career.

Finally, I could not have survived this process were it not for my amazing wife Julie, the light and joy in everything I do and who has worked, played, laughed, cried, and prayed with me through this journey, and our four sweet children, Ammon, Lucy, Zachary, and Russell, the last three of whom were born in the wilderness of our graduate school years. Because of them, my work has meaning.

ABSTRACT

The motivation of this thesis is the study of markets in which consumers are under-informed concerning the quality of any given product and in which the quality of consumers also matters to producers of products. This study has resulted in a primary application paper, comprising the first chapter which focuses on the market for training lawyers, as well as a second technical chapter exploring theory which can prove useful in analyzing these markets.

The first chapter is based on the observation that the number of lawyers being produced at high cost combined with the relative lack of job options has recently created significant concern. In order to partially explain this phenomenon, I propose a game of incomplete information modeling the strategic interaction between law schools as they compete for potential students. The information asymmetries come from the fact that any given law school is better informed about the quality of its education than its potential students. Using a change in market information structure generated by student placement reporting requirements, I use the model to estimate the dynamic effect of increased information on distributions of tuition rates, incoming student ability, class sizes, and the rate at which law schools open and potentially close. Using these estimates, I show that there have not necessarily been too many law schools or students, but rather an equilibrium enforced mismatch between students and their optimal schooling choices. The new information has acted as a forced collusion mechanism to partially overcome this mismatch, which has differentially

decreased school welfare, strictly increased student welfare, and resulted in a positive total welfare gain of $685 million.

The second chapter provides a thorough exploration of the microeconomic foundations for the multi-variate linear demand function for differentiated products that is widely used in industrial organization. A key finding is that strict concavity of the quadratic utility function is critical for the demand system to be well defined. Otherwise, the true demand function may be quite complex: Multi-valued, non-linear and income-dependent. The solution of the first order conditions for the consumer problem, which we call a local demand function, may have quite pathological properties. We uncover failures of duality relationships between substitute products and complementary products, as well as the incompatibility between high levels of complementarity and concavity. The two-good case emerges as a special case with strong but non-robust properties. A key implication is that all conclusions derived via the use of linear demand that does not satisfy the law of Demand ought to be regarded with some suspicion.

PUBLIC ABSTRACT

The motivation of this thesis is the study of markets in which consumers are under-informed concerning the quality of any given product and in which the quality of consumers also matters to producers of products. This study has resulted in a primary application paper, comprising the first chapter which focuses on the market for training lawyers, as well as a second technical chapter exploring theory which can prove useful in analyzing these markets.

The first chapter is based on the observation that the number of lawyers being produced at high cost combined with the relative lack of job options has recently created significant concern. In order to partially explain this phenomenon, I propose a game of incomplete information modeling the strategic interaction between law schools as they compete for potential students. The information asymmetries come from the fact that any given law school is better informed about the quality of its education than its potential students. Using a change in market information structure generated by student placement reporting requirements, I use the model to estimate the dynamic effect of increased information on distributions of tuition rates, incoming student ability, class sizes, and the rate at which law schools open and potentially close. Using these estimates, I show that there have not necessarily been too many law schools or students, but rather an equilibrium enforced mismatch between students and their optimal schooling choices. The new information has acted as a forced collusion mechanism to partially overcome this mismatch, which has differentially

decreased school welfare, strictly increased student welfare, and resulted in a positive total societal benefit of $685 million.

The second chapter provides a thorough exploration of the foundations of one of the standard models in the theoretical literature for studying markets in which products are differentiated, not only by how good they are, but also by how they contrast with each other. We show that some of the standard assumptions made on this type of models do not necessarily hold and that the violation of these assumptions can result in untrustworthy results.

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INFORMATION ASYMMETRY IN PROFIT-GENERATING GRADUATE EDUCATION MARKETS: A STRUCTURAL APPROACH TO LAW SCHOOLS

1.1 Introduction

Over the last decade, there has been an influx of evidence suggesting a persistent disconnect between expected and actual returns to attending law school. In 2010, the projected national market surplus for lawyers exceeded 27,000, with all but two states displaying oversupply [19]. Although a somewhat coarse measure, using total number of bar-exam passers (53,508 in 2009, 54,448 in 2010, and 55,387 in 2011 [53]) compared to projected total number of annual job openings for bar-qualified workers over the years 2010-2015 (26,239) is nevertheless telling. Further, there has been a consistent increase in both number of schools operating, with 8-20 new schools per decade and no schools closing down and enrollment increasing by roughly twenty-thousand students nationwide per decade [1]. Consequently, while the American Bar Association (ABA) predicts 440,000 new law graduates between 2008 and 2018, the Bureau of Labor Statistics (BLS) predicts 240,400 lawyer jobs created during that time [45].

By classical reasoning, this production behavior should be associated with an increase in the expected value of going to law school, either through a decrease in the price of attending school or through an increase in expected post-graduation earnings. Neither seems to have been the case. Nominal tuition has grown at an average rate of 6.7% per year since 1993, 2.6 and 1.8 times more than private and public 4-year undergraduate institutions, respectively [13]. In 2013, the average education-based debt upon graduation from law school was over $80,000, with 83% of graduates from ABA-accredited schools finishing with student debt [72].

Wages have not improved in real terms over this period, although they have not dropped either. [68] show stable real wages for lawyers over the past two decades, with a high mean of $100-$140 thousand. However, these data are for lawyers reporting to the BLS, which is not necessarily indicative of graduates from law schools in general. Besides the usual aggregate reporting bias (people happy with their salaries are more likely to report), lawyer jobs are not necessarily available to graduates of any law school. Consider, as an example, the case of Shell Oil [33]. Shell hires in-house attorneys to handle various legal issues. Shell is certainly not one of the top lawyer positions in the country, and yet if a student graduated from a fourth-tier school, they will only consider her application at all if she was in the top 5% of her class. This is one example of a broader phenomenon, that while students from top tier schools might be able to get a job as a lawyer, this option might not even be available to students from lower tier programs.

An answer to this apparent market failure comes in light of the information structure inherent in this market. For the majority of law students, the primary goal of attending law school seems to be to make more money (for example, in 2010 only five percent of students went in to public interest in favor of more profitable sectors [52]). As such, the primary school quality indicator relevant to a student should be job placement. However, until 2010, the placement number reported each year to the ABA, which governs official school-level statistics in the United States, was percentage employed nine months after graduation. This statistic does not specify if those jobs were in high paying law firms, the low skill service industry, or even at the graduate's law school with a part time position around the nine month marker. There were other secondary statistics reported, such as proportion placed in a law job or in business, but those were also fairly uninformative with respect to the quality of work.

Given this information structure, a student was left to observe job placement and impute wages from available data on lawyers, either from the BLS or from school reports to the US News. However, the BLS data are highly skewed towards graduates from top-tier programs and US News reports have suffered from underreporting and misrepresentation. A student interested in a school might think the tuition payments are reasonable given a 95% placement rate in jobs 9 months after graduation with lawyers making $120,000 on average per year. The danger in this reasoning was highlighted by an interview with the former Dean of the New York Law School who asserted that, while NYLS students expect to be making $160,000 per year upon graduation, the same as graduates from Yale or Harvard, they will actually be earning a median wage of $35-75,000 (which could be inflated as well, given that this figure was based on a 26% reporting rate of graduates) [65].

In 2010, in response to pressure from law school graduates and the media [64], the ABA changed placement reporting standards for accredited law schools. Rather than a few vague categories, schools were now required to report a battery of specific placement types, such as “number of students employed full time, short term in a law firm with 101-250 employees” or “number of students employed part time short term in a state or local clerkship”. With these new reporting standards, students could look at any given school and much more accurately infer their expected post-graduation wages.

In analyzing the time periods before and after the change in this market’s information regime, I find significant positive and normative effects. First, using raw data and reduced form results, I show suggestive evidence that students have responded to the increased information through willingness-to-pay. Schools in lower quality brackets have had to drop tuition rates in response to a drop in the marginal valuation of their degree. Lower quality schools have also lowered their standards for admissions as measured by the LSAT scores and undergraduate GPAs of their incoming cohorts.

The relative elasticity of LSAT scores to undergraduate GPA in the incoming class combined with inelastic class sizes suggests a significant rank premium affecting a school’s decision making process. To capture the rank premium, to recover student preference parameters and to allow for school shut-down and welfare calculation in counterfactual analysis, I propose a dynamic game of incomplete information modeling the interaction of schools as they compete for student enrollment.

The dynamic game and corresponding structural estimates provide values for both producer (school) and consumer (student) surplus. Schools are affected differentially by the new information policy. Top schools benefit from the quality separation as their degrees are revealed to be highly lucrative. For schools in the lower half of the top tier or in the second or third tiers, the extra information negatively affects welfare through policy adjustments tailored to maintain rank premium.

Lower third tier and fourth tier schools actually benefit from the increased information. This seemingly counterintuitive result stems from the fact that, under the previous information regime, a substantial subpopulation of lower ability potential students was being crowded out by higher ability students.

High ability students incorrectly believed that attending a low quality law school would yield higher returns than their outside option (going straight into the labor force). These students thus were incentivized to attend low quality programs. Using the incorrect expectations, schools were able to enroll enough higher ability students to both generate a sufficient revenue stream and maintain their relative standing among other schools. This behavior by schools resulted in excluding lower ability students from enrollment, who, based on their outside option, would have actually benefited from a low quality law degree.

The new information regime acts as an exogenously imposed collusion mechanism. All lower quality schools are forced to simultaneously reveal that their product is of lower value than many of their previous students' outside options. Since all schools act simultaneously, there is little rank effect since ranks are relative. Further, all low quality schools now have access to the previously underserved subpopulation of lower ability students, thus actually increasing enrollment and consequent tuition revenue.

The aggregate change in producer surplus is negative with a mean estimate of -$212 million. Much of the loss to producer surplus is recovered by students as they receive a product, either through going to law school or opting out of the market, consistent with their preferences and abilities. The aggregate consumer surplus change is $575 million, resulting in a total surplus gain of $363 million.

The idea that students have incorrect expectations concerning returns to schooling has been the subject of several previous studies. [48] discusses this phenomenon, explaining how students often can have post-education wage expectations different from reality. [5] estimate expectations of the return to choosing a particular major. They show that, not only do expected earnings matter when students choose a major, but students' expectations with respect to major-specific earnings are often wrong. This inconsistency between schooling expectations and reality can partially explain why students are willing to pay for a degree in a low-return field, with schools using these expectations to extract tuition rent.

The problem of using information frictions between students and schools to inflate profits is indicative of a broader phenomenon. That is, 1) information asymmetry with respect to product differentiation can have serious effects on proper function of market and 2) market function can be restored when the information asymmetry is removed. These two points are part of the quality disclosure literature[1]. In the case of law schools, schools have had more information than students concerning the

[1]For an excellent review of the quality disclosure literature, see [17]

quality of their product (law degree). [18] show in the market for health care that when patients are better informed concerning the quality of any specific hospital, prices and general treatment both improve while hospitals also change the type of risk they are willing to take on. This change in healthcare behavior gives evidence that hospitals can use information asymmetry to affect prices in a similar way to law schools. [37] show that consumers choose health plans consistent with published information available, but inconsistent with some important unpublished indicators of quality. Health plan providers can extract rent based on the information asymmetry between themselves and their clients.

While previous work has shown how information can affect prices and demand for a product, this is the first paper, to my knowledge, to document a systemic mismatch between consumers and producers created by information frictions. This mismatch is due in large part to the two-sided nature of the legal (or any) education market in which the quality of the consumer also matters to the producer. The paper also gives encouraging evidence that information availability can provide a market-based approach to at least partially mitigating this mismatch.

1.2 Information Structure

Information in the market for training lawyers is largely governed by the ABA. The primary function of the ABA is to set accreditation standards for law schools in the United States. If a school is not ABA accredited, its students may not (with the exception of California) take the bar exam. Thus, for a school to be viable, it must be ABA accredited. One of the primary tools the ABA uses to make accreditation decisions is its yearly school questionnaire, which is published each year jointly with the Law School Admissions Council (LSAC) as part of the "ABA-LSAC Official Guide to ABA-Approved Law Schools" [46]. This questionnaire provides statistics on various attributes of a school such as Curriculum, JD Enrollment and Ethnicity, GPA and LSAT Scores, and Grants and Scholarships. For a sample of a pre-2010 questionnaire, see Appendix B.1.

Much of the information reported in the ABA-LSAC report might be interesting or important for reasons pertaining to social issues (ethnicity and gender of students) or to educational atmosphere (student to teacher ratio or number of professional librarians). However, based on the prevailing motivation to attend law school, the indicator of most concern to a student deciding which, if any, school to attend should be expected wages after graduation.

In reports issued before 2010, placement success was primarily measured by the number of students placed in a job nine months after graduation. There were also refinements given, including number employed in law firms, in business and industry, in government, in public interest, as judicial clerks, or in academia. However, these refinements gave little information with respect to salaries. A student employed in a law firm could make anywhere from $30-130 thousand a year. An employee in "business" could likewise be earning a high salary on a management track at a large corporation or minimum wage working in a low-skill sector.

Since the traditional ABA-LSAC report was mostly uninformative concerning post-graduation wages, if a potential student wanted to go to law school, she would have to make a judgement based on the data provided for the market for lawyers in general and extrapolate from the data provided by a law school for the ABA-LSAC report. She might see, for example, a low-ranked school that places 85% of its students in a job nine months after graduation, see BLS report that lawyers make $120,000 per year and reason that it would be a good investment to take on $100,000 in student debt to get that degree.

This extrapolation does not, however, fit the reality of the lawyer market. Oyer and Schaefer show that, not only do graduates from the top 20 law schools comprise the majority of partners and associates in the top major law firms in the United States [57], but graduates from the top 10 schools make 25% more money than those from top 11-20 schools and 50% more than those from schools ranked from 21-100 [56].

An alternative to the BLS wage reports is the Starting Salaries [52] report from the National Association for Law Placement (NALP). This report has been produced in an attempt to decrease the information gap between student expectation and reality by reporting wage statistics for each of the reported categories in the ABA-LSAC report. However, since the NALP does not have the same influence as the ABA, it has not been able to maintain substantial reporting rates. For example, while over 40,000 lawyers passed the bar exam in 2010, there were only 18,398 respondents in total who responded to the NALP survey. Further, those reporting are generally also those making more money [65].

While schools have generally been aware of the disparity between students' wage expectations based on available data and the realities of the market [65], they have been able to use their asymmetrically high information set to extract rent from ever more students in the form of tuition revenues as new schools have opened, class sizes have grown, and tuition rates have increased.

In an attempt to mitigate this issue, in 2010 the ABA modified its reporting requirements. Rather than fifteen vague categories, schools were now required to report on 144 specific placement types. Rather than "Employed," a school would now have to report various types of employment, such as "Employed - Bar Passage Required, Full Time, Short Term." Rather than "Employed in a Law Firm," a school would now have to report on the size of the firm, such as Solo, 2-10, 11-25, 251-500, etc. The firm size reports are especially informative given information made available by the NALP. Besides average starting salaries in general industries, the NALP also reports starting salaries at law firms conditional on firm size. Table 1.1 is an example report from 2011 [51]. As stated before, these numbers are likely biased. However,

Firm Size	2-10	11-25	26-50	51-100	101-250	251 or More
Salary	73,000	73,000	86,00	91,000	110,000	130,000

Table 1.1: Median Lawyer Starting Salary per Firm Size (2011)

given the motivation behind the reporting bias is aversion to reporting and getting reports on low salaries, the bias should either be uniform across category types or more severe for lower-sized firms, generating lower wages generally. The difference in reported wages between small firms and large firms should therefore be a lower bound on the actual spread. The lower bound is still substantial, with the difference in earnings expectations between the largest and smallest firms reaching almost $60,000. Because of the reporting change, students debating law school attendance post-2010 can use the newly reported categories combined with these wage reports to make a better informed decision about which, if any, law school to attend.

Besides the ABA and the NALP, the US News and World Report provides one more major source of information. Although the US News does not provide a large number of school characteristics for public consumption, it does serve as the primary data aggregator in this market through its primary instrument, the US News and World Report Graduate School Rankings [72].

To construct its rankings for law schools, the US News utilizes data required for the ABA-LSAC report as well as several of its own measures. The most notable variables with regard to salary expectations are percentiles for starting salaries of law school graduates. While these measures should be the most informative to students, they have traditionally possibly exacerbated the information asymmetries due to low reporting standards, with documented evidence of schools reporting their graduates earning as much as $100 thousand more on average than they actually were [65].

1.3 Data

The market for training lawyers is characterized by two general sets of players: schools and students. To estimate parameters relavent to both sets, I utilize both school-level and student-level data. For initial results showing effects in the market in Section 1.4, I will only use school-level data. Individual-level data will be necessary for identifying welfare effects in Section 1.5.

1.3.1 School Level

School-level data were taken from the start of academic years 1998-2013, which are designated as the years 2000-2015 for both the ABA-LSAC report and the US News and World Report rankings data. The US News data includes both the rankings and extra data used to for the rankings not included in the ABA-LSAC report. Summary statistics are reported in Table B.1.

The variables reported represent student quality and school characteristics. Quality is given by two measures. The measure is the US News Rank. While US News rankings certainly do not give a perfect quality separation measure between schools, it does give a reasonable approximation of tiers. For example, #1 ranked Harvard clearly has a better law program than the #144 ranked South Texas College of Law.

The second quality variable not only measures quality separation, but also explicitly captures the new information made available to students concerning school quality. I define this measure as a school's *placement ratio* which is constructed using the new information available with the post-2010 ABA/LSAC reports.

I first take a weighted sum of graduate placement in law firms, weighted by firms size according to the 2011 NALP report in Table 1.1. I normalize the highest possible wage to unity, representing a “full” placement. Each subsequent category is weighted as the fraction of the highest possible wage. For example, each placement in a firm with 51-100 employees would be weighted by (91,000 / 130,000), or 0.7. For solo firms, I assume the same wages as the next two smallest firm categories. I put zero weight on placements in firms of unknown size.

After generating the weighted placement, I divide it by raw placement numbers in law firms. Since I normalize the highest possible wage to unity, this ratio will always lie strictly on the unit interval. As is shown in Table B.1, the mean and median are close at 0.64, indicating that the average school is placing its students at jobs with starting salaries only 60% of the top salaries. The best placing school is gaining its graduates 97% of the top salaries on average and the worst placing schools are achieving 33%.

For the placement ratio to be useful, it must span all the time periods available in the data. However, by definition, it cannot be calculated earlier than 2010. I use ratio persistence to motivate an imputation solution. A basic AR(1) model yields an AR coefficient of 0.915 with a standard error of 0.015. Thus, I impute the ratio for all previous years as that calculated for 2011 and designate this variable as each school’s persistent type.

Tuition is the price of attending law school while Freshmen is the size of the incoming class. This class is characterized by its average undergraduate GPA and LSAT scores. Schools also award grants and have other room/board expenses, cost of books, large or small faculties relative to student body size, acceptance statistics, and Bar examination passage rates at first attempt.

The variable for tuition is constructed from the sticker price for a full-time out-of-state student for schools that price discriminate based on residency and standard full-time tuition for schools that do not. I discuss the use of sticker price more in depth in Section 1.5. Since the tuition reported in any given ABA-LSAC report actually corresponds to the tuition paid by the previous year's students, I lag tuition by one year. The lag aligns payments with the academic year represented by the rest of the report.

Table B.1 also shows summary statistics for 25th and 75th percentiles of reported private sector salaries. The mean 25-75th percentile spread is about \$56-89,000 which is somewhat higher than the median spread of \$48-80,000. These suggest a possible median salary in the range of \$64-72,000, which is about \$10-20,000 lower than the salaries implied by the transparency ratio. And once again, although better than the NALP, reporting rates are somewhat low, with mean and median rates of 61% and 64%, respectively. Since schools are aware that students are highly influenced by the rankings developed by this report, the salary reports are once again likely biased upwards.

1.3.2 Student Level

Student-level data were taken from Law School Numbers (LSN) [44], an online resource for potential law students. LSN was created primarily as a forum for law school applicants to report on application, admission, and enrollment decisions as they go through the process of possibly attending law school. LSN makes these reports publicly available as a crowd-sourced database on the state of law school applications and admissions.

A unit of observation in the dataset extracted from the LSN reports is a school-student pair, with one observation for each pair. Each observation includes the year of the potential match, the student's profile, consisting of her LSAT score and undergraduate GPA[2], the name of each school she applied to, and the result of that application, including whether or not the student was accepted (or waitlisted) and the student's decision (whether or not she decided to matriculate).

Various summary statistics for the LSN dataset are reported in Tables B.2, B.3, and B.4. Table B.2 provides student summaries conditional on one of three stages of the school admissions process: application, admission, and matriculation. Tables B.3 and B.4 summarize the entire student sample by ability quartile. In all tables, statistics are provided for student samples under the old and new information regimes.

From Table B.2, it seems that the applicant skill distribution remains practi-

[2]GPA is reported for both raw degree GPA, the number reported on the student's diploma, and the LSAC-adjusted GPA, which is the one used by law schools for admission decisions.

cally unchanged under the new information regime. The biggest difference between the two samples is in admission and matriculation probabilities. In the treatment period, probability of admission increases by roughly 0.06 while probability of matriculation decreases by 0.06 percent. Although small, this is indicative of a phenomenon that will be studied more in depth later in the paper. Specifically, schools face smaller applicant pools under the new information regime and students have a lower valuation of certain types of law degrees, thus decreasing matriculation probability.

The information effect is further emphasized in Tables B.3 and B.4. While once again, the distribution of applicant type remains virtually unchanged across information regimes, admission and matriculation probabilities change again, with most of the difference observed in the second quartile of students. Once again, this difference indicates a decrease in options for middle to high quality schools as well as a shift in expectations for middle to high ability students.

1.4 Evidence of Information Effects

Behavior of law schools with respect to students has evolved according to the information structure as well as market indicators. Figures B.1 and B.2 provide visualization of the primary variables of interest. These variables include tuition rates, number of students, incoming class profiles (undergraduate GPA and LSAT scores), number of applicants, grant awards (both number and size), and starting salaries of graduates (25th and 75th percentiles).

1.4.1 Conditional Quartiles

Each variable is divided into quartiles based on school quality. In Figure B.1, quality of a given school is measured by its rank as assigned by the US News and World report. As previously discussed, this is a standard quality measure in this market. In Figure B.2, I use the placement ratio as defined in Section 1.3.1.

Since the placement ratio was not observed before 2010, behavior of students and schools conditional on the ratio should change based on being in a post-2010 world. The population thus becomes a policy treatment group with treatment being the new school-idiosyncratic information administered starting in 2010. Ideally, the data would include a control group as well, unaffected by this extra influx of information. Although there is no strict control group, I can utilize the fact that the top schools would be expected to place almost all their students in the most competitive jobs. Thus, the extra information would technically have a negligible effect on these programs. These schools could thus be considered a quasi-control group and the treatment could be considered as given post-2010 in some varying amount over the compact interval [0, 1], with 0 indicating a full dose and 1 indicating no treatment.

US News ranking as a quality measure does not *explicitly* capture the information effect of 2010. *Implicitly*, the same school behavior stemming from the extra information in 2010 should be observed with respect to rank as well since rank and placement ratio are highly correlated, with a correlation coefficient of -0.75 for the years in which placement ratio can be directly observed.

1.4.2 Variable Evolution

There are two main features of Figures B.1 and B.2. The first and most obvious is the aggregate drop in the market in 2010. This drop stems from two sources. The first is the great recession of 2008. While many industries were affected immediately by the market downturn, the effect on the market for lawyers lagged roughly two years behind the rest of the economy. This effect is born out primarily in legal salaries, as can be observed primarily in the 75th percentile of salaries and somewhat in the 25th percentile as well.

While the effects of the recession could have a trickle down effect on school and student behavior, such as application rates and quality of students going to law school, it is likely that this is not the primary driver behind the corresponding aggregate shocks in these other variables. This is due to the fact that the recession-based shock affected all industries, not just lawyers. Without a stable outside option, substitution effects should be small.

The second source of this aggregate drop in the market, however, was idiosyncratic to the market for lawyers. Between 2010-2011, the legal market experienced an aggregate opinion shift regarding law degree profitability. Stemming from the seminal New York Times article by David Segal, “Is Law School a Losing Game?” [64], a sentiment developed in the United States that many students were graduating from law school, apparently without a way to pay back loans incurred during schooling. Segal, and many authors following him, contended that going to law school might not in fact be profitable on average. The ensuing series of articles, blog posts, etc.

sparked an aggregate downturn in demand for law school. This is likely one of the primary causes of the drop in applications to law school beginning in 2010.

The second feature of note in Figures B.1 and B.2 beyond the aggregate shocks is the separation in several variables between quantiles after 2010. The most obvious is undergraduate GPA of a given school. In both Figure B.1 and Figure B.2, the lower the quality of a school, the more it had to drop its threshold for the quality of students it would admit. The same separation holds for Tuition as well, although more prominently in Figure B.2.

While it is clear that the separation holds for GPA, it seems possible upon visual inspection that it might also be occurring in the other variables. This could be of concern when trying to identify the effects of information. Specifically, if there is a significant separation of salaries conditional on quality after 2010, it could be that the recession effects are primarily driving the altered school behavior. The same holds for the aggregate dropped expectations of the return to law degrees and number of applicants to law school.

To test for the presence of quality conditional post-2010 separation, I propose the following specification

$$y_{it} = \beta_0 + \beta_1 R_{it} + \beta_2 Post2010_t + \beta_3 (R_{it} * Post2010_t) + \gamma X_{it} + \varepsilon_{it}, \tag{1.1}$$

with R either rank or ratio and y_{it} either one of the primary school outcome/choice variables including class size and tuition (quantity and price) and quality of students willing to attend (GPA and LSAT scores) or one of the other market variables (starting salaries or number of applicants). X is a vector of controls from the set

{{Rank}, {Tuition, Median Undergraduate GPA, Median LSAT}, {Median Grant, Percent Receiving Grants, Room/Board Expenses, Cost of Books}, {Student/Faculty Ratio, Number Accepted, Acceptance Rate, Bar Passage Rate in Jurisdiction}}. Note the subsetting of X. These subsets correspond to models controlling for 1) rank (in the case of $R =$ ratio), 2) school choice variables, 3) cost variables, 4) miscellaneous quality controls. Each model will be estimated separately as well as once with all four sets of controls.

For each possibility for y, β_3 is the primary parameter of interest. Since an increasing value of ratio and a decreasing value of rank both indicate an increase in quality, the sign of β_3 should be opposite between the two. I will proceed discussing the case of *ratio*, but with the understanding that the converse should also hold for *rank*.

I first address the question of either application or salary separation to determine if they might confound identification of the desired information effect. Consider first the model with $y =$ number of applications. The results are given in Tables B.6 and B.5. Notice that most of the model specifications result in an estimate of β statistically indistinguishable from zero. The exceptions in both cases are when admission rates are included as controls. However, this could yield artificially significant results because of the high collinearity between number of applications and application rates.

The more worrisome results are in Tables B.8, B.7, B.10, and B.9 which correspond to 25th and 75th percentile salaries. With $R =$ ratio, β_3 is significant and positive for both percentiles under every model specification. However, for $R =$ rank, the significance almost completely disappears. If both the ratio and rank specifications had yielded a significant interaction coefficient, there could have been strong evidence that salaries themselves are driving the separation in the other variables. However, the disparity between the two sets of models suggests that the connection between the ratio models and salary separation is part of some process outside of the effect connecting ratio and rank. As long as both the ratio and rank models yield the correct values for β_3, this would provide evidence that the information effect is indeed being identified.

If the information treatment is effective, β_3 should be positive for (y = Tuition). That is, β_3 represents the immediate shift in students marginal valuation of a degree from any given law school based on increased transparency due to information restructuring.

For GPA, LSAT and Class size, β_3 depends on a school's dynamic consideration. A school with a decreased reputation could maintain class size and drop the quality of students it accepts. This way, it can maintain immediate revenue. However, lowering student quality also hurts its future rank, which decreases its future revenue. The elasticity of GPA, LSAT, and Class size will depend on how much a school could gain or be hurt in the long or short run. The estimates for Model (1.1) for dependent variables Tuition, Class Size, Undergraduate GPA, and LSAT are given in Tables B.12 B.14 B.18 and B.16.

As expected, Tuition responded significantly to the information shock. For the baseline estimate with no controls, the treatment coefficient indicates that a school with a 10 percentage point higher placement ratio will have a $1.6 thousand higher marginal valuation on the part of its students. This increases when controlling for Rank to $2.7 thousand, but drops to between $600 and $1.2 thousand for other control specifications.

Controls are based on incoming class profile (Freshman Class Size, Undergraduate GPA, and LSAT scores), cost structure (Median Grant, Percent Receiving Grants, Room/Board Expenses, and Cost of Books), and other quality controls (Student/Faculty Ratio, Number of Students Accepted, Acceptance Rate, and Bar Passage Rate in Jurisdiction). Incoming class profile seems to account for much of the extra variation in tuition rates (highest effect on R^2), as well as part of the covariance between the treatment effect and Tuition.

The model only controlling for freshman profiles is the only model that causes the significance of the treatment effect to fall below the 0.001 level. However, the significance jumps back when including the other controls as well.

In the final estimate, most of the variables are highly significant. The three exceptions are Post-2010, the level-effect for being in treatment period; Freshmen, the size of incoming class; and Bar Passage Rate. Class size is likely highly correlated with some of the other controls, making its insignificance in the full model not surprising. Bar Passage Rate was never significant, which is surprising. Post-2010 is not only most insignificant across specifications, but also changes signs frequently.

The treatment level effect is likely insignificant for two connected reasons. First, the other variables in the dataset account for much of the variation in Tuition. Second, the treatment period is relatively short. This makes the fact that the treatment effect is so significant even more surprising. In the final model with all controls included, $\beta_3 = 9112.45$, suggesting that a school with a 10 percentage point higher placement ratio would optimally be able to charge \$911.25 higher price for its product.

The coefficient estimates for the class size and student profile models shed light on the profit structure for law schools. First, as is seen in Table B.14, class size seems to be completely inelastic with respect to the information shock. The ratio-treatment interaction coefficient is insignificant in every model specification. LSAT scores are similarly rigid. While the coefficients are all technically positive, none are significant.

Schools are statistically more flexible with the undergraduate GPA scores of incoming classes, although the substantive reaction is small. For the baseline specification with no controls, the ratio-treatment coefficient is 0.17. This coefficient implies that a school that dropped 50% in its expected returns would have to lower its GPA standards by roughly 0.09 GPA points. The change could also be as small as 0.045 GPA points. Across every specification, though, this change is significant and is on average about 0.07.

Few of the controls are significant predictors of GPA and are motivated, not by causation, but because of joint indication of school or student quality. For example, students with higher GPAs will likely have higher LSAT scores. Thus, the LSAT coefficient is significant. GPA is also understandably negatively correlated with acceptance rate, since in order to increase the student quality threshold, a school must be more selective in accepting students.

1.4.3 Discussion

The relative elasticity of tuition compared to class size is indicative of inelastic supply with respect to demand. The change in information in 2010 would represent a demand shift for any given school. As is well known, if the supply is relatively inelastic with respect to demand, we would expect to see a large change in market price but little to no change in the quantity being produced.

Inelasticity of supply indicates a high marginal cost of producing students. In the case of law schools, this would arise not in direct marginal cost, but in the forgone rank premium associated with enrolling a higher number of students. As a school admits the marginal student, it must lower its quality threshold to allow a marginally less qualified student to attend. In doing so, although the school gains in current tuition, it loses future profits related to the drop in prestige.

The argument of a high rank premium is further born out in the relative flexibility of GPA compared to LSAT scores. LSAT scores are not only considered a more important prestige measure for a school, but they are also weighted 40% more in the US News ranking formula. Given that schools apparently find it optimal to maintain class size, they would rather lower GPA standards than the more valuable LSAT standards.

1.5 Structural Framework

The reduced-form evidence in Section 1.4 suggest that the new information regime has changed the way that students view any given school as well as how schools can behave with respect to students. It also highlighted that schools seem to make dynamic decisions with respect to payoff-relevant variables like class size and quality. However, the discussion thus far has been agnostic concerning the normative effects of the new information regime. The question remains, has this influx of information been good for the economy? If so, in what way and for whom? To determine societal welfare effects, it is necessary to impose additional structure.

As stated, the reduced-form evidence suggests a dynamic decision-making process. A school's dynamic considerations should be with respect to its own evolution as well as to the evolution of its competitors. Each time a school tries to attract students, it is competing for those matriculations with several other schools of similar quality. As such, to model the dynamic considerations of a school, it is necessary to include the strategic effect.

Modeling this market as a dynamic game also has the benefit of allowing identification and estimation of structural payoff-relevant parameters otherwise unattainable. As was discussed in the previous section, the behavior of schools suggests a high rank premium. This should affect schools not only though tuition revenue, but also through donations and grants, a significant source of revenue but unobservable in the data. The structure imposed in the following section will allow for identification of parameters associated with the rank premium and allow for calculation of producer (school) surplus. The extra structure will also provide identification for the utility for individual students from attending law school as well as their outside option. Thus, we can also analyze consumer (student) surplus.

Structural estimates are also necessary for positive dynamic analysis concerning entry and exit of schools. As stated earlier, between 8-15 new law schools have opened nationally per decade for many years. There has, however, been no exit. The estimates of Section 1.4 suggest that profitability for schools, especially low quality schools, could decrease. For some schools who entered the market with the expectations of high revenue payments, the profitability drop could lead to shut-down.

In order to capture all the relevant player characteristics, I propose a school choice model based on the application/admission game in [28], but extended to a dynamic framework in the tradition of [49] and [23]. The dynamic extension allows for explicit consideration of rank effects as well as entry/exit decisions.

1.5.1 Players

The market for training lawyers is characterized by two sides of players: schools and students. Schools act as competing oligopolistic firms producing their good, a law degree, for their consumers, prospective students. Schools are infinitely lived and make decisions in discrete time periods (one year per period), discounting future returns at rate β. Any given student only participates once for one period in the market.

Schools, each consisting of a tuition office and an admissions office, are differentiated by their rank which acts as a proxy for relative quality. In this application, rank will be operationalized by a school's rank as measured by the US News and World Report [72]. A more in-depth discussion of the use of US News rankings was given in Section 1.3. A rank of zero indicates that the school is not operating (that is, it has not yet opened or has closed). In any period, the state is represented by the tuple (R, g). R is a vector of length $\overline{N}$, where $\overline{N}$ is the maximum possible number of operating schools and $R_j \in R$ is the rank of school j in the current period for any $j \in J$, the set of currently operating schools. The remaining state variable g is a demand growth term that will be discussed later. Each school is endowed with a fixed capacity $\overline{Q}_j$ with $\overline{Q}_j > 0$ and $\sum_{j=1}^{\overline{N}} \overline{Q}_j < 1$.

The rank vector is proxy for the production technology available to any given school. That is, a school ranks represents what quality of a product the school can produce. In the case of law schools, quality of a program is assumed to be the expected financial returns to getting a degree. Although some students do enter law school on more philanthropic grounds, given that only five percent of 2010 law graduates went in to public interest in favor of more profitable sectors [52], the assumption that students attend law school in order to purchase a better wage distribution seems reasonable.

The other state tuple element, g, is a time-specific demand shifter. This will grow according to a degenerate process and captures the persistent exogenous effects on aggregate tuition growth in law schools. This is in large part due to an increase in federal funding and loans to law students and is connected with the perceived benefits of attending law school in general [54].

There is a continuum of students, normalized to unit measure. Students differ in ability endowment A, which is unobserved but correlated with a student's LSAT score and undergraduate GPA, given by the tuple $(LSAT, GPA)$. I assume A is distributed $N(f_A(LSAT, GPA), \sigma_A^2)$, with $\partial f_A/\partial LSLAT > 0$, $\partial f_A/\partial GPA > 0$, and $\partial^2 A/\partial LSAT \partial GPA > 0$.

In order for student i to apply to school j, she must pay the application cost $C(\cdot)$, which is increasing in number of applications.

1.5.2 Stage Game

In each period, students and schools play an application/admission game in the spirit of [28]. The timing of the stage game is as follows: first, each school announces a tuition level to which it must commit (which gives the game a Bertrand-type competition structure, since tuition represents the price in this market); second, students make application decisions while schools simultaneously choose admission policies; and third, students learn about admissions results and make enrollment decisions.

1.5.2.1 Student Payoffs

The preferences of student i with ability A with respect to attending school j of rank R is given by the random indirect utility function

$$u_{ij} = \overline{u}(A_i, R_j, I) + \epsilon_{ij} \tag{1.2}$$

with $\overline{u}(\cdot)$ the average preference for a student with ability A to attend a school with rank R and $\epsilon_{ij} \sim N(0, \sigma_u^2)$ student i's idiosyncratic preference for school j. The realization of ϵ occurs after admissions decisions have been made. The variable I is binary and represents the information regime in place, with $I = 0$ indicating the reporting system in place before 2010 and $I = 1$ the system with more detailed placement reports which the ABA implemented after 2010.

I assume that all students pay the same sticker price tuition rate, or that there is no price discrimination in this market. This assumption ignores scholarship behavior, an important aspect of the market, and is made due to data limitations. Given the available data, integration of price discrimination is possible directly through the mechanism in [28]. [24] also provides an alternate approach to utilizing price discrimination based on an auction-theoretic model. However, since the primary goal of the analysis at hand is to study the welfare changes from the introduction of a new information regime, the only cause of potential bias would come from a systematic change in price discrimination behavior based on the regime change. While the data in Figure B.1 suggest that there is unlikely to be such a change, potential biases from this omission will nevertheless be discussed in Section 1.7.

For tuition profile $t \equiv \{t_j\}_{j \in J}$, the ex-post payoff to student i for attending school j is

$$U_{ij}(t) = u_{ij} - t_j. \tag{1.3}$$

1.5.2.2 School Payoffs

Period profits for an individual school are derived from tuition revenue as well as yearly donations and fixed costs. Tuition revenue is given by

$$\tilde{\pi}_j = \int_0^{Q_j} t_j di \tag{1.4}$$

with Q_j the incoming class size for school j.

Donation revenue comes mostly in the form of alumni giving as well as earnings from private and public endowments/grants and are a function of the school's rank. While there are effectively zero explicit marginal costs for schools (although there are implicit marginal costs through reputation effects, which are discussed below), fixed costs are substantial and vary according to quality of a school. Higher quality schools generally offer higher faculty salaries, have better and more expensive resources that require maintenance and replacement, etc.

Although donations and fixed operation costs are not separately identifiable, since they are both functions of rank I can identify donations net of operating costs with a net donation function. I parameterize the net donations with the quadratic function

$$D(R_j; \delta) = \delta_1 R_j + \delta_2 R_j^2. \tag{1.5}$$

If fixed costs are greater then than yearly donations, D will be negative. There is possibly also some fixed cost δ_0 associated with running a school regardless of rank. The decision to exclude δ_0 is based on data availability, since I do not observe cases in which schools shut down but do not leave the market. Without such variation, I cannot identify δ_0. However, as long as δ_0 is independent of the information regime, its exclusion will not bias the final results with respect to change in welfare.

An inactive school deciding to open independently draws a private information fixed entrance cost κ at the beginning a period from the distribution $N(\mu_\kappa, \sigma_\kappa^2)$. Incumbent schools also at the beginning of a period receive a privately observed independent scrap value draw ϕ from distribution $N(\mu_\phi, \sigma_\phi^2)$. Thus, a school i participating in action a receives the period profit adjustment

$$\Phi(a_j; \kappa_j, \phi_j) = \begin{cases} -\kappa_j, & \text{if the school is a new entrant} \\ \\ \phi_j, & \text{if the school exits} \end{cases} \tag{1.6}$$

The period returns for a school with any given action can be thus defined by the profit function given by Equations (1.4), (1.5), and (1.6) as

$$\pi_j = \tilde{\pi}_j + D(r_j; \delta) + \Phi(a_j; \kappa_j, \phi_j) \tag{1.7}$$

1.5.2.3 Information

After applicant i has applied to school j, school j receives a private signal $\nu_{ji} \sim N(0, \sigma_\nu^2)$ of student ability which enters into the expected ability of student i additively as

$$E[A_i | LSAT_i, GPA_i, \nu_{ij}] = f_A(LSAT_i, GPA_i) + \nu_{ij}. \tag{1.8}$$

Thus, students' private information consists of latent type A and idiosyncratic shock ε_{ij} while a school's private information consists of ν_{ij} and either κ_j if school j is a potential entrant or ϕ_i if the school is an incumbent.

1.5.2.4 Application, Admissions and Enrollment

This subsection adapts the application, admissions and enrollment process of [28] to the current market and thus will closely follow her exposition[3]. I will thus solve the student's problem given tuition levels as well as the college admission's problem. To simply notation, define $X_i = (LSAT_i, GPA_i)$, $S \equiv (t, I, (R, g))$ and $\varepsilon_i \equiv (\varepsilon_{ij})_{j \in J}$.

Normalizing a student's outside option to zero, the value to any admitted student i is given by

$$w_i(O_i, A_i, \varepsilon_i | S) = \max\{0, \max_{j \in O_i} U_{ij}(t)\} \tag{1.9}$$

with O_i defined as the set of schools j admitting student i. Define the optimal enrollment strategy as $d(O_i, A_i, \varepsilon_i | S)$.

Given the probability $p_j(A_i, X_i | S)$ of student i being admitted to school j, the value of an application portfolio Y for student i is

$$W(Y, A_i, X_i | S) \equiv \sum_{O \subseteq Y} \Pr(O | A_i, X_i, S) E[w(O, A_i, \varepsilon_i | S)] - C(|Y|) \tag{1.10}$$

with the expectation over the shock to the value of attending school j and $|Y|$ the number of applications. The probability that set O of colleges admits student i is

$$\Pr(O | A_i, X_i, S) = \prod_{j \in O} p_j(A_i, X_i | S) \prod_{j' \in Y \setminus O} (1 - p_{j'}(A_i, X_i | S)). \tag{1.11}$$

Finally, the student's application problem is given by

$$\max_{Y \subseteq \{1, \ldots, J\}} \{W(Y, A_i, X_i | S)\} \tag{1.12}$$

---

[3]The empirical strategy will be different than in [28], but the theoretical results are utilized to ensure a well-behaved stage game.

with optimal application strategy $Y(A_i, X_i|S)$.

1.5.3 Dynamics and Timing

When deciding which students to admit, a school must take two sets of opposing forces into account. The first is the immediate revenue effect. If a school wants to admit more students given an applicant pool, it must lower its standards relative to student quality. The immediate effect of such a quality drop is a gain in tuition revenue. However, the school will also face a loss dynamically corresponding with the state (rank) evolution process.

The rank transition probability for the set of N active firms is the function

$$P : R^2 \times \mathbb{R}^{\overline{N}} \times [120, 180]^{\overline{N}} \times [0, 4]^{\overline{N}} \times \mathbb{R}^{2\times\overline{N}} \rightarrow [0, 1], \tag{1.13}$$

defining the probability $P(R, R', t(R), F_{LSAT_M}, F_{GPA_M}, F_A)$ that the state will transition from rank vector R to R' given tuition vector t and the length-$\overline{N}$ vector of incoming class distributions F with corresponding median LSAT scores, median GPA, and distribution of ability. Note that the transition of R_j for any school j is dependent on the entire vectors R, t and F since ranks are relative.

Function (1.13) is based on the ranking process used by the US News to rank schools. The ranking system explicitly takes into consideration Tuition, LSAT, and GPA as well as various types of reputation. These reputation measures as well as other persistent characteristics are captured by the previous period's rank. Note the implication of Equation (1.13) on future profits. A lower rank will lead to fewer students in the next period, less flexibility with tuition, and lower future donations.

While schools benefit in the short run from dropping admissions criteria, this can be counterbalanced by the long-run effects from the corresponding rank drop.

Schools will also be explicitly competing against other schools in this admissions process. Specifically, in setting its optimal enrollment policy, school j must take into account that any student i is also considering enrolling in all other schools in the enrollment set O. This type of competition has the flavor of quality-local competition suggested by [3] since O_i will likely contain schools of similar rank.

The timing of the dynamic aspect of the game is therefore as follows. Potential entrants, that is, schools deciding whether or not to open, draw from a distribution of fixed entrance costs. Incumbent schools decide, based on fixed costs and draws of profit shocks, whether to close down. After entrance/exit decisions have been made, staying incumbents and entrants compete in the application-admission game detailed in Section 1.5.2 for matriculating students. After competition concludes, investment matures, entry and exit occurs, then finally rankings are updated.

1.5.4 Strategies, Value Functions, and Equilibrium

For computational and theoretical tractability, I restrict the set of school strategies to be Markovian, anonymous and symmetric. This is a standard assumption in the literature (see, for example, [49], [23], [16], and [62]). Note that this implies that the index j through Section 1.5.2.4 only matters inasmuch as it indicates the rank of school j.

A strategy for school j is two-fold. The first part is a mapping, conditional on tuition and information structure, from components of the school-student match into a binary admission outcome. The mapping is given by

$$\sigma_{j1} : (R_j, \nu_{ij}, X_i | t_j, I) \to \{0, 1\}. \tag{1.14}$$

The second part of a schools strategy involves tuition setting and entry and exit decisions, all of which I denote as a_j. The remaining strategy is therefore

$$\sigma_{j2} : ((R_j, g), \xi_j | I) \to a_j \tag{1.15}$$

with ξ_j defined as the vector of private information held by school j. Define σ_j to be the vector $(\sigma_{j1}, \sigma_{j2})$ and $\sigma = (\sigma_j)_{j=1,\ldots,\overline{N}}$.

Notice that σ_{j1} and σ_{j2} include only R_j, not R. This does not mean, however, that other schools are not taken into account in the strategic behavior of school j. Rather, since R defines a strict ordinal separation, knowledge of R_j is sufficient to infer the state of school j relative to all other schools as well.

Let the parameters in Equation (1.7) be denoted by θ. The incumbent and potential entrant school value functions are given, respectively, by

$$\begin{aligned} & V_j((R, g); \sigma, I, \theta) \\ & = \max\left\{\pi_j + \max\left\{\phi_j, \beta \int E_{\xi_j} V_j((R', g'); \sigma, I, \theta, \xi_j) dP(R'; R, \sigma, I)\right\}\right\} \end{aligned} \tag{1.16}$$

and

$$\begin{aligned} & V_j((R, G); \sigma, I, \theta) \\ & = \max\left\{0, \beta \int E_{\xi_j} V_j((R', g'); \sigma, I, \theta, \xi_j) dP(R'; R, \sigma, I) - \kappa_j\right\}. \end{aligned} \tag{1.17}$$

The equilibrium can now be defined in two stages, first by the Application-Admission Equilibrium of the stage game and second by the full markov-perfect equilibrium.

Definition. *Given tuition profile t, information regime I and rank vector (R, g), a symmetric, anonymous application-admission equilibrium, denoted $AE(S)$ is the vector $(d(\cdot|\cdot), Y(\cdot|\cdot), \sigma_{j1}(\cdot|\cdot), p(\cdot|\cdot))$ such that*

1. *$d(\cdot|\cdot)$ is an optimal enrollment decision*
2. *Given $p(\cdot|\cdot)$, $Y(\cdot|\cdot)$ is an optimal college application portfolio*
3. *For every j, given $(d(\cdot|\cdot), Y(\cdot|\cdot), p_{-j}(\cdot|\cdot))$, $\sigma^*_{j1}(\cdot|\cdot)$ is an optimal admissions policy, and $\sigma^*_{j1}(\cdot|\cdot) = \sigma^*_{j'1}(\cdot|\cdot)$ if $R_j = R_{j'}$*
4. *$p_j(\cdot|\cdot) = \int \sigma^*_{j1}(\cdot|\cdot)\Phi(0, \sigma^2_\nu)$*

The full equilibrium is now defined as follows

Definition. *A symmetric, anonymous, markov-perfect equilibrium for the market for training lawyers is the vector $(\sigma^*_j, d(\cdot|\cdot), Y(\cdot|\cdot), \sigma_{j1}(\cdot|\cdot), p(\cdot|\cdot))$ such that*

1. *For every t, $(d(\cdot|\cdot), Y(\cdot|\cdot), \sigma_{j1}(\cdot|\cdot), p(\cdot|\cdot))$ constitutes an AE(t)*
2. *For every j, given σ^*_{-j2}, $V_j((R, g); \sigma^*_1, \sigma^*_{-j2}, \sigma^*_{j2}, I, \theta) \geq V_j((R, g); \sigma^*_1, \sigma^*_{-j2}, \tilde{\sigma}_{j2}, I, \theta)$ for $\tilde{\sigma}_{j2} \neq \sigma^*_{j2}$, V as defined in Equations (1.16) and (1.17), and $\sigma^*_{j2} = \sigma^*_{j'2}$ if $R_j = R_{j'}$*

1.6 Empirical Strategy

To estimate the utility and profit parameters in the model outlined in Section 1.5, I use the two-step estimator developed by [7] (BBL)[4] for dynamic games. BBL is one of several papers[5] that use the conditional-choice probability innovation

[4]See [62], [25], and [76] for some applications of the BBL estimator.

[5]Others include [2], [58], and [59].

in [35] to identify and estimate structural parameters in dynamic games. In the BBL case, the researcher assumes that all observed players are playing a specific equilibrium. Thus, behavior observed in the market corresponds to the optimal policy functions associated with that equilibrium. The researcher flexibly estimates the reduced-form policy functions and uses those estimates to generate a value function (in this case, Equations (1.16) and (1.17)) through forward-simulation corresponding to the market equilibrium.

Estimation is based on the equilibrium definition, that unilateral deviations from equilibrium play are suboptimal. By perturbing the optimal policy functions, the researcher can forward-simulate off-equilibrium value functions for a specific school. Since these should be lower than the value function generated by the optimal policy functions, the researcher can construct an estimator that minimizes the profitable deviations from the optimal policy.

1.6.1 Identifying Assumptions

For identification to hold, I need to make several assumptions on the market. The first involves equilibrium selection.

Assumption 1. All players play the same equilibrium in all markets.

This assumption is standard in the literature and could be viewed as an "empirical refinement" on the set of equilibria being played. While [16] prove existence of Markov-perfect, symmetric equilibria given for games similar to the one currently being played by schools, there are generally many possible such equilibria. Assumption 1 posits that, while many equilibria are possible, one is being played in the market and that is the equilibrium which we estimate. It also assures that the policy function being estimated is indeed the equilibrium policy function.

It is also necessary that schools make future plans based on the information structure now in place. This leads to Assumption 2

Assumption 2. Players assume that information structure is permanent.

The final assumption allows for identification of the distribution of scrap values, since no exit is yet observed in the data.

Assumption 3. Let $\underline{R}$ be the least profitable ranking for a school. The mean and variance of the distribution of scrap values are such that

1. $\mu_\phi = \{\mu_\phi : \Pr(a = \text{exit}|R_j = \underline{R}, \text{No info}) < 0.01\}$
2. $\sigma^2_\phi = \sigma^2_\kappa$

While Assumption 3 is primarily for identification, it also has theoretical motivation. Part 1 is based on the observation that schools have not traditionally left the market. It states that under the previous "No information" regime, it is very unlikely that even an lowest ranked – and thereby generally least profitable – school in the least lucrative state in the United States will exit. It is still a possibility, as it would be in real life under extreme circumstances, but highly unlikely. Part 2 of Assumption 3 is necessary for Part 1 to be identifying and has the intuitive interpretation that capital depreciates at a constant rate with respect to initial cost.

1.6.2 Application-Admission Game

The application-admission game consists of three separate decision functions. Two of the functions are based on the decision of a student with ability signal $(GPA, LSAT)$ to 1) apply to any given school with profile $(Rank, Tuition)$ and then 2) whether to enroll in one of her admitting schools and, if so, which one. The third function is based on each school's decision whether or not to admit the applicant. These decisions will also be based on the information regime (ie. if the new information treatment has been administered) and current year through the exogenous demand growth g. Thus, the outcome of application-admission game is based on functions for the three steps of the game

$$f_k(LSAT_i, GPA_i, Rank_j, Tuition_j, treat_{\{0,1\}}, year) \to (0,1) \tag{1.18}$$

for $k \in 1, 2, 3$ and a student i and school j.

It is theoretically probable and empirically likely that there is a significant amount of both nonlinearity and interaction between variables in f_k. Although a nonparametric estimator based on spline or kernel fitting would capture the nonlinearity, the number of variables alone, not including the interactions, would be prohibitive due to the curse of dimensionality. To allow for nonparametric estimation with such a large feature space, I estimate the conditional probability function $\hat{f}_k$ with gradient boosted classification trees[6].

Classification trees are a subset of decision trees in which the final outcome is discrete. Decision trees divide the set of explanatory variables into J hypercubes

[6] For a more in-depth discussion, see [31], especially Ch. 9-10.

according to an optimal criterion function and uses some type of mean estimate of the outcome variable as the predicted value conditional on the values in each hypercube. The final result generated by a decision tree is either a conditional mean estimate (for continuous outcomes) or a conditional probability function or classification rule (for discrete outcomes) defined over the space spanned by the explanatory variables.

Boosting is an ensemble method for decision trees that utilizes the fact that a combination of weak learners (in this case, trees) can be used to improve predictive power [63]. The boosting algorithm is an iterative process that fits a small tree, generates pseudoresiduals for the predictions of that tree, then fits a tree to those residuals again. This process is continued until some M number of iterations has been completed. Each new tree is added to the previous estimate, but weighted by a shrinkage parameter λ, which determines the learning rate of the algorithm. Gradient boosting is a modification of boosting that allows for a general loss function used in solving for the optimal prediction value in any characteristic space hypercube [27]. For binary outcomes, [26] show boosting is a close approximation to an additive logistic regression.

Optimal values for J, M and λ are generally determined through K-fold cross-validation. Following [31] Chapter 7, K-fold cross-validation consists of splitting the dataset in to K distinct subsets fitting the estimator K different times. The kth estimation repetition is fit to the data set after removing the kth partition. The K estimates of the prediction error are then combined and values of the vector (J, M, λ) are determined that minimize a these errors using some standard norm (for example, MSE). K-fold cross-validation is in essence a way of taking advantage of the training-set/testing-set approach while using the entire data-set.

For this specific application, however, the parameters recommended by K-fold cross-validation underperform due to the need to forecast based on out-of-sample years. Since K-fold cross-validation randomly subsets the observed data, it "optimally" over-interacts year with the other covariates, thus leading to poor future predictive power. As such, I will utilize a more simple tree structure.

Although boosting decision trees improves their accuracy, it also confounds much of a tree's interpretability. This loss is not concerning in the current setting since the goal of the BBL first-stage estimator is to flexibly predict behavior so as to accurately simulate market evolution.

The estimates of f_k can be used to simulate school-level outcomes, including class profile (median LSAT and undergraduate GPA) and class size. One approach to incorporate this method into the broader BBL estimator is to run this market simulation once per period. Since this must be repeated each period for each simulation of every value function, the ensuing computational burden makes this approach infeasible. To circumvent the problem, I use the estimated functions $\hat{f}_k$ to simulate outcomes for median LSAT and undergraduate GPA and incoming class size (see Appendix A.2 for details). This results in the vector of outcome functions

$$\tilde{f}(Rank_j, Tuition_j, treat_{\{0,1\}}, year) \rightarrow ([120, 180], [0, 1], \mathbb{R}^+)' \tag{1.19}$$

Note that student profile has been integrated out from these functions. Thus, the outcome in each period for a school j with profile $(Rank_j, Tuition_j)$ can be predicted using $\tilde{f}$. While f_k will not be used any more for estimating the school's problem, f_3 will still be necessary for identifying u_{ij}.

The focus on $\tilde{f}$ puts these estimates in the spirit of the endogenous threshold literature (for example, [6]). While not explicitly setting student quality thresholds, a school with rank R_j and tuition T_j is in essence deciding the quality of students it would like to accept. The difference here is that the tuition decision also *simultaneously* determines class size given optimal enrollment behavior.

In each function in the vector $\tilde{f}$, tuition and rank are taken as given. These are determined by the tuition policy function and state evolution function, respectively.

1.6.3 Tuition Policy Function

The tuition policy function represents the primary investment decision on the part of a school, in that tuition in the current period combined with the state vector R stochastically determines next period's state. The reduced form tuition function

$$f_T(Rank_j, treat_{\{0,1\}}, year) \rightarrow \mathbb{R} \tag{1.20}$$

can be estimated flexibly with a gradient boosted regression tree, which is similar to a classification tree but with primary difference being the use of the least-squares loss function. I include *year* to control for demand growth effects on tuition through the state variable g and allow for policy heterogeneity across information regimes.

1.6.4 State Evolution

To estimate the transition probabilities defined in Equation (1.13), I try to mimic the actual U.S. News ranking process as closely as possible. This process consists of two steps. The first step involves constructing a score for each school, and independent of other schools, based on many persistent characteristics as well as current period policy-dependent attributes. The second step normalizes these scores to discrete ranks in the sequence $1, 2, \ldots$ based on relative standing.

To capture the first scoring step, I estimate the simple tobit model

$$R'_j = f(\psi_0 + \psi_1 R_j + \psi_x x_j + \varepsilon_R) \tag{1.21}$$

bounded at one below and with $x \equiv (t, LSAT_M, GPA_M)$. The persistence in this estimate is captured by the parameter ψ_1. Next, I use the standard tobit expected value to simulate the scoring step as follows

$$\tilde{R}' = \Phi\left(\frac{X\hat{\beta}+\hat{\varepsilon}}{\hat{\sigma_R}}\right)\left[X\hat{\beta}+\hat{\varepsilon}+\hat{\sigma_R}\lambda(X\hat{\beta}+\hat{\varepsilon})\right] + 1 - \Phi\left(\frac{X\hat{\beta}+\hat{\varepsilon}}{\hat{\sigma_R}}\right) \tag{1.22}$$

with λ defined as the Inverse Mill's ratio, $X \equiv [R, x]$, and $\hat{\varepsilon}$ draws from $N(0, \hat{\sigma}^2_R)$. Ranks are then normalized to positive integers based on relative score.

1.6.5 The BBL Estimator

As in BBL, I exploit the fact that the structural parameters linearly enter in to Equations (1.16) and (1.17) to construct the value function for an active firm remaining in the market as

$$V_j((R,g);\sigma,I,\theta) = \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Psi_j(\sigma,(R_t,g_t),\varepsilon_t|(R_0,g_0)=(R,g))\right]\cdot\theta = \mathbf{W}_j((R,g);\sigma)\cdot\tilde{\theta} \tag{1.23}$$

with the basis functions taking the form

$$\Psi_j = [\tilde{\pi}_j, r_j, r_j^2]'$$

and $\tilde{\theta} \equiv [1, \delta_1, \delta_2]$. By linearity, W_j can be simulated up to structural paraters $\tilde{\theta}$. The structural coefficients for the donation function can be estimated as usual with the BBL estimator based on Equation (1.23). I simulate $K = 1250$ of these alternate policies with 10 simulation runs per policy.

Denoting an off-equilibrium policy $\tilde{\sigma}_j$, Definition 1.5.4 gives the condition

$$W_j((R,g);\sigma_j^*,\sigma_{-j}^*)\cdot\tilde{\theta} \geq W_j((R,g);\tilde{\sigma}_j,\sigma_{-j}^*)\cdot\tilde{\theta} \tag{1.24}$$

for all off-equilibrium policies. For the current application, I use a standard-normal additive perturbation to the estimates of school policy functions outlined in Section 1.6.3.

Following BBL, Equation (1.24) can be rewritten as

$$m(\tilde{\sigma}_j;\tilde{\theta}) = [W_j((R,g);\sigma_j^*,\sigma_{-j}^*) - W_j((R,g);\tilde{\sigma}_j,\sigma_{-j}^*)]\cdot\tilde{\theta}. \tag{1.25}$$

The desired m-estimator can now be written as

$$\min_{\tilde{\theta}} Q_n(\tilde{\theta}) = \frac{1}{K}\sum_{k=1}^{K} 1(m(\tilde{\sigma}_{j,k};\tilde{\theta}) > 0)m(\tilde{\sigma}_{j,k};\tilde{\theta})^2, \tag{1.26}$$

thus searching for parameter values that minimize profitable deviations from the optimal policy. To solve for these parameters, I use the numerical optimization package KNITRO [12] via the Python API `ktrinterface` [21].

To estimate the distribution of entrance costs and scrap values, I could exploit the fact that there has been no observed exit in this market to simulate an approximate value function with no exit included in the simulation. The approximate value function could then be used as a basis for a method of simulated moments estimator.

Details are given in Appendix A.1. However, this will prove unnecessary as 1) the change in the value of production will not be enough for entry/exit candidates (poorly (potentially-)ranked schools) to change their behavior and 2) post-treatment data are not yet extensive enough to yield either exits or significant change in entrance rates.

1.7 Estimates

The BBL estimator proceeds in two steps. The first step is to estimate the application-admission game functions, the tuition policy function, and transition probabilities between states. The second step will be to construct an M-estimator from simulated value functions leveraging the definition of a Markov-perfect equilibrium to get estimates for Equation (1.5) and for the distribution parameters for Equation (1.6).

1.7.1 BBL First Stage

To estimate the application-admissions game parameters, for each function f_k in Equation (1.18), I ran 100 boosting stages at a learning rate of 1.0, and a maximum tree depth of 1. Relative importance of each contributing variable is reported in Figure B.3.

The relative importance of each variable per game stage is telling. In predicting application probability, school rank is more than twice as important as the next explanatory variable, Tuition. It is clear that school profile matters more for application decisions than student profile. This is consistent with the observation that schools face similar applicant profile distributions through all ranges of Rank.

Year has little relative effect on predicting outcomes at any stage of the application-admission game. The only time, however, in which it does not have the lowest importance is in predicting application probability. This is consistent with the model in which any yearly effect independent of the information treatment is due to an exogenous growth in demand for law school, which would manifest itself primarily in application probability, the closest proxy to demand in this game.

The most important variable in explaining admission decisions is applicant LSAT scores. This is consistent with the rank premium argument in Section 1.4, that schools care more about LSAT scores of their students because LSAT scores have a higher weight in the US News Rankings than GPA, which contributes less than Rank to admission decisions. Rank is understandably important since better schools are more selective and tuition, being of second order importance at this stage to schools, has an even lower relative effect.

Finally, matriculation decisions are based first and foremost on undergraduate GPA of the applicant. This is to be expected, since GPA is the primary indicator in the data of outside options for students. While LSAT scores matter for schools with respect to rank effects, they make little difference in employment options outside of law for potential students. Thus LSAT has a lower relative effect than both GPA and school rank. It is surprising that Tuition has such low relative importance in predicting matriculation probability. This is likely due to the fact that rank is the primary predictor of post-graduation wages. Thus, the primary interaction should be between expected wages conditional on going to law school (associated with rank) and outside option (associated with GPA).

To give further confirmation of the implied directions of effects from the boosted estimates, Tables B.19, B.20 and B.21 show corresponding logit models with similar, albeit less flexible, estimates. The direction on the coefficients seem to match the interpretation of the results already discussed, although the amount of interaction makes interpretation difficult as well.

As discussed in Section 1.6.2, the equations in (1.18) are used to simulate estimates of the vector of equations (1.19). Outcomes reported in Figures B.4 and B.5 for treat=0, 1, respectively. Although somewhat uninformative by themselves, the comparison of the relative importance plots across information regimes shows an increase in the relative importance of rank, consistent with the newly induced quality separation.

The estimate of the tuition function defined in Equation (1.20) is reported in Figure B.6. The primary relationship of note in Figure B.6 is how little importance the treatment indicator is assigned in estimator, which is initially surprising given the analysis thus far. However, the relative importance result can be understood in the context of the estimator itself. The flexibility of the boosted regression tree allows the year variable to capture most of the level effect which otherwise would have been allotted to the treatment dummy. As will be shown in Section 1.8.1, the high importance placed on year will not confound the information regime effect. On the contrary, the information effect will dominate the year effect in the estimator, thus removing the tuition trend effect. While this will decrease base producer surplus estimates, it will not change the numbers for *change* in producer surplus since the growth term g is constant across information regimes.

Due to the few number of years in the sample, there is not enough data to recover a reliable estimate of entrance probability of schools or the change in that probability based on the information switch. As such, I will compute the model without entrance, simulate the market both with and without the information, and compare the difference in positive and normative outcomes. It will be shown that these results indicate changes in value functions for producers such that change in entry/exit behavior should not be optimal. Thus, the analysis should still be informative.

1.7.2 BBL Second Stage

Having obtained first-stage estimates, I can simulate the market up to parameter values in order to build the minimum distance estimator outlined in Section 1.6.5. The primary parameters that needed to be estimated were those associated with the rank premium in the donation function in Equation (1.5). The estimated values are

$$(\hat{\delta}_1, \hat{\delta}_2) = (14.78, -0.16). \tag{1.27}$$

Since rank gets higher as schools get worse (a rank of 1 indicates the best school), this indicates the concavity expected. Further, net donations become negative for a high enough rank (approximately 90), showing that for lower quality schools, grants and donations are outweighed by the cost of school operation. This also gives the intuitive result that donations, grants, alumni giving, etc. are much more prevalent among higher quality programs.

1.8 Market Dynamics and Welfare

Having completed the estimation, I can now turn to analyzing the market under both information regimes. To do so, I forward simulate market interactions between schools and students for T periods in the future, first under the counterfactual assumption that the reporting system never changed and second under the current reporting system.

1.8.1 Market Dynamics

Dynamics in the simulated markets are given in Figures B.8-B.12. In each figure, the solid and dotted lines indicate the markets simulated under the previous and current information systems, respectively. Cells represent quantiles of the reported variable.

Demand in Figure B.8 changes very little for the bottom half of schools. In the 65th-75th quantiles, demand drops slightly, then starkly in the 85th quantile. Behavior then changes drastically as demand actually increases for the top 95th percentile of schools. The intuition behind this result is based on the new relative profitability information available to students, revealing that top tier schools are in fact very lucrative. The responses to new information will be discussed in depth in Section 1.8.2.

Median LSAT scores of incoming students in Figure B.9 drop slightly for most schools, consistent with the elasticity results in Section 1.4. Once again, the top quantile of schools exhibits different behavior as they are able to attract even better students than without the extra information.

The primary purpose for Figure B.10 is to provide a consistency check for the rank evolution estimator. In the data, there is little variation over time in rank and this should be evident in the market evolution. Consistent with market behavior, Figure B.10 shows a high persistence in rank.

Figure B.11 shows the evolution of tuition over this period. In this case, the estimate loses power because of the estimator. Rather than capturing the time trend directly, the boosted regression tree fits the years non-parametrically, attributing the difference in pre and post treatment periods to a level effect. The growth term is lost. However, while this will have implications for the stand-alone producer surplus measure, the separation based on school quality is still estimated, which can be seen by the increasing gap between the treatment and non-treatment tuition paths. Thus, the difference in producer surplus can still be accurately inferred.

The final set of time path graphs is given in Figure B.12 for median undergraduate GPA. As was suggested in Section 1.4, lower quality schools in general have to decrease their admissions criteria significantly. Due to rank premium effects, dropping GPA is the more profitable option over dropping LSAT requirements.

1.8.2 School Welfare

The change in school welfare can be retrieved by comparing the value functions with and without the new information regime. This is given by the function difference

$$\Delta V(R) = V(R|info = 1) - V(R|info = 0) \tag{1.28}$$

with V as defined in Equations (1.16) and (1.17). The estimate of ΔV is shown in Figure B.7.

Dividing schools into four tiers in equidistant segments conditional on rank, the estimates in Figure B.7 suggest four approximate categories of schools to consider.

Category 1: The top half of the first tier (positive change)

Category 2: The bottom half of the first tier through the top half of the third tier (negative change)

Category 3: The bottom and top halfs of the third and fourth tiers, respectively (positive change)

There are two competing forces in effect causing the differences in welfare changes across the categories. The first effect is based on behavior of continuing students. The information regime change induces a increased observed separation between schools of various ranks. Rather than mere ordinal differentiation, ranks now provide a cardinal measure for a school of any given rank. Not surprisingly, the top schools are revealed to be highly lucrative for students. Students then respond with higher demand which in turn results in higher returns.

The separation effect decreases sharply as rank drops, turning negative at approximately the 25th rank. At this point and on, schools are now revealed to be of lower quality that previously believed. While the difference in welfare continues to descend, it does so at a decreasing rate as the second force, the expansionary effect, applied.

The expansionary effect also comes as a consequence of the drop in expected outcome of schooling from any given school, but is generated by students previously not participating in this market. Before the change in reporting standards, a substantial number of potential students were willing to attend law school and would have benefited from the education, but were unable to attend based on their sub-standard admissions profile. As a lower quality school is revealed as such, higher ability students with better outside options are no longer interested in attending its program. Lower ability students, however, have lower outside options and might therefore benefit from attending such a school. Thus, around rank 60, the elasticity of demand from lower quality students outweighs the corresponding elasticity from higher quality students and revenue begins to increase as the school opens up to this previously under-served market.

One might ask at this point, if there was previously this entire subpopulation willing to attend law school at high tuition prices and law schools had room, why were they not admitted earlier? Why did schools wait until the change in information structure to change their admissions behavior? The answer is in the rank premium. For an individual school to admit this class of student, it would have to drop its admissions criteria unilaterally. Since the ranking system is relative, all other schools would immediately improve relative to the deviating school, thus revealing the deviating school to be of lower quality than previously believed. Through the rank premium, the cost of deviation would outweigh the benefits of immediate tuition revenue. Thus, not deviating corresponds with equilibrium behavior.

In this context, the new information regime acts as an exogenous enforcement mechanism for deviation. As all schools in a certain range are revealed to be of lower quality than previously believed, none can enroll as many higher quality candidates and so all offer enrollment to the lower-quality student pool. Since the deviations are en masse, ranks remain fixed and schools maintain rank profits while simultaneously increasing stage profits. Eventually, the expansionary effect dominates and profits increase.

Another surprising implication of the producer surplus results is that quality disclosure on the part of schools occurred because of regulatory pressure rather than by voluntary action. Indeed, the process of the best producers voluntarily revealing their quality, thus putting pressure on the next-best firm to disclose and so-on until all firms have disclosed their quality, is a process called “unraveling,” the presence or absence of which has been investigated in the quality-disclosure literature.

Conditions under which voluntary quality disclosure is not optimal are given in [17]. The primary reason at play in the current case is costliness of disclosure. As first discussed in [29] and [38], only high quality producers will disclose their quality when the disclosure process itself is costly. In the case of law schools, disclosure involves an extra battery of alumni surveys with all the accompanying administrative structure required to administer the surveys. In order to comply with post-2010 requirements, many schools were required to hire extra administrative staff or invest in other ways in generating the report.

An additional factor involved in the voluntary disclosure decision is the verifiability of independent reports. While schools were free to report student placement, no regulatory body was previously in place to verify the veracity of the report. This lack of oversight led to many of the problems in this market previously discussed, such as students attending a school with artificially inflated expectations with regard to post-graduation earnings.

Besides the information regime change, the remaining salient feature during this time period is the aggregate recession in the United States beginning in 2008. This is especially relevant since the recessionary effects on the lawyer labor market did not begin to apply in force until 2009-2010. The expected dip is in fact clearly observed in Figure B.1, raising the concern that some of the welfare effects estimated could be due to business cycle effects rather than informational effects.

The recession could indeed have affected law school surplus, even differentially based on school quality. If the drop in expected wages were uniform across rank, then the change in producer surplus could be similar to the estimate in Figure B.7. Substitution effects would cause lower quality students to attend higher quality schools, thus once again forcing the lower quality schools to open up to a lower ability student pool.

There are two primary issues with this possibility. First, this story does not explain the sharp increase in producer surplus in higher ranges, a feature characteristic of the information effect. Further, wage changes do not seem to be uniform. In fact, the biggest drops seem to be in wages expected from going to higher quality schools, especially in the lower quantiles. In fact, the business cycle would not necessarily bias the welfare results at all, since it is not clear that the corresponding gains and losses would not simply cancel each other out, resulting in a net zero effect. So, while it is possible that the welfare changes in Figure B.7 were exacerbated by business cycle effects, it seems information still plays the primary role in the total change in producer surplus.

Since the entire school population is observed each period, the total change in producer surplus can be calculated using Equation (1.28) and is given by

$$\Delta PS = \sum_{j=1}^{\overline{N}} \Delta V(\hat{R})_j \approx -\$211 \text{ million} \tag{1.29}$$

which amounts to slightly more than a 5% drop, with $\hat{R}$ denoting the empirical state vector R at any given period (in this case, 2013).

1.8.3 Student Welfare

Computing student surplus at this stage is more involved than computing school surplus. While I computed school welfare through value function simulation, I could not directly retrieve consumer surplus at each period because of the computational trick in Section 1.7.1 integrating out the student side from GPA, LSAT, and matriculation functions.

To retrieve student welfare, I go through a similar simulation process as that outlined in Section 1.7.1 and detailed in Appendix A.2, but with some differences. The first difference is that, since students have no dynamic considerations themselves with respect to the game and since the game is anonymous and symmetric, the entire population over all time periods can be simulated simultaneously. The second difference, once again by symmetry, is that the entire game does not need to be simulated, only the student relevant outcomes at each stage. This includes admission and matriculation probabilities and expected rank and tuition of the school conditional on matriculation.

The third difference is implied by the second, specifically that now, all information about both sides of the potential match is still included in the matriculation function f_M (no integration of school-side characteristics). This can be used to identify u_{ij} by noting that the probability of matriculation at a given school is the probability that the value of attending is greater than zero, given by the equation

$$Pr(U_{ij}(t) > 0) = f_M(LSAT_i, GPA_i, Rank_j, Tuition_j) \equiv f_{Mij} \tag{1.30}$$

with $U_{ij}(t)$ defined in Equation (1.2). The proof of identification based on Equation (1.30) is given in Appendix A.3.1 and further estimation details are given in Appendix A.3.2.[7]

There are three primary types of change with respect to student welfare. The first follows from change in tuition rates, which can either be positive or negative depending on the school. The second component changing welfare is the substitution effect. Whether students go to a better school under the new information regime or stop going to school all together, the change in student profiles indicates that in some way, the type of school any given student will (or won't) attend will change as a result of the new information.

The final factor affecting student surplus is connected to the second. Besides

[7]Application costs are not currently incorporated into either computation or welfare calculations. Since application costs are relatively low, this should not have a substantial effect on welfare results.

students who were going to law school who substitute into better schools or away from law school in general, schools have also opened themselves up to a new previously underserved subpopulation. This is evidenced in the significantly decreased admissions profiles, especially in lower quality schools. These schools previously would have never admitted these lower-ability students. Under the new information regime, while higher ability students would no longer be willing to attend lower quality schools, the marginal cost might actually be smaller than the benefit relative to outside options for lower-ability students.

The total change in consumer (student) surplus is the net positive amount

$$\Delta CS = \int \Delta W_i dG(i) \approx \$896 \text{ million} \tag{1.31}$$

corresponding with a 0.5% increase, with W_i as defined in Equation (1.9) and $\Delta W_i \equiv W_i(I = 1) - W_i(I = 0)$.

1.8.4 Total Welfare

The total welfare change due to the change in the information regime is positive and is given by Equation (1.32) as

$$\Delta TS = \Delta SS + \Delta PS = \$685 \text{ million} \tag{1.32}$$

The implications of this result are intuitive. In markets characterized by informational friction with regard to producer quality separation, producers can use the information asymmetry to extract rent from consumers. In the case that the quality of the consumer matters to the producer as well, the information frictions also lead to systemic mismatch as consumers are matched with inefficiently low-quality products. The fact that the total welfare change was positive gives evidence that a market based approach, in which the only intervention involves enforcing honesty in reporting, can at least partially mitigate the mismatch problem.

1.9 Conclusion

The lawyer glut has generated significant concern, both for legal education and more broadly for the general higher education market. I have provided evidence to suggest that that information asymmetry between students and their prospective schools led to a systemic mismatch between the two sides of this market. I have further argued that this mismatch was at least partially mitigated by an exogenous influx of quality-separating information concerning school quality.

While there are some clear weaknesses due to the nature of the treatment, primarily that there was no control group against which to estimate treatment effects more precisely, the evidence still suggests the further use of this type of information policy in other similar markets. One possible application would be to tailor the estimators used in this paper to the undergraduate market in order to analyze the recent information policy imposed on undergraduate programs in the United States. This would further our understanding of markets with this type of information asymmetry as well as motivate further application of similar policy.

CHAPTER 2
ON THE MICROECONOMIC FOUNDATIONS OF LINEAR DEMAND FOR DIFFERENTIATED PRODUCTS

2.1 Introduction

The emergence of the modern theory of industrial organization owes much to the development of game theory. Due to its privileged position as the area where novel game theoretic advances found their initial application in an applied setting, industrial organization then served as a further launching ground for these advances to spread to other areas of economics. Yet to explain the success of industrial organization in reaching public policy makers, antitrust practitioners, and undergraduate students, one must mention the role played by the fact that virtually all of the major advances in the theory have relied on an accessible illustration of the underlying analysis using the convenient framework of linear demand.

While this framework goes back all the way to Bowley [9], it received its first well-known treatment in two visionary books that preceded the revival of modern industrial organization, and yet were quite precise in predicting the intimate link to modern game theory: Shubik [66] and Shubik and Levitan [67]. Then early on in the revival period, Dixit [15], Deneckere [14] and Singh and Vives [69] were among the first users of the linear demand setting. Subsequently, this framework has become so widely invoked that virtually no author nowadays cites any of these early works when adopting this convenient setting.

Yet, despite this ubiquitous and long-standing reliance on linear demand, the present paper will argue that some important foundational and robustness aspects of this special demand function remain less than fully understood.[1] Often limiting consideration to the two-good case, the early literature on linear demand offered a number of clear-cut conclusions both on the structure of linear demand systems as well as on its potential to deliver unambiguous conclusions for some fundamental questions in oligopoly theory. Among the former, one can mention the elegant duality features uncovered in the influential paper by Singh and Vives [69], namely (i) the dual linear structure of inverse and direct demands (along with the clever use of roman and greek parameters), (ii) the duality between substitute and complementary products and the invariance of the associated cross-slope parameter range of length one for each, and (iii) the resulting dual structure of Cournot and Bertrand competition. In the way of important conclusions, Singh and Vives [69] showed that, under linear demand and symmetric firms, competition is always tougher under Bertrand then under Cournot. In addition, were the mode of competition to be endogenized in a natural way, both firms would always prefer to compete in a Cournot rather than in a Bertrand setting. (Singh and Vives [69] inspired a rich literature still active today). Subsequently, Häckner [30] showed that with three or more firms and unequal demand intercepts, the latter conclusion is not universally valid in that there are parameter ranges for which competition is tougher under a Cournot setting, and that consequently some

[1]One is tempted to attribute this oversight to the fact that industrial economists' strong interest in linear demand is not shared by general microeconomists (engaged either in theoretical or in empirical work), as evidenced by the fact that quadratic utility hardly ever shows up in basic consumer theory or in general equilibrium theory.

firms might well prefer a Bertrand world. Hsu and Wang [36] show that consumer surplus and social welfare are nevertheless higher under Bertrand competition for any number of firms.

With this as its starting point, the present paper provides a thorough investigation of the micro-economic foundations of linear demand. Following the aforementioned studies, linear demand is derived in the most common manner as the solution to a representative consumer maximizing a utility function that is quadratic in the n consumption goods and quasi-linear in the numeraire. When this utility function is strictly concave in the quantitites consumed, the first order conditions for the consumer problem do give rise to linear demand, as is well known. Our first main result is to establish that this is the only way to obtain such a micro-founded linear demand. In other words, we address the novel question of integrability of linear demand, subject to the quasi-linearity restriction on candidate utility functions and find that linear demand can be micro-founded in the sense of a representative consumer if and only if satisfies the strict Law of Demand in the sense of decreasing operators (see [32]), i.e., if and only if the associated substitution/complementarity matrix is positive definite. As a necessary first step, we derive some general conclusions about the consumer problem with quasi-linear preferences that do not necessarily satisfy the convexity axiom. In so doing, we explicitly invoke some powerful results from the theory of monotone operators and convex analysis (see e.g., [73] and [32]), as well as a mix of basic and specialized results from linear algebra.

We also observe that strict concavity of the utility function imposes significant restrictions on the range of complementarity of the n products. For the symmetric substitution matrix of Häckner [30], we show that the valid parameter range for the complementarity cross-slope is $(-\frac{1}{n-1}, 0)$, which coincides with the standard range of $(-1, 0)$ if and only if there are exactly two goods ($n = 2$). In contrast, the valid range for the cross parameter capturing substitute products is indeed $(0, 1)$, independently of the number of products, which is in line with previous belief. We also explore the relationship between the standard notions of gross substitutes/complements and the alternative definition of these relationships via the utility function. Here again, the conclusions diverge for substitutes and for complements as soon as one has three or more products. All together then, the neat duality between substitute and complementary products breaks down in multiple ways for the case of three or more products.

Another point of interest is that, in the case of complements, as one approaches from above the critical value of $-\frac{1}{n-1}$, the usual necessary assumption of enough consumer wealth for an interior solution becomes strained as the amount of wealth needed is shown to converge to infinity! This further reinforces, in a sense that is hard to foresee, the finding that linear demand is not robust to the presence of high levels of inter-product complementarity!

Since many studies have used linear demand in applied work without concern for microeconomic foundations,[2] it is natural to explore the nature of linear demand

[2] A classical example appears in Okuguchi's [55] early work on the comparison between Cournot and Bertrand equilibria, which is discussed in some detail in the present paper.

when the strict concavity conditions on utility do not hold. In other words, we investigate the properties of the solution to the first order conditions of the consumer problem. For simplicity we do so for the n-good fully symmetric case (i.e., all off-diagonal terms of the substitution matrix are equal). Due to the lack of strict concavity, this will be only a local extremum (with no global optimality properties), which we term a local demand function. We find that, depending on which violation of strict concavity one allows, several rather unexpected exotic phenomena might arise. Demand functions might then fail the Law of Demand, even though each individual demand might remain downward-sloping in own price. For another parameter violation, individual demands might even be upward-sloping in own price (i.e., Giffen goods with a linear demand). In particular, we explicitly solve for the global solution of the utility maximization problem with a symmetric quadratic utility function that barely fails strict concavity, and show that the resulting demand can be multi-valued, highly non-linear and overall quite complex even for the two-good case. (Though similar effects will apply, higher values of n appear to be intractable as far as a closed-form solutions are concerned).

As a final point, we investigate one special case of linear demand with a local interaction structure. This is characterized by the fact that the representative consumer is postulated as viewing products as imperfect substitutes if they are direct neighbors in a horizontal attribute space and as unrelated otherwise. Though intended as a model of vertical differentiation, the well-known model of the car industry due to Bresnahan [10] has the same local interaction structure.

This paper is organized as follows. Section 2 gathers all the microeconomic preliminaries for general quasi-linear preferences. Section 3 specializes to quadratic utility and investigates the integrability properties of linear demand. Section 4 narrows consideration further to symmetric quadratic utility for tractability reasons. Section 5 explores the relationship between the notions of gross substitutes/complements and the alternative definition of these relationships via the utility function. Finally Section 6 offers a brief conclusion.

2.2 Some Basic Microeconomic Preliminaries

In this section, we work with the two standard models from the textbook treatment of consumer theory, but allowing for general preferences that are quasi-linear in the numeraire good, but do not necessarily satisfy the convexity axiom (i.e, the utility function is not necessarily strictly quasi-concave). The main goal is to prove that Marshallian demands are decreasing in the sense of monotone operators [32], and thus also decreasing in own price.

2.2.1 On Consumer Theory With Quasi-Linear Utility

Let $x \in R^n_+$ denote the consumption levels of the n goods and $y \in R_+$ be the numeraire good. The agent is endowed with a utility function $U : R^n_+ \longrightarrow R$ over the n goods and the numeraire y appears in an additively separable manner in the overall utility. The agent has income $m \geq 0$ to spend on purchasing the $(n+1)$ goods.

The utility maximization problem is, given a price vector $p \in R^n_+$ and the numeraire price normalized to 1,

$$\max U(x) + y \tag{2.1}$$

subject to (throughout, "$\cdot$" denotes the usual dot product between vectors)

$$p \cdot x + y \leq m. \tag{2.2}$$

We shall refer to the solution vector (i.e., the argmax) as the Marshallian demands, denoted $(x^*(p,m), y^*(p,m))$ or simply (x^*, y^*). We shall also use the notation $D(p) = (D_1(p), D_2(p), ..., D_n(p))$ for this direct demand function since the argument m will be immaterial in what follows.

The (dual problem of) expenditure minimization is (with u being a fixed utility level)

$$\min p \cdot x + y \tag{2.3}$$

subject to

$$U(x) + y \geq u.$$

We shall refer to the solution vector as the Hicksian demands $(x^h(p,u), y^h(p,u))$ or simply (x^h, y^h). We shall also use the notation $D^h(p)$ for this direct demand function since the argument u will not matter below. Recall that the (minimal) value function is the so-called expenditure function, denoted $e(p,u)$.

The following assumption is maintained throughout the paper.[3]

[3]Smoothness is assumed only for convenience here, and is not critical to any of the conclusions of the paper.

(A1). *The utility function U is twice continuously differentiable and satisfies* $U_i \triangleq \frac{\partial U_i}{\partial x_i} > 0$, *for* $i = 1, 2, ..., n$.

Since U is not necessarily strictly quasi-concave, the solutions to the two problems above, the Marshallian demands (x^*, y^*) and the Hicksian demands (x^h, y^h), may be correspondences in general.[4] By Weirstrass's Theorem, both correspondences are non-empty valued for each (p, m).

2.2.2 On the Law of Demand

In standard microeconomic demand theory, though not always explicitly recognized, the downward monotonicity of multi-variate demand is usually meant in the sense of monotone operators (for a thorough introduction, see [73]). This is a central concept in the theory of demand aggregation in economics [32] as well as in several contexts in applied mathematics [73]. We begin with its definition and a brief summary of some simple implications.

Let S be an open convex subset of R^n and F be a function from S into R^n. We shall say that F is (strictly) aggregate-monotonic if F satisfies (here "$\cdot$" denotes

[4]It is important in this paper to allow for utility functions that do not satisfy the ubiquitous quasi-concavity assumptions since we shall be concerned in some parts of this paper with maximizing quadratic, but non-concave, utility functions.

dot product) [5]

$$[F(s) - F(s')] \cdot (s - s') \leq (<) 0 \text{ for every } s, s' \in S. \tag{2.4}$$

This notion of downward monotonicity is quite distinct from the more prevalent notion of monotonicity in the coordinate-wise (or product) Euclidean order that arises naturally in the theory of supermodular optimization and games ([70], [75]). Nonetheless, for the special case of a scalar function, both notions boil down to the usual notion of monotonicity, and thus constitute alternative but distinct natural generalizations.

The following characterization of aggregate monotonicity in this context is well known. Let $\partial F(s)$ denote the Jacobian matrix of $F(s)$, i.e., the ijth entry of the matrix $\partial D(s)$ is $\partial_{ij} F(s) = \frac{\partial F_i(s)}{\partial s_j}$, which captures of the effect of a change in the price of the jth good on the demand for the ith good. This is a well-known result; for a proof, see e.g., Vainberg [73] or [Hildenbrand [32], Appendix].

Lemma 1. *Let S be an open convex subset of R^n and $F : S \longrightarrow R^n$ be a continuously differentiable map. Then the following two properties hold.*

(i) F is aggregate-monotonic if and only if the Jacobian matrix $\partial F(s)$ is negative semi-definite.

(ii) If the Jacobian matrix $\partial F(s)$ is negative definite, then F is strictly aggregate-monotonic.

[5] In the mathematics literature, functions with this property are simply referred to as monotone functions (or operators). The choice of the terminology "aggregate-monotonic" is ours, and is motivated by two considerations. One is that this is the standard notion of monotonic demand in aggregation theory in economics. The other is a desire to distinguish this monotonicity notion from the more prevalent one of coordinatewise monotonicity.

In Part (ii), the equivalence between the two strict notions need not hold. There are examples of strictly aggregate-monotonic maps with a Jacobian matrix whose determinant is not everywhere non-zero.

An important direct implication of Lemma 1 is that the diagonal terms of $\partial F(s)$ must be negative. However, this monotonicity concept does not impose restrictions on the signs of the off-diagonal elements of $\partial F(s)$. In contrast, monotonicity in the coordinate-wise order requires that every element of the Jacobian $\partial F(s)$ be (weakly) negative.

Definition. *The Marshallian demand $D(p)$ satisfies the (strict) Law of Demand if $D(p)$ is (strictly) aggregate-monotonic, i.e., for any two price vectors p and p', D satisfies*

$$[D(p) - D(p')] \cdot (p - p')(<) \leq 0 \tag{2.5}$$

In classical consumer theory, this property is well-known not to hold under very general conditions on the utility function, but sufficient conditions that validate it are available, see Hildenbrand [32] for details and discussion.

Consistent with Lemma 1, a demand function that satisfies the Law of Demand necessarily has the property that each demand component is downward-sloping in own price (i.e., the diagonal elements of the Jacobian matrix are all ≤ 0). In other words, no good can be a Giffen good. In addition, as Lemma 1 makes clear, the Law of Demand entails significantly more restrictions on the demand function.

The following general result reflects a key property of demand that constitutes the primary motivation for postulating a quasi-linear utility function in industrial organization. This result will prove very useful below.

Proposition 1. *Under Assumption A1, the Marshallian demand $D(p)$ satisfies the Law of Demand.*

Proof. We first prove that the Hicksian demand satisfies the Law of Demand. In the expenditure minimization problem, the expenditure function $e(p, u)$, as defined in (2.3), is the pointwise infimum of a collection of affine functions in p. Hence, by a standard result in convex analysis (see e.g., [61], Theorem 5.5 p. 35), for an arbitrary such collection, $e(p, u)$ is a concave function of the price vector p. Since the Hicksian demand $D^h(p)$ is the gradient of $e(p, u)$, i.e, $\frac{\partial e(p,u)}{\partial p_i} = D_i^h(p) = x_i^h$ (in other words, this is just the standard Hotelling's Lemma), it follows from a well-known result in convex analysis, which characterizes the subgradients of convex functions [60], that $D^h(p)$ satisfies (2.5).

Since the overall utility is quasi-linear in the numeraire, it is well known that the Marshallian demand inherits the properties of the Hicksian demand. Hence $D(p)$ too satisfies the Law of Demand (2.5). □

Recall that in the standard textbook treatment of these monotonicity issues, the utility function is assumed to be strictly quasi-concave. The main advantage of using the given general results from convex analysis is to bring to light the fact that quasi-concavity of the utility function is not needed for this basic result.

2.3 The Case of Quadratic Utility

In this section, we investigate the implications of the general results from the previous section that hold when we specialize the utility function U to be a quadratic function in problem (2.1). Along the way, we also review and build on the basic existing results for the case of a concave utility.

Using the same notation as above, the representative consumer's utility function is now given by (here "′" denotes the transpose operation)

$$U(x) = a'x - \frac{1}{2}x'Bx \tag{2.6}$$

where a is a positive n-vector and B is an $n \times n$ matrix. Without loss of generality, assume B is symmetric and has all its diagonal entries equal to 1.

2.3.1 A Strictly Concave Quadratic Utility

For this subsection, we shall assume that the matrix B is positive definite, which implies that the utility function is strictly concave. This constitutes the standard case in the broad literature in industrial organization that relies on quadratic utility.

It is well known that such a utility function gives rise to a generalized Bowley-type demand function. We allow a priori for the off-diagonal entries of the matrix to have any sign, although different restrictions will be introduced for some more definite results. Thus, this formulation nests different inter-product relationships, including substitute goods, complementary goods, and hybrid cases.

The consumer's problem is to choose x to solve

$$\max\{a'x - \frac{1}{2}x'Bx + y\} \quad \text{subject to} \quad p'x + y = m \tag{2.7}$$

As a word of caution, we shall follow the standard abuse of terminology in referring to the demand function at hand as linear demand, although a more precise description would clearly refer to it as being an affine function whenever positive and zero otherwise.

The following result is well known (see e.g, [4]), but included for the sake of stressing the need to make explicit the following basic assumption.

(A2). *The primitive data in (2.7) satisfy* $B^{-1}(a-p) > 0$ *and* $pB^{-1}(a-p) \leq m$.

As will become clear below, this Assumption is needed not only to obtain an interior solution to the consumer problem (in each product), but also to preserve the linear nature of the resulting demand function.

Lemma 2. *Assume that (A2) holds and that the matrix* B *is positive definite. Then the inverse demand is given by*

$$P(x) = a - Bx \tag{2.8}$$

and the direct demand is

$$D(p) = B^{-1}(a-p) \tag{2.9}$$

Proof. Since the utility function is quasi-linear in y, the consumer's problem (2.1) can be rewritten as $\max\{a'x - \frac{1}{2}x'Bx + m - p \cdot x\}$. Since B is positive definite, this maximand in strictly concave in x. Therefore, whenever the solution is interior, the usual first-order condition with respect to x, i.e., $a - Bx - p = 0$, is sufficient for global optimality. Solving the latter matrix equation directly yields the inverse demand function (2.8). It is easy to check that this solution is interior under Assumption (A2), as the part $B^{-1}(a-p) > 0$ says that each quantity demanded is strictly positive, and the part $pB^{-1}(a-p) \leq m$ simply says that $p \cdot D(p) \leq m$, i.e., that the optimal expenditure is feasible.

Since B is positive definite, the inverse matrix B^{-1} exists and is also positive definite (see e.g., [50]). Inverting in (2.8) then yields (2.9). □

At this point, it is worthwhile to remind the reader about three hidden points that will play a clarifying role in what follows. The first two points elaborate on the tacit role of Assumption (A2).

Remark 1. *In the common treatment of the derivation of linear demand in industrial organization, one tacitly assumes that the representative consumer is endowed with a sufficiently high income. The main purpose of Assumption (A2) is simply to provide an explicit lower bound on how much income is needed for an interior solution. We shall see later on that when Assumption (A2) is violated, the resulting demand is not only non-linear, it is also income-dependent. Thus income effects are then necessarily present, a key departure from the canonical case in industrial organization.*

The second point explains the abscence of income effects, and thus captures the essence of a quasi-linear utility.

Remark 2. *Suppose we have a solution (2.8) and (2.9) for some m such that $pB^{-1}(a-$*

$p) \leq m$. Then it can be easily shown that, for every $m' > m$, the solution of the consumer problem is still given by (2.8)-(2.9).

The third point explains the need for the *strict* concavity of U.

Remark 3. *The reason one cannot simply work with a quadratic utility function that is just concave (but not strictly so) is that, then, a matrix B that is just positive semi-definite (and not positive definite) may fail to be invertible. One immediate implication then is that the direct demand need not be well defined (unless one uses some suitable notion of generalized inverse).*

It is well-known that when B is positive definite, direct and inverse demands are both decreasing in own price (see e.g., [4]). In fact, we now observe that a stronger property holds.

Corollary 1. *If the matrix B is positive definite, both the inverse demand and the direct demand satisfy the strict Law of Demand, i.e., (2.4).*

Proof. This follows directly from Lemma 1, since the Jacobian matrices of the inverse demand and the direct demand are clearly B and B^{-1} respectively, both of which are positive definite. □

The Law of Demand includes joint restrictions on the dependence of one good's price on own quantity as well as on all cross quantities. It captures in particular the well known property that own effect dominates cross effects.

2.3.2 Integrability of Linear Demand

In this subsection, we consider the reverse question from the one treated in the previous subsection. Namely, suppose one is given a linear inverse demand function of the form $D(p) = d - Mp$, where d is an $n \times 1$ vector and M is an $n \times n$ matrix, along with the corresponding inverse demand. The issue at hand is to identify minimal sufficient conditions on d and M that will guarantee the existence of a utility function of the form 2.1, a priori satisfying only continuity and quasi-linearity in the numeraire good, such that $D(p)$ can be obtained as a solution of maximizing that utility function subject to the budget constraint (2.2)?

The framing of the issue under consideration here is directly reminiscent of the standard textbook treatment of integrability of demand, but there are two important distinctions. In the present treatment, on the one hand, we limit consideration to quasi-linear utility, but on the other hand, we do not a priori require the underlying utility function to reflect convex preferences. The latter point is quite important in what follows, in view of the fact that one of the purposes of the present paper is to shed light on the role that the concavity of the quadratic utility function (or lack thereof) plays in determining some relevant properties of the resulting linear demand function. The second distinction from the textbook treatment is that the starting primitives here include both the direct and the inverse demand functions. It turns out that this is convenient for a full characterization.

Proposition 2. *(i) Let there be given a linear demand function $D(p) = d - Mp$ with $d_i \geq 0$ for each i, along with the corresponding inverse demand $P(\cdot)$. Then there* *exists a continuous utility function* $U : R_+^n \longrightarrow R$ *such that* $D(p)$ *can be obtained by solving*

$$\max\{U(x) + y\} \quad \textit{subject to} \quad px + y \leq m$$

if and only if M *is positive definite and Assumption A2 holds.*

Then the desired U *is given by the strictly concave quadratic function (2.6) with* $B = M^{-1}$*, and both the demand and the inverse demand function satisfy the strict Law of Demand.*

Proof. Part (i) The "if" part was already proved in Lemma 2, with being the quadratic utility given in (2.6).

For the "only if" part, recall that by Proposition 1, every direct demand function that is the solution to the consumer problem when U is continuous and quasi-linear in the numeraire good (but not necessarily quadratic) satisfies the Law of Demand. Therefore, via Lemma 1, the Jacobian of $(d - Mp)$, which is equal to M, must be positive semi-definite.

Now, since both direct and inverse demands are given, the matrix M must be invertible, and hence has no zero eigenvalue. Therefore, since M is Hermitian, M must in fact be positive definite. This implies in turn that the system of linear equations $Ma = d$ possesses a unique solution a with $a_i > 0$, i.e., such that $a = M^{-1}d$. Finally, identifying M with B^{-1} yields the fact that the demand function can be expressed in the desired form, i.e., $D(p) = d - Mp = M(a - d) = B^{-1}(a - d)$, as given in (2.9).

Inverting the direct demand $D(p)$ yields the inverse demand (2.8). Integrating the latter yields the utility function (2.6), which is then strictly concave since the matrix B is positive definite.

Finally, the fact that both $D(p)$ and $P(x)$ satisfy the strict Law of Demand, then follows directly from Corollary 1. □

The main message of this Proposition is that any linear demand that is micro-founded in the sense of maximizing the utility of a representative consumer necessarily possesses strong regularity properties. Provided the utility function is quasi-linear (but not even quasi-concave a priori), the linear demand must necessarily satisfy the Law of Demand, and originate from a strictly concave quadratic utility function.

This clear-cut conclusion carries some strong implications, some of which are well understood, including in particular that (i) the demand for each product must be downward-sloping in own price (i.e., no Giffen goods are possible), and (ii) demand cross effects must be dominated by own effects.

On the other hand, the following implication is remarkable, and arguably quite surprising.

Corollary 2. *If a quadratic utility function of the form given in (2.6) is not concave, i.e., if the matrix B is not positive semi-definite, then this utility could not possibly give rise to a linear demand function.*

We emphasize that this conclusion holds despite the fact that the utility function is concave in each good separately (indeed, recall that the matrix B is assumed to have all 1's on the diagonal). The key point here is that joint concavity fails.

This immediately raises a natural question: What solution is implied by the first order conditions for utility maximization in case the matrix B is not positive semi-definite, and how does this solution fit in with the Corollary? This question is addressed in the next section, in the context of a fully symmetric utility function, postulated as a simplifying assumption, as in Singh and Vives [69], Häckner [30] and others.

2.4 Symmetric Non-Concave Quadratic Utility

In this section, we investigate a common specification of linear demand in industrial organization along the lines suggested by the results of the previous section. We also elaborate on the question raised there about the meaning of first order conditions when the substitution matrix fails to be positive semi-definite.

2.4.1 A Common Special Case

A widely used utility specification for a representative consumer foundation is characterized by a fully symmetric substitution/complementarity matrix, i.e. one in which all cross terms are identical for all pairs of goods and represented by a parameter $\gamma \in [-1, 1]$ (e.g., [69] and [30]). The substitution matrix is thus

$$B = \begin{bmatrix} 1 & \gamma & \dots & \gamma \\ \gamma & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \gamma \\ \gamma & \dots & \gamma & 1 \end{bmatrix}, \tag{2.10}$$

which can be reformulated as $B \equiv (1-\gamma)I_n + \gamma J_n$, where I_n is the $n \times n$ identity matrix and J_n is the $n \times n$ matrix of all ones.

It is common in the literature to postulate that the meaningful range for the possible values of γ is a priori $[-1, 1]$, with $\gamma \in [-1, 0)$ corresponding to (all goods being) complements, $\gamma \in (0, 1)$ to substitutes, and $\gamma = 0$ to independent goods. While we begin with $[-1, 1]$ being the a priori possible range, we shall see below that for the case of complements, further restrictions will be needed.

As previously stated, concavity of U is sufficient for the first-order condition to provide a solution to the consumer's problem. It turns out that for the special substitution matrix at hand, concavity of U can easily be fully characterized.

Lemma 3. *The quadratic utility function in (2.6) with B as in (2.10) is strictly concave if only if $\gamma \in (-\frac{1}{n-1}, 1)$.*

Proof. For U to be concave, it is necessary and sufficient that B be positive semi-definite. To prove the latter is equivalent to showing that all the eigenvalues of B are positive. To this end, consider

$$B - \lambda I_n = \begin{bmatrix} 1-\lambda & \gamma & \dots & \gamma \\ \gamma & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \gamma \\ \gamma & \dots & \gamma & 1-\lambda \end{bmatrix}$$

$$= \gamma J_n + (1 - \lambda - \gamma) I_n$$

By the well known matrix determinant lemma, we have

$$\begin{aligned} \det[B - \lambda I_n] &= \det[\gamma J_n + (1 - \lambda - \gamma) I_n] \\ &= (1 - \lambda - \gamma)^{n-1}[1 - \lambda + (n-1)\gamma] \end{aligned} \tag{2.11}$$

The solutions of $\det[B - \lambda I_n] = 0$ are then $\lambda = 1 - \gamma$ and $\lambda = 1 + (n-1)\gamma$. Since a priori $\gamma \in [-1, 1]$, by simple inspection, these solutions are > 0 if and only if $-1/(n-1) < \gamma < 1$. □

The following observation follows directly from the Proposition and the results of the previous section.

Corollary 3. *Given a linear demand* $D(p) = B^{-1}a - B^{-1}p$ *with* B *as given in (2.10),* $D(p)$ *can be derived from a quadratic utility function of the form (2.6) if only if* $\gamma \in (-\frac{1}{n-1}, 1)$*, in which case both* $D(p)$ *and the corresponding inverse demand satisfy the Law of Demand.*

It follows that the range of values of the parameter that validate a linear demand function is not $(0, 1]$, but rather $(-\frac{1}{n-1}, 1)$. One important direct implication is that there is a fundamental asymmetry between the cases of substitutes and complements. For substitutes, the valid range is indeed $(0, 1)$, as is widely believed, and this range is independent of the number of goods n. However, for complements, the valid range is $(-\frac{1}{n-1}, 0)$, which monotonically shrinks with the number of goods n, and converges to the empty set as the number of goods $n \longrightarrow +\infty$.

This Corollary uncovers an exceptional feature of the ubiquitous two-good case, as reported next.

Remark 4. *The special case of two goods* ($n = 2$) *is the only case for which the valid range of the parameter* γ *for a concave utility, and thus for a well-founded demand, i.e.* $(-\frac{1}{n-1}, 1)$*, is equivalent to the interval* $(-1, 1)$*, as commonly (and correctly) believed (e.g., [69]).*[6]

---

[6]Actually, in industrial organization, it is not uncommon to find studies that postulate the valid range as being the closed interval $[-1, 1]$, instead of the open $(-1, 1)$.

Before moving on to explore the properties of the solutions of the first order conditions when the latter are not sufficient for global optimality, we report a remarkable result about the hidden regularizing effects of strict concavity.

Proposition 3. *Consider the quadratic utility function in (2.6) with B as in (2.10). As $\gamma \downarrow -\frac{1}{n-1}$, the level of income required to obtain an interior linear demand function converges to ∞.*

Proof. We first derive a simplified version of Assumption (A2) for the case where the matrix B as in (2.10). As $a_i = a > p_i = p$, one clearly has $x > 0$.

To check that $px \leq m$, first note that $b_{ii} = 1$ and $b_{ij} = \gamma$ (for $i \neq j$). In addition, for the matrix B^{-1}, each diagonal element is equal to $1 + \frac{(n-1)\gamma^2}{(1-\gamma)[1+(n-1)\gamma]}$, and each off-diagonal term is $\frac{-\gamma}{(1-\gamma)[1+(n-1)\gamma]}$. Therefore, upon a short computation, $p \cdot x = \frac{np(a-p)}{1+(n-1)\gamma}$. The latter fraction converges to $+\infty$ as $\gamma \downarrow -\frac{1}{n-1}$ (since its numerator is > 0).

Since Assumption (A2) requires that $p \cdot x = \frac{np(a-p)}{1+(n-1)\gamma} \leq m$, the conclusion follows. □

This Proposition is a powerful criticism of the assumption that the representative consumer is endowed with a sufficient income level to allow for an interior solution to the utility maximization problem, in cases where the products under consideration are strong complements (i.e., for γ close to the maximum allowed value of $-\frac{1}{n-1}$). If one needs to require infinite wealth to rationalize a linear demand system for complements, then perhaps it is time to start questioning the well-foundedness of such demand functions. Put differently, perhaps industrial economists have been overly valuing the analytical tractability of linear demand.

We next move on to other pathological features that might emerge in the absence of a strictly concave utility function.

2.4.2 The Solution to the First Order Conditions

Continuing with our investigation of the robustness of the linear demand specification, we now address the following key issue: What properties are satisfied by the solution implied by the first order conditions for utility maximization in case the utility is given by (2.6), but with a matrix B in (2.10) that is not positive semi-definite? The answer, as we shall justify in some detail below, is that although the said solution looks exactly like the familiar linear demand function, it is actually not the true global solution to the utility maximization problem, due to the lack of concavity of the utility function.

We know from Lemma 3 that for U not to be concave requires exactly the following assumption, which we make throughout this subsection.

(A3) *The parameter* γ *satisfies* $-1 < \gamma < -1/(n-1)$.

In such a case, it is obvious that the first order conditions for utility maximization continue to give rise to a *candidate solution*, which looks just like the standard linear inverse demand (2.8). However, no longer being a priori the actual global argmax of the consumer problem (due to the absence of concavity of the consumer's objective function), we shall refer to (2.8) as a local inverse demand function in this context. We stress that this just an extremum of the consumer problem, and not the actual demand function. The true demand function in such cases will actually be non-linear and quite complex, as illustrated in an example below.

In light of Assumption (A3) and Lemma 3, this candidate solution need not, a priori, be invertible so as to yield a corresponding direct demand function. Thus the first issue we tackle is the invertibility of the matrix B for this local demand, as given in (2.10) but without positive definiteness. Due to the special structure at hand, we obtain a full characterization via a closed-form inverse.

Lemma 4. *As long as $\gamma \neq 1$ and $\gamma \neq -\frac{1}{n-1}$, the matrix $B = (1-\gamma)I_n + \gamma J_n$ is non-singular and has the inverse*

$$B^{-1} = \frac{1}{1-\gamma}\left[I_n - \frac{\gamma}{(n-1)\gamma+1}J_n\right]. \tag{2.12}$$

Proof. The proof follows from the identity that the inverse of of a matrix of the form $aI_n + bJ_n$ takes the form $\tilde{a}I_n + \tilde{b}J_n$. The unknown coefficients can be identified directly by setting the product of B and B^{-1} equal to I_n as follows (removing n subscripts for notational convenience):

$$\begin{aligned}
&[(1-\gamma)I + \gamma J][aI + bJ] = I \\
\Rightarrow\ &(1-\gamma)IaI + (1-\gamma)IbJ + \gamma JaI + \gamma JbJ = I \\
\Rightarrow\ &(1-\gamma)aI + (1-\gamma)bJ + \gamma aJ + \gamma bnJ = I \\
\Rightarrow\ &(1-\gamma)aI + [(1-\gamma)b + \gamma a + \gamma bn]J = I,
\end{aligned}$$

the third step following from the fact that $J_nJ_n = nJ_n$. Letting $a = \frac{1}{1-\gamma}$, all that remains to be done is solve for b with

$$(1-\gamma)b + \frac{\gamma}{1-\gamma} + \gamma b n = 0,$$

which has the solution

$$b = -\frac{\gamma}{(1-\gamma)[(n-1)\gamma+1]},$$

as long as $\gamma \neq 1$ and $\gamma \neq -\frac{1}{n-1}$. This concludes the proof. □

To recapitulate, for all values of $\gamma \in [-1, 1]$ other than $\gamma = 1$ and $\gamma = -\frac{1}{n-1}$, the solution of the first order conditions for utility maximization yields a well-defined local inverse demand and direct demand of the forms (2.8) and (2.9). However, the main results of the present paper indicate that this pair cannot be the actual solution to the utility maximization problem unless $\gamma \in (-\frac{1}{n-1}, 1)$! In other words, whenever $\gamma \in [-1, -\frac{1}{n-1}]$, the utility maximization problem here is a well-defined non-concave quadratic optimization problem for which the first order conditions do not yield an actual solution, due to the absence of concavity.[7]

Nonetheless, it is worth investigating the properties of the local inverse and direct demand functions, despite the fact that they do not arise from the actual solution to the consumer problem. In particular, any study that postulates a demand function with a B matrix that is not positive definite might be viewed as a local demand function in the present sense. One such example appears in Okuguchi [55], and is reviewed below.

[7] In the general theory of quadratic programming, this feature is well known to arise when suitable second order conditions do not hold (see e.g., [11]).

The following results will underscore the importance of strict concavity of the utility function for consumer theory even in the quadratic case.

The first set of properties of the local direct demand function already contain some major departures from familiar characteristics in industrial organization.

Proposition 4. *Under Assumption (A2), the following hold.*

(i) Though not aggregate-monotonic, the local inverse demand function is such that the price of each good is downward-sloping in own quantity.

(ii) If $\gamma \in [-1, -\frac{1}{n-2})$*, the local direct demand of every good is decreasing in own price, even though* D *does not satisfy the Law of Demand overall.*

(iii) If $\gamma \in (-\frac{1}{n-2}, -\frac{1}{n-1}]$*, the local direct demand of every good is increasing in own price, i.e., all goods are Giffen goods.*

Proof. (i) It is obvious that the local inverse demand function $P(x) = a - Bx$ is such that each price is downward-sloping in own quantity since all the diagonal entries of B are 1's.

(ii)-(iii) Since the slope of the demand for each good is determined by the diagonal elements of B^{-1}, using (2.12), the following condition is equivalent to a downward-sloping (upward-sloping) demand curve:

$$1 - \frac{\gamma}{(n-1)\gamma + 1} > (<) 0. \tag{2.13}$$

Upon a simple computation, using Assumption (A3), the two desired conclusions follow (the details are left out). □

2.4.3 Two Examples

Here, we shall solve explicitly for the demand function corresponding to a quadratic utility function in the two-good case when the underlying utility function is not strictly concave. This is meant as an example to illustrate the fact that without strict concavity, the resulting true demand function can be quite complex and non-linear. The restriction here to two goods is due to tractability. This example will further highlight the importance of strict concavity when working with a linear demand. Of particular interest is that violations of concavity for a quadratic utility easily lead to drastic departures from the usual properties one spontaneously associates with linear demand.

Example 1. *Here we consider a substitution parameter $\gamma = -1$. We find that even though this is a boundary case, the optimal solution departs in substantial ways from the familiar linear demand function.*

For inverse demand function $p_i = 1 - x_i - rx_j$, with $\gamma = -1$, the underlying utility function is

$$U = x_0 + x_1 + x_2 - 0.5x_1^2 - 0.5x_2^2 + x_1x_2.$$

Without loss of generality, assume that $p_1 < p_2$. If $p_1 \geq 1$, substituting from the budget constraint $x_0 = m - p_1x_1 - p_2x_2$ into U yields

$$U = m + (1 - p_1)x_1 + (1 - p_2)x_2 - 0.5(x_1 - x_2)^2.$$

Hence the demand is $x_1 = x_2 = 0, x_0 = m$.

Now we let $p_1 < 1$, and consider the Lagrangian:

$$L = x_0 + x_1 + x_2 - 0.5(x_1 - x_2)^2 - \lambda(x_0 + p_1x_1 + p_2x_2 - m).$$

We need to compare three cases to determine the optimal demand.

(i) Choose $x_0 = m$, so $x_1 = x_2 = 0$, with $U = m$.

(ii) Choose $x_0 = 0$,so $p_1x_1 + p_2x_2 = m$. Then substitute $x_2 = (m - p_1x_1)/p_2$ into the utility function to obtain

$$U = m + (1 - p_1)x_1 + (1 - p_2)(m - p_1x_1)/p_2 - 0.5[x_1 - (m - p_1x_1)/p_2]^2.$$

a) The optimal demand can be obtained by the first-order condition as the second-order condition is easily seen to hold. If $m \geq (p_2 - p_1)p_1/(p_1 + p_2)$, we have

$$x_1 = \frac{m}{p_1 + p_2} + \frac{(p_2 - p_1)p_2}{(p_1 + p_2)^2} \text{ and } x_2 = \frac{m}{p_1 + p_2} - \frac{(p_2 - p_1)p_1}{(p_1 + p_2)^2}.$$

This leads to the corresponding $U = \frac{2m}{p_1+p_2} + \frac{(p_2-p_1)^2}{2(p_1+p_2)^2}$.

b) If $m < (p_2 - p_1)p_1/(p_1 + p_2)$, the demands are $x_1 = m/p_1$ and $x_2 = 0$, and then $U = m/p_1 - 0.5(m/p_1)^2$.

(iii) Choose $x_0 \in (0, m)$, and x_1 and x_2 must satisfy the first-order conditions: $1 - x_1 + x_2 = p_1$, and $1 - x_2 + x_1 = p_2$. This is possible only if $p_1 + p_2 = 2$. The budget constraint implies $x_1 = 0.5[m - x_0 + p_2(1 - p_1)]$, $x_2 = 0.5[m - x_0 + p_1(1 - p_2)]$ where $p_1 + p_2 = 2$. This yields $U = m + (p_2 - p_1)^2/8$.

Finally the true demand function must be chosen from the solutions (i) – (iii), which yield the highest utility, depending on the parameters m, p_1 and p_2.

It can be shown that if $p_1 + p_2 = 2$*, (iii) is indeed the optimal demand (equally good as (iia)). However, if* $p_1 + p_2 < 2$*, it is dominated by (iia) and cannot be the true demand.*

This demand function may be discontinuous. For instance, let $p_1 = \frac{3}{4}, p_2 = 9/4, m = 3/8$*. Then case (iii) is not relevant. As* $m = (p_2 - p_1)p_1/(p_1 + p_2)$*, both (ii.a) and (ii.b) can apply as well as (i). All three cases yield the same utility of* $3/8$*, so all these three demand functions are valid.*

If p_1 *falls marginally, (ii.a) is valid, but not (ii.b). The utility given by (ii.a) rises while that in (i) remains constant. So we should have* $x_1 = m/p_1 = 0.5$ *and* $x_2 = 0$*.*

However, if p_1 *rises marginally, (ii.b) applies, but not (ii.a). But the utility given by (ii.b) falls, while that in (i) remains constant. Then the true demand becomes* $x_1 = x_2 = 0$*. In this case the demand for good 2 remains continuous, but not for good one.*

To obtain a linear demand function, we assume prices are sufficiently low such that $p_1 + p_2 \leq 2$*.*

This implies some of x_1 *or* x_2 *will be demanded, so case (i) is ruled out. Furthermore we assume there is sufficient income (*$m \geq (p_2 - p_1)p_1/(p_1 + p_2)$*), so that (ii.b) is excluded. Then we only have two cases of (ii.a) and (iii) left.*

If $p_1 + p_2 < 2$*, the solution (ii.a) dominates (iii). The demand functions are non-linear, and the income effect exists. Different from normal cases of concave utility functions, the income effect never disappears regardless of how high the income* *is. This is because two goods are perfect complements, the marginal utility of income can be kept above 1, so* x_0 *is never consumed.*

If $p_1 + p_2 = 2$*, (ii.a) and (iii) become identical when* $x_0 = 0$*. The demand for* x_1 *and* x_2 *will be lower in (iii) when* $x_0 > 0$*, but the utility is same (for both* x_1 *and* x_2 *to be positive,* x_0 *cannot be equal to* m *unless both* p_1 *and* $p_2 = 1$*). Even in this case, (iii) cannot be a truly linear demand. First,* $p_1 + p_2 = 2$ *implies prices cannot change independently. Secondly,* x_1 *and* x_2 *cannot be determined as both depend on* x_0*. So a linear demand is not feasible.*

Given these conditions, the demand is a continuous function as long as $p_1 + p_2 < 2$*, but becomes an upper-semi continuous correspondence along the boundary* $p_1 + p_2 = 2$*.*

The second example appears in a classic study in the literature on the comparison between Cournot and Bertrand equilibria.

Example 2. *Okuguchi [55] uses the following demand specification to show that equilibrium prices may be lower under Cournot than under Bertrand.*

Consider the symmetric inverse/direct demand pair (for $i \neq j$*) :*

$$p_i = \frac{1}{8}(2 + x_i - 3x_j) \text{ and } x_i = 1 - p_i - 3p_j \tag{2.14}$$

Two violations of standard properties stand out: (i) The inverse demand is upward-sloping, (ii) the two products appear to be complements in the inverse demand function, but substitutes in the inverse demand function.

The candidate utility function to conjecture as the origin of this demand pair *is clearly*

$$U = \frac{1}{8}(2x_1 + 2x_2 - 3x_1x_2 + 0.5x_1^2 + 0.5x_2^2) + x_0.$$

This is not a concave function. In fact, in contrast to our treatment so far, this utility function is actually strictly convex (and not concave) in each good separately, though not jointly strictly convex.

It can easily be shown by solving the usual consumer problem with this utility function that the resulting demand solution is not the one given in (2.14). The true solution includes some of the same complex features encountered in the previous example (the solution is not derived here for brevity).

This confirms what the results of the present paper directly imply for this demand pair, namely that it cannot be micro-founded in the sense of maximizing the utility of a representative consumer.

Therefore, this demand pair is essentially invalid, and thus the fact that it leads to Bertrand prices that are higher than their Cournot counterparts does not a priori constitute a valid counter-argument to the well known positive result under symmetry [74][4].

2.4.4 Cournot and Bertrand Oligopoly

Here we consider the standard models of Cournot and Bertrand oligopolies with linear (fully symmetric) demand, and linear costs normalized to zero. The issues already discussed concerning the relationship between concavity of the underlying utility function and the extent of product differentiation arise for oligopoly as well.

To understand the role of the concavity of the utility function, we also consider local demand functions that are not necessarily global solutions to the consumer problem. After characterizing the concavity of each firm's profit function in own action, we will show how strategic substitutes or complements arise for different values of γ.

Under Assumption A2, the profit functions for firm i under Cournot and Bertrand competition are respectively

$$\Pi_i^C(q) = q_i(a - b_i \cdot q) \tag{2.15}$$

$$\Pi_i^B(p) = p_i b_i^{-1}(a - p) \tag{2.16}$$

where $b_i$and b_i^{-1} are the ith row of B and B^{-1} respectively. We allow a priori $\gamma \in (-1, 1)$ since we wish to investigate the behavior of firms' reaction curves even when the demand and inverse demand are only valid in a local sense.

Lemma 5. *For the profit functions in* (2.15) *and* (2.16)

- $\Pi_i^C(q)$ *is strictly concave in own output* q_i *if and only if* $\gamma \in \left(-\frac{1}{n-1}, 1\right)$.
- $\Pi_i^B(p)$ *is strictly concave in own price* p_i *if and only if* $\gamma \in \left(-1, -\frac{1}{n-2}\right) \cup \left(-\frac{1}{n-1}, 1\right)$.

Proof. The proofs of concavity come directly from the second-order conditions of Equation (2.15) and Equation (2.16), which are straight forward to derive for firm i as $-2b_{ii}$ and $-2b_{ii}^{-1}$, respectively. That these terms are negative is easy to see from the fact that the diagonal elements of B are 1 and B^{-1} are given by Equation (2.12). The details are omitted. □

Häckner [30] imposes similar restrictions on the range of γ, as second order conditions when investigating the properties of firms" reaction curves.

As pointed out in Singh and Vives [69], Bertrand reaction curves should be upward-sloping under substitutes and downward-sloping under complements and the converse should hold for Cournot. While this is clearly the case under Cournot competition since the off-diagonal elements of B are simply γ, it is not necessarily true for Bertrand.

Proposition 5. *Under Bertrand competition, reaction curves are*

(i) always upward-sloping for substitutes ($\gamma > 0$), and

(ii) downward sloping for complements ($\gamma < 0$) if and only if $\gamma > -1/(n-2)$.

Proof. The reaction function for firm i under Bertrand competition is given by

$$p_i = \frac{B_i^{-1}a}{2B_{ii}} - \frac{B_{-i}^{-1}p_{-i}}{2B_{ii}}$$

with $-i$ indicating that the ith element has been removed. We want to show that

$$\frac{\partial p_i}{\partial p_j} = -\frac{B_{ij}^{-1}}{2B_{ii}} = \frac{\gamma}{2[(n-1)\gamma + 1 - \gamma]} < 0$$

For $\gamma > 0$, the result holds trivially since $\gamma < 1$. For $\gamma < 0$. For $\gamma < 0$, all that is required is that $(n-1)\gamma + 1 > 0$, which implies the result. □

Proposition 5 highlights the fact that excessive complementarity, i.e., $-1 \leq \gamma < -1/(n-2)$, is also not compatible with a standard property of the behavior of firms in price competition with complements and more than three firms, namely strategic substitutes.

2.5 Gross Substitutes/Complements vs Substitutes/Complements in Utility

The purpose of this subsection is to explore the relationship between the standard notions of gross substitutes/complements and the alternative definition of substitute/complement relationships via the utility function. The main finding argues that two products may well appear as substitutes in a quadratic utility function, even though they constitute gross complements in demand. On the other hand, we also establish that when all goods are complements in a quadratic utility function, then any two goods necessarily appear as gross complements in demand as well.

We begin with the formal definitions of the underlying notions, along with some general remarks.

Definition. *(a) Two goods, i and j, are said to be gross substitutes (gross complements) if $\partial x_i^*/\partial p_j = \partial x_j^*/\partial p_i > (<)0$*

(b) Two goods, i and j, are said to be substitutes (complements) in utility if the utility function U has increasing differences in (x_i, x_j), or for smooth utility, if $\partial^2 U(x)/\partial x_i \partial x_j \leq (\geq 0)$ for all x.

Here, Part (a) is a standard notion in microeconomics. On the other hand, though a useful and well defined notion, part (b) is not as widely used in demand theory. The following remark will prove useful below.

Remark 5. *One can also define substitutes (complements) with respect to the inverse demand function, in the obvious way: i and j are substitutes (complements) if*

$\partial P_i/\partial x_j = \partial P_j/\partial x_i < (>) 0$. *However, since the inverse demand is simply the gradient of the utility function here, this new definition would simply coincide with part (b).*[8]

It is generally known that for two-good linear demand, the two definitions are equivalent, namely two goods that are gross substitutes (complements) are always substitutes (complements) in utility as well, and vice versa (see e.g., [69]). On the other hand, this is not necessarily the case for three or more goods, as we now demonstrate.

Example 3. *Consider a quadratic utility function $U(x) = a'x - \frac{1}{2}x'Bx$ with any strictly positive vector a and*

$$B = \begin{bmatrix} 1 & 3/4 & 0.5 \\ 3/4 & 1 & 3/4 \\ 0.5 & 3/4 & 1 \end{bmatrix}.$$

It is easy to verify that this matrix is positive definite, so that U is strictly concave. Hence the first order conditions define a valid inverse demand function, and we are thus in the standard situation. It is also easy to check that the inverse of B is

$$B^{-1} = \begin{bmatrix} 7/3 & -2 & 1/3 \\ -2 & 4.0 & -2 \\ 1/3 & -2 & 7/3 \end{bmatrix}$$

Recall that the slopes of both inverse and direct demand functions do not depend on the vector a (though both intercepts do depend on a). Hence, invoking the above Remark,

[8] In other words, one always has directly from the first order conditions

$$\frac{\partial^2 U(x)}{\partial x_i \partial x_j} = \frac{\partial P_i}{\partial x_j} = \frac{\partial P_j}{\partial x_i}.$$

one sees by inspection that all goods are substitutes in utility (or according to inverse demand), including in particular goods 1 *and* 3*. On the other hand, the latter two goods are clearly gross complements (according to* B^{-1}*).*

This possibility is actually quite an intuitive feature, as we shall argue below by providing a basic intuition for it. Nonetheless, this might well appear paradoxical at first sight because we tend to be over-conditioned by observations that hold clearly for the standard two-good case, but are actually not fully robust when moving to a multi-good setting (similar counter-examples are easy to construct whenever $n \geq 3$).

The intuition behind this switch is quite easy to grasp. Assume for concreteness that we consider an exogenous increase in p_3. This leads to a lower demand for good 3, but a higher demand for goods 1 and 2 through substitution. The latter effect impacts good 2 relatively more than for good 1 (due to a constant of .75 for 3-2 versus .5 for 3-1). A second effect is that the large increase in the consumption of good 2 ends up driving down that of good 1 (as the two are substitutes in utility). The overall effect of the increase in p_3 is a decrease in the consumption of both goods 3 and 1, which thus emerge as gross complements.

Consider next a three good utility function with all goods being complements in utility instead. Adapting the foregoing intuition to this case will make it clear that any two goods will then emerge as gross complements too. In fact, we now prove that this constitutes a general conclusion for the n-good case.

Proposition 6. *Consider an* n*-good concave quadratic utility* U *that is supermodular in* x *(i.e.,* $\frac{\partial^2 U(x)}{\partial x_i \partial x_j} \geq 0$ *for all* $i \neq j$*). Then all the goods are gross complements.*

Proof. Since $\frac{\partial^2 U(x)}{\partial x_i \partial x_j} \geq 0$ for all $i \neq j$, all the off-diagonal elements of B are negative. Since B is positive semi-definite, it follows from a well known result (see e.g., Mc Kenzie, 1960) that all the off-diagonal elements of B^{-1} are positive. This is turn implies directly, via (2.9), that all goods are gross complements. □

In this case, all the reactions to a given price change move is the same direction, in a mutually reinforcing manner, so complementarity in utility across all goods always carries over to gross complementarity between any two goods.

2.6 Linear Demand With Local Interaction

In this section, we introduce one more alternative form of the substitution matrix B that may be of interest in particular economics applications. Specifically, we suggest a particular substitution matrix based on product similarities, place it in context of the broader study of linear demand, and highlight interesting properties of the resulting inverse and direct demands.

Consider a consumer with a preference ordering over goods based on their similarities. The main idea consists in capturing the intuitive notion that the closer products are in their characteristics, the closer substitutes they ought to be. Specifically, the consumer has preferences over n goods horizontally differentiated along one dimension, with the goods uniformly dispersed over a compact interval in that dimension. Without loss of generality, let $i = i, \ldots, n$ represent the order of the products over the interval. Consider a quadratic utility function (2.6) where B is now a Kac-Murdock-Szegö matrix, that is, a symmetric n-Toeplitz matrix whose ij-th term is

$$b_{ij} = \gamma^{|i-j|}, \quad i, j = 1, \ldots, n. \tag{2.17}$$

As such matrices were first defined in Kac, Murdock, and Szegö [39], we will refer to this as the KMS model. We focus on the case $\gamma \in (0, 1)$, so that all products are substitutes. In this specification, the price of any good j responds to changes in the quantity of every other good j with a magnitude that decreases with the distance between i and j in characterisitic space.

It is well known[9] that this matrix is positive-definite for $\gamma \in (0, 1)$ and has the inverse[10]

$$B^{-1} = \frac{1}{1-\gamma^2} \begin{bmatrix} 1 & -\gamma & 0 & \ldots & 0 \\ -\gamma & 1+\gamma^2 & -\gamma & \ldots & 0 \\ & \ddots & \ddots & \ddots & \\ 0 & \ldots & -\gamma & 1+\gamma^2 & -\gamma \\ 0 & \ldots & 0 & -\gamma & 1 \end{bmatrix}. \tag{2.18}$$

While indirect demand facing a given firm is a function of all other goods, direct demand is only a function of the two adjacent substitutes for interior firms, and one adjacent substitute for the two firms at the edges. As an example of a demand system with such structure in empirical industrial organization, consider the vertically differentiated model for the automobile industry due to Bresnahan [10], with equal quality increments. The key idea in this model is to capture the intuitive fact that a given car is in direct competition only with cars of similar qualities.

From our previous general results, we easily deduce this demand system is

---

[9]See, for example, Horn and Johnson, Section 7.2, Problems 12-13 [34]

[10]Due to the location of two products at the end points of the segment (that can thus have only one neighbor instead of two), direct demand is no longer fully symmetric.

well-defined for all $\gamma \in (0, 1)$.

Corollary 4. *Both inverse and direct demand in the KMS model satisfy the strict Law of Demand, i.e. Equation* (2.4)*.*

Proof. The proof follows directly from Corollary 1, since the matrix B is positive definite. □

This result has different implications for oligopolistic competition between firms (when each firm sells one of the varieties), depending on the mode of competition. Under Cournot competition, each firm competes with all other firms, but reacts more intensely to those whose products are more similar to its own. In contrast, under Bertrand competition, each firm directly competes only with its one or two adjacent rivals, i.e., those with very similar products (with respect to horizontal differentiation). With respect to those similar firms, previous results still hold. Specifically, as in Singh and Vives [69], the Bertrand reaction curve for a firm with respect to its direct neighbors is upward sloping.

Proposition 7. *In the KMS model, each firm i price competes only with its closest substitutes, $i+1$ and $i-1$. With respect to these two rivals, firm i's reaction curve is upward sloping.*

Proof. Reaction curves can be derived as in Proposition 5, yielding the derivative (here, b_{ij}^{-1} is the ij-th term of the matrix $(1-\gamma^2)B^{-1}$)

$$\frac{\partial p_i}{\partial p_j} = -\frac{b_{ij}^{-1}}{2b_{ii}} = \begin{cases} \frac{\gamma}{K} & \text{for } |i-j| = 1 \\ 0 & \text{for } |i-j| \neq 1 \end{cases}$$

with $K = 2 > 0$ for boundary firms and $K = 2(1+\gamma^2) > 0$ for interior firms. The conclusion follows from the fact that $\frac{\gamma}{K} > 0$. □

The KMS model thus highlights another interesting lack of duality between oligopolistic price and quantity competition, which is a result of a lack of duality between indirect and direct demands. When firms compete over price, a type of local strategic interaction takes place in that each firm *directly* takes into account the behavior of only their direct neighbors (though in equilibrium, every firm's action will still end up indirectly being a function of all the rivals' actions). However, when firms compete over quantity, they directly take into consideration the behavior of all the other firms (as in the standard case).

2.7 Conclusion

This paper provides a thorough exploration of the theoretical foundations of the multi-variate linear demand function for differentiated products that is widely used in industrial organization. For the question of integrability of linear demand, a key finding is that strict concavity of the quadratic utility function of the representative consumer is critical for the resulting demand system to be well defined. Without strict concavity, the true demand function may be quite complex, non-linear and income-dependent. In addition, the solution of the first order conditions for the consumer problem, which we call a local demand function, may have quite pathological properties.

The paper uncovers a number of failures of duality relationships between substitute products and complementary products, as well as the incompatibility of high levels of complementarity and concavity. The two-good case often investigated since the pioneering work of Singh and Vives [69] emerges as a special case with strong but non-robust properties.

A key implication of our results is that all conclusions and policy prescriptions derived via the use of a linear multi-variate demand function that does not satisfy the law of Demand ought to be regarded a priori with some suspiscion.

APPENDIX A
PROOFS AND ESTIMATION DETAILS

A.1 Entry/Exit Distributions

I utilize Assumption 3 and Equation (1.6) to construct the equation

$$\Pr(\phi_i \leq V(\underline{r}, \underline{s})) = 0.999$$
$$\Rightarrow \Phi(V(\underline{r}, \underline{s}); \mu_\phi, \sigma_\kappa^2) = 0.999 \tag{A.1}$$

The rest of the parameters can be estimated as follows

1. Use approximate value function and predicted entrances to estimate entrance costs
2. Use the moment equality from Equation (A.1) and, replacing σ_κ^2 with $\hat{\sigma}_\kappa^2$ from Step 1, estimate μ_ϕ with the Method of Simulated Moments
3. Update the approximate value function based on Step 2
4. Iterate until $|\hat{\mu}_{\phi,\tau+1} - \hat{\mu}_{\phi,\tau}| < \epsilon$

with τ the iteration count. I can then estimate the distribution of entrance costs using predicted probabilities of entrance based on first-stage BBL estimates.

A.2 Stage Game Outcome Simulation Subroutine

The required task is to simulate the outcomes of the application-admission stage game for a representative population of applicants and schools. The final goal of the simulation is to produce the function $f_{treat=0,1}(outcome|Rank, Tuition)$, with f a boosted random forest defined for either the period before ($treat = 0$) or after

($treat = 1$) the information regime chance and *outcome* one of three variables: 1) median LSAT scores of matriculants, 2) median undergraduate GPA of matriculants, and 3) total number of matriculants.

The outcome functions are simulated for a given year y in the set of possible years defined by $treat$ as follows:

1. Draw $\hat{n}$ applicants with replacement from all reported applicants in y
2. Select $n \times J$ data points $(Rank_j, Tuition_j, LSAT_i, GPA_i, treat, year)_{i \in n, j \in J}$ corresponding to each applicant/school combination to generate the simulation dataset
3. For each application, simulate admission based on predicted probability of a positive admission decision
4. For each applicant with a non-zero number of admissions, simulate the decision to enroll at all based on the highest rank of admitting schools
5. For each enrolling student, simulate the school to attend based on the highest predicted probability of attendance among the admitting schools
6. Repeat K times

Each of the simulation runs results in an incoming class profile, consisting (among other things) of median LSAT and GPA scores and class size. Finally, average the results across the K runs to get the dataset to which the f can be fit, in this case with a gradient boosted regression tree.

A.3 Student Preferences

The preferences for students of any given observed quality must be calculated in order to determine total welfare effects of the information regime change. The next two sections outline identification and estimation of a student's utility for attending any given law school as well as her corresponding outside option.

A.3.1 Identification of Student Utility

Lemma 6. *Equation* (1.30) *is sufficient to identify the value for student i of attending school j.*

Proof. Beginning with Equation (1.30) and, for ease of notation, defining $\overline{u}_{ij} \equiv \overline{u}(A_i, R_j, I)$ and $f_{Mij} \equiv f_M(LSAT_i, GPA_i, R_j, t_j)$ and with $\tilde{\Phi}$ and $\tilde{\Phi}^{-1}$ denoting the normal distribution with variance σ_u^2 and quantile functions, respectively,

$$
\begin{aligned}
& & Pr(U_{ij}(t) > 0) &= f_{Mij} \\
&\Rightarrow & Pr(\overline{u}_{ij} + \varepsilon_{ij} - t_j \geq 0) &= f_{Mij} \\
&\Rightarrow & Pr(\varepsilon_{ij} \geq t_j - \overline{u}_{ij}) &= f_{Mij} \\
&\Rightarrow & \tilde{\Phi}(t_j - \overline{u}_{ij}) &= f_{Mij} \\
&\Rightarrow & (t_j - \overline{u}_{ij}) &= \tilde{\Phi}^{-1}(f_{Mij}) \\
&\Rightarrow & \overline{u}_{ij} &= t_j - \tilde{\Phi}^{-1}(f_{Mij})
\end{aligned}
$$

which implies identification given σ_u^2 since f_{Mij} is non-parametrically identified in the data and t_j is observed. Finally, σ_u^2 can be identified using variation within groups of applicants with the same application profile and best-choice school. □

A.3.2 Estimation Details for Student Preferences

To compare estimates based on the two information regimes, I sample a population of students for each year $1, \ldots, \overline{T}$, with $\overline{T}$ the cutoff year for forward simulation. Each student i is drawn from the empirical distribution $\hat{F}(LSAT_i, GPA_i|treat)$, with the number of draws equal to the average number of applicants in a given year conditional on being in the treatment period or not. The size of the applicant pool is taken from the ABA-LSAC End-of-Year summary of law school applicants [47].

To determine matriculation probability, I use estimates of admission and matriculation probabilities to first simulate the binary admission outcome for student i (did admission happen?) and second, calculate the expected rank of a matriculating school j, should the student matriculate. I then estimate the tuition for school j using the tuition policy function estimate. Finally, I use the estimates from the application-admission game to determine the probability of student i matriculating to school j. This probability constitutes f_{Mij}.

APPENDIX B
QUESTIONNAIRES, TABLES AND FIGURES

B.1 Questionnaires: Pre and Post Regime Change

The following are representative questionnaires given to schools before and after the 2010 change in reporting requirements.

Official ABA Data

The University of Akron School of Law

150 University Avenue
Akron, OH 44325-2901
Phone: 800.4.AKRON.U or 330.972.7331; Fax: 330.258.2343
E-mail: lawadmissions@uakron.edu; Website: www.uakron.edu/law

ABA Approved Since 1961

The Basics

Type of school	Public
Term	Semester
Application deadline	3/1
Application fee	$0
Financial aid deadline	3/1
Can first year start other than fall?	No
Student to faculty ratio (fall)	16.5 to 1
Does the university offer: housing restricted to law students?	No
graduate housing for which law students are eligible?	Yes

Faculty and Administrators

	Total		Men		Women		Minorities	
	Fall	Spr	Fall	Spr	Fall	Spr	Fall	Spr
Full-time	25	25	17	16	8	9	2	3
Other Full-time	0	0	0	0	0	0	0	0
Deans, librarians, & others who teach	4	4	3	3	1	1	0	0
Part-time	27	21	14	8	13	13	1	0
Total	**56**	**50**	**34**	**27**	**22**	**23**	**3**	**3**

Curriculum

	Full-time	Part-time
Typical first-year section size	41	58
Is there typically a "small section" of the first-year class, other than Legal Writing, taught by full-time faculty	No	No
If yes, typical size offered last year	0	0
# of classroom course titles beyond 1st year curriculum	84	
# of upper division courses, excluding seminars with an enrollment: Under 25	59	
25-49	27	
50-74	26	
75-99	4	
100+	0	
# of seminars	21	
# of seminar positions available	539	
# of seminar positions filled	195	82
# of positions available in simulation courses	578	
# of simulation positions filled	309	155
# of positions available in faculty supervised clinical courses	264	
# of faculty supervised clinical positions filled	73	19
# involved in field placements	55	13
# involved in law journals	35	17
# involved in moot court or trial competitions	40	11
# of credit hours required to graduate	88	

JD Enrollment and Ethnicity

	Men		Women		Full-time		Part-time		1st-year		Total		JD Degs. Awd.
	#	%	#	%	#	%	#	%	#	%	#	%	
African Amer.	13	3.9	12	5.0	16	4.6	9	4.1	11	6.1	25	4.4	10
Amer. Indian	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0	0
Asian Amer.	8	2.4	4	1.7	8	2.3	4	1.8	6	3.3	12	2.1	1
Mex. Amer.	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0	0
Puerto Rican	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0	0
Hispanic	7	2.1	4	1.7	7	2.0	4	1.8	7	3.9	11	1.9	2
Total Minority	28	8.5	20	8.4	31	8.9	17	7.8	24	13.3	48	8.4	13
For. Nation.	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0	0
Caucasian	254	76.7	194	81.5	272	77.7	176	80.4	132	72.9	448	78.7	135
Unknown	49	14.8	24	10.1	47	13.4	26	11.9	25	13.8	73	12.8	42
Total	331	58.2	238	41.8	350	61.5	219	38.5	181	31.8	569		190

Tuition and Fees

	Resident	Nonresident
Full-time	$12,896	$20,332
Part-time	$10,462	$16,411

Living Expenses

Estimated living expenses for singles

Living on campus	Living off campus	Living at home
$13,134	$13,134	$13,134

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The University of Akron
150 University Avenue
Phone : 330-972-7331
Akron, OH 443252901
Website : www.uakron.edu/law

Section of Legal Education and Admissions to the Bar
AMERICAN BAR ASSOCIATION

EMPLOYMENT SUMMARY FOR 2013 GRADUATES

EMPLOYMENT STATUS	FULL TIME LONG TERM	FULL TIME SHORT TERM	PART TIME LONG TERM	PART TIME SHORT TERM	NUMBER
Employed - Bar Passage Required	86	0	12	0	98
Employed - J.D. Advantage	21	0	4	0	25
Employed - Professional Position	18	0	3	1	22
Employed - Non-Professional Position	0	0	3	0	3
Employed - Undeterminable	0	0	0	0	0
Pursuing Graduate Degree Full Time					1
Unemployed - Start Date Deferred					1
Unemployed - Not Seeking					2
Unemployed - Seeking					8
Employment Status Unknown					6
Total Graduates					166

LAW SCHOOL/UNIVERSITY FUNDED POSITIONS	FULL TIME LONG TERM	FULL TIME SHORT TERM	PART TIME LONG TERM	PART TIME SHORT TERM	NUMBER
Employed - Bar Passage Required	0	0	0	0	0
Employed - J.D. Advantage	0	0	0	0	0
Employed - Professional Position	0	0	0	0	0
Employed - Non-Professional Position	0	0	0	0	0
Total Employed by Law School/University	0	0	0	0	0

EMPLOYMENT TYPE	FULL TIME LONG TERM	FULL TIME SHORT TERM	PART TIME LONG TERM	PART TIME SHORT TERM	NUMBER
Law Firms					
Solo	9	0	0	0	9
2 - 10	30	0	8	0	38
11 - 25	11	0	0	0	11
26 - 50	8	0	0	0	8
51 - 100	3	0	0	0	3
101 - 250	1	0	0	0	1
251 - 500	2	0	0	0	2
501 +	2	0	0	0	2
Unknown Size	3	0	0	0	3
Business & Industry	35	0	4	0	39
Government	18	0	5	0	23
Pub. Int.	2	0	1	0	3
Clerkships - Federal	0	0	0	0	0
Clerkships - State & Local	0	0	1	0	1
Clerkships - Other	0	0	0	0	0
Education	1	0	1	1	3
Employer Type Unknown	0	0	2	0	2
Total	125	0	22	1	148

EMPLOYMENT LOCATION	STATE	NUMBER
State - Largest Employment	Ohio	122
State - 2nd Largest Employment	New York	4
State - 3rd Largest Employment	District of Columbia	2

Employed in Foreign Countries	2

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B.2 Tables and Figures

	Mean	SD	Min	Median	Max	n
year	1999.50	8.08	1986.00	1999.50	2013.00	5376
Rank	136.35	69.03	1.00	178.00	195.00	5376
Ratio	0.64	0.12	0.33	0.61	0.99	5344
Tuition	22.77	7.01	0.56	22.61	44.11	3583
Students	612.32	273.25	32.00	555.00	1771.00	3754
Undergrad GPA	3.41	0.21	2.68	3.41	3.91	2410
LSAT	157.40	5.78	141.00	157.00	173.00	3101
Median Grant	8581.57	5894.01	294.00	7500.00	61737.00	3207
Percent Grants	45.66	19.37	1.00	43.98	100.00	3487
Room/Board Expenses	13.45	3.02	0.60	13.22	31.39	3710
Cost of Books	1345.44	482.37	33.00	1250.00	4474.00	2301
Student/Faculty Ratio	15.78	8.05	6.80	15.20	251.00	2947
Accepted	788.47	461.11	101.00	703.50	4174.00	2416
Acceptance Rate	38.68	15.24	4.34	37.00	88.18	3806
Bar Passage Rate	82.33	11.38	0.75	84.50	100.00	3197
Private Sector 25% Salary	49.41	21.67	4.01	42.00	128.43	3110
Private Sector 75% Salary	78.85	26.65	32.54	73.43	490.97	3110
% Private Sector Reporting	61.18	20.70	0.00	64.00	100.00	2656

Table B.1: Summary Statistics

Info	Subset	LSAT	GPA	n_{Apps}	Pr(Admit)	Pr(Matric)	n
Treat=0	Applicants	161.79	3.46	9.43	0.86		8835
		(7.85)	(0.4)	(6.27)	(0.35)		
	Admitted	162.43	3.48	10.13		0.5	7600
		(7.49)	(0.38)	(6.15)		(0.5)	
	Matriculants	163.12	3.49	10.63			3780
		(7.55)	(0.39)	(6.64)			
Treat=1	Applicants	161.55	3.45	10.3	0.92		3242
		(8.63)	(0.4)	(6.53)	(0.27)		
	Admitted	162.01	3.46	10.79		0.44	2989
		(8.35)	(0.4)	(6.44)		(0.5)	
	Matriculants	163.15	3.48	11.29			1325
		(8.32)	(0.4)	(6.67)			

Table B.2: Stage Game Student Profile Summaries

Info	Pctl.	LSAT	GPA	n_{Apps}	pr(Admit)	pr(Matric)
Treat=0	0-25th	158.26	2.91	9.11	0.8	0.38
		(7.73)	(0.27)	(6.6)	(0.4)	(0.49)
	25-50th	161.01	3.39	9.38	0.85	0.42
		(7.55)	(0.09)	(5.95)	(0.36)	(0.49)
	50-75th	163	3.65	9.56	0.88	0.43
		(7.35)	(0.07)	(5.72)	(0.32)	(0.5)
	75-100th	164.91	3.89	9.64	0.91	0.48
		(7.2)	(0.08)	(6.67)	(0.29)	(0.5)
Treat=1	0-25th	157.28	2.89	10.2	0.9	0.39
		(8.53)	(0.27)	(7.64)	(0.3)	(0.49)
	25-50th	160.34	3.37	10.32	0.91	0.36
		(8.17)	(0.1)	(6.39)	(0.28)	(0.48)
	50-75th	163	3.65	10.42	0.94	0.42
		(7.94)	(0.07)	(5.98)	(0.24)	(0.49)
	75-100th	165.54	3.89	10.33	0.93	0.47
		(7.59)	(0.08)	(6.06)	(0.25)	(0.5)

Table B.3: Summary by GPA Quantile

Info	Pctl.	LSAT	GPA	n_{Apps}	pr(Admit)	pr(Matric)
Treat=0	0-25th	151.5	3.29	8.56	0.75	0.33
		(4.13)	(0.42)	(6.51)	(0.43)	(0.47)
	25-50th	159.23	3.42	9.14	0.87	0.41
		(1.96)	(0.39)	(6.14)	(0.33)	(0.49)
	50-75th	164.5	3.5	9.89	0.89	0.44
		(1.71)	(0.36)	(5.92)	(0.31)	(0.5)
	75-100th	170.89	3.6	10.17	0.92	0.51
		(3.3)	(0.34)	(6.25)	(0.28)	(0.5)
Treat=1	0-25th	150.59	3.26	8.97	0.86	0.33
		(4.69)	(0.42)	(6.75)	(0.35)	(0.47)
	25-50th	159.15	3.4	10.14	0.93	0.35
		(1.99)	(0.38)	(7.06)	(0.26)	(0.48)
	50-75th	164.98	3.52	11.44	0.95	0.43
		(1.96)	(0.37)	(6.5)	(0.22)	(0.5)
	75-100th	171.72	3.62	10.83	0.96	0.52
		(3.05)	(0.33)	(5.61)	(0.2)	(0.5)

Table B.4: Summary by LSAT Quantile

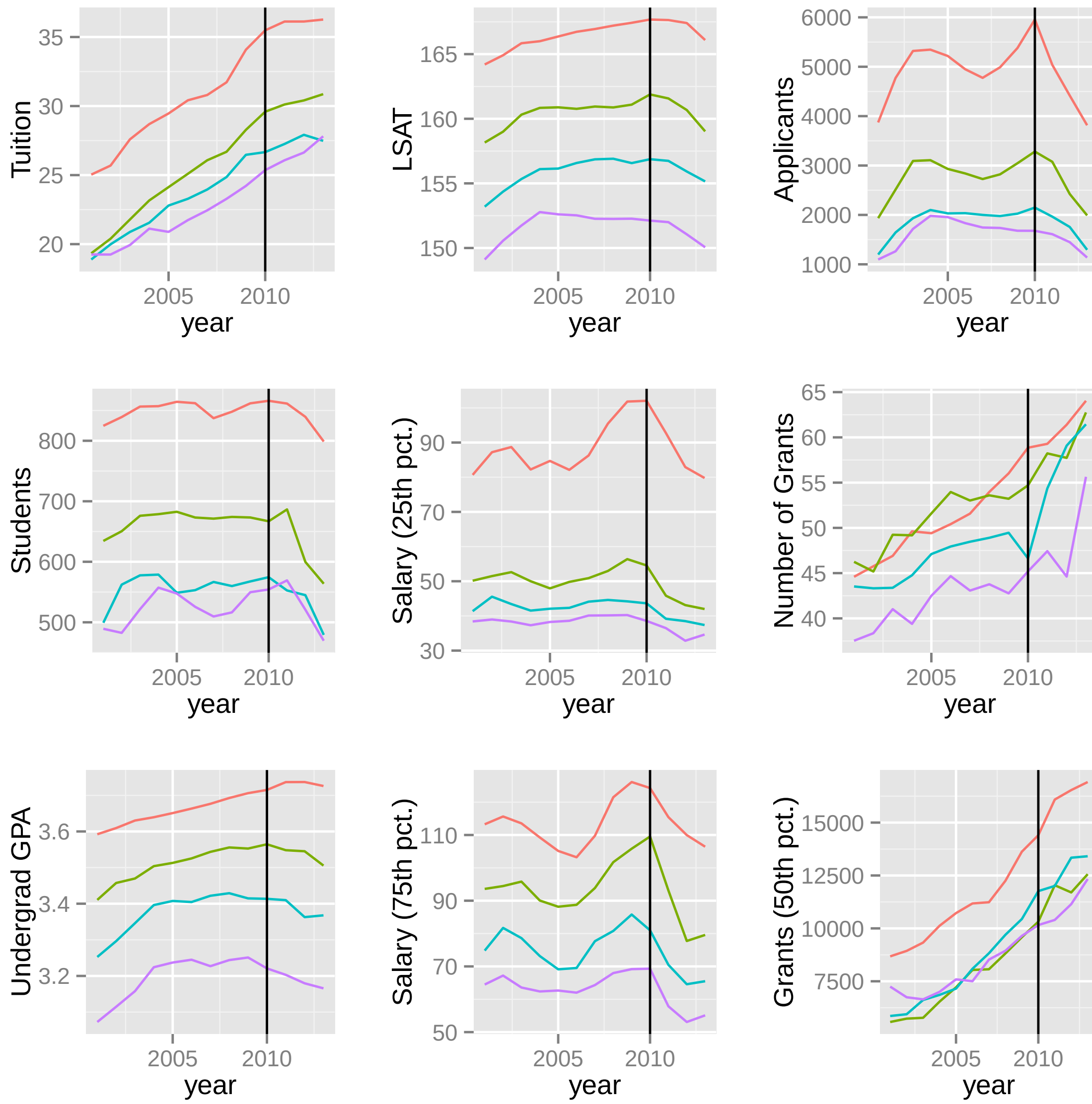

Figure B.1: Evolution by Rank Quantile

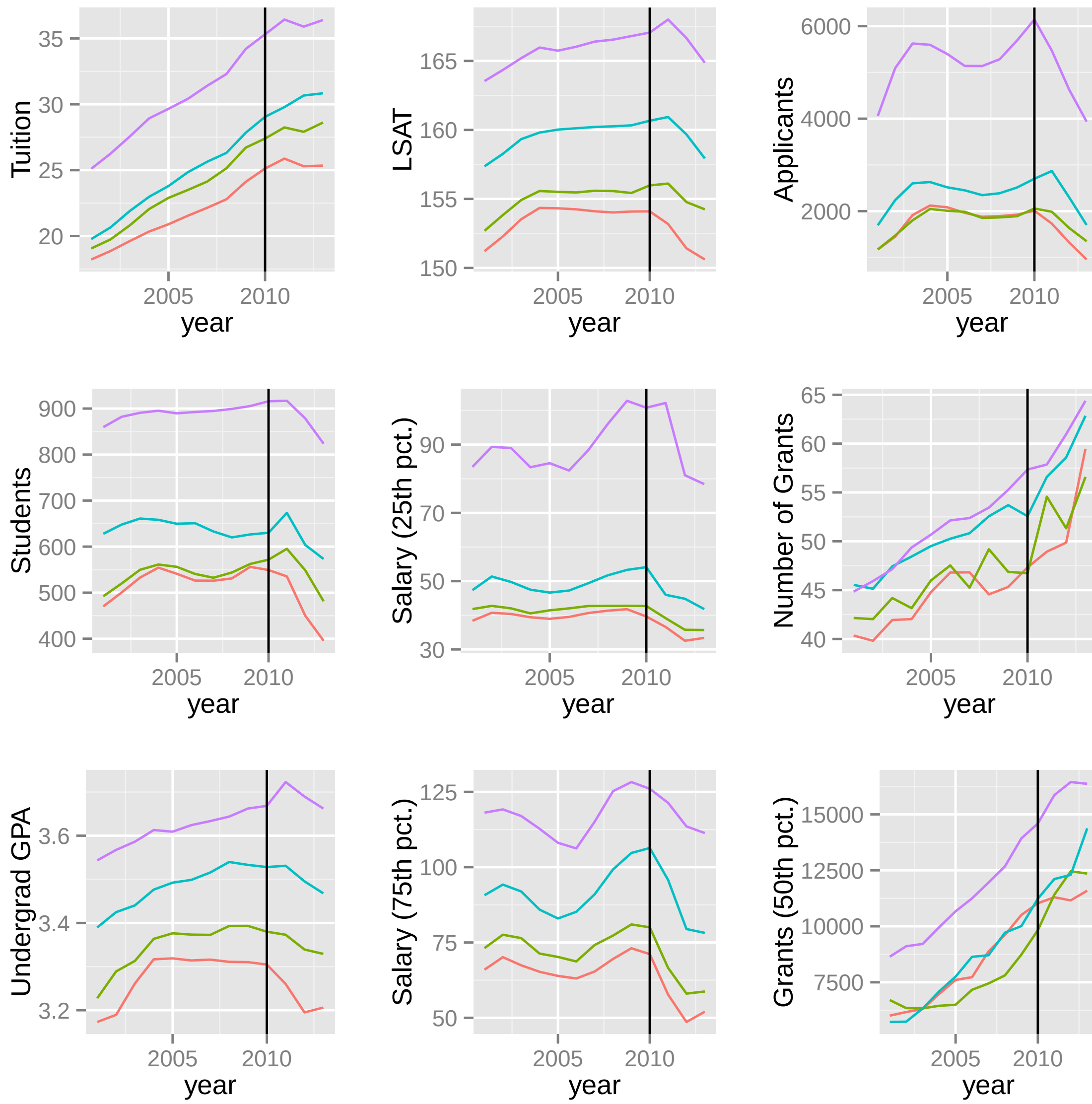

Figure B.2: Evolution by Ratio Quantile

	Model 1	Model 2	Model 3	Model 4	Model 5
(Intercept)	4395.37***	−24364.93***	1891.22***	3135.48***	−10013.07***
	(71.39)	(1538.59)	(181.30)	(177.28)	(1317.14)
Rank	−19.84***	2.86**	−18.52***	−6.32***	1.27
	(0.67)	(1.01)	(0.64)	(0.53)	(0.80)
Post-2010	−144.84	−422.12***	−229.45	−305.15***	−379.44***
	(127.43)	(88.66)	(120.00)	(65.32)	(64.77)
Post-2010 * Rank	0.79	1.00	−0.49	4.04***	3.78***
	(1.16)	(0.78)	(1.08)	(0.60)	(0.59)
Tuition		43.41***			22.02***
		(4.75)			(4.46)
Students		3.40***			1.18***
		(0.09)			(0.09)
Undergrad GPA		−985.65***			86.82
		(203.56)			(155.59)
LSAT		170.66***			74.29***
		(10.18)			(8.50)
Median Grant			0.03***		−0.01*
			(0.01)		(0.00)
Percent Grants			−8.02***		−2.01*
			(1.60)		(0.90)
Room/Board Expenses			245.49***		20.42**
			(10.75)		(6.64)
Cost of Books			−0.65***		−0.15***
			(0.06)		(0.03)
Student/Faculty Ratio				−4.81	−16.86**
				(5.59)	(5.74)
Accepted				2.82***	2.10***
				(0.04)	(0.05)
Acceptance Rate				−62.21***	−50.70***
				(1.51)	(1.72)
Bar Passage Rate				0.98	−4.01*
				(1.69)	(1.79)
R^2	0.35	0.74	0.51	0.84	0.88
Adj. R^2	0.35	0.74	0.51	0.84	0.88
Num. obs.	2419	2263	2261	2353	2118

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.5: Difference-in-differences: Applicants vs Rank

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
(Intercept)	-4100.60^{***}	-2041.75^{***}	-21232.93^{***}	-2243.07^{***}	-606.97^{**}	-5829.84^{***}
	(179.96)	(283.33)	(1554.56)	(298.74)	(201.79)	(1250.96)
Ratio	10310.60^{***}	8090.12^{***}	2921.56^{***}	5899.45^{***}	4849.74^{***}	4045.57^{***}
	(273.55)	(359.51)	(292.94)	(356.64)	(179.64)	(206.13)
Post-2010	-547.66	-360.87	146.07	-578.34	623.66^{***}	724.93^{***}
	(343.35)	(337.95)	(243.73)	(321.77)	(166.72)	(170.78)
Post-2010 * Ratio	585.33	335.55	-700.30	500.97	-952.34^{***}	-1128.91^{***}
	(520.12)	(511.74)	(367.76)	(483.30)	(249.08)	(253.58)
Rank		-6.76^{***}	4.06^{***}	-9.41^{***}	0.68	3.65^{***}
		(0.73)	(0.99)	(0.72)	(0.49)	(0.74)
Tuition			36.59^{***}			6.62
			(4.74)			(4.23)
Students			3.30^{***}			0.98^{***}
			(0.09)			(0.09)
Undergrad GPA			-822.66^{***}			311.51^{*}
			(200.82)			(145.51)
LSAT			135.99^{***}			27.09^{**}
			(10.63)			(8.30)
Median Grant				0.02^{***}		0.00
				(0.00)		(0.00)
Percent Grants				-5.52^{***}		-0.16
				(1.51)		(0.85)
Room/Board Expenses				197.31^{***}		9.60
				(10.43)		(6.21)
Cost of Books				-0.59^{***}		-0.13^{***}
				(0.06)		(0.03)
Student/Faculty Ratio					-1.44	-16.01^{**}
					(4.93)	(5.38)
Accepted					2.65^{***}	2.23^{***}
					(0.03)	(0.05)
Acceptance Rate					-55.42^{***}	-50.49^{***}
					(1.35)	(1.59)
Bar Passage Rate					-1.21	-3.75^{*}
					(1.49)	(1.67)
R^2	0.46	0.48	0.75	0.58	0.88	0.89
Adj. R^2	0.46	0.48	0.75	0.58	0.88	0.89
Num. obs.	2389	2389	2236	2232	2324	2091

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.6: Difference-in-differences: Applicants vs Ratio

	Model 1	Model 2	Model 3	Model 4	Model 5
(Intercept)	70.48***	−459.10***	57.85***	78.28***	−394.15***
	(0.63)	(23.72)	(2.30)	(3.95)	(28.09)
Rank	−0.20***	0.05**	−0.29***	−0.19***	0.03
	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
Post-2010	5.82***	−8.24***	−3.98**	−3.33*	−7.61***
	(1.35)	(1.33)	(1.49)	(1.42)	(1.36)
Post-2010 * Rank	−0.10***	0.02	−0.01	0.03*	0.03*
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Tuition		0.52***			0.70***
		(0.07)			(0.09)
Students		0.01***			0.01***
		(0.00)			(0.00)
Undergrad GPA		−5.02			1.07
		(3.11)			(3.30)
LSAT		3.19***			2.77***
		(0.16)			(0.18)
Median Grant			0.00***		0.00
			(0.00)		(0.00)
Percent Grants			−0.11***		−0.11***
			(0.02)		(0.02)
Room/Board Expenses			1.99***		0.45**
			(0.14)		(0.14)
Cost of Books			0.00***		0.00**
			(0.00)		(0.00)
Student/Faculty Ratio				−0.32**	−0.22
				(0.12)	(0.12)
Accepted				0.01***	0.00
				(0.00)	(0.00)
Acceptance Rate				−0.49***	−0.12**
				(0.03)	(0.04)
Bar Passage Rate				0.02	−0.12**
				(0.04)	(0.04)
R^2	0.42	0.65	0.54	0.56	0.67
Adj. R^2	0.42	0.65	0.54	0.56	0.67
Num. obs.	3110	2184	2200	2284	2074

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.7: Difference-in-differences: Salary (25th pct.) vs Rank

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
(Intercept)	−39.16***	−18.40***	−303.20***	−34.57***	−19.12***	−243.57***
	(1.40)	(2.05)	(20.01)	(3.01)	(3.79)	(23.54)
Ratio	135.84***	113.98***	112.34***	130.49***	127.85***	113.33***
	(2.10)	(2.61)	(3.69)	(3.56)	(3.32)	(3.82)
Post-2010	−35.20***	−36.11***	−19.17***	−24.07***	−17.58***	−16.67***
	(3.11)	(3.02)	(3.07)	(3.22)	(3.11)	(3.18)
Post-2010 * Ratio	53.25***	53.34***	21.73***	30.31***	23.14***	19.59***
	(4.68)	(4.55)	(4.62)	(4.82)	(4.62)	(4.71)
Rank		−0.06***	0.09***	−0.07***	−0.01	0.08***
		(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
Tuition			0.19**			0.19*
			(0.06)			(0.08)
Students			0.01***			0.00**
			(0.00)			(0.00)
Undergrad GPA			2.06			7.34**
			(2.55)			(2.71)
LSAT			1.63***			1.26***
			(0.14)			(0.16)
Median Grant				0.00***		0.00
				(0.00)		(0.00)
Percent Grants				−0.05**		−0.05**
				(0.02)		(0.02)
Room/Board Expenses				0.88***		0.14
				(0.10)		(0.12)
Cost of Books				0.00***		0.00*
				(0.00)		(0.00)
Student/Faculty Ratio					−0.23*	−0.19
					(0.09)	(0.10)
Accepted					0.01***	0.00***
					(0.00)	(0.00)
Acceptance Rate					−0.29***	−0.13***
					(0.03)	(0.03)
Bar Passage Rate					−0.04	−0.12***
					(0.03)	(0.03)
R^2	0.67	0.69	0.77	0.75	0.76	0.78
Adj. R^2	0.67	0.69	0.77	0.75	0.75	0.78
Num. obs.	3083	3083	2159	2173	2257	2049

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.8: Difference-in-differences: Salary (25th pct.) vs Ratio

	Model 1	Model 2	Model 3	Model 4	Model 5
(Intercept)	103.24***	−128.55***	79.94***	104.59***	−55.34
	(0.79)	(27.69)	(2.51)	(4.17)	(31.83)
Rank	−0.23***	−0.16***	−0.30***	−0.28***	−0.21***
	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
Post-2010	10.04***	−5.51***	−3.76*	−0.44	−3.97*
	(1.69)	(1.55)	(1.63)	(1.50)	(1.54)
Post-2010 * Rank	−0.14***	−0.03*	−0.05***	−0.01	−0.02
	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
Tuition		0.76***			0.61***
		(0.08)			(0.11)
Students		0.02***			0.01*
		(0.00)			(0.00)
Undergrad GPA		−21.82***			−14.89***
		(3.63)			(3.74)
LSAT		1.73***			1.10***
		(0.18)			(0.21)
Median Grant			0.00***		0.00
			(0.00)		(0.00)
Percent Grants			0.01		0.04
			(0.02)		(0.02)
Room/Board Expenses			2.49***		0.80***
			(0.15)		(0.16)
Cost of Books			0.00***		0.00
			(0.00)		(0.00)
Student/Faculty Ratio				0.69***	0.94***
				(0.13)	(0.14)
Accepted				0.02***	0.01***
				(0.00)	(0.00)
Acceptance Rate				−0.41***	−0.31***
				(0.04)	(0.04)
Bar Passage Rate				−0.07	−0.06
				(0.04)	(0.04)
R^2	0.40	0.62	0.57	0.61	0.66
Adj. R^2	0.40	0.62	0.57	0.61	0.65
Num. obs.	3110	2184	2200	2284	2074

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.9: Difference-in-differences: Salary (75th pct.) vs Rank

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
(Intercept)	−3.15	48.54***	−57.33*	38.61***	57.30***	14.63
	(2.33)	(3.29)	(28.18)	(4.20)	(5.10)	(31.96)
Ratio	125.46***	70.99***	41.42***	59.67***	62.34***	44.35***
	(3.51)	(4.19)	(5.19)	(4.97)	(4.47)	(5.19)
Post-2010	−36.83***	−39.13***	−23.26***	−29.55***	−15.21***	−19.14***
	(5.18)	(4.84)	(4.33)	(4.50)	(4.18)	(4.32)
Post-2010 * Ratio	57.41***	57.68***	23.50***	33.25***	20.11**	21.18***
	(7.79)	(7.29)	(6.50)	(6.73)	(6.22)	(6.40)
Rank		−0.15***	−0.16***	−0.21***	−0.20***	−0.20***
		(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
Tuition			0.64***			0.40***
			(0.08)			(0.11)
Students			0.02***			0.00
			(0.00)			(0.00)
Undergrad GPA			−18.24***			−11.61**
			(3.59)			(3.68)
LSAT			1.05***			0.43*
			(0.19)			(0.21)
Median Grant				0.00**		0.00
				(0.00)		(0.00)
Percent Grants				0.04*		0.06**
				(0.02)		(0.02)
Room/Board Expenses				1.93***		0.65***
				(0.15)		(0.16)
Cost of Books				0.00***		0.00
				(0.00)		(0.00)
Student/Faculty Ratio					0.73***	0.96***
					(0.12)	(0.14)
Accepted					0.02***	0.01***
					(0.00)	(0.00)
Acceptance Rate					−0.30***	−0.32***
					(0.03)	(0.04)
Bar Passage Rate					−0.10**	−0.06
					(0.04)	(0.04)
R^2	0.39	0.47	0.64	0.61	0.65	0.67
Adj. R^2	0.39	0.47	0.63	0.60	0.65	0.67
Num. obs.	3083	3083	2159	2173	2257	2049

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.10: Difference-in-differences: Salary (75th pct.) vs Ratio

	Model 1	Model 2	Model 3	Model 4	Model 5
(Intercept)	25.84***	−117.42***	13.16***	25.66***	−94.16***
	(0.19)	(6.35)	(0.54)	(1.06)	(6.10)
Rank	−0.04***	0.05***	−0.03***	−0.03***	0.03***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Post-2010	8.73***	6.09***	3.60***	5.01***	2.74***
	(0.43)	(0.37)	(0.36)	(0.39)	(0.31)
Post-2010 * Rank	−0.01***	−0.01*	−0.01**	−0.01*	−0.01*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Students		0.01***			0.00***
		(0.00)			(0.00)
Undergrad GPA		−3.32***			−0.84
		(0.90)			(0.76)
LSAT		0.92***			0.69***
		(0.04)			(0.04)
Median Grant			0.00***		0.00***
			(0.00)		(0.00)
Percent Grants			0.04***		0.04***
			(0.00)		(0.00)
Room/Board Expenses			0.66***		0.33***
			(0.03)		(0.03)
Cost of Books			0.00**		0.00
			(0.00)		(0.00)
Student/Faculty Ratio				−0.29***	−0.17***
				(0.03)	(0.03)
Accepted				0.01***	0.00***
				(0.00)	(0.00)
Acceptance Rate				0.00	0.06***
				(0.01)	(0.01)
Bar Passage Rate				0.01	−0.02
				(0.01)	(0.01)
R^2	0.41	0.56	0.63	0.55	0.74
Adj. R^2	0.41	0.56	0.63	0.55	0.74
Num. obs.	3583	2265	2157	2235	2118

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.11: Difference-in-differences: Tuition vs Rank

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
(Intercept)	6.32***	16.31***	−103.20***	3.77***	12.92***	−80.05***
	(0.55)	(0.82)	(6.60)	(0.92)	(1.30)	(6.25)
Ratio	22.62***	12.11***	8.73***	13.75***	17.07***	7.51***
	(0.83)	(1.03)	(1.30)	(1.10)	(1.15)	(1.06)
Post-2010	1.09	0.14	2.49*	−0.87	−0.65	−0.64
	(1.32)	(1.28)	(1.09)	(0.99)	(1.07)	(0.89)
Post-2010 * Ratio	10.95***	11.52***	4.35**	5.55***	7.10***	4.25**
	(1.99)	(1.92)	(1.64)	(1.48)	(1.58)	(1.31)
Rank		−0.03***	0.05***	−0.01***	−0.01	0.03***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Students			0.01***			0.00*
			(0.00)			(0.00)
Undergrad GPA			−2.49**			−0.17
			(0.90)			(0.76)
LSAT			0.78***			0.55***
			(0.04)			(0.04)
Median Grant				0.00***		0.00***
				(0.00)		(0.00)
Percent Grants				0.05***		0.04***
				(0.00)		(0.00)
Room/Board Expenses				0.53***		0.28***
				(0.03)		(0.03)
Cost of Books				0.00*		0.00
				(0.00)		(0.00)
Student/Faculty Ratio					−0.30***	−0.16***
					(0.03)	(0.03)
Accepted					0.01***	0.00***
					(0.00)	(0.00)
Acceptance Rate					0.03**	0.06***
					(0.01)	(0.01)
Bar Passage Rate					0.00	−0.02
					(0.01)	(0.01)
R^2	0.40	0.44	0.57	0.67	0.61	0.75
Adj. R^2	0.40	0.44	0.57	0.66	0.60	0.75
Num. obs.	3556	3556	2238	2130	2208	2091

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.12: Difference-in-differences: Tuition vs Ratio

	Model 1	Model 2	Model 3	Model 4	Model 5
(Intercept)	782.08***	−470.53	492.18***	245.03***	−679.61*
	(8.81)	(375.37)	(31.42)	(40.77)	(316.08)
Rank	−1.47***	−1.24***	−1.97***	−1.49***	−1.08***
	(0.07)	(0.25)	(0.11)	(0.12)	(0.19)
Post-2010	3.92	−140.69***	−22.73	−38.81**	−12.72
	(20.17)	(21.44)	(20.79)	(14.96)	(15.56)
Post-2010 * Rank	−0.37*	0.45*	0.12	0.48***	0.10
	(0.18)	(0.19)	(0.19)	(0.14)	(0.14)
Tuition		18.23***			4.45***
		(1.09)			(1.07)
Undergrad GPA		−322.21***			0.07
		(49.21)			(37.38)
LSAT		11.84***			5.03*
		(2.47)			(2.04)
Median Grant			0.00		0.00***
			(0.00)		(0.00)
Percent Grants			−2.27***		−2.17***
			(0.28)		(0.21)
Room/Board Expenses			38.44***		10.84***
			(1.86)		(1.58)
Cost of Books			−0.10***		−0.02**
			(0.01)		(0.01)
Student/Faculty Ratio				16.29***	16.61***
				(1.28)	(1.33)
Accepted				0.40***	0.35***
				(0.01)	(0.01)
Acceptance Rate				−3.81***	−2.31***
				(0.35)	(0.41)
Bar Passage Rate				1.07**	0.78
				(0.39)	(0.43)
R^2	0.14	0.32	0.34	0.64	0.67
Adj. R^2	0.14	0.32	0.34	0.64	0.67
Num. obs.	3754	2265	2263	2351	2118

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.13: Difference-in-differences: Students vs Rank

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
(Intercept)	−107.71***	37.35	−10.87	147.70**	−143.86**	−326.16
	(23.83)	(36.55)	(384.86)	(54.49)	(51.58)	(320.51)
Ratio	1106.69***	954.31***	419.58***	478.41***	497.33***	317.27***
	(35.99)	(46.24)	(71.98)	(65.06)	(45.88)	(52.36)
Post-2010	−71.93	−85.41	−107.77	−112.74	15.89	−67.95
	(57.14)	(57.00)	(60.28)	(58.71)	(42.56)	(43.74)
Post-2010 * Ratio	106.93	114.67	22.61	155.34	−24.23	104.43
	(86.36)	(86.07)	(91.04)	(88.20)	(63.57)	(64.94)
Rank		−0.40***	−0.94***	−1.14***	−0.74***	−0.89***
		(0.08)	(0.24)	(0.13)	(0.13)	(0.19)
Tuition			16.36***			2.71*
			(1.12)			(1.08)
Undergrad GPA			−301.69***			17.04
			(49.30)			(37.29)
LSAT			6.86**			1.22
			(2.63)			(2.13)
Median Grant				0.00		0.00***
				(0.00)		(0.00)
Percent Grants				−2.08***		−1.93***
				(0.27)		(0.21)
Room/Board Expenses				34.61***		9.72***
				(1.90)		(1.58)
Cost of Books				−0.09***		−0.02*
				(0.01)		(0.01)
Student/Faculty Ratio					16.99***	16.65***
					(1.26)	(1.33)
Accepted					0.38***	0.36***
					(0.01)	(0.01)
Acceptance Rate					−3.10***	−2.25***
					(0.34)	(0.41)
Bar Passage Rate					0.81*	0.72
					(0.38)	(0.43)
R^2	0.24	0.25	0.33	0.36	0.66	0.68
Adj. R^2	0.24	0.25	0.33	0.36	0.66	0.68
Num. obs.	3725	3725	2238	2234	2322	2091

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.14: Difference-in-differences: Students vs Ratio

	Model 1	Model 2	Model 3	Model 4	Model 5
(Intercept)	164.57***	134.11***	162.68***	163.53***	141.22***
	(0.12)	(1.47)	(0.31)	(0.49)	(1.39)
Rank	−0.07***	−0.06***	−0.09***	−0.07***	−0.06***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Post-2010	2.89***	−0.76***	0.40*	0.88***	−0.37*
	(0.26)	(0.18)	(0.21)	(0.18)	(0.17)
Post-2010 * Rank	−0.04***	0.00	−0.01***	0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Tuition		0.20***			0.19***
		(0.01)			(0.01)
Students		0.00***			0.00*
		(0.00)			(0.00)
Undergrad GPA		7.06***			4.84***
		(0.39)			(0.39)
Median Grant			0.00***		0.00*
			(0.00)		(0.00)
Percent Grants			0.00		0.00
			(0.00)		(0.00)
Room/Board Expenses			0.21***		0.05**
			(0.02)		(0.02)
Cost of Books			0.00*		0.00***
			(0.00)		(0.00)
Student/Faculty Ratio				−0.07***	0.00
				(0.02)	(0.01)
Accepted				0.00***	0.00
				(0.00)	(0.00)
Acceptance Rate				−0.10***	−0.08***
				(0.00)	(0.00)
Bar Passage Rate				0.05***	0.03***
				(0.00)	(0.00)
R^2	0.70	0.89	0.86	0.88	0.92
Adj. R^2	0.70	0.89	0.86	0.88	0.92
Num. obs.	3101	2265	2256	2345	2118

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.15: Difference-in-differences: LSAT vs Rank

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
(Intercept)	134.04***	150.70***	129.57***	155.23***	156.81***	136.84***
	(0.41)	(0.48)	(1.44)	(0.50)	(0.59)	(1.39)
Ratio	35.95***	18.31***	8.49***	10.84***	9.02***	6.96***
	(0.62)	(0.61)	(0.56)	(0.60)	(0.53)	(0.52)
Post-2010	−5.21***	−5.77***	−1.55**	−2.29***	−0.96*	−0.32
	(0.88)	(0.68)	(0.48)	(0.54)	(0.49)	(0.45)
Post-2010 * Ratio	8.36***	8.29***	1.21	3.18***	2.34**	0.22
	(1.34)	(1.03)	(0.73)	(0.81)	(0.73)	(0.67)
Rank		−0.05***	−0.05***	−0.08***	−0.06***	−0.05***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Tuition			0.15***			0.14***
			(0.01)			(0.01)
Students			0.00**			0.00
			(0.00)			(0.00)
Undergrad GPA			6.91***			4.93***
			(0.37)			(0.37)
Median Grant				0.00***		0.00**
				(0.00)		(0.00)
Percent Grants				0.01***		0.00
				(0.00)		(0.00)
Room/Board Expenses				0.12***		0.02
				(0.02)		(0.02)
Cost of Books				0.00		0.00***
				(0.00)		(0.00)
Student/Faculty Ratio					−0.07***	0.00
					(0.01)	(0.01)
Accepted					0.00***	0.00**
					(0.00)	(0.00)
Acceptance Rate					−0.09***	−0.08***
					(0.00)	(0.00)
Bar Passage Rate					0.04***	0.03***
					(0.00)	(0.00)
R^2	0.61	0.77	0.90	0.88	0.90	0.92
Adj. R^2	0.61	0.77	0.90	0.88	0.90	0.92
Num. obs.	3071	3071	2238	2227	2316	2091

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.16: Difference-in-differences: LSAT vs Ratio

	Model 1	Model 2	Model 3	Model 4	Model 5
(Intercept)	3.69***	0.86***	3.65***	3.45***	1.07***
	(0.01)	(0.16)	(0.01)	(0.02)	(0.18)
Rank	0.00***	0.00***	0.00***	0.00***	0.00***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Post-2010	0.07***	0.07***	0.06***	0.07***	0.06***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Post-2010 * Rank	0.00***	0.00***	0.00***	0.00***	0.00***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Tuition		0.00***			0.00
		(0.00)			(0.00)
Students		0.00***			0.00
		(0.00)			(0.00)
LSAT		0.02***			0.01***
		(0.00)			(0.00)
Median Grant			0.00***		0.00*
			(0.00)		(0.00)
Percent Grants			0.00***		0.00***
			(0.00)		(0.00)
Room/Board Expenses			0.00*		0.00
			(0.00)		(0.00)
Cost of Books			0.00***		0.00**
			(0.00)		(0.00)
Student/Faculty Ratio				0.00*	0.00
				(0.00)	(0.00)
Accepted				0.00***	0.00***
				(0.00)	(0.00)
Acceptance Rate				0.00***	0.00**
				(0.00)	(0.00)
Bar Passage Rate				0.00***	0.00***
				(0.00)	(0.00)
R^2	0.75	0.79	0.75	0.78	0.80
Adj. R^2	0.75	0.79	0.75	0.78	0.80
Num. obs.	2410	2265	2256	2344	2118

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.17: Difference-in-differences: Undergrad GPA vs Rank

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
(Intercept)	2.71***	3.71***	0.69***	3.64***	3.48***	0.92***
	(0.02)	(0.02)	(0.16)	(0.03)	(0.03)	(0.19)
Ratio	1.06***	−0.01	−0.17***	0.02	−0.03	−0.16***
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Post-2010	−0.18***	−0.09**	−0.06*	−0.11***	−0.09***	−0.09***
	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Post-2010 * Ratio	0.30***	0.18***	0.14***	0.19***	0.19***	0.16***
	(0.06)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Rank		0.00***	0.00***	0.00***	0.00***	0.00***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Tuition			0.00**			0.00
			(0.00)			(0.00)
Students			0.00***			0.00
			(0.00)			(0.00)
LSAT			0.02***			0.02***
			(0.00)			(0.00)
Median Grant				0.00***		0.00*
				(0.00)		(0.00)
Percent Grants				0.00***		0.00***
				(0.00)		(0.00)
Room/Board Expenses				0.00**		0.00
				(0.00)		(0.00)
Cost of Books				0.00***		0.00*
				(0.00)		(0.00)
Student/Faculty Ratio					0.00*	0.00
					(0.00)	(0.00)
Accepted					0.00***	0.00***
					(0.00)	(0.00)
Acceptance Rate					0.00***	0.00***
					(0.00)	(0.00)
Bar Passage Rate					0.00***	0.00***
					(0.00)	(0.00)
R^2	0.42	0.75	0.79	0.75	0.78	0.80
Adj. R^2	0.42	0.74	0.79	0.74	0.78	0.79
Num. obs.	2380	2380	2238	2227	2315	2091

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.18: Difference-in-differences: Undergrad GPA vs Ratio

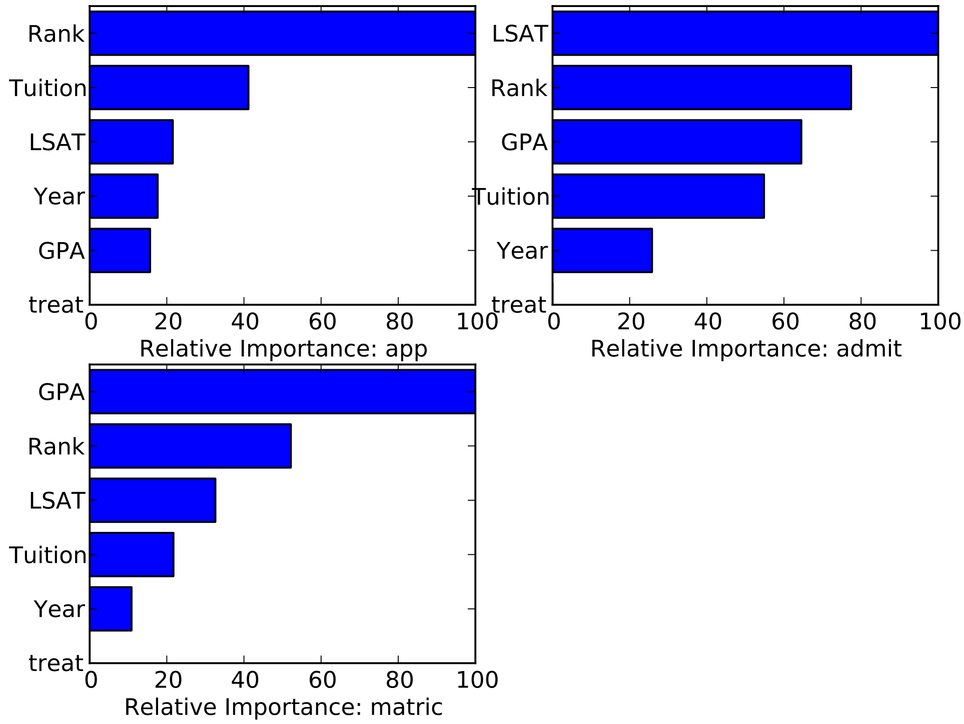

Figure B.3: Application-Admission Game: Variable Importance

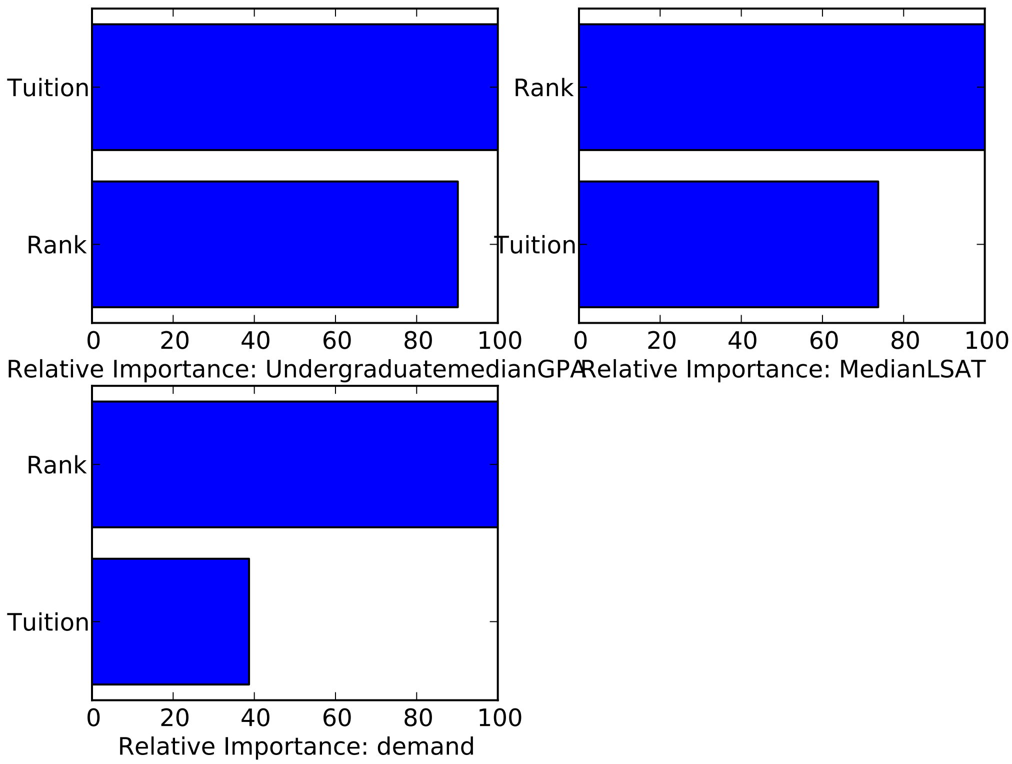

Figure B.4: Application-Admission Outcome Functions (treat=0): Variable Importance

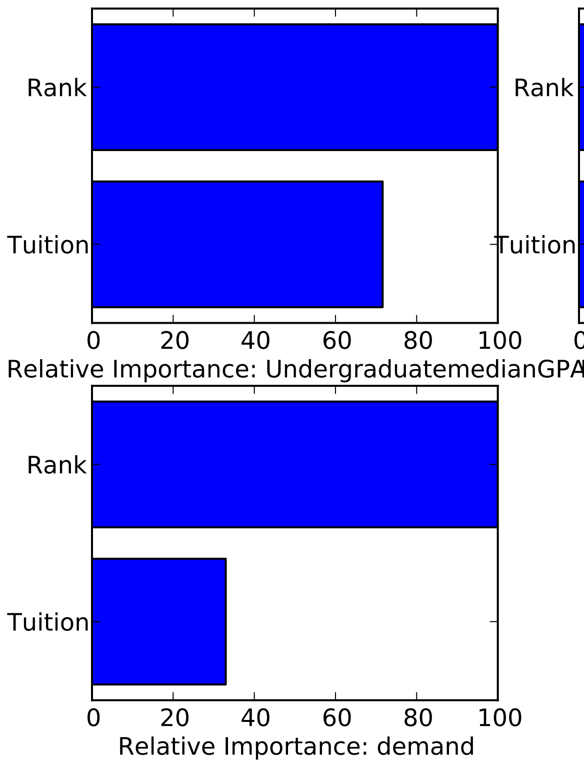

Figure B.5: Application-Admission Outcome Functions (treat=1): Variable Importance

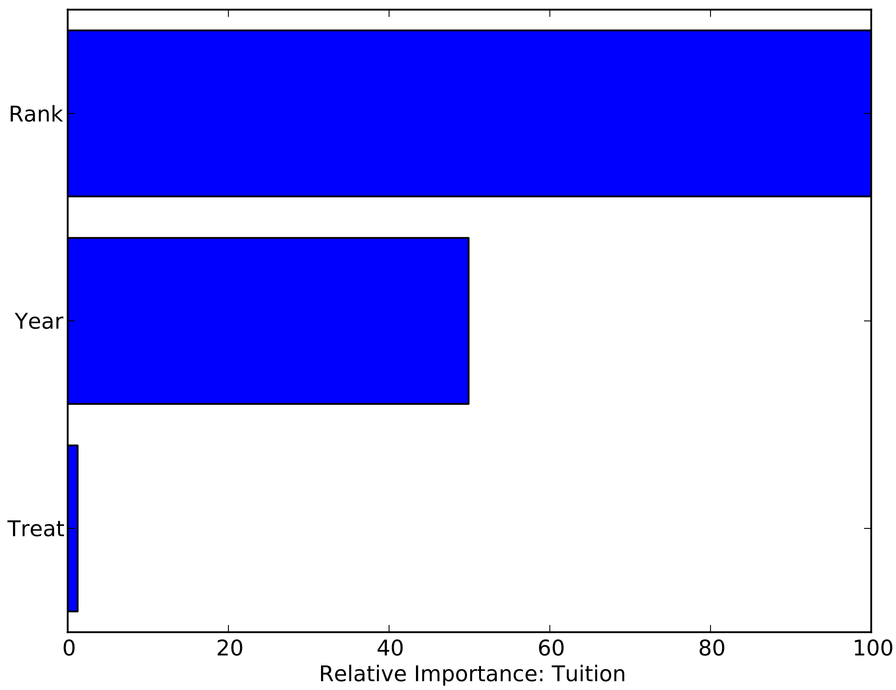

Figure B.6: Tuition Policy Function: Variable Importance

	Model 1	Model 2	Model 3	Model 4	Model 5
(Intercept)	-0.08^{***}	-0.09^{***}	4.88^{***}	5.00^{***}	4.64^{***}
	(0.00)	(0.00)	(0.20)	(0.20)	(0.20)
OverallRank	-0.00^{***}	-0.00^{***}	-0.00^{***}	0.00^{***}	-0.00^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Tuition	0.00^{***}	0.00^{***}	0.00^{***}	0.01^{***}	0.00^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
LSAT	0.00^{***}	0.00^{***}	0.00^{***}	0.00^{***}	0.00^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
LSDAS_GPA	-0.00	-0.00	0.00	0.00	0.05^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)
treat		-0.01^{***}	0.00	-0.03^{***}	-0.10^{*}
		(0.00)	(0.00)	(0.00)	(0.05)
year			-0.00^{***}	-0.00^{***}	-0.00^{***}
			(0.00)	(0.00)	(0.00)
OverallRank:Tuition				-0.00^{***}	
				(0.00)	
OverallRank:treat				0.00^{***}	
				(0.00)	
Tuition:treat				0.00^{**}	
				(0.00)	
OverallRank:Tuition:treat				-0.00^{*}	
				(0.00)	
LSAT:LSDAS_GPA					-0.00^{***}
					(0.00)
LSAT:treat					0.00^{*}
					(0.00)
LSDAS_GPA:treat					0.02
					(0.02)
LSAT:LSDAS_GPA:treat					-0.00
					(0.00)
R^2	0.04	0.04	0.04	0.05	0.04
Adj. R^2	0.04	0.04	0.04	0.05	0.04
Num. obs.	2155488	2155488	2155488	2155488	2155488
RMSE	0.22	0.22	0.22	0.22	0.22

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.19: Logit: Applications

	Model 1	Model 2	Model 3	Model 4	Model 5
(Intercept)	-5.22^{***}	-5.16^{***}	-24.31^{***}	-30.10^{***}	-21.70^{***}
	(0.04)	(0.04)	(1.72)	(1.71)	(1.74)
OverallRank	0.01^{***}	0.01^{***}	0.00^{***}	-0.00^{***}	0.00^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Tuition	-0.00^{***}	-0.01^{***}	-0.01^{***}	-0.03^{***}	-0.01^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
LSAT	0.03^{***}	0.03^{***}	0.03^{***}	0.03^{***}	0.02^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
LSDAS_GPA	0.25^{***}	0.25^{***}	0.25^{***}	0.27^{***}	-0.28^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.08)
treat		0.11^{***}	0.07^{***}	0.14^{***}	-1.49^{**}
		(0.00)	(0.00)	(0.03)	(0.47)
year			0.01^{***}	0.01^{***}	0.01^{***}
			(0.00)	(0.00)	(0.00)
OverallRank:Tuition				0.00^{***}	
				(0.00)	
OverallRank:treat				-0.00^{*}	
				(0.00)	
Tuition:treat				-0.00	
				(0.00)	
OverallRank:Tuition:treat				-0.00	
				(0.00)	
LSAT:LSDAS_GPA					0.00^{***}
					(0.00)
LSAT:treat					0.01^{**}
					(0.00)
LSDAS_GPA:treat					0.68^{***}
					(0.14)
LSAT:LSDAS_GPA:treat					-0.00^{***}
					(0.00)
R^2	0.20	0.21	0.21	0.24	0.21
Adj. R^2	0.20	0.21	0.21	0.24	0.21
Num. obs.	113781	113781	113781	113781	113781
RMSE	0.44	0.44	0.44	0.43	0.44

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.20: Logit: Admissions

	Model 1	Model 2	Model 3	Model 4	Model 5
(Intercept)	1.43***	1.44***	7.53***	8.97***	7.61***
	(0.05)	(0.05)	(1.77)	(1.79)	(1.81)
OverallRank	−0.00***	−0.00***	−0.00***	0.00***	−0.00***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Tuition	−0.00***	0.00	0.00	0.01***	0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
LSAT	−0.01***	−0.01***	−0.01***	−0.01***	−0.01***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
LSDAS_GPA	−0.06***	−0.06***	−0.06***	−0.07***	−0.15
	(0.00)	(0.00)	(0.00)	(0.00)	(0.10)
treat		−0.03***	−0.02***	−0.10**	−0.40
		(0.00)	(0.01)	(0.03)	(0.57)
year			−0.00***	−0.00***	−0.00**
			(0.00)	(0.00)	(0.00)
OverallRank:Tuition				−0.00***	
				(0.00)	
OverallRank:treat				0.00**	
				(0.00)	
Tuition:treat				0.00	
				(0.00)	
OverallRank:Tuition:treat				−0.00	
				(0.00)	
LSAT:LSDAS_GPA					0.00
					(0.00)
LSAT:treat					0.00
					(0.00)
LSDAS_GPA:treat					0.04
					(0.16)
LSAT:LSDAS_GPA:treat					−0.00
					(0.00)
R^2	0.02	0.02	0.02	0.03	0.02
Adj. R^2	0.02	0.02	0.02	0.03	0.02
Num. obs.	50200	50200	50200	50200	50200
RMSE	0.30	0.30	0.30	0.30	0.30

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table B.21: Logit: Matriculations

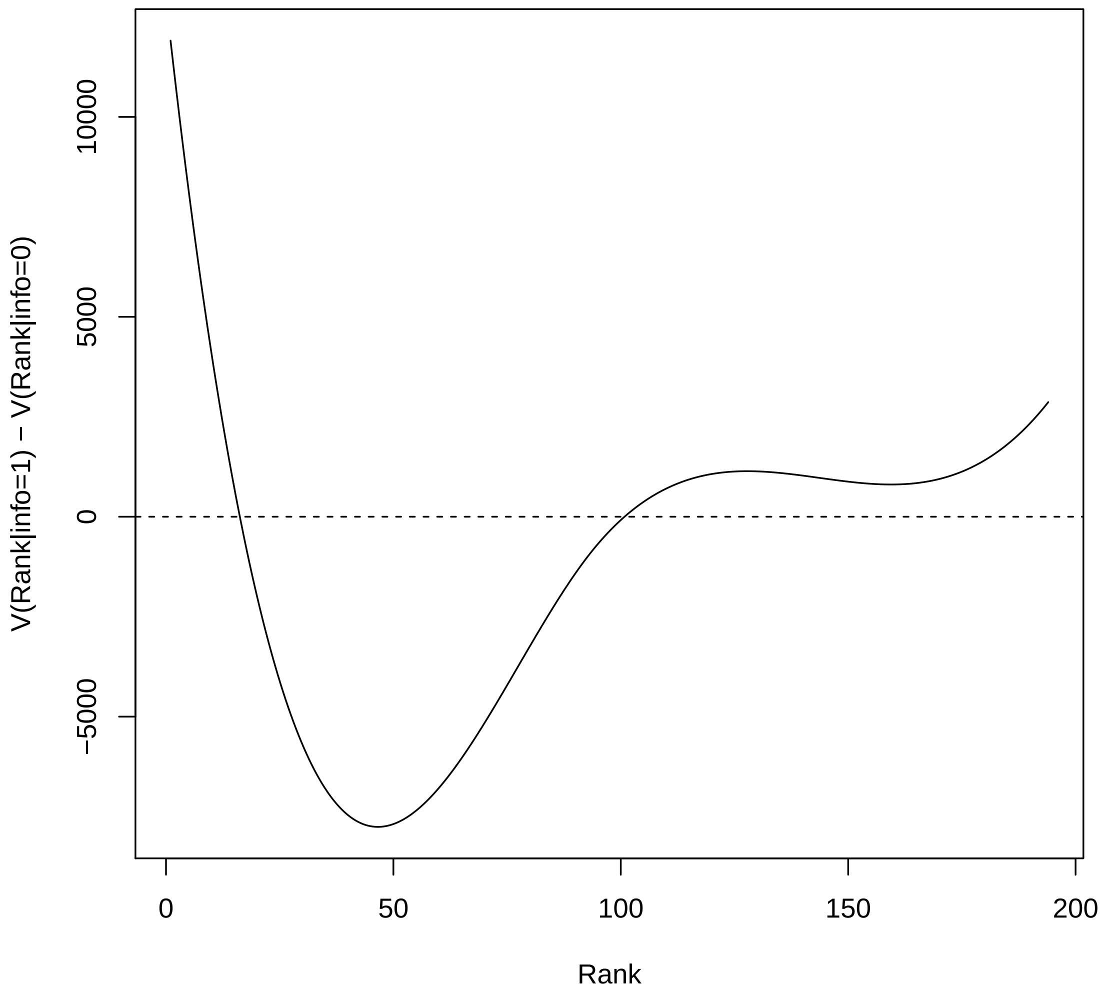

Figure B.7: Change in producer surplus function: $\Delta V(R)$

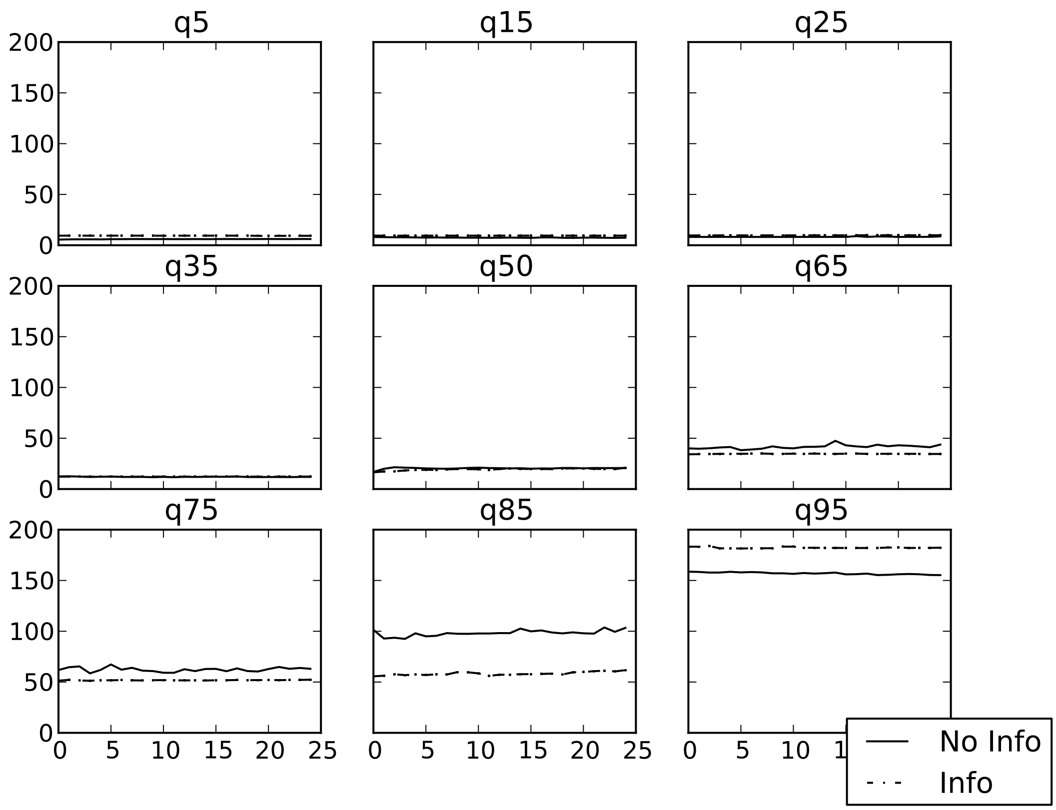

Figure B.8: Demand Quantiles

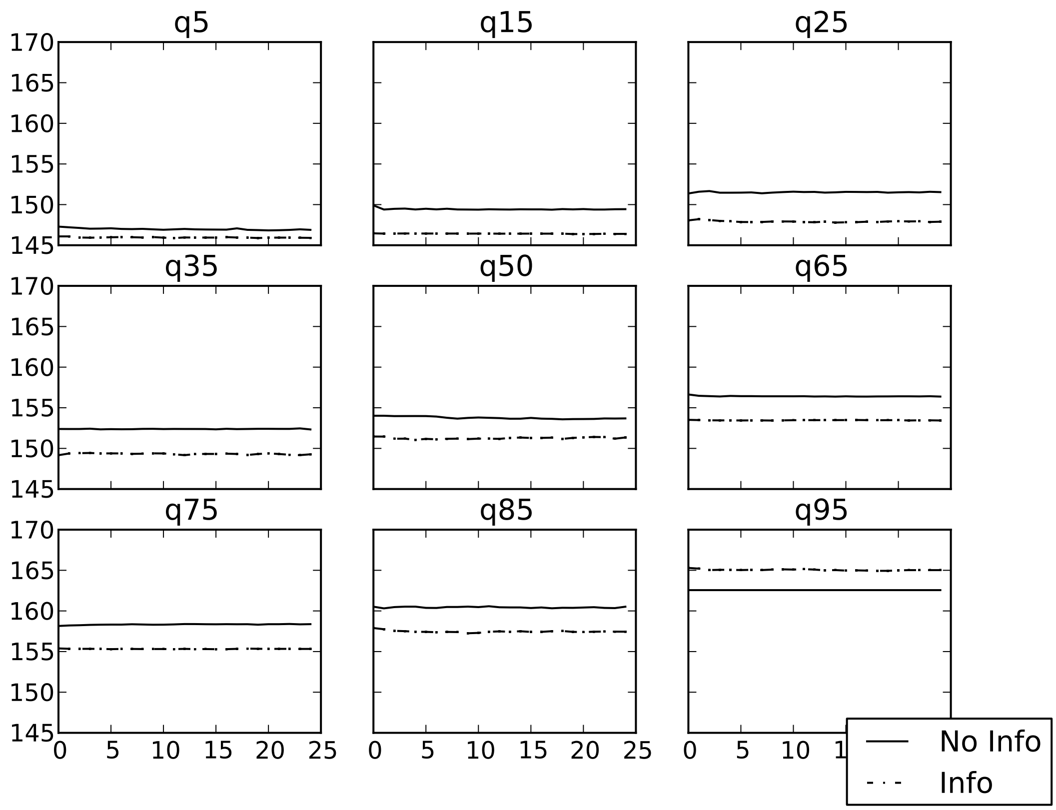

Figure B.9: LSAT Quantiles

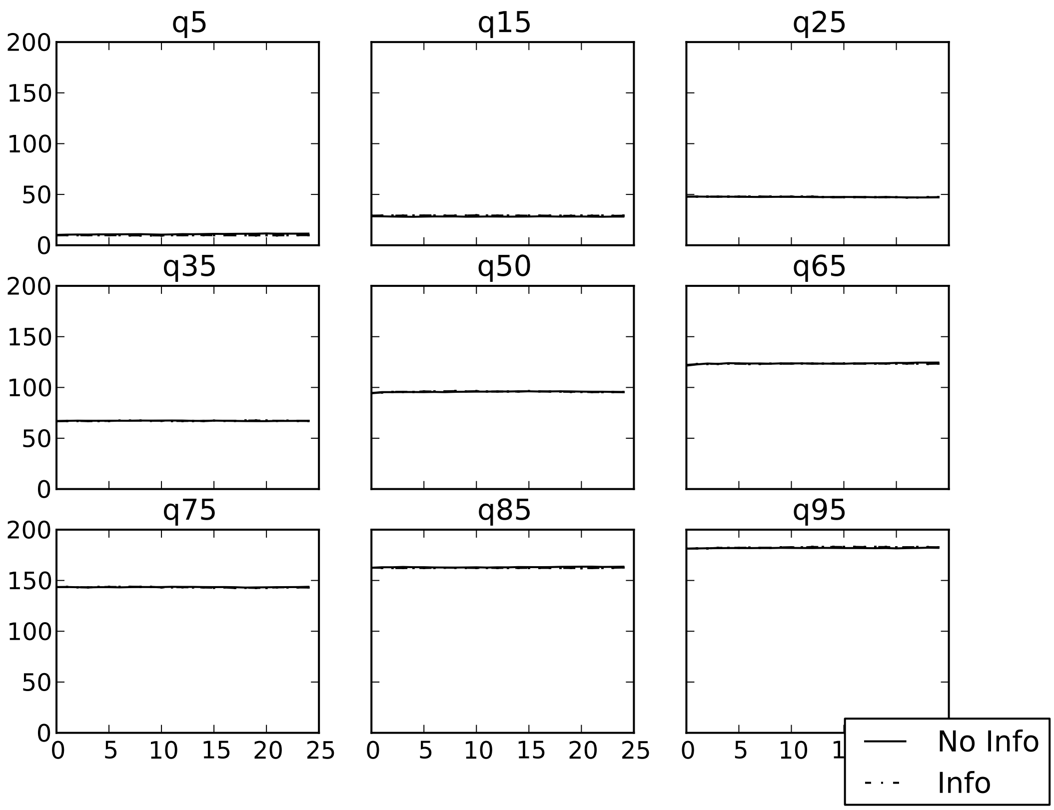

Figure B.10: Rank Quantiles

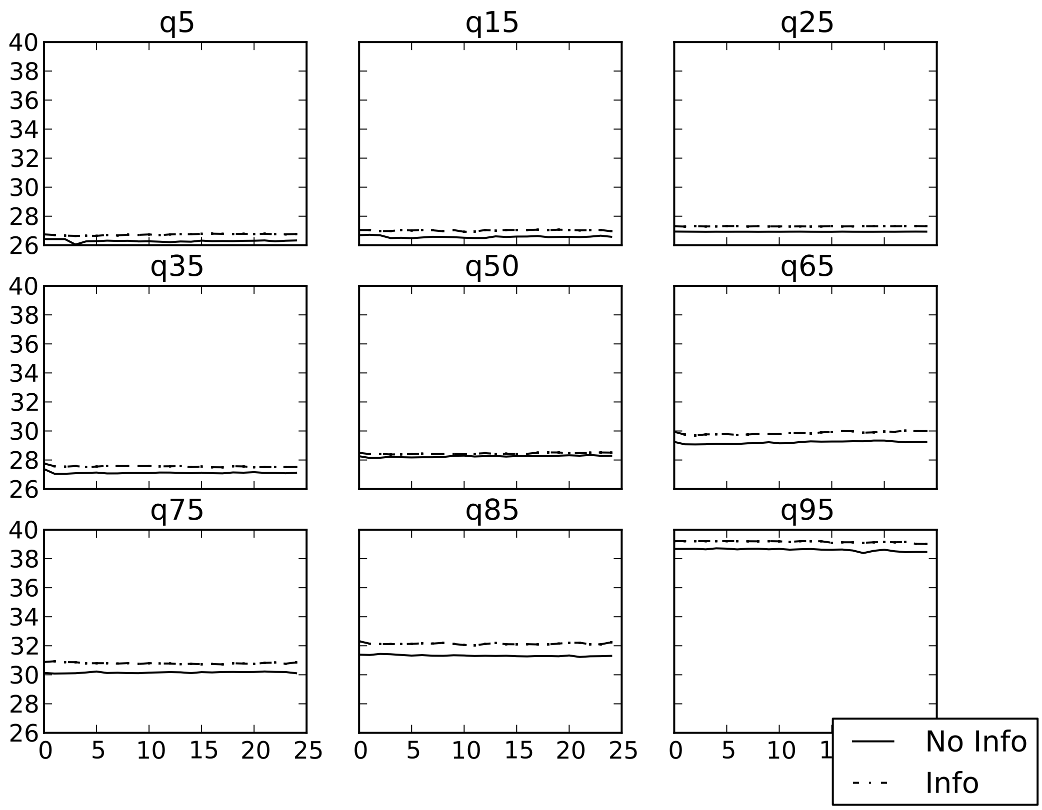

Figure B.11: Tuition Quantiles

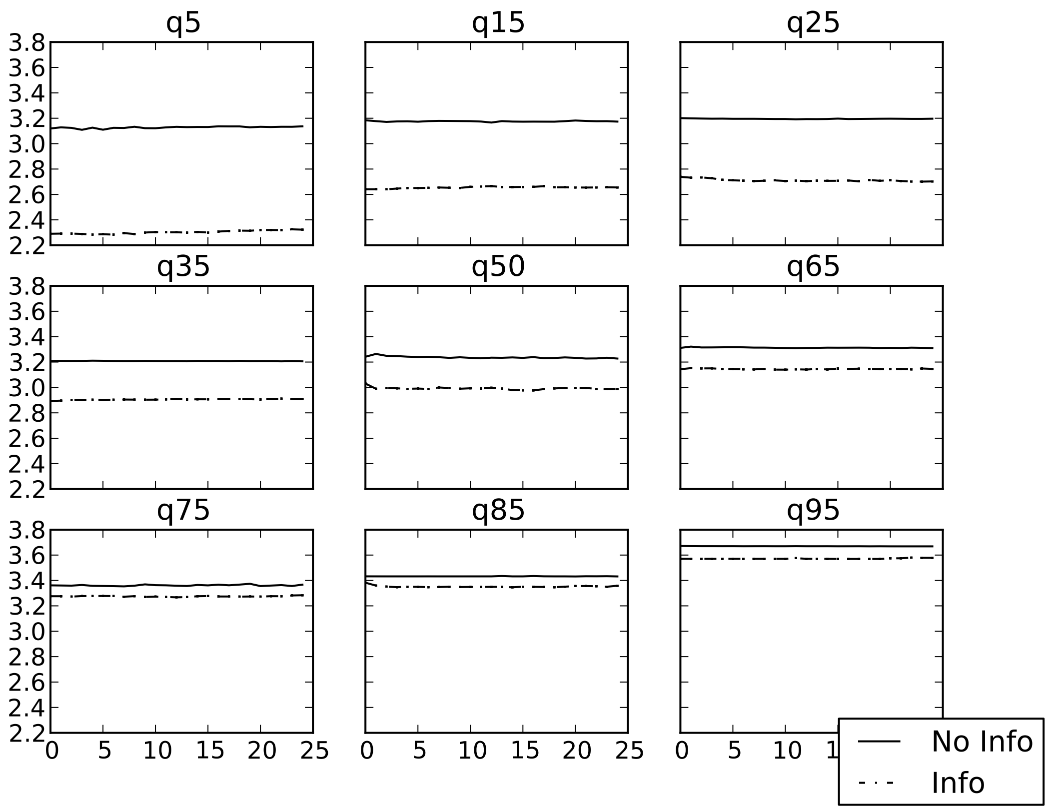

Figure B.12: GPA Quantiles

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