## USE OF ANALOGIES IN FLUID MECHANICS

by

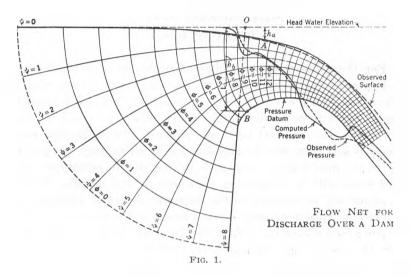
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For the solution of a number of problems in fluid mechanics and hydraulic engineering scientists and engineers have worked out devices which are very efficient because they enable solutions of complex cases to be made rapidly which otherwise could be accurately solved only at great labor and expense. In its broader aspect, a model in any form may be considered to be an analogy, the performance of the model being analogous to that of the prototype. Since model tests will be considered elsewhere in the bulletin, however, this paper will be confined to the narrower aspect of analogies, that in which the apparatus used is not a model of the structure or situation for which a solution is desired. In some cases it is hard to tell where the model ends and analogies, from this more restricted viewpoint, begin, since all the commonly used analogies have some features in common with the prototypes. No attempt therefore will be made to distinguish rigidly between analogies and models, but the approach, rather, will be to illustrate some of the analogies and to explain how they may be used for the solution of fluid flow.

One of the most useful analogies in the solution of fluid flow problems is the flow net. The flow net consists of a series of intersecting lines resembling a fish net, composed of two families of lines which intersect each other at right angles. In hydraulic applications, one set of these lines represent the paths of flow of the water particles and the other set are lines of constant potential.

The theory of the flow net, like so many other scientific things, had its beginnings many years ago. but its application to hydraulics is comparatively recent, probably being an outgrowth of its use in the aerodynamic field. Fig. 1 is a flow net for the flow of water over a dam. One set of the lines represents the average direction of flow of the particles of water and the other set is related to the potential. As will be seen, these families of lines are at right angles to each other. In such a net the polygons are curvillinear rectangles, all with the same ratios of width to length.

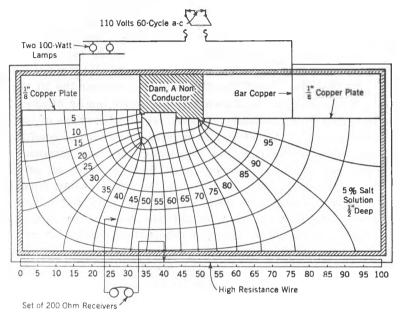
The flow net can be most readily applied to the solution of hydraulic problems of flow which can be analyzed with sufficient ac-



curacy in two dimensions. It assumes a perfect fluid and therefore cannot be accurately applied to conditions where hydraulic friction or turbulence are appreciable. It usually can be applied to cases where water is accelerating, such as approaching an orifice or flowing over the top of a dam, but not to cases where the water is decelerating, since under these conditions separation usually occurs. It is also readily applied to the flow of water through granular media. It can be applied to some three dimensional problems but the procedure is rather complex.

Flow nets can be constructed graphically by a cut-and-try process. The one in Fig. 1 was developed in that way. It is said that in a short time one can learn to work them out rapidly, but since the apparatus for doing so electrically can be constructed so easily and cheaply, it hardly pays to do so. The electrical ap-

paratus for constructing flow nets has been called by a number of names, but probably the best is simply the "electric analogy." Perhaps the cheapest form is that shown in Fig. 2, which is set up to develop the net for flow under a dam on a permeable foundation. It consists of a carefully leveled flat bottom glass tray filled about



ELECTRIC ANALOGY APPARATUS
Fig. 2.

½-inch deep with a 5 per cent salt solution. Two copper plate electrical terminals are placed in this solution and connected through incandescent lamps to a source of 110 volt, alternating current in such a way as to give about 10 to 15 volts potential between the terminals. One terminal represents the upstream bed of the river and the other the downstream bed, with a piece of insulating material between cut to represent the shape of the contact of the dam with the river bottom. A Wheatstone bridge, in the form of a high resistance wire over a meter stick, is connected at each end to one of the terminals. The sliding contact of the bridge is connected through ear phones to a probe which can be moved from place to place in the water. Suppose the sliding contact on the bridge is

placed 40 per cent of the length of the bridge wire from the end connected with the terminal representing the upstream river bed. The potential at this point is then 40 per cent of the drop between the terminals away from the potential of the upstream river bed terminals. If the probe is moved around in the salt water a current will pass through the headphones and cause a humming, except when the probe is at a point where the potential is the same as the point of contact on the bridge current, or in other words, when the probe is at a point of 40 per cent potential drop between the terminals. By moving the probe around, a line can be developed all points of which have this same 40 per cent potential. In like manner the 10, 20, 30 per cent, etc., lines can be drawn in. The other family of lines can then be drawn by a cut-and-try process, because the shape of one set of lines is known. Another way is to reverse positions of conducting and insulating boundaries and, by using the new terminals in the same manner as the previous ones, find the other set of lines by the electrical process. In this net the lines leading from upstream to downstream river beds are the flow lines and follow the direction of the path the water would take beneath the dam if the foundation material were equally pervious throughout and the contact of the dam with the bed was perfect. The others are lines of constant piezometric pressure in the foundation material. The accuracy of the results obtained, as applied to any actual case, depends upon how closely the actual conditions agree with the assumption of homogeneity and perfect contact. From such flow nets engineers can tell a great deal regarding the safety of the foundations of dams on pervious foundations. The cost of this apparatus, not including the time required to make it, is less than \$10.00.

Other forms of this apparatus use very thin tinfoil in place of the water tray and recently a carbon coated cardboard has been placed on the market. With this material the current from a couple of dry cells is sufficient and an induction coil and ear phones may be used for detecting the current. Where much of this work is to be done, a sensitive galvanometer on which the needle swings both ways is better since it indicates which side of the neutral line the probe is located. Although it is more expensive, it will soon pay for itself if much work is done. The apparatus which we have at the hydraulic laboratory uses the carbon coated cardboard and the galvanometer, and, in addition, a pantograph to locate the position of the probing point. The example which we will demonstrate here

is the flow under a row of sheet piling. This piling is simulated by removing a slit from the cardboard which corresponds to the insulating medium of Fig. 2. We will now draw out the 25, 50, and 75 per cent potential lines. Another form of apparatus which will produce flow nets is that developed by Hele Shaw<sup>1</sup> and consists of parallel transparent plates, between which a fluid is flowing. The path of the particles, as shown by the introduction of coloring matter, will give the flow lines of the net. This form of apparatus can be used to solve many of the same problems as the electric analogy but for most of them the electric analogy is more convenient. For the problem of flow through earth dams, however, the handling of the potential relations where the water surface intersects the downstream face of the dam is difficult, and the parallel plate analogy is very effective. Model tests of earth dams, which will reproduce accurately to scale the capillary rise which takes place in the prototype, using actual soil, are difficult to construct and it is also difficult to secure homogeneous pervious material in the model. In the parallel plate analogy the capillary conditions can be adjusted to fit the prototype by varying the spacing between the plates, or if need be, by using a more viscous fluid than water. The apparatus which we have here represents the flow through an earth dam. The paths of flow through the dam are shown by the color bands. Note how the fluid rises from capillary attraction at the upper end and then flows down again toward the lower side of the dam. We often hear it said that water will not flow uphill. Of course we know it will if pumped up or drawn up in a siphon, but one rarely sees a case of it flowing uphill in an open channel starting from practically quiet water.

As previously stated, the distinction between an analogy and a model is hard to draw. This device has merely replaced the granular medium of the earth dam model with another form of channel which also will produce laminar flow and is therefore very close to being a model of an earth dam.

The parallel plate analogy has been used by Mr. Nelson of the Army Engineers here at Iowa City for working out problems connected with the filling ports of navigation locks.

## BIBLIOGRAPHY

(1) Shaw, Hele. Engineering, July 16, 1897; April 8, 1898, et seq.