FLUID FLOW THROUGH POROUS MEDIA

by

Morris Muskat
Gulf Research & Development Co.
Pittsburgh, Pa.

As a conduit for fluid flow, a column of porous material might well be considered as a perfect example of what a conducting channel should not be—for if the conduit is to represent mainly the means for confining the fluid stream to a restricted path, the addition of the porous medium will superimpose such an enormous flow resistance as to change even the order of magnitude of the head required to maintain a predetermined flux through the system.

Nevertheless, porous bodies constitute media for fluid transmission in so many phases of industrial activity that they merit thorough study. The science of fluid filtration is nothing more than a direct application of the principles of fluid flow through porous media. Certain methods of oil refining involve the same type of problem. The chemical industry is using towers and columns of packings of granular materials in an increasing range of applications. Textile scientists are finding a knowledge of the fluid-transmission properties of woven materials to be of major significance in evaluating their products. The fluid conductivity of ceramic materials has long been recognized as an important characteristic requiring close control. The failure of the civil engineer properly to provide for the leakage of water through or underneath a dam which he may construct is conclusive and damning evidence of his incompetence. The hydrologist who does not clearly understand the percolation of water through soils can properly evaluate neither water run-off conditions in watersheds, nor the requirements of irrigation projects. And the petroleum engineer who does not fully appreciate the role played by the porous rock in controlling the production of oil and gas from petroleum-bearing sands may well convert a key to economic and military victory into the sealed doom of irreversible defeat and human misery.

From the strictly scientific point of view, the problem of fluid flow through porous media seems to offer but little interest, since, at least in the region of viscous flow, the whole subject is covered in principle by the well-known Stokes-Navier equations of classical hydrodynamics. The fact that the solution of these equations for the multiply-connected maze of capillary channels and interstices is a hopelessly futile analytical task is a reflection of the limitations of the mathematician, rather than an indication of a flaw in the physical description of the problem.

From the practical standpoint, however, the physical implications of this mathematical complexity—namely, the intricate geometrical network of zero-velocity boundaries—constitute the unique feature of the flow through porous materials. Indeed, it is because of the tremendous internal boundary surface of a porous medium as compared to its volumetric capacity that the fluid resistance is so much greater than in open channels of equivalent cross section. An estimate of these relative fluid resistances is provided by the following considerations:

Poiseuille's formula for the average velocity of flow of a liquid of viscosity μ through a tube or pipe of cross section A under a pressure gradient dp/dx is:

$$U_0 = -\frac{A}{8\pi\mu}\frac{dp}{dx} \tag{1}$$

In the case of flow through a porous medium, Darcy's law states that the average macroscopic velocity is independent of the total cross section of the channel, if linear, and is given simply by

$$U = -\frac{k}{\mu}\frac{dp}{dx} \tag{2}$$

where k is the "permeability" of the medium, which is, in fact, best defined by Eq. (2) itself. Now physical and dimensional considerations indicate that the permeability k should be expressible as:[1]

$$k = \frac{c\varepsilon^3 d^2}{(1-\varepsilon)^2} \tag{3}$$

where ε is the porosity (fractional void space of the medium), d is

[1] This is the original Kozeny relation. It is not presented here as a quantitative expression for the value of k, but rather merely as a means for predicting its order of magnitude.

an average diameter of the constituent particles, and c is again a constant of proportionality. Experiments on unconsolidated granular materials substantiate the approximate validity of Eq. (3), and give for c a value of the order of 0.01 when the units in Eqs. (1) and (2) are expressed in the absolute c.g.s. system. Hence,

$$\frac{U_0}{U} = \frac{A(1-\varepsilon)^2}{8\pi c \varepsilon^3 d^2} \sim \frac{3A(1-\varepsilon)^2}{\overline{A}\varepsilon^3} \tag{4}$$

where $\overline{A}$ is the effective particle cross section. Thus for a porosity of 0.4, $U_0/U \sim 17\ A/\overline{A} = 17\ d_p^2/d^2$, where d_p is the diameter of the open pipe. Hence if a 1-inch-diameter pipe be filled with particles of 0.1-inch diameter to a porosity of 40 percent, the flow resistance will be increased by a factor of 1700. Clearly the contrast will become even greater if the pipe diameter used in the comparison be increased or the grain diameter of the equivalent porous medium be taken smaller. In fact, in many of the practical situations involving fluid flow through porous media the average particle diameters are even less than 0.01 inch.

A practical illustration of the tremendous difference in flow resistance between open pipes and equivalent columns of porous materials is provided by a situation frequently arising in the production of oil from loose or easily-disintegrating oil-bearing sands. If a well drilled into such formations is not properly completed or provided with strainers, sand may flow into the well bore. Because the well bore then becomes in effect a column of porous material, rather than an open conduit, in series with the producing formation, the flow from the latter may become seriously reduced. In particular, calculation shows that even if the permeability of the sand in the well bore is 10 times as great as that of the main sand body, the resultant flow resistance of a 25-foot-thick stratum will be increased by a factor of 10 merely by the sanding up of the open hole opposite the producing section.

By considering the porous aggregate macroscopically as a continuous medium for fluid transmission, a very important simplification results. The detailed microscopic structure with its enormous internal surface area can then be averaged or smoothed out, as it were, and replaced dynamically by a continuous and uniform medium with a fluid transmission coefficient, or permeability k, as defined in Eq. (2). The effective flow boundaries of the system

are then no longer the myriads of internal particle surfaces, but rather those defining the overall extension of the porous medium. Darcy's law, as given by Eq. (2), can then be generalized from its one-dimensional analogy with that for pipe flow—namely, Eq. (1) —and expressed in vector form as

$$\bar{v} = -\frac{k}{\mu} \nabla p \tag{5}$$

where for simplicity the medium is assumed to be isotropic, and the fluid is not subjected to body forces.

This equation represents the macroscopic equivalent of the Stokes-Navier equations, in which the porous body is considered as a continuum, and $\bar{v}$ represents the vector volume flux per unit area of the medium. The structure of the medium is characterized by the integrated transmission coefficient or permeability k. Although, in principle, it is to be considered as a function of the space coordinates, it is locally independent of the nature of the fluid as long as the latter is in a single phase. The only property of the latter which is of dynamical significance is segregated in the viscosity factor, μ.

The validity of these considerations has been verified by numerous direct and indirect experiments. However, two basic limitations are to be noted. The first relates to the concept of k as an invariable constant, for a particular medium, independent of the nature and state of the single fluid phase. Recent careful experiments have revealed that, in the case of gases flowing at low pressures through "tight" media, the observed value of k is higher than that for liquids. This is apparently due to molecular slippage at the solid particle boundaries, and, as is to be expected, decreases with increasing pressure and decreasing molecular mean-free-path. Indeed, on extrapolation of the experimental data to infinite pressures, the apparent permeability reduces to that observed for the liquid phase. However, from a practical point of view these effects are generally of little importance and do not need to be taken into account.

A more fundamental question pertaining to Eq. (5) concerns its range of validity. By analogy with well-established hydraulic experience, the proportionality between the velocity and pressure gradients—i.e., the so-called "viscous" region—breaks down at
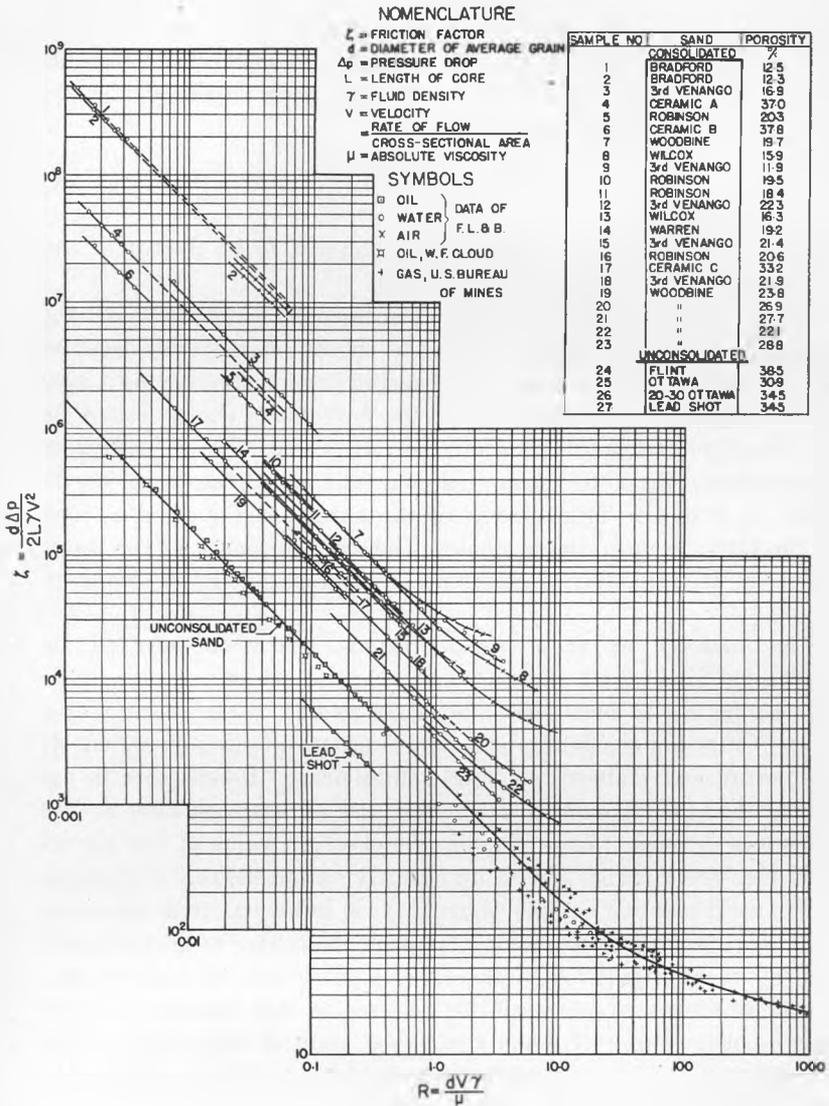
FIG. 1.—CHART OF FRICTION FACTOR ζ AS A FUNCTION OF THE REYNOLDS NUM-
BER R FOR THE FLOW OF HOMOGENEOUS FLUIDS THROUGH VARIOUS TYPES
OF CONSOLIDATED AND UNCONSOLIDATED POROUS MEDIA.

high velocities. When this occurs, the flow is said to be "turbulent," and the pressure or hydraulic gradient varies approximately as the square of the velocity. As in the case of the usual hydraulic systems the transition between the viscous and turbulent regions is determined by the Reynolds number, $dv\rho/\mu$, where ρ is the fluid density and d the average pore or grain size. However, whereas for pipes the transition occurs at Reynolds numbers of between 1000 and 2000, in the case of porous media the continual alternation from converging to diverging microscopic flow channels results in a transition beginning at Reynolds numbers of the order of 1. Moreover, the change to turbulent flow develops gradually, rather than abruptly, because of the broad range in pore sizes occurring in porous media. Data [1][1] illustrating this behavior for both consolidated and unconsolidated sands are plotted in Fig. 1. Fortunately, in most problems arising in the oil industry the fluid flow through oil-or gas-bearing strata is in the viscous region.

With the basic dynamical equation, such as Eq. (5), established, one need only add the equation of continuity and equation of state defining the thermodynamic character of the flow to obtain a complete system of hydrodynamic equations. For incompressible liquids, in isotropic and homogeneous media, one thus finds La-Place's equation in the pressure distribution. This simple result has permitted the solution of a great variety of specific problems of practical interest. In cases where the liquid compressibility becomes of importance and transient effects must be considered, the combination of Darcy's law and the equation of continuity leads to the Fourier heat-conduction equation with the liquid density as the dependent variable. Because of the extensive investigations of this equation already available, it is here, too, possible to carry through a large number of applications to problems of practical interest. On the other hand, when the fluid phase is gaseous, a parabolic non-linear equation in the gas density or pressure is obtained, and closed analytical solutions cannot readily be found. Nevertheless, for many cases of practical importance it is possible to derive approximate solutions of the governing differential equation of sufficient accuracy to be of value in making predictions and controlling industrial operations and development.

As a whole, therefore, the subject of viscous flow of single-

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[1] References appear at the end of the article.

phase fluids through porous media may be considered to be reasonably well worked out [2]. Of course, many special problems still remain to be solved, as, for example, certain types of free-surface flow systems, with or without the influence of capillary effects. These, however, involve mainly questions of analytical manipulation rather than the development of new physical principles.

While it is difficult to imagine, on the basis of the general program of this conference, why any one other than the author would be interested in the flow of multiphase or heterogeneous fluids through porous media, this subject will be briefly reviewed for the sake of completeness. From the point of view of the oil industry the subject of homogeneous fluid flow has represented mainly an unfortunately necessary preliminary to the problems of the flow of gas and liquid mixtures through porous media. The basic fact underlying the mechanics of oil production is that in the great majority of oil-producing reservoirs the oil is expelled from the oil-bearing rocks primarily by virtue of the evolution and expansion of the gas originally dissolved in the oil. One of the major goals of research on fluid flow through porous media by the oil industry has therefore been the understanding of the flow of gas, oil, and water mixtures through sands so as to aid in the efficiency of oil recovery. That this goal is of commercial significance will be readily appreciated when it is realized that the expulsion efficiency of the dissolved gases is so limited that even when all the gas originally dissolved in the oil is exhausted and dissipated, only 25 to 50% of the oil initially present in the rock will have been recovered. A situation such as this certainly merits serious study and attention.

At first thought, the development of a dynamical description of the microscopic chaos permeating a porous medium carrying a multiphase fluid as a gas and liquid would certainly seem to be a futile and hopeless ambition. At least an appeal to classical hydrodynamics would imply here an even higher order of *naïveté* than in the case of the flow of a single fluid phase through the complex of interstices of a porous rock. And indeed the lesson of the latter problem teaches that only the macroscopic approach can offer promise of obtaining practical results.

Even when accepting the macroscopic point of view in the treat-

ment of the general problem of fluid flow, the question immediately arises, in the consideration of multiphase fluids, as to how the state of such a fluid is to be defined and described. While the question as posed in this form may appear to be quite natural from an intuitive standpoint, it turns out that a more fruitful procedure is to consider the individual fluid phases of the mixture separately. That is, instead of combining the separate phases into a single complex pseudo-gas or pseudo-liquid with a composite or effective viscosity and equation of state, the identity of each phase is retained and its passage through the medium is followed in detail as if it were merely flowing in parallel with the other phases. The interaction and interplay between these parallel fluid streams is given expression by generalizing the original concept of the permeability as representing a fixed, integrated transmission coefficient determined solely by the microscopic grain structure of the porous medium. Rather, even the local microscopic permeability of the medium is now visualized as being dependent on the local volumetric distribution of the separate phases. This volumetric distribution is expressed in terms of the fractional saturations of the free pore space of the medium contributed by the individual phases. To each individual phase is then attributed a variable permeability which depends on the local phase saturation distribution.

These physical considerations may be crystallized analytically into a set of simultaneous representations of Darcy's law for the various phases, namely:

$$\overline{v}_i = - \frac{k_i}{\mu_i} \nabla p \qquad (6)$$

where $\overline{v}_i$ is the vector volume flux of the ith phase, of viscosity μ_i, and k_i is the permeability to the ith phase. The permeabilities k_i are functions of all the phase saturations s_i, which in turn are now to be considered as functions of position and as dependent variables, in addition to the common fluid pressure p. The application to Eqs. (6) of the equation of continuity for each individual fluid component then completes the construction of the hydrodynamic equations.

This procedure may be illustrated by the case of a mixture of gas, oil, and water flowing through a sand. For convenience it will

be assumed that the gas retains its molecular identity whether dissolved in the liquid phases or flowing as a free-gas phase. The three components will be indicated by the subscripts, g, o, and w. The expansion of the oil and water due to the gas dissolved in them will be denoted by β_o and β_w, and the solubility of the gas in the oil and water by S_o and S_w. The gas density will be represented by ρ, the porosity by ε, and the time variable by t. It may then readily be verified that the combination of Eqs. (6) with the three equations of continuity for the individual phases leads to:

$$
\left.
\begin{aligned}
\nabla \cdot &\left[\left\{ \frac{S_o k_o}{\beta_o \mu_o} + \frac{S_w k_w}{\beta_w \mu_w} + \frac{\rho k_g}{\mu_g} \right\} \nabla p \right] \\
&= \epsilon \frac{\partial}{\partial t} \left\{ \frac{S_o S_o}{\beta_o} + \frac{S_w S_o}{\beta_w} + \rho s_g \right\} \\
\nabla \cdot &\left(\frac{k_o}{\beta_o \mu_o} \nabla p \right) = \epsilon \frac{\partial}{\partial t} (s_o / \beta_o) \\
\nabla \cdot &\left(\frac{k_w}{\beta_w \mu_w} \nabla p \right) = \epsilon \frac{\partial}{\partial t} (s_w / \beta_w) \\
& s_g + s_o + s_w = 1
\end{aligned}
\right\}
\tag{7}
$$

It will be clear that the Eqs. (7) can be readily extended to include even more than two immiscible liquid phases. The analytical complexity will then, of course, become even greater than that already inherent in the three Eqs. (7). On the other hand, many cases of practical interest may permit simplifications. Thus, as the solubility of natural gases in water is much lower than in oil, the former may generally be neglected (S_w set $= 0$) when both oil and water are present in the system. Or, only one liquid phase may be present, and the equation for the other liquid phase may be dropped entirely.

Before Eqs. (7) or their equivalents can be considered to represent a complete formulation of the hydrodynamics of multiphase fluid flow through porous media, the form of the permeability coefficients k_i must be specified. This is, in fact, the central problem of the experimental study of such flow systems, from the point of view presented here. Because of the difficult and laborious technique involved in this type of experimentation, complete investigations have been carried through for only a small number of porous media. From a study of these it appears that while the

quantitative details will depend on the exact structure of the medium, the broad features will be essentially the same, at least for large groups of materials.

An illustration of the type of relationship obtained between the permeabilities k_i and the fluid saturation is that shown in Fig. 2 for unconsolidated sands carrying water and carbon dioxide [3]. Here the ordinates are the observed permeabilities for each of the two phases, expressed as percentages of the homogeneous or single-phase fluid permeability. The abscissas indicate the local liquid saturation as a percentage of the pore space. It will be observed that the permeability to the liquid phase k_l drops rapidly as the gas begins to displace it, and, in fact, falls to half of the single-phase value when the gas content of the sand is only 19 percent. Moreover, it becomes quite negligible when the liquid saturation falls to $30 - 40$ percent of the pore space. The gas permeability, however, remains zero until the free-gas saturation rises to a certain minimum value, the gas remaining locked, in effect, within the pores of the rock. This minimum saturation has been termed the ''equilibrium saturation for the gas'' and the corresponding liquid permeability as the ''equilibrium permeability.'' The latter is thus the maximum value which can be maintained in steady-state conditions of flow, and the equilibrium gas saturation represents the minimum value under which steady-state conditions can be maintained.

When the gas saturation exceeds its equilibrium value, the gas permeability becomes non-vanishing, and moreover grows rapidly with continued increase in gas saturation. Finally, when the liquid saturation has fallen to $10 - 20$ percent, the gas permeability becomes substantially equal to that of the liquid-free rock.

These empirical results can be readily interpreted in terms of the nature of the microscopic flow channels established and maintained by the individual phases. The differential behavior between the gas and liquid phases is largely the consequence of the fact that with respect to the internal surface of the porous medium the gas is a non-wetting phase and the liquid is the wetting phase. It is therefore to be expected that the gas bubbles will tend to occupy and block off the open parts of the pore space and thus markedly cut down the permeability to the liquid phase even when it is present in quite low concentrations. On the other hand, the liquid

phase will tend to fill out first the small and sharp-angled recesses
of the medium, which, even when free of liquid, do not contribute
appreciably to the flow of gas through the system. It is for this
reason that the gas permeability substantially reaches its homo-
geneous fluid value even though 10 — 20 percent of the total pore
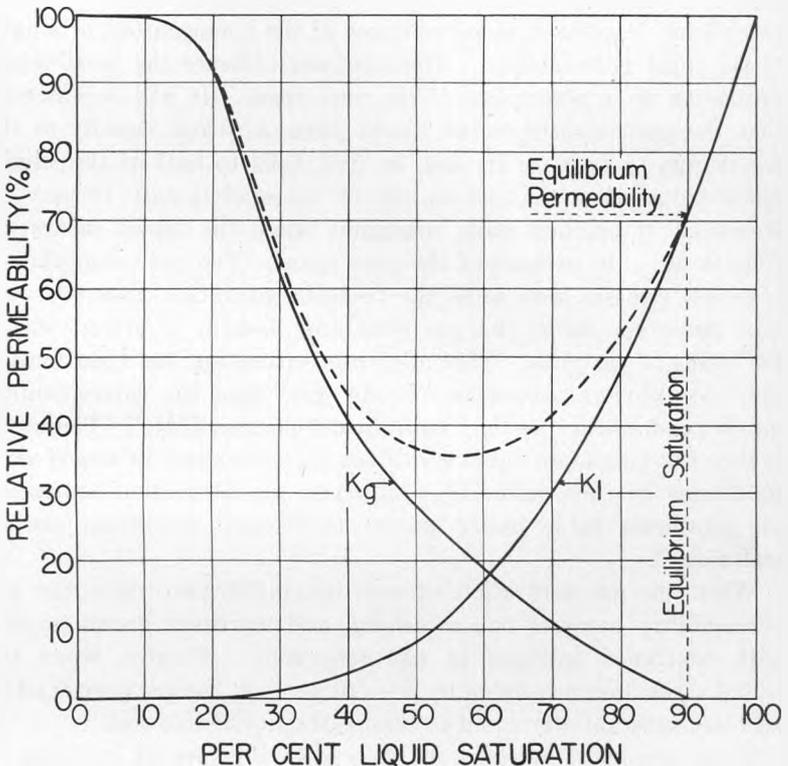space is occupied by liquid.

FIG. 2.—RELATIVE PERMEABILITIES FOR THE GAS AND LIQUID PHASES FLOW-
ING SIMULTANEOUSLY THROUGH AN UNCONSOLIDATED SAND.
(K_g = permeability to gas phase, K_1 = permeability to liquid
phase. Relative permeability is expressed in percentage of homo-
geneous fluid value. Dashed curve gives resultant
permeability for both phases.)

These considerations are substantiated by the fact that the gen-
eral features of the curves of Fig. 2 characterize any pair of wet-
ting and non-wetting phases, such as oil and water. If the latter

preferentially wets the sand, it will play the role of the single-liquid phase in the above discussion. And the role of the gas in the latter will be taken here by the oil. With this analogy the variations of the permeabilities for the oil and water will be similar to and can be interpreted in the same manner as those for gas and liquid.

It is to be noted that the sum of the permeabilities of a porous medium to the individual phases of the multiphase flow system is, in general, less than that of the same porous medium to a single phase or homogeneous fluid, except for low saturations of the wetting phase, which, as already mentioned, does not materially obstruct the flow of the non-wetting phase. This is shown directly by the dashed curve of Fig. 2, which is a plot of the sum of the two solid-curve ordinates. The compressibility of the gas is not a primary factor in determining the additional energy dissipation and increased fluid resistance in the multiphase systems, for, as previously indicated, the same type of permeability-saturation curves is obtained with oil and water as with gas and liquid. The major factors are the pore structure of the medium and the in-
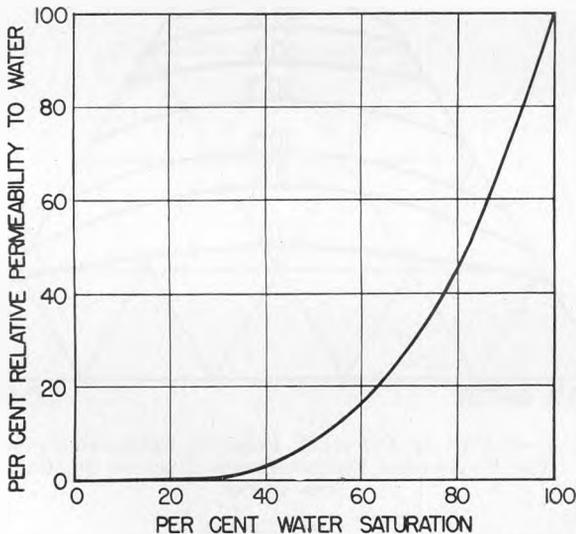

FIG. 3.—RELATIVE PERMEABILITY TO WATER AS A FUNC-
TION OF THE WATER-SATURATION IN A POROUS MEDIUM
CARRYING OIL, GAS, AND WATER.

herent immiscibility of the two phases, which, in turn, results in the complex and fluctuating network of fluid interfaces.

Unfortunately only one type of porous medium—a group of unconsolidated sands—has been studied experimentally with heterogeneous fluids consisting of three phases—namely, gas, oil, and water [4]. While the quantitative results of these experiments undoubtedly have no general validity, it seems very likely that the qualitative features will be typical of many porous media. Fig. 3 shows the observed variation of the permeability of the water phase, which was found to be essentially independent of the distribution of the other two, and hence is plotted directly against the water saturation. This independence of the water permeability of the composition of the other phases is undoubtedly due to the fact that it
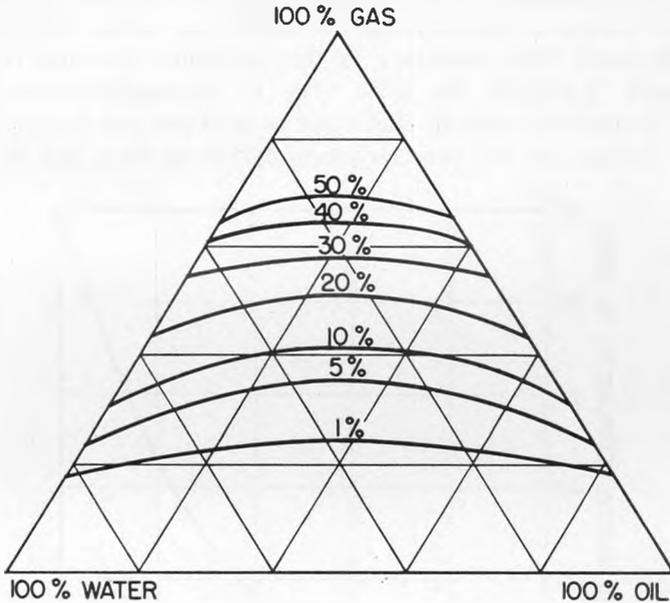

FIG. 4.—CURVES OF CONSTANT RELATIVE PERMEABILITY TO THE
GAS PHASE FOR A POROUS MEDIUM CARRYING OIL, GAS,
AND WATER.

is here, too, the wetting phase. In fact, these data for the three-phase systems agree closely with those shown in Fig. 2 for the water in two-phase systems.

In the case of the gas phase there is some variation of the gas permeability with the composition of the other two. This is indicated by the curvature of the constant-permeability curves in the triangular diagram of Fig. 4. It will be noted that the gas permeability decreases as either of the other two is added to the system. The reverse effect, together with an asymmetry, was observed for the permeability to the oil phase, as shown in Fig. 5.
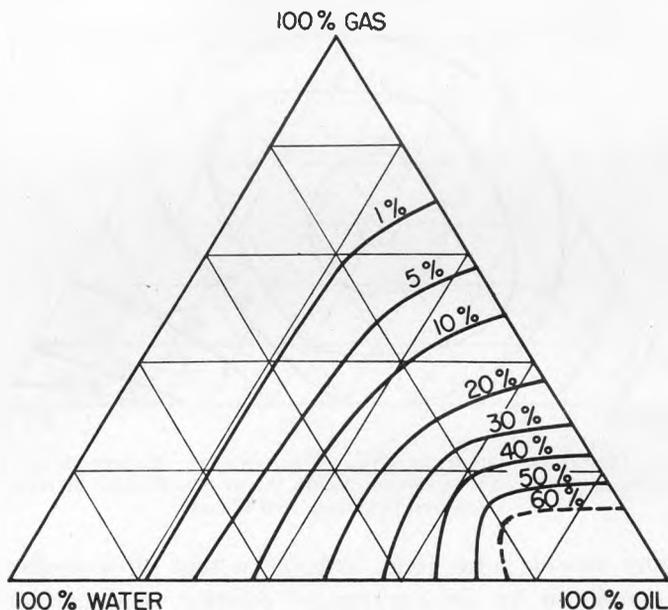

FIG. 5.—Curves of Constant Relative Permeability to the Oil Phase for a Porous Medium Carrying Oil, Gas, and Water.

With respect to the fluid distribution within the composite flow system, these studies show that if the free-gas saturation exceeds some 35 percent, the flow stream will be almost entirely gas. Appreciable water flow will generally develop when the water saturation becomes greater than about 50 percent, and for a clean oil flow to result the oil saturation must be at least 50 percent. To obtain an appreciable flow of each component simultaneously, the saturation distribution must lie within a narrow range, with gas saturations below 30 percent. In this range the composite flow
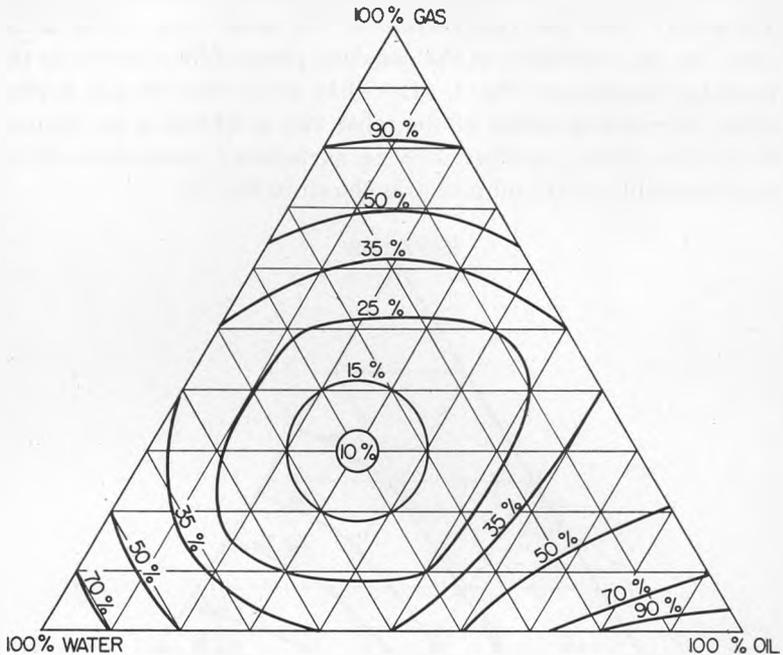
FIG. 6.—CURVES OF CONSTANT TOTAL PERMEABILITY EXPRESSED AS PER-
CENTAGES OF THE HOMOGENEOUS FLUID VALUE IN A POROUS MEDIUM
CARRYING OIL, GAS, AND WATER.

resistance may be seven times as great as that for a single phase
fluid, as shown by the contours of constant total permeability
plotted in Fig. 6. In the case of two-phase flow, the maximal flow
resistances were less than three times as great as for the single-
phase system, as indicated in Fig. 2.

These are the essential empirical results of investigations on
fluid flow through porous media. It is realized that the subject at
the best lies only on the fringe of what is generally considered to
be the field of fluid resistance. Moreover, the terminology that has
been used in this paper is admittedly unorthodox from the point
of view of the hydraulic engineer. Nevertheless, it is hoped that
this review will stimulate additional interest in a technical subject
which, regardless of its exact classification, is of vital importance
in many of our daily activities, both in war and peace.

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