

THE HYDRAULIC PROBLEM OF CAVITATION IN PUMPS

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The hydraulic problem of cavitation concerns the relation between cavitation, defined as local vaporization in a rapid stream of liquid, and the general flow conditions. The hydraulic problem therefore is distinct from the problem of cavitation erosion and from that of the physical nature of cavitation.

The hydraulic problem of cavitation in pumps and turbines has been the subject of a large number of papers and discussions (see Bibliography) and has thereby reached a state of development in which apparently different considerations can be recognized as logically consistent and interrelated parts of a single system of theoretical reasoning. It is the purpose of this paper to demonstrate the uniformity and completeness of this system, relative to the present incomplete knowledge about the underlying physical phenomena.

SIMILARITY CONSIDERATIONS

The flow conditions in centrifugal pumps and related machines are such that their prediction by theoretical means is very difficult if not impossible. That is, of course, also true for the theoretical prediction of the flow conditions leading to cavitation. This situation has led to the general adoption of similarity considerations as the most reliable form of attack on the hydraulic problem of cavitation as well as on most other hydraulic problems connected with this type of machinery.

Similarity considerations of this kind are based on the answer to the following simple question: If the performance characteristics of a given machine are known from tests for one particular speed of rotation, what conclusions can be drawn regarding the performance of the same machine at different speeds of rotation and regarding the performance of other machines which are geometrically similar to the first machine?

This question can be answered in a general way only if the fluid passages are the same or geometrically similar to those of the machine tested and if, as well, the flow pattern in these passages is at least approximately similar to that under test conditions.

In a system of fixed conduits the flow conditions remain similar if all fluid velocities are changed proportionally to each other. (The exactness of this definition depends on the exactness with which the fluid velocities are defined. In the following the turbulent velocity fluctuations will not be taken into account.) In a pump or turbine there are always two types of velocities to be considered: The fluid velocities and the peripheral velocities of the rotating parts. The latter always change proportionally to each other (assuming only one rigid rotor), but the fluid velocities can change proportionally to each other only if their changes are proportional to those of the peripheral velocities of the rotating parts (note the relation between relative and absolute fluid velocities). The kinematic condition of similarity therefore requires that the ratio between the peripheral velocities of the runner and the fluid velocities have the same value at all corresponding points of the machines compared. If D denotes any representative dimension (such as the diameter of the runner), Q the rate of flow, and N the revolutions per minute, the fluid velocities will be proportional to Q/D^2 while the peripheral velocities of the runner are proportional to ND . The kinematic condition of similarity of the flow therefore may be expressed in the form:

$$\frac{Q/D^2}{ND} = \frac{Q}{D^3N} = \text{constant} \quad (1)$$

For example, this condition is satisfied if the speed and capacity of one pump are changed proportionally.

The kinematic condition, however, is not sufficient to insure similarity of the flow pattern. It is also necessary that the forces acting on the fluid be similarly distributed, because otherwise the fluid would be forced off the similar paths within the fluid passages in spite of the fact that the kinematic conditions in themselves permit similarity of the flow. In order to obtain simple relations it is customary to consider only the mass force—i.e., to disregard in particular the viscosity forces. This practice has been well con-

firmed by a large number of tests. Under this assumption all corresponding head differences h in the pump must be proportional to the square of the fluid velocities $(Q/D^2)^2$ and to the square of the peripheral velocities of the runner $(ND)^2$, which leads to the following conditions of dynamic similarity:

$$\frac{hD^4}{Q^2} = \text{constant, and } \frac{h}{D^2N^2} = \text{constant} \quad (2)$$

Only if the condition of kinematic similarity (Eq. (1)) and one of the conditions of dynamic similarity (Eq. (2)) are satisfied is it possible to have similar flow conditions in geometrically similar fluid passages of pumps. (The two conditions of dynamic similarity (Eq. 2) are interdependent if the condition of kinematic similarity is also satisfied.)

The conditions of similarity embodied in Eqs. (1) and (2) also answer the original question as to the conclusions which can be drawn regarding the operating characteristics of a machine if the operating conditions of a similar machine under similar flow conditions are given. For instance, if h is interpreted as the total pump head H , it is seen that Eq. (2) expresses the well-known fact that the pump head varies as the square of the speed of rotation provided the capacity is changed according to the first power of the speed of rotation as stated by Eq. (1).

If one desires to know which operating conditions can be satisfied by similar pumps under similar flow conditions independently of the size of the machine, it is clear that the size expressed by the diameter D has to be eliminated from the conditions stated in Eqs. (1) and (2). This leads to the condition:

$$N_s = \frac{ND^3}{Q} \left(\frac{Q^2}{HD^4} \right)^{\frac{3}{4}} = \frac{N\sqrt{Q}}{H^{\frac{3}{4}}} = \text{constant} \quad (3)$$

In other words: As long as the specific speed (N_s) remains constant, the operating conditions (N , Q , and H) can be satisfied by changing only the size of the machine, leaving the geometric shape of the fluid passages and the flow conditions in these passages unaltered.

For cavitation considerations it is necessary to remember that the preceding conditions must be satisfied with respect to all pres-

sure or head differences in the pumps and not only with respect to the total pump head H . When cavitation takes place, the pressure somewhere inside the pump must have dropped to the vapor pressure of the fluid pumped. If the similarity of the flow is to be maintained under cavitation conditions, the extent of the cavitation zone relative to the fluid passages must remain unchanged. This is possible only if the differences between the surrounding pressures and the vapor pressure which prevails in the region of cavitation follow the same laws (Eqs. (2) and (3)) which hold for all other pressure or head differences in the pump under similar flow conditions.

The pressure or head having the most directly observable effect on cavitation is the total head at the pump inlet, including the atmospheric pressure (if the system is subjected to it) and including the velocity head at the point of measurement. The difference between this total inlet head and the vapor pressure of the fluid (pressure at the point or region of cavitation) shall be designated by H_{sv} .

In order to maintain similarity of flow with respect to cavitation, it is necessary that H_{sv} satisfy the conditions of similarity stated in Eqs. (2) and (3). The oldest method of meeting this requirement consists in making H_{sv} proportional to the pump head H , which is already known to satisfy Eqs. (2) and (3). This leads to the Thoma law of similarity for cavitation in pumps or turbines [1, 2, 3, 4]:¹

$$\sigma = \frac{H_{sv}}{H} = \text{constant} \quad (4)$$

It is, however, equally permissible to introduce H_{sv} directly into the Eqs. (2) and (3) in place of the general pressure difference h . Since cavitation is most likely to occur in the low-pressure regions of the machine—i.e., at the inlet of the pump impeller—it is natural to use the inlet or “eye” diameter D_E of the impeller as the representative dimension in Eqs. (1) and (2). By these substitutions one arrives at the following kinematic condition for similarity of inlet flow and cavitation [5]:

¹ References appear at the end of the article.

$$\frac{Q}{D_E^3 N} = \text{constant} \quad (1a)$$

and the corresponding dynamic conditions for similarity of inlet flow and cavitation:

$$\frac{H_{sv} D_E^4}{Q^2} = \text{constant}, \quad \text{and} \quad \frac{H_{sv}}{N^2 D_E^2} = \text{constant} \quad (2a)$$

Again, by eliminating the dimension D_E from the foregoing expressions, one arrives at the following requisite for the inlet operating conditions permitting similarity of inlet flow and cavitation in similar inlet passages of pumps [3]:

$$S = \frac{N \sqrt{Q}}{H_{sv}^{3/4}} = \text{constant} \quad (3a)$$

Because of its close relation to the specific speed, the cavitation parameter S has been called the suction specific speed [6].

The suction specific speed describes the inlet flow conditions in a manner similar to the Thoma sigma (see Eq. (4)) with the difference that a constant value of sigma will permit similar inlet flow and cavitation conditions only if at the same time the specific speed is held constant, while a constant value of S alone will satisfy this condition. From this the conclusion may be drawn that values of S which are significant for the cavitation behavior of pumps will not change very rapidly as a function of the specific speed of the pump, while corresponding values of sigma must be strongly influenced by changes in the specific speed, because they depend on the pump head H . This expectation was surpassed by the empirical fact that the critical S values of pumps are statistically independent of the specific speed of the pump so far as can be judged from available data. The suction specific speed may therefore be considered as a criterion which is practically independent of the specific speed of the pump.

The fact that the preceding group of similarity relations is complete can be proven by the dimensional analysis of the operating conditions N , Q , and H , together with the impeller inlet diameter D_E , which represents the linear dimensions of the inlet passages. If the pump head H is taken into account, one obtains by the same process corresponding expressions having the form of Eqs. (1), (2), and (3), and the head ratio sigma = H_{sv}/H , or other ratios

between any two head differences in the machine. Fundamentally different combinations, however, cannot be formed out of the variables considered.

CAVITATION PARAMETERS BASED ON EXPERIMENTAL DATA

While the foregoing similarity considerations have thus reached a certain finality, they cannot answer in this form the problem of the design of a new impeller if test results with a geometrically similar impeller are not available. This question is fundamentally different from that proposed at the beginning of this paper and therefore requires a different form of attack.

Since the flow conditions cannot be described readily by theoretical means, the most reliable methods are again based on similarity considerations; but this time they pertain to the flow conditions in closely defined regions of the machine, rather than to the machine or major portions of the machine (inlet passages) as a whole.

It has been stated before that Eqs. (1) and (1a) express the ratio between the fluid velocities and the peripheral velocities of the rotating parts. At that time it was not necessary to state which velocities were being considered. If the velocities are selected specifically, Eqs. (1) and (1a) describe the geometric form of the flow in a certain region of the machine. This is particularly easy with respect to the inlet flow conditions of the impeller. In the

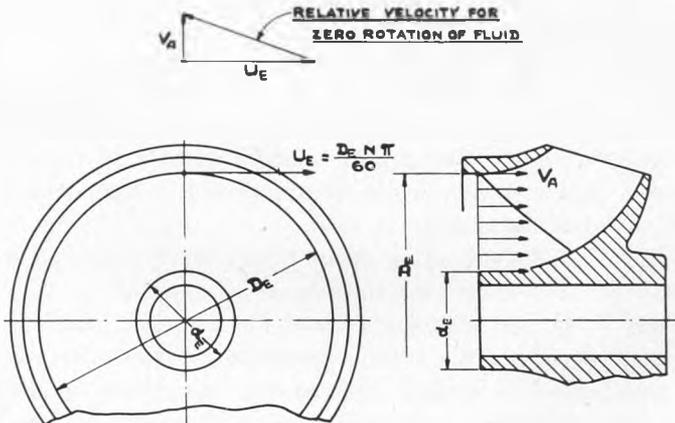


FIG. 1.

inlet or eye of the impeller (Fig. 1) the axial component of the average fluid velocity is:

$$V_A = \frac{4Q}{\pi D_E^2} \frac{1}{(1 - d_E^2/D_E^2)} \quad (5)$$

while the peripheral velocity is:

$$U_E = \frac{\pi N}{30} \frac{D_E}{2} \quad (6)$$

The ratio between the through-flow velocity V_A and the peripheral velocity U_E is therefore:

$$\frac{V_A}{U_E} = \frac{Q}{D_E^3 N} \frac{240}{\pi^2} \frac{1}{(1 - d_E^2/D_E^2)} \quad (7)$$

which differs from the left side of Eq. (1a) only by a constant containing the hub-diameter ratio d_E/D_E of the inlet cross section. For zero rotation of the fluid in the eye, the ratio V_A/U_E is the tangent of the angle between the relative fluid velocity and the peripheral direction at the eye diameter, considering average velocities only. This angle is of course closely related to the inlet vane angle at that diameter.

Using the velocities V_A and U_E in place of the corresponding expressions for Q , N , and D , the kinematic condition for similarity of inlet flow and cavitation is:

$$\frac{V_A}{U_E} = \text{constant} \quad (8)$$

Expressed in words, this condition requires that the velocity diagram formed by the average fluid velocity in the eye and the peripheral velocity of the eye remain similar. This statement includes also the case of prerotation of the fluid, since for geometrically similar inlet passages the rotational component will be proportional to the axial component V_A of the fluid velocity.

The dynamic conditions of similarity of inlet flow and cavitation may be expressed by means of V_A and U_E in the form:

$$\left. \begin{aligned} \frac{H_{sv}}{V_A^2/2g} &= \frac{H_{sv}D_E^4}{Q^2} \frac{(1 - d_E^2/D_E^2)^2\pi^2}{16} 2g = \text{constant} \\ \text{and} \\ \frac{H_{sv}}{U_E^2/2g} &= \frac{H_{sv}}{N^2D_E^2} \left(\frac{60}{\pi}\right)^2 2g = \text{constant} \end{aligned} \right\} \quad (9)$$

Expressed in words, this condition requires that the ratio of the total inlet head above the vapor pressure to the velocity head of the axial through-flow velocity in the eye and to the velocity head of the peripheral velocity of the eye remain constant.

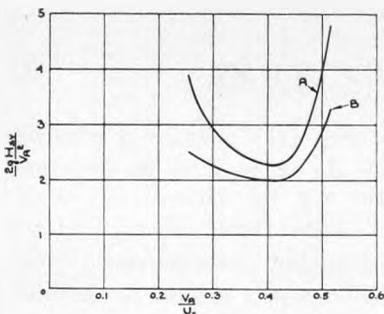


FIG. 2.

If the values of H_{sv} which are significant for the cavitation behavior of a given pump (beginning of cavitation or complete cavitation break-down) have been determined experimentally for different capacities and thereby for different values of V_A/U_E , it is possible to plot corresponding significant values of $2gH_{sv}/V_A^2$ and $2gH_{sv}/U_E^2$ as functions of the dimensionless

capacity V_A/U_E . Such diagrams have been introduced by Sedille [7] and by Gongwer [5], and are shown in Figs. (2) and (3).

These cavitation curves give a good picture of the cavitation behavior of a pump. Considering that a simple stationary nozzle would cavitate at practically constant values of $2gH_{sv}/V_A^2$ (this value being somewhat greater than unity due to a lack of uniformity in velocity distribution), the curvature of the lines in Fig. 2 may be interpreted as an indication of the varying influence of the impeller vanes on the cavitation behavior. According to this, the location of the minimum of these curves indicates the operating condition at which the vanes have the least disturbing effect on the flow through the eye. This interpretation appears to be confirmed by comparisons of the corresponding value of V_A/U_E with the inlet vane angles [5].

On the other hand the values of $2gH_{sv}/U_E^2$ are directly propor-

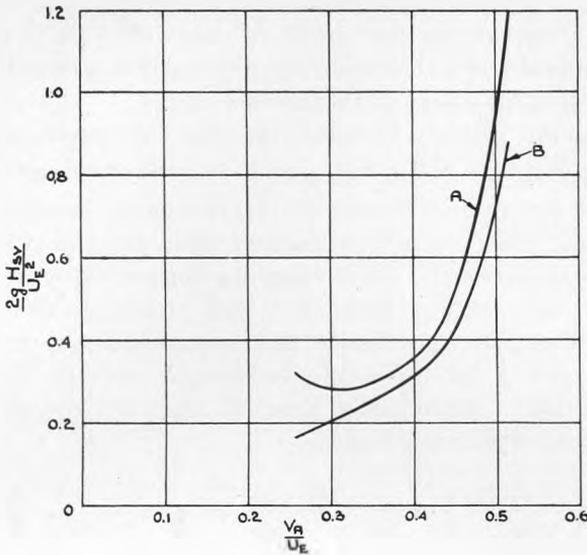


FIG. 3.

tional to the total inlet head at constant speed of rotation. The curves for $2gH_{sv}/U_E^2$ as a function of V_A/U_E are therefore dimensionless representations of the corresponding critical values of H_{sv} as a function of the capacity of the pump at constant speed (Fig. 3).

By plotting a number of curves such as shown in Figs. 2 and 3 from test results with pumps of different inlet passages, one obtains empirical values of $2gH_{sv}/V_A^2$ and $2gH_{sv}/U_E^2$ for different values of V_A/U_E . In order to derive from these data the desired information regarding the eye diameter and the inlet vane angle, it is necessary to relate these parameters to the given inlet operating conditions, H_{sv} , Q , and N . This relation can be expressed by means of the suction specific speed S which has the same classifying significance for the design of the inlet passages as the specific speed N_s with respect to the hydraulic design of the machine as a whole. The suction specific speed can be calculated from the inlet operating conditions before drawing any conclusions as to the design of the pump. If any one of the three parameters V_A/U_E , $2gH_{sv}/V_A^2$,

and $2gH_{sv}/U_E^2$ is empirically known as a function of S , one is able to derive from existing test curves as shown on Figs. 2 and 3 the other parameters and therefrom the previously-mentioned information regarding the design of the impeller inlet.

Plotting the relations between the four cavitation parameters V_A/U_E , $2gH_{sv}/V_A^2$, $2gH_{sv}/U_E^2$, and S on separate diagrams, however, does not make full use of the theoretical relationship between them. Since the new expressions (Eqs. (8) and (9)) for the conditions of similarity differ from the former expressions (Eqs. (1a) and (2a)) only by constant factors, it is clear that Eqs. (8) and (9) must have practically the same relation to the suction specific speed S as previously established between Eqs. (1a), (2a), and (3a). Introducing Eqs. (5) and (6) into the suction specific speed, it is found that:

$$\text{or } \left. \begin{aligned} S &= \left(\frac{V_A^2/2g}{H_{sv}} \right)^{3/4} \frac{U_E}{V_A} \frac{30 (2g)^{3/4}}{\sqrt{\pi}} \sqrt{1 - \frac{d_E^2}{D_E^2}} \\ S &= \left(\frac{U_E^2/2g}{H_{sv}} \right)^{3/4} \sqrt{\frac{V_A}{U_E}} \frac{30 (2g)^{3/4}}{\sqrt{\pi}} \sqrt{1 - \frac{d_E^2}{D_E^2}} \end{aligned} \right\} \quad (10)$$

It is thus clear that the suction specific speed S is definitely determined by the ratio of the axial fluid velocity in the eye to the peripheral velocity of the eye V_A/U_E , and by the ratio of one of the two velocity heads in the eye, $V_A^2/2g$ or $U_E^2/2g$, to the total inlet head above the vapor pressure, H_{sv} . This almost self-evident relation, considering the verbal definition for S given in connection with Eq. (3a), permits the construction of a useful nomographic chart—for instance, in the form given in Fig. 4. (The curved lines shown in this nomogram represent theoretical equations for cavitation limits to be discussed later.)

If any two of the four variables represented on this chart are calculated from test results corresponding to a significant state of cavitation, it is possible to represent these results by a point on the diagram. This point gives simultaneously the values of all four cavitation parameters, V_A/U_E , $2gH_{sv}/V_A^2$, $2gH_{sv}/U_E^2$, and S , for the test conditions in question. In this respect the new diagram has a decided advantage over the previously-discussed diagrams which give the relation between only two of the four variables considered.

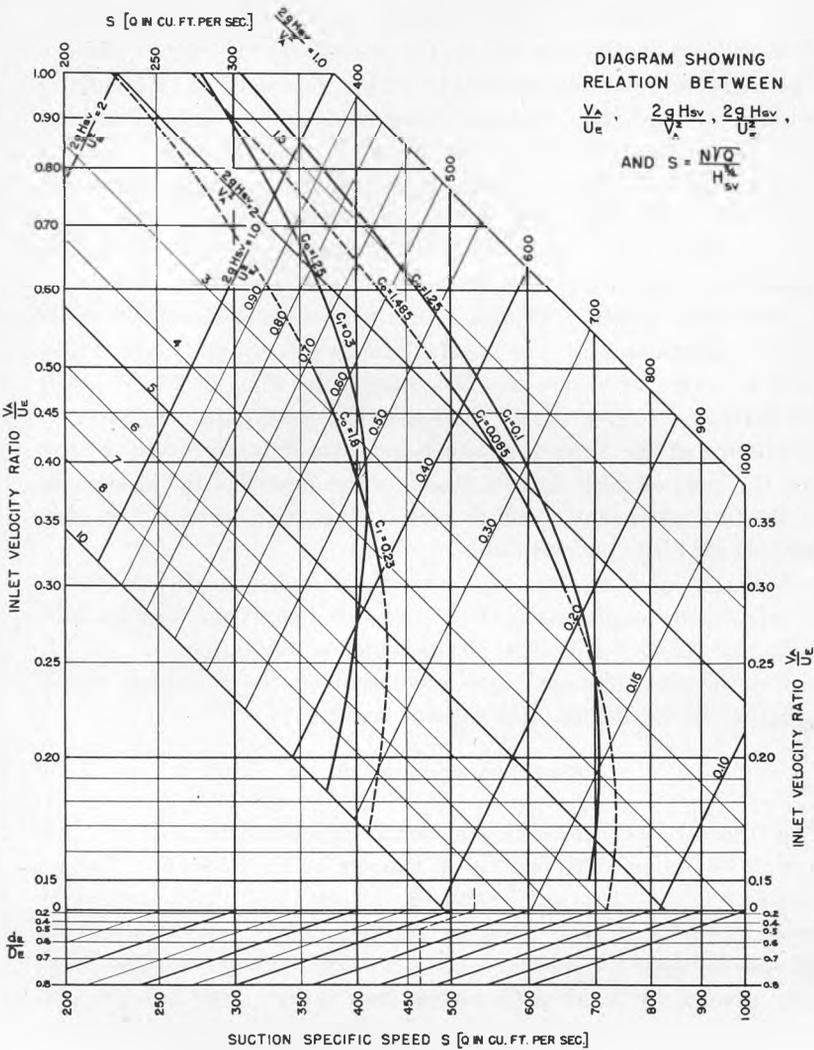


FIG. 4.

THEORY OF INLET FLOW AND CAVITATION CONDITIONS

In the first part of this paper the hydraulic problem of cavitation was treated on the basis of a general comparison between pumps with similar flow conditions in geometrically similar inlet passages. In the second part the general similarity relations, de-

rived in the first part, were applied specifically to the through-flow velocity in the eye and to the peripheral velocity of the eye. The ratios of velocities and heads, which were derived originally as criteria of similarity, became dimensionless—i.e., generalized expressions of test results applied to the flow conditions in the eye of the impeller. These test data were found to permit the determination of the eye diameter and to some extent the inlet vane shape of new impellers, as long as the values of the parameters V_A/U_E , $2gH_{sv}/V_A^2$, $2gH_{sv}/U_E^2$, and S remain within the range of existing experimental results. This is, of course, merely a repetition of the initial statement that test results can be generalized only within such a range for which the flow conditions remain similar—such similarity now referring to the local flow conditions. In order to go outside of the field of existing test data it is necessary to form (on the basis of these data) a theory which describes the mechanism of the flow sufficiently well to permit an extrapolation of the existing test data into untried fields.

A theory of inlet flow and cavitation conditions will consist of a quantitative explanation of the pressure drop from a pump inlet with zero velocity (forebay) to the region of cavitation.

The theories discussed here are based on the following simple equation for cavitation in pumps or turbines:

$$H_{sv} = C_E \frac{V_A^2}{2g} + \Delta h \quad (11)$$

The first term on the right side clearly describes the pressure drop due to the axial fluid velocity in the eye of the impeller. The coefficient C_E for pumps is generally larger than unity, expressing local increases of the absolute through-flow velocities over V_A . (Losses between the inlet and the eye of the impeller are also taken into account by making C_E larger than unity. For turbines this coefficient may be smaller than unity, because the draft-tube losses tend to increase the pressure in the throat of the runner.) In the following, the fluid will be assumed to have no absolute rotation before entering the impeller, because the available experimental information is not sufficient to cover this case (see the theoretical investigation by Spannhake [2]).

Since the first term expresses the pressure drop without the influence of the impeller vanes, the second term Δh is defined as ex-

pressing the pressure drop which is due to the presence of the vanes. The principal object of the theory is therefore the determination of the vane pressure drop Δh , because the coefficient C_E may be expected to stay within reasonable limits (say between 1.2 and 1.8).

One of the oldest attacks on this problem [1] consisted simply of making Δh dimensionless through division by the total head H of the machine. Eq. (11) thereby assumes the form:

$$\frac{H_{sv}}{H} = \sigma = C_E \frac{V_A^2}{2gH} + \frac{\Delta h}{H} \quad (12)$$

The ratio $\Delta h/H$ has been given the symbol K_c . This parameter is quite analogous to σ and has to be determined from experimental data. Eq. (12) therefore constitutes a theory only to the extent that it separates the pressure drop due to V_A from the remainder of the total pressure drop, but no attempt is made to explain this remaining pressure drop Δh .

Two different forms of reasoning have been used to determine Δh theoretically:

In the first place, Δh may be expected to vary in proportion to the square of the maximum velocity of the fluid relative to the inlet vane tips. Assuming no absolute prerotation, this velocity is determined by the relation $v^2_{relative} = V_A^2 + U_E^2$, so that

$$\Delta h = C_1 \frac{v^2_{rel}}{2g} = C_1 \frac{V_A^2 + U_E^2}{2g}$$

By substitution into Eq. (1) one obtains the relation

$$H_{sv} = C_E \frac{V_A^2}{2g} + C_1 \frac{v^2_{rel}}{2g} \quad (13)$$

or

$$H_{sv} = C_E \frac{V_A^2}{2g} + C_1 \frac{V_A^2}{2g} + C_1 \frac{U_E^2}{2g}$$

resulting finally in an expression of the form

$$H_{sv} = C_0 \frac{V_A^2}{2g} + C_1 \frac{U_E^2}{2g} \quad (14)$$

wherein

$$C_0 = C_E + C_1$$

It is desirable to bring this result into a dimensionless form, permitting its comparison with empirical values of the cavitation parameters derived by the preceding similarity considerations. Dividing by $U_E^2/2g$ one obtains

$$\frac{2gH_{sv}}{U_E^2} = C_0 \left(\frac{V_A}{U_E} \right)^2 + C_1 \quad (15)$$

which corresponds to a relation proposed by Sedille [7], while division by $V_A^2/2g$ yields

$$\frac{2gH_{sv}}{V_A^2} = C_0 + C_1 \left(\frac{U_E}{V_A} \right)^2 \quad (16)$$

Eq. (15) is represented by a series of parabolas on the diagram in Fig. 3 [7] while Eq. (16) gives a corresponding series of general hyperbolas on the diagram in Fig. 2. Both series of curves become identical when plotted on the general diagram in Fig. 4. On the basis of a fairly large number of accurate cavitation tests made at the California Institute of Technology with Francis-type centrifugal pumps, Gongwer [5] found $C_E = 1.4$ and $C_1 = 0.085$ ($C_0 = 1.485$) for the lowest points of the breakdown [2] curves of $2gH_{sv}/V_A^2$ as shown in Fig. 2. From the same data Gongwer also derived a curve for safe operating limits, which satisfies Eq. (14) with the coefficients $C_0 = 1.8$ and $C_1 = 0.23$. These cavitation-limit curves as proposed by Gongwer are plotted on the general diagram of Fig. 4, with the experimentally-determined portion shown in solid lines and the theoretical extrapolations in broken lines.

For pumps and turbines of high specific speeds (propeller type), Δh was found to vary appreciably as a function of the vane area or the ratio of overlapping (see Fig. 5). This has led to the second method for calculating Δh —namely, on the basis of the average pressure difference between the two sides of the vanes which is necessary to support the pressure rise from the inlet to the discharge side of the impeller [2, 8]. For reasons of simplicity this rise in static pressure shall be assumed to be equal to the total pump head H . (Actually this pressure rise is equal to H plus the losses in the pump minus the increase in velocity head through the runner. The two corrections partly cancel each other). Under this

assumption the vane-pressure differences and in particular the pressure reductions at the lower side of the vanes Δh_v are proportional to H divided by the ratio of overlapping $j = a/b$, because the pressure difference H is acting axially on an area which is pro-

COAXIAL VANE SECTIONS OF DIAMETER d , DEVELOPED

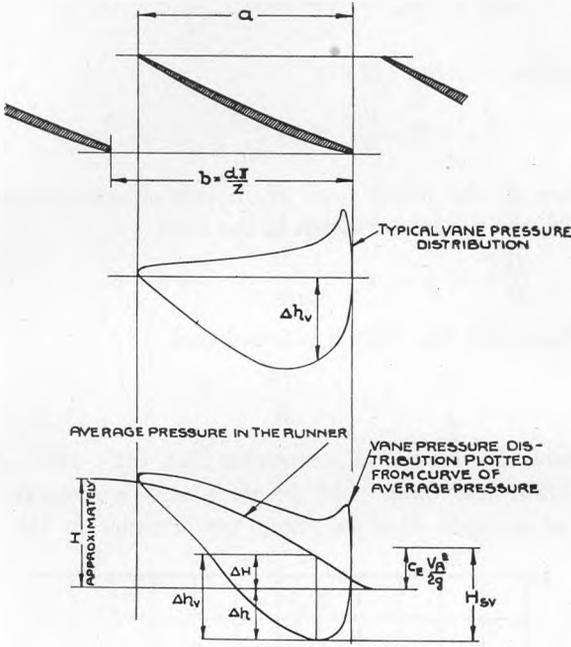


FIG. 5.

portional to b (Fig. 5) while the vane-pressure differences counter-act only over an area which is proportional to a . Hence

$$\Delta h_v = \frac{C_v H}{j} \tag{17}$$

where C_v is a dimensionless coefficient to be determined from test results.

At the point of the absolute pressure minimum on the vane surface, however, the average pressure in the runner has already risen above the pressure in front of the vane by an unknown part

ΔH of the total pressure rise H (See Fig. 5). This rise in average of the machine. Eq. (11) thereby assumes the form:

$$\Delta H = C_H H \quad (18)$$

where the coefficient C_H is again to be derived from test data.

Referring to Fig. 5 it is seen that

$$\Delta h = \Delta h_v - \Delta H = H \left(\frac{C_v}{j} - C_H \right) \quad (19)$$

By substitution into Eq. (11)

$$H_{sv} = C_E \frac{V_A^2}{2g} + H \left(\frac{C_v}{j} - C_H \right) \quad (20)$$

The presence of the pump head H in this equation suggests the use of the Thoma parameter sigma in the form

$$\frac{H_{sv}}{H} = \sigma = C_E \frac{V_A^2}{2gH} + \frac{C_v}{j} - C_H \quad (21)$$

By comparison with Eq. (12) it is found that

$$\frac{C_v}{j} - C_H = \frac{\Delta h}{H} = K_c \quad (22)$$

Fig. 6 shows the results of comparing Eqs. (12), (21), and (22) with cavitation test data. The points plotted correspond to the beginning of a rapid drop in pump performance at the capacity

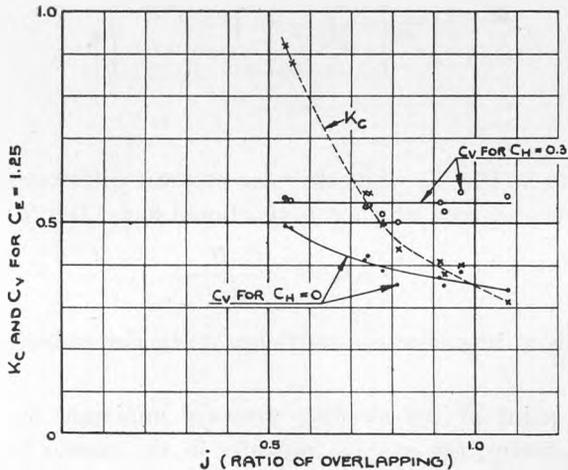


FIG. 6.

which was believed to give the most favorable inlet conditions. Points marking the first hydraulic indications of cavitation (safe-limit points) were also determined but did not form a sufficiently consistent curve to permit the desired evaluation.

Eqs. (12) and (21) were evaluated using for C_E a value of 1.25. In order to check whether the average pressure rise in the runner ΔH should be taken into account, results for C_v were plotted also for $C_H = 0$. It was found, however, that $C_H = 0.3$ gave the most consistent results, indicating that according to this evaluation the absolute pressure minimum in the runner occurs in a region where the average pressure has already risen by nearly one-third of its total increase. This conclusion, however, must be considered as very preliminary since considerably more test results are required to support a theoretical equation with three independent empirical constants.

There exists still another useful and instructive way of dealing with the basic Eq. (20). If U is the peripheral velocity of the runner at its high-pressure side, the total head H may be expressed in the form

$$H = \frac{U^2}{2g} \frac{1}{\Phi^2} \quad (23)$$

where Φ is defined as usual as the ratio of the peripheral velocity of the runner to $2gH$. Substituting Eq. (23) into Eq. (20) one obtains

$$H_{sv} = C_E \frac{VA^2}{2g} + \frac{1}{\Phi^2} \left(\frac{C_v}{j} - C_H \right) \frac{U^2}{2g} \quad (24)$$

or, with reference to Eq. (22),

$$H_{sv} = C_E \frac{VA^2}{2g} + \frac{K_c}{\Phi^2} \frac{U^2}{2g} \quad (24a)$$

For axial-flow machines, to which the foregoing considerations mostly apply, one can substitute U_E for U , expressing merely the fact that the peripheral velocities at the inlet and discharge sides of such runners are the same. By this substitution Eq. (24a) becomes

$$H_{sv} = C_E \frac{VA^2}{2g} + \frac{K_c}{\Phi^2} \frac{U_E^2}{2g} \quad (25)$$

This equation, however, has exactly the same form as the previously derived Eq. (14) and could be made identical with the latter if one equated the corresponding coefficients

$$C_0 = C_E, \text{ and } C_1 = \frac{K_c}{\Phi^2} = \frac{(C_v/j) - C_H}{\Phi^2} \quad (26)$$

Eq. (14) is therefore somewhat more general than originally assumed, covering for axial-flow runners also the cases where the average vane-pressure difference controls the pressure drop at the vanes. In this case its coefficients have to be determined according to Eq. (26), where the change in C_0 is of course practically insignificant while C_1 becomes a function of Φ and of the ratio of overlapping. Considering the results shown in Fig. 6 and the usual characteristics of axial-flow runners with respect to Φ , one finds that according to Eq. (26) C_1 may be expected to vary from 0.4 to well below 0.1. On the other hand, values of C_1 below 0.1 are actually not to be expected, because the average vane-pressure differences are in such cases (large Φ , and low K_c values) relatively so small that the relative velocity, not the average vane-pressure difference, becomes the controlling factor for the pressure drop at the vanes. In other words the determination of Δh based on the average vane-pressure difference goes over into its determination by the relative velocity. The relation between Eqs. (14) and (25) as expressed by Eq. (26) and the given experimental data seem to indicate that this transition takes place without a break.

It is not surprising that the two forms of calculating Δh have led finally to the same fundamental relation of Eqs. (14) and (25), because the pump head on which the second term of Eq. (25) is based may be calculated from the relative velocity and the lift coefficient of the vane sections [9]. Eq. (14) may therefore be considered as expressing the results of both hydraulic theories of cavitation discussed in this paper. Reasonable results of this equation as derived from Eq. (26) and the data given in Fig. 6 are plotted on the diagram in Fig. 4, using $C_0 = 1.25$, $C_1 = 0.1$ and 0.3. It should be repeated that these results as well as those represented by the curve $C_0 = 1.485$, $C_1 = 0.085$ (Gongwer), apply only to the cavitation breakdown at approximately the most favorable inlet flow conditions.

The restriction just mentioned is significant regarding the

present state of the hydraulic theory of cavitation. In the first place, the cavitation behavior away from the most favorable inlet flow conditions (see Fig. 2) has not yet been described theoretically in a satisfactory manner. Gongwer's interesting attempt [5] based on a theoretical investigation by Betz and Petersohn [10] cannot yet be considered entirely conclusive. The application of the method by Betz and Petersohn to axial-flow runners appears to be even more promising but may be analytically somewhat complicated. No serious investigation has been published as yet concerning the cavitation conditions at very low capacities for which the inflow is known to depart radically from its normal characteristics.

Secondly, no reliable method is available as yet for calculating safe cavitation limits outside of the range of available test data. The scatter of corresponding test points is usually far greater than that of points pertaining to the cavitation break-down (i.e., rapid drop of the hydraulic characteristics). In particular, it is the difficult problem of finding experimentally usable definitions for safe limits of cavitation under various field conditions which will have to be solved before decided theoretical progress in this direction can be expected. This problem, however, clearly exceeds the limits which were initially adopted for this paper.

The relative completeness and effectiveness of simple similarity considerations thus appear to be in marked contrast to the limited results obtained so far by theoretical predictions of cavitation limits outside of the range of existing test data. This contrast, nevertheless, is in agreement with the present state of fluid mechanics of centrifugal pumps and related machinery, and cannot be removed by vague assumptions but only by persistent experimental and theoretical research.

On the other hand it may be well to consider whether the same similarity considerations, which are serving so effectively with respect to the hydraulic problem of cavitation in pumps and turbines, may not be applied equally well to other problems. The answer to this question is definitely affirmative. Relations such as expressed by the diagram in Fig. 4 can be applied to all axial-flow machines with respect not only to cavitation but also to other operating characteristics. To do so it is only necessary to substitute for H_{sv} the total head H (changing S into N_s) or any other

head difference which is significant for the operation of the machine. This generalization seems to be promising not only with regard to axial-flow pumps or turbines but also when applied to marine propellers.

In closing, it is necessary to stress the fact that similarity considerations with regard to cavitation are based on certain assumptions as to the physical nature of this phenomenon which are confirmed only by the field application of the conclusions derived. This confirmation, however, may not always be sufficient. It has already been mentioned that effects of changes in the Reynolds number have not been taken into account. Another fact which the writer believes to be of importance concerns the influence of the time during which a particle of the liquid remains in the low-pressure zone. The preceding similarity considerations are valid only if the time required for the formation and growth of the vapor bubbles is negligibly small with respect to all hydraulic phenomena involved.

While experience has not as yet given any indications regarding a scale or time effect on cavitation, even a remote possibility of such an effect is sufficiently serious to warrant a careful investigation.

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