PRESSURE DISTRIBUTION AND CAVITATION ON SUBMERGED BOUNDARIES

John S. McNown

University of Iowa, Iowa City, Iowa

One of the basic problems of fluid flow is the determination of the pressure distribution around a given boundary, as knowledge of this one variable makes possible the solution of many problems which may be encountered. Forces on sea and air craft, incidence of cavitation, velocity distribution, and expenditure of energy at a transition, to list only a few, are all related to this fundamental determination. For many of these investigations it may be desirable to evaluate pressure-integral terms such as lift or righting moment directly by dynamometry, as pointed out by Dr. J. W. Daily [1], but for others, such as those related to cavitation, a more complete picture is obtained from the continuous pressure variation. It is interesting to note that several of the papers presented at the Third Hydraulics Conference [2], [3] have dwelt on this topic. Design of a conduit inlet and analysis of dynamic stability are appropriate illustrations of the uses of pressure distributions in providing fundamental design data for a wide range of practical applications. It should be worthwhile, therefore, to give consideration to the available procedures and their application to practical problems. Although the methods are familiar ones (since analysis, adaptation of previous experimental work, and specific model studies must serve here as in many other problems), their specific application to this problem will be treated in some detail with particular emphasis on procedures which are not in general use. The problem is certainly a broad one, and it is the general aspects that will be given primary consideration in the following discussion. However, specific illustrations will be drawn from a study of the pressure distribution around head forms conducted by the Iowa Institute of Hydraulic Research.

Analytical methods will be discussed first, because they should

receive primary consideration in any problem and are nevertheless often slighted. If a problem is amenable to analytical treatment, an analysis may yield a complete and general solution, and will, in any event, provide a valuable guide for experimentation and systematization of results. A few years ago engineers were reluctant to use the results of hydrodynamical studies, both because of the mathematical complexities, and because the results seldom applied to the problems of that time. However, velocities of ships and heights of dams have been greatly increased, and consequently higher efficiencies, lower drags, and negligible cavitation are now required. As a result, boundaries must be well streamlined and are therefore more amenable to theoretical treatment than those encountered formerly. The understanding which the present-day engineer has of analytical methods may be greatly increased, not only by improving his basic training, but also through translation of much that is highly mathematical into more easily understandable terms.

An illustration of this gap between theory and application was encountered by the writer in making a comparison between experimentally determined pressure distributions around a series of semi-ellipsoidal head forms and the theoretical results for irrotational flow around symmetrical ellipsoids. Reference to classical treatments of this problem [4], [5] yielded expressions in elliptical coordinates necessitating a complicated transformation before a practical result could be obtained. The variation of velocity, as eventually simplified for the case of ovary ellipsoids (longitudinal axis greater than the maximum diameter) was expressible in familiar terms:

$$v = v_0 \frac{e^2}{1 - A} \sin \theta \qquad (1)$$

The quantity v_0 is the approach velocity, e is the eccentricity of the ellipse ($e = \sqrt{a^2 - b^2}/a$), A is a function of e alone and hence is a constant for a given boundary form,

$$A = \frac{1 - e^2}{2e} \ln \frac{1 + e}{1 - e}$$

and θ is the angle between the direction of flow and a line normal to the profile. While these expressions are still somewhat involved, it is because the problem is a difficult one; the mathematician re-

sorted to elliptical coordinates because he was forced to do so by the complexity of the problem. However, the final result is general and has been reduced to readily usable terms. The evaluation of the quantity $e^2/(1 - A)$ is a relatively simple matter, and once tabulated or plotted would yield a simple and direct solution for the velocity. Finally the pressure-head variation may be expressed in the form

$$\frac{h - h_0}{v_0^2/2g} = 1 - \left(\frac{e^2 \sin \theta}{1 - A} \right)^2 \tag{2}$$

The results expressed in Eq. (2) were compared with the measured values, both to determine the zone of usefulness of the theory and to assist in interpolation and systematization of the family of experimental curves. The dimensionless pressure term is plotted against r/r_0 in Fig. 1 so that the curves have the same

FIG. 1—COMPARISON OF EXPERIMEN-
TAL AND THEORETICAL PRESSURE-
DISTRIBUTION CURVES FOR THE EL-
LIPSOIDAL SERIES (ASSUMING COM-
PLETE ELLIPSOIDS FOR THEORETICAL
DETERMINATIONS).

spread along the abscissa. The agreement is seen to be excellent for the longer ellipsoids, becoming progressively poorer for the shorter forms as would be expected. The lack of agreement near the point of tangency for the longer forms is primarily due to the difference in boundary geometry between the assumed complete ellipsoidal form and the actual conditions of a half-ellipsoid mounted on a cylindrical shaft. The more general disagreement for the shorter forms, on the other hand, is the result of viscous effects in producing separation and rotational flow. It may be concluded that the analytical results would be very useful for ellipsoids with axis ratios of 2:1 or greater.
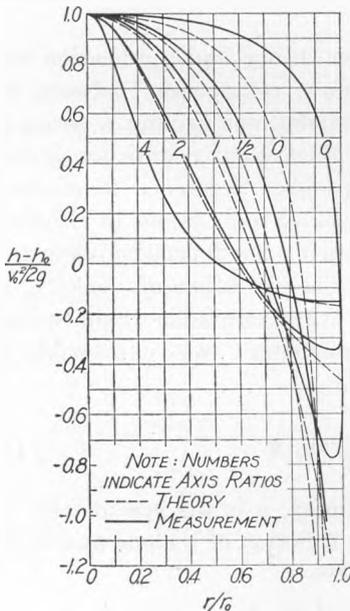
Other problems which have been solved by similar means include flow past a number of mathematically defined boundaries in either

two or three dimensions, flow in a vortex field, and the determination of contraction coefficients for a wide variety of efflux problems [6]. It is reasonable to conclude, therefore, that engineers should not overlook the possibilities of hydrodynamical analysis in the event that the effects of viscosity are relatively unimportant.

A distinctly different analytical method, introduced by Professor von Karman [7], is approximate but adaptable to a wide variety of boundary forms. If a number of mathematical sources and sinks are combined with a field of uniform velocity, it is possible to adjust their intensities or strengths so that one stream line of the resulting flow approximates a pre-determined boundary form. Once these strengths are determined, a stream function can be written from which the velocity and pressure along the boundary can be computed. An elementary example of this approach results from the combination of uniform flow with a point source. The stream line which passes through the stagnation point a short distance upstream from the source describes what is known as a half body, a natural flow boundary which can be modified at will by the introduction of additional sources. For every condition which is imposed upon the boundary, a source of unknown strength must be added. The various strengths may then be determined by solution of the corresponding number of simultaneous linear equations. In addition to point sources and uniform flow, both line sources and doublets may be used, as expressions have been derived for the stream function and the velocity components for each of these.

This method is applicable to many problems of two- and three-dimensional flow around faired boundaries. Solutions with six degrees of freedom were prepared at the Iowa Institute for the 4:1 and 2:1 ellipsoidal forms, each with a cylindrical afterbody. The results, which are presented in Fig. 2, illustrate excellent agreement between theory and experiment for these forms, and hence demonstrate that viscous effects are indeed of minor importance. The work involved, while laborious, is slight compared with the construction and testing of a model, as one familiar with routine computational procedure should be able to prepare a solution of the type presented in Fig. 2 in a day or two. Furthermore, the analytical background can be reduced to terms and concepts which are compatible with the mathematical training given all engineers.

One further application of this technique is of practical sig-

nificance to civil engineers and is therefore deserving of further consideration. As there is little real difference between flow with internal and external boundaries, it should be possible to adapt the technique described in the preceding chapters to a study of
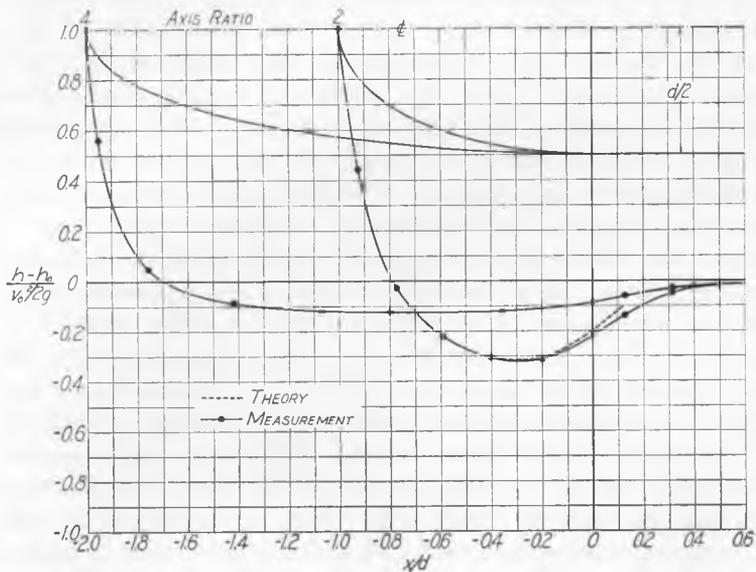

FIG. 2—COMPARISON OF EXPERIMENTAL AND THEORETICAL PRESSURE-DISTRIBU-TION CURVES FOR THE 4:1 AND 2:1 ELLIPSOIDS WITH CYLINDRICAL AFTERBODY.

flow through a well-rounded conduit inlet. In addition to the methods of the preceding case, vortex pairs may be used to produce the desired boundary stream lines. While this procedure may be limited to two-dimensional forms, it is potentially capable of providing worthwhile design information.

The writer well realizes that many engineers would prefer the building of a model to utilization of the analytical methods which have been outlined in the foregoing. Their reasons would probably be (1) lack of familiarity with the procedure, (2) absence of satisfactory treatments in the literature, (3) antipathy to all mathematical procedures, and (4) distrust of results obtained in this manner. As all of these arguments are directly traceable to the extreme scarcity of analytical treatments of this type in the engineering literature, one of the primary purposes of this discussion

is to stimulate interest so that the usefulness of the method can be demonstrated through the presentation of other examples, and so that the boundary of applicability can be more clearly defined. It can not be emphasized too strongly that problems within the range of the analytical assumptions can be solved far more economically by analytical means than by any other, and that such a study may be combined with experimentation to good advantage.

It is well known that a large proportion of problems encountered in engineering design do not satisfy the assumptions essential to theoretical analysis and that experimentation is required to obtain practical results. Analytical procedures are not a panacea for all problems, but represent an effective approach to one type. However, even with the experimentation, it is important that certain analytical tools be used, the principal ones being the methods of dimensional analysis and dimensionless representation. Only in this way can the maximum benefit be derived from a series of experiments, both in the application to a specific problem and in the subsequent adaptation to other problems of similar nature. As data of the type presented in Fig. 1 and in other papers in this volume [2], [3] are collected and published, the engineering profession is accumulating a valuable backlog of extremely useful design information, so that the amount of experimentation required in a given instance may often be reduced materially. On the other hand, fragmentary data presented in an unsystematic fashion will yield little of use for subsequent investigations.

For present considerations, the distribution of pressure may be expressed as a function of the Reynolds number, the Froude number, the cavitation number, and the boundary geometry. In functional form:

$$\frac{p - p_0}{\varrho v_0^2/2} = \phi \left(\mathbf{R,\ F,\ K,}\ \frac{b}{a}\ ,\ \frac{c}{a},\ \text{................} \right) \quad (3)$$

Variation of the Reynolds number $\mathbf{R}$ changes the flow pattern through viscous influence upon the pattern of separation. For a consideration of completely submerged flow the Froude number $\mathbf{F}$ may be omitted. The cavitation number $\mathbf{K}$, which is less widely known, has received acceptance in many cavitation studies and will be discussed further herein. Linear ratios of the type b/a and c/a

serve to define the type of boundary and the variations which are being studied.

Like the Reynolds number, the cavitation number may take different forms for different types of flow. For flow past a submerged boundary

$$\mathbf{K} = \frac{p_0 - p_v}{\varrho v_0^2 / 2}$$

and **K** is therefore a measure of the proximity of the centerline pressure to the vapor pressure in units of the dynamic pressure, the zero subscripts denoting reference conditions in a region undisturbed by the boundary under study. If the overall pressure is reduced by lowering p_0, or if the local minimum pressures are reduced by increasing v_0, the tendency for cavitation to occur is increased. It follows that a reduction in **K** indicates a net increase in the tendency toward cavitation.
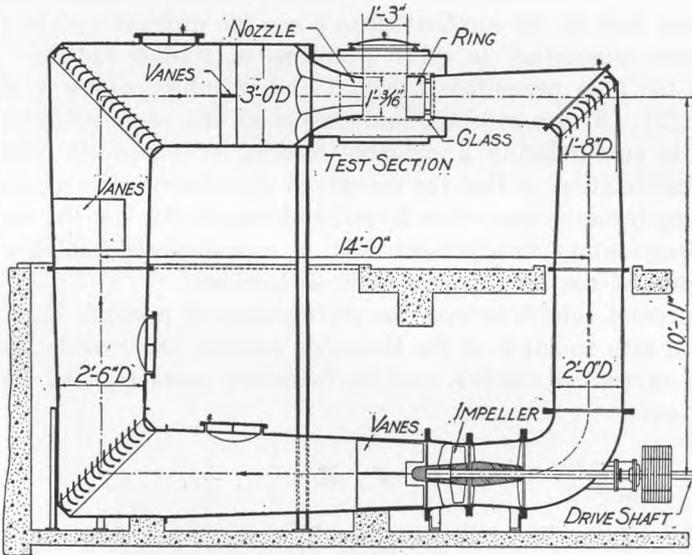

FIG. 3—VERTICAL SECTION THROUGH TUNNEL.

If $\mathbf{K}_i$ is defined as the value of **K** for conditions of incipient cavitation in flow around a given boundary, all values of **K** greater than $\mathbf{K}_i$ will yield a pattern of flow which is unaffected by cavitation. As **K** is reduced below $\mathbf{K}_i$, however, the effects of cavitation

become more and more pronounced. In this effective range, the value of **K** indicates the extent of cavitation and governs conditions of similarity for model and prototype for this phenomenon.

A brief description of certain features of the water tunnel used in an experimental study of the type described by Eq. (3) is pertinent to a discussion of the results obtained therefrom. Completed in the spring of 1944, the water tunnel at the Hydraulics Laboratory of the Iowa Institute of Hydraulic Research is similar in functional design to water tunnels at other research laboratories and to many wind tunnels. A 24-inch Fairbanks-Morse propellor pump provides a discharge from 8 to 35 cubic feet per second, and since the cross-sectional area of the jet is one square foot, the velocity may be varied from 8 to 35 feet per second. The pressure in the test section is controlled by a vacuum pump and a secondary circuit to provide pressures from +30 to —30 feet of water.

The axis of the tunnel has the form of a vertical rectangle, as shown in Fig. 3. The test section has an octagonal housing with windows of $3/4$-inch tempered plate glass on either side and access openings at the top and bottom. The throat velocity is indicated by a differential gage and controlled by varying the speed of the pump. The pressure in the test section is determined by a second mercury differential gage which indicates the difference between the pressure at the wall of the test section and that in the atmosphere.

As the studies in the tunnel have consisted of the determination of pressure distributions around several systematic series of bodies of revolution, the mounting shown in Fig. 4 was installed for the various models. A hollow brass shaft serves as a support for the interchangeable head forms and contains the copper piezometer leads which pass to the rear and through the side of the test section. A manifold and a third differential gage provide means for determining as many as twenty-four piezometric heads along the body.

Since completion, the tunnel has been used for a study requested by the David Taylor Model Basin, first through the National Defense Research Committee, and thereafter by direct contract with the Bureau of Ships of the U. S. Navy. The series of tests conducted under this contract on the ellipsoidal head forms already mentioned provides an excellent illustration of the variation in pressure with the Reynolds number, the cavitation number, and the boundary

geometry. The series ranges from a 4:1 ellipsoidal form through the hemispherical, shorter ellipsoidal, and blunt head forms to concave ellipsoidal profiles.

The effect of systematic variation of the boundary geometry for the ellipsoidal series is clearly demonstrated in Fig. 5; in this study
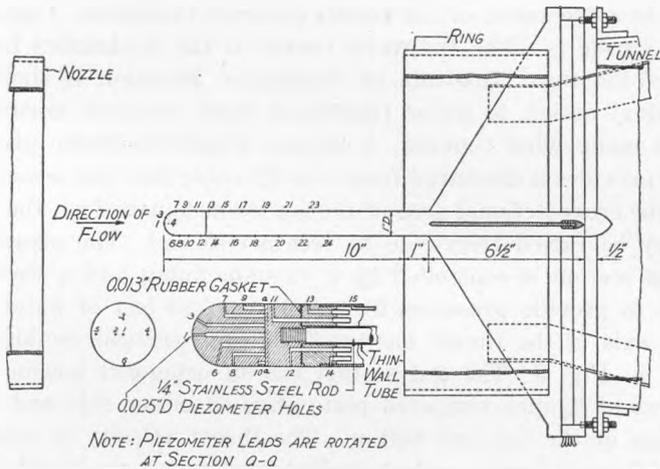

FIG. 4—DETAILS OF SHAFT, MOUNTING, AND TYPICAL HEAD.

the axis ratio of the ellipsoids was varied. The forms tested as indicated by the solid lines at the top of the figure had the following axis ratios: 4:1, 2:1, 1:1 (hemispherical), ½:1, 0:1 (blunt), —½:1, (concave), and —∞:1 (hollow cylindrical). The experimental data yielded the corresponding solid curves for longitudinal pressure distribution around the various forms. The dotted lines are interpolated values which were prepared from a series of interpolation plots and with considerable assistance from the analytical results illustrated in Fig. 1.

The effect of the Reynolds number on the hemispherical head form, which is representative of this series, is shown in Fig. 6. The pressure distribution was found to change with variation in the Reynolds number for **R** less than 2×10^5, but further increase above this value indicated no further change. Therefore an upper critical Reynolds number exists for a given boundary corresponding in effect to the limiting value of **R** for fully developed turbulence within a uniformly roughened pipe. The numerical value
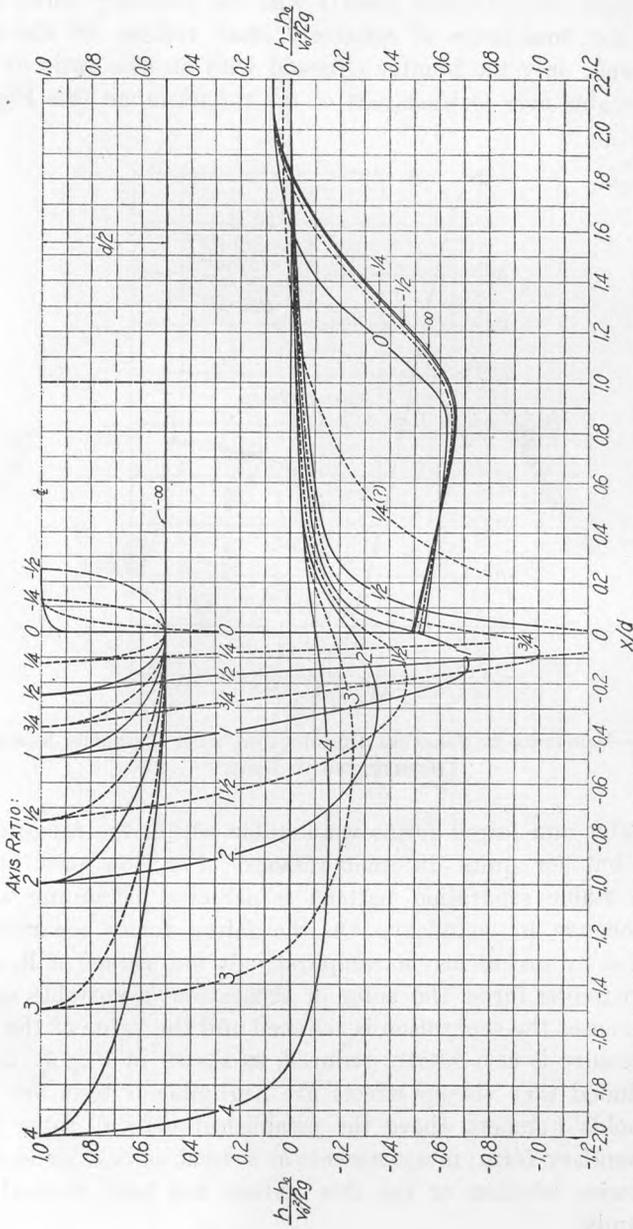
FIG. 5—LONGITUDINAL DISTRIBUTION OF PRESSURE ON ELLIPSOIDAL HEAD FORMS AS A FUNCTION OF AXIS RATIO.

of this upper critical varies greatly with the boundary form, being greatest for boundaries of relatively small radius. Of the ellipsoidal series, only the blunter ellipsoid with an axis ratio of $\frac{1}{2}$:1 was not stable over at least part of the tunnel range (see Fig. 5).
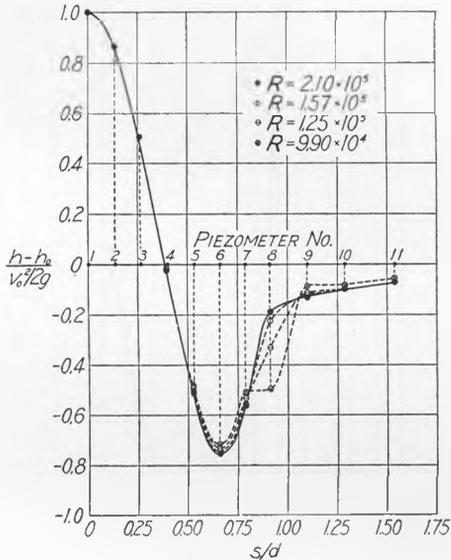

FIG. 6—VARIATION IN PRESSURE DISTRIBUTION WITH REYNOLDS NUMBER. (HEMISPHERICAL HEAD.)

The blunter and faired forms were stable within the range of the tunnel, but for quite different reasons. For flow past blunter forms a stable separation pattern is achieved beginning at the abrupt change in boundary. On the faired forms separation is eliminated by turbulence at comparatively low values of **R**, while for short-radius forms the point of separation is unstable so that as **R** increases the separation is reduced and the value of the minimum pressure is also greatly reduced, as shown in Fig. 7. It may be concluded that viscous effects are negligible if tests are made at Reynolds numbers above the established critical value for a given boundary form; measurements at several values should suffice to determine whether or not this critical has been reached in a model study.

The effect on the pressure distribution of varying the cavitation

parameter, although similar in many respects to that resulting
from variation of the Reynolds number, is perhaps more easily in-
terpreted. Over most of the practicable range of hydraulic phe-
nomena, occurrences of hydraulic significance are entirely indepen-
dent of vapor-pressure effects, since only in the event that a local
pressure drops to the vapor pressure does variation of the cavita-
tion parameter produce any significant change. Below the critical
value of K_i for any boundary form, however, cavitation effects be-
come apparent to a degree which is indicated by the numerical
value of $K_i — K$. This dependence is clearly demonstrated for
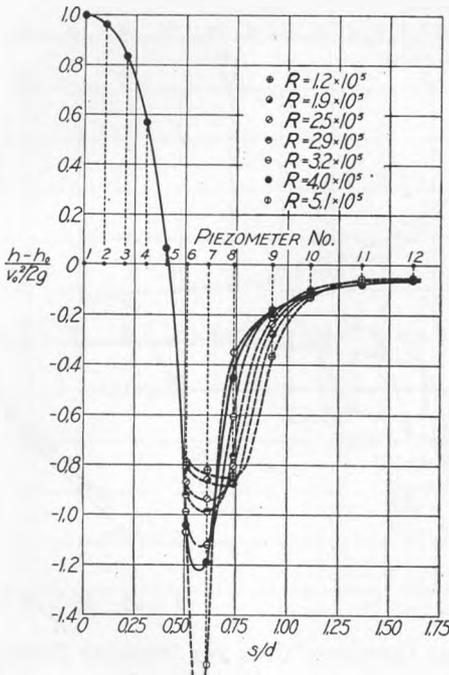the hemispherical head form in **Fig. 8.**

Fig. 7—Variation in Pressure Distribution with Reynolds Number.
(½:1 Ellipsoidal Head.)

The half-profile of the boundary form together with the cavi-
tation-free pressure distribution is shown at the top of the figure.
This pressure curve is reproduced in developed form as the curve

marked $K \geq 0.82$. Variation of K in the range $K > 0.82$ then has no effect on the distribution and the value for incipient cavitation on this head form may be defined as $K_i = 0.82$. Since the distribution of pressure head referred to the centerline pressure head is plotted in units of velocity head, and since the value of K is a measure of centerline pressure head relative to the vapor pressure head in the same units, it follows that the numerical value of the negative pressure head can never exceed the value of K. This is borne out by the family of curves and by the curve of minimum pressure vs. K in the lower right-hand corner. As K is reduced,
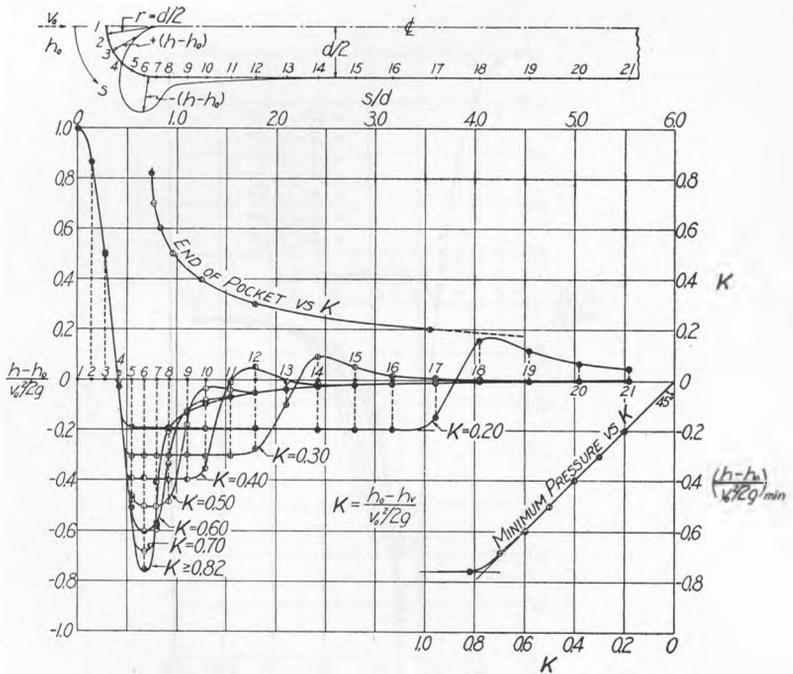

FIG. 8—EFFECT OF CAVITATION UPON THE PRESSURE DISTRIBUTION AROUND A CYLINDRICAL BODY WITH HEMISPHERICAL HEAD.

the two values become numerically equal and the vapor pocket extends over a distance which increases systematically with K as shown in the plot of K vs. the location of the end of the vapor pocket.

Perhaps the most surprising feature of this and other similar families of curves is that the curve at the right is not simply two straight lines, one sloping at 45° between $\mathbf{K} = \left(\dfrac{h - h_0}{v_0^2/2g}\right)_{\min}$ and $\mathbf{K} = 0$ indicating that $h_{\min} = h_v$ and the other horizontal for all values of $\mathbf{K} > \left(\dfrac{h - h_0}{v_0^2/2g}\right)_{\min}$. This would fit the popular conception of the incidence of cavitation, i.e., that the centerline
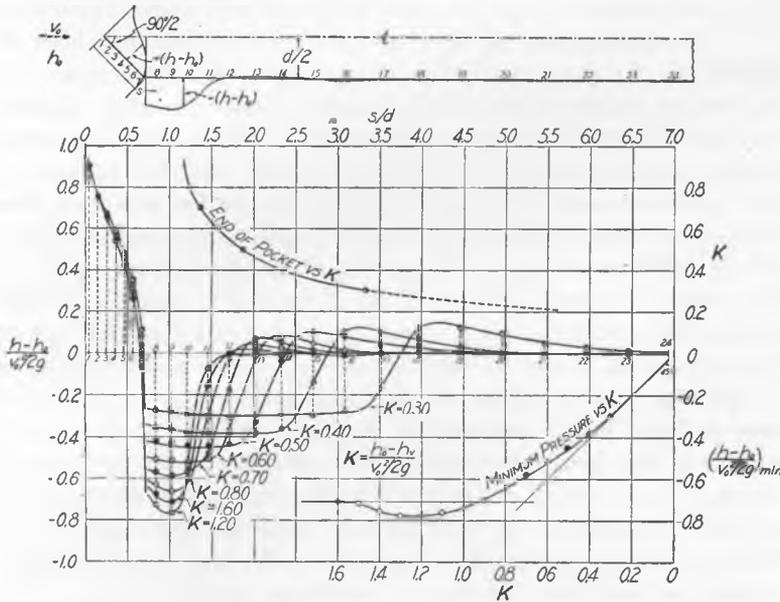

Fig. 9—Effect of Cavitation Upon the Pressure Distribution Around a Cylindrical Body with 90° Conical Head.

pressure may be reduced until the pressure on the boundary is equal to vapor pressure, and a further reduction results simply in cutting off the lower part of the curve. Although this simplified picture is approximately correct for the hemispherical head, for the other types of boundary it is greatly in error as shown in Fig. 9. Although not a part of the ellipsoidal series, the results for the 90° conical head are presented because they demonstrate the radical departures from an over-simplified picture of the onset of cavitation which may be encountered. In this instance $\mathbf{K}_i = 1.6$ while the minimum pressure ratio was approximately —0.7, indicating that cavitation

effects occurred while the pressure around the boundary was nearly one velocity head above the vapor pressure. Furthermore, the boundary pressure approached the vapor pressure only for very small values of **K**.

The explanation of this phenomenon is relatively simple. Cavitation first occurs at points of minimum local pressure and these are often within the vortices along the boundary of the separation zone. Hence, vapor may form and cause a change in the flow pattern while the pressures along the boundary are well above vapor pressure. An interesting variation of this phenomenon has been described by Mr. Locher [8] in which severe cavitation took place in a region unaffected by solid boundaries where the mean pressure was never below one-half an atmosphere. The vapor was formed within vortices along the boundary between the high-velocity jet and the driven water within a jet pump. As the jet expanded, these vortices eventually reached the conduit wall, remaining sufficiently intense for pronounced pitting of the conduit walls to occur in this region. It should be kept in mind, therefore, that the pressure distribution around a given boundary does not necessarily permit even a reasonably accurate prediction of the incidence of cavitation.

Although the experimental data presented in the foregoing discussion have been restricted to the case of flow around a solid boundary, the similarity between flow patterns which are bounded internally and others which are bounded externally should enable a broader application of results than might be visualized at first. That is, since many points of similarity exist between a flow around a flat disk and flow through a diaphragm orifice in a pipe line, there should be a marked similarity between flow around the various head forms and flow through boundary transitions in a conduit. An illustration of this point arises from a comparison of the effect of radius of rounding upon the minimum pressure along a given boundary. The trend established for the ellipsoidal series is seen to be similar indeed to that for two-dimensional conduit inlets as presented by Dr. Rouse [2]. In each set of experiments the most extreme pressure condition occurred for a moderately small radius of curvature.

Three methods for determining the pressure distribution around a given boundary have been described in some detail. Analytical methods, either exact or aproximate, may be applied to bodies of

easy curvature with worthwhile results. However, much remains to be done in bringing the analytical methods within reach of engineers without highly specialized training, not because the procedure is difficult, but because of the wide gap between published theory and practical applications. The second method, which is the adaptation of previous experimentation to a given problem, presupposes a fund of systematic data upon which one can draw. That such a fund is in the process of being accumulated is evidenced by several of the papers presented in these Proceedings, and it is to be expected that such data may be applied to other than strictly geometrically similar bodies. Specific model studies, as the third basic method, should be carried out in a systematic manner, both to provide the desired results and to add to the fund of useful data. If the maximum benefit is to be derived from such studies, dimensionless representation should be used and dictates of dimensional analysis should be observed.

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