

APPROACHES TO THE STUDY OF THE MECHANICS OF BED MOVEMENT

by

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INTRODUCTION

Transportation of sediment along the beds of natural and artificial water courses, with the associated changes in conformation of the stream bed, has long been one of the most perplexing and yet fascinating types of occurrences confronting the hydraulic engineer. Probably in no field of hydraulics have there been more confusing and even diametrically opposite statements made in the course of the past several decades. It is believed, however, that at the present time a number of focal points which will lead to sound fundamental analysis are being reached as a result of careful study of the scattered fragmentary data regarding both field and laboratory observations.

Because of the many variables and great complexity of stream bed movement it is quite logical that pioneer efforts at analysis should be of a quite empirical nature. In time the many variables will be isolated and evaluated as regards their individual influences upon the general phenomena of bed movement. The turbulent flow of homogeneous liquids in smooth circular pipes in itself has provided considerable complexity of analysis. More recently, gratifying analytical approaches have been made to the general problem of fluid turbulence, in particular as regards flow through pipes of idealized roughness consisting of spherical, uniform-size grain projection; however even here there is still concern over the detailed treatment in case of roughness of types different from the idealized. The approach remains empirical although rationalization is a rapid trend.

It is recognized that the individual influences making up the mechanics of bed movement must be carefully analyzed separately in

order to obtain a comprehensive analysis of the various occurrences. Procedures involving dimensional analysis are to be recommended. Studies of this character have been outlined recently for sediment transportation at meetings of technical societies as well as at this conference. The basic knowledge of the theory of turbulence, as presented by Prandtl, von Kármán, Taylor, and others and verified in part by experimental studies, has been used as a basic framework and expanded to encompass sediment movement. Possibly one of the outstanding pioneer efforts in this direction is that of Shields¹ in his discussion of the application of the principles of the mechanics of similitude and turbulence research to bed-load movement.

Like most problems in the field of mechanics, in the case of bed-load movement there are different approaches, all of which are essential to the complete analysis of the problem. The more rational studies are concerned with the detailed analysis of the internal mechanism of the physical occurrences. Another approach involves a bulk analysis in which the internal mechanism is given less consideration. Although the latter is usually considered more empirical, it ordinarily forms the basis for practical engineering application. To be sure, the entire mechanics of the problem is necessary for its clear interpretation and understanding.

For further discussion here attention will be focused on various observations which emphasize important considerations in the study of bed movement from the practical point of view rather than on the detailed mathematical formulation of the physical conceptions.

Bed-load movement and suspended load, of course, are not absolutely independent of each other. It should be pointed out, however, that one cannot generalize analytically regarding their mutual interrelationship. Here incidental reference will be made to suspended-sediment distribution in flowing water with a view of showing at least qualitatively the relation of the two modes of sediment movement.

CHARACTER OF RIVER SEDIMENTS

River bed sediments in general seem to be characterized by a dominant intermediate size particle and diminishing quantities of particles at both sides in size of the dominating intermediate grade. Typical examples are given in Figs. 1 and 2. On the other hand the material carried in suspension by rivers seems to show a quite uniform gradation from fine to very fine particles, ordinarily with no dominant size.

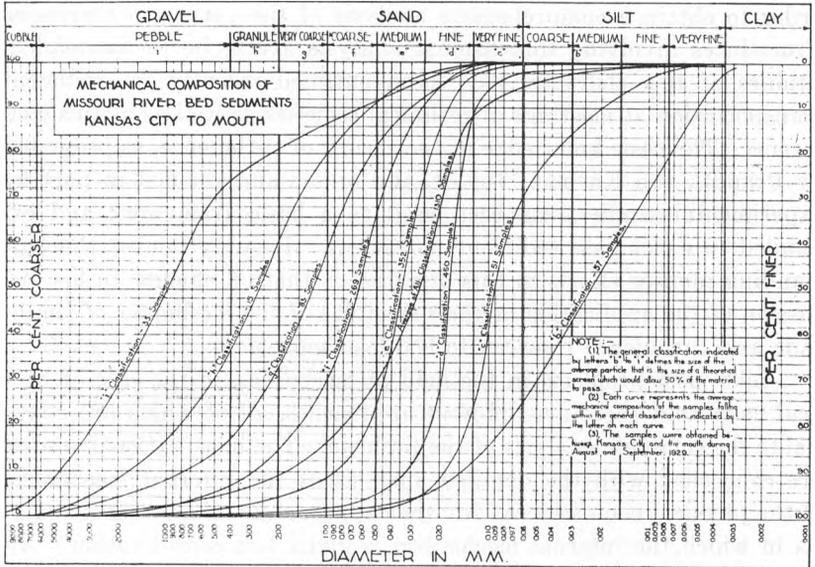


FIG. 1.

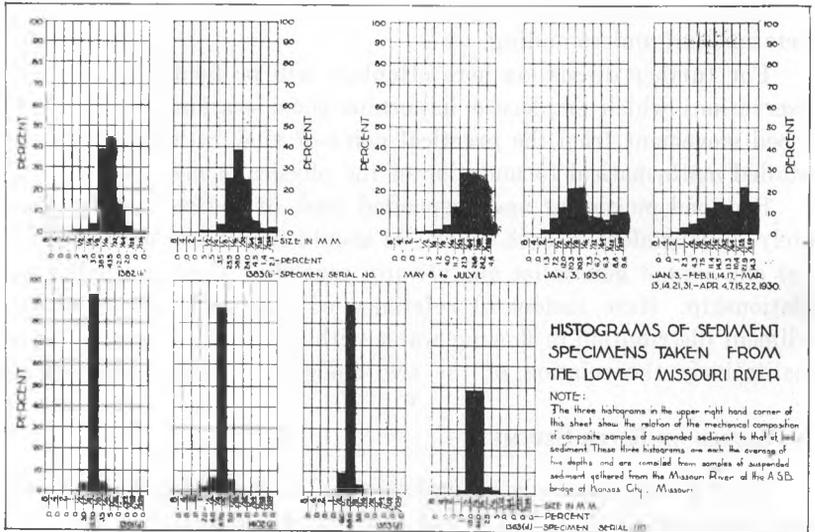


FIG. 2.

A point of interest relative to the nature of bed material and suspended material in most rivers is that the two seem to have little rela-

tion to each other as regards mechanical composition, at least when the entire bulk is considered. Thus, some streams have beds composed largely of coarse gravel while the normal suspended load of these streams consists of finely divided particles. A comparison of Fig. 3, showing the mechanical composition of suspended load in the Missouri River, with Fig. 1, showing the nature of the bed materials, is a typical illustration of this statement. It is to be pointed out in

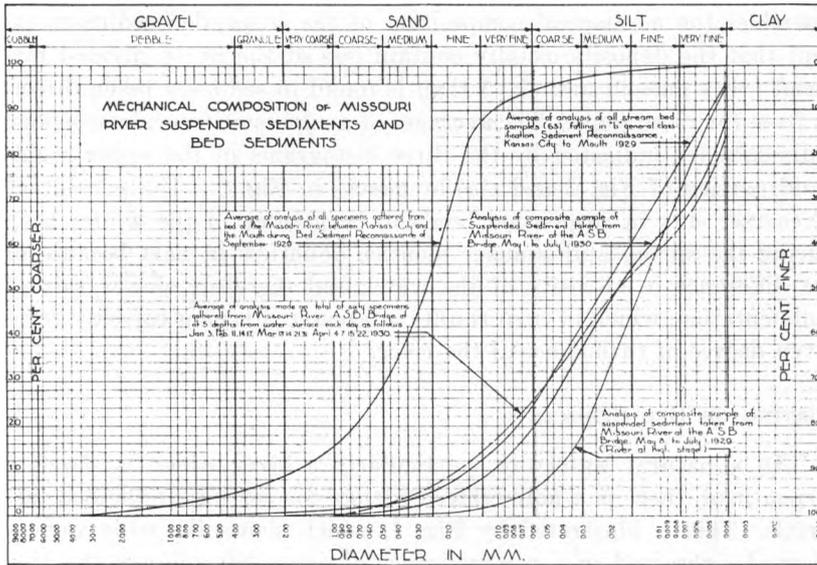


FIG. 3.

this connection that the curve indicated by classification "b" (Fig. 1) is made up largely of the analyses of samples taken from slack water areas where suspended material would deposit. This material has a mechanical composition corresponding closely to that of the suspended load.

Reference is again made to Fig. 2, showing typical sediment specimens taken from the lower Missouri River. Most specimens gathered from the stream bed are of the well-sorted type, as indicated by specimen serial nos. 1391, 1402, 1353, and 1363, which is a type to be expected where the scouring capacity of the stream has remained constant for a prolonged period of time. They are characterized by a preponderance of a single grade of material. Such specimens are

found primarily in smooth stretches of channel subject to normal velocities. A further type, also illustrated in Fig. 2 and referred to as "channel silt," includes finely divided sediments which probably have been deposited at least partly from a state of suspension. This type is also indicated by the "b" classification shown in Fig. 1. It occurs primarily downstream of sand bars which project above the water surface. Specimens 1382 and 1383 (Fig. 2) are samples which fall into this group. The mechanical composition of these sediments resembles the mechanical composition of the suspended sediments except that the deposits usually contain less of the finely divided material (very fine silt and clay) than is found in sediment taken directly from the river water. The averages of analyses of typical suspended sediments are indicated by the three histograms in the upper right-hand portion of Fig. 2 and also by curves in Fig. 3. The percentage of coarser silts in suspension tends to be greater in the winter than during the spring and summer months, doubtless in part because of the difference in viscosity of the water and therefore slower rate of sedimentation, which in turn results in greater carrying capacity for a given degree of turbulence of the river.

SORTING OF BED SEDIMENTS

In a natural waterway, particularly a river which is strongly serpentine, there is a noticeable tendency for sorting of the bed materials. This is illustrated by Fig. 1, which shows the wide variety of grades obtained in a reconnaissance of some 400 miles of the Missouri River. In straight stretches of river there is a marked tendency toward uniformity in grade as illustrated by specimens 1391, 1402, 1353, and 1363 in Fig. 2. It is also indicated particularly by classifications "d" and "e" in Fig. 1.

In making experiments to determine transportability of sediments by means of the usual laboratory procedure with arbitrarily chosen mechanical compositions of the sediment, great care must be exercised to avoid complications arising as a result of sorting. It is possible that much of the inconsistency of data obtained in laboratory studies is the result of this situation. The situation is illustrated by Figs. 4 and 5. Fig. 4 shows the results of a study with sediment of a fairly uniform size and an average diameter of 4 mm. In this case the sedimentary material was added uniformly at the upstream end of the flume; the rate of feeding was slightly greater than for an

equilibrium condition of the initial slope, the slope, therefore, gradually increasing by the process of aggradation. At the end of the run specimens were gathered from the stream bed at distances of 2, 4, 8, 12, 16, 20 and 24 feet measured from the initial point at which the sediment was fed into the channel. It is interesting in this case that (contrary to the usual conception) the coarser material is being transported more rapidly than the finer, the latter being predominantly present near the headwater end of the flume. A surprisingly uniform gradation took place from one end of the flume to the other, despite the fact that the original form was of quite uniform size particles. It

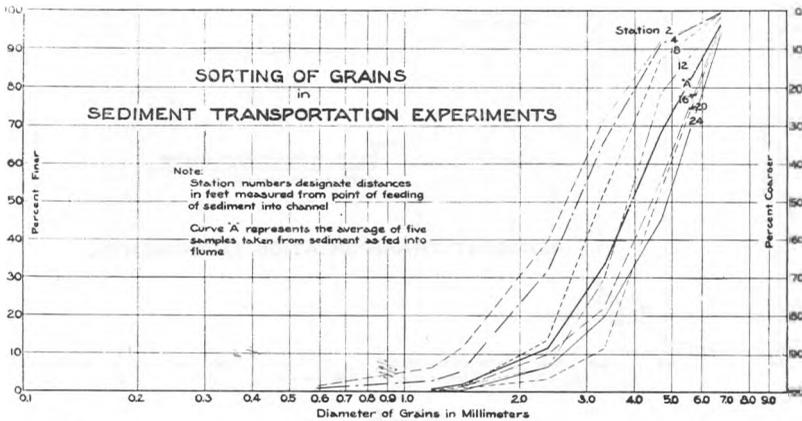


FIG. 4.

will be noted also that the tendency was to produce a sediment of more predominant intermediate grade as the sediment moved downstream. This corresponds to observations in actual rivers as illustrated by Figs. 1 and 2.

Similar occurrences have been observed in laboratory studies with material of finer mechanical composition. Depending upon the flow conditions and type of material, segregation upon deposition may take place either from fine to coarse (from one end of the channel to the other) or the reverse. Much additional study is required both in field and laboratory before any definite conclusions can be drawn as to the laws governing this occurrence.

In laboratory studies the maintenance of uniformity of mechanical composition throughout the channel can probably best be accomplished by avoiding aggradation in the course of the experiment.

Preferably, a slight amount of degradation should take place so that little of the material added at the headwater end is deposited in traveling through the flume. With a control of this character much greater consistency in the result has been obtained. Fig. 5 indicates the uniformity in mechanical composition of a typical river sediment when this control (slight amount of degradation) was followed. The specimens were gathered from the surface of the bed to a depth not exceeding $\frac{1}{4}$ inch in a manner similar to that involving the results presented in Fig. 4. Even here a close examination of the surface of

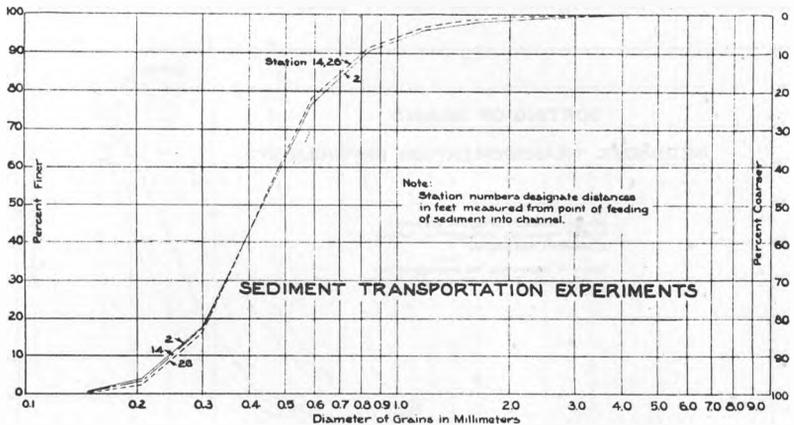


FIG. 5.

the bed revealed a slight amount of sorting, though very much localized; there is evidence of different mechanical composition of the material in the crest and valley of the sand riffles.

The sorting of bed sediments is quite complex, not being confined to the separation into different particles over large areas, but also over quite small areas and even in the vertical. For example, a close analysis of the distribution of size of particles over a small ripple only a few inches long indicates segregation which is quite noticeable. Moreover, it will be observed in many cases that the material varies in size with elevation within the traveling stratum for a given rate of movement by stream traction. Careful observations and mechanical analyses of this situation made at the University of Minnesota for a given sediment indicate a quite systematic variation in mechanical composition from top to bottom of the transported layer. In fact, by systematic progress of an experiment it was possible to determine quite

accurately the thickness of the moving layer. The difference in mean size of particle from top to bottom of the moving layer was found to approach 100 per cent variation in case of natural river sands, the coarsest in this instance being near the surface and the finest at the line of separation between the moving layer and that undisturbed.

ANALYSIS OF RIVER BED CHANGES

A number of fundamental principles which relate primarily to bulk flow conditions and bed-load movement make possible the estimation of variations in the stream bed for changes in flow conditions. A movable-bed channel in contrast to a rigid-bed channel tends toward a condition of equilibrium within itself which is dependent upon bed load for different conditions of flow. In the case of a fixed bed, variations in cross-section along the stream length are reflected by variations in elevation of the water surface. In the case of movable-bed streams these changes are reflected in the change in shape of the stream bottom as well as in the water surface. Disregarding tributary inflow for the moment, the river channel for a given discharge approaches an equilibrium condition in which the flow past various sections remains constant and also the sediment load remains constant.

An approach to the analysis of such equilibrium conditions for a river requires a formulation of a law defining approximately the rate of bed-load movement for various cross-sections. For straight stretches of river it is believed that some degree of success has been achieved in this regard at least in determining the proper order of magnitude of this load or the relative quantity for different types of sands ordinarily encountered. The procedure is based upon the general acceptance of the du Boys relation when supplied with suitable transportation characteristics for the particular sediment concerned, and the adoption of a suitable open-channel flow formula to define the water discharge. Experience has shown that the Manning formula gives satisfactory results over a quite wide range of flow conditions with and also without bed-load movement. Various forms of equation have been set up by accepting these two more or less empirical relations. Both of the relations have been verified over quite a wide range of conditions and to this extent at least should be acceptable.

The following forms of the relation have proved useful in studying the effect of artificial structures upon the regimen of rivers and have been checked in a number of different ways by comparison with

actual river investigations and by laboratory experiments. These have been derived by the writer with the aid of the foregoing considerations and are here presented without algebraic proof. For the idealized case of a rectangular channel of infinite breadth (side effects negligible) the relation takes the form

$$G = \Psi \left[S^{1.4} / C^{1.2} \right] Q^{3/5} \left[Q^{3/5} - Q_0^{3/5} \right] \quad (1)$$

where G is the quantity of sediment transported along the stream bed in pounds per unit width of channel, Q is the actual water discharge per unit width of channel, Q_0 is the discharge per unit width of channel (for a slope S) at which sediment transportation begins, C is the coefficient in an open-channel flow formula of the form $v = C R^{2/3} S^{1/2}$, and Ψ is the "sediment characteristic," an experimental coefficient depending upon the size, specific gravity, and mechanical composition of the sediment.

The foregoing equation may be written in a number of other forms for use in the solution of practical problems. The equation is of general applicability but requires the experimental determination of the parameters Ψ , C , and Q_0 . The values of Ψ and Q_0 depend upon the mechanical composition (variation in size of grains as determined by sieve analysis or elutriation), specific gravity, and form of the sediment particles; while C depends upon the character of the sediment and also the rugosities of the channel itself independent of the roughness due solely to the mechanical composition of the sediment.

The theory has been expanded mathematically to explain the equilibrium conditions for channel-contraction works. The principle involved includes the considerations explained in the foregoing and, in addition, the fact that the sediment transported through the contracted section of the stream, as well as the water discharge, must be the same as through the uncontracted section. Contracting the stream results in greater power for erosion in the beginning but finally a state of equilibrium must be reached in which there occur both a change in slope and change in depth of stream. Algebraically the depth in the contracted section in terms of the flow conditions in the uncontracted section may be expressed as follows for the case of a rectangular channel:

$$d_2 = \frac{d_1}{(1-\alpha)^{3/7}} \left\{ -\frac{\tau_0}{w S_1} + \left[\left(\frac{\tau_0}{w S_1} \right)^2 + \frac{4d_1 \left(d_1 - \frac{\tau_0}{w S_1} \right)}{(1-\alpha)} \right]^{1/2} \right\}^{3/7} \quad (2)$$

$$\left[2 \left(d_1 - \frac{\tau_0}{w S_1} \right) \right]^{3/7}$$

where d_2 = the water depth at the stretch of channel which has been contracted; d_1 = the water depth before contraction (or the depth upstream and downstream of the contracted region); τ_0 = the transporting force (tangential force of the water against the stream bed per unit area of bed) at which sediment transportation begins ($\tau_0 = w d S$); S_1 = the natural slope of the hydraulic gradient of the stream in the uncontracted part; α = the amount of contraction, that is, $B_2 = (1-\alpha)B_1$, B_2 and B_1 being the breadths of the channel, respectively, at the contracted and normal sections.

It is of interest to compare equation (2) with the results of studies recently presented by W. M. Griffith² for a special case. Griffith from an entirely empirical approach (based upon extensive field observations) arrived at the equation

$$d_{m2} = d_{m1} \left(\frac{B_1}{B_2} \right)^{0.637} \quad (3)$$

which indicates the relation of depth at the constricted section to depth at the normal section, B_2 and B_1 being corresponding breadths.

In the way of comparison, again considering equation (2), if the critical tractive force τ_0 mathematically approaches zero or is negligible compared to the tractive force and therefore velocity which actually exists in the channel, the equation reduces to

$$\frac{d_2}{\tau_0 \rightarrow 0} = \frac{d_1}{(1-\alpha)^{9/14}} \quad (4)$$

which when taking into account the relation $B_2 = (1-\alpha)B_1$ can be written as

$$\frac{d_2}{\tau_0 \rightarrow 0} = d_1 \left(\frac{B_1}{B_2} \right)^{9/14} \quad (5)$$

or, expressed with a decimal exponent,

$$d_2 = d_1 \left(\frac{B_1}{B_2} \right)^{0.642} \quad (6)$$

which agrees almost exactly with Griffith's equation (3). A further equation paralleling (1) for the case of constant discharge can be written in the form

$$G = \Psi Q^{2/3} C^{-4} v^{7/3} [v^{7/3} - v_0^{7/3}] \quad (7)$$

In the case of readily transported sediment the critical transporting force, that is, the traction force at which movement of the bed sediment begins, is very small; consequently, for appreciable velocity of flow the stream, v_0 approaches zero, at least as compared to the value of v . This would be the case of relatively steep slopes or fine detritus. Thus,

$$G = K_1 v^{14/13} \quad (8)$$

$v_0 \rightarrow 0$

where

$$K_1 = \Psi Q^{-2/3} C^{-4} \quad (9)$$

As regards the non-silting, non-eroding condition, that is, for the condition of a stable channel, it is important to recognize that both the sedimentary load and the water discharge respectively passing various sections in a continuous channel must be constant. The writer desires to emphasize at this point of the discussion that it is the bed load which certainly must be of primary importance in defining the stability of a channel in contrast to the frequently expressed conception that it is the suspended load. Of course, in the case of some streams the concentration of the coarser constituents of suspended load in the stream is likely to have some relation to the amount of bed load being moved. Inasmuch as the suspended material, for the most part, continues as such throughout its course, it is quite important to recognize that the bed sediment in a stream, having a loose granular material forming its bed, is most sensitive to sedimentation and erosion by changes in velocity. Therefore, the foregoing analysis which has been developed for bed-sediment transportation apparently might be used in a special form for determining the non-silting, non-eroding velocity for a given set of conditions. For a comparatively finely divided sediment the critical transporting force, and hence the critical velocity v_0 , approaches zero. Furthermore, if neither silting nor erosion is to take place in a canal system, the amount of sediment transported per unit volume of water must be the same at all sections of

the canal. Therefore, the ratio of the amount of sediment carried to the water discharge must be constant, or, expressed in symbols

$$\frac{G}{Q} = K \text{ (a constant)} \quad (10)$$

With these considerations in mind, the foregoing equation may be rewritten, as a special case, as follows:

$$K = \Psi \frac{1}{Q^{5/3} C^4} v^{14/3} \quad (11)$$

Since the discharge per unit width of channel is $Q = v d$,

$$K = \Psi \frac{1}{v^{5/3} d^{5/3} C^4} v^{14/3} \quad (12)$$

or

$$v^{9/3} = \frac{K C^4 d^{5/3}}{\Psi} \quad (13)$$

from which

$$v = \frac{K^{1/3} C^{4/3}}{\Psi^{1/3}} d^{5/9} \quad (14)$$

and

$$v = c d^{0.56} \quad (15)$$

Again this equation resembles the equation obtained by a purely empirical approach by Kennedy and others. More recently, a form based upon experimental studies was determined to be as follows by Griffith,²

$$v = c d^{0.57} \quad (16)$$

The writer's formula (15) has the advantage in that the value c can be computed from a number of parameters defining the sedimentary and hydraulic characteristics of the stream. The value of c computed by equation (16) gives values which for usual conditions correspond quite well to those found by a number of investigators in the study of the canals of India. It is to be noted that the value of c is a function of the sediment transportation characteristic, the sediment load, the water discharge, and the roughness of the channel. The "constant" c in the special equation here presented is thus seen to vary with the sedimentary and hydraulic characteristics of the channel. It

would appear, therefore, that if c is taken as a constant, the formula is applicable only to a very special combination of circumstances. This is probably the reason for some embarrassment which some investigators have experienced in applying the Kennedy formula and similar relations of this form. Kennedy realized this himself, although his formula was derived purely empirically, for he says, "Strictly speaking, a separate value of v_0 should obtain at each season of the year, and for each class of sand carried in each reach of a canal."

According to equation (14) the value of c varies as the $4/3$ power of the discharge coefficient C , but is affected merely as the cube root of K and inversely as the cube root of Ψ ; hence, considerable variation may take place in both K and Ψ without materially changing the value of c . However, a relatively small variation in the coefficient C will have noticeable influence upon the value of c in the stable-channel formula. Thus, there is comparatively little variation in c despite the large variations which might occur in Ψ and K for different channels.

In special cases where the equilibrium conditions are greatly different from those corresponding to the channels on which the Kennedy formula was based, the value of c can be expected to deviate considerably from unity. Thus, for quite coarse bed materials and relatively steep slopes, values of c have been determined by the writer amounting to several times the value usually found applicable to irrigation canals.

CHANNEL CONTRACTION WORKS

In order to show the applicability of the writer's theory, here presented in abstract, reference is made to Figs. 6 and 7. Fig. 6 represents the results of a study to determine the transportation characteristics of a particular type of sand gathered from the natural river channel of one of the Mississippi River tributaries. The mechanical composition is indicated in the lower right-hand corner of this figure. From laboratory study and analysis of the data, the following characteristics were found: $\Psi = 100,000$ and $\tau_0 = 0.010$. Fig. 7 shows the results of experimental observations upon the situation for a contracted channel. The amount of contraction was 30 per cent, that is $\alpha = 0.30$ in equation (2). The sand was originally placed in the channel at a definite slope and a definite rate of flow passed through the channel for a period of time sufficiently long to allow equilibrium to set in, that is, stability against erosion (not absence of sediment transportation). Sediment was added continually to the headwater end of the

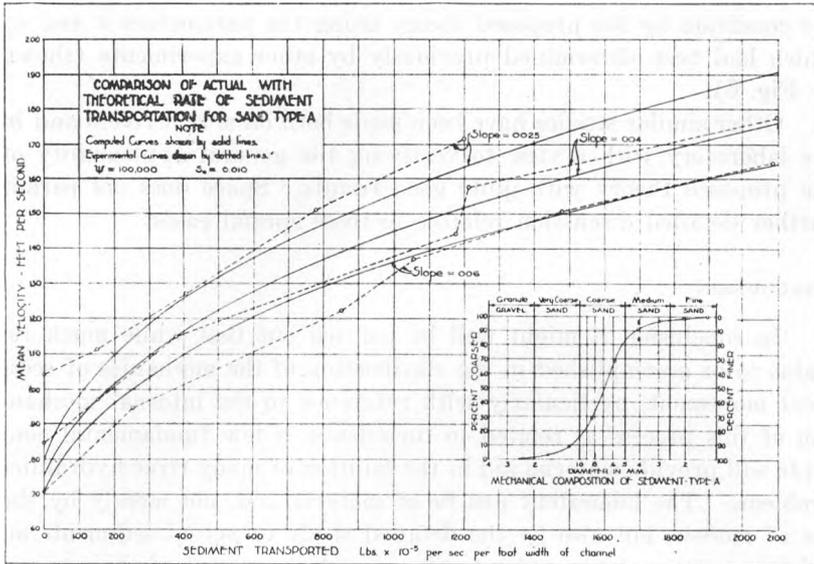
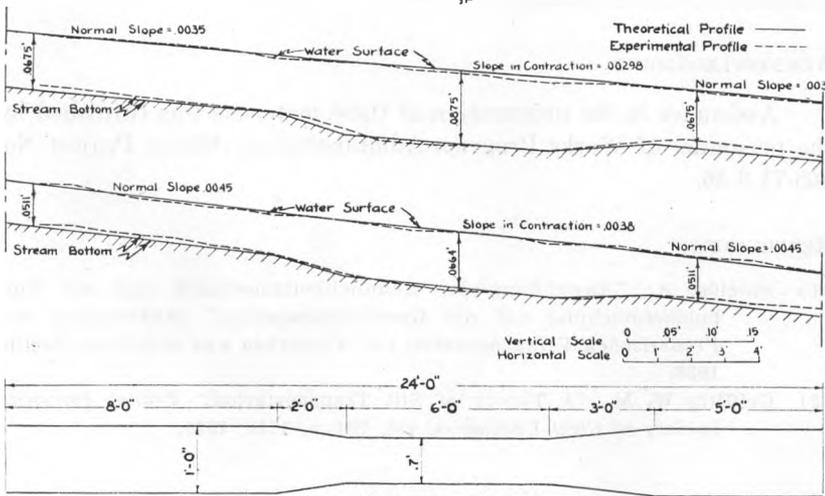


FIG. 6.
 Comparison of Theoretical and Experimental Profiles for Sand Type A



Plan View of Channel
 FIG. 7.

flume to replace that transported through the channel in order to maintain the equilibrium. The figure shows the results of measurements for two typical runs. Superimposed are the computed values of

the condition by the proposed theory using the parameters Ψ and τ_0 , which had been determined previously by other experiments (shown by Fig. 6).

Other similar studies have been made both on actual rivers and in the laboratory with a view to verifying the general applicability of the proposed theory with quite good results. Space does not permit further detailed discussion relative to these special cases.

CONCLUSION

In conclusion it might well be pointed out that while much remains to be accomplished in the clarification of the mechanics of sediment movement, particularly with reference to the internal mechanism of this process as related to turbulence, a few fundamental concepts will provide material aid in the solution of many river-hydraulics problems. The laboratory can be of material aid, not merely by the use of models, but also by the detailed study of actual sediments on a fairly large scale, in order to obtain a clear concept of their transportability. In any case, at present, close correlative studies of natural conditions in the river are indispensable if gross errors in analysis are not to be made.

ACKNOWLEDGMENT

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