WAVES IN THE ATMOSPHERE AND APPLICATION TO FORECASTING

by

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Although waves in the atmosphere are for the most part invisible to an observer peering upward, their influence on the activities of Man is greater than that of the largest waves possible on the sea-surface. The periods of atmospheric waves range from a fraction of a second [1][1] to several days and even longer. Among the multitude of waves the following are important because of their association with significant weather phenomena: (1) gravitational waves (or Helmholtz waves) having wave lengths from 0.5 to 5 kilometers and periods of the order of minutes and hours; (2) cyclone waves (or Bjerknes waves) having wave lengths between 500 and 3000 kilometers and periods of hours and days; and (3) long waves in the Westerlies having wave lengths from 3000 to 9000 kilometers and periods of days and weeks.

Gravitational Waves

Waves of the gravitational type form at horizontal surfaces of discontinuity in density and wind velocity, where the lower layer is of the greater density. The waves are of the mixed shearing and gravitational type with the former effect tending to render the waves unstable while the latter tends to stabilize them.

The orbital planes of the vibrating particles are vertical. Since these waves are of small wave length, they can be analyzed without taking into account the rotation of the earth [2, 3].

The effect of these waves on weather is quite trivial. If the air is sufficiently humid, the ascending particles may become saturated, condense, and form a cloud. The descending particles of cloud will dissipate because of the heat generated by the adia-

[1] References appear at the end of the article.

batic compression of the air. Thus the cloud system may be in the form of alternate rolls of cloud and clear space—the well-known "billow-cloud." These are not likely to produce precipitation, because the strong gravitational stability at the discontinuity surface above the cloud prevents air particles from penetrating upward and forming thick rain-producing clouds. However, as Jacobs pointed out [4], this type of phenomenon, if it occurs at low levels, may account for cloud-ceiling fluctuations of considerable importance to aircraft operations. Shown below are some observations of such a phenomenon made at San Diego, California. The ceiling fluctuations were measured by means of a vertical searchlight beam and an observer sighting at the illuminated cloud patch through a clinometer at a fixed distance from the searchlight.

Date	Time	No. Waves	Average Period	Aver. wave length	Average Ceiling	Average Amplitude
Jan. 15, 1936	7:10 P.M. to 8:45 P.M.	6	15.8 min.	(1900 m.)	185 m.	23 m.
Feb. 22, 1936	6:35 P.M. to 10:55 P.M.	7	37.1	4450	158	35

CYCLONE WAVES

Helmholtz also suggested that the extratropical cyclones originate as small disturbances on sloping surfaces of discontinuity (fronts) in the atmosphere. Later, during and after the first World War, the Scandinavian meteorologists, led by V. Bjerknes, studied the problem from both a theoretical and an observational viewpoint. Their efforts in the latter field showing the existence, structure, and life history of the cyclone waves on the "Polar Front" were far more successful than their accomplishments in the theoretical analysis of the phenomenon [5]. The mathematical problems involved were exceedingly difficult even with the simplifying assumption of neglection of the 2nd and higher-order terms of the "perturbed" velocities. Nevertheless, it was found that unstable waves of wave length and velocity similar to those observed in the atmosphere can originate on frontal surfaces [6]. This class of wave is characterized by wave lengths between about 500 and 3000 kilometers, so that the motion of air particles is influenced by the earth's rotation. Since the orbital motion is practically horizontal, the shearing instability is greater than gravitational stability, thus resulting in unstable waves whose amplitude increases until the wave cyclone "occludes," intensifies, and becomes transformed into a vortex as illustrated in Fig. 1.

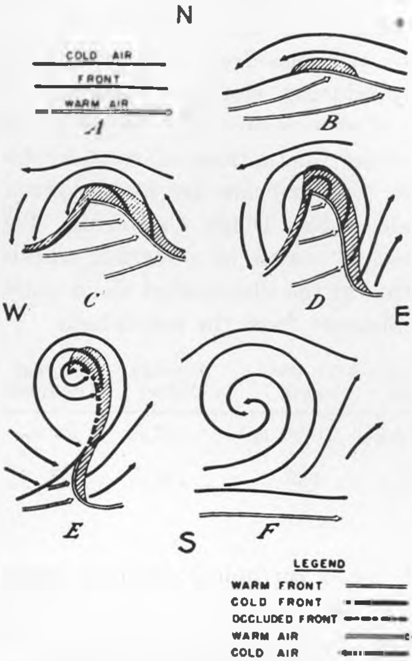

FIG. 1.—SCHEMATIC HORIZONTAL REPRESENTATION OF THE GRADUAL DEVELOPMENT AND ULTIMATE OCCLUSION OF A POLAR-FRONT WAVE.

This is the type of travelling atmospheric disturbance that produces the day-to-day weather changes affecting regions outside of the tropics. Along a portion of the Polar Front there may be an entire "family" of wave cyclones at various stages of development, as illustrated in Fig. 2.

The task of the daily forecaster then is to predict the motion and development of these individual wave cyclones as they progress with the Polar Front. Any method that could predict several days in advance the location, orientation, and intensity of the Polar Front itself would obviously define regions whose weather would be under the influence of the individual cyclone waves. It is believed that individual segments of the Polar Front occur in the troughs of the long waves in the Westerlies.

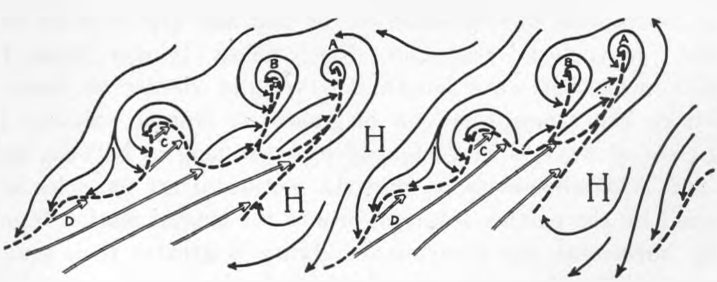

FIG. 2.—CYCLONE FAMILIES.

Long Waves in the Westerlies

High above the earth's surface flows a great current of air from west to east, energy of which is derived from the poleward decrease in air temperature. This current of about 3000 miles in width is known as the "Westerlies." Its axis moves north and south with the sun; in winter in North America it is found near the Canadian border and in summer it shifts farther north.

Although this current is called the "Westerlies," it is rather rare for the winds to remain wholly west-east in direction over one spot for any appreciable length of time, since waves of length varying from 3000 to 9000 kilometers are found in the current, most of them moving eastward, and some westward.

J. Bjerknes, son of V. Bjerknes, pioneered in the analysis of these waves [7], and was followed by C. G. Rossby [8], who extended this work by pointing out the fundamental effect of the earth's rotation on the origin and maintenance of the waves.

Rossby's theory can be described briefly in the following manner: Under certain restrictions we can assume that each moving column of air moves so as to maintain its absolute vorticity—that is, the sum of the local vorticity caused by the earth's rotation and the vorticity of the air column relative to the earth. The former vorticity increases with the sine of the latitude. The relative vorticity may be considered as made up of two terms—shear of the Westerlies and curvature of the Westerlies. Near the axis of the Westerlies we can neglect the shear and consider the relative vorticity to be expressed in terms of curvature alone. When an air column moves from the southwest with cyclonic (counter-clockwise) curvature, it moves over regions which are characterized by increasing cyclonic vorticity; to maintain the constant absolute vorticity the relative cyclonic vorticity of the air column must therefore decrease, and so the trajectory must show decreasing cyclonic curvature, then no curvature, and finally anticyclonic (clockwise) curvature as illustrated in Fig. 3. After a while the air column is moving from the northwest with anticyclonic curvature (vorticity) over regions of the earth which successively have smaller and smaller cyclonic vorticity. Thus the relative cyclonic vorticity of the air column must increase so that once again curvature of the trajectory changes sign. This process goes on until

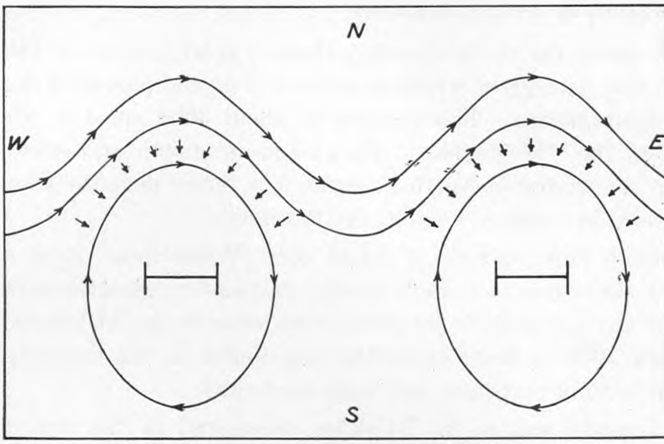

FIG. 3.—LONG WAVES IN THE WESTERLIES.

there exists a series of waves extending downstream from some region over which the initial cyclonic torque was given to the Westerlies. These critical regions are the coastal regions which possess strong west-east thermal contrasts such as the coast lines. The wave length of these waves is found to be

$$L_s = 2\pi\sqrt{\frac{U}{\beta}}$$

wherein U is the undisturbed velocity of the Westerlies, measured positively from west to east, and β is the variation with latitude of the vorticity due to the rotation of the earth; $\beta = \frac{2\Omega \cos \phi}{R}$, in which Ω is the angular velocity of the earth's rotation, ϕ is the latitude, and R is the radius of the earth.

When the waves themselves move, their velocity, c, is given by

$$c = U - \frac{\beta L^2}{4\pi^2} = U\left[1 - \left(\frac{L}{L_s}\right)^2\right]$$

Hence, given a definite value of the Westerly velocity U corresponding to a stationary wave length, L_s, then waves of length $L < L_s$ move from west to east, and waves of lengths $L > L_s$ move from east to west. For $U = 12$ meters per second or 25 miles per hour, the stationary wave length is 5000 kilometers (3000 miles).

These waves are of fundamental importance in weather forecasting because at their troughs they induce a flow of cold air from north and warm air from the south. This strong contrast in thermal properties causes a "front" to form, along which individual wave cyclones may originate and move. It is particularly important to note that, although the initial trough of the Westerlies may be caused by the strong thermal contrasts existing at coastlines, the successive troughs formed downstream may create their own thermal contrasts and fronts even over regions which originally had no west-east thermal contrast.

In order to remove the transitory effects of the daily cyclone waves and show the existence of the waves in the Westerlies, mean maps are drawn for a five-day period. Such a mean map drawn

Fig. 4.—Five-day Mean Map at 10,000 Feet, Feb. 15-19, 1941.
(Isobars are labelled 300 mbs. too high)

FIG. 5.—FIVE-DAY MEAN MAP AT SEA-LEVEL, FEB. 15-19, 1941.

for the period Feb. 15-19, 1941, at a height of 3 kilometers (10,000 ft.) is shown in Fig. 4. At northern latitudes there exists a large wave with a trough at 180°, a ridge at 130° W, and another trough at 70°W, corresponding to a length of about 5500 kilometers at 50°N. At lower latitudes in the same longitudinal sector there exist two complete waves of length about 3500 kilometers. The mean sea-level map for the same period is shown in Fig. 5 and the accompanying temperature and precipitation anomalies in Fig. 6 [9].

The central and eastern portions of the country are normal or below normal in temperature since they are under influence of cold air coming from the north behind the trough located off the coast. Since a good portion of this air comes from the cool, moist regions of the North Atlantic Ocean, it is not very cold for the winter season especially in the Northeastern States. The air mass, however, has deposited moderate to heavy amounts of precipitation in the region of the Great Lakes and Appalachian Mountains. The western portion of the country has mostly above-normal temperature because the air has come from the tropical portions of the Pacific Ocean; it has also produced moderate and heavy precipitation amounts in the Southwestern States. The long band of moderate to heavy precipitation in the Midwest is caused by the Pacific Air being forced over the wedge of dense Polar Air flowing from Canada.

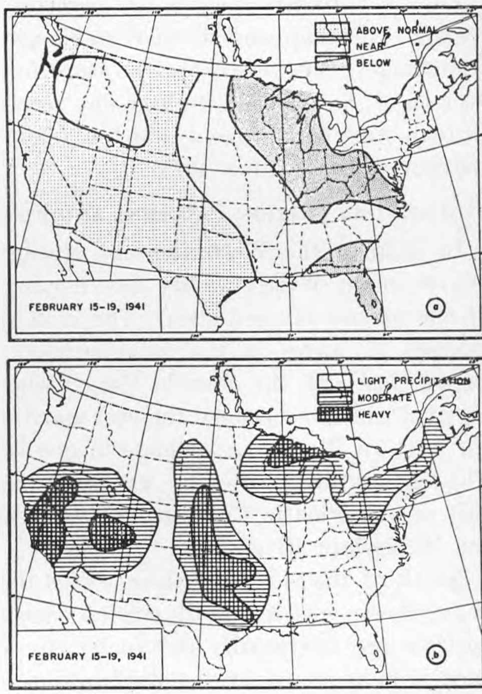

FIG. 6.—TEMPERATURE AND PRECIPITATION ANOMALIES, FEB. 15-19, 1941.

The precipitation anomaly divisions into heavy, moderate, and light for each State are so chosen that each class has 1/3 probability of occurrence; the temperature departures are divided into much above, above, normal, below, and much below, and are defined on the basis of past records to have probabilities ⅛, ¼, ¼, ¼, and ⅛, respectively. For Iowa in February the class limits for five-day precipitation amounts are 0.25 and 0.06 inches,[1]

[1] Heavy, $>$ 0.25 inches; Moderate $<$ 0.25 inches, $>$ .06 inches; Light $<$ .06 inches.

while for Louisiana, where the normal rainfall is heavier, the limits are 0.90 and 0.35 inches. The February class limits for temperature in Iowa are ± 17.4° F and ± 4.8° F,[2] while for Louisiana, where the temperature range is smaller, the limits are ± 12.5° F and ± 3.5° F.

Perfect forecasts receive 100 percent, forecasts based on chance receive 0 percent, and worse-than-chance forecasts receive negative percentages. On this basis, the score for the temperature forecasts over the U.S. made by the five-day forecast section of the Weather Bureau was + 48 percent and for precipitation + 16 percent from October, 1940, to June, 1941.

SUMMER CIRCULATION PATTERNS AND FORECASTING

In summer the Westerlies lose strength and shift to the north. Waves in the Westerlies are slow moving and tend to remain fixed at one or two favored sites. The most favored site for location of troughs of waves in the neighborhood of North America is the region just off the Pacific Coast, where, because of the strong thermal contrast between the cool maritime air and the hot air over the western Plateau, a cyclonic torque is applied to the Westerlies. This generates a series of waves downstream. The next trough east of the Pacific Coast trough is located normally just east of the Mississippi River.

South of the ridges of these waves there exists a series of large anticyclonic vortices as illustrated schematically in Fig. 3. These vortices are frictionally driven by the Westerlies and their existence is best shown by the distribution of moisture. The easterly branch of the eddy brings in dry air from the north while the westerly branch transports moist air to the north. As individual particles in the free air move, they maintain approximately constant entropy (at least for a period of several days in absence of evaporation and condensation); consequently, it is useful to trace the motion of air particles by plotting water-vapor concentration on a surface of constant entropy. These surfaces in general slope upward from south to north. In Fig. 7 is shown the normal summer circulation pattern as represented on the isentropic chart of

[2] Much above, temperature departure $> 17.4°F$; Above, temperature departure < 17.4, > 4.8; Normal, temperature departure < 4.8, > -4.8; Below, temperature departure < -4.8, > -17.4; Much below, temperature departure < -17.4.

which the potential temperature, θ, is 315° A. The dashed lines are contours expressed in meters, the solid lines represent the specific humidity given in grams of water vapor per kilogram of air, and the arrows fly with the wind at speeds increasing with the number of feathers.

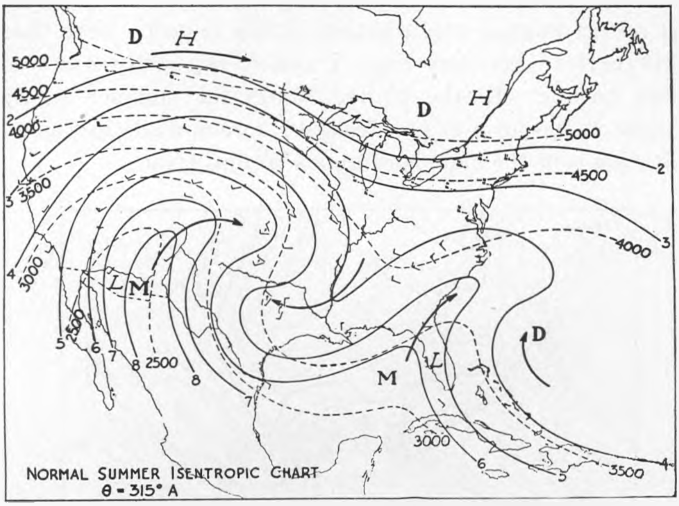

FIG. 7.—NORMAL SUMMER CIRCULATION PATTERN.

Two large eddies are shown, one located east of the Pacific Coast trough and centered in the southern Great Plains, and the other centered off the southeastern Atlantic States. The location of the latter eddy is determined by the wave in the Westerlies, which was initially induced by the cyclonic torque off the Pacific Coast and which depends in length on the speed of the Westerlies and on the latitude, as described above.

By their rearrangement of the moisture distribution, these large eddies exert a dominating influence on the summer rainfall regime over the United States. Under the moist current showers are frequent, while under the dry currents fair, dry weather prevails. These relationships follow not only from presence or absence of moisture but also from the fact that the moist air is rising and cooling while the dry air is sinking and warming, as indicated by the winds and contour lines.

The similarity of the precipitation-percentage pattern to the circulation pattern is shown in Fig. 8. This chart is based on precipitation amounts observed at several hundred stations, most of whose records are of 50-year duration. The percentage of the annual precipitation falling in August minus 8.5 percent (the percentage of the number of days in August to 365) is plotted to show those regions which receive more or less than the proportional amount of the annual precipitation in this month. Note the similarity in pattern between Figs. 7 and 8, especially south of the Canadian border. In the United States the summer rainfall is principally in the form of non-frontal showers, while in Canada the precipitation is of the more continuous frontal type.

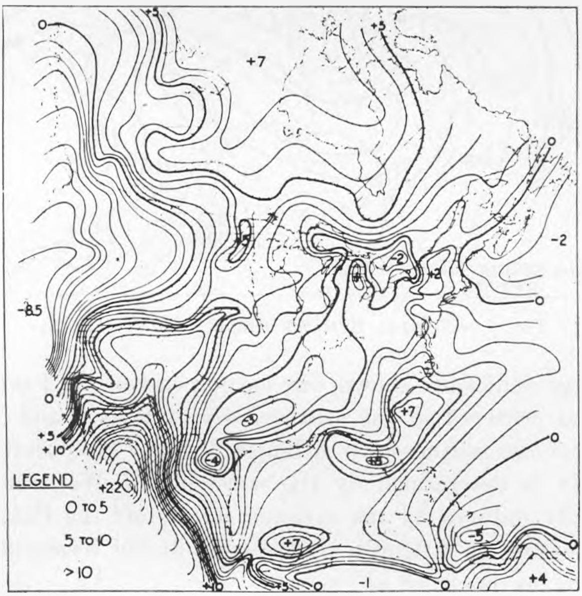

FIG. 8.—PERCENTAGE OF THE NORMAL ANNUAL PRECIPITATION (MINUS 8.5%) OCCURRING IN AUGUST.

The lines in Fig. 8 should not be interpreted as lines of equal precipitation amount. In the Southeast, under the eastern moist current, the summer precipitation amounts are nearly twice as great as those in the Midwest, which is under the central dry current. However, these latter precipitation amounts are greater than those under the western moist current. The reason for this is that this moist current has been lifted 2000 meters over the Mexican and the Western Plateau and so has lost much of the total amount of moisture in the vertical column. Although the number of days with precipitation is larger in summer over the Plateau, the amount of precipitation falling in any one shower is smaller than that falling in an average Midwestern shower. However, in absence of topographical differences such as exist between the Midwest and the Southeastern States, the precipitation amounts as well as percentages are definitely larger for the latter region. This is all the more remarkable since the circulation pattern existing on the sea-level weather offers no apparent explanation for this vast differ-

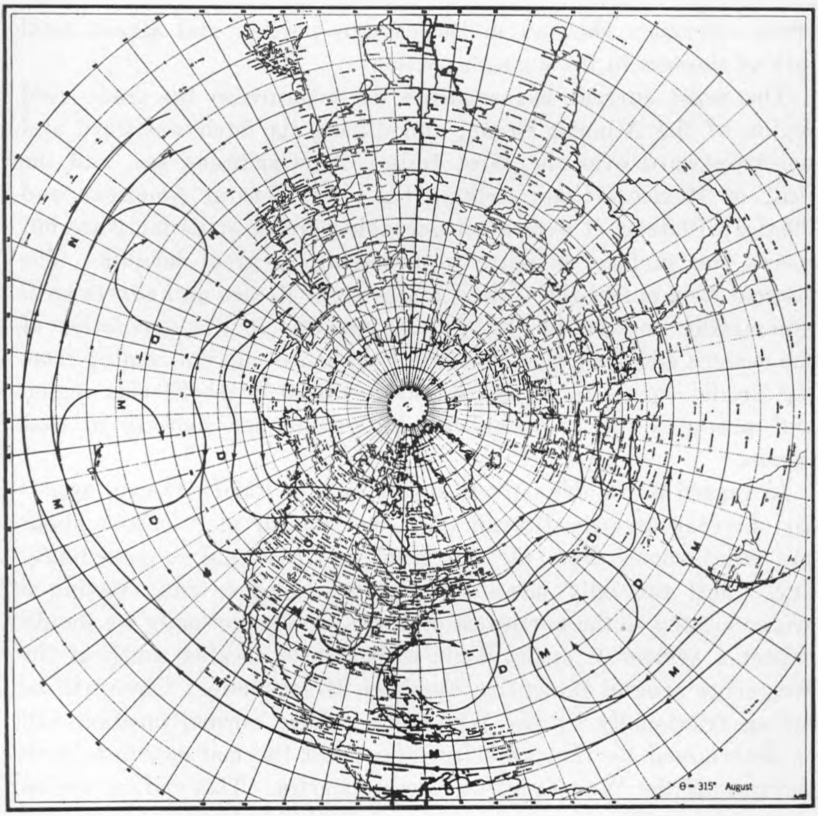

FIG. 9.—MEAN SUMMER POSITIONS OF THE ANTICYCLONIC EDDIES ON THE 315° ISENTROPIC SURFACE.

ence in rainfall regime. The solution is to be found in the upper-air circulation pattern which differs markedly from the sea-level pattern.

The almost total lack of precipitation in the Pacific States is another remarkable feature of the summer rainfall pattern. This phenomenon is explained by a current of dry air descending from great heights over the Pacific Ocean, as illustrated in Fig. 9. Off the West Coast this dry current comes under influence of the cyclonic torque and curves cyclonically. It thus enters the Western States as a dry southwesterly west flowing side by side with a moist current which has had an entirely different life history. In Arizona the boundary between these two currents is characterized by numerous afternoon showers in the eastern portion and almost total lack of showers in the western portion.

The moist current has acquired its moisture in the trade-wind region of the Atlantic Ocean, and during its fetch westward and northwestward over the West Indies, the Caribbean sea, and the Gulf of Mexico. Upon striking the lofty Central American and Mexican Plateau, it loses a considerable portion of its moisture but forces the surface moisture to penetrate to great heights. The current then enters the United States from Mexico and curves anticyclonically as seen in Fig. 9. In response to the circulation of the eastern eddy, another portion of the moist current coming from the Trades curves to the north and northeast in the Florida region and produces the frequent and heavy summer showers in that region.

In regard to the dry current in the eastern Pacific Ocean, upper-air observations over Hawaii show that the air aloft (above about one mile) comes from the northeast and east and is exceedingly dry. This can only mean that there is another eddy having a center located to the northwest of Hawaii. Theoretically we should expect a system of anticyclonic eddies to be located south of the Westerlies around the entire Northern Hemisphere. These will be driven frictionally by the Westerlies. Their normal position will be determined by the powerful land-ocean thermal contrasts such as exist off the West Coast of North America. This critical region fixes the location of several waves and eddies downstream and so exerts a controlling influence on the summer rainfall pattern over the United States. The west coast of Europe and North Africa is undoubtedly another control sector which fixes the positions of several eddies downstream. The eddies shown in the schematic picture in Fig. 9 were located by all available aerological data plus consideration of the theoretical lengths of the waves in the Westerlies.

The eddies occasionally wander from their normal positions for long periods of time; and when they do, the summer rainfall pattern becomes greatly changed from the normal.

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