SESSION ON

OPEN CHANNEL FLOW

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BACKWATER CURVES IN THEORY AND PRACTICE

by

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Introduction

Development of the theory of backwater curves has proceeded at an increasing rate since its inception about a hundred years ago. Until the beginning of the twentieth century, few engineers in this country even knew of the existence of backwater curves. In the present stage of the development of our national resources, however, a knowledge of the subject has become a necessary part of the engineer's equipment.

If one examines the literature of backwater curves, one finds a wide diversity in the type of treatment. Indeed, there might be said to be two or more schools of thought, each using different nomenclature and methods of computation. For example, the title of this paper includes the term "backwater curves" which is intended to refer generically to the various possible longitudinal water-surface curves of steady, gradually-varied flow in open channels. To many engineers, however, the term "backwater curves" refers to a certain type of longitudinal water-surface curve of steady non-uniform flow which occurs where a stream having a mild slope flows into a lake or pool. If we do agree upon the name of our curve we may still differ as to the method to be used for computing its shape. The present paper is not intended to heighten the conflict over these matters, but rather to assist in reconciling the different viewpoints by pointing out the advantages of each line of attack on the various problems which may arise.

Criteria of Flow

A necessary preliminary to a general knowledge of backwater curves is a good understanding of the criteria which give rise to the different cases. The first of these criteria is the normal, or neutral, depth. For a given channel shape, roughness, slope, and discharge, there is a depth at which the water will flow with its surface parallel to the bottom. (It is assumed that the slope is not so great as to give rise to flow in a series of traveling waves.) Actually, uniform flow at normal depth is only possible in a prismatic channel, but at every section in a non-uniform channel there is an imaginary normal depth at which the given quantity would flow if the section remained constant and the grade continued uniform for a sufficient distance. For uniform flow at the normal depth, the slope of the water surface is just sufficient to overcome friction.

The other criterion of importance is Belanger's critical depth. The depth is critical when the total head is a minimum for the given discharge and channel shape. An equivalent definition which is more convenient for computation is that the flow is critical when the velocity head is one-half the average depth. If the depth is less than the critical depth, the flow is said to be rapid. The velocity is super-critical, and downstream conditions are unable to affect the flow for any appreciable distance upstream. At the crest of a broad-crested weir, the flow passes through the critical depth as the water accelerates to super-critical velocity. Raising the tailwater level has no appreciable effect on the discharge over the weir until the submergence reaches the level where the flow is critical. This is not true of a sharp-crested weir or of a flat-crested weir not sufficiently broad to hold the jet in a fixed position.

If the depth is greater than critical, the flow is said to be tranquil. The velocity is sub-critical, surface waves are able to travel upstream, and downstream conditions do affect the flow.

CLASSIFICATION OF BACKWATER CURVES

In channels with bottom sloping downward in the direction of flow, eight markedly different surface curves may form, in addition to the special case of uniform flow at normal depth. The shape of the curve which will form in any given case will depend upon whether the normal depth is less than, equal to, or greater than the critical depth, and upon upstream or downstream conditions. If the channel bottom is level, or rises in the direction of flow, there is no normal depth, and the critical depth alone serves to distinguish four additional cases.

The twelve possible cases were first classified and described by Professor S. M. Woodward.[1] They are shown in Fig. 1, with the

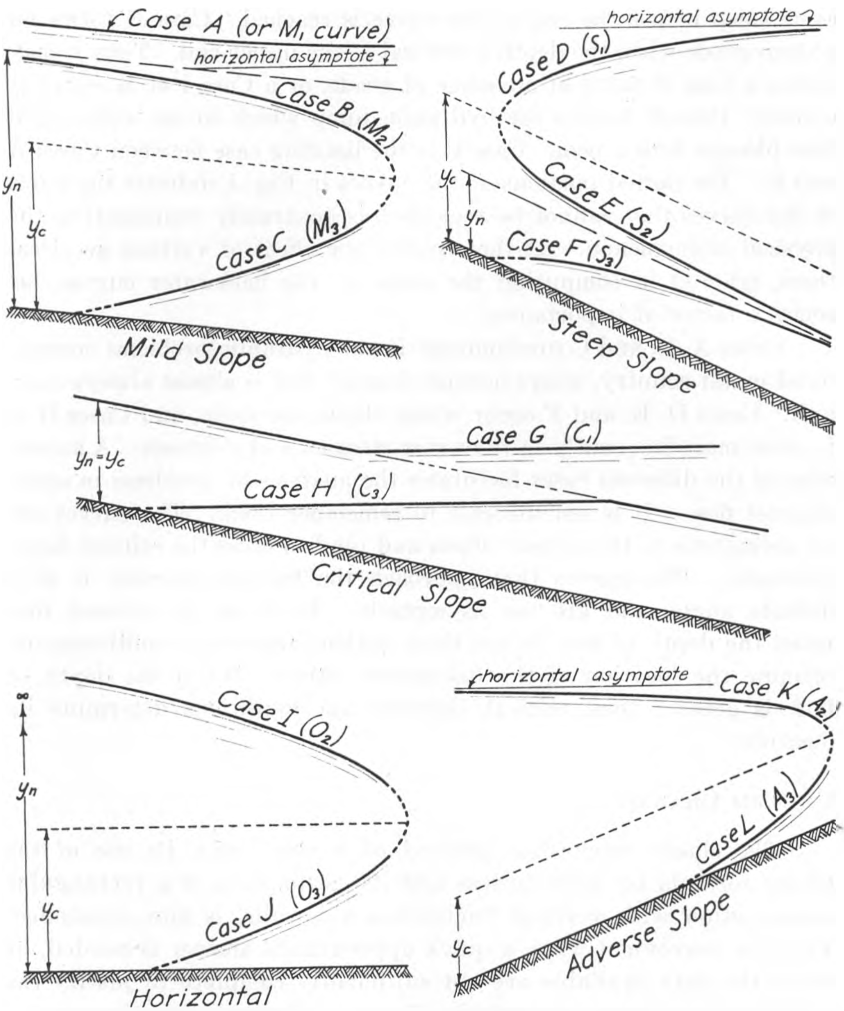

FIG. 1.—BACKWATER CURVES—LONGITUDINAL PROFILES OF GRADUALLY-VARIED STEADY FLOW IN OPEN CHANNELS. Flow is from left to right. Vertical scales are greatly distorted.

letters he assigned and the scheme later introduced by Professor Bakhmeteff[2] and completed by Professor Rouse.[3] The Case A, or M_1 curve is the one commonly known as the backwater curve, and the Case B, or M_2 curve is the one commonly known as the drop-down curve. Cases C, F, H, J, and L occur where water flows out from under a gate. Of these, only Case F can continue indefinitely to the right. The others must terminate in a hydraulic jump, or the channel bottom must become steep before the end of the curve is reached. Case E forms on a steep grade when the depth is critical at the upper end. Thus it may follow a Case B curve at a change of grade, or a Case I or K curve at a crest. Case D follows the hydraulic jump which forms when rapid flow plunges into a pool. Case G is the limiting case between Cases A and D. The dotted portions of the curves in Fig. 1 indicate the parts of the curves that cannot be regarded as accurately representing the physical phenomenon. At these points the effect of vertical accelerations, ignored in computing the shape of the backwater curves, becomes a factor of importance.

Cases A, B, and C predominate in the hydraulic problems encountered in flat country, where normal channel flow is almost always tranquil. Cases D, E, and F occur where slopes are steep, and Cases G to L occur most frequently in tidewater streams and channels. A knowledge of the different cases facilitates the analysis of problems in open-channel flow. It is not difficult to remember them. The curves are all asymptotic to the normal depth and tend to cross the critical depth vertically. The curves that approach the bottom intersect it at a definite angle, and are not asymptotic. It should be noticed that when the depth of flow is less than critical, upstream conditions determine the location of the backwater curve. When the depth of flow is greater than critical, downstream conditions determine its location.

UNIFORM CHANNELS

The classic integration method of Bresse,[4] with its use of the Chezy formula for friction loss and its assumption of a rectangular cross-section with vertical frictionless sidewalls, is now outmoded. Yet it is convenient when a quick approximate answer is needed, or when the data available are not sufficiently complete to justify the use of a more accurate method. Tables of Bresse's function are included in several standard textbooks on hydraulics. N.Y.A. students at the Hydraulic Institute have computed a new table, prints of which may be obtained at nominal cost by writing to the author.

With the method developed by Professor Bakhmeteff,[5] friction may be evaluated by either the Manning or Kutter formulas. The channel shape is taken into account by means of an empirical exponential approximation. The integration is not complete, however, in that the effect of variation of velocity head must be considered separately. This is not a serious disadvantage when the depth is quite a bit greater than the critical. The effect of variation of velocity head is usually insignificant for the common M_1 curve, as the velocity head itself is comparatively small. Moreover, the tendency of divergent flow to be turbulent justifies, to some extent, neglect of the regain of velocity head in this case. In practice, it is best to omit or include velocity head changes according to whichever will give the safest result for the problem at hand.

The method presented by Nagaho Mononobe[6] is a complete integration method, in which velocity-head changes are taken into account and friction is evaluated by means of the Manning formula. Unfortunately, however, his charts are difficult to use, and until accurate tables of his function are available, the carefully prepared tables of Professor Bakhmeteff are to be preferred.

The two cases of backwater curves in horizontal channels are very easily solved by the direct method given by Professor Bakhmeteff. Tables that facilitate the computation of backwater curves in channels of adverse slope are given by Arthur E. Matzke.[7]

All of the cases of backwater curves in uniform channels may be computed by means of step methods. Step methods are simpler in theory though more laborious in application. They do not require the use of tables, and are preferred by those who usually work with non-uniform channels. If one has many backwater curves to compute in uniform channels, however, he will find it distinctly to his advantage to use the integration methods. In using the step method for uniform channels, the tedious cut-and-try computations necessary in non-uniform channels may be eliminated by solving for lengths along the curve corresponding to increments of depth.[8]

NON-UNIFORM CHANNELS

All river channels, and many artificial channels, are non-uniform. In such channels, the various cases of the backwater curves lose their mathematically regular shape, and the special case of uniform flow at normal depth is no longer an ideal straight line. Step methods are clearly indicated here. The particular step method to be used depends upon the accuracy required, the funds available, and the relative cost of the different methods, including the cost of getting the data. A few general remarks applying to all the methods will be in order first.

If the depth of flow is greater than critical, the step computations should progress upstream; if it is less than critical, they should progress downstream. In order to predict the water-surface curve in complicated cases, it is desirable to be acquainted with the twelve different types of backwater curves, and to bear in mind the possibility of a hydraulic jump.

It is sometimes necessary to compute a backwater curve through a stretch of river that is not near a point of control, so that no definite information as to what elevation to start with is available. In this case all that is necessary is to assume an elevation some distance downstream from the reach, if the flow is deeper than critical, and carry the computations upstream to the beginning of the reach. The error will tend to disappear, as shown in Fig. 2. To make sure that the error has become small enough to neglect, another computation should be made, starting at a quite different elevation. If the two curves converge closely at the lower end of the reach, the correct elevation there has been found, and the computations may now be carried up through the reach. Fig. 2 serves as a warning as to what would happen if the computations had been started above the reach instead of below. In figuring the wrong direction, any small error will tend to make the curve deviate further and further from its correct location. Even the difference between the true value of friction loss and its value as approximated by the Manning or Kutter formula is an error large enough to eventually affect the slope of the curve. Unless each such error is followed by others exactly compensating for its effect, the curve will inevitably diverge from its true location.

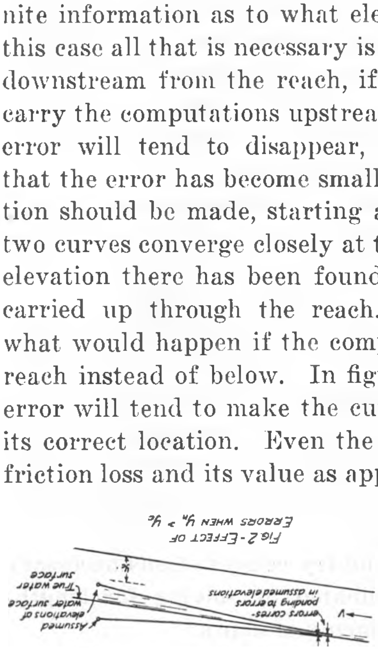

FIG. 2.

When a step method is to be used for what is essentially one of the cases of backwater curves modified by irregularities in the channel, care must be taken not to use steps that are too long. The true average slope through the step has to be approximated by some sort of a numerical average, giving rise to a systematic error if the radius of curvature of the water surface profile is continuously changing in the same direction.

In computing backwater curves by step methods, the effect of change of velocity head may be omitted if the velocity head is small, or if the change accompanies a decrease in velocity due to a sudden expansion of the cross-section. In considering the changes of velocity head it is reasonable to assume that decrease of velocity head cannot cause the surface to slope upward in the direction of flow. In case of doubt, it is well to make the assumption that will yield results on the safe side.

The data necessary for the step computations may include (1) detailed information as to the topography and hydraulic roughness of the bed and banks of the stream, or (2) records of actual profiles and discharges, preferably for free river conditions previous to the construction of the dam.

If conditions of the problem dictate the use of survey data, there is still opportunity for considerable difference in the computation procedure. One may plunge immediately into the cut-and-try computations, as in the example given in Chapter XI of Part VII of the Miami Conservancy District Technical Reports, or one may take time for preliminary computations to eliminate as many of the variables as possible. Two excellent articles describing different methods of expediting this work are those by H. R. Leach[9] and J. C. Stevens.[10]

The method which uses as data actual measured profiles of the stream was first described by C. I. Grimm.[11] In his original paper Grimm used the slope at the lower end of the step as an approximation to the average slope through the reach. Under this assumption the method is direct, but it involves a systematic error which may become large unless the steps are quite short. The error may be more conveniently minimized by finding the average slope through the reach by successive approximations or by means of the nomographs developed by I. H. Steinberg.[12] The changes of velocity head due to channel irregularities are automatically taken into account by the Grimm method, while changes of velocity head due to change in the general slope of the backwater curve are neglected. Its application should be restricted to cases where the slopes of the backwater curves are not far different from the slopes of the measured profiles. The simplicity and convenience of the method recommend it for further study.

No matter which step method is used, the diagram given at the end of the article by H. R. Leach and elaborated in King's Handbook[13] is valuable if many backwater computations are to be made for the same reach. Its preparation takes time, but once it is completed the labor of subsequent computations is reduced to a minimum.

VERIFICATION

Few measurements verifying the theory of backwater curves are available. The paper by Nagaho Mononobe describes laboratory tests in several small channels of different shapes and roughnesses. A comparison of computed and measured curves on the Skunk River is given by Albion Davis.[14] Both report good agreement between the computed and observed curves. It is to be hoped that increasing use of the Grimm method will make available additional comparisons. More are needed, for we know very little about the rate of loss of energy in gradually-varied flow.

The writer has cited a few of the outstanding articles which have been published on the subject of backwater curves. It is certain that the profession has made notable progress since the turn of the century, when the question, "Is there back-piling of water above dams?" was the subject of a controversy in the technical press.

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