ATMOSPHERIC TURBULENCE AND THE MEASUREMENT OF EVAPORATION [1]

by

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Some of the difficulty attendant on the measurement of evaporation arose from the original formulation of the problem. Evaporation was thought of as the reverse of precipitation, and since precipitation could be measured in a straightforward manner by collecting samples in rain gages it seemed reasonable to seek analogous methods for evaporation—to allow water to evaporate freely from an evaporation gage back into the atmosphere and to measure the loss. The water caught in rain gages does provide a measure of the precipitation falling upon a natural surface, but water lost from an evaporimeter provides only a rough measure of evaporation from reservoirs or other free water surfaces and no measure at all from land surfaces. Depth of precipitation is almost exclusively a function of the physical qualities of the atmosphere. But the depth of water evaporated from a surface is a function of the physical properties not only of the atmosphere but also of the evaporating surface.

Nearly a century ago George Marsh said:

"There is one branch of research which is of the utmost importance . . . but which, from the great difficulty of direct observation upon it, has been less successfully studied than almost any other problem of physical science. I refer to the proportions between precipitation, superficial drainage, absorption, and evaporation. Precise actual measurement of these quantities upon even a single acre of ground is impossible . . ."

Since that time much progress has been made in the measurement

of precipitation and runoff, and recently a great deal of attention has been given to the problems involved in the measurement of infiltration into different soils and under different forms of soil management. But evaporation has continued to resist measurement, despite the fact that its determination has become increasingly necessary as measurements of rainfall, runoff, and infiltration have been improved.

In recent years investigations on the vertical distribution of temperature, moisture, and wind velocity in the lower layers of the atmosphere have resulted in notable advances in our understanding of the mechanism of the transfer of moisture, heat, and momentum through these layers. This, in turn, has led to a reexamination of the old problem of evaporation measurement, with the result that it is now coming to be regarded as a transfer problem rather than one of sampling.

Water-vapor molecules will move away from an evaporating surface by the ordinary process of diffusion. But diffusion is so slow that molecules would return to the surface of the liquid nearly as rapidly as they leave it, and evaporation would be an extremely unimportant process if the water vapor were not removed from the vicinity of the liquid surface in some other way. Actually, the process of turbulent mixing is so much more important than molecular diffusion in carrying water molecules away from an evaporating surface that it may be considered almost wholly responsible for evaporation. In the ·turbulent layer the mixing process depends on the shearing stresses associated with the roughness of the ground and with wind velocity, and increases as the values of these factors increase. The mixing process also depends upon the stability or density structure of the air, the intensity of mixing diminishing, under otherwise constant conditions, as stability increases. The vertical motion of the air in the turbulent layer tends to establish an adiabatic distribution of temperature and a uniform admixture of water vapor, and thus to eliminate differences in moisture concentration. If moisture is neither added to nor withdrawn from the turbulent layer, its moisture content quickly becomes uniform throughout.

On the other hand, water vapor emitted from an evaporating surface is transported upward and scattered throughout the turbu-

lent layer. Thus, as long as a stream of water vapor flows upward
into the turbulent layer the moisture concentration will be highest
at its base and diminish upward, and a moisture gradient directed
upward will be established. Such a gradient can be maintained
only so long as moisture continues to be added below. Soon after
evaporation ceases, the moisture is distributed uniformly through-
out the layer, and the moisture gradient is thereby destroyed.
Similarly, if water vapor is abstracted from the base of the layer
by condensation in the form of dew or frost, the moisture concen-
tration there will be reduced; and so long as removal of moisture
continues, a moisture gradient directed downward will be main-
tained. The greater the intensity of turbulent mixing, the greater
is the tendency toward the establishment of uniformity of moisture
concentration and the greater the evaporation or condensation re-
quired to maintain a constant gradient upward or downward.
Similarly, with a given rate of turbulent mixing, the greater the
rate of evaporation, the steeper will be the moisture gradient.

Air flowing along the surface of the ground encounters a fric-
tional resistance and is slowed down. The effect of this resistance
is felt for hundreds of feet aloft. Within this zone of frictional
influence each successively higher layer of air moves faster than
the one immediately beneath so that shearing stresses are set up
between them, which result in the upward and downward displace-
ment of small masses of air, each tending to preserve the horizontal
velocity that it had previously. If a mass moves upward to a faster
moving layer, in the process of mixing it will tend to retard the
air in the higher layer. Similarly, a mass of air moving downward
will tend to speed up the air in the lower layer. Thus, the differ-
ence in velocity between two adjacent levels tends to be equalized as
horizontal momentum is transferred downward and ultimately
dissipated at the ground. The tendency toward equalization of
velocity depends on the rate of vertical mass interchange, or the
intensity of turbulent mixing.

The greater the differences in velocity between adjacent layers,
or the stronger the shear, the greater is the tendency for turbulence
to develop and to reduce the shear. If there were no external
influences helping to maintain the differences in wind velocity at
different levels, turbulent mixing would eventually equalize the

velocities in all levels and the air would move as a solid. However, since the supply of momentum in the upper air is practically inexhaustible, and since momentum is constantly being converted into heat and lost at the ground surface, wind velocities in different levels are never completely equalized. Thus, although turbulence tends to destroy shear, nevertheless the strongest mixing at a fixed level near the ground is associated with the strongest shear.

The dependence of vertical differences in velocity on the intensity of turbulent mixing is important in the measurement of evaporation because it enables us to determine the rate of mixing in the atmosphere by measuring the vertical gradient of wind velocity.

The intensity of turbulent interchange or the Austausch coefficient is related to the wind gradient by the following equation:

$$A = \frac{\rho k_0^2 z (u_2 - u_1)}{\log_e \left(\dfrac{z_2}{z_1} \right)} \tag{1}$$

Moisture is transferred upward through the atmosphere from an evaporating surface by the same turbulent motion that causes a downward transfer of momentum, and it can be shown that

$$E = - A \frac{dq}{dz} \tag{2}$$

in which E is evaporation, dq/dz is the rate of the change of moisture concentration with respect to height, and A is again the Austausch coefficient. Since it is reasonably assumed that the Austausch for momentum and matter are the same, the value of A in Eq. (1) can be substituted in Eq. (2):

$$E = - \frac{\rho k_0^2 z (u_2 - u_1)}{\log_e \left(\dfrac{z_2}{z_1} \right)} \frac{dq}{dz} \tag{3}$$

After solving for dq/dz and integrating between levels z_2 and z_1 in the turbulent layer, the general equation for evaporation is obtained:

$$E = \frac{\rho k_0^2 (q_1 - q_2)(u_2 - u_1)}{\left(\log_e \dfrac{z_2}{z_1} \right)^2} \tag{4}$$

The formula giving evaporation in inches depth per hour is

$$E = \frac{1.34 \; k_0{}^2 P (q_1 - q_2) \; (u_2 - u_1)}{(T + 459.4) \left(\log_e \dfrac{u_2}{u_1} \right)^2} \tag{5}$$

where k_0 is the universal turbulence constant, P the pressure in inches of mercury, q_2 and q_1 the specific humidity in grams of moisture per kilogram of air, u_2 and u_1 the mean wind velocities in miles per hour at heights z_2 and z_1, and T the temperature in degrees Fahrenheit.

The accuracy with which evaporation is determined depends on the accuracy of measurement of each of these terms. Both moisture and wind appear as differences that are usually quite small. Proportionately small errors in observation of moisture concentration or wind velocity can result in large percentage errors in the differences and consequently in the final computation of evaporation. Likewise, k_0 enters the equation as a squared term; thus any error in the determination of its value is exaggerated in the evaporation computation.

Neither atmospheric humidity nor wind velocity is easy to measure; and of the two, surprisingly enough, it appears that the latter is the more difficult. It is quite as important to know the vertical structure of velocity with low wind speeds as with high. But meteorologists have never given much attention to the measurement of low velocities, and, consequently, anemometers are designed to withstand velocities at the upper limit of the scale but are so insensitive that they stop altogether at low velocities. The frictional resistance and inertia of the familiar whirling-cup anemometer are such that correction factors obtained for steady flow in a wind tunnel do not yield true mean values for the varying velocities encountered in the free atmosphere. Furthermore, it is not easy to find a number of anemometers whose starting speeds, stopping speeds, and running speeds are identical. Thus, when simultaneous measurements are made with a number of anemometers at different levels, in order to define a velocity gradient, the difficulties are multiplied and there may be considerable error, particularly in the low-velocity range. Going downward in the wind profile, velocities below the running speeds

of the anemometers are increasingly frequent; this is an added source of error.

Because of these difficulties, there are in existence few good observations of wind velocity at various levels near the ground, in spite of the importance of a knowledge of wind structure in the determination of the characteristics and magnitude of turbulent mass interchange. Most recent studies of the structure of the wind near the ground have made use of observations published by Hellmann, Shaw, Wüst, Best, and Sverdrup, which are neither very numerous nor free from instrumental error. To obtain a precise law of vertical wind distribution in the layer near the ground it is necessary to have observations taken throughout the day and at various seasons of the year, providing accurate measurements at various velocities, during different conditions of atmospheric stability, and over different kinds of surfaces.

Most workers have used either a power law or a logarithmic law in attempting to express the relationship between wind velocity and height. The power law has usually taken the form

$$u = u_1 \, z^a$$

where u is wind velocity at height z and u_1 is wind velocity at unit height. The logarithmic law is of the type

$$u = \frac{\log z - \log z_0}{\log a}$$

where $\log z_0$ is the ordinate intercept and $\log a$ is the slope of the straight line obtained by plotting $\log z$ as ordinate and u as abscissa. The weight of observational evidence has led to a preference for the logarithmic law. Furthermore, the theoretical formulation developed by the Prandtl school requires that the vertical distribution of wind velocity follow a logarithmic law in the part of the turbulent layer where shearing stress, τ, and density, ρ, may be assumed constant.

Available observations are not sufficiently accurate to determine the precise form of the vertical distribution of wind. There is a suggestion, however, in observations being made currently that neither power nor logarithmic law adequately describes the distribution, but that a power of the velocity other than the first power is proportional to the logarithm of height. For unstable conditions

u^2 plotted against log z gives a very close approach to a straight line, and as stability increases the appropriate power of the velocity diminishes. It appears, therefore, that an improvement in the law of vertical wind distribution is achieved by combining logarithmic and power laws:

$$u^p = \frac{\log z - \log z_0}{\log a}$$

The exponent p is believed to vary between 2.0 with fully developed turbulence and some value less than 1.0 when turbulence reaches its smallest actual value. Should additional observations confirm the tentative conclusion that in fully developed turbulence the square rather than the first power of the velocity varies with the logarithm of height, it will be necessary to re-examine and revise the theory of fluid turbulence as developed by the Prandtl school. This is an important task for fluid mechanics.

In the theoretical treatment of turbulence, Prandtl, von Kármán, Rossby, and others have introduced the concept of the mixing length. The mixing length is considered as the vertical distance traveled by the eddies from the level where they originate to the level where they lose their individuality and mix with the turbulent fluid surrounding them. This "path of mixing" is related to the size of the eddies; the larger the diameter of an eddy, the greater the distance it may travel before it disintegrates. Apparently, too, the size of an eddy is directly proportional to the distance from the boundary surface at which it forms. Thus the mixing length is proportional to the height. This relationship is shown by the expression

$$l = k_0 z$$

in which k_0 is a constant of proportionality, known as the "universal turbulence constant," or von Kármán's constant.

The constant of proportionality, k_0, relates the mixing length, l, to height, z. Its numerical value has been determined mainly from measurements of velocity distributions of water in smooth and rough pipes and from wind-tunnel measurements, and has variously been found to be 0.36, 0.38, and 0.40. A recent study of flow of water in open channels has yielded values of k_0 ranging between 0.22 and 0.43 depending on the geometry of the channel.

The constant, k_0, can be determined in a straightforward manner for flow in wind tunnels and pipes because independent measurements of the shearing stress may be made in terms of the pressure distribution as well as the velocity distribution. Since no such straightforward procedure can be used in the free air, k_0 has never been determined for the atmosphere, and thus its appropriate numerical value is uncertain.

There is a strong suspicion that the currently adopted value of k_0 of 0.38 is too low. We have made observations on moisture and wind and have computed the evaporation for several locations. The computations are all of the right order of magnitude but seem to be smaller than one might expect.

Over a meadow in Arlington, Va., during 10 months in 1939, the computed total evapo-transpiration was 11.97 inches. The precipitation for the same period was 26.34 inches. Taking account of condensation in the amount of 1.81 inches also, the runoff for the period is computed to be 16.18 inches. The runoff for the same period from the 8.5-square-mile drainage basin of the North River a few miles east of Washington was 13.85 inches, according to Geological Survey measurements. Precipitation is not identical in the two areas and they are not strictly comparable in other respects. On the basis of the comparison the evaporation at Arlington might reasonably be 2.33 inches or 19 percent greater than its computed value. An increase in the value of k_0 from 0.38 to 0.416 would have given the larger value of evaporation.

During the summer of 1941 we operated an evaporation station over a cornfield at the National Agricultural Research Center in Maryland. The computations of evapo-transpiration appear to be reasonable but might have been somewhat larger.

In February, 1942, we installed a station over the water surface of Lake Corpus Christi, Texas. The evaporation data only for the month of March are as yet available but these values also are smaller than were expected.

It will be possible, presumably, to check the value of k_0 empirically by determining the evaporation from the surface of a reservoir, where the loss can also be determined independently by direct measurement of inflow and outflow. However, a more direct determination would be preferable.

An equation developed by Ertel for determining the Austausch coefficient from short-time fluctuations of any physical property of the atmosphere, such as momentum, temperature, specific humidity, or dust, suggests a possible way to determine k_0. Ertel's Austausch equation does not contain the proportionality factor, k_0. Thus, if it is equated to Prandtl's expression for Austausch it should be possible to determine the value of k_0 from measurements of the short-time fluctuations of wind velocity. This is a question that might profitably be examined by someone actively working in the field of atmospheric turbulence. At any rate it is most important to obtain a more reliable value of k_0 so long as Prandtl's theoretical formulation is followed.

It is clear that much remains to be done in the study of atmospheric turbulence, but enough has already been done to show that it is going to be possible to measure the evaporation from any natural surface. As the turbulence theory is revised, and as instruments for more accurate measurement of wind velocity and humidity are developed, the computations of evaporation should become more reliable. The future promises that measurements of the transfer of moisture to the atmosphere from all types of geographic surfaces will become available, and that they will provide information on the moisture requirements of various crops and types of natural vegetation, and on the effectiveness of various moisture-conserving practices. With the accumulation of this information the interrelations of climate, hydrology, and agriculture will be more clearly understood.