

SUSPENSION OF SEDIMENT IN UPWARD FLOW

by

HUNTER ROUSE

Professor of Fluid Mechanics

Consulting Engineer, Iowa Institute of Hydraulic Research

Engineers have long been aware of the fact that the percolation of a fluid through a granular material will displace the material in the direction of flow if restraining forces are not sufficiently great. The quicksand phenomenon is a well known instance of such conditions as found in nature, while the method of washing rapid-sand filters by reversing the normal direction of flow represents an engineering application of the same principle; in either case the drag exerted by upward flow through an initially compact bed tends to exceed the immersed weight of the bed material, thereby producing an increase in bed porosity and a corresponding reduction in drag. Although in such examples of upward flow the material is rarely expanded to a point at which the grains are no longer in mutual contact, it is not unreasonable to presume that ever higher degrees of expansion would be obtained with increasingly higher rates of flow — the particles ultimately becoming so dispersed as to be in a state of complete suspension within the rising fluid. It is with the analysis of this phase of the phenomenon that the present paper is concerned.

Once bed material is displaced by such flow, it is apparent that the coefficient of permeability k in the Darcy relationship $Q = k A dh/dz$ will no longer equal that of the initially compact, uniformly mixed material, but will increase considerably as the interstices within the material grow in size. Moreover, as the expansion of the bed reduces the frictional restraint upon the individual particles, they will not only begin to go into suspension, but will at the same time assume positions commensurate with the resistance which they offer to the flow. In other words, finer or lighter particles will be carried higher than coarse or heavy ones, until a state of complete suspension displays almost perfect stratification of material according to size and density. If the upward flow is then abruptly

stopped, the material will settle to yield an even more perfectly stratified bed.

Since the Darcy coefficient k is at best an empirical factor, known values of which apply specifically to the case of pure percolation, a more rational approach to the case of complete suspension is to be preferred. Let it be assumed, therefore, that the immersed weight of the particles in suspension is exactly balanced by the drag exerted upon them by the upward flow. The external forces upon the elementary "free body" of the fluid-solid mixture shown in Fig. 1 must then also be in equilibrium, such forces involving the

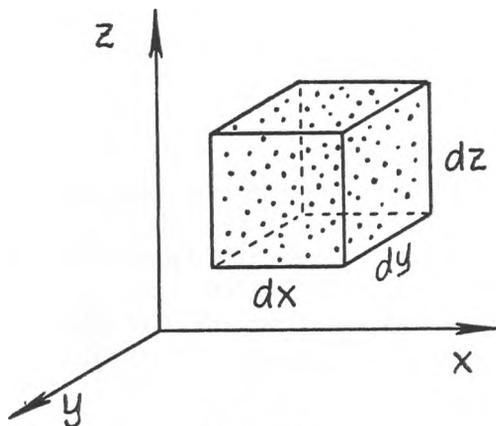


Figure 1.

weight of the mixture and the normal and tangential stresses upon the faces of the element. Designating by γ_s the specific weight of the solid matter, by γ the specific weight of the fluid itself, and by e the ratio of the volume of solids to volume of mixture, it follows that the weight of the element will have the magnitude $[e(\gamma_s - \gamma) + \gamma] dx dy dz$. If conditions are statistically similar at all points in any horizontal plane, the mean tangential stresses will be equal to zero over each of the faces. Normal stresses in the horizontal directions will then in themselves yield a state of equilibrium, while in the vertical direction the weight of the element will be balanced by the difference in pressure between the lower and upper faces. That is, expressing the change in pressure intensity with elevation as dp/dz , it will be seen that

$$- [e(\gamma_s - \gamma) + \gamma] dx dy dz - \frac{dp}{dz} dz (dx dy) = 0$$

whence

$$-\frac{dp}{dz} = \gamma + e(\gamma_s - \gamma)$$

Evidently, the rate of decrease of pressure intensity with elevation at any point is equal to the specific weight of the suspension at that point. Or, defining the concentration of the suspension as the difference between its specific gravity and that of the fluid alone (i.e., $c = e(\gamma_s/\gamma - 1)$),

$$-\frac{dp}{dz} = \gamma(1 + c)$$

In case the fluid is a liquid, the foregoing equation may be written in the more significant form

$$-\frac{dh}{dz} = c \tag{1}$$

in which the quantity h represents the manometric head $p/\gamma + z$ within the suspension—that is, the height to which the liquid would rise in an open manometer tube connected to a piezometer at the point in question. Eq. (1) therefore states that the rate of decrease of manometric head in the vertical direction is equal to the local concentration of the suspension. More practically expressed, the difference in manometric head between any two levels is directly proportional to the immersed weight of the material held in suspension between these two levels.

Although the foregoing relationship was obtained by assuming a state of static equilibrium to prevail between the immersed weight of the material and the drag exerted by the flow, this effectively presumes a kinematic balance between the settling velocity w of the material and the local velocity v of the upward flow. The settling velocity of the individual granules probably varies somewhat with the concentration of the suspension, but as a first approximation it may be considered proportional to the velocity of fall of a single representative particle in still liquid; thus, $w = K_1 v$. The local velocity of flow may likewise be expected to vary with concentration (or vice versa), but in a manner which is more readily evaluated. Representing by Q the rate of upward flow past a horizontal section of total area A , the local upward velocity should be directly proportional to the rate of flow and inversely proportional to the area of the reduced flow section. If the mean particle diameter is desig-

nated by d , and the mean spacing of the particles by L , the area of the total section may be expressed as $K L^2$ and that of the reduced section as $K (L^2 - K_2 d^2)$. Therefore, introducing the nominal velocity of flow $V = Q/A$ and combining the several proportionalities, it appears that

$$w = K_1 v = K_1 \frac{Q}{K(L^2 - K_2 d^2)} = K_1 \frac{KVL^2}{K(L^2 - K_2 d^2)} = \frac{K_1 V}{1 - K_2 \frac{d^2}{L^2}}$$

Similarly, the concentration c may be expressed in terms of the parameters d and L , in combination with the specific weights of fluid and solid, through the following proportionality:

$$c = e \left(\frac{\gamma_s}{\gamma} - 1 \right) = K_3 \frac{d^3}{L^3} \left(\frac{\gamma_s}{\gamma} - 1 \right)$$

Therefore,

$$\frac{d^2}{L^2} = \left(\frac{c}{K_3 \left(\frac{\gamma_s}{\gamma} - 1 \right)} \right)^{2/3}$$

Substitution of this expression in the foregoing relationship for w then yields the general equation

$$c = K_3 \left(\frac{\gamma_s}{\gamma} - 1 \right) \left(\frac{1 - K_1 V/w}{K_2} \right)^{3/2}$$

Finally, combining constants and introducing Eq. (1).

$$c = - \frac{dh}{dz} = A \left(1 - B \frac{V}{w} \right)^{3/2} \quad (2)$$

This equation is evidently in accord with the earlier statement that the finer material is carried toward the top of a zone of suspension, since c can decrease with elevation only if w also becomes smaller. Indeed, since the gradient dh/dz is a measure of c , one might now conclude that measurements of manometric head at various levels in such upward flow would provide a quantitative means of determining the fall-velocity characteristics of the suspended material. The import of such a conclusion cannot be over emphasized, for fall-velocity characteristics are comparable—and in some cases preferable—to size characteristics obtained by physical analysis. Certain features of the derivation, however, must not be overlooked. The coefficient A is governed by the shape and relative density of the material and perhaps even by viscous effects upon the pattern of flow around the individual particles; only if these factors are

essentially the same for all particle sizes may A be expected to retain a constant magnitude. The factor B , on the other hand, embodies the proportionality between the rate of settling and the local velocity of the upward flow; since turbulence can produce a state of suspension without any upward flow whatever, the magnitude of B must be expected to vary with the degree of turbulence present in the flow.

To what extent such influences will affect the practicability of this relationship can only be determined through experimental tests, a series of which were conducted at the University of Iowa in 1939-40 by Warren DeLapp and described in "Sediment Behavior in Upward Flow," a thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the Department of Mechanics and Hydraulics. Duplicate studies were made in a glass cylinder 12 in. in diameter and 19 in. high and in a Lucite cylinder $1\frac{3}{4}$ in. in diameter and 54 in. high, each provided with means of measuring the manometric head over practically the entire height of cylinder. The granular material investigated consisted of quartz sand artificially graded to yield a size-frequency distribution closely following the normal error curve, 2% being finer than 0.07 mm. and 2% coarser than 0.8 mm.

A typical series of measurements made in the larger cylinder is shown in Fig. 2, the ordinate scale representing elevation above the fine-mesh screen which served to support the material at zero discharge, and the abscissa scale the difference between the manometric head within the water-sediment mixture and the head within the clear water above. Prior to the beginning of manometer readings, the entire bed was brought into suspension, and thereafter allowed to settle in stratified form. Measurements at a very low rate of flow then yielded the bottommost curve, the form of which is typical of pure percolation through a bed of decreasing grain size — that is, the Darcy permeability k grows smaller as the interstices are reduced in size, the slope — dh/dz attaining its maximum value at the bed surface. At the next higher rate of flow, however, a reversal in curvature is noted in this region, indicating that the topmost material has been carried into suspension. Continued increase in the discharge brings more and more material into suspension, at the same time increasing the drop in head through that portion of the bed which has not yet begun to expand. Finally, however, the form of the curve indicates that the entire bed has been carried into sus-

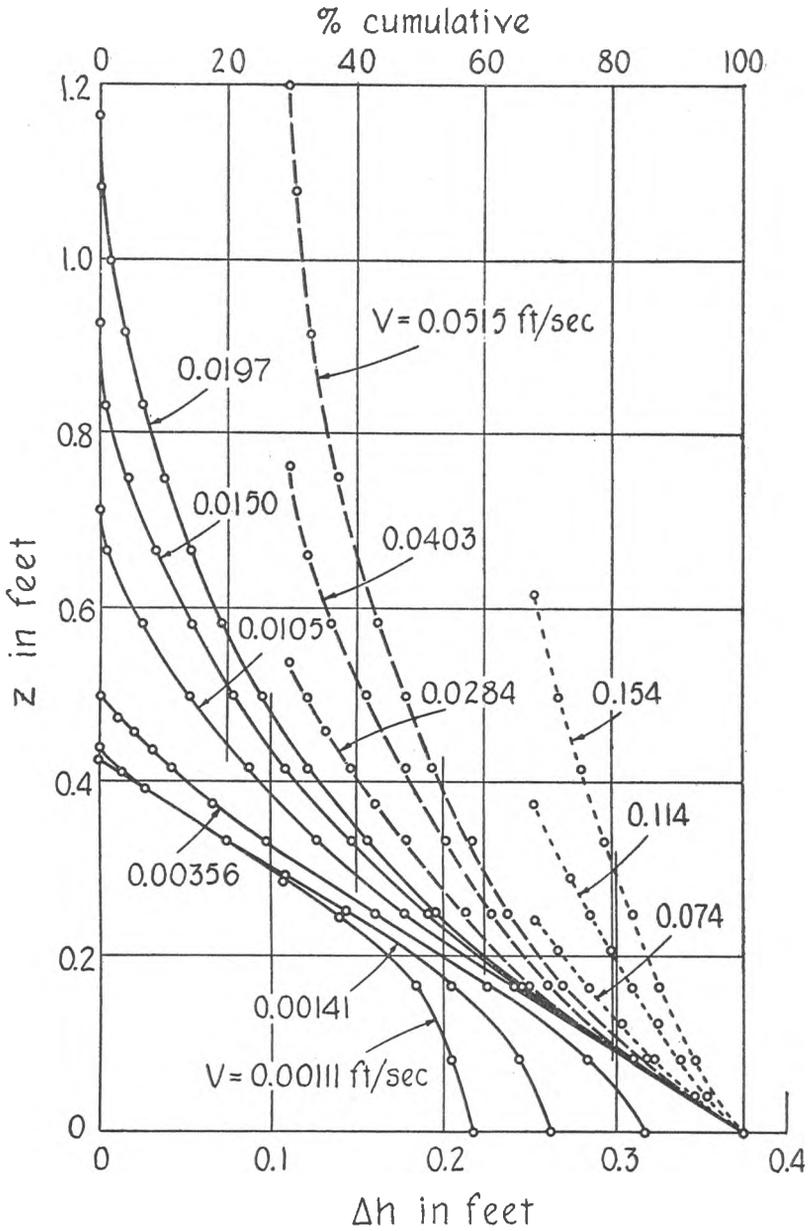


Figure 2.

pension, further increase in the rate of flow producing no further change in the head at the base of the column. This is in complete accord with Eq. (1), for the entire weight of the material is now supported by the upward flow. Indeed, supplementary tests proved that the addition or removal of known amounts of material invariably changed the manometer reading by the amount computed on the basis of Eq. (1). For this reason the broken curves in Fig. 2, obtained at such high rates of flow that the finer material had to be removed to avoid being carried over the top of the container, have arbitrarily been displaced to make the readings at zero elevation coincide.

From samples withdrawn at various elevations and at various rates of flow, it was possible to determine, by timing the fall in still water of a hundred or more representative particles of each sample, characteristic values of the quantity w appearing in Eq. (2). Measurement of the slope of $h:z$ curves at points represented by the

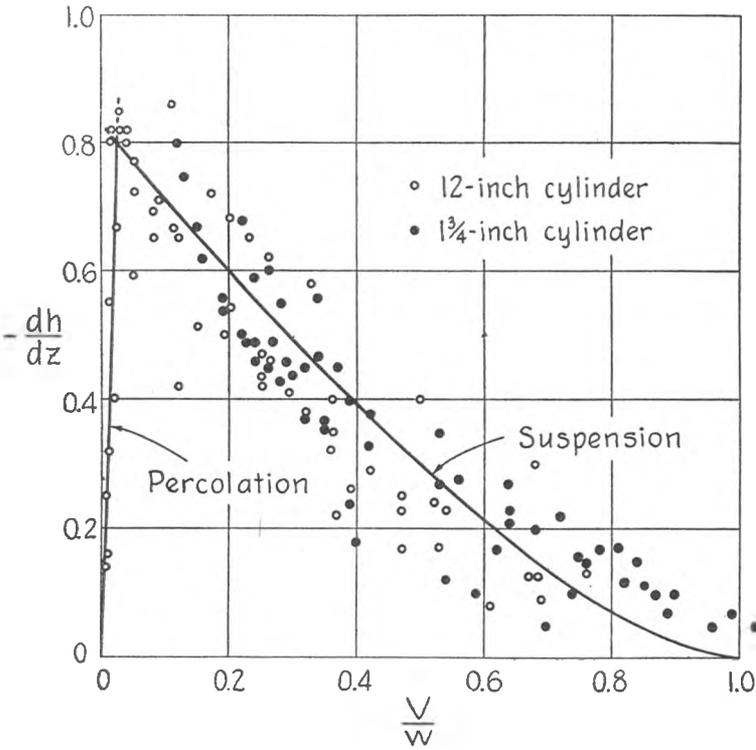


Figure 3.

samples then yielded the corresponding gradient $-dh/dz$, a plot of which against the ratio V/w is shown in Fig. 3. The straight line passing through the origin averages points obtained under conditions of pure percolation, its upper limit indicating that expansion of the material begins at a value of $-dh/dz$ of about 0.8, regardless of grain size. Once the material is in suspension, however, this gradient rapidly decreases in magnitude, finally approaching the limit zero as the nominal velocity of flow approaches the normal settling velocity of each different size of grain. Although there is appreciable scatter of points, they follow the general trend of the plotted curve (refer to Eq. (2)), for which $A = 0.83$ and $B = 1.0$. Nevertheless, it was found that the points lying above this curve invariably corresponded to the coarser material, and points below to the finer; in other words, the coefficient B , arbitrarily chosen as unity, actually varies with the fall velocity of the various grain sizes. Reference to the derivation of Eq. (2) will show that this indicates incomplete proportionality between w and v , a variation attributed to the effects of turbulence upon the suspension. Indeed, a definite pattern of eddies was noticeable through the transparent walls of the cylinders, apparently due neither to imperfect stilling of the approaching flow nor to the wake behind each individual

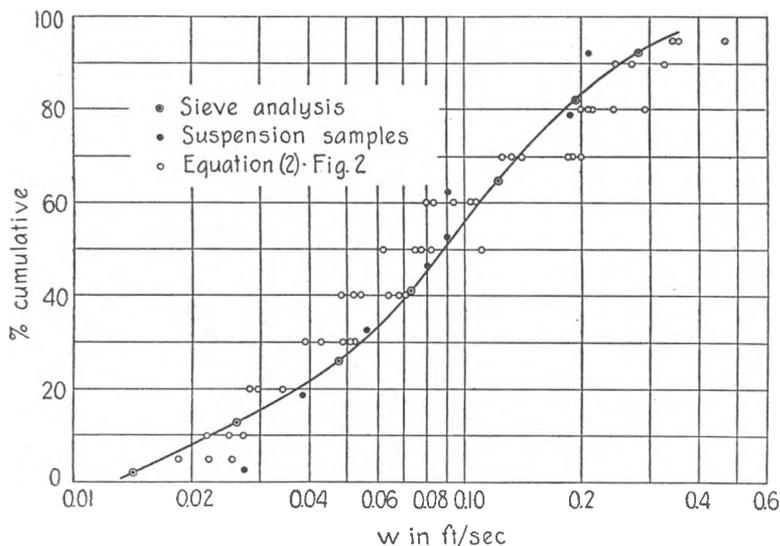


Figure 4.

grain, but rather to characteristic momentary fluctuations in the density of the water-sediment mixture.

Despite the probable lack of precision which such tendencies would produce if the method were used for sediment analysis, an effort was made to compare the results so obtained with the known characteristics of the original material. Using the size of sieve openings as a measure of the grain diameter and assuming the grains to be spherical in shape, points on the cumulative fall-velocity curve shown in Fig. 4 were computed. A second series of values was obtained by correlating the measured fall velocities of the several suspension samples with the corresponding points on the cumulative scale of Fig. 2. The third series resulted from measuring the slope $-dh/dz = c$ of the various curves of Fig. 2 for successive cumulative values, the corresponding magnitudes of w being obtained from the following form of Eq. (2).

$$w = \frac{V}{1 - 1.13 \left(-\frac{dh}{dz} \right)^{2/3}}$$

Although considerable deviation of the points from one another is again evident, the consistent trend of the measurements as a whole indicates that Eq. (2) embodies the basic principles of sediment suspension in upward flow.

Whether or not further investigation of the method will yield means of predicting quantitatively the effect of turbulence upon the sediment distribution, the use of upward flow as a means of segregating the various sediment grades is of immediate value. Indeed, the method has already been found effective in the preparation of highly sorted separates for use in other phases of sediment research, the desired grade being removed by siphon during settlement after the flow has been stopped. A somewhat similar procedure suggests itself as a possible improvement upon such methods of physical analysis as those of Wiegner and Crowther, based upon the theory of Odén, in which the sediment is first brought into a uniform state of suspension by thorough mixing of the entire vertical column of fluid. In other words, segregation by upward flow would replace the initial stirring process, and the w -frequency distribution would then be obtained from the temporal variation in manometric head near the base of the column during settlement of the material as a series of layers graded from the outset according to velocity of fall.