# THE INFLUENCE OF AIR ENTRAINMENT ON FLOW IN STEEP CHUTES

# L. STANDISH HALL

# East Bay Municipal Utility District Oakland, California

# Synopsis

During the past 20 or 30 years, the construction of major hydraulic structures has focused attention on the need for more information relating to the phenomenon of air entrainment and its influence on the design of steep chutes. In the preparation of plans for smaller structures, engineers in the past have relied on judgment and on a liberal allowance for a factor of safety, but nevertheless, in many instances the divergence between design and actual performance has been sufficiently great to require the repair or remodeling of a structure. With the expanding construction of large and important hydraulic structures, it has been increasingly apparent that design methods must be placed on a more rational basis to create assurance that the flow will actually conform with the design assumptions.

From actual observation, the writer has found no cases in which the designed channel cross section was inadequate solely as a result of improper allowance for air bulking, since the generous freeboard usually allowed compensated for the deficiencies between the design and actual performance in this respect. In all cases investigated, the failure of chutes to carry the desired discharge was principally due to the indirect effect of air bulking, which caused the computed velocities of the water to fall short of actuality. Excess velocities in a channel as a result of air entrainment have lead to the overtopping of side walls at (1) sharp horizontal curves, (2) sharp vertical curves, and (3) jump basins at chute terminals. In all these cases, the calculation of velocities with the provision for air entrainment will yield better conformity between the design and actual results.

Calculations of the wetted area must be modified with respect to the application of normal frictional resistance, which is altered as a result of air entrainment, so as to produce higher velocities than would be calculated from the application of conventional flow formulas. In all cases where water flowing in a steep chute is allowed to accelerate over a sufficient distance, air is mixed with the water and the phenomenon of "white water" results. Very few volumetric measurements of the air which is mixed with the water under these flow conditions have been made. In general, model tests are futile; as sufficient velocity cannot be attained to create the phenomenon. Adequate measurements of velocities and entrainment of air on actual chute structures by the author in 1936 and 1937 and by the U.S. Bureau of Reclamation in 1938 [1] have been previously described. Theoretical analysis of these data leads to the development of the formulas used in this paper for flowing water in steep chutes with the inclusion of an allowance for the effect of air entrainment. An example is given of the application of these formulas to the design of steep chutes.

# CHARACTERISTICS OF HIGH VELOCITY FLOW

Definition of Chute and Illustrations of Use: A chute is a structure used for conveying water down steep inclines under free-surface conditions. This definition excludes falls or drops, or flow through pipes under pressure, but includes many other types of structures. Most types of ogee-shaped over-flow dams can be classed as chutes; discharge through tunnels may be included, in cases where it is anticipated that the tunnel will flow only partly full. The chute is used in many instances to convey water from a feed canal to a reservoir; in hydroelectric practice, irrigation, and municipal water supply, the chute is used to convey waste water from a main conduit down a steep slope to a nearby stream channel; in irrigation, the chute is also used to convey water from canals on bench lands to canals or laterals on bottomlands.

The principal design requirement for all chute outlet structures is that the energy of the released water will be dissipated so as to render the works free from injury by erosion. This may be accomplished by the hydraulic jump, by a nearly horizontal jet discharging on a water surface, or by means of various types of diffuser blocks or sills at the toe of the chute. In the past, the dissipation of the high velocities has often proved a problem not yielding to solution by means of ordinary hydraulics. Rapid flow in steep chutes is subject to mechanical aeration as a result of the frictional resistance at the wetted perimeter and at the surface exposed to the air. With the possible exception of wide chutes, the effect of the wind velocity on air entrainment can be neglected. In all cases of high-velocity flow there exists a strong down draft of air parallel with the current. Air drawn into the water naturally continues in the direction of flow after escape. The side walls of the chute provide protection from cross winds, except in cases of unusual channel width.

Mechanics of Air Entrainment: In narrow chutes, the entrainment of air gradually encroaches on the jet from the side walls starting near the head of the chute until the whole water prism is composed of white water |2]. In wide channels the entrainment of air appears to be generated after a certain velocity has been attained. With extremely smooth entrance conditions such as occur on the downstream face of the ogee-section overflow dam, entrainment occurs immediately at the piers but in the intervening space, entrainment is delayed until the turbulent boundary layer occupies the entire depth; it may also occur in the turbulent wakes of piers.

Effects of Air Entrainment on Chute Operations: The phenomenon of air entrainment develops velocities greater than those computed by the usual hydraulic formulas using the normal coefficients of frictional retardation. For example, in a concrete chute for which values of Kutter's n of 0.012 to 0.013 would be used, it has been found that the actual terminal velocities using the water cross section indicated the use of values of n from 0.008 to 0.010. In addition, the bulking of the flow by the added mixture of air increased the water prism. Since the design of energy-dissipation structures is predicated on the computation of the correct velocity, the excessive increase in this factor in chute discharge results in marked variation between design and performance necessitating costly remodeling or repairs to remedy the faulty structural conditions.

Added freeboard is required on chute side walls to handle the expanded water cross section, although the proper allowance for air entrainment introduces a serious problem only in those designs in which it is planned to provide for larger volumes of flow in

rough-surfaced channels. Meager data available prior to 1936 indicated that for small concrete chutes, having velocities ranging from 15 to 40 ft. per sec., the entrained air occupied from 15% to 35% of the cross section [3]. Measurements of a masonry-lined chute in Austria give air entrainment ranging from 60% to 80% [4]. The use of such large percentages for air bulking in design would make large spillway chutes for the discharge of river floods very costly. For the design of such structures, more research is still badly needed to determine the actual performance of air and water mixtures.

Secondary Effects: Unsatisfactory entrance conditions may introduce flow patterns requiring rectification as a result of excessive air entrainment or the production of cross waves [5]. Short-radius horizontal curves or an abrupt narrowing of the channel width by too short a transition section at the inlet below the point developing shooting velocities will produce cross waves that result in great variance with the design, and these cross waves may persist through the length of the chute.

On unbanked horizontal curves of sharp radius the water will largely follow the outer wall, leaving the inner floor of the curve dry. The flow along the outer wall will exhibit maximum wave points, often resulting in unpredictable flow patterns in the channel below the curve.

In steep channels the water often tends to flow in a series of pulsations designated as "roller waves" or "slugs", since uniform flow at normal depth is unstable. In extreme cases the deepest part may become much deeper than that indicated by the standard formulas for uniform flow. Once this flow condition develops, it may easily result in overtopping of the side walls. A complete analysis of this phenomenon is not available, but studies that have been made indicate that the slope must be approximately four times the critical slope [6, 7]. It is possible that pulsations or waves will occur if the friction slope in any portion of the channel tends to exceed the bed slope. These surges differ from pulsations which normally occur in turbulent flow [8]. The pulsations in steep chutes are frequently of this latter type in an exaggerated form or a transitional stage between normal pulsating flow and true "slug" flow.

The increase of velocities as an indirect result of air entrain-

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ment often leads to design failure by the overtopping of side-walls at sharp horizontal or vertical curves. For satisfactory operation, the vertical curves in short chutes must be designed for a flow trajectory equal to the theoretical velocity generated under the action of gravity for a vertical fall between the inlet and the point of vertical curvature. Horizontal curves must be banked for proper velocity of flow or else the height of the side walls must be increased to provide for a maximum pressure rise on the outside wall of about twice as high as it would be for a flow at the same velocity on a banked curve [9]. The cross waves generated on unbanked curves usually continue in the straight channel below the curve, often with undesirable effects.

# THEORY OF FLOW IN CHUTES

Basic Assumptions: A rational formula for the calculation of air entrainment must be equally applicable to small chutes and to spillway channels of large capacity. To be of practical application, such a formula should satisfy three requirements: (1) It should conform as nearly as practicable to established methods of hydraulic calculations; (2) it should be of sufficient simplicity to be readily applied; and (3) it should yield results that are in reasonable conformity with the measured tests of air entrainment.

The mechanics by which the air enters the water is only partially understood. Entrainment results from excessive turbulence and small surface waves generated by the relative velocity between air and water. Channel roughness is one of the important factors, and hence, entrainment of air usually starts along the side walls. The mechanism that carries the air downward against its natural tendency to rise, is the turbulent motion of the water. Equilibrium for a given kineticity is only dynamic and air is continually entrained to replace that escaping. Changes in the kineticity, or the ratio of velocity head to potential head, are quickly reflected by changes in the percentage of entrained air. These changes are not instantaneous, there being a lag between changes in kineticity and air entrainment.

Personal observation and data collected from tests point to certain factors influencing air entrainment: (1) The velocity of the water, which depends on the slope of the energy grade line and the hydraulic radius; (2) the roughness of the sides and bottom of the channel; and (3) the width of the channel.

For the computation of flow in a rectangular channel three formulas are required [Eq. (1), (2), and (3)]. By the first equation the ratio of water in an air-water mixture flowing at high velocity may be determined.

Since the retardation factor depends on the perimeter wetted by the mixture, and because it is more convenient for purposes of computation to use a cross-section assuming no air entrainment, the relation between n and  $n_c$  in the Manning formula is found by Eq. (2).

The length of travel in the chute required to gain assumed increments in velocity can be computed for rectangular chutes by Eq. (3). For trapezoidal sections and sections of variable width, Eq. (1) still applies, but other equations must be used in place of Eqs. (2) and (3) [10, 11].

$$s = \frac{1}{1 + \frac{KV^2}{gR_c}}$$
(1)  
$$\frac{n_c}{n} = \left(\frac{R_c}{R_o}\right)^{2/3} = \left[s\left(1 + \frac{2KV^2}{bg}\right)\right]^{2/3}$$
(2)  
$$\frac{dl}{dV} = \frac{\frac{\alpha V}{g} - \frac{q}{V^2} \cos \theta}{\sin \theta - \frac{n_c^2 V^2}{2.21 R_c^{4/3}}}$$
(3)

Nomenclature:

$$b - Width$$

g — Acceleration due to gravity

- l Distance along channel
- K Coefficient of air entrainment
- n Computed roughness coefficient to be used with  $R_0 \& V_0$
- $n_c$  Computed roughness coefficient to be used with  $R_c \& V_o$

sVP.

- $P_c$  Computed perimeter for a section of water only
- $P_{o}$  Observed perimeter for a section of air and water q Discharge of water per unit width of channel

$$R_c$$
 — Computed hydraulic radius =  $\frac{V}{VP_c}$ 

$$R_{\circ}$$
 — Observed hydraulic radius =

V or  $V_{\circ}$  — Observed velocity average in a section

- α Mean-velocity-head coefficient to obtain average kinetic energy in section
- $\theta$  Angle of channel bottom with horizontal in degrees
- s Ratio of water in a mixture of air and water, or the relative density

Hydraulic Jump Basins: An advantage of determining the calculated areas of solid water is realized in the design of jump basins, which in many cases form the chute terminal. Here, the values of  $y_c$  and  $A_c$  together with the actual water velocity  $V_o$  at the toe of the chute should be used in the energy and momentum equations of the hydraulic jump [12]. If the velocity-distribution factor  $\alpha$ is used in the energy equation, a similar factor  $\beta$  based on the summation of  $V^2$  over the cross section should be applied to the momentum equation. This factor is smaller than  $\alpha$ , which has values of about 1.07 to 1.10; corresponding values of  $\beta$  ranging from 1.04 to 1.07 are generally applied.

The entrainment of air increases the velocity at the terminal of a chute, thereby increasing the kinetic energy at the entrance to a jump basin. Many illustrations could be eited of the unsatisfactory operation of jump basins at the end of chutes due to the water velocity being greater than that for which the structure was designed. In tests made in the laboratory at the writer's suggestion, air introduced to a stream discharging under a sluice-gate did not increase the depth of the mixture, but rather caused the mixture to travel at a higher velocity, while maintaining the same depth as without air. This increase in velocity resulted in the movement of the jump downstream [13].

Calculation of the jump basin dimensions, by using the true velocity in cases of air entrainment, will avoid the inadequate lengths and depths encountered in many constructed designs.

In many cases, other methods for energy dissipation prove more economical than the hydraulic jump. For small quantities of flow, baffle boxes or labyrinths are often used; and with larger discharges, diffuser blocks or sills are frequently employed. The use of the jet is gaining in popularity, its most recent application on a large structure being at the spillway of the Fontana Dam in Tennessee [14, 15].

Vertical and Horizontal Curves: Another of the problems in the design of chutes is the determination of the path of the water around vertical and horizontal curves. Concave vertical curves present no difficulty in design, except for stress created by the centrifugal force. Convex vertical curves must not be made sharper than the trajectory that would be followed by the high velocity water under the action of gravity. If the flume bottom is steeper than this trajectory, the water will leave the bottom and may rise above the top of the side-walls as observed at Hat Creek and Kittitas chutes. The trajectory will be a parabola tangent to the slope of the chute.

In cases where it is desired to have the jet follow the profile alignment in the design of convex vertical curves on chutes, the bottom profile should be computed on the basis of the theoretical velocity determined from the total vertical drop, or on a value 1.2 times the mean velocity determined from the energy gradient, whichever value is the smaller.

On vertical curves too sharp for maximum velocities, the bottom layers of water adhere to the floor of the chute, whereas the upper layers separate from the water prism, throwing the stream into spray. Air resistance gradually slows the velocity of the top spray causing the upper surface of the jet ultimately to conform to the chute profile. The increase in the retardation factor in such cases indicates a possibility of dissipating energy by this means. The same principle is utilized to some extent in discharging water from a chute bucket above the surface of the discharge pool. The throwing of the jet into the spray assists in dissipating the energy in addition to its further dissipation by eddies and turbulence in the pool itself. The writer has found very few vertical curves in chutes that were properly designed so as to permit the trajectory of the water to follow the vertical profile at all discharges. Again, this is evidence of insufficiently large design velocities.

Since water traveling in the chutes is always at less than the critical depth, changes in horizontal direction are more difficult to make and, if possible, should be avoided. If a horizontal curve becomes necessary, it should preferably be inserted near the head of the chute before the water has attained a high velocity. In cases of unusual entrance conditions, model tests should be made to test the design prior to actual construction.

The concept of the flow regimen on horizontal curves has been greatly clarified by the studies of Professors Ippen and Knapp [9], the theory of flow differing for banked and unbanked curves. Banked cross sections are recommended on wide channels due to the tendency of the water to crowd to the outside channel wall on unbanked curves. Superelevation of the outer wall of the curve affords a better distribution of the flow and less extraneous disturbance in the channel downstream from the horizontal curve.

# Values of Constants to be Used in Design.

In beginning the design of a steep chute by the method described in the preceding section, it is first necessary to assume values for n, K, and  $\alpha$ . Values to be assigned to  $\alpha$  have already been discussed. Once air entrainment occurs with supercritical flow, values of the retardation coefficient n for the cross section occupied by the mixture are the same as those values established as a result of many measurements of discharge with subcritical flow. Values of the coefficient K have been ascertained from the test on the Kittitas chute made by the Bureau of Reclamation and from three tests made by the writer.

Retardation Factors: Chutes have been constructed of timber, metal, masonry, or concrete. A wide range of surfaces may result. The late Robert E. Horton assigned values of n for use in either the Kutter or the Manning formulas [16, 17].

For the four steep chutes used for air-entrainment tests, the values of the retardation factor using the observed areas and velocities conform, in general, with the range normally used for the appropriate surface. In Table 1 the most representative values of n and  $n_c$  for both the Manning and the Kutter formulas are tabulated for the four chutes. Exceptions were noted near inlets where unusual values were recorded as the result of entrance conditions. Similar high values occurred below the convex vertical curves on Hat Creek and Kittitas chutes due to the spreading of the cross section of the jet as the water left the bottom of the chute. Also, on the sharp horizontal curves in the South Canal chute an increased friction loss was noted.

Comparison of the surfaces in the test chutes and the corresponding retardation factors with other data leads to the question: Are the values of n computed on steep chutes using observed areas and

velocities exactly equal to those obtained on similar surfaces with subcritical flow? Eliminating the values from the steep slopes of the Kittitas chute and of Hat Creek chute, it appears that the values of n are slightly lower than those generally applied to the surfaces described. This decrease of n may result from air entrainment or it may be due to another feature of chutes in constant operation noted by the writer and referred to by Scobey [18]. Algae, moss, and insect larval growth are not scoured off by the high velocities, but apparently thrive on the excess air resulting from mechanical aeration. These growths tend to smooth the channel surface, a fact of considerable significance to the designer.

## TABLE NO. 1

COMPARISON OF RETARDATION FACTORS IN STEEP CHUTES

	Man	ning	Kut	ter	Man	ning	Kut	ter
Chute	п	,	7	ı	n,		$n_{c}$	
	From:	To:	From:	To:	From:	To:	From:	To:
Hat Creek Chute:	Fair concre	ete wit	h pronou	inced f	form ma	rks and	l tie	
	wires proje	ecting						
$ heta=23^\circ \ 30' \ { m to} \ 27^\circ$ :	35' 0.010	0.013	0.013	0.013	0.008	0.010	0.008	0.010
$\theta == 34^{\circ} \ 45'^{*}$	0.012	0.025	0.018	0.024	0.009	0.017	0.009	0.017
Rapid Flume: Plan	ned wood v	with lo	ngitudin	al bat	tens.			
	0.010	0.012	0.011	0.012	0.008	0.010	0.008	0.010
South Canal Chute:	Concrete,	rough	and pitt	ted, ag	gregate	fragme	nts expo	osed.
Straight or gentle								
curves	0.012	0.013	0.012	0.013	0.011	0.012	0.011	0.012
Sharp curves	0.013	0.014	0.013	0.015	0.011	0.013	0.012	0.013
Kittitas Chute: Co	ncrete, sid	es smo	oth, bot	tom ro	ugh and	I pitted	1.	
$\theta = 10^{\circ} \ 12'$	0.015	0.016	0.015	0.016	0.014	0.015	0.014	0.015
$\theta = 33^{\circ} \ 10'^{*}$	0.019	0.021	0.019	0.021	0.014	0.015	0.014	0.015

\* Trajectory of jet departs from profile of channel bottom on sharp convex vertical curve between upper and lower slopes. Large volume of air entrained at this point from which jet does not fully recover before end of chute.

More tests are needed on steep chutes, with complete measurement of velocities as well as cross sections, in order that the friction slope may be determined with sufficient accuracy to permit computation of additional values of n with high-velocity flow, in addition to those available at the present time. The velocity, the hydraulic radius, and the slope of the energy gradient all enter into the determination of n, and, hence, any observational errors in ascertaining any of these quantities will affect the computed values of n.

Coefficient of Air Entrainment: The values of K are reasonably consistent with the general condition of roughness of the respective channels. In Rapid Flume the wood construction results in the lowest value of the coefficient. Of the concrete channels, Hat Creek has a lower value than Kittitas chute, although the results from these two chutes are in close accord. The South Canal chute with its rough concrete bottom and sinuous alignment has the highest value of K.

Values of K for ashlar masonry or rubble concrete have not been determined. Tests were planned by the writer in 1941 on the former type of construction, but due to the war these were never completed. It appears that the value of K increases with the roughness of the surface and, based on the available tests supplemented by personal observations, the following values of K are suggested for use:

Surface	Values of K
Plank flumes without battens	0.003 to 0.004
Neat cement surfaces	0.003 to 0.004
Cement mortar or average concrete	0.004 to 0.006
Rough concrete or smooth dressed ashlar	0.008 to $0.012$
Rough ashlar or smooth cement-rubble	0.015 to 0.020

More experiments over a wide range of channels are needed to confirm the magnitude of the factor K. The straight-line relation of the air-water ratio to a dimensionless parameter indicates its application to all ranges of velocity in channel sizes from the smallest to the largest. Until such time as a more appropriate formula may be developed, Eq. (1) may serve as a framework upon which subsequent data may be tested.

Variation of s with V, K, and  $R_c$ : The nature of the relationship between s and V as expressed in Eq. (1) is difficult to visualize because of the fact that four variables are involved. A series of curves has been drawn for values of K equal to 0.005, 0.010, and 0.020, showing the relation between s and V for representative values of  $R_c$ . From a study of these curves, shown in Fig. 1, it is apparent that for constant values of V and  $R_c$ , the percentage of

water in the mixture decreases as the coefficient K is increased. Also, for constant values of K and V, the percentage of water increases as  $R_c$  increases. Finally, with K and  $R_c$  constant, the percentage of water decreases as the velocity increases. These relationships are also apparent from the examination of Eq. (1), but the influence of the changes in the various factors upon the percent of water in the mixture is best visualized from the graphical presentation.



FIG. 1-THE RATIO S AS A FUNCTION OF V, Re, AND K.

# Example of Design Procedure

In order to illustrate the results that may be obtained by the application of the design methods for steep chutes described in this paper, an example will be presented. The computations have been made for a discharge of 100 cu. ft. per sec. in a rectangular chute, a channel width of 8 ft. and a slope of  $33^{\circ}$  10' (the same dimensions and slope as the steep portion of the Kittitas chute.

Water Surface Profile: The chute is assumed to start at the main canal where the steep slope  $(S_2 = 0.547)$  joins the mild upstream slope  $(S_1 = 0.001)$ , so that critical flow will be obtained at this junction of the slopes; it is further assumed that entrance conditions will create no air entrainment or wave patterns at the head of the chute.

Computations have been made for assumed n-values of 0.010,

0.012, 0.015, 0.017, and 0.020, and the corresponding K values of 0.003, 0.005, 0.005, 0.010, and 0.020.

The characteristics of this channel are first computed for the assumed values of n and K. Values of  $A_c$ ,  $R_c$ ,  $V_o$ , and  $V_o^2/gR_c$  are computed from the discharge and the channel dimensions. Values of s are computed from Eq. (1) and the ratio  $n_c/n$  from Eq. (2). These factors are all plotted graphically as functions of the depth  $y_c$ . It should be noted that the channel characteristics differ with every channel depending on the discharge, the dimensions and shape of the cross section, and the values of n and K chosen for the design.

The initial depth at the critical section should be increased if  $\alpha$  is assumed to be greater than unity. For the assumed discharge and channel width in this case, the initial depth is 7.99 ft. In Table 2 an example is given showing the flow characteristics and computations for the values n = 0.015 and K = 0.005.

Starting at the critical section, the values in columns 4 to 13 in Table 2 are taken from the computed channel characteristics either directly or indirectly. The value of  $n_c$  is taken only to the nearest 0.0005 near the head of the chute and is decreased from the initial value of 0.015 at the critical section to a value of 0.009 with uniform flow. Columns 14 and 15 are obtained from Eq. (3). The friction slope in column 14 is the last term in the denominator on the righthand side of the equation. Values of V are taken in sufficiently small increments to assure accuracy; in this example, 10% of V is used. The area of the mixture  $A_0$  in column 18 is obtained by dividing the computed area in column 4 by the percent of water in the mixture from column 10, and values of  $y_0$  and  $R_0$ in columns 19 and 20 are taken from the curves of channel characteristics by substituting values of  $A_{0}$ ,  $y_{0}$ , and  $R_{0}$  for the values of  $A_{c}$ ,  $y_{c}$ , and  $R_{c}$ . The computations are continued until the friction slope equals the bed slope. In the case where n has been assumed 0.010 and K = 0.003, the friction slope reaches an equilibrium value only with a length of 2,255 ft. of channel. With a low ratio of water in a chute over 500 ft. in length, probably the terminal velocity would be attained when the air friction operated as a retarding factor on the mixture of foam and spray.

Assuming the chute to have a length of only 400 ft., the comparison of flow characteristics at the chute terminal for the five

	CHUTE	
	STEEP	
	¥	
	N	
TABLE NO. 2	CHARACTERISTICS	S ON NOTTOWING
	FLOW	4
	OF	
	CALCULATION	

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(1)	(2)	(3)	(4)	(2)	(9)	(1)	(8)	(6)	(10)	(11)	(12)	(13)	(14)	(15)	(91)	(11)	(18)	(10)	(20)
Station	Depth	y, cos	Area 0 A.	$\begin{array}{c} \mathrm{Hyd} \\ \mathrm{Rad} \\ \mathrm{Rad} \\ R_{\mathrm{e}} \end{array}$	Vel- ocity	1.05H v 1.05F2	Energy Head $y_{,\cos\theta}$ -1.05 $H_{,\cos\theta}$	Kine- tlcity	Water 8	% Air 1—8	-T2 9Re	$n_{\rm c}$	Fric- tion Slope	al dF	Др	lb	Obs. Area	Obs. Depth 3',	Obs. Hyd. Rad.
0+00	7,99	6.69	63.9	2.66	15,65	3.99	10.60	0.60	98.6	1.4	2.9	0.015	0.0067	0.146	1.56	0.2	64.8	8.10	2.68
0+00.2	7.28	6.10	58.3	2.58	17.16	4.80	10.90	0.79	98.3	1.7	3.5	0.015	0.0084	0.387	1.72	0.7	59.2	7.40	2.60
6-00+0	6.63	5.55	53.0	2.49	18,88	5.81	11.36	1.05	97.8	2.2	4.5	0.015	0.0107	0.598	1.89	1.1	54.2	6.78	2.52
0+02.0	6.01	5.03	48.1	2.40	20.77	7.03	12.06	1.40	97.4	2.6	5.6	0.015	0.0136	0.818	2,08	1.7	49.4	6.17	2.43
0+03.7	5.47	4.58	43.8	2.31	22.85	8.51	13.09	1.86	96.7	3,3	7.0	0.015	0.0181	1.030	2.28	2.4	45.3	5.66	2.34
0+06.1	4.97	4.17	39.8	2.22	25.13	10.30	14.47	2.47	95.8	4.2	80.00	0.0145	0.0207	1.238	2.51	3,1	41.5	5.19	2.26
0+09.2	4.53	3.79	36.2	2.12	27.64	12.46	16.25	3.28	95.0	5.0	11.2	0.0145	0.0266	1.468	2.76	4.1	38.2	4.77	2.17
0 + 13.3	4.11	3.69	32.9	2.03	30.40	15.07	18.76	4.08	93.5	6.5	14.1	0.0145	0.024	1.713	3.04	5.2	35.4	4.40	2.09
0 + 18.5	3.75	3.14	30.0	1.94	33.44	18.23	21.37	5.80	92.0	8.0	17.9	0.0145	0.044	1.984	3,34	6.6	32.7	4.08	2.02
0+25.1	3.40	2.84	27.2	1.84	36.78	22.04	24,88	12.7	90.0	10.0	22.8	0.0145	0.057	2.291	3.68	8.4	30.2	3.78	1.94
0 + 33.5	3.09	2.59	24.7	1.75	40.46	26.69	29.28	10.30	87.5	12.5	29.1	0.0145	0.074	2.658	4.05	10.8	28.3	3.53	1.87
0 + 44.3	2.81	2.35	22.5	1.65	44.51	32.30	34.65	13.74	84.3	15.7	37.3	0.014	0.090	3.063	4.45	13.6	26.7	3.34	1.82
0+57.9	2.55	2.13	20.4	1.57	48.96	39.08	41.21	18.35	80.8	19.2	47.6	0.014	0.118	3.625	4.90	17.8	25.3	3.16	1.77
0+75.7	2.32	1.94	18.6	1.47	53.86	47.30	49.24	24.38	76.3	23.7	61,4	0.0135	0.14.	4.265	5.39	23.0	24.3	3.04	1.73
7.76+0	2,11	77.L	16.9	1.39	59.25	57.23	59.00	32.33	7.17	28.3	78.5	0.013	0.175	5.121	5.92	30.3	23.5	2.94	1.70
1 + 29.0	1.92	1.61	15.4	1.30	65.17	69.24	70.85	43.01	66.2	33.8	101.6	0.0125	0.213	6.296	6.52	41.0	23.2	2.90	1.68
1+70.0	1.74	1.46	14.0	1.22	71.69	83,80	85.26	57.40	61.0	39.0	130.8	0.012	0.259	8.059	LT.7	27.8	22.8	2.85	1.67
2+27.8	1.59	1.33	12.7	1.14	78.86	101.41	102.74	76.26	54.8	45.2	169.4	0.0115	0.315	11.026	7.89	87.0	23.2	2.90	1.68
3+14.8	1.44	1.20	11.6	1.07	86.75	122.69	123.89	102.24	48.0	52.0	218.0	0.011	0.382	17.079	8.68	148.2	24.0	3.00	1.72
4+63.0	1.31	1.10	10.5	0.99	95.43	148.48	149.58	134.98	41.0	59.0	286.0	0.010	0.420	24.449	9.54	233.2	25.5	3.19	1.78
6 + 96.2	1,19	1.00	9.5	0.92	104.97	179,67	180.67	179.671	35.5	64.5	372.0	6000.0	0.505	81,380	6.73	547.7	26.8	3.35	1.82
12 + 43.	9 1.12	0.94	9.0	0.87	111.70	203.43	204.37	216.41	31.0	69.0	445.0	600.0	0.547	Inf.	0	Inf.	28.9	3.61	1.90
n = 0. a = 12	015 1 5 cm. ft	Sin 0	== 0.5	47 C	08 0 =	= 0.837	Q = 001.15	1,000 c	3u. ft. ] m Eq.	per sec (3). C	. b =	= 8.0 ft W is tal	ten as 10	% of T	. excep	t for 1	the las	t two e	ntries
	-	A	5		1				T	/ - /				C LUL AN	-				

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assumptions is given in Table 3. The velocities are about equal for the three high values of n, but increase with the two lower values. The depth of the mixture increases as the coefficient of air entrainment increases.

#### TABLE NO. 3

# COMPARISON OF TERMINAL DEPTHS AND VELOCITIES IN A STEEP CHUTE WITH VARIOUS SURFACES

-	Length on slo Bottom	pe 400 feet width 8.0 f	; Inclination eet; Rectang	33° 10'; S ular Cross	in $\theta = 0.5$ Section	47;
		Discharge	e 1,000 cu. ft	. per sec.	Mirturo	Fristian
		Depth	Velocity	Water	Depth	Slope
n	K	y <sub>c</sub>	Vo	S	<i>y</i> <sub>o</sub>	S
0.010	0.003	1.20	104.5	47.5	2.52	0.270
0.012	0.005	1,23	101.5	37.5	3.28	0.315
0.015	0.005	1.36	91.5	43.2	3.15	0.400
0.017	0.010	1.36	92.0	28.0	4.86	0.410
0.020	0.020	1.37	91.0	16.5	8.30	0.410

Note: Bottom slope  $S_1$  greater than friction slope S at terminal of chute, i.e., velocity still increasing.

### Conclusions

It cannot be repeated too frequently that the velocity of an airwater mixture on steep chutes is greater than that computed for water alone by the conventional hydraulic formulas. The inadequate length of many constructed jump basins is evidence that the actual velocities with entrained air are greater than the design velocities. Very few vertical curves in chutes are designed to permit the trajectory of the water to follow the vertical profile at all discharges. Again this is evidence of insufficient design velocities. The construction of horizontal curves on steep chutes has often ended in failure of the structure by overtopping of the outer wall, partly because of excess velocities, and partly because of inadequate knowledge of the performance of water on banked and unbanked curves.

The method of calculation of flow characteristics in steep chutes described in this paper will yield results which are in reasonable conformity with actual velocities and the percentage of entrained

air. The computations are subject to the usual uncertainties of hydraulic design resulting from the selection of values of the retardation coefficient, and in this instance an additional coefficient of air entrainment. For the selection of the proper coefficient for the latter factor, the assumed data are admittedly meager and more observations are needed over a wider range of channel sizes, discharges, and degrees of surface roughness.

A uniform definition of the observed depth should be adopted. The definition used in this paper as the average maximum and minimum depth to the base of the flying spray appears to be rational. Improvements in the techniques of measuring high velocities are needed in order to reduce the necessity of judicial interpretation of field observations.

The possibilities for theoretical research and laboratory investigation are probably not exhausted. Further laboratory studies of the action of the hydraulic jump below a sluice gate with the introduction of air at the downstream edge of the gate could be made to advantage. More studies are also desirable to determine the best means for dissipation of energy by different types of jump basins and types of jets.

Admittedly, the data collected on the chutes tested cover a relatively small range of conditions, and it has been necessary to extrapolate the coefficients on the basis of observation and subjective analysis. However, it is the writer's opinion that the theoretical approach utilized is sound, and the coefficients may be used judiciously in the design of steep chutes until more precise coefficients or formulas are developed by future investigations.

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