

PRINCIPLES OF TOWING-TANK RESEARCH

by

L. A. Baier

University of Michigan

Ann Arbor, Michigan

NORTH AMERICAN TOWING TANKS

The importance of the contributions made possible by the towing tank in the field of naval architecture and to the study of fluid mechanics in general is indicated by the number of towing tanks now in operation. There are approximately fifty active tanks throughout the world, divided almost equally between strictly government-operated naval plants and those under private or technical group ownership. Twelve of these tanks are located in North America, distributed as follows:

National Research Council, Ottawa, Canada, 1930. This is a high speed tank (392 x 9 x 6 feet) originally designed for seaplane float and flying boat hull tests. Ship-model work, necessitated by the rapid expansion of the Dominion's naval forces, is being undertaken, but the narrow basin width restricts the length of the usual model to about 5 feet. Propulsion of the towing car is by cable. A larger tank is now under construction and will be in operation in the near future.

National Advisory Committee for Aeronautics, Langley Field, Va., 1930. This tank (1980 x 24 x 12 feet) was lengthened in 1937 to 2880 feet and the maximum car speed increased from 60 to 80 miles per hour. The work in this tank is devoted almost exclusively to model tests on seaplane floats and hulls, although its high speed has been utilized by the Navy Department in the testing of fast surface craft. The car operates on eight pneumatic rubber-tired wheels, each wheel being driven by a 75-horse-power electric motor.

Stevens Institute of Technology, Hoboken, N. J., 1935. The principal work done at this tank (100 x 9 x 4.5 feet) is with sail

and power yachts, and models 3 to 5 feet in length can be handled. Models are towed by an overhead, monorail combined trolley and dynamometer, readings being noted by the operator while keeping pace with the travel. By means of multiple drive pulleys and resistance stops selected speeds are available and need not be measured during the run. The equipment is unique in the use of lateral dynamometers for sailboat tests, heeled and with yaw or leeway. A 75 foot square tank is now under construction for maneuvering tests by means of small self-propelled models.

Newport News Shipbuilding and Dry Dock Company Hydraulic Laboratory, Newport News, Virginia, 1933. In this small tank (56 x 8 x 4 feet) ship models are towed by means of falling weights, insuring constant propelling forces. Auxiliary weights shorten the time required for acceleration and automatic records are taken of model speed. Preliminary investigations of hull form are facilitated and the final design confirmed by tests with a 20-foot model at the Navy Department tank. The standard length of model is 4 feet and considerable work has been done with friction planes of this length.

U. S. Navy Experimental Model Basin, Navy Yard, Washington, D. C., 1899. This is the oldest tank in the country (472 x 42.7 x 14.6 feet) and, together with the University of Michigan tank, has contributed to the profession much of the pioneer research data. The models are of wood, 20 feet in length, and may be self-propelled. Research work on rolling, eddy-making, etc., is conducted in a small tank (80 x 7 x 4 feet) built in 1933, using the gravity type of propulsion.

U. S. Navy, David W. Taylor Model Basin, Carderock, Md., 1941. This \$3,500,000 laboratory has just been completed and now is in operation with one towing carriage. It includes a shallow-draft (303 x 51 x 10 feet) and a deep-water tank (963 x 51 x 22 feet) in tandem, totaling 1266 feet; an additional high speed tank (1168.3 x 21.1 x 10 feet) parallel to the deep basin; a smaller utility tank (142.5 x 10 x 5.5 feet, gravity drive) in the substructure; these together with a large turning basin, variable-pressure tunnels, material-testing and applied-mechanics laboratory, shops, and offices, make this establishment the last word in testing facilities.

University of Michigan Naval Tank, Ann Arbor, Michigan, 1906.

While primarily part of the educational equipment of the Department of Naval Architecture and Marine Engineering, this tank (350 x 22 x 9.5 feet) has been continuously active in research work and in fulfilling the commercial requirements of ship designers and operators. Models are of paraffin wax or wood and usually 10 feet to 12 feet in length. Some 25 papers have been contributed by the staff and form an important source of design data. An adjustable false bottom 140 feet in length is available for shallow-water tests, and extensive research work in this field for the government and private firms has been concerned with barges, flotillas, towboats, and paddle wheels. In 1938 the Department of Naval Architecture and Marine Engineering proposed an organization of American tank directors which is now known as the American Towing Tank Conference. The annual meetings have been successful in stimulating professional progress and standardizing technical details, and its reports are published in the Transactions of the Society of Naval Architects and Marine Engineers.

HISTORICAL RESUME

The possibility of using models to study phenomena of interest in the full-scale prototype has long been known and utilized by scientists and engineers. In the late 16th century Sir William Gilbert, the English physicist, investigated the directional forces of the earth's magnetic field through study with a small sphere of lodestone. In the field of ship resistance, experiments by de Bordo in 1763 established the square law of velocity, and further experiments were undertaken by the Marquis de Condorcet and the Abbé de Bossut in 1775, using a gravity-type rig in a basin 100 x 52 x 6 feet. A few years previously in 1768 our own versatile Ben Franklin satisfied his curiosity concerning the increase in resistance of barges operating in shoal water through the use of tank tests. Small models were towed the length of a miniature tank by means of the gravitational pull on a shilling attached to a line from the bow, and running over a sheave at the tank end. By comparing the increase in time required by the model to traverse a given distance, as the water was successively shoaled, he obtained reliable estimates of shallow-water amplification of drag. During the next

100 years Beaufoy, Chapman, Scott Russell, and others experimented and theorized in the field of ship resistance, but little was achieved until the advent of the elder Froude. In 1868 William Froude propounded the Law of Comparison and laid the foundation for our present technique. In 1870 the British Admiralty authorized the construction of the experimental tank at Torquay in accordance with Froude's proposals, and under his direction and imagination came forth the seeds from which have been harvested the solutions of complex problems of hull form, appendages, and propellers. In this country pioneer development was carried on and stimulated by Admiral Taylor at Washington and Sadler and Bragg at Ann Arbor.

REQUIREMENTS FOR SHIP-MODEL TESTING

The essential requirements for ship-model testing may be summed up briefly as follows:

Proper Relationships Between the Dimensions of the Model and the Length and Cross Section of the Basin.

The length of the tank depends on the designed car speed, acceleration, and deceleration. The distance of steady run during which readings are taken is a function of the time required by the various observers and the time necessary for the flow pattern to become stabilized. The minimum width of the tank is frequently considered as 15 times the beam of the model, and the cross section as 100 times the model midship area. Assuming d as representing the mean depth of a towing basin, the relationship $V = \sqrt{gd}$ may be used to approximate the maximum speed at which waves can be propagated.

Standardization of the Model Surface.

This is an important function frequently neglected. Paraffin wax models are particularly subject to considerable change in surface characteristics due to "weathering," and some paint finishes show decided differences in resistance after two or three hours of immersion.

Uniform Movement of the Model Through the Water.

Slight irregularities in track levels, variations in voltage, and temperature changes in drive motors produce local accelerations

and decelerations which seriously affect dynamometer measurements.

Control of the Physical Condition of the Tank Water.

The several factors to be emphasized under this heading are: first, elimination of currents and waves; second, freedom from surface dust and oil scum, algae growth, and temperature strata; and third, the maintenance of an internal condition conducive to turbulent flow. The latter problem has received considerable attention from tank directors in recent years and various methods are used to stimulate turbulent flow at the lower speeds. It has long been assumed that the Reynolds number, $R = VL/v$, alone served as a criterion to indicate the point at which a natural turbulent flow could be expected. Abroad, the minimum value of R is generally taken at 3×10^6 . Experience at Michigan, however, suggests that the volume of the tank water also is a critical factor. As tank size is decreased, the value of R at which turbulent flow becomes general is also reduced. In addition, methods used to encourage turbulent flow artificially can be successfully employed at much lower values of R in small tanks than is possible in the larger basins.

Suitable Dynamometers to Measure and Record Pertinent Data. Shop Facilities for the Accurate Construction of Models and Fittings.

PRESENT-DAY FUNCTIONS OF TOWING TANKS

Before considering the theory of model testing, it might be well to note some of the present-day functions of the basins. The principal concern of these establishments, of course, is with problems involving fluid mechanics in the broad field of ship design. In general the investigations can be grouped into research work and specific commercial problems.

Resistance

1. Prediction of power for a given form and speed.
2. Systematic research over a comprehensive range of forms and speeds to accumulate data suitable for future estimation of resistance.

3. Determination of the effect on resistance of variation in one or more form characteristics.
4. Measurement of the resistance of hull appendages.
5. Study of the influence of shoal water and canal walls.
6. Production of artificial waves and the study of their effect on power, propulsive efficiency, steering, and stability.
7. Investigation of the frictional resistance of planks, and the augment of resistance due to various types and conditions of roughness.
8. Tests on the suitable arrangement of barges into fleet flotillas.

Propulsion

1. Open-water tests of propellers and paddle wheels involving a wide range of characteristics.
2. Self-propelled model tests covering the interaction of propeller and hull.
3. Cavitation studies through use of variable-pressure hydraulic tunnels.
4. Measurement of basic wake behind models.
5. Influence of Kort nozzles, contra-rudders, and fins on propulsive efficiency.
6. Backing tests at various speeds of advance.

Miscellaneous

1. Location of stream-line flow to facilitate proper location of struts, bilge keels, bossings, and condenser scoops.
2. Investigation of torque, turning moments, turning circles, heel, decelerations, and other rudder and steering data.
3. Determination of wave profiles along model sides to aid in location of side paddle wheels and proper design of forward sections.
4. Rolling experiments to evaluate effect of form and appendages on the period and amplitude.
5. Calibration of current meters.

6. Correction of yaw in towed barges.
7. Change of trim and squat underway.
8. Vibration of hulls.
9. Dynamic balance of sailboats, heeled, and making leeway.
10. Measurement of lift, drag, trim, longitudinal hydrodynamic stability, and spray characteristics of model seaplane hulls and pontoons.
11. Submerged tests of submarines, double superstructures, wing sections, etc.
12. Odd experiments in some detailed phase of fluid mechanics requiring the available asset of controllable speed of advance through a reasonable length of undisturbed water.
13. Correlation of model predictions and ship-trial results.

MODEL LAWS

The laws of model testing are but a part of the general principle of similitude, which underlies the science of physics. The treatment and analysis of model tests involving dynamic loading are far more complicated than in the case of statically-loaded elastic structures, since perfect similarity of motion occurrences is very seldom obtainable. The mechanics of similitude, however, can be applied within practical limitations compatible with the required accuracy of results.

The following three conditions must be fulfilled for any two systems of model and prototype:

(a) *Geometrical Similarity*. This concerns form and is independent of time-motion occurrences or involved forces. The ratio of all homologous linear dimensions must be constant, and all corresponding angles equal. The scale factor is usually identified by λ , and the relationships of the familiar ship form characteristics are listed below, denoting model and ship by the subscripts 'm' and 's', respectively (see p. 68 for list of symbols):

$$\frac{L_s}{L_m} = \frac{B_s}{B_m} = \frac{H_s}{H_m} = \frac{(K_s)_s}{(K_s)_m} = \lambda$$

$$\frac{S_s}{S_m} = \lambda^2$$

$$\frac{\nabla_s}{\nabla_m} = \frac{\Delta_s}{\Delta_m} = \lambda^3$$

Based on dimensional homogeneity, the following dimensionless quantities or 'numerics' can be derived from the above:

$$\frac{L}{B} = r_1 \quad \frac{L}{H} = r_2 \quad \frac{B}{H} = r_3 \quad \frac{L}{K_s} = r_4$$

$$\frac{L}{\sqrt{A_{\infty}}} = r_5 \quad \frac{S}{\nabla^{2/3}} = r_6 \quad \frac{S}{L^2} = r_7 \quad \text{etc.}$$

The various form coefficients of the hull are likewise non-dimensional. In addition, the ratio of the cross-sectional area of the basin to A_{∞} must be held constant or at least numerically equal to 100 or greater to ensure freedom from scale effect.

(b) *Kinematic Similarity.* Comparing two geometrically similar systems, kinematic similarity for two motion occurrences is attained when, under the influence of physical forces, the various similarly-situated particles involved freely trace similar flow patterns in corresponding time periods, the ratio of all homologous time periods being constant. Hence, in addition to $L_s/L_m = \lambda$, it is required that $T_s/T_m = \lambda$

(c) *Dynamic Similarity.* If the flow patterns around two geometrically similar bodies are kinematically similar, then the two flows also are dynamically similar if the corresponding fluid forces on the model and prototype are proportional at any instant. Hence, the ratio of the physical forces acting on any two homologous mass particles must be proportional to the products of their respective homologous masses and their corresponding accelerations, since the basic dynamic law of Newton states that force is equal to mass times acceleration. The resultant accelerating force may be composed of a system of several forces of different degrees of in-

fluence, which indeed prevents the exact solution of the majority of hydraulic occurrences by means of models.

From the above we have the following relationships :

$$\frac{M_s}{M_m} = \lambda_M \quad \frac{A_s}{A_m} = \lambda_A \quad \text{and} \quad \frac{F_s}{F_m} = \lambda_F$$

or

$$\lambda_F = \lambda_M \times \lambda_A$$

$$\lambda_F = \lambda_M \times \lambda \times \lambda_T^{-2} \tag{1}$$

This is the qualifying equation between the four fundamental non-dimensional ratios involved in the treatment of similar motion occurrences. Now if λ is arbitrarily fixed and λ_M is determined by the choice of fluid mediums, then λ_T , the time ratio, depends on the value of λ_F for the particular force under examination. Hence, it is necessary to set up an empirical expression for λ_F in terms of each particular force :

$$\lambda_F = \frac{F_s}{F_m} = \frac{F_{1s}}{F_{1m}} = \frac{F_{2s}}{F_{2m}} \quad \dots \dots \dots \text{etc.} \tag{2}$$

However, from Eq. (2) additional non-dimensional ratios result as follows :

$$\lambda_{F1} = \frac{F_s}{F_{1s}} = \frac{F_m}{F_{1m}} \tag{3}$$

$$\lambda_{F2} = \frac{F_s}{F_{2s}} = \frac{F_m}{F_{2m}} \tag{4}$$

These formulae can be rearranged as shown below :

$$\lambda_F = \frac{\lambda_M \times \lambda}{\lambda T^2} \tag{1}$$

$$\lambda_F = \frac{F_s}{F_{1m} \times \lambda_{F1}} \tag{5}$$

$$\lambda_F = \frac{F_s}{F_{2m} \times \lambda_{F2}} \tag{6}$$

Equating Eqs. (1) and (5),

$$\lambda T^2 = \frac{\lambda_M \times \lambda}{F_s} F_{1m} \times \lambda_{F1} \tag{7}$$

Equating Eqs. (1) and (6),

$$\lambda_T^2 = \frac{\lambda_M \times \lambda}{F_s} F_{2m} \times \lambda_{F_2} \quad (8)$$

Assuming that, of the four fundamental ratios in Eq. (1), λ and λ_M are fixed, it becomes apparent from Eqs. (7) and (8) that if more than one type of physical force (F_2, F_3 , etc., in addition to the inertia force F) influence the motion occurrences, then difficulties will arise in the use of models due to the contradictory requirements of the time ratios. This will become evident as the principal forces involved in hydraulic experiments are considered.

The principal hydraulic forces are:

$$\begin{aligned} F &= \text{Inertia force} = \rho \times L^2 \times V^2 && MLT^{-2} \\ F_1 &= \text{Viscous force} = \mu \times L^2 \times V/L && MLT^{-2} \\ F_2 &= \text{Gravity force} = \rho \times L^3 \times g && MLT^{-2} \\ F_3 &= \text{Elastic force} = E \times L^2 && MLT^{-2} \\ F_4 &= \text{Surface-tension force} = \sigma \times L && MLT^{-2} \end{aligned}$$

Eq. (2) can now be employed to pair the above forces.

(a) Considering first inertia and viscous forces:

$$\lambda_F = \frac{F_s}{F_m} = \frac{\rho_s L_s^2 \left(\frac{L_s}{T_s}\right)^2}{\rho_m L_m^2 \left(\frac{L_m}{T_m}\right)^2} = \frac{\rho_s}{\rho_m} \lambda^2 \left(\frac{V_s}{V_m}\right)^2 \quad (9)$$

$$\lambda_F = \frac{F_{1s}}{F_{1m}} = \frac{\mu_s L_s^2 \frac{V_s}{L_s}}{\mu_m L_m^2 \frac{V_m}{L_m}} = \frac{\mu_s}{\mu_m} \lambda^2 \frac{V_s L_m}{V_m L_s} \quad (10)$$

Equating Eqs. (9) and (10) and collecting all model magnitudes on the right side and those for the prototype on the left,

$$\frac{\rho_s}{\mu_s} \times \frac{V_s^2}{V_s} \times L_s = \frac{\rho_m}{\mu_m} \times \frac{V_m^2}{V_m} \times L_m$$

or

$$\frac{V_s L_s}{\nu_s} = \frac{V_m L_m}{\nu_m} = \text{a constant or 'numeric'} \quad (11)$$

From Eq. (3) this same constant can be derived dimensionally:

$$\lambda F_1 = \frac{F}{F_1} = \frac{MA}{\mu L^2 V / L} = \frac{ML}{\rho v T^2 L^2 V / L} = \frac{VL}{v}$$

This constant, known as the Reynolds number, or **R**, therefore must be maintained for dynamic similarity. For large values of **R**, inertia forces predominate in comparison with viscous forces. When **R** is small the reverse is true.

It is now possible to determine the model characteristics if the size of model (λ) and the type of medium (λ) are chosen, when viscous forces are dominant:

$$L_m = \frac{L_s}{\lambda} \tag{12}$$

$$M_m = \frac{M_s}{\lambda^3} \times \frac{\rho_m}{\rho_s} \tag{13}$$

$$V_m = \lambda V_s \times \frac{v_m}{v_s} \tag{14}$$

$$F_{1m} = \frac{\mu_m}{\mu_s} \times \frac{F_{1s}}{\lambda} \frac{V_m}{V_s} \tag{15}$$

$$T_m = \frac{T_s}{\lambda^2} \times \frac{v_s}{v_m} \tag{16}$$

When inertia force predominates, the model characteristics can be likewise determined. Eqs. (12) and (13) will again apply, but

$$V_m = \lambda V_s \sqrt{\frac{F_m}{F_s} \times \frac{\rho_s}{\rho_m}} \tag{17}$$

$$F_m = \frac{F_s}{\lambda^2} \times \frac{\rho_m}{\rho_s} \times \left(\frac{V_m}{V_s} \right)^2 \tag{18}$$

$$T_m = \frac{T_s}{\lambda^2} \sqrt{\frac{\rho_m}{\rho_s} \times \frac{F_s}{F_m}} \tag{19}$$

(b) In a like manner inertia and gravity forces can be paired:

$$\lambda_F = \frac{F_{2s}}{F_{2m}} = \frac{\rho_s}{\rho_m} \times \frac{L_s^3}{L_m^3} \times \frac{g_s}{g_m} \tag{20}$$

Equating Eqs. (9) and (20) and collecting terms,

$$\frac{V_s^2}{L_s g_s} = \frac{V_m^2}{L_m g_m} = \text{a constant or 'numeric'} \quad (21)$$

Or, from Eq. (4) by dimensional analysis,

$$\lambda_{F2} = \frac{F}{F_s} = \frac{MA}{Mg} = \frac{V^2}{gL} = \text{a constant}$$

This constant is known as the Froude number F and must be maintained for dynamic similarity when gravitational forces are involved. The model characteristics in terms of the prototype are, in addition to Eqs. (12) and (13):

$$V_m = \frac{V_s}{\lambda^{1/2}} \sqrt{\frac{g_m}{g_s}} \quad (22)$$

$$F_{2m} = \frac{F_{2s}}{\lambda^3} \times \frac{\rho_m}{\rho_s} \times \frac{g_m}{g_s} \quad (23)$$

$$T_m = \frac{T_s}{\lambda^{1/2}} \sqrt{\frac{g_s}{g_m}} \quad (24)$$

Since g is essentially constant it is commonly assumed that

$$V_m = \frac{V_s}{\lambda^{1/2}} \quad (22a)$$

$$F_{2m} = \frac{F_{2s}}{\lambda^3} \times \frac{\gamma_m}{\gamma_s} \quad (23a)$$

$$T_m = \frac{T_s}{\lambda^{1/2}} \quad (24a)$$

It should now be noted that if both viscous forces and gravity forces are involved in addition to the inertia force, the earlier prediction of incompatibility in the time-rate demands is clearly shown by Eq. (11) wherein $\frac{VL}{\nu}$ is required to be held constant and by Eq. (21) wherein $\frac{V^2}{gL}$ must likewise be held constant. Again, Eq. (16) demands that $T_m = \frac{T_s}{\lambda^2} \times \frac{\nu_s}{\nu_m}$ whereas Eq. (24) requires that

$T_m = \frac{T_s}{\lambda^{1/2}} \sqrt{\frac{g_s}{g_m}}$. This problem was first solved practically by

Froude in his application of the Law of Comparison.

(c) In like manner inertia and the elastic force due to compressibility will be considered. Since the elastic force is of little importance in ship model testing, it will be sufficient merely to derive its 'numeric':

$$\lambda_{F_3} = \frac{F}{F_s} = \frac{MA}{E \times \text{area}} = \frac{FT^2L}{LT^2L^2E}$$

$$\lambda_{F_3} = \frac{\rho V^2}{E} \tag{25}$$

This 'numeric' is known as the Mach number **M** and is frequently stated as V/V_c or $(V/V_c)^2$ where V_c is the velocity of sound in the fluid medium at a given temperature. This follows from the physical law that $E = \rho V_c^2$; hence

$$\frac{\rho V^2}{E} = \frac{V^2}{V_c^2} = \text{a constant} \tag{25a}$$

(d) The next pair of forces is inertia force and surface-tension force. Surface-tension, designated by σ , is involved in the energy loss due to "tension" in the elastic skin over curved surfaces; if the radius of curvature is small, as in very small models, this factor may influence motion phenomena:

$$\lambda_{F_4} = \frac{F}{F_s} = \frac{MA}{\sigma L} = \frac{ML}{\sigma LT^2} = \frac{\rho V^2 L}{\sigma} \tag{26}$$

This constant, or 'numeric', is known as the Weber number **W** and the model characteristics become, in addition to Eqs. (12) and (13),

$$V_m = \lambda^{1/2} V_s \sqrt{\frac{\sigma_m \rho_s}{\sigma_s \rho_m}} \tag{27}$$

$$F_{4m} = \frac{F_{4s}}{\lambda} \times \frac{\sigma_m}{\sigma_s} \tag{28}$$

$$T_m = \frac{T_s}{\lambda^{1/2}} \sqrt{\frac{\rho_m \sigma_s}{\rho_s \sigma_m}} \tag{29}$$

The total-resistance 'numeric' of a motion occurrence will be a function of all the contributing 'numerics':

$$C_t = \frac{R_t}{\rho S V^2 / 2} = \phi (\Pi_1, \Pi_2, \Pi_3, \dots) \quad (30)$$

This can readily be shown by applying the principle of dimensional analysis to a statement of the factors involved:

$$R_t = \phi (\rho, L, V, \mu, g, \sigma, E, L/K_s, r_1, r_2, r_3, \dots)$$

If V does not approach V_c and if the model size is not too small, the elastic and surface-tension forces can be neglected. Other minor energy losses, such as dissipation of heat, can also be neglected and their effect absorbed in the experimental factor. Therefore,

$$R_t = \Sigma K (\rho^a L^b V^c \mu^d g^e) \quad (31)$$

$$MLT^{-2} = (ML^{-3})^a (L)^b (LT^{-1})^c (ML^{-1} T^{-1})^d (LT^{-2})^e$$

$$(M) \quad 1 = a + d$$

$$(L) \quad 1 = -3a + b + c - d + e$$

$$(T) \quad -2 = -c - d - 2e$$

from which

$$a = 1 - d$$

$$b = 2 - d + e$$

$$c = 2 - d - 2e$$

and d and e can be determined only by experiment:

$$R_t = \Sigma K (\rho^{1-d} L^{2-d+e} V^{2-d-2e} \mu^d g^e)$$

or

$$\frac{R_t}{\rho L^2 V^2} = \phi \left(\frac{VL}{v}, \frac{V^2}{gL}, \frac{L}{K_s}, r_1, r_2, r_3, \dots \right)$$

Substituting $\rho/2$ for ρ and S for L^2

$$C_t = \frac{R_t}{\rho S V^2 / 2} = \phi \left(\frac{VL}{v}, \frac{V}{\sqrt{gL}}, \frac{L}{K_s}, r_1, r_2, r_3, \dots \right) \quad (32)$$

This drag coefficient is non-dimensional with respect to units, but in practice is no longer so with respect to the scale factor λ , since no fluid is available for the model with a sufficiently low value of v to reconcile the conflicting requirements that the Reynolds number and the Froude number be simultaneously held constant for both model and prototype.

APPLICATION OF THEORY

The application of the foregoing theory to three types of hydraulic models will now be considered.

(a) *Submerged forms.* Since the predominate physical force is viscous,

$$C_t = \frac{R_t}{\rho S V^2 / 2} = \phi \left(\mathbf{R}, \frac{L}{K_s}, r_1, r_2, r_3, \dots \right) \quad (33)$$

and dynamic similarity is possible. If the model is tested in a fluid having the same density and viscosity as the prototype medium, however, an inspection of Eq. (14) indicates practical difficulties, for

$$V_m = \lambda V_s \quad (34)$$

$$F_m = F_s \quad (35)$$

(b) *Submerged Plates, 90° to the Motion.* Obviously the eddy-making forces predominate and the viscous forces are negligible; hence Eq. (33) reduces to

$$C_t = \frac{R_t}{\rho S V^2 / 2} = \phi (r_1, r_2, r_3, \dots) \quad (36)$$

where r represents aspect ratios of the plate. Referring to Eq. (18), when $\rho_s = \rho_m$ and $V_m = V_s$, then

$$F_m = \frac{F_s}{\lambda^2} \quad (37)$$

and the forces are proportional to the respective areas.

(c) *Surface Vessels.* The motion of partly submerged forms is affected by all the forces previously discussed. Neglecting, for convenience, the minor forces, Eq. (32) applies to the total resistance of these bodies. The method invoked to make this formulation of practical value was first proposed by William Froude, and is universally used.

The first assumption is that the total resistance function can be broken down into two independent functions

$$C_t = \frac{R_t}{\rho S V^2 / 2} = \phi_1 \left(\frac{VL}{v}, \frac{L}{K_s}, r_1, r_2, r_3, \dots \right) + \phi_2 \left(\frac{V}{\sqrt{gL}}, \frac{L}{K_s}, r_1, r_2, r_3, \dots \right) \quad (38)$$

$$C_t = \frac{R_f}{\rho S V^2 / 2} + \frac{R_r}{\rho S V^2 / 2} \quad (39)$$

$$C_t = C_f + C_r \quad (40)$$

or

$$R_t = R_f + R_r \quad (41)$$

The second assumption is that C_f can be calculated for any form and surface condition. Having measured R_t , and hence C_t , for the model in the tank, C_f for the model can be deducted, leaving C_r , or the coefficient of residual resistance.

The third assumption is that C_r is a constant for both model and ship at corresponding speeds or equal values of V/\sqrt{gL} . That is, the Law of Comparison is applied to the residual resistance of the model in order to obtain R_r for the ship or prototype.

At corresponding speeds

$$V_m = V_s / \lambda^{1/2}$$

and

$$C_r = (C_t)_m - (C_f)_m = (C_t)_s - (C_f)_s$$

then

$$(C_t)_m - (C_t)_s = (C_f)_m - (C_f)_s \quad (42)$$

and since

$$\frac{R}{\rho S V^2 / 2} \propto \frac{R}{\Delta} = M^0 L^0 T^0$$

then

$$\left(\frac{R_t}{\Delta} \right)_m - \left(\frac{R_t}{\Delta} \right)_s = \left(\frac{R_f}{\Delta} \right)_m - \left(\frac{R_f}{\Delta} \right)_s \quad (43)$$

The total resistance of the ship can be derived from

$$(R_t)_s = \frac{\rho_s}{2} S_s V_s^2 \left[\frac{R_{t_m}}{\frac{\rho_m}{2} S_m V_m^2} - \left((C_f)_m - (C_f)_s \right) \right] \quad (44)$$

or

$$(R_t)_s = (R_f)_s + \left[\left((R_t)_m - (R_f)_m \right) \frac{\rho_s g_s}{\rho_m g_m} \lambda^3 \right] \quad (44a)$$

The significance of Eq. (42) is clearly indicated in the idealistic graph shown in Fig. 1. It will be seen that, in order to obtain C_t for the ship from the C_t of the model, any curve parallel to the C_f curve will serve equally as well, since the absolute value of C_r is unimportant in this extrapolation. It is therefore obvious that the

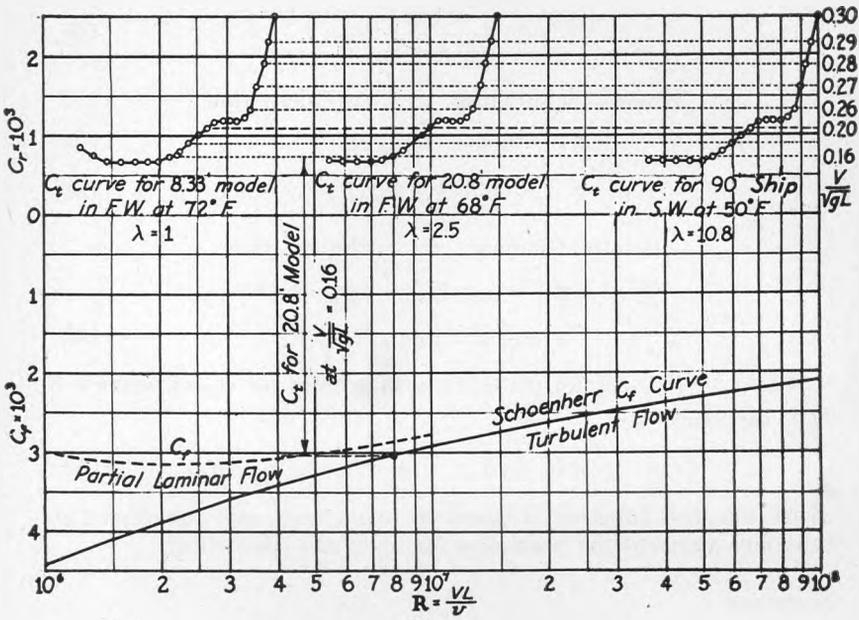


FIG. 1.

slope of the C_f curve is the controlling factor for accurate extrapolation. It would be desirable, however, if a single C_f curve were universally adopted by all tanks in order to facilitate test comparisons.

It is not within the purpose of this paper to discuss the questions of frictional resistance, surface roughness, self-propulsive technique, scale effect, or the final determination of power for ships. It might be useful, however, to list those C_f formulations which the author has found most accurate as extrapolators. In the following, the terms "laminar," "transition," and "turbulent" indicate flow conditions in the boundary layer, and C_f is the mean and C'_f the local coefficient:

Blasius — laminar flow

$$C_f = 1.327 R^{-1/2} \tag{45}$$

Prandtl — transition flow

$$C_f = 0.074 R^{-0.20} - 1700 R^{-1} \tag{46}$$

Schoenherr — turbulent flow

$$\frac{0.242}{\sqrt{C_f}} = \log_{10} (R + C_f) \tag{47}$$

$$C'_f = \frac{0.558 C_f}{0.558 + 2\sqrt{C_f}} \quad (48)$$

Prandtl-Schlichting — turbulent flow

$$C_f = 0.455 (\log_{10} R)^{-2.58} \quad (49)$$

$$C'_f = (-0.65 + 2 \log_{10} R)^{-2.30} \quad (50)$$

Schultz-Grunow — turbulent flow

$$C_f = 0.427 (-0.407 + \log_{10} R)^{-2.64}$$

$$C'_f = 0.370 (\log_{10} R)^{-2.584} \quad (52)$$

It may be that the ultimate friction formula for vessel forms will be of the type

$$C_f = \phi_1 (R, L/K_s) + \phi_2 (R_1) + \phi_3 (R_2) \quad (53)$$

where the first function is based on plank tests, and the other two functions augment the resistance for form and edge effect.

NOTATION

*	dimensionless for a consistent system of units	
Π	a numeric	*
L	unit of length	<i>foot</i>
M	unit of mass	<i>slug</i>
T	unit of time	<i>second</i>
F	unit of force = MA	MLT^{-2}
L	length on water line	L
L_{EFF}	effective length of underwater form	
	= $\frac{\text{area of submerged centerline plane}}{\text{mean draft}}$	L
B	maximum moulded beam at water line	L
H	moulded draft	L
A_{Σ}	area of maximum transverse section	L^2
S	wetted surface area of under-water form	L^2
∇	volume of displacement	L^3
Δ	displacement or buoyancy in long tons, salt water. (subscript F.W. for fresh water.)	MLT^{-2}
V	mean velocity in feet per second	LT^{-1}
V_c	velocity of sound	LT^{-1}

$\frac{V}{\sqrt{gL_{EFF}}}$	Froude number (F)	II
$\frac{VL_{EFF}}{\nu}$	Reynolds number (R)	II
A	linear acceleration	LT ⁻²
g	acceleration of gravity	LT ⁻²
ρ	mass density	ML ⁻³
μ	coefficient of absolute viscosity	ML ⁻¹ T ⁻¹
ν	coefficient of kinematic viscosity = μ/ρ	L ² T ⁻¹
σ	coefficient of surface tension	MT ⁻²
γ	specific weight = ρg	ML ⁻² T ⁻²
E	coefficient of elastic modulus (stress)	ML ⁻¹ T ⁻²
λ	linear ratio of ship to model	*
R _t	total resistance	MLT ⁻²
R _f	friction resistance	MLT ⁻²
R _r	residual resistance = R _t - R _f	MLT ⁻²
R/Δ	resistance per ton displacement	*
C _t	coefficient of total resistance = $\frac{R_t}{\rho SV^2/2}$	*
C _f	coefficient of mean friction resistance = $\frac{R_f}{\rho SV^2/2}$	*
C _f '	coefficient of local friction resistance = $\frac{dR_f}{\rho ds V^2/2}$	*
C _r	coefficient of residual resistance = C _t - C _f = $\frac{R_r}{\rho SV^2/2}$	*
K _s	measure of surface roughness	L
$\frac{L_{EFF}}{K_s}$	coefficient of roughness	*
φ	any function	
K	any constant	

REFERENCES

International Conference of Tank Superintendents, The Hague, July 13 and 14, 1933.

Congrès International des Directeurs de Bassins, Paris. Octobre, 1935.

Internationale Tagung der Leiter der Schlepptversuchsanstalten, Berlin, May, 1937.

- Saunders, Harold E., *C. and R. Bulletin No. 7*, Navy Department, Washington, December, 1932.
- Schoenherr, K. E., "Resistance of Flat Surfaces Moving Through a Fluid," *Transactions Society of Naval Architects and Marine Engineers*, 1932.
- Lackenby, H., "Re-Analysis of William Froude's Experiments on Surface Friction and their Extension in the Light of Recent Developments," *Transactions Institution of Naval Architects*, 1937.
- Skin Friction Committee's Report, *Transactions Institution of Naval Architects*, 1925.
- Schultz-Grunow, F., "New Frictional Resistance Law for Smooth Plates," *Luftfahrtforschung*, Vol. 17, No. 8, August 20, 1940, *N.A.C.A. T.M. No. 986*.