

THE TRANSPORTATION OF SAND IN PIPES

I. FULL-PIPE FLOW

By

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Problems posed by the movement of granular materials by fluid media are encountered in many phases of engineering endeavor. The elimination of unwanted sediment from drainage systems, the dredging of channels, the suspension of catalysts in "fluidized beds," and the transport of coal in pipe lines are just a few of the applications requiring a fundamental knowledge of the laws governing fluid-solid mixtures. The impetus for the investigation described in this paper was provided by the problems arising in the design of drainage systems. The study was therefore limited to a small phase of the general problem. The results, however, are general and should be useful to workers in these many related fields.

One of the most perplexing aspects of the design of drainage systems is with regard to the proper allowance to be made for the presence of sediments in the flow. Too often undesirable but unavoidable accumulations of sediment seriously reduce the ability of the system to perform its intended function by constricting the cross section and by increasing the effective roughness of the pipe. Removal of these sediments is a costly operation and should be avoided by designing the system to carry the expected load, or by providing easily cleaned sand traps.

Unfortunately, the criteria upon which proper design must be based have not been determined. Although conditions at the site will indicate the type, size-distribution and quantity of sediments, the available hydraulic gradient, and the average and maximum runoff, the effect of these variables on the performance of any proposed design has not as yet been assessed.

There is at present a paucity of information either theoretical or experimental pertaining to the problem. Previous experiments on the transport of sand in pipes have been primarily concerned

with dredging and, therefore, with suspended load moving at high velocities. In the case under consideration, however, the velocities and sediment concentrations are relatively low. The major portion of the sediment is transported adjacent to the bottom or bed and only a small percentage is carried in suspension. The problem becomes one of bed-load transport and its relation to the geometric and hydraulic properties of the conduit and flow. A comprehensive experimental program designed to evaluate this relationship was undertaken at the Iowa Institute of Hydraulic Research under the sponsorship of the Bureau of Public Roads. This paper presents the principal results of that portion of the investigation which treated with conduits flowing full. A detailed account of this study is available in the dissertation [1] submitted by the writer to the Graduate College of the State University of Iowa in partial fulfillment of the requirements for the Ph. D. An extension of this study to include the general case of the pipe flowing partially full is presented by Dr. Ambrose [2] as the second part of this paper.

Present transport theory is to a large degree embodied in a series of bed-load equations which have been determined empirically or semi-empirically for open channels. Although the quantitative agreement between them is poor, they are useful as a qualitative indication of the fundamental parameters involved in the transport process. The majority of these equations are expressible in the form

$$S = \varphi \left[\frac{q_s}{q}, \frac{\Delta\gamma}{\gamma}, \frac{y}{d}, S_c \right] \quad (1)$$

in which

- S is the hydraulic slope or energy gradient,
- q_s is the absolute volume rate of sand discharge per unit width,
- q is the volume rate of flow per unit width,
- γ is the specific weight of water,
- $\Delta\gamma$ is the difference in specific weight of water and sand,
- d is the mean diameter of the sand grain,
- y is the mean depth of flow, and
- S_c is the critical energy slope at which movement begins.

An analogous expression for a pipe flowing full must reflect the dissimilarities between pipe flow and open-channel flow. The slope S of the bed is necessarily equal to the piezometric gradient for uniform flow in an open channel, whereas the two are independent in pipe flow. Therefore, the pipe equation must include both S and S_p , the

slope of the pipe, as independent parameters. A further dissimilarity is encountered in the concept of a limiting energy slope designated as S_c — the slope for which the force exerted by the flowing water on the particles comprising an inert bed just overcomes the resistance to motion. However, there are many flow conditions for which no sand bed exists. The slope S_L is therefore defined as the limiting value of the slope for which the ratio of sediment transport to the rate of flow (Q_s/Q) approaches zero

$$S_L = \lim_{Q_s/Q \rightarrow 0} [S]$$

As stated above, two limits can occur, one in which an inert bed of sand deposits on the bottom of the pipe and for which the limiting slope is dependent upon the critical shear of the sand, and one in which all of the sediment is swept from the pipe and the limiting slope is simply that for sediment-free pipe flow. The transport equation for a pipe thus has the form

$$S = \varphi \left[\frac{Q_s}{Q}, \frac{d}{D}, \frac{\Delta\gamma}{\gamma}, S_L, S_p \right] \quad (2)$$

in which

- Q_s is the absolute volume rate of sand flow,
- Q is the volume rate of fluid flow, and
- D is the pipe diameter,
- S_p is the slope of the pipe.

In addition to the bed-load equation the flow is necessarily characterized by the familiar resistance equation

$$S = \frac{f}{4R} \frac{V^2}{2g} \quad (3)$$

in which f is the resistance coefficient and R is the hydraulic radius. The velocity in this equation is taken as that in the constricted section if deposition is present, and R is the mean hydraulic radius. The resistance coefficient will depend upon the material in the bed, the shape of the bed, the Reynolds number, and the geometry of the cross-section. Equation (3) can be expressed in the functional form

$$\frac{Q}{D^{2.5} \sqrt{gS}} = \varphi \left[\frac{y_s}{D}, \frac{d}{D}, R, \frac{k}{D} \right] \quad (4)$$

in which

y_s is the depth of sand in the pipe,

R is the Reynolds number, and

k is the equivalent roughness height for the pipe.

Equations (1) and (4) can be combined to eliminate the energy gradient, reducing to

$$\frac{Q}{D^{2.5}\sqrt{g}} = \varphi \left[\frac{Q_s}{q}, \frac{y_s}{D}, \frac{\Delta\gamma}{\gamma}, \frac{d}{D}, R, \frac{k}{D} \right] \quad (5)$$

or to eliminate the discharge, giving

$$S = \varphi \left[\frac{Q_s}{D^{2.5}\sqrt{g}}, \frac{\Delta\gamma}{\gamma}, \frac{y_s}{D}, \frac{d}{D}, R, \frac{k}{D} \right] \quad (6)$$

These equations are sufficient to describe the bed-load transport once their form is known. The experiments conducted by the writer were designed to determine these functional relationships.

The primary experimental equipment employed in the investigation is illustrated schematically in Fig. 1. The pipe was a sixty-foot length of lucite tubing 6 inches in diameter, and was supported in adjustable cradles mounted on an I-beam. A portion of the experiments was conducted using a 2-inch lucite pipe of proportionate length. The slope of the pipe was made adjustable by means of a pulley system. Water was supplied from a constant-head tank, and the discharge and depth of flow were controlled by gates at the upstream and downstream ends of the pipe. The flow was measured either by means of an orifice meter or by direct readings of weight during selected time intervals. The energy gradient was measured by means of piezometers placed at five-foot intervals along the pipe. Three grades of uniform quartz sands (0.25, 0.58 and 1.62 mm) were employed. The sands were fed into the upstream end of the pipe by a mechanically operated sand feeder and were trapped at the discharge end. For each run the fluid and sand discharges were established at predetermined rates and so maintained until equilibrium was reached; this condition was determined from successive readings of the piezometers, successive measurements of the bed, and visual observation.

The energy gradient is plotted as a function of the transport-discharge ratio in Fig. 2. It is seen that there are three distinct regions; one in which the head loss is primarily dependent upon the transport discharge ratio Q_s/Q , a transition region, and a limit-

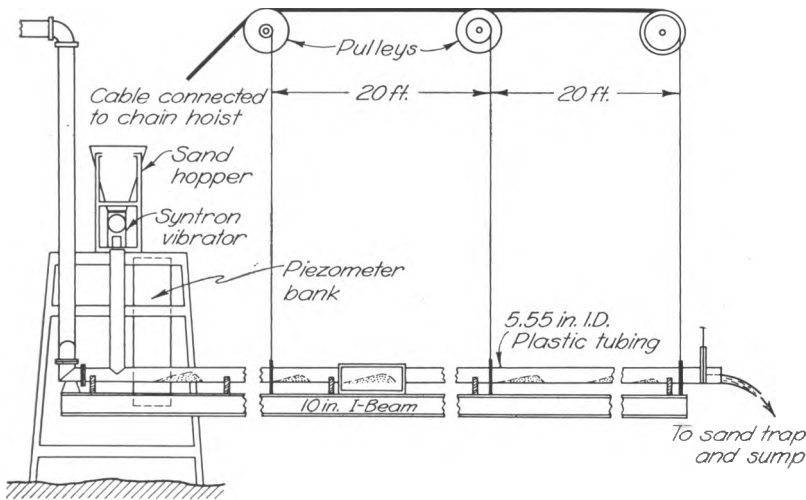


FIG. 1. EXPERIMENTAL EQUIPMENT EMPLOYED IN THE INVESTIGATION.

ing region in which the gradient is independent of the sediment transport and approaches the limiting slope.

A relationship for the limiting slope can be determined analytically from an analysis of the mechanics of flow. If it is presumed (after White [3]) that the force exerted on a particle by the flowing water which is just sufficient to cause its motion, is proportional to the immersed weight of the particle, and that the average shear in the pipe is related to the critical shear for that condition by the relationship

$$\tau_{av.} = \tau_c \varphi(y_s/D) \tag{7}$$

It follows from elementary mechanics that, for the limiting slope S_L

$$\tau_{av.} = \gamma S_L R = \tau_c \varphi(y_s/D) \propto \Delta \gamma d \varphi(y_s/D) \tag{8}$$

Introduction of the resistance equation leads to the relationship

$$\frac{fV^2}{(\Delta \gamma / \rho) d} = \varphi(y_s/D) \tag{9}$$

Here V is the mean velocity in that area of the pipe which is unobstructed by sand,

$$V = \frac{4Q}{\pi D^2 (1 - A_s/A_o)}$$

in which A_s is the obstructed area of the pipe and A_o is the entire area of the pipe. Algebraic manipulation of Eq. (9) yields an expression in a form of primary interest to designers (i.e., an expression

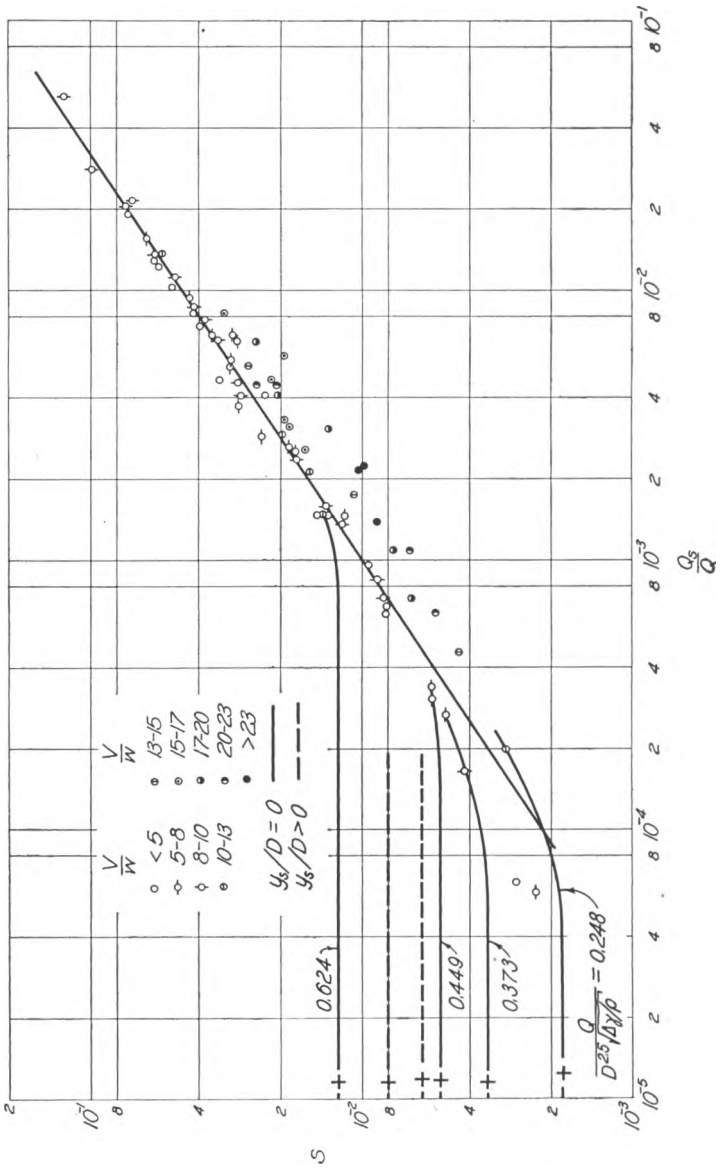


FIG. 2. ENERGY GRADIENT IN TERMS OF THE TRANSPORT-DISCHARGE RATIO.

for the amount of sand which will remain in the pipe when the flow is so reduced that the limit of movement is reached).

$$y_s/D = \varphi \left[\frac{Q}{D^2 d^{0.5} \sqrt{(\Delta\gamma/\rho)d}}, \frac{d}{D} \right]$$

The validity of this relationship is shown in Fig. 3. The experimental points in this figure were determined by filling the pipe to a predetermined level with sand and increasing the flow rate until movement was observed. Experimental difficulties precluded measurement of the slope in these experiments but it is readily seen that S_L is given by Eq. (8) if $y_s/D > 0$, and becomes that for sediment-free pipe flow if $y_s/D = 0$. Limiting values for which $y_s/D > 0$ are shown as dotted lines in Fig. 2, and several limiting values for clear-pipe flow are shown as solid lines.

Figure 2 indicates the existence of a short transition from a value of the slope which is constant to one which can be approximated by the equation

$$S = 0.606 \frac{\Delta\gamma}{\gamma} \left[\frac{Q_s}{Q} \right]^{2/3} \tag{10}$$

The considerable scatter of the points about the line of this equa-

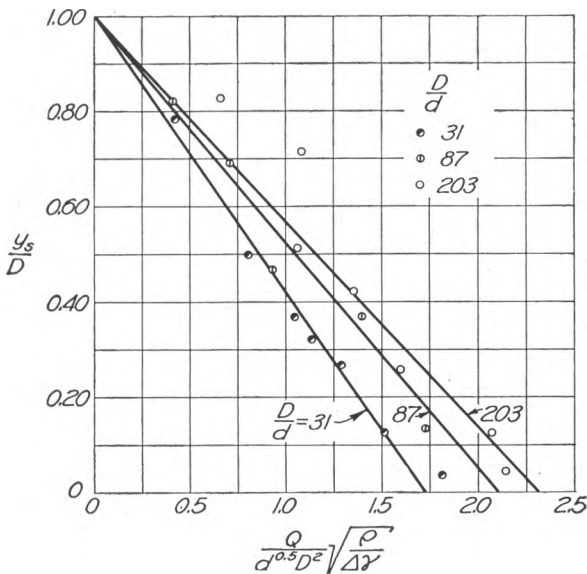


FIG. 3. FUNCTIONAL RELATIONSHIP BETWEEN RATIO OF DEPTH OF SAND TO PIPE DIAMETER AND A PARAMETER OF WATER DISCHARGE.

tion is attributable to the bed configuration, which in turn appears to be a function of the parameters $\frac{Q_s}{Q}$, $\frac{Q}{D^{2.5}\sqrt{\Delta\gamma/\rho}}$, and $\frac{V}{w}$ (the ratio of the mean velocity of flow in the unobstructed area to the fall velocity of the sediment particle). If all other parameters are held constant and the transport-discharge ratio Q_s/Q is varied, the shape of the sand bed changes in the following manner. If Q_s/Q is equal to zero, the pipe line will either be free of sand or will contain an inert bed. Which of these conditions will occur depends upon the magnitudes of the fixed parameters and the history of transport. If Q_s/Q is just greater than zero, isolated dunes will appear in the pipe or will be superposed upon an inert bed. As the ratio is increased, the dune spacing decreases until the dunes join together. A further increase in Q_s/Q causes the dunes to lengthen and to flatten until the bed is perfectly plane.

Although the bed configuration for each of the sands evolved through the same pattern, the value of Q_s/Q at which a given change in form occurred was different for each grain size (i.e., a

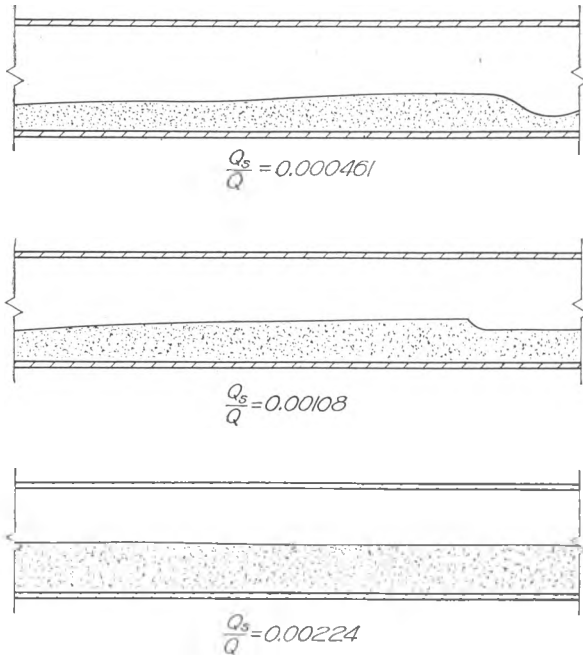


FIG. 4. TRACINGS OF BED PROFILES FOR SELECTED RATIOS OF SEDIMENT TRANSPORT TO WATER DISCHARGE.

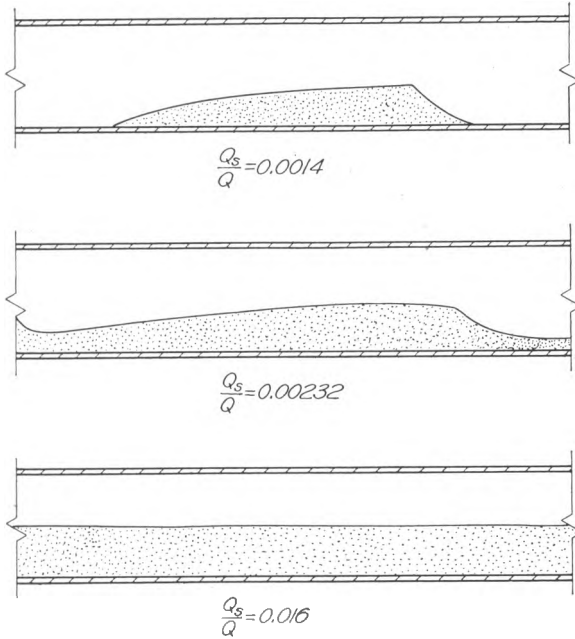


FIG. 5. TRACINGS OF BED PROFILES FOR SELECTED RATIOS OF SEDIMENT TRANSPORT TO WATER DISCHARGE.

dependence upon the parameter V/w was indicated). Figures 4 and 5 are tracings of photographs of the bed which illustrate this geometric transition. It appears from these figures that the higher the value of V/w the greater is the tendency for the sediment to be lifted into suspension and the less the tendency for it to travel in a series of dunes. Examination of Fig. 2 shows that the scatter in the points can be explained in terms of this parameter and its effect upon the geometry of the deposited sediments.

It should be noted that the form of Eq. (10) is the same as for the great majority of bed-load equations if the shear is well above the critical. These equations reduce to the form

$$S \propto \frac{\Delta\gamma}{\gamma} \left[\frac{Q_s}{Q} \right]^n$$

in which n is an exponent that varies from $\frac{1}{2}$ (Shields) to $\frac{2}{3}$ (Schoklitsch, Meyer-Peter [4]). This variation probably arises from variations in the range of experimentation and from the approximate nature of the exponential function.

A relationship for the quantity of sediment which will be deposited in the pipe can be obtained by combining Eq. (10) with the Darcy-Weisbach equation. This relationship is illustrated in Fig. 6.

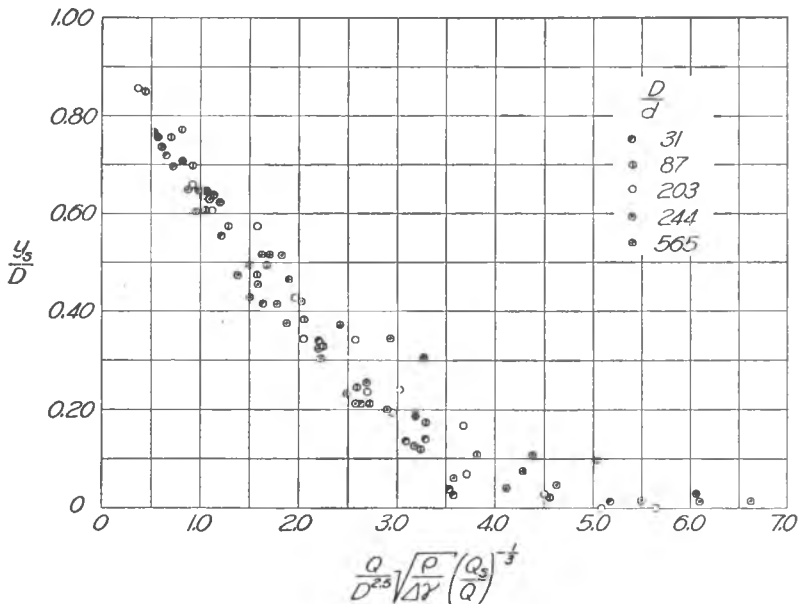


FIG. 6. RELATIONSHIP BETWEEN RATIO OF DEPTH OF SAND TO PIPE DIAMETER AND A PARAMETER OF SEDIMENT CONCENTRATION.

$$y_s/D = \varphi \left[\frac{Q}{D^{2.5} \sqrt{\Delta\gamma/\rho}} \times \left(\frac{Q_s}{Q} \right)^{-1/3}, \frac{d}{D}, R, \frac{k}{D} \right]$$

Although the results of this investigation are not extensive enough to cover the entire range of parameters to be encountered in drainage design, certain limits are definitely indicated. The designer must insure that the great majority of the expected flows

will be such that the parameter $\frac{Q}{d^{0.5} D^2 \sqrt{\Delta\gamma/\rho}}$ exceeds 2.5. If such

is not the case a permanent deposit of sediment will gradually accumulate in the pipe. If the drainage area is one which contains large quantities of clay or other materials of a cementing nature, then the sediments must be kept in motion at all times (*i.e.*,

$\frac{Q}{D^{2.5} \sqrt{\Delta\gamma/\rho}} \times \left(\frac{Q}{Q_s} \right)^{-(1/3)}$ should exceed 5.0). Other design criteria

may be devised and it is hoped that the material presented herein will be a useful guide in the solving of the perplexing problems arising in the design of storm drains.

THE TRANSPORTATION OF SAND IN PIPES

II. FREE-SURFACE FLOW

By

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In an extension of the investigation described by Dr. Craven in the preceding paper, the program of study was generalized to the extent of giving primary consideration to the transport of sand in a pipe under free-surface conditions. Although the case of full-pipe flow is again discussed, it is treated as a limiting condition for free-surface flow. As is required for the more general phenomenon, emphasis is placed on the geometry of flow rather than on sediment concentration.

It is quite evident that the transport of sediment by free-surface flow in a non-rectangular channel is an exceedingly complex phenomenon. The usual simplification of the flow pattern made by imposing the conditions of steadiness and uniformity can be applied only in a statistical sense to sediment transport. In the general case, the configurations of bed and fluid surface, as well as the characteristics of flow, change continuously with time and distance along the channel; consequently, it becomes necessary to deal with averages with respect to both time and distance for each of the variables involved.

The variables pertinent to this case are:

1. For the fluid: Q — volume rate of flow
 y — average depth above bed
 γ — specific weight
 ν — kinematic viscosity
2. For the sediment: Q_s — absolute volume rate of transport
 d — nominal grain diameter of uniform sediment
 y_s — average depth of bed
 γ_s — specific weight

3. For the pipe: D — diameter
 k — linear measure of roughness
4. General: g — acceleration of gravity
 S — energy gradient = hydraulic gradient = slope
of bed = pipe slope

It will be noted that the list omits certain variables which in other situations may be important. Specifically, there is no measure of the bed configuration, nor is an entrainment quantity defined. Although the existence of a “critical discharge”, “critical slope”, or “critical shear” which precisely defines the beginning of entrainment is not precluded, any variables upon which such a quantity would depend are already present in the above list. Perhaps the most realistic way of viewing the effect of the initiation of movement in the present case is to consider the fact that, in decreasing to a very low value, the transport rate ultimately has a negligible effect on the resistance to flow.

As is true of the entrainment characteristic, the form of the bed is not independent but depends upon the listed variables and no others if uniform sediment is assumed.

Because of the many variables which must be related and the extreme complexity of the mechanics of transport, it is understandable that a rigorous derivation of the general transport function has not been found possible. Even in the simplified case of two-dimensional flow, all transport equations have been derived by the use of certain assumptions. In each instance, the assumptions — whether gross or concerned with details of the mechanics — exert a real influence on the quantitative relationships. So it is that all present transport equations are at best semi-empirical. In order to advance the present state of knowledge, then, experimental investigation appears necessary.

Dimensional consideration of the above-mentioned variables will yield functional forms for the necessary relationships. Two relationships to define the phenomenon are required, as a minimum: (a) a transport function, relating the transport to the geometry and to other characteristics of flow, channel, and sediment; and (b) a discharge function, relating the discharge to the resistance to flow from sand bed and pipe wall. Because of the large number of variables, the possible forms of these two functions are numerous. Fortunately, past experience affords a guide to the more significant forms.

Earliest investigations of the primary processes of sediment transport — namely, scour and deposition — indicated the importance of the mean velocity of flow and the geometry of the cross section in these processes. Although in later years attention has been diverted to the details and minutiae of the transport phenomenon (perhaps in some measure due to vagaries resulting from oversimplification of the velocity-transport relations), most investigations in this field again center attention on the velocity-geometry-transport relation.

The mean velocity of flow can be written in the functional form

$$V = (Q/D^2)\varphi[y/D, y_s/D]$$

since the discharge and mean cross-sectional area of flow are necessary to its definition. (The symbol φ stands for “function of”.) Inasmuch as the overall flow-transport process is predominantly a gravitational phenomenon it appears logical to combine Q_s and g to form a term for transport with the dimensions of velocity. The only such combination possible is $Q_s^{1/5} g^{2/5}$. On the basis of these considerations, the transport function becomes

$$\frac{Q/D^2}{Q_s^{1/5} g^{2/5}} = \varphi [y/D, y_s/D, d/D, k/D, s_s, \mathbf{R}] \quad (1)$$

in which s_s is the specific gravity of the sediment and is equal to γ_s/γ , and \mathbf{R} is a Reynolds number term which effectively indicates the role of viscosity.

The form of the discharge function is obtained by writing the Chézy equation as

$$\frac{Q}{A\sqrt{gRS}} = \sqrt{f/8}$$

in which f is the Darcy-Weisbach resistance coefficient, A is the cross-sectional area of flow, and R is the effective hydraulic radius. Since in the present situation,

$$\begin{aligned} A &= D^2 \varphi [y/D, y_s/D] \\ R &= D \varphi [y/D, y_s/D, d/D, k/D, \mathbf{R}] \\ f &= \varphi [y/D, y_s/D, d/D, k/D, \mathbf{R}] \end{aligned}$$

the above expressions can be combined to yield the desired discharge function,

$$\frac{Q}{g^{1/2} D^{5/2} S^{1/2}} = \varphi [y/D, y_s/D, d/D, k/D, \mathbf{R}] \quad (2)$$

The functional relationships indicated in Eqs. (1) and (2) remain

to be determined experimentally. The experimental program designed for this purpose was carried out in three parts.

In addition to the general case, two limiting conditions will at once be noted. If for a given equilibrium condition the slope of the pipe or the discharge is decreased, or if the sediment load is increased, the geometry of the cross section must also change progressively until the capacity for transport is once again equal to the sediment load. Whatever the initial cause of the unbalance, the result will be deposition of part of the bed load in the upstream section of the pipe as the system attempts to increase the bed slope and thereby restore the transport capacity. If the pipe is relatively short and the unbalance slight, equilibrium may be restored with non-uniform flow conditions. On the other hand, if the pipe is long or the unbalance marked, deposition will continue until the pipe is under pressure. Eventually, a uniform-flow system is obtained for which the hydraulic gradient is no longer fixed by the pipe slope. Craven [1] has shown that the independent effect of pipe slope is negligible for this range ordinarily encountered in practice, however. For uniform flow, the geometry is simplified in that the depth of sand bed is always equal to the difference between the pipe diameter and the depth of flow. That is to say, the characteristic equation of the first limiting condition is $y_s/D = 1 - y/D$. Hence, the average y/D value is adequate to define the mean size and shape of the flow cross section.

Logically, the second limiting condition occurs as the ultimate effect of unbalance of equilibrium opposite to the first case. Increase of discharge or pipe slope, or decrease of sediment load will bring about entrainment of the inert bed material. Since the thickness of the bed is finite, the pipe will be swept clear of sediment if the unbalance is sufficiently great. Without an inert bed, the mean geometry is again defined by the average value of y/D . Further unbalance merely results in an advanced stage of transport — that of suspension. However, the limiting condition of primary interest in the maintenance of sediment-free pipes is that for sediment moving in flume traction without deposition but for which a slight increase in sediment load or decrease in discharge or slope causes deposition. For the purpose of identification, the slope for this condition will be referred to as the slope of impending deposition S_{id} . The characteristic equations for this rather arbitrarily defined limiting condition are $y_s/D = 0$ and $S = S_{id}$.

The equipment employed was essentially that described in the preceding paper. The test pipe was modified by the installation at the upstream end of an open head-box with baffles and a vertical sluice gate, and by the provision of opening at five-foot intervals along the pipe for measuring depths with a point gage. A special piezometer system and manometer board permitted remote measurement of both slope and depth simultaneously. A complete description of the equipment will be found in the writer's dissertation [2].

The procedure varied with the particular type of flow. In the investigation of Case I ($y_s/D = 0$; $S = S_{id}$), the desired rates of sand and water flow were set with the pipe inclined at a slope which would maintain the sand in constant motion. The slope was slowly decreased until the sand grains began to deposit, whereupon the sand feed was diverted to clear the pipe. The slope was then increased approximately 1 percent and sand was again fed into the pipe. If no deposition appeared, the desired condition was considered established; otherwise, the procedure was repeated.

For Case II ($y_s/D = 1 - y/D$), the procedure was similar to that described in the preceding paper. Modification of the equipment facilitated the determination of the equilibrium and the measurement of the hydraulic gradient.

The procedure for the general condition designated as Case III ($0 < y_s/D < 1 - y/D$), consisted in first establishing the approximate depth of bed required for full-pipe flow, then setting the rates of water and sand flow for the run, and finally adjusting the pipe slope to obtain the equilibrium condition. The slope of the water surface was kept equal to that of the pipe and bed. The method of determining if the flow had become stabilized varied with the rate of sand transport. For the highest rates of sand feed, changes in bed and water surface occurred quite rapidly if the system was out of balance. Visual observation was sufficient to determine whether or not uniform conditions existed for such runs. For runs with lower rates of sand transport it was necessary to measure the rate of sand discharge from the pipe and to match this to the rate of feed by adjustment of the pipe slope. Timed samples of the sediment discharge, caught in sieves of appropriate mesh size, were weighed immersed in a calibrated bottle. The length of time for a representative sample varied from about 3 to 30 minutes depending upon the rate of sand flow and the configuration of the bed. At least two dunes were allowed to pass from the pipe, and the sam-

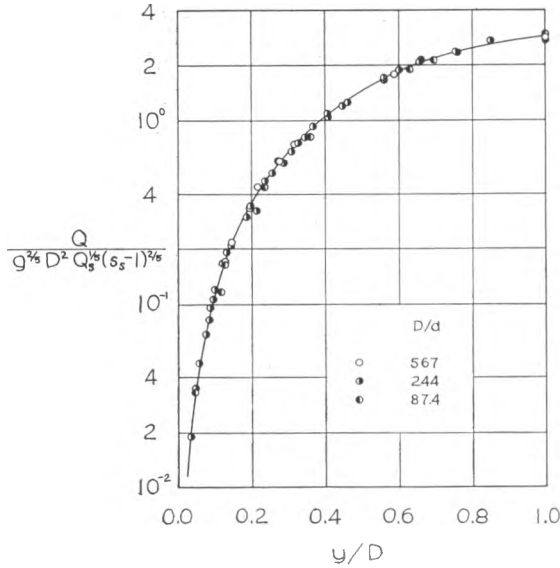


FIG. 1. TRANSPORT FUNCTION FOR IMPENDING DEPOSITION (CASE I).

pling was started and stopped when the leading edges of the succeeding dunes passed the same section in the pipe. A complete tabulation of the experimental data is to be found in the writer's dissertation. Results of preliminary tests to determine the resistance characteristics of the plastic pipe were compared with the Kármán-Prandtl resistance equation for smooth pipes. The test-pipe resistance conformed to the equation throughout the range of experimentation.

Noteworthy in Figs. 1 and 2 is the fact that for all three cases the transport function appears to be dependent solely upon the mean geometry. No effect of d/D is discernible although extension of the range for Case II over eighteen-fold is made by incorporating Craven's data for the 1.62-mm sand in the 2-inch pipe. No effect on these or other geometrically related functions is shown by the bed configuration in spite of the extreme range, from a perfectly flat bed, through low dunes fourteen times the pipe diameter in length, to separate dunes of height more than one-third of the pipe diameter moving along the otherwise bare pipe.

The specific gravity of the sand was not varied. The term $(s_s - 1)^{2/5}$ in the transport function is included on the basis of

qualitative agreement with the Shields transport equation [5] since Shields was notably successful in correlating data for sediments of widely varying specific gravities.

Although k/D was not varied it seems reasonable to assume that k/D will play no significant role in the given transport function. This assumption is supported both by the character of the function and by the lack of effect of varying d/D . A definite trend with d/D is to be noted in all other functions which are inherently dependent on resistance and for which there exists an inert sand bed.

It must be noted that the investigation was not carried into the range of negligible transport and it is not to be expected that the functional relationships which are developed herein will remain unchanged under such circumstances. Rather, it is to be expected that the role of viscosity will be displayed as transport becomes insignificant. Although this role has not been fully assessed, the work of Shields indicates that the parameter

$$\frac{d\sqrt{gys}}{\nu} \sim \frac{d}{\delta'}$$

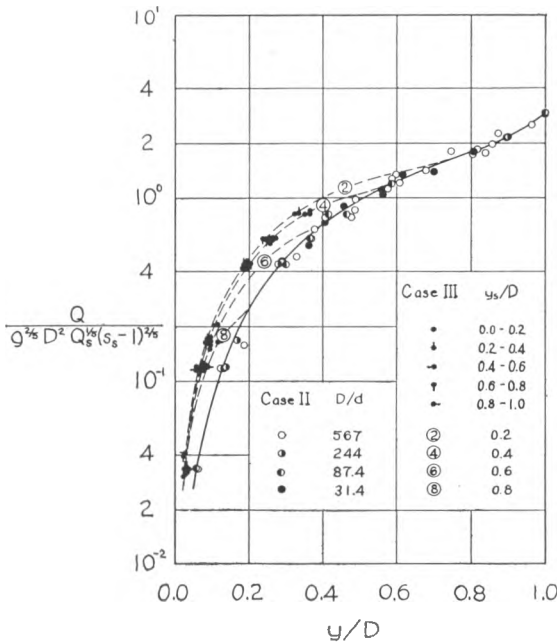


FIG. 2. TRANSPORT FUNCTION FOR INERT BED (CASES II AND III).

governs both the limit of significant transport and the form of the bed at this limiting condition (δ' is a length dependent upon the relative magnitude of viscous forces adjacent to the bed). Qualitative confirmation of the importance of this parameter was shown in the present study by the fact that a radical change in bed form occurred for d/δ' approximately equal to unity and a consequent decrease in transport efficiency resulted.

A comparison of the three cases is relevant. For all three cases a continuous increase of the transport function with y/D is shown. The maximum value

$$\frac{Q}{g^{2/5} D^2 Q_s^{1/5} (s_s - 1)^{2/5}} \approx 2.9$$

indicates that no deposition will occur for higher values of this parameter whatever the case may be. This is very important in maintaining sediment-free pipes.

For Cases I and II the curves are directly defined, y/D being the only geometrical variable required. Because it was not found pos-

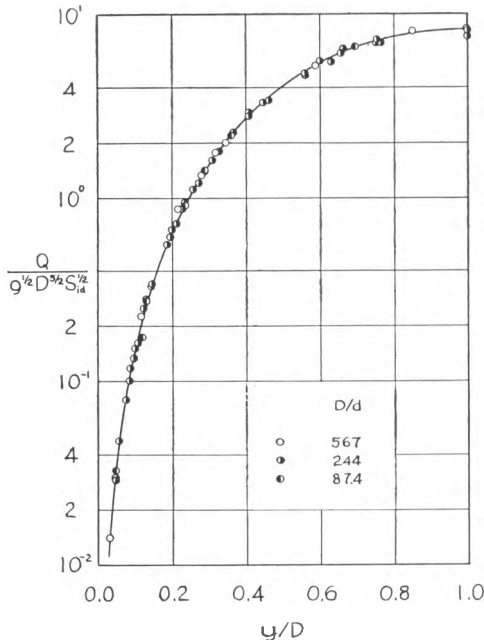


FIG. 3. DISCHARGE FUNCTION FOR IMPENDING DEPOSITION (CASE I).

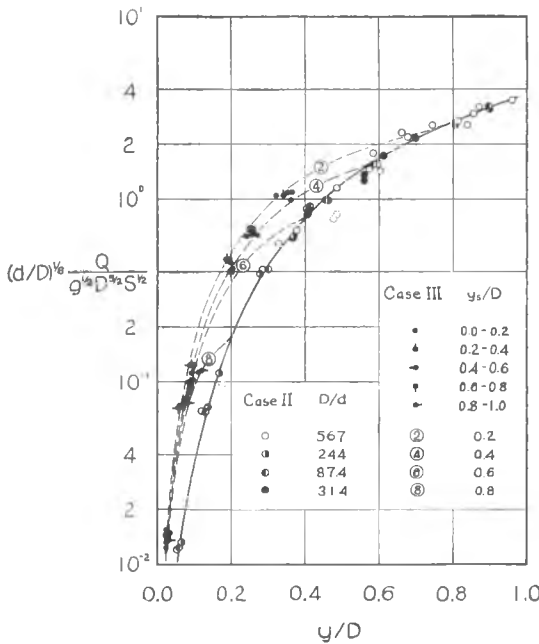


FIG. 4. DISCHARGE FUNCTION FOR INERT BED (CASES II AND III).

sible to control completely the depth of bed for Case III, the curves of constant y_s/D are necessarily drawn by interpolation. Location of the upper terminal point for each curve of $y_s/D = \text{constant}$ is facilitated by the requirement that for $y_s/D + y/D = 1$ the full-pipe (Case II) curve be joined. For $y/D \rightarrow 0$, the curves of $y_s/D = \text{constant}$ show a single functional trend. This is to be expected since for very small depths of flow a two-dimensional flow pattern is approached regardless of the thickness of bed.

It will be noted that as y_s/D approaches zero the shape of the curves of constant y_s/D approaches that of the Case I curve, but for the same depth of flow the values of the transport function are higher for the former. This apparent discrepancy results from the fact that a limiting condition is being approached from opposite directions and the two processes are not directly reversible. Other investigators of this special range of transport have also reported this phenomenon. Once sediment has deposited, an increase in transport ability is required to carry away the deposit over that which

was required to maintain the same sediment in continuous motion before deposit.

The correlation of the data for the discharge function is shown in Figs. 3 and 4. As illustrated by Fig. 4, it is necessary to include a factor in the function which represents the effect of d/D in order to obtain good correlation if there is an inert sand bed. The approximation of this effect by the power relationship $S \sim (d/D)^{1/4}$ appears entirely adequate for the range of the investigation. Since no effect of d/D is to be discerned for Case I, this is interpreted to mean that the parameter d/D is an indication of the relative roughness of the bed. Such an effect is absent if there is no bed or if roughness itself has no influence on the relationship.

Again, a discrepancy appears to be indicated between Case I and Case II as y/D approaches unity. This apparent discrepancy is resolved in the light of the foregoing discussion. The limiting condition is approached in Case I with transport occurring by flume traction — the movement of particles over a relatively smooth fixed surface. For Case II the mode of transport is by stream traction; that is, by movement of particles over other temporarily stationary particles. The latter is naturally a less efficient process. Evidently some sort of transition function exists very near the limiting condition in which the effect of d/D is variable. The investigation indicates that the transition must be very short and that it is apt to be unstable.

The reader will recognize the possibility of various combinations of the transport and discharge functions to provide graphs useful in the solution of design problems which fall within the scope of this investigation. Examples of such solutions are given in the writer's dissertation.

This study, in dissertation form, was submitted by the writer to the Graduate College of the State University of Iowa in partial fulfillment of the requirements for the Ph. D. Degree in Hydraulics. The experimental program was partially sponsored by the Bureau of Public Roads and the Iowa State Highway Commission. The writer is indebted to the staff of the Iowa Institute of Hydraulic Research for their cooperation, to Dr. Hunter Rouse under whose direction the investigation was completed, to Mr. Carl Izzard of the Bureau for his practical suggestions, and to Mr. Emmett Laursen of the Institute for his counsel.

DISCUSSION

Mr. S. Kolupaila complimented the Iowa Institute of Hydraulic Research on their studies of sediment transport. Earlier investigations, by E. W. Lane, F. T. Mavis, A. A. Kalinske, and H. Rouse, and the recent work by the authors have contributed much to the progress in this complicated field of fluid mechanics.

He outlined the problem of sand movement in pipes as either for transport (dredging and hydraulic fill), or for flushing or preventing deposition of material. The first problem is limited to the determination of the friction loss or of the most effective concentration of sand to be transported. The second involves the estimation of an impending transport, and deals with full and partly-filled pipes, with or without deposition of silt. The authors have primarily contributed to the solution of the second problem. Their diagrams represent different combinations of the slope and both discharges — of water and of silt — as functions of relative depths of both. Certainly, these curves are restricted to the investigated size of grains of the experimental sand, but they can be successfully adopted for the drainage channels.

Mr. Kolupaila also presented a short revue of the recent European research in this field. In studies similar to those by G. W. Howard, E. J. Dent, and M. P. O'Brien, several authors have discussed a problem of transportation of dredged material (see Winkel [6]). To the usual Darcy formula is applied the factor

$\left(\frac{p + s_s}{p + 1}\right)^2$ in which $p = Q/Q_s$ and s_s is the specific gravity of sand.

G. W. Howard [7] developed several equations of the type $h_f = m v^x$, in which the constants m and x depend on the concentration. R. Durand [8] found similar relationships in his investigations in the Neyrpic Laboratory in France: friction losses increase with concentration and the size of grains.

Another approach is based on a critical velocity, defined as that at the limit between carrying and depositing silt. This velocity is assumed to be the most economical. Durand presented a diagram of the variation with the velocity of the ratio of Q/Q_s to the spatial concentration. The critical velocity is determined as that for which a reversal of curvature takes place. From a combination of the Du Boys and Manning formulas, Popescu [9] derived a transport equation for trapezoidal channels.

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