

EXPERIMENTAL STUDIES OF LIQUID TURBULENCE

by

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INTRODUCTION

More than half a century ago, Osborne Reynolds reported his series of classical experiments and analyses which completely differentiated between viscous and turbulent flow of fluids. Since that time many investigators have worked in the laboratory studying the turbulent flow of liquids and gases, particularly in conduits of various types. In the main most of the investigations having engineering application have been concerned with the outward effect of turbulence, with little or no attention paid to the inner mechanism of the turbulence.

As the science of aeronautics began to develop, the engineers and physicists engaged in this field of work began to see that many problems of air flow over surfaces could not be adequately or completely solved, or properly investigated in the laboratory, unless more was known of the mechanics of turbulent motion. Thus the study of turbulence received a tremendous impetus, and it has been the work of aeronautics engineers, and those interested in aeronautics, that has advanced our knowledge of turbulence to its present stage.

Reviewing hurriedly the important work done to date on fluid turbulence, we find first the valuable contributions of Prandtl and his associates in Germany starting about 1910 and continuing to this day. Prandtl's work pertained to problems of fluid friction and the turbulence mechanism. A good summary of all this work is contained in a paper by Rouse.¹ Prandtl was the first to try to give a physical conception to turbulence by the introduction of his idea of the "mixing length," and by drawing an analogy between the haphazard fluid masses whirling about in turbulent flow and the molecules in a gas. In England, Taylor² wrote a paper in 1915 on the "eddy diffusion"

in the atmosphere which presented some very fundamental ideas regarding the diffusion characteristics of turbulence. Von Kármán,^{3,4} first in Germany and now in the United States, has extended the theories of Prandtl and has made many notable contributions to the mathematics of fluid turbulence. Dryden⁵ and his co-workers at the U. S. Bureau of Standards, and the research engineers at the Langley Field laboratories are making great progress in the experimental work on air turbulence.

Since the hydraulic engineer is primarily concerned with water in turbulent motion, it should be apparent that a knowledge of the mechanism of turbulence is of importance. Of course, in certain hydraulic problems such knowledge would be important only from an academic point of view, because these particular problems can be properly and adequately solved by the empirical methods and approximate analyses which look only at the outer effects of the turbulence. However, there are certain other problems, which this report is to discuss in detail, that cannot be properly solved or understood unless the inner workings of the turbulence are considered.

Problems of energy dissipation and transformation, both where energy dissipation is to be a minimum and where energy is to be dissipated as quickly and effectively as possible, require a knowledge of the mechanics of turbulence. The problem of suspended-material transportation is very closely associated with turbulence; in fact its complete solution and understanding is dependent entirely on our knowledge of turbulence. Sedimentation is another problem requiring a knowledge of the turbulence mechanism. In model studies it is necessary to have some measure of turbulence in order that dynamically similar conditions may be approached. It seems quite apparent that future advances in hydraulics will be in the direction of increased knowledge of the turbulence mechanism.

WHAT IS TURBULENCE?

Before the discussion proceeds any farther it would be well to define the term "turbulence." By injecting a dye or some other material into moving water, or by watching smoke coming from a chimney, a very haphazard mixing process is seen, which we refer to as turbulence. Fundamentally it might be said that a fluid at any point is in a state of turbulence if the direction and magnitude of the ve-

locity vary irregularly with time. This variation is relatively rapid and cannot be predicted except in the probability sense.

These variations in velocity are in general caused by the whirling about in the fluid of masses or eddies of various sizes. It is these eddies which transfer momentum, mass, heat, etc., from one point of the fluid to another, and also cause the high rate of energy dissipation associated with turbulence.

True turbulence as it exists in pipe or channel flow is such that there is no periodic variation of the velocity. The phenomena observed immediately behind a grid or some body or object around which the liquid flows, is not true turbulence because of the regularity of the size and formation of the eddies. This type of flow is better called "vortex motion." This motion eventually breaks down into true turbulence.

The most convenient way to deal with turbulence, both analytically and experimentally, is to represent the varying velocity vector at any point by three components U , V , and W along the axes x , y , and z . The value of U at any instant can be represented then as $(\bar{U} \pm u)$ where \bar{U} is the mean velocity along the x -axis and u is the fluctuating part. The other two components are represented as $(\bar{V} \pm v)$ and $(\bar{W} \pm w)$. As will be shown later the components u , v , and w in true turbulence are each quite random; that is, there is no periodicity, for instance, of u with respect to time. The velocity variations do not occur in any regular cycle. In fact, the velocity variations follow what is known in statistics as the normal error-frequency law.

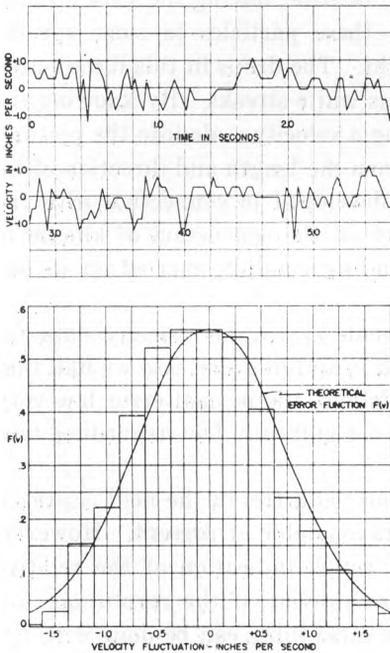
MEASURING VELOCITIES IN TURBULENT FLOW

In air-flow studies in wind-tunnels the standard piece of equipment used for measuring the fluctuating velocities is the hot-wire anemometer. The hot-wire technique has been very well developed by the aeronautics engineers, and recent developments indicate the possibility of determining all three components of the fluctuating velocity, u , v , and w , by use of a special type hot-wire anemometer. The quantities measured are the values of $\sqrt{u^2}$, $\sqrt{v^2}$, and $\sqrt{w^2}$, which in statistics are referred to as the "standard deviations" of a fluctuating quantity. Of course, the arithmetic mean values of u , v , and w are zero; also, if the flow is in the x -direction, V and W are zero. The root-mean-square values of the turbulent velocity fluctuations are a meas-

ure of the turbulence intensity. By use of the hot-wire apparatus, it is possible to measure the values of turbulence intensities directly. A detailed discussion of the apparatus and procedure employed is given in the National Advisory Committee for Aeronautics Reports.⁵

Our experimental studies of turbulence in this laboratory have been made practically entirely by use of the photographic method, using 16 mm. motion picture film. In one of our experiments a fine jet of dark red color was injected through a fine hypodermic needle and motion pictures taken of the color stream close to the tip of the needle-

tube. An approximate idea of the transverse velocity, v , was obtained by calculating from each frame of the motion picture the value of v by obtaining the value of the transverse spread Y at a distance x of about 1 inch. Knowing the mean velocity of flow in the x -direction, U , the value of v was calculated from the expression YU/x . The value of x was sufficiently small so that the stream of color did not deviate appreciably from a straight line between the point at which Y was measured and the tip of the needle. Typical data for v obtained in this way are shown in Fig. 1. Since the time between individual frames of the motion picture film was about $1/25$ of a second, values of v



FIGS. 1 AND 2.

were obtained for each such interval of time.

A statistical analysis of such velocity data is shown in Fig. 2. The value of \bar{U} in this case was 0.65 ft. per sec. and $\sqrt{v^2}$ was .0575 ft. per sec. The significant point is that the velocities are statistically distributed according to the normal error law which is expressed thus:

$$F(v) = y = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{v^2}{2\sigma^2}} \tag{1}$$

$$\sigma = \sqrt{(V - \bar{V})^2} = \sqrt{v^2}$$

In this case \bar{V} is zero and $V = v$. The quantity $f(v)dv$ indicates the proportion of time that the velocity v will lie between the values v and $v+dv$. The area under the $f(v)$ curve from minus infinity to plus infinity is, of course, unity.

Using another photographic technique it was possible to measure the values of U for short intervals of time and thus determine $\sqrt{\overline{u^2}}$. This technique consisted of mixing into the flowing water droplets of a mixture of carbon tetrachloride and benzine, having the same specific gravity as water, and illuminating these particles in some specific plane by use of a beam of parallel light. The drops in this illuminated plane show up on photographic film as white streaks. By knowing the speed of the camera, or by introducing a velocity scale into the picture, it is possible to determine U and V from the length and direction of individual streaks. This method was developed in connection with the A.S.C.E. Hydraulic Research Project on the conversion of kinetic to potential energy in a circular expanding conduit, carried on in our laboratory.

Statistical analyses have been made of various velocity data for both of the velocity components u and v , and in every case we find that the frequency distribution diagram follows the normal error law very closely. This is a fundamental and a significant fact regarding true turbulence.

The measurement of instantaneous velocities in the field is practically impossible with any instruments available at present. However, the Price current meter will give a rough indication of the relative magnitude of the velocity fluctuations present, if the revolutions are read for short intervals of time. The same thing can be done with the miniature current meters used in model studies.

Although the photographic method of analysing turbulence velocities requires a great amount of time, the results obtained should be reliable. The development of instruments for measuring the fluctuating velocities, both in the laboratory and the field, is most desirable, in order that the investigations of liquid turbulence may be less time-consuming.

MEASURING DIFFUSION IN TURBULENT FLOW

The intensity of the turbulence as measured by the root-mean-

square value of the velocity fluctuation about the mean is one of the important parameters characterizing turbulence. Another parameter of significance is called the "coefficient of diffusion." This particular parameter is of importance in the study of the relation of suspended material concentration and turbulence. Diffusion in a turbulent fluid can be compared with the process of molecular diffusion, although turbulence diffusion is much more intense.

The diffusing power of turbulence is due to the eddies which travel and whirl about from point to point in a haphazard fashion. These fluid masses can convey from one point to another heat, matter, momentum, energy, etc. The existence of a mean velocity gradient, such as $d\bar{U}/dy$, in water which is in turbulent motion indicates that there is a transference of momentum. The rate of the transfer of momentum is a measure of the force or shear existing between adjacent layers of liquid with different mean velocities. Osborne Reynolds showed that this shear can be represented by: $\tau = \rho \overline{uv}$, where \overline{uv} is the mean product of the simultaneous values of u and v , and ρ is the unit density. Prandtl transformed this expression for shear into a more usable form by introducing a length factor l , such that $u = l d\bar{U}/dy$. The formula for shear is then:

$$\tau = \rho \overline{vl} \frac{d\bar{U}}{dy} \quad (2)$$

The quantity \overline{vl} is a measure of the transfer power of the turbulence; in this case the transfer of momentum is considered.

In the problem of suspended material distribution in a stream we have the turbulence eddies transferring sediment. The sediment, of course, tends to settle due to gravity; thus the concentration tends to be higher at the bottom than towards the surface. Under equilibrium conditions the amount transferred upwards by the turbulence must be equal to that transferred down by the turbulence plus that which falls down by gravity. The *net* transfer due to turbulence must then be equal to the rate of falling by gravity. The fundamental equation describing this phenomenon is:

$$\overline{vl} \frac{dN}{dy} = -cN \quad (3)$$

N = concentration of sediment

c = velocity of fall of sediment size considered.

The derivation of Eq. 3 is given in various publications; O'Brien probably first introduced it to American engineers. Note that the quantity $\overline{v'l}$, which will be designated by ϵ , occurs again, in this case it being a measure of the transfer power of the turbulence in regard to sediment. This term is called the diffusion coefficient. Though definite experimental evidence does not exist proving it, there seems to be no particular reason why this coefficient of diffusion is not of the same magnitude for all diffusion processes.

This coefficient ϵ is the product of a velocity and a length factor. The velocity is, of course, dependent on the intensity of the turbulence, and the length factor depends on the so-called "scale of the turbulence" or the average size of the eddies.

In suspended material studies the value of ϵ is computed from the shear and velocity-gradient relationship. Usually some drastic assumptions must be made as to the value of the shear at various points in a channel or river. In order to study directly the diffusion characteristics of the turbulence, particularly the variation of the diffusion coefficient throughout the section of an open channel, laboratory experiments were made by which it was possible to get a direct measure of the diffusing power. The purpose of these experiments was primarily to check the validity of Eq. (2), for computing ϵ in regions where the velocity gradient approached zero.

The channel in which experiments were made was 2.5 ft. wide and the water depth was 1.0 ft. The channel was not long enough for the normal velocity distribution to become established, so the desired shape of velocity distribution was obtained by use of various baffles upstream. Data were taken for various mean velocities and at different vertical sections across the channel. The method of taking data is as follows:

At various points at the selected vertical section, droplets of carbon tetrachloride and benzine, having the same specific gravity as the water, were injected into the water through a fine hypodermic type of needle-tube. The drops were black and could readily be photographed against a white illuminated background. The spread of these drops transversely, that is, "up and down," was obtained for a distance of some 12 inches downstream from the point of injection. From about 400 frames of motion picture film at various selected distances downstream from the point of injection, the position of the drops transverse to the direction of flow was obtained. At any section, x inches

downstream from the point of injection, the value of $\overline{Y^2}$ and $\sqrt{\overline{Y^2}}$ was obtained where Y is the distance above or below the horizontal line through the point of injection. Of course, the arithmetic mean of the various values of Y is zero if the mean direction of flow is along the x -axis.

For the case of molecular diffusion the relation between $\overline{Y^2}$ and x is:

$$\overline{Y^2} = \frac{2Kx}{U} \tag{4}$$

where K is the coefficient of molecular diffusion and \overline{U} is the mean velocity. Note that $\overline{Y^2}$, or the mean square spread of the particles, varies directly with the distance, x . Obviously, there is no reason to suppose that the above relation will apply to turbulent diffusion, the principle reason being that in molecular diffusion the size of molecular paths is of a very small order of magnitude compared to the distances x and $\sqrt{\overline{Y^2}}$ observed. In turbulent diffusion the eddying fluid masses have the rôle played by the molecules, and these eddies may be of the same order of magnitude as observed values of $\sqrt{\overline{Y^2}}$ and x ; therefore there must be a difference in the two diffusion phenomena particularly for the smaller values of x . Experimental data show this to be true.

In Fig. 3 are shown values of $\overline{Y^2}$ and $\sqrt{\overline{Y^2}}$ plotted against x as

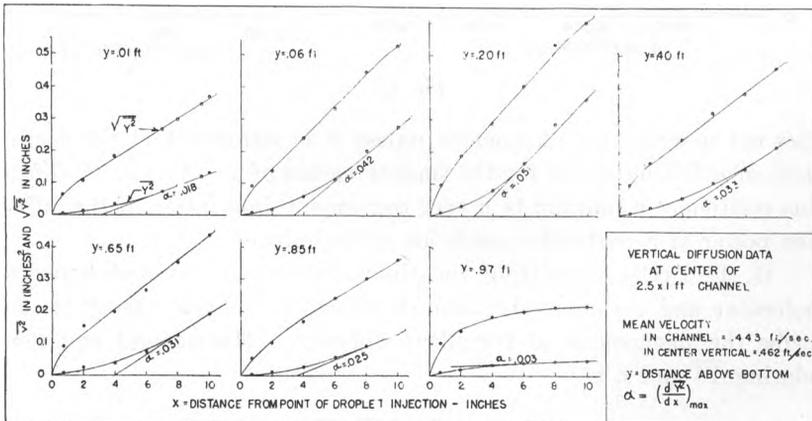


FIG. 3.

were obtained in the 2.5 ft. channel, with a water depth of 1 foot and a mean velocity of .443 ft. per sec. for the whole channel. The data were obtained for seven points in the vertical section in the center of the channel; the mean velocity at that section was .462 ft. per sec. The velocity distribution is shown in Fig. 4. Note that $\overline{Y^2}$ varies parabolically with x for small values of x , and gradually approaches a straight line variation for greater downstream distances. Therefore, Eq. (4)

DISTRIBUTION OF DIFFUSION COEFFICIENT
'D' AND MEAN VELOCITY 'U' IN
CENTER VERTICAL OF CHANNEL

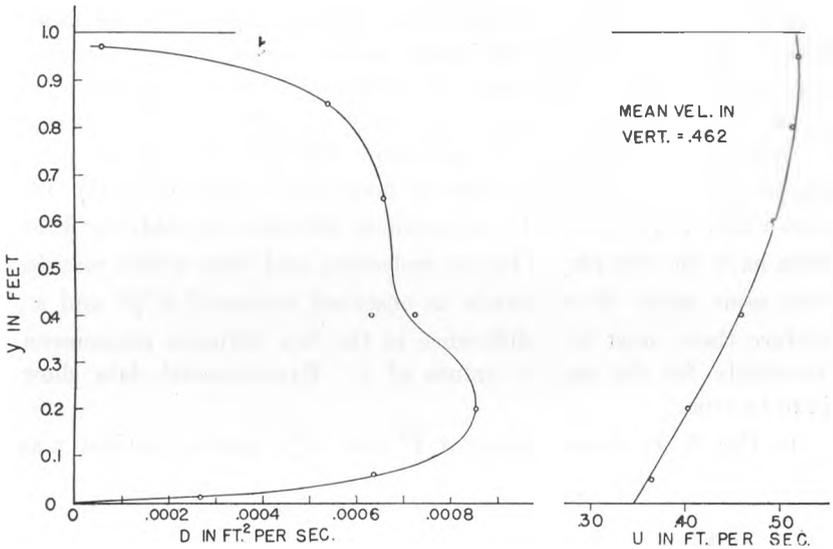


FIG. 4.

does not describe the phenomena unless it is assumed that the coefficient of diffusion varies for the smaller values of x . A variable diffusion coefficient would not be a very convenient description of the diffusion power of a particular condition of turbulence.

G. I. Taylor,⁸ realizing the dissimilarity that existed between molecular and turbulent diffusion developed a different theory to describe the phenomena of turbulent diffusion. His general equation relating $\overline{Y^2}$ and x is:

$$d\overline{Y^2}/dx = 2(\overline{v^2}/\overline{U}) \int_0^x R dx \quad (5)$$

The term R is the so-called "correlation coefficient" between the velocity, v , of a particle at one point, and its velocity, v_x , after a distance of travel of x . It is defined thus: $R = \overline{v v_x} / \sqrt{\overline{v^2}}$. Numerically, R can have any value from $+1$ to -1 . The concept of the correlation coefficient is very useful. Logic indicates, and experiments verify, that the value of R should be near to $+1$ for small values of x . In other words, depending on the size of the fluid masses involved, the velocity of a particle will be very much the same at the beginning and end of an interval of time, if that interval is relatively small. However, as the interval becomes large, which means as the particle travels farther, chances are that on the average the velocities at the beginning and end of the interval will be less and less related. For larger values of x , R becomes zero.

For values of x when R becomes zero the quantity $\int_0^x R dx$ is a constant. Taylor ⁹ then designated the quantity $(\overline{v^2}/U) \int_0^{x'} R dx$ as the diffusion coefficient, where x' is the distance downstream where R becomes zero. Calling this coefficient D , Eq. (5) then becomes for all values of x greater than x'

$$\overline{Y^2} = \frac{2Dx}{U} \quad (6)$$

This is similar to Eq. (4) which applies to molecular diffusion. The quantity D can be calculated if $\overline{Y^2}$ is plotted against x . Note that D is also equal to $(d\overline{Y^2}/dx)U/2$, where the slope, $d\overline{Y^2}/dx$, is determined at the point when R becomes zero, which is when Eq. (6) applies, or in other words, when the relationship between $\overline{Y^2}$ and x is linear. In Fig. 3, the quantities $(d\overline{Y^2}/dx)_{max}$ are designated as α . The approaching of a linear relationship between $\overline{Y^2}$ and x for the larger values of x is quite apparent.

The coefficient D seems to be the most logical measure of the diffusing power of turbulence. It is undoubtedly proportional to the coefficient, $\epsilon = v\bar{l}$, occurring in Eqs. (2) and (3), though not necessarily equal. The variation of the computed value of D in the center vertical of the channel for which diffusion data are shown in Fig. 3, is indicated in Fig. 4. The shape of this curve in general corresponds to that of the ϵ curves which have been determined for very wide rivers in which the shear can be computed by the simple relation, $w y S$, where

w is the unit weight of fluid, y the depth, and S the slope of the river. No such computation was made for our experimental channel since, due to the side effects, shear computations must be made using various assumptions, and therefore no attempt was made to calculate ϵ using the shear and velocity gradient equation, (Eq. (2)).

Theoretically, since the transverse velocities are statistically distributed according to the normal error law, the concentration of particles, which are weightless in water, downstream from the point of their origin, should also be according to the normal error law. In other words, the quantity $f(Y)$ which indicates the relative number of particles which occur between the values Y and $Y+dY$ is given by the following expression:

$$f(Y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\bar{Y}-Y)^2}{2\sigma^2}} \quad (8)$$

$$\sigma = \sqrt{(\bar{Y} - Y)^2}$$

Theoretically, the mean value of Y should be zero if the flow is in the x -direction, the particles are of the same specific gravity as water, and a sufficient number of readings are taken. Usually in the experimental data \bar{Y} was not always exactly zero; however the deviation was slight.

In Fig. 5 are shown the frequency diagrams for various values of Y at different distances downstream from the point of origin of the droplets, $x=0$. The smooth curves represent Eq. (8), and the values of $\sqrt{\bar{Y}^2}$ are those shown in Fig. 3 for the point in the central vertical, $y=0.40$ ft. above the bottom of the channel. In general, the experimental data check the theory very well. Using Eq. (8) and knowing the variation of $\sqrt{\bar{Y}^2}$ with x it is possible to compute the concentration of particles any distance downstream from some point at which the concentration is known or assumed. In hydraulics this idea or concept has wide application in sedimentation studies.

An interesting method of measuring diffusion in liquid turbulence was used by Richardson.¹⁰ The method in general involved the injection of dye into a water stream from a line source. The relative concentration of the dye downstream was determined by use of photoelectric cells. The entire apparatus was quite involved, although the \bar{Y}^2 measurements were in general similar to those obtained by the author.

EQUATION OF SMOOTH CURVES:

$$f(y) = \frac{1}{\sqrt{2\pi} \sigma_y} e^{-\frac{(\bar{y}-y)^2}{2\sigma_y^2}}$$

$$\sigma_y = \sqrt{(\bar{y}-y)^2}$$

X = DISTANCE DOWNSTREAM FROM POINT OF DROPLET INJECTION.

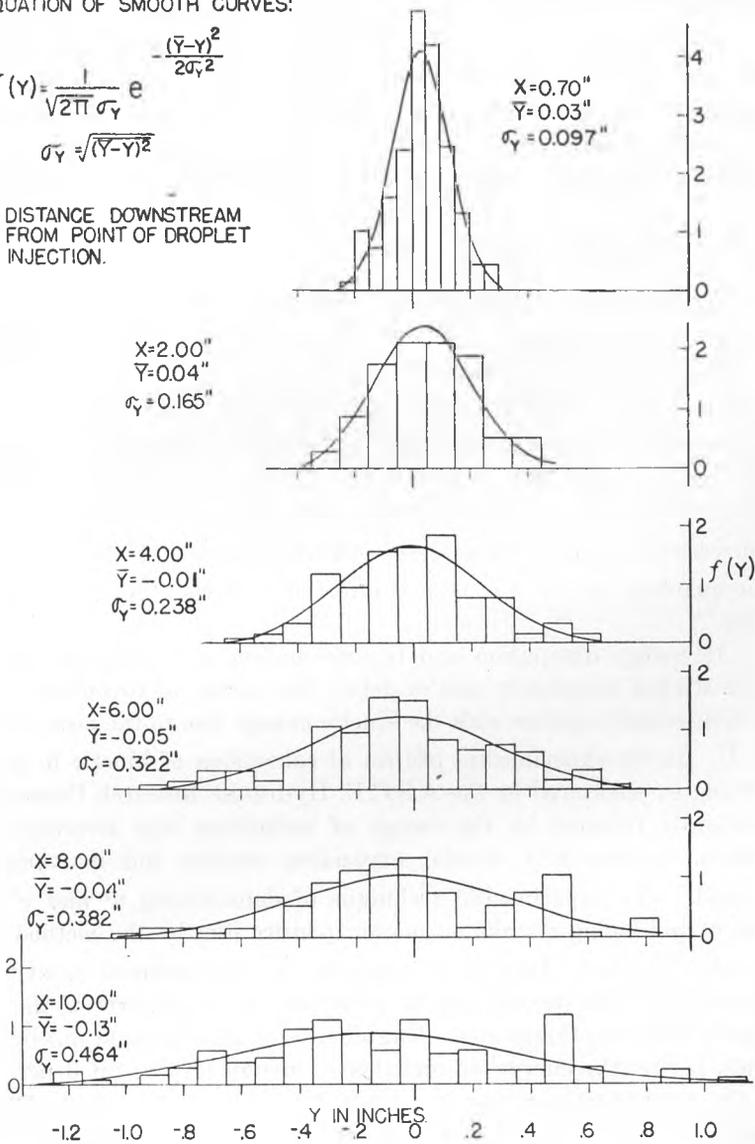


FIG. 5.

Two general methods of diffusion measurements in air flow have been reported. The first method is that of Schubauer¹¹ using diffusion of heat, and more recently Towle, Sherwood, and Sedar¹² used the method of dif-

fusion of carbon dioxide and hydrogen in an air stream. In all these air experiments the investigators assumed that the laws of molecular diffusion applied, even for the \bar{Y}^2 measurements at small downstream distances. Such an assumption probably is more correct for air turbulence than liquid turbulence because the eddies in a wind-tunnel air stream are relatively much smaller than in large water channels.

ENERGY CONSIDERATIONS

An important phrase used in talking about turbulence is that of "energy of turbulence." When water flows in turbulent motion the total kinetic energy at any point per pound of water flowing is not simply $\bar{U}^2/2g$. The total kinetic energy per pound of water flowing at any instant is $q^2/2g$, where q is the magnitude of the velocity vector at any instant and is equal to $(U^2 + V^2 + W^2)$. Using our adopted designation for U , V and W , and assuming the mean flow is in the x -direction, the mean value of q^2 is then equal to $(\overline{U^2 + u^2 + v^2 + w^2})$. The quantity $(\overline{u^2 + v^2 + w^2})/2g$ is referred to as the "energy of turbulence."

If energy dissipation and transformation in a turbulent flow is to be studied completely and in detail, the energy of turbulence must be determined together with the kinetic energy due to the mean velocity, \bar{U} . In the experimental project of conversion of kinetic to potential energy sponsored by the A.S.C.E. Hydraulic Research Committee, previously referred to, the energy of turbulence was determined at various sections in a circular expanding conduit and its variation studied. The experimental technique of determining $\overline{u^2}$ and $\overline{v^2}$ was that of producing streaks on motion picture film by the method previously described. Because of symmetry $\overline{w^2}$ was assumed equal to $\overline{v^2}$. The data on this project will be presented in the reports of the Hydraulic Research Committee. One significant item brought out by this study is that the energy of turbulence, though only about 3 per cent of the total kinetic energy in a straight conduit, may rise to 30 per cent of the total kinetic energy in, for instance, a 15° expansion from a 3-inch to 5-inch circular conduit.

When the energy of turbulence is determined together with the other forms of energy, it is quite apparent that there is no such thing as a sudden dissipation of energy in flowing water. For instance, in

such flow phenomena as the sudden expansion or the hydraulic jump, the concept is sometimes conveyed that there is a very sudden loss of energy, a so-called "shock loss." This is not true in any respect. The mean kinetic energy in such cases is converted into energy of turbulence which is then gradually dissipated downstream. In fact, if the *total* energy gradient were plotted along a conduit having a sudden expansion there would be no sudden drop in this gradient at any point beyond the expansion. The extra dissipation of energy brought about by the sudden expansion is a very gradual process—that is, if by dissipation we mean dissipation of energy into heat, which is what true dissipation is so far as fluid flow is concerned.

Whether a fluid is in viscous or turbulent motion it should be kept in mind that it is viscosity which produces energy dissipation. In turbulent flow most of the potential energy is first converted into turbulent energy which is then dissipated into heat by the action of viscosity on the whirling masses of fluid. To avoid excessive dissipation of energy in hydraulic structures, turbulence must be kept to a minimum. An experimental evaluation of the energy of turbulence makes possible definite quantitative studies.

Model studies of structures involving large changes of turbulent energy must be made with the fact in mind that scale also applies to the relative size of the turbulent eddies if true similarity is to be obtained. The "scale of the turbulence" in general must be kept in proportion, and also the per cent of turbulence, which for one velocity component can be designated as $100(\sqrt{\overline{u^2}})/\overline{U}$, should be kept the same in model and prototype.

SUMMARY AND CONCLUSIONS

To summarize, the salient points discussed in the paper are:

(1) The results of the work of aeronautics engineers in the study of fluid motion have permeated into hydraulic investigations, with the result that the inner mechanics of liquid turbulence is beginning to receive intensive laboratory study.

(2) The important hydraulic engineering problems which are dependent for their complete understanding on a more thorough knowledge of the turbulence mechanism are: (a) suspended material transportation in rivers, (b) scouring in canals and rivers and sedi-

mentation in reservoirs, and (c) energy dissipation and transformation processes.

(3) A proper concept of turbulence, which an engineer can apply quantitatively, is essential if the study of turbulence is to be of practical use. An attempt to present such a concept is made, by introducing and defining the terms: intensity of turbulence, scale of turbulence, and energy of turbulence.

(4) Experimental data regarding the measurement of velocities in turbulent flow by motion picture photography are presented. Two general methods were used: that of the color stream issuing from a fine tube, and the method of "streak" pictures.

(5) A statistical analysis of velocity fluctuations, both parallel and normal to the direction of flow, showed that the velocity fluctuations in true turbulence are statistically distributed according to the normal error law.

(6) The coefficient of diffusion for turbulence was directly determined by experimental means using motion picture photography to record the necessary data. The results obtained checked the theory of turbulent diffusion developed by G. I. Taylor.

(7) A proper understanding of the diffusion or transfer characteristics of turbulence is indispensable in solving the problems of suspended material transportation, sedimentation, and scouring.

(8) Energy of turbulence is a form of kinetic energy present in turbulent flow. A complete experimental study of the processes of energy dissipation and transformation, such as occur in expanding conduits and streams, necessitates that the energy of turbulence be calculated in order that a true picture of the phenomena occurring may be obtained.

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