ON THE STUDY OF SHAPE RESISTANCE

by

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I have been invited by the Iowa Institute of Hydraulic Research to render my views on the subject of hydraulic shape resistance. This subject struck me right away as being particularly timely and wisely chosen. For in the fluid-motion research of the last decade we have been treated to so much progress on many other fascinating subjects — such as compressibility, and boundary layers (even several kinds of the latter making their appearance both separately and in combination), not to mention shock waves and laminar boundary wing sections — that, in the excitement about all this, I am afraid shape resistance has somewhat been forgotten. It is right that there again appear a paper about it, be it only to remind us that there is still such a thing as shape resistance. For when the need is greatest and the pilot takes to his parachute and bails out as a last chance to save his life, he can in no way do without it.

The aerodynamic brakes of dive bombers depend likewise on shape resistance. Without it, on the other hand, streamlining would be unnecessary and the friction drag could be greatly cut down. The profile drag of airfoils could likewise be reduced, for the limitation of the lift capacity of wings is closely tied up with their shape resistance. For all of this, shape resistance stands in utter contempt with the engineers, notwithstanding its usefulness just alluded to. Here, as so often happens, good services are more quickly forgotten than unfriendly actions. The physicist has no excuse for such bad feeling. To him, the phenomenon of shape resistance is, or should be, attractive and instructive, a thrilling and inspiring manifestation of the laws of fluid mechanics.

Before going further into this, shape resistance has to be formally introduced. It is not always easy to separate it strictly from friction effects. There are border cases where this is very difficult indeed. But what could we accomplish if we would permit the abandonment of proper and useful distinctions and classifications on account of inconvenient border cases? There are enough cases where the shape resistance is clearly defined. It is this portion of the entire problem of flow resistance, often its major portion, which is directly associated with the interaction of the mass effects and the pressure effects of the fluid, to the exclusion of its immediate friction effects. That is to say, shape resistance is an incident of such features of the flow of actual fluids as resemble and are dominated by the laws of the motion of perfect fluids.

This definition gives the lie to the so-called hydrodynamic paradox. The discoverer of that paradox failed to receive any assistance from his mathematical analysis of the motion of perfect fluids in an attempt to compute the flow resistance, in that he succeeded only in computing the absence of such resistance. So he made the best of it, and reported his failure to the world as a paradox. But absence of resistance does not follow at all if we do not expect the fluid to be too perfect, but are satisfied with a perfection within reasonable limits.

All physical fluids exhibit viscosity, and it is going rather far to stipulate that their frictional effects shall be considered unnoticeable within the fluid, so that the forces transmitted from one fluid particle to an adjacent particle are always and necessarily at right angles to the common boundary through which they are transmitted. That is the assumption defining perfect fluids. The framer of the hydrodynamic paradox went beyond that and further assumed the law to be retroactive and extreme. He also assumed the friction effect to be negligible at the external fluid boundaries, at the walls of immersed objects or of the fluid container. This, however, is not a good representation or idealization of what happens or can reasonably be expected to happen, for the friction forces are proportional not only to the magnitude of the viscosity, but also to the crowding of velocity differences within the fluid. Near the walls is great crowding of velocity differences, and the smaller the viscosity the larger the crowding, so that very noticeable friction forces must be expected at the walls regardless of whether the viscosity is ever so small or not.

Aerodynamics, at an early stage of its development, abandoned therefore the exaggerated assumption of absence of friction even along the walls, and in that manner obtained numerical results correctly representing important features of the facts, and of great value for practical application. The fluid is permitted to glide layer over layer where it has once been separated by an obstacle, or, more generally, where it once has been in contact with a solid surface. Such gliding has, however, always to be a sideways gliding only, or, as the mathematicians put it, the vortex lines have to coincide with the stream lines in steady flows, and in unsteady flows they have to move with the fluid. Any other kind of gliding would require friction effects within the fluid, which effects are still assumed negligible.

In some cases the flow may be steady, offering at all times the same picture to the eye of the observer. The airfoil flows which have been so intensely and successfully studied during the last decades form outstanding examples of this. Their trailing vortex system represents a gliding of adjacent air layers of the type referred to in the preceding paragraph. In this particular case the shape resistance has received the name of "induced drag," and the induced drag of technically employed airfoils is not much larger than the absolute minimum consistent with the lift of the airfoil under the prevailing conditions.

The total induced drag of more than one airfoil has not necessarily to be associated with a total lift or side force of any definite amount. Consider, for instance, the combination of a pair of opposing airfoils, the one in normal position and the other inverted, both airfoils being otherwise equal and in symmetric position. The two lifts, one positive and the other negative, then cancel each other, so that no total lift remains. But the two induced drags sum up. Here then we have a genuine shape resistance not accompanied by any total lift or side force.

The two airfoils may be bodily connected with each other by members not producing any lift. Also, an airfoil may be bent into a loop such as a circle, forming then a wing-section-contoured ring, with variable angle of attack. The angle of attack may vary periodically, according to a sine law say, changing sign every 90 degrees, circle. Such an airfoil ring does not have to remain a child of or every sixty degrees, or any other even fraction of the complete phantasy. It can easily be made and studied experimentally. It will give rise to a substantially steady fluid motion and will exhibit a genuine shape resistance of measurable magnitude.

The fluid motion and its shape resistance can be predicted for the wing-section-contoured ring with moderate angles of attack because it is possible to see beforehand where the split fluid stream will reunite again. Experience and common sense agree that this will be at the trailing edge. With a blunt solid forming an obstacle in a flowing medium, the situation is different. In this case it is not known where the place of reunion of the fluid will be. Unless all fluid reunites practically at one point, this must be a line. With a simple solid, without perforation like a ring, this line must not enclose any area. Otherwise there would be fluid forming a flow system by itself with loop-shaped streamlines, like fluid moving within a closed container. That would not be in harmony with energy considerations. A body of immovable fluid behind a solid would, on the other hand, not be in equilibrium, and might also involve vortex lines at its boundary which did not move with the fluid.

The question arises at this point whether every such line, simple or branched, but not enclosing any area, is a theoretically possible line of reunion. The question arises, whether a distribution of vortices exists which trails from such a line and makes it in turn the reunion line. From analogy with the wing theory, it is likely that this question has to be answered in the affirmative. A mathematical investigation of the question would be welcome.

Assuming for the time being that all such lines can be reunion lines, and that their shape and location determine the distribution of the strength of the trailing vortices, it would appear that an infinite variety of such steady quasi-potential flows is possible, each being associated in general with a different shape resistance. This kind of shape-resistance flow is dependent on the presence of local lift or side forces. They cannot be very large. It is therefore not likely that a simple blunt body derives a considerable shape resistance from a steady quasi-potential flow.

Before proceeding to unsteady quasi-potential flows, it remains to be mentioned that any quasi-potential flow relies on friction effects for coming into being. Once the flow has been established, it can maintain itself in the absence of any further friction effects, but not in the presence of each and every friction effect, as we shall immediately see.

The relation between friction as the cause and quasi-potential flow as the effect fits very imperfectly into the usual picture of physics. Unlike the usual workings of the ordinary laws of physics, cause and effect are here too disproportionate; friction forces and mass forces are not in team work but rather in the relation of the driver to the horse. Physics deals ordinarily and preferably with happenings which are altogether in proportionate relation. Notwithstanding the classical simplicity of the basic laws of fluid motion, the fluids seem to exhibit in the most primitive way the working of cause and effect as we are accustomed to find in biology, not to go as far as human actions or the fate of nations. Even there, cause and effect are never suspended, but are a necessary ingredient of conception and thinking. But, the higher the order of manifestation, the more do cause and effect recede to the humble station akin to afterthought, a principle of arrangement not to be relied on for the prediction of events. Strange, that the motion of dead, homogeneous fluid so distinctly exhibits the weak significance of cause and effect.

Not too much hope should therefore be entertained for ever learning how to genuinely compute the shape resistance in all cases. Its simple physical laws do not do much good, because they do not work in a simple manner. Physics is successful only where both the laws and occurrences are simple.

If the fluid friction is essential in electing out of the infinitely many theoretically possible steady potential flows about a blunt body the particular one that takes place, it may easily continue to play its role of controlling large forces through very small ones. A series or plurality of quasi-potential flows may alternate more or less gradually. If the rate of change is fast enough, the unsteady quasi-potential flows are different from the steady ones, and the number of the possible motions is thus again infinitely increased.

Any change would be impossible as long as all vortex lines strictly move with the fluid at all places, for the velocity at the reunion points is zero, and, strictly speaking, the surface fluid particles cannot cross this point. This is a mathematical truth which is not in keeping with common sense. To reconcile conditions, it is not necessary to give up Helmholtz's theorem entirely. A very feeble relaxation is sufficient, the fluid velocity in the vicinity of the reunion line being practically zero, and vortex transit there requiring next to no friction forces. This is therefore the region most and genuinely exposed to the working of small friction effects. That is to say, the line system of the reunion of the fluid may easily move, steadily and continuously, from one position to the next. The line must be supposed to wander, yielding to friction effects hard to follow up. Unsteady quasi-potential flows with wandering reunion lines is therefore the picture which has been formed for the description of the essential character of fluid motion associated with shape resistance.

This most general type of quasi-potential shape-resistance flow is still to be considered perfect, in that in the first place no energy dissipation or creation of frictional heat is considered to take place. All energy is primarily used for supplying kinetic energy. There come into being local and more often total side forces or lift forces, periodically changing in direction and magnitude. Their components in the direction of flow form the shape resistance. The effective fluid motion relative to the blunt body is inclined to the path of flow, and hence the side forces and lift forces, being at right angles to the local flow, are not at right angles to the path of motion, but have resistance components of the nature of the ordinary induced drag.

If we consider that for a circular cylinder or a flat strip moving sideways, for instance, the shape resistance is great enough to give average resistance pressures larger than the dynamic pressure, it appears that the local and temporary lift coefficients must be enormous. They must be much larger than a drag coefficient of unity because in the first place they act during only a portion of the entire time, say during one-half of it, and because in the second place only a component of the lift or side force forms the resistance, say one-half again in magnitude. Such rough estimate would suggest lift coefficients having a magnitude of 4 or more. A wing section having permanently or steadily a lift coefficient of 4 would exceed in lift capacity all wing sections technically used. Here then the shape-resistance flow shows something outstanding, commanding our astonishment. The mechanism of momentum exchange between a simple fluid and a blunt body seems to be very worthy of intense study.

The great discoveries in physics of the past came from the study of phenomena which seemed utterly unprofitable and unusable. No such study failed to contribute something useful and instructive. Weighing, measuring, observing along paths already trodden hard is surely necessary, but that is not science nor is it basic, and the efficiency of such routine work is relatively small. It is often fishing where most of the fishes have already been caught, grazing where most of the grass has already been eaten. The true places of fluid research in this country should divorce themselves more than they do now from such routine measurement, often performed with new and costly equipment which nevertheless was already out of date before it was completed, and had long ago served its purpose. Basic and scientific experimental research knows no permanent and standard test equipment. Let idle and academic scientific curiosity have its way more than it now does. As to shape resistance and quasi-potential flow, find out what happens under controlled laboratory conditions, for which the sensitive and nervous flow pattern, ever so ready to change and to accommodate itself to external influences of small magnitude, is a most interesting subject of study.

Of course, unscientific and thoughtless weighing and measuring will not get anybody very far. Each test has to be prepared and followed up by profound scientific and mathematical analysis. That is hard and thankless work, but its outcome may be profound. One apparently minor discovery may change an entire continent. The outcome of the present war may be vitally influenced or even decided by a development relating to fluid motion which started from such apparently unprofitable playfulness of some genuine scientists and physicists.

With that in mind, and being unwilling to come before you with empty hands, so to speak, with nothing but generalities (true, perhaps, but unfruitful by themselves) may I, by way of closing this paper, ask for a little more of your attention. May I take the opportunity to present an improvement of a mathematical tool helpful in dealing with shape resistance, as a minor but more tangible contribution to the science of hydraulics.

Shape resistance is intimately interwoven with interchange of momentum, or of moment of momentum, and of energy. There exist theorems or mathematical rules by the use of which the momentum, the moment of momentum, and the kinetic energy of bodies of fluid in motion can be computed by an integration through such regions only where the motion has vorticity. That is a great simplification and there should be opportunity for using these integrals in the study of shape resistance.

Only recently did I arrange these theorems, and allied ones, in systematic order, and also a few new ones that may be of use [1].[1] The following relates specifically to that paper.

These theorems give the quantity in question in the form of simple integrals, the integrand containing the radius vector "r" as factor—namely, the vector connecting a point of origin with the point to which the integration refers. These are three-dimensional integrals, space integrals, and surface integrals through surfaces that may extend in any fashion through the three-dimensional space. The fluid motion is supposed to be general throughout; no assumption restricting the motion to a two-dimensional flow is made.

In the application of these integrals, two-dimensional problems readily make their appearance. They are much simpler, and still may be general enough to bear out an essential point. Two-dimensional flows can easily be depicted on paper; they can be more easily comprehended mentally by the optical picture they present. The question arises, therefore, whether the three-dimensional integrals just referred to possess or lead to analogous two-dimensional theorems for use with two-dimensional flows, and, if so, to which, because the application of a three-dimensional integral to a two-dimensional flow is cumbersome, and the advantage of the restriction to two dimensions becomes immediately lost.

These integrals in question were obtained by tentatively writing down expressions containing the radius vector and the rotation or the divergence of the velocity, being otherwise of correct geometric dimensions. The differentiation occurring was then executed on r. Previous to executing the differentiation, the correctness of the

[1] Reference appears at the end of the article.

transformation occurring does not depend on whether r denotes the radius vector or any other vector field.

Let it now be assumed that the vector field v of the velocity be plane and two-dimensional, and that a new quantity $\bar{r}$ be introduced as a substitute for r, $\bar{r}$, denoting the corresponding axial or two-dimensional radius vector—namely, the projection of any radius vector r on the plane of the two-dimensional flow. Three different kinds of differentiation occur, the dyadic, the vectorial, and the scalar product of $\triangle$ by r or by $\bar{r}$, giving respectively the derivation dyadic (or its conjugate), the rotation, or the divergence.

The rotation of the common radius vector is zero. So is the rotation of the axial or two-dimensional radius vector. In this respect, therefore, there is no difference between the three-dimensional and the two-dimensional case.

The dyadic differentiation of the common radius vector gives the identity dyadic, which, when multiplied by any vector, more particularly by the vector v, gives that vector, or v again. The derivation dyadic of the axial radius vector is not the identity dyadic, but it can serve as such with respect to the two-dimensional flow in question, because when multiplied thereby it likewise gives v again. In that respect then, too, the inquiry looks promising.

The divergence, at last, of the radius vector r is equal to 3. The divergence of the axial radius vector $\bar{r}$ is only 2. The mathematical development has accordingly to be scrutinized, paying particular attention to how far the change from 3 to 2 will affect the result.

Beginning with the expression of the momentum by means of the divergence

$$dS\, v = \int dS\, \nabla \cdot v\, r + \int do \cdot v\, r$$

only the dyadic product of r and ∇ occurred. Hence the theorem can be directly applied to two dimensions, without any change in form. There too, the momentum is equal to the static moment of the divergence and of the flux.

It is different with the vector moment of the rotation. $(\nabla ; \bar{r} \cdot v - v \nabla \cdot \bar{r})$ is $-v$ and is not $-2v$, because the divergence is now 2 and not 3. It follows that in two dimensions the momentum equals the entire vector moment of the rotation and of the roll and not one half of that, is in three dimensions.

The surface integrals of the two end planes cancel each other for both momentum integrals.

The several theorems about the computation of the moment of momentum do not make use in their derivation of any divergence of r. It follows that the substitution of the axial radius vector for the general radius vector is permissible without modification of the theorems. The two-dimensional integration does not include the two plane end faces, and the outcome is the component of the moment of momentum at right angle to these faces.

Preceding at last to the integral for the kinetic energy, there occurs, in Eq. (23) of the above paper, the following difference:

$$- v \cdot \nabla \; ; r \cdot v + \tfrac{1}{2} \, (\nabla \cdot r) \, v \cdot v$$

the first term containing the identity diadic and giving simply $- v \cdot v$, the second term containing the divergence and giving 3/2 $v \cdot v \cdot$ Summing up the factor of $v \cdot v$ appears therefore ½, the difference between 3/2 and 1.

Changing now to the two-dimensional flow, and to the axial radius vector, both terms of the above difference become equal. The first term remains the same, and the second term receives the factor ½ times 2, or likewise unity. Hence the proof is not transferable to the two-dimensional case, and it follows that the energy theorem (26) of the above paper holds only in three-dimensional statement.

It results then that one of the two momentum theorems changes its factor, the moment of momentum theorems survive in form, but the energy theorem cannot be used at all with the axial radius vector.

REFERENCE

[1] Munk, Max M., "On some vortex theorems of hydrodynamics." *Journal of the Aeronautical Sciences,* Jan. 1942.