EVALUATION OF BOUNDARY ROUGHNESS

by

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Somewhat over a decade ago Professor von Kármán published an extremely significant analysis of the velocity distribution and resistance of turbulent flow past smooth and rough boundaries. This analysis has had a profound influence upon fluid mechanics throughout the world, but, despite considerable publicity in engineering journals, it has seen little application to hydraulic design. Current literature, for instance, continues to reflect adherence to exponential resistance formulas, and evidences at times a complete misunderstanding of the roles played by viscosity and boundary roughness.

There is probably a three-fold reason for this situation. First, the apparent complexity of the analysis does not of itself attract the interest of the practical mind. Second, the experiments substantiating the analysis involved artificial boundary roughness which was not geometrically similar to the roughness of commercial material. And, third, the analysis did not originally include that phase of resistance which is most frequently encountered in practice, in which the boundary is, in effect, neither completely smooth nor completely rough. In this paper the writer seeks to overcome each of these three difficulties, in the hope of providing information of immediate value to the engineer.

It matters little whether the resistance equation for uniform flow is written in the Darcy-Weisbach form

$$h_f = f \frac{L}{4R} \frac{V^2}{2g} \tag{1}$$

or in the Chezy form

$$V = C \sqrt{RS} \tag{2}$$

for the resistance coefficient f and the discharge coefficient C are evidently related through the expression

$$\frac{1}{\sqrt{f}} = \frac{C}{\sqrt{8g}} \tag{3}$$

The factor f (and therefore C as well) has long since been shown to depend only upon the Reynolds number of the flow and the geometry of the conduit boundaries. In the case of uniform flow through a conduit of circular cross section, the relative roughness of the boundary is the only geometrical parameter involved. If such roughness is very slight—as is true of glass or drawn metal tubing—the Reynolds number $\mathbf{R} = VD/\nu$ is the sole resistance parameter, and above the critical limit $\mathbf{R} \approx 2000$ the coefficients f and C will vary according to the relationship

$$\frac{1}{\sqrt{f}} = -0.8 + 2 \log \mathbf{R}\sqrt{f} = \frac{C}{\sqrt{8g}} \tag{4}$$

This (see Fig. 1) is the von Kármán resistance function for smooth pipes [1], the constants having been determined by Nikuradse

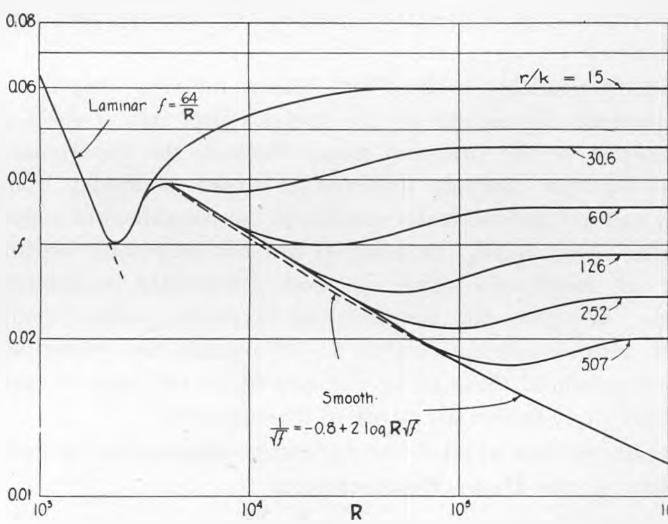

FIG. 1.—THE NIKURADSE EXPERIMENTS ON UNIFORM-SAND ROUGHNESS.

[2] and closely checked by other experimenters [3]. If the roughness is relatively great, on the other hand, viscous effects in the

boundary zone will be negligible at high Reynolds numbers, and the coefficients f and C will depend only upon the relative roughness—i.e., the ratio of a linear measure k of the boundary irregularities to the radius r of the pipe:

$$\frac{1}{\sqrt{f}} = 1.74 + 2 \log \frac{r}{k} = \frac{C}{\sqrt{8g}} \tag{5}$$

This is the von Kármán resistance function for rough pipes [1], the constants having been determined by Nikuradse [4] by means of pipes artificially roughened through coatings of uniform sand grains of diameter k. Since the Reynolds number is not involved in this equation, it evidently refers to the horizontal portions of the resistance curves shown in Fig. 1.

The validity of Eq. (5) for Nikuradse's artificial roughness was shown by plotting against r/k on semi-logarithmic paper the values of $1/\sqrt{f}$ for the horizontal portions of the curves of Fig. 1, the six points falling along the straight line of the equation as seen

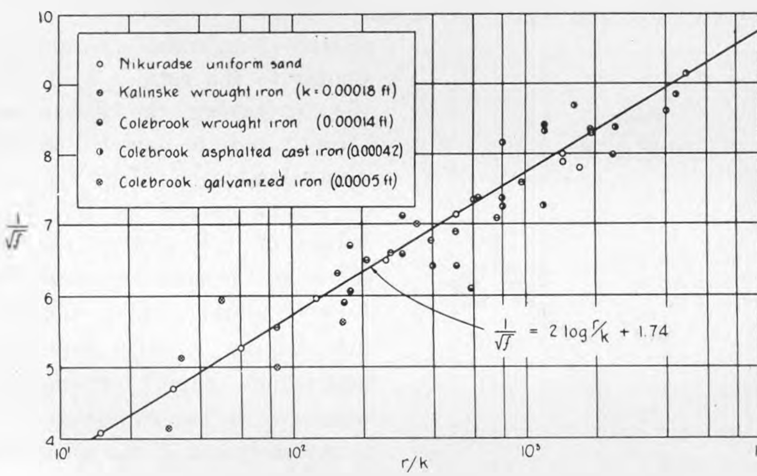

FIG. 2.—CORRELATION OF TESTS ON ARTIFICIAL AND NATURAL ROUGHNESS.

in Fig. 2. The validity of the same equation for the case of natural roughness was first indicated in this manner by Kalinske [5] through an analysis of Kessler's tests [6] on new wrought-iron pipe of various sizes (see Fig. 2). As further indication of the correctness of this function for commercial roughness of comparable types, the writer has adapted to this form of diagram values obtained by Colebrook [7] in an analysis of measurements by a great number of experimenters on new wrought-iron, galvanized-iron, and asphalted cast-iron pipe. Despite the appreciable scatter of the results shown in Fig. 2 (probably due in large measure to variations in fabrication, experimentation, and evaluation of test results), the points unquestionably follow the trend of the original functional relationship.

If the familiar Manning formula is written in the Chezy form, it will be seen that

$$\frac{1}{\sqrt{f}} = \frac{1.49}{\sqrt{8g}} \frac{R^{1/6}}{n} = \frac{C}{\sqrt{8g}} \tag{6}$$

The factor n is evidently a measure of boundary roughness, since it is invariably tabulated solely as a function of boundary composition; in the case of wood-stave penstocks, for instance, n is normally assumed to have a value between 0.010 and 0.013, and for concrete between 0.012 and 0.016. As a matter of fact, the ratio $R^{1/6}/n$ is a relative-roughness parameter similar to the ratio r/k of Eq. (5), n having the dimension $(\text{feet})^{1/6}$ and the factor 1.49 the dimension of $\sqrt{g}$. Were Eq. (6) as nearly correct as Eq. (5), values of r/k plotted against $R^{1/6}/n$ for the same magnitude of f or C would yield a straight line having a 6:1 slope on logarithmic paper. From the nature of the two equations, this is obviously out of the question, and the deviation of the Manning formula from the actual function may be judged from Fig. 3. Although empirical formulas of the Kutter-Manning-Bazin type may well continue to fill a useful purpose in

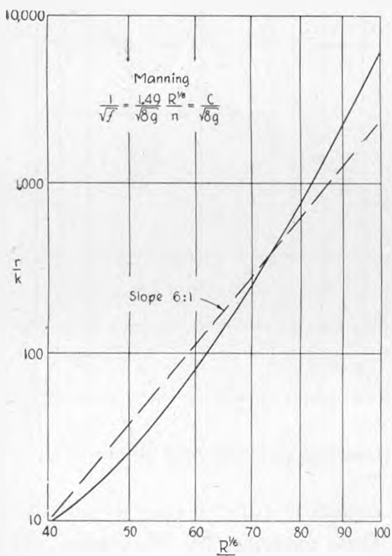

FIG. 3.—RELATIONSHIP OF THE VON KARMAN AND MANNING ROUGHNESS EQUATIONS.

the field, two facts should now be apparent: Such formulas from their very nature embody the influence of boundary roughness alone, and hence do not apply to that zone of motion in which viscous effects are of appreciable magnitude. And since they are far from exact except over a limited relative-roughness range, they may be expected, when used in connection with research in allied fields, to introduce inherent errors not present in Eq. (5).

In reducing measurements of commercial pipe resistance to the form of Eq. (5), one must evaluate an effective absolute roughness k in terms of Nikuradse's sand-grain diameter, extrapolating when necessary to the zone in which the function becomes independent of **R**. This has often been considered impracticable because of apparent differences in functional form in the intermediate zone. In the case of the uniform-sand roughness, to be sure, each of the family of curves shown in Fig. 1 is essentially similar to the others, beginning to deviate from the smooth-pipe curve as the nominal thickness δ of the laminar boundary film ($\delta = 65.6\, r/\mathbf{R}\sqrt{f}$) approaches the order of magnitude of the boundary roughness k. Evidently, the boundary irregularities contribute in no way to the resistance so long as they are fully contained within the boundary zone of laminar motion. This is clearly seen by plotting the quantity $1/\sqrt{f} - 2 \log r/k$ against the ratio of $\mathbf{R}\sqrt{f}$ to r/k for all curves of Fig. 1, as shown in Fig. 4; the horizontal portions of these

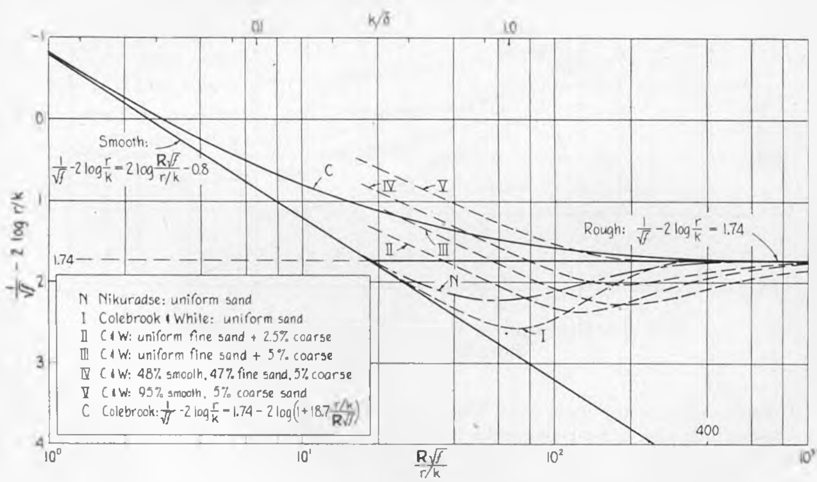

FIG. 4.—TRANSITION FUNCTIONS FOR DIFFERENT ROUGHNESS PATTERNS.

curves are superposed at the ordinate 1.74, from which limit a single curve extends to the sloping line representing the following adaptation of Eq. (4), the smooth-pipe function:[1]

$$\frac{1}{\sqrt{f}} - 2 \log \frac{r}{k} = -0.8 + 2 \log \frac{\mathbf{R}\sqrt{f}}{r/k} \qquad (7)$$

On the other hand, experiments by Colebrook and White [8] on artificial roughness of non-uniform character yield a different transition curve for each roughness pattern, as shown by curves I—V on the same plot. It is only reasonable, of course, to expect the larger boundary irregularities to disturb the laminar motion at the wall long before the smaller ones become effective, leading to the conclusion that each statistical combination of surface protuberances should produce its own characteristic transition function. Nevertheless, it was shown by Colebrook [7] that all three types of commercial pipe listed in Fig. 2 are characterized by essentially the same transition curve, for which he obtained the relationship

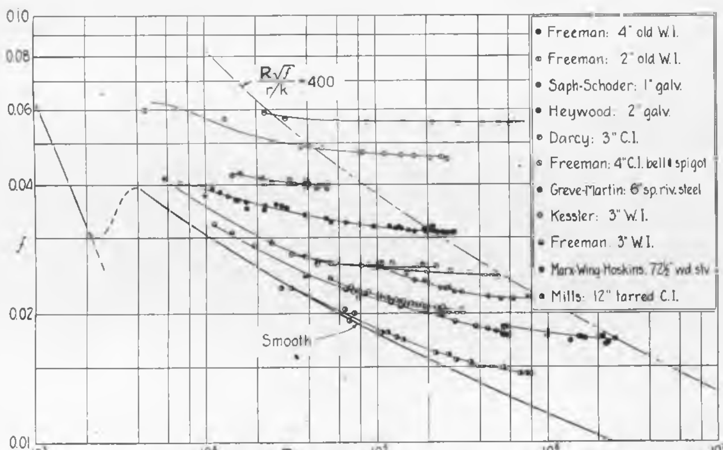

FIG. 5.—RESISTANCE DATA FOR VARIOUS COMMERCIAL BOUNDARY MATERIALS.

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[1] Noteworthy is the fact that Nikuradse found the transition curves to have as asymptote the Blasius equation $f = 0.3164/\mathbf{R}^{1/4}$ when plotted as in Fig. 1, and the von Kármán smooth-pipe equation when plotted as in Fig. 4. The magnitude of the discrepancy may be judged from the deviation of the broken line from the full line in Fig. 1.

$$\frac{1}{\sqrt{f}} = 1.74 - 2 \log\left(\frac{1}{r/k} + \frac{18.7}{\mathrm{R}\sqrt{f}}\right) \tag{8}$$

This expression will be found to approach Eqs. (4) and (5) as limits as r/k and $\mathrm{R}\sqrt{f}$, respectively, become very large; it is plotted in Fig. 4 in the alternative form

$$\frac{1}{\sqrt{f}} - 2 \log \frac{r}{k} = 1.74 - 2 \log\left(1 + 18.7 \frac{r/k}{\mathrm{R}\sqrt{f}}\right) \tag{9}$$

These equations are obviously too complex to be of practical use. On the other hand, if the function which they embody is even approximately valid for commercial surfaces in general, such extremely important information could be made readily available in diagrams or tables.

Contrary to Colebrook's conclusion, O'Brien, Folsom, and Jonassen [9]—far from finding the same transition curve for different boundary materials—asserted that they could not obtain a represenative curve for even the same material in different pipe sizes. In the effort to show that such variation is beyond practical significance, however probable its existence may be, the writer has plotted in Fig. 5 a series of measurements by various authorities on various types of commercial conduit [3, 6, 10, 11, 12, 13, 14, 15], the data having been selected on the one hand for range

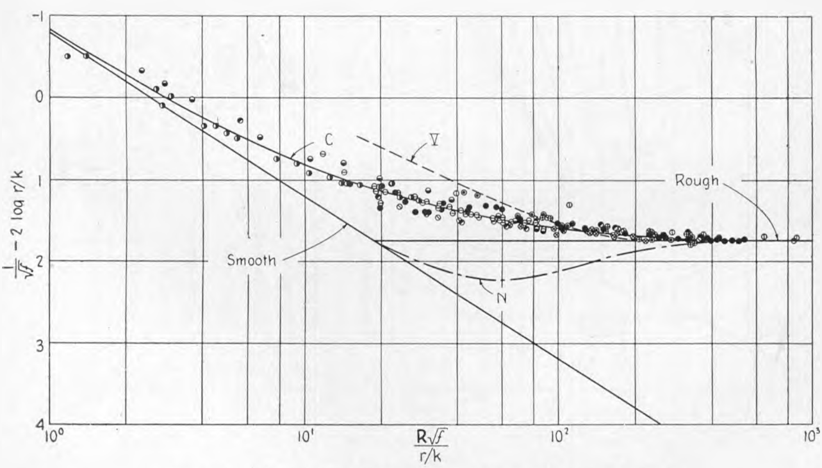

FIG. 6.—TRANSITION FUNCTION FOR DATA OF FIG. 5.

of Reynolds number and on the other for range of f. On this familiar f:**R** type of diagram, the several curves show to some extent the variations in form so characteristic of the maze of

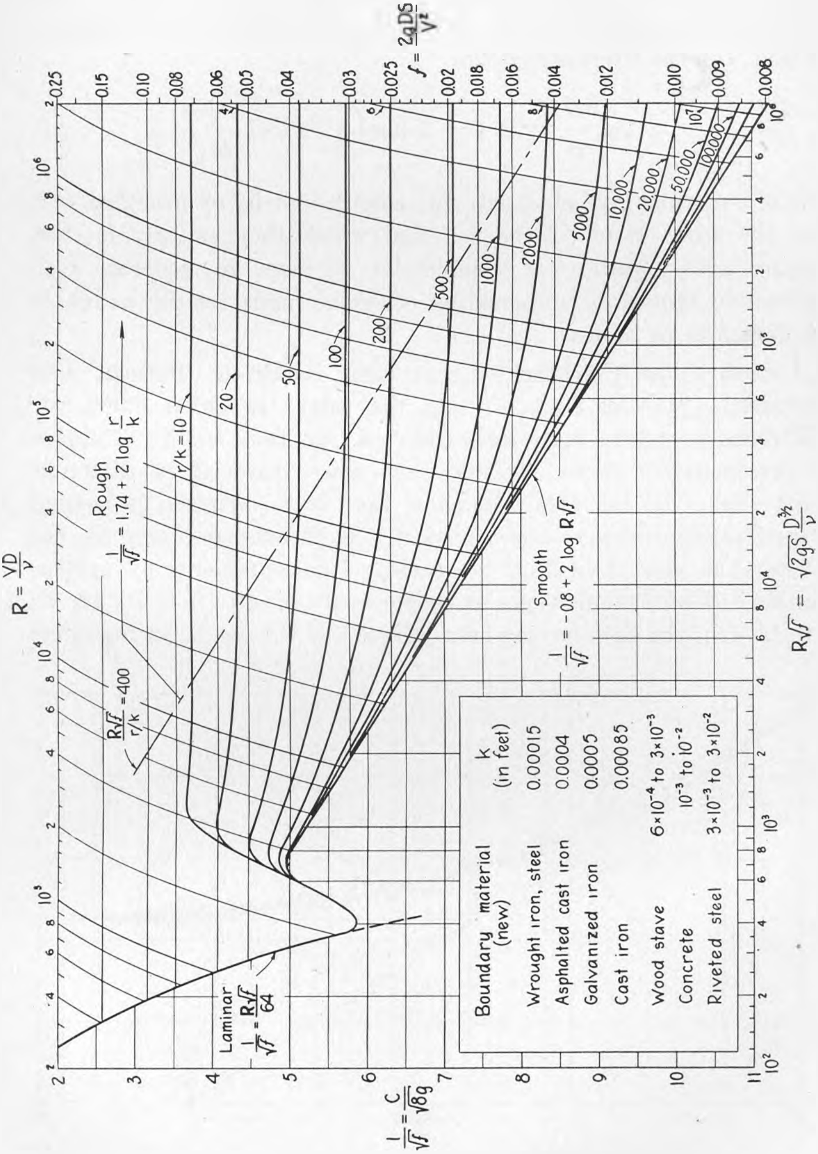

FIG. 7.—PROPOSED RESISTANCE DIAGRAM FOR COMMERCIAL PIPE.

experimental values often published in the past. However, it is at once evident that all curves tend to become more and more nearly horizontal as they approach the line $\mathbf{R}\sqrt{f}\ k/r = 400$, which corresponds to the value on the abscissa scale of Fig. 4 at which the rough-pipe law may be considered to begin. Indeed, when the several series of points are reduced to the form of Fig. 4 through extrapolation according to Eq. (8), their deviation from the plotted curve of Colebrook is evidently not much greater than the experimental scatter of the individual measurements in any one series, as may be seen from Fig. 6. Only in the case of the spiral-riveted pipe, which is of a basically different nature from the other boundary materials, is a somewhat different transition (such as Curve V) apparent, yet even here the departure from the general trend is not serious.

In the light of such results, the writer recommends the use of Eq. (8) as a close approximation to the actual resistance law. And in the light of the roles played therein by the parameters $1/\sqrt{f}$ and $\mathbf{R}\sqrt{f}$ of von Kármán's original analysis, the writer further recommends the use of these parameters as graph-paper coordinates in place of the customary f and $\mathbf{R}$. A diagram of this nature is shown in Fig. 7, in which the latter scales have been included for convenience, the basic vertical and horizontal scales nevertheless determining the form of the system of transition curves. These, it will be noted, are all geometrically similar—an obvious advantage over the $f:\mathbf{R}$ type of plot. Moreover, as shown by the alternative forms of the ordinate and abscissa parameters, values on the left-hand scale are directly proportional to (in American units about 1/16 of) the Chezy C, while in the bottom scale the velocity of flow does not appear. Such a diagram therefore permits the evaluation of either the velocity of flow or the hydraulic gradient in either the Darcy-Weisbach or the Chezy equation. Involving, if not the actual transition function, at least a close approximation thereto, this diagram may hence be used for design purposes with greater confidence than a purely empirical system of curves (such, for instance, as that formerly proposed by Pigott [16]). In addition, its efficacy in determining the magnitude of k from experimental data on new types of pipe will be apparent from the fact that it is no longer necessary either to extrapolate or to reduce the data to the forms of Figs. 2 and 6—but simply to plot the points on Fig. 7 and read directly the magnitude of r/k.

The engineer, in considering the general applicability of such a diagram, will probably ask at least one of the following questions: What value of k should be used for this or that specific surface? Does the diagram apply only to continuous sections of conduit, or is the effect of joints included? Is the transition function valid for pipes which have long been in service? And what application can the diagram have to conduits of other than circular cross section—in particular to open channels? Although full and precise information is by no means yet at hand, qualitative indications safely permit the following encouraging answers to be given at this time.

As may be noted from the table on Fig. 7, values of k (except for steel, approximately after Colebrook [7]) have been determined for five common types of pipe surface in new condition. Listed thereafter are three boundary materials which are likewise in common use, but which, unlike the others, vary considerably in absolute roughness with method of fabrication. More precise tabulation of the corresponding values of k is not consistent with existing experimental data [17]. However, lest the engineer, accustomed to the relatively small variation in the Kutter-Manning n for such surfaces, regard the tabulated ranges of k as exorbitant, the fact must be emphasized that a three-fold change in n (say from 0.01 to 0.03) corresponds roughly to a thousand-fold change in k. In other words, appreciable inaccuracies in the evaluation of k will not seriously affect either f or C.

So far as the influence of joints is concerned, it must be noted that connections of various types and frequencies are represented in the data of Figs. 2, 5, and 6, without marked variation in the form of the transition function. Both spacing and form of coupling may conceivably alter the effective magnitude of k to an appreciable degree, however, and systematic experimental study of this phase of the resistance problem is highly desirable.

Aging of the pipes, on the other hand, generally involves a continuous increase of the effective relative roughness k/r, corresponding to a continuous upward progression across the family of curves of Fig. 7 at a rate depending upon the conduit material and the fluid transmitted (it should be noted that such a trend will follow a vertical course if the hydraulic gradient remains constant, but will follow the Reynolds-number curves in the case of constant discharge). For evaluating the effect of age upon pipe capacity, Colebrook and White [18] have proposed an exponential expression which, in terms of boundary roughness, reduces to the following simple form [7]:

$$k = k_0 + \alpha t \tag{10}$$

The term k_0 represents the initial roughness of the new conduit, k the roughness to be expected after the time interval t, and α the rate of roughness growth. Determination of k_0 and α evidently involves at least two successive series of resistance measurements, their evaluation proceeding most conveniently according to the method herein recommended.

For reasons not at once obvious, the foregoing pages of this paper have dealt entirely with flow through uniform conduits of circular cross section. As shown by Keulegan [19], nevertheless, the von Kármán relationships for smooth and rough boundaries are fully as applicable to both closed and open conduits of other cross-sectional forms, provided only that the proper adjustment for form effect is made in the numerical coefficients. Such adjustment is, to be sure, never considered in using a formula of the Manning type, and is actually very small, unless the section departs considerably from the circular. The diagram of Fig. 7 should therefore yield as a first approximation the resistance characteristics of channels of moderate width-depth ratios, and should in any event indicate the general form of the resistance law for all conduits of uniform section.

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