LEONARDIAN FLUID MECHANICS

I - HISTORY OF KINEMATICS

II - INCEPTION OF MODERN KINEMATICS

By

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Sponsored by

National Science Foundation

and

National Endowment for the Humanities

IIHR Monograph No. 112

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Iowa City, Iowa 52242-1585

August 1991
TAKE TO KINEMATICS,
IT WILL REWARD YOU;
IT IS MORE FECUND THAN GEOMETRY;
IT ADDS A FOURTH DIMENSION TO GEOMETRY

Advice of Chevyshev to Sylvester.
ADDENDUM
IIHR MONOGRAPH 112

It takes a few weeks for monographs like this to be printed. In this case, those weeks were spent in a lecture trip. I took the opportunity of visiting several libraries to expand, if possible, the view of what kinematics is nowadays. Everywhere I found very co-operative librarians and thus, in a short time, I was able to collect more information useful for my project on the History of Kinematics. I am thankful to all of them. This addendum would have not been possible without their help and understanding. I am also indebted to persons who interacted with me during my lectures and periods of contact in different institutions; through their questions and critical comments they helped me very much.

1. FURTHER READING

During this trip, I read the *Eye and the Brain* by Richard Gregory [1990]. I found the chapter on vision of movement really fascinating. There is a hint of such an application of kinematics in Hartenberg 1964 [see comment on page 28 of this monograph]. Surely, this is a topic that must be included in a comprehensive history of kinematics. Therefore, the studies like those of Helmholtz in his *Physiological Optics* must be taken into account. It seems amazing to me that velocity may be sensed by humans without involving a simultaneous measurement of a time interval.

2. COMPUTATIONAL KINEMATICS

I have referred to the importance of computational kinematics in this monograph. This is an old branch of kinematics, but it has been enhanced by the use of modern computers. During a seminar at the Massachusetts Institute of Technology, I had the opportunity of learning about computational experimental kinematics aimed at gaining an understanding of arrays of vortices in general and also in connection with fish-like propulsion. I thought that Ampère would have enjoyed watching at such an elegant realization of the study of motion in the way defined in the famous essay in which he introduced the notion of modern kinematics.

3. APPLICATIONS OF KINEMATICS

On page 62 of this monograph, I have included lists of different areas of human endeavor in which kinematics is the object of study and/or finds applications. In these lectures, due in part to having left at home the corresponding slides, I took a different approach. Similarly, to Keynes notions of positive and negative analogies [Macagno 1986, in this Addendum], I decided to define positive and negative lists, and ask the persons in the audience to make their own list of those fields in which kinematics has no application. The reader may want to try it also.
4. ADDITIONAL BIBLIOGRAPHY


Contains comments on the general rule for the composition of movements.


See especially Chapter Seven, which deals with vision of movement. The entire book makes for a very enjoyable reading. Anyway, unless you read the first six chapters, it is difficult to profit from the reading of the seventh chapter.


This book deals with aesthetics of movement, dancing and choreography. [See also Souriau 1983]


MUYBRIDGE, Eadweard 1955. *The Human Figure in Motion*. Dover Publ., New York (Introduction by Robert Taft.)


Contains a selection of 183 plates from the 11-volume work, Animal Locomotion, first published in 1887.


Contains contributions by W.D. Marks, H. Allen and F.X. Dercum.


MUYBRIDGE, Eadweard 1984. The Male and Female Figure in Motion: 60 Classic Photographic Sequences. Dover Publ., New York.


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LEONARDO DA VINCI’s SCIENTIFIC AND TECHNICAL KINEMATICS
PREFACE

This monograph contains work for what I expect to constitute two separate volumes. On the one hand, I believe that a comprehensive history of kinematics has never been written; on the other hand, it seems necessary to write separately the history of the movement that led to the establishment of mathematical kinematics as a discipline of a standing similar to that of geometry. The idea of a geometry in motion is already in Leonardo da Vinci and in embryonic form in older writings, but it flourishes only much later in the nineteenth century and it acquires full realization in our century.

I have chosen to integrate my initial efforts on the history of kinematics into the series Leonardian Fluid Mechanics for several reasons, of which the most compelling to me is that it was the study of Leonardo’s manuscripts that led me to examine the entire history of kinematics because so much of Leonardo’s writings and drawings is in fact about the study of motion in general and also in fields other than mechanics of fluids. In second place is the immediacy one feels when the time left is running shorter and shorter, and one needs to put on record what seems a valuable effort. If enough time is given to me, then the two volumes will be ready in the near future; if not, at least the basic plan will be available to other scholars who may want to bring it to fruition.
GENERAL IDEAS

Kinematics is the name for the study of motion that was introduced by Ampère in his famous essay on the classification of sciences. The intent of Ampère was to create a new discipline in which motion would be studied without regard to the forces involved, that should be taken into account in dynamics. He was certainly responding to an existing trend, and he knew very well that the role of kinematics would be like that of a bridge connecting geometry and physics. I will discuss in detail this historical moment in the second part of this monograph. What we need to establish now is that kinematical studies have existed much before Ampère, and even before Greek science. Kinematics, unlike its sister Geometry, is not reducible to a branch of Mathematics, although the movement initiated by Ampère has made modern kinematics very much into an exact science.

Apparently, a comprehensive history of kinematics has never been written. Following an investigation of this question for some years, I want to take some initial action by summarizing my findings and offering a preview of such a history. If we take into account the classical trilogy: geometry, kinematics, and dynamics, it is quite remarkable that both geometry and dynamics have been the object of important historical writings, while kinematics has being considered only in some technical publications or in isolated chapters, always too specific, dealing with the history of mechanics.
The bias one finds in men who live a life of dedication to a given profession, or to a branch of a profession, is pervasive and is found in many works on kinematics. In an extensive bibliography of kinematics published two decades ago [de Groot 1970], we find almost nothing that does not relate directly or indirectly to the kinematics of machines and mechanisms. By chance (or mistake?) there seem to be there only one entry which is on kinematics of continua. There is absolutely nothing on the mathematical kinematics that one can find in many other fields, in spite of the bibliography having started as an effort to cover "pure" kinematics. I tested the bibliography by trying to find authors like Lagrange, Euler, Muybridge, Truesdell, but they were not included. I think, the reason is simply that the kinematics of deformable bodies was never contemplated by de Groot; of course, his was the right to choose the areas to be covered. I say all this not as a harsh criticism of such a formidable accomplishment, but only to make the point that it is difficult to agree on what is the thematic of kinematics, be it "pure" or "applied".

One can place kinematics somewhere between geometry and dynamics, but it should be much closer to the latter than to the former. From the point of view of dimensions, geometry implies only length, while kinematics requires length and time, and dynamics needs to consider length, mass and time. One must be careful, however, not to take a simplistic view, suggested by either dimensional analysis, or purely theoretical studies of displacements of geometric figures, be they rigid or deformable. It may sound paradoxical, but kinematics cannot be comprehended unless one knows dynamics, and even
more than that, unless one knows physics. This is not surprising, because even the full understanding of geometry requires a knowledge of physics.

Kinematics is a science that deals with motion in the most general ways possible. We can see a water wave propagating while the water itself stays very much where it was before the wave was produced, but there are other waves which travel as well, heat waves, pressure waves, etc. There can obviously be motion of matter, but there is also motion of form and other properties. It is not clear if the notion of what constitutes the science of kinematics enjoys a consensus even among those who are specialists in some of its aspects. For instance, in the case of mechanical engineers, one finds the notion that kinematics is only about particles and rigid bodies, or chains of rigid bodies [see, e.g., 11.1 in Engineering Mechanics by I.H. Shames 1960]. Kinematics is much less than mechanics in some ways, and much more in other ways. It is not easy to describe with a few sentences what kinematics is. I will try to give an overall picture of my views in this chapter, preceding the statement of the scope of this project, and the methodology which I believe is appropriate.

Study and application of motions in technology is a very old human activity. The historical developments are documented to a certain extent, but we should harbor no doubt that there existed heuristic and intuitive notions of kinematics developed and abstracted during those technological developments about which we know only the product, the machine or the device, but do not know the thinking associated with it. There is also much complex kinematics in dancing and in ballet, but what is scientific or even proto-scientific knowl-
edge in such activities?. Let us remember that also animals can perform very complex kinematic feats but obviously know no kinematics as a science, either empirical or rational, although something like geometry and kinematics must be encoded in their brains. We know, from photographic and from cinematographic studies, how complex can be just the normal walking of animals, not to speak of other much more complex motions like those of predator and prey, and courting evolutions. Why can one empirically devise and execute very complex motions, but it takes a very long time to come up with a theory for even very simple motions.

Nowadays we are used to the notion that technology is helpless without science, but we know that historically technology came first and prepared the ground for the emergence of the first scientific notions. Therefore much kinematics and dynamics must have existed before any theory was available. And in fact, there in technology was the source of the science that was going to systematize, under the name of rational mechanics, much empirical knowledge. But from the point of view of planning the study of the history of kinematics, I think it preferable to study first the development of theoretical kinematics, then of its applications, and only after that investigate the roots of all that knowledge. I tend to agree with Truesdell [1977] in saying that the study of the history of a science without a theoretical system is, if not impossible, at least difficult and full of pitfalls. Because in the case of kinematics, we have such a system, it seems preferable to begin with theoretical developments together, or followed by the study of the applicative developments.
The sources of kinematics, as those of geometry, are in the long historical process of development of empirical knowledge. It is believed that Thales visited Egypt and, having had the opportunity of learning there empirical geometry, developed such knowledge further and began to give it a rational basis, a theoretical system of thinking. We know that in Greece there were philosophers who challenged the scientific approach to geometry; apparently they did not inhibit progress, and they may have had a role in making the geometers much more careful in their postulations and proofs. However, the critical attitude may have worked much better in the case of the study of motion, i.e. of kinematics. It is also true that, in hindsight, we may see that kinematics needed much more empirical and experimental knowledge before it could be distilled into the science that Ampère recognized as ready to be systematized.

In the long process of developing the science of motion, one source lies in the observation of natural motion and flow phenomena, another in the constant invention of motion for our own body, for objects we handle, for devices we operate. I believe that one of the first to make such a connection between machines and mechanisms of early times and theoretical kinematics was Franz Reuleaux [See Weihe 1942]. There is also a passage in the writings of Henri Poincaré which reveals the tremendous importance of perceiving keenly the world around us, who live at the interfaces earth-air and air-water, to make possible the development of a science like geometry (and I would add, kinematics). I will use at this point a section of a recent paper [Macagno 1991]
commenting the ideas of Poincaré about the understanding of geometry and kinematics.

The difficult process of the human mind in understanding and properly describing motion was described by Henri Poincaré [1895]. To appreciate Poincaré's argument, just remember the tremendous mental efforts spent to cope with Zeno's paradoxes which are about extremely simple kinds of motion. It is a sobering experience, for anybody who believes to know his or her geometry, to try to describe in the simplest possible way the general motion of a rigid body from any one position to another. This, as it could be expected, was achieved sooner than the description of a motion in which the body experiences changes in shape. At the time Poincaré wrote his comments, the study of the geometry of deformation was still a rather young discipline [see, e.g., Helmholtz 1858]. Poincaré pointed at the great difficulty in following the motion of a body with important changes in shape; he noted that the great discovery in this field was the ability to follow the motion of small elements, that such an idea is very complex and could only have appeared relatively late in the history of science. It could, moreover, not have arisen, if the observation of solid bodies had not already taught us to distinguish and describe changes of position amounting to rigid motion. Poincaré concluded: "If, therefore, there were no solid bodies in nature, there would be no geometry." And without geometry, I would add, no kinematics.

Even an initial survey of the historical developments of the studies of motion reveals immediately how little of permanent value and how much
wasted effort was contributed by those who chose to approach motion in a purely speculative way, without resorting to observations like the astronomers did from the beginning or to experimentation and invention as many early engineers also did. Probably, geometry could emerge only after surveyors and engineers for long centuries had made the perception and understanding of fundamental common traits almost obvious to any reflective mind. Then it became possible to formulate postulates and construct a logico-deductive system of thinking. But the great adventure of adding the variable time to geometry proved to be a formidable task not easily accomplished. It is often said that we have an advanced technology because of science, but perhaps it is the other way around, that science emerges after great technological progress. Once a body of science is available, more technology takes place and again a new chapter in science is added, and so on and so forth. At least, this seems to be the pattern until recent times in which the acceleration of all processes may have fundamentally changed the interactions.

In the development of geometry, even today, there is a persistent trend aimed at excluding motion from the subject. This appears to match the requisite of not considering force in kinematics, but this does not constitute a perfect analogy. Although not usually mentioned explicitly, there is a basic role for motion in geometry. It has been unavoidable from the very beginning. There is an interesting experience that anybody with some knowledge of geometry can live, and it is to try to avoid any reference to motion in discussing geometric questions. For instance, try to explain how to construct, in elementary geometry, a regular polyhedron without any reference to motion.
Another example: when we define bodies of revolution we customarily use the kinematics of rotatory motion. The point is that motion is indeed unavoidable in geometry, and it should be recognized also as useful, and at the same time troublesome, because it begs the question of preservation of rigidity in all geometrical operations. On the other hand, anybody who has attempted the study of motion has found unavoidable the clever use of geometry, or otherwise has failed in contributing anything of permanent value.

In kinematics there has been very important activities of an experimental nature concerning not only the study of models and the visualization of flows, but also that of objects like the human figure and the bodies of animals. There are many other phenomena that require the use of the methods of experimental physics, and I will devote a chapter to discuss experimental kinematics. Although Greek science was never able to make much progress in kinematics, we must take into account some inroads that were undoubtedly of historical significance. For instance, on composition of motions, Aristotle was right when he stated that a body, actuated by two motions that are such that the distances traveled in the same time are in constant proportion, will move along the line of a parallelogram the length of whose sides are in constant ratio to each other. [Dugas 1955]. If the ratio of the velocities varies, the body will describe a curve. I find this a very interesting passage on kinematics.

We must take into account that Greek science was approached in different ways and that the loss of most of the documents results in a biased view. But, according to Clagett [1955], three fundamental views were important and had
lasting effects. Motion was important in each of them, although its study may have been ill-conceived oftentimes, or abandoned to tackle instead of easier static phenomena. The first, the so-called material view of nature lead to the atomic theory of Leucippus and Democritus, and thus motion was given a prominent place through the conception of a ceaseless agitation of the atoms. The second approach was mathematical (Plato and Pythagoras). The third approach was the Aristotelian which can be classified as one of compromise in which matter and form are considered as bound in an inextricable way. Perhaps, the wide scientific interests of Aristotle led him inevitably in this direction. Matter is in constant change, and so is form.

In Aristotelian physics, motion ought exist because there is a center toward which all bodies tend to move, then because there is already a density stratification in the world, i.e. a series of concentric spheres of different density, we may observe centrifugal trends when air is placed in water or centripetal effects when water is placed. But if there would exist vacuum, no natural motion could exist; i.e. no tendency towards a natural place. Aristotle also said that if a body would be set in motion in vacuum it would not possibly come to rest. The reason being that why would it stop at one place rather than at another? [Dugas 1955].

René Dugas, when detecting some incipient notions of a more general view of time dependent phenomena in Aristotle’s Physics and his Treatise on Heavens, expressed that the great philosopher did not differentiate, regarding change, mechanical concepts from others with more general significance.
Indeed, why should the study of motion be confined to mechanics? In a broad sense, motion and change are everywhere in the universe. It is not only use of analogical language to speak of velocity (or rate of change) in chemical or biological phenomena, or really in any kind of phenomena. Mathematically, it is the time rate of change of any variable property that provides the common thread.

In the preface to his history of mechanics, René Dugas explains that Hellenistic dynamics failed because it did not know how to account for the resistances to motion and it lacked a precise kinematics of the accelerated motion. Having treated and developed so well geometry, there was no ability to think in terms of time dependent geometry. Thus they were quite good in statics and sterile in dynamics, not having been able to invent kinematics. During a long time, only the astronomers did what amounts to observational work on celestial kinematics, and they did it quite well, although the relative motion they were describing was not the simplest.

If history has shown something is that we in fact got from researchers and thinkers closer to technology most of the original and fruitful ideas, clear notions and useful results. Clagett [1955] has called attention to the notions of Strato the Physicist (?-287 ?). Some of his work was attributed to Aristotle, but their views were in fact quite different. I will include a summary of Strato's concepts where I think it belongs, i.e. in the section of scientific and engineering kinematics.
Except for empirical advances, after Greek science came to a halt, there was little activity in kinematics [See Pedersen et al. 1990]. But late in Medieval times, in the schools of Oxford and Paris, the theoretical study of kinematics of very simple motions saw a brilliant period, but this movement in turn had little projections for a long time. [See the important studies of Clagett 1948, 1950, 1959].

Kinematics was revived by Leonardo when, using scanty notions and concepts received from medieval science, he developed his own body of knowledge and built a *sui generis* kinematics with striking resemblance in some areas with the kinematics of centuries after him rather than to that before him. Unfortunately, Leonardo only left unpublished material, and a new interruption was unavoidable. Leonardo developed his own rudimentary transformation geometry, an aspect of his work that did not escape the keen eye of Hermann Weyl [1952] although he did not research the matter fully. The vast work of Leonardo in proto-kinematics has been studied by M. Macagno, and much of what is summarized in this monograph about Leonardo's *geometria che si prova col moto* has already been exposed in her contributions [Macagno M. 1987, 1990, 1991]. She has looked at Leonardo with the eyes of an applied mathematician, and she has seen as geometry in motion some aspects that here are considered as incipient kinematics in some cases, or as true kinematics in others.

There are studies in Leonardo's manuscripts, that show him as a student of motion in the roles of artisan and artist, engineer and scientist. He studied the
shaping of different materials by the means available in art workshops of his time [See, e.g. the Codex Atlanticus]. This appears to have been at the root of his excellent understanding of the geometry of deformation. Of course, there is a great distance between Leonardo and Euler and Lagrange, but in several ways he is the first to dwell on what we call now the Lagrangian and Eulerian views of the motion of deformable bodies [Macagno 1987]. As an engineer and scientist, he studied in a large variety of contexts, pathlines of one particle, of systems of particles, of points of rigid and deformable bodies. In some cases we see him examining how an artist can change manually the shape of the material with which he works. In others he examines deformation of figures and elements thereof much in the way that was done by scientists of the nineteenth century, as Helmholtz and Bertrand. In others he determines path and velocities for elements of mechanisms as a mechanical engineer. His studies of flow of water, air and many other substances (like, sap, blood, mud, etc.) is remarkable and takes hundreds if not thousands of folios in his manuscripts. His kinematics of fluids is partially synthesized already in a number of papers and monographs [See Macagno 1989c]. There is still a great need for modern-approach studies of other areas of kinematics in Leonardo's manuscripts (e.g., elastic bodies).

The period dominated by Galileo and Newton saw the beginning of mechanics as a science which, for macroscale phenomena, is still used today. Whatever kinematics was developed then, until Euler and Lagrange included, was embodied in the studies of statics and dynamics. The historical study of kinematics for this period must tackle the problem of extracting from the avail-
able documentation what belongs to kinematics. This task exists also for many more recent writings in which the authors did not care to separate findings in kinematics from their treatment of wider or deeper problems.

As one examines books on mechanics there is a period in which their title is *Rational Mechanics*. The books I used as a student for my first course in mechanics were Levi-Civita [1918] and Cisotti [1925] both with that title. A few words are needed to explain what is meant by rational mechanics. Truesdell defined what he considered as rational mechanics [Truesdell 1958]; paraphrasing his definition, one could say as well *rational kinematics is a part of mathematics*. He added that mechanics is a mathematical science; I would rather say that it is a physico-mathematical science, which I believe is more in line with the first users of the terminology (*mécanique rationnelle, meccanica razionale*). In what (for once) I agree wholeheartedly with Truesdell is in his dictum that mathematics is also a science of experience, of that kind of experience that happens in our brains, because as he put it, *who would think that an oscilloscope is a more precise instrument than the brain?*.

The notions of analysis and synthesis in kinematics must also be studied. Although apparently originated by mechanical engineers, the idea of moving from analysis to synthesis in the kinematics of mechanisms and machines [Reuleaux 1875, Hartenberg 1964] is nowadays present in all fields of application, and in a crude way must have been present also in previous empirical developments as made evident by the ever increasing efficiency of kinematic
devices through history. In tracing the historical developments of kinematics, and of any of its branches, synthesis deserves as much attention as analysis.

In kinematics, synthesis is defined as follows: systematic approach to the design of a mechanism to perform a given function. According to Ferguson this was at first explored by Reuleaux 1876 [see Introduction to Dover Edition and Ch. XIII]. Reuleaux made the point that one must master analysis before engaging in synthesis. There is also in Reuleaux an approach which departs from the approach of kinematics in Paris, but this seems to be essentially in aspects that pertain to the consideration of efforts and forces, which are by definition beyond kinematics.

The role of some kind of knowledge of kinematics in representational art is perhaps less obvious than in technology, but, once we know how to look at the work of artists, the evidence is also overwhelming. For instance, the perception of flow among artists begins very early in history, and some "rheograms" are five millennia old, while the first important scientific "rheograms" are five centuries old [Macagno 1984/5]. Of course, artistic kinematics (meaning mobiles, animations, devices of kinetic art) is much more recent, and is probably inspired by scientific and technical kinematics.

Kinematics on its own seems to begin in late Medieval and early Renaissance times, it flourishes briefly and secretly in Leonardo da Vinci, it becomes an exact science with Galileo and followers, it only gets its modern
name and powerful mathematical apparatus about two centuries ago. It continues to be researched and developed very actively in our times.

In this chapter, I have tried to show that in kinematics there are a number of facets that are worth examining and that can be taken as the basis for a scheme for a historical study. In this first attempt, I shall use those activities that appear to me as the four main walls of the great building of kinematics: empirical, theoretical, experimental, and applied.
SCOPE

Now that an overall picture of kinematics has been delineated, it seems warranted to advance some ideas about a plan of action. The history of kinematics I envision should encompass art, science and technology. But the main initial thrust should be aimed at establishing the history of kinematics as a science, from its origins to rather recent times. The main reason for this choice is that I believe that the history of physico-mathematical developments can be least subjected to subjective influences. Other aspects, that are more susceptible of biases, should be investigated afterward.

Some developments in geometry are important in the history of kinematics, and this is evident in the terminology which we find in some areas of geometry; terms like "bodies of revolution", "translations", "screw", "glides", etc. The only term that seems to elicit optical phenomena rather than a common type of motion phenomena is "reflection". But as mathematics progressed, there were other areas of this science that became useful in the study of motion. One should recognize that the boundaries between mathematics and kinematics were not always clearly traced. However, we should not recognize any study as belonging really to kinematics unless a clear association of space and time is present.
Men surely have always had a visual and muscular perception of motion, of velocity, and acceleration, but it is fascinating that it took such a long time to examine those perceptions and generate notions in a useful abstract manner. I believe that, in due time, we should write a history that it is not only based on written documents about the analysis of motion. The generation of ideas, concepts, ability to produce motions physically, etc., should also be investigated and traced through all kinds of evidence and the archaeological material available. For me, the deep roots of kinematics are in the long period of empirical kinematics, and in the somewhat briefer period of experimental kinematics that preceded the mathematical approach to motion. The learning experience of Leonardo, who was an autodidact par excellence, can serve as a relatively recent model to theorize on how humanity went from very empirical activity to a profoundly theoretical knowledge as we can witness in kinematics.

In what follows, I will discuss first the appropriate methodology for this project, and then summarize the views of the different facets of kinematics that I have developed in the several years of work in this endeavor.
APPROACH AND METHODOLOGY

The question of the adequate approach to, and methodology of, a history of kinematics is not obvious and surely should not be that of traditional history of science. The adequate way to proceed must be considered carefully. One of the important parameters of the historical study of any science is the knowledge of the present status of that science in its most advanced forms. If such a parameter is replaced by an elementary or popularized form, at a level similar to that of the period studied, we may seriously fail to use the adequate approach. I would propose the criterion that it is not advisable to engage into the history of a discipline for which one is not able to provide a good critical description of its present status, and a reasonable prediction of where it is going.

Several concrete examples to illustrate the above argument can be given. One could think that since geometry was Euclidean for millennia, it is not necessary to know non-Euclidean geometry to study the geometry of say the Medieval times, or the Renaissance times. But the fact is that for a long time already we have had a much better perspective of the fifth postulate thanks to the works of Lobachevsky, Gauss, Boliiay and followers. But we do not need to consider non-Euclidean geometry to find striking examples. R. Marcolongo [1932], a distinguished classic mathematician, despite careful studies, was not able to see that Leonardo da Vinci developed the basics of transformation geometry, while Hermann Weyl [1952], a contributor to modern geometrical
developments, only needed browsing the manuscripts to find material enough to formulate a theorem in group theory which he named after Leonardo.

Within my own experience, I can cite other striking examples, in the case of hydrostatics [Macagno 1985, Dijksterhuis 1957], the analysis of Archimedes work that is done by fluid-mechanicists is certainly much more penetrating than that of men who know only elementary physics as taught in high-school or in a general course of university physics, even if they are distinguished in other fields. If one knows about tensorial calculus one can do an incisive analysis of writings on hydrostatics, and one can see where Archimedes failed, and also where Leonardo da Vinci (with an entirely different approach) failed also. It is interesting to realize that hydrostatics was decoupled from dynamics by Archimedes but not by Leonardo, who rarely failed to associate statics and dynamics. Another example is the history of the classic study of Newtonian fluids, which is much better understood by those who are familiar with non-Newtonian fluids. Some invoke the danger of anachronism to object to the use of the best present knowledge to examine that of the past, and one can grant that there are dangers and extreme care is needed. But the informed person may try to avoid some pitfalls, the un-informed person is simply blind to part of the picture he wants to examine, and he has no idea of what he can do to avoid mistakes.

An adequate methodology in the study of the history of kinematics requires that one be familiar not only with theoretical studies, but also with experimental techniques, because contrary to geometry, kinematics has important components
which are observational, experimental, and technical. Therefore, if one is not also an experimentalist one is really handicapped for this project. Of course, the knowledge of the science in its present status is a necessary but not sufficient condition; one must discover how to analyze and synthetize its entire historical development.

In the case of the history of kinematics there is a particular problem that must be considered carefully, because, as an independent discipline, is much younger than what can be considered its sister, geometry. Therefore one must trace kinematical developments that are usually buried between two historical layers, one is usually geometrical, the other is mechanical, or physical, or chemical or biological. It is a pity that a very early book on history of geometry has been lost, because, perhaps that early historian faced a problem of this type, when trying to extract geometry from writings on different subjects in which it was integrated. As one examines documents, one must be aware that not always the authors have clearly delineated what is geometry, kinematics and dynamics in their writings, an approach rarely used [Macagno-Landweber 1958]. The easiest case, from this point of view, seems to be in the field of mechanisms and machines, and to a certain extent in mechanics of the continuum, because for both there exist already some specific historical studies which can serve as models. [See the corresponding section in Bibliographical References].

To further illustrate the dire need for scientists to take an interest in the history of kinematics, I can cite many examples, but perhaps the consideration of the Hadamard conditions for surfaces and layers of discontinuity
[Predvotilev 1962] is most fit for that purpose. Studying such a work one can gain a view that seems reserved to scientists because of a relatively high mathematical language barrier. However, it seems that if contribution like Hadamard's were left out from a history of kinematics much would be lost just because of being difficult to analyze and evaluate by generalists. To name just a few fields in which such conditions are applied, I can mention acoustics, optics, mechanics of particles, explosions, combustion, aerodynamics, hydrodynamics, shock loading and unloading in solids.

A field in which some models for the methodology to be used exist is that of the kinematics associated to early astronomical observations and studies [Schiapparelli 1874, Neugebauer 1962]. This is not an area without amazing features, because if one compares the observational means of those times with modern instrumentation in this discipline, it seems almost unbelievable that so much was accomplished in the description of the motion of stars and planets. One interesting problem is why, if the motions of celestial bodies were (even with old geocentric theories) not more complex than that of some mechanisms, the degree of sophistication in the description of the motion was so different. One reason may be that in the case of mechanisms and machines there was no interest in describing motions accurately, but in physically producing them. Be as it may, we find quite a different approach needed in these two areas, because almost from its very inception, astronomy offers more advanced theoretical kinematics than that of mechanisms. Perhaps only the study of the level and floods of the Nile constituted a match to the astronomical studies in the neighboring lands of Mesopotamia and Greece.
It seems necessary to point out that the history of kinematics should be the history of motion rather than that of the natural or man-made systems in which such motions are found, or that of the points of view adopted (motion relative to the Earth, or relative to the Sun, for instance). Because of this, it makes little difference that an elliptical motion is accomplished by a planet or by a material point in a mechanism. Only if the study of motion has peculiarities in one discipline or another there is an interest in establishing a differentiation. This circumstances may be found in many cases; waves in water and in elastic media may have a unity in the fundamental equations, but then the mathematical and experimental techniques used in the actual study of the corresponding kinematics may be quite different, and had had a different historical development [Tokaty 1971, Timoshenko 1953 should be consulted]. A similar situation occurs in kinetic theories as discussed below.

Another point that certainly requires knowledge of science rather than any other else, is that the introduction of mass does not make necessarily some notions strictly dynamical, and outside the field of kinematics, they may remain strongly and fundamentally kinematical. For instance, when one discusses the formulation of the equations for conservation of volume in fluid mechanics one is still very much in the realm of kinematics, but such domain is not abandoned even when the equations for conservation of mass are formulated; the two treatments are very similar, the mass distribution becoming the less significant part of the statement. [See Appendix I of Macagno 1989a]. Even the discussion of momentum flux and kinetic energy flux are fundamentally kinematical. One could say, in general, that there are terms in the equa-
tions of dynamics which are essentially kinematical. When is it that one is
dealing essentially with kinematics, in spite of being in the field of dynamics,
and when not, is a matter that requires careful examination, and of course, it
may depend on the definitions adopted. In turbulence theory, for instance, the
so-called Reynolds stresses present this kind of problem.

In the study of kinetic theories of the different states (solid, liquid, gas),
and in the studies of mass transport processes, we find a role for kinematics
which has to be traced carefully by scientific historians. If one examines the
basic definitions and formulations in the theory of mass transport by fluid
flows, for instance, the notion of velocity for a mixture of different species
cannot be introduced without taking into account the different densities
involved [Bird 1963]. There is an analogy with a simpler problem, that of
centers of gravity as they are defined and mathematically determined in some
courses of calculus. The matter is treated in a purely geometric manner,
although it is really a problem in physics, especially if the mass-density dis-
tribution is not uniform. When the density is not constant it cannot be easily
ignored.

In the definition of velocity in a mixture of different species, either the
common mass density or the molar concentration are used. Thus the velocity
vector at a point is defined as \( \mathbf{v} = \sum_i \left( \rho_i \mathbf{v}_i \right) / \sum \rho_i \), where \( \rho \) is used to
indicate mass density. The notion of mass flux is involved, but it is more of a
kinematical notion than a dynamical one. Or we could say that an incursion in
a third dimension, that of mass, becomes necessary to define velocity (a typical kinematical quantity if there is one) in this context.

Finally, there is the still more subtle question of when kinematics can be decoupled from dynamics, and when not. This is important because, if one is aware of this possibility, kinematics in cases of coupling becomes a much more delicate subject to study, not only *per se* but also in its historical development. In the Chapter on Theoretical Kinematics, more will be said about this question.

I hope that this discussion is helpful in any future study, my own or others'. If a history of kinematics is to be written, it seems that some pitfalls should be avoided that can be detected in the history already written for other disciplines. I hope also that there is a challenge here to engineers and scientists not to relinquish to less prepared scholars the responsibility for writing the history of kinematics or that of any other engineering or scientific discipline. If necessary, we can learn about history and its methods, a task that seems easier than that of historians learning what we know about engineering and science.
EMPIRICAL KINEMATICS

Reuleaux has influenced modern thinking about what are mechanisms, and his wide conceptions show in my including devices with solid, flexible, fluid parts as mechanisms. That does not mean, of course, that anything relating to fluid flow is considered to be a mechanism in this monograph. But the boomerang is regarded as a mechanism in spite of consisting of a single solid body; the other element is the air. The lasso is a mechanism because in spite of being basically a single piece it has originally two elements one of which is built at one extreme by a properly done knot. I even go as far as envisioning as part of kinematics the operations involved in kneading, and in knitting and weaving. I believe that for a long time all this was in a purely, but not at all crudely, empirical basis. Nowadays, for example, we have engineers designing mechanisms to knit in three dimensions which require a sophisticated knowledge of theoretical kinematics.

Each successful mechanism or machine of pre-historic or early historic times is in itself a study in motion of an empirical nature, but unavoidably related to notions and thoughts about displacements, velocities and accelerations. I am not saying that the inventors of the boomerang or the shadoof had ideas similar to ours about velocity or acceleration, but their sensorial perceptions related to our modernly defined properties of motion were surely as keen, or more, than ours. It seems interesting to attempt to reach a comprehension of the kinematics of early projectiles, machines, tools and instruments.
with an eye to the kinematics involved. The student of the history of kinematics would do well to construct models of different devices and mechanisms and operate them with his hands; it may be very revealing to discover how keenly can one sense acceleration in a muscular rather than a visual way [Macagno M. 1987]. No matter how long was the way from skilful early production of certain motions to our scientific notions about them, it seems a more reliable and correct source than the persistent obscure and confusive lucubrations about motion by a number of philosophers, although, there are no doubt some important philosophical contributions, like Zeno's paradoxes, who made permanently valid and disturbingly challenging contributions and stimulated an ever more rigorous codification of our knowledge; however paradoxes are not sources of invention and creativity in the acquisition of new knowledge of the physical world and its phenomena and in the processes of invention.

Reuleaux [1875] believed that motion is an easier and more accessible idea to early cultures than force [ See p. 224 of Reuleaux 1876 ]. He considered the fire-drill as the first machine contrary to the then generalized belief that it was the lever. He thought that we believe the lever to be the first machine because we think that the first goal was to overcome great resistances. He believed that what first attracts the open consciousness, is really the accompanying motion. He noted that children are much more attracted by motions than by the forces implied in those same motions. Reuleaux concluded that separation of the idea of force from that of motion is a very difficult mental operation and because of that we find it occurring late and gradually. Reuleaux believed that the un-eradicable fascination with perpetual motion is rooted in
the initial fascination with all kinds of motion. Reuleaux [1876. p. 222] thought that rectilinear motion (blow-tube, arc and arrow) is not common among very old peoples, however primitive such motion may appear to us. I would like to see these questions investigated more thoroughly.

Reuleaux's section on ancient machines is extremely interesting. He listed the following as machines invented in antiquity: rollers to move great stones, carriages for war and transport, water wheels, toothed wheels, pulleys, certain kinds of levers. He also made a list of "basic mechanical components:"

1. the eye-bar type of link called crank in kinematics (vague connection with what is today called this way). Sometimes called lever.
2. the wheel, including toothed wheels.
3. the cam
4. the screw
5. the ratchet (intermittent motion devices).
6. the tension-compression organs (chains, strings, belts, hydraulic lines).

Hartenberg [1964] agrees that all the six Reuleaux's "basic mechanical components" were already invented in antiquity and put to use before the Christian era.

We can trace the above kind of empirical kinematics even further back in time if we examine the work of archaeologists who document the existence of different mechanisms in different cultures all over the world. The popular
idea that civilizations can be measured by having known or not the wheel is an indication of this possibility. However, in my opinion, there are inventions that are perhaps cleverer than the wheel, like the boomerang, the picota, the sling, the shadoof, the blow-tube, the arc and arrow, the "boleadoras" (a lariat with stone balls) which are very ingenious kinematical inventions and require much more kinematical skills to use them properly. Of course, the wheel is probably the more versatile kinematic element we possess. There is kinematics not only in utilitarian inventions; we find it in toys, games, dance there is also much of kinematics analyzed and synthesized by our first computer, the brain, with mainly the aids of eye and hand. I would go as far as to say that there is much kinematics to be studied in the motions performed and created by animals, but this would overextend an already too vast program.

An interesting aspect of inborn kinematics has been pointed out by Hartenberg [1964, p. 117-8]: we are experts at integrating velocities and making predictions of where a body with which we can collide will be in the next few seconds. This is vital in crossing busy, or not so busy, streets. There is no dynamics involved, pure kinematics. This ability to avoid collision, or to escape danger by estimating velocities must be as old as mankind, must in fact be inherited from even older than human ancestors.

For the history of kinematics one should study the mechanisms found in early cultures from the point of view of their motion, leaving to others a systematic study of the origins, development and skillful use of mechanisms in different particular cultures. For instance, in the case of the Egyptian shadoof
[as represented in Fig. 171 Reuleaux 1875] one does not immediately have the elements to evaluate it. The basic idea seems relatively simple, but it does not appear as a device to which it is enough to provide only muscles. It would surely be difficult to design a robot that would operate this system with many degrees of freedom as efficiently as the many unknown men who have used it for such a long time. I believe this is a way of judging some mechanisms. The mechanical clock is a rather recent invention but the fact that it is enough to provide a weight or a winding spring to have it running by itself tells a story of effective but less complex kinematics, free from instabilities. I have constructed a model of a shadoof and tried my hand at it and I recommend the reader to do the same. Or try the "boleadoras" which seem simpler to make. Again, I would like to see a robot throwing successfully the boleadoras as I saw it done as a child in the Argentinean pampas Mechanisms which still need an alert brain to function well are of a category different from those for which more or less brute force is enough.

During the long period of empirical kinematics, it is highly plausible to assume that motion was many times conceived regardless of considerations of force or power. I suppose that some mechanisms were abandoned because not enough power could be made available or because there were no materials appropriate for the construction. This happened to a number of Leonardo’s "inventions", but some of them were good in the long term. The great difficulties to develop some mechanisms and machines should not be underestimated; Friedrich Klemm [1954] has discussed this question and has quoted the writings of some inventors which narrated such difficulties.
The idea of disregarding the forces involved recurred more than once in the history of kinematics. Mechanisms were considered in some periods as the devices which essentially were designed to change one force into another, rather than one motion into another. That it is really better to begin by studying motion disregarding the forces both for teaching and research had to be rediscovered more than once. Then there is the question of what one should expect of a good kinematicist, his powers of analysis or his powers of synthesis?. Design and invention require always synthesis rather than analysis. Thus, of the remote past, we know the results of synthetical work and very little, if anything, of the analytical work. Should we conclude that our ancestors did not have a way of analyzing motion?

There are obviously a number of mechanisms in our own bodies, in addition to much more complex physiological systems. Mechanisms in which there are chains of linked bars are too analogous to arms and legs [Fischer 1909] not to have resulted from such inspiration. The oar could have had as ancestor the arm and hand which can also be used for propulsion not only in swimming but also in rowing. If one accepts Reuleaux's classification of mechanisms and includes not only those made of solid pieces but consider the blowers for instance as mechanisms with fluid components, then the possibility of producing air flow with our mouth may very well be behind the invention of blowers. Probably the chain of links of so many mechanisms invented by different cultures started by the experience gained in producing motions with our own bodies or its parts.
Some rotary motions were produced with human hands, like the alternative rotation of a rod by putting it in between the hands and moving them alternatively back and forth. This is a very important kinematic feat because it was useful to drill holes and to produce fire [Tylor 1870]. Geiger [1871] believed that rotary motion was the first that mankind produced by means of what could be called a machine. I am not sure that this is true, although Geiger adduced very good reasons in support of his idea. In one case the rod is rotated alternatively with the hands directly, and in another by means of a string or a cord that makes a loop around the rod.

Probably it was easier to conceive and develop alternating rotary motion than continuous rotation in one direction. The wheel is less universal among different peoples than some device to make fire. Natural water currents produce circulation or vortices at some places and one can presumably see a piece of wood rotating endlessly when caught in whirlpool. But it seems a long way to arrive at the point of designing and mounting a water wheel or a wind wheel. We also know that the potter wheel is a rather old invention. At some time circular, more or less uniform motion, must have been conceived and devices invented that produced it. The water wheel could be kept in motion for as long as there was a steady flow of water propelling it. The potter wheel was probably kept in more less steady motion by either the feet of the artist or by the hands of an assistant. The ways by which a mechanism was kept in motion is extremely interesting. In this study there is a theme, that I visualize as hovering over all the great variety of devices and gadgets, namely, what
kinds of motions were conceived and put into existence one way or the other?. If circular motion was already known by some people, did they convert it into rectilinear motion of some kind? Did they develop other conversions of a given motion into another?. Look at the motions rather than to the mechanical devices.

When carriages were developed, their wheels would accomplish rotation and translation; the carriage itself mainly translation, but occasionally it should change direction and rotate. For the two carriage wheel, the change in direction presented little difficulty, but a more complex kinematics characterizes a four-wheel carriage which can easily turn. Two-wheel chariots were used in Western Asia, Egypt, Greece for war at rather early times. Perhaps they came from farther east, India, China. They are also mentioned in the Bible. Reuleaux believes that a predecessor of the wheel may have been the roller under a heavy stone block. To me this looks, kinematically, very much like the rod rotating between two hands. The alternative rotation was used in a later development which seems related to the potter's wheel, the lathe. Both in the East and the West this happened with different arrangements, but kinematically with the same type of motion.

To clarify a basic point, I would like to look at the piston and cylinder pump as a device that converts an alternative rectilinear motion into a unidirectional motion (not uniform of course) of a water column. However, when a centrifugal pump is used, a rotatory motion is converted into a rectilinear nearly uniform motion of a water column. This is an application of Reuleaux's
criterion of not restricting kinematics to rigid bodies, or linked rigid bodies; it was, in fact, adopted in ancient empirical kinematics. The use of flexible or fluid elements as integral parts of motion converters is very old. Note that swimming is based on creating a certain fluid motion in order to propel our body in a given direction. We must study mechanisms and chains or assemblies of mechanisms from the point of view of a chain of motions conceived already by our ancestors as part of their technology, science and art.

Modern kinematics has taught us that a solid body, which can be considered as rigid, can only perform a combination of translation and rotation. For planar motions, there is a simplification, because any displacement can always be obtained by means of just a rotation. Note that translation may be considered as a rotation around a center at infinity. In space motions, any displacement is equivalent to one by a screw or helical motion, i.e., generally composed of a rotation and a translation. This general theorem does not exclude, of course, descriptions of a specific kind for particular types of motion.

When we consider bodies like strings, belts, springs, or fluids like water and air, to the roto-translatory components an additional, much more complex component of motion, must be considered. Even solid bodies may have to be studied as deformable bodies, as in the case of studying the vibrations of a building or a machine, for example. There is a well developed theory of deformable bodies which will be considered in the chapter on theoretical kinematics, but from the point of view of empirical kinematics, an understanding
of deformation surely existed for some primitive devices and mechanisms to have been developed and perfected.

Some mechanisms are challenging if one tries to reconstruct a plausible path toward their invention. A good example is the screw-nut mechanism when used as a device to convert rotary motion into linear motion, for example. Reuleaux was puzzled by when the screw-nut combination made its appearance and why is predominantly right-hand rather than left-hand screw, although in antiquity there are both [Reuleaux, p. 222-223]. The screw-gear combination is another puzzling motion converter. Turbines are in this category. And also centrifugal pumps.

Kinematics is a field in which the engineer came in the wake of ingenious men and women, and then in the wake of engineers came mathematicians who finally erected a new dynamic geometry as the old static geometry had been erected more than twenty centuries before from measuring, architecture, mapping and surveying.

In the following table, I have attempted to catalog the most important motion conversions in the long empirical period, which I believe continues to this day, because inventors still proceed in the same way; at least those many without a formal scientific or engineering training. Just one typical illustration is included for each item in the table.
<table>
<thead>
<tr>
<th>Rigid elements</th>
<th>Potter wheel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexible only</td>
<td>Lasso</td>
</tr>
<tr>
<td>Rigid-flexible</td>
<td>Pulleys and cord</td>
</tr>
<tr>
<td>Rigid-liquid</td>
<td>Oar, rudder</td>
</tr>
<tr>
<td>Rigid-gas</td>
<td>Boomerang</td>
</tr>
<tr>
<td>Flexible-fluid</td>
<td>Sails</td>
</tr>
<tr>
<td>Liquid-gas</td>
<td>Air pump with liquid piston</td>
</tr>
</tbody>
</table>
THEORETICAL KINEMATICS

Using just a single symbolic mathematical expression, we can say that theoretical kinematics is about

\[ x_i = x_i(X, t). \]

What is meant by this is that, for each case of motion, we have a procedure to calculate the coordinates \( x_i \) of a set of material points, given the vector \( X \) that identifies each point (\( X \) can be the initial-position vector) and the time \( t \).

For the case of the continuum, we can find in Truesdell [1954a] more precise specifications, but the expression is applicable to any set of any number of particles starting with one and ending at infinity. The above expression is of great generality, because it is applicable to either discrete systems or continua, and to all kind of models, deterministic or stochastic. For example, we can consider it as the representation of a particle that follows Newton's first law or of one that performs a random walk, or anything in between. It should be understood also that our symbolic expression may represent any discontinuities that we may want or need to include in our kinematic model; for instance quantic jumps in coordinates at any given time.

In kinematics, we are concerned not only with the positions of a point at certain times, but also with the displacement accomplished and the distance traversed in a certain time interval. Historically, it took a long time to gradually introduce other functions of time like those for velocities and accelera-
tions. For some problems, even higher derivatives have been introduced, like the rate of change of the acceleration called jerk, and even other higher order derivatives. [Hartenberg 1964]. Resal [1862] called this quantity the *suracceleration*, but he noted that it was Transon, in 1845, who introduced the notion. It should be realized that displacement is not the same as distance traversed; if a point describes a circle 20 cm in length to return to the same position, the displacement is zero, the distance is 20 cm. The corresponding angular displacement and distance are zero and 360°, respectively.

In this chapter, I am not attempting a systematic summary of the history of theoretical kinematics, not even a survey; rather, I intend to visit briefly a few highlights here and there along a period of about twenty five centuries. I have selected such material with an eye to those particular aspects that elicit some comments I believe useful for future work in this project.

The first steps in mathematical kinematics were accomplished for the motion of a single point representing either a small particle, or that of a rigid body undergoing a pure translation. Later, the point could be in curvilinear motion either free or subject to some constraints as exemplified in one case by a projectile and in the other by a pendulum. In general terms and for a planar motion we can replace our above general expression by

\[ x_1 = F(t), \quad x_2 = G(t). \]
In theory, one can always eliminate the time \( t \) between the two equations and obtain the equation

\[
x_2 = f(x_1)
\]

which is the pathline of the point in question. It is interesting to realize that this elimination of \( t \) entails a significant loss of information about the motion of the point. If we study, e.g., Eudoxus model for planetary motion, we discover that there was a successful attempt to determine the corresponding pathline for a simple version of the model. This led to a curve defined by the intersection of a cylinder and a sphere. This was certainly interesting because one could visualize the path, but by itself it suppressed any information about how the particle moved along it. To better understand this point let us assume that we have

\[
x_1 = R \cos pt, \quad x_2 = R \sin pt.
\]

Then, elimination of \( t \) leads to

\[
x_1^2 + x_2^2 = R^2,
\]

which is the equation of a circumference of radius \( R \). But this last equation by itself carries no information with which we could, for example, determine the acceleration of the given motion. The last equation does not even tell us where the particle was at the initial time \( t = 0 \).
Once differential calculus was developed, it became rather easy to calculate velocities and accelerations when the parametric equations were known. In the last example, we only need to determine the vectors $d\mathbf{x}_i/dt$ and $d^2\mathbf{x}_i/dt^2$. We can quickly find out that, in the given motion, the speed is constant but not the velocity (vector); the acceleration (vector) is centripetal. Once the kinematics of a point is developed, the kinematical theory of a rigid body, and that of chains of rigid bodies or mechanisms may seem to become readily accessible. The history of this aspect is interesting because it is full of fascinating developments. In the planar motion of mechanisms, we have come to consider a plane attached to each link. As we usually refer the motion to a link that remains fixed, its plane is usually chosen to refer to it the motions of the other planes. Any figure (including its plane) in planar motion rotates instantaneously around a center. If we think of two planes, one rotates relative to the other around the instantaneous center. The names of Descartes and Bernoulli [1742] are related to the discovery of this center.

Although translation is thought to be understood by everybody, it is perhaps warranted to say a word of clarification. In transformation geometry and in kinematics it does not have precisely the same meaning than in common language. As we know, this happens with many words (e.g. moment, work, vortex, circulation, etc.). A set of points (those of a rigid body or those of a fluid!) is performing a translation if the velocity vectors for all the points are identical. Be mindful that this does not exclude the existence of acceleration, and higher derivatives of the velocity. The pathlines of a body performing a translation do not necessarily have to be straight lines.
To give just one example of a more recent finding from mechanical engineering kinematics, Aronhold [1872] and Kennedy [1886] discovered independently a still more interesting theorem: The instantaneous centers of any three links having planar motion lie on the same straight line [Hartengerg 1963]. Other notions are those of poles and centrods. In planar motion, if a figure suffers a change of position (a given displacement), in which it does not remain parallel to itself, it can only be reduced to a finite rotation around a point which is called the pole. If we make a succession of displacements, there will be a succession of poles, they can be marked on the fixed plane and on the moving plane, thus yielding a fixed and a moving polygon. If the motion is continuous and not by steps, we generate a fixed and a moving curve, they are called centrods. In the case of wheel rolling on a pavement, or two gears, the centrods are simple curves. This knowledge may be useful in studying rather old discussions of circles rolling on a straight line [see, e.g. Heath 1921 and Clagett 1959 on one of Heron's problems].

The first mathematical descriptions of fluid flows are rather recent. Relatively speaking, not much exists before Euler and Lagrange, although one should not neglect the many efforts (many successful) of Leonardo da Vinci to describe the motions of air and water using almost purely geometrical procedures. [Macagno, Series of IIHR Monographs]. The students of flow between the two Leonardos (da Vinci and Euler) usually treated the problems they considered as specific mechanical problems, and the historian faces the task of extracting from their writings what was actually purely kinematical.
This is probably easier after Euler and Lagrange, although some scholars ignored the didactical facet of the appeal of Ampère, and did not separate kinematics from dynamics.

Almost always, a discipline can be approached in many different ways; this is especially true of fluid mechanics, and I believe of continua mechanics also. Already Euler and Lagrange realized that there were two fundamental ways of describing flow, and more generally any motion with deformation. In fact one can use the two points of view also for rigid motions, but is generally not necessary. We refer nowadays to the Lagrangian and the Eulerian descriptions, although, as already indicated, they can be found in geometric representations in Leonardo da Vinci's manuscripts [Macagno 1991]. The history of the use of this two representations of flow and transport phenomena is very interesting and also very much influenced by the type of problems our Western civilization has considered important at different times. But returning to the different ways of describing motions, we find the emergence of families of lines like the pathlines, the streamlines, the streaklines, the fluid lines, to mention the most important ones. Water in motion, like other fluids, which seem so amorphous when at rest, becomes highly structured to the eyes of any hydrodynamicist. Artists have captured the many faces of water, but only scientists have been able to depict the infinite variety of lines and surfaces that can be seen in the midst of water bodies in motion.

In fluid mechanics, the notion of vorticity has its own history. It is rather trivial in two-dimensional flows, because this is a property that seems to have
its full meaning only in three-dimensional space. Once the essential vectorial character of vorticity was recognized, the notion of vortex lines and vortex tubes was bound to emerge, as it did. But not until Truesdell published his masterful study of the kinematics of vorticity [Truesdell 1954a], was it possible to realize the full significance of this property of rotational flows.

The kinematics of stochastic movements has a shorter history, but it is not as recent as some who speak about chaos are inclined to believe. Two samples should suffice. One, the studies of turbulent fluctuations [see, to begin, Reynolds 1894], and two, the work on random vibrations of machines and buildings [Crandall 1963].

I hope these few examples emphasize the importance for the historian of kinematics of being knowledgeable about modern kinematic theory. Not until we are aware of such theory, can we have the ability to put in the right perspective the great difficulties experienced historically to develop a rational approach to the study of motion. We can also realize that not many decades ago there were still important gaps and pitfalls; and who knows if some do not exist even today in our knowledge of motion. This awareness is essential in order to recognize that the science of motion progressed slowly but cannot be divided in a period that is scientific and one that is not scientific at any point in its history. Let us not define as true science our level of scientific knowledge, or that of one or two centuries ago.
It is interesting to consider what must have been looked upon as the advanced theory at some remote times in the past. The example I have chosen for this combines kinematics and dynamics. The author of *Mechanical Problems* (Aristotle?) [Dugas 1955] regards the law of the lever as a consequence of the notable virtues of the circumference; the reasoning seems to be that something remarkable can be expected from something still more remarkable. Continuing the argument: In a lever with a fixed point, the others describe different arcs of circumference; the idea of opposites is then thrown in, and we have a philosophical piece handed to us to justify the theory of the lever. We read that "The properties of the balance are related to those of the circle and the properties of the lever to those of the balance. Ultimately most of the motions in mechanics are related to the properties of the lever." [Dugas 1955, p. 19].

It appears that Aristotle tried to present motion in such a way that would circumvent the criticism of the Eleatic school which denied the possibility of the existence of motion and change. So, he considered motion as the actualizing of the potential. Change is a process going from potential existence to actual existence (from one mode of existence to another). Aristotle considered that there was one physics for sublunar regions, and another for the heavens. The circular motion was considered to be the natural motion for celestial bodies.

Although Aristotle discusses in detail when a motion is faster than another by considering the space traversed and the corresponding time, he never
arrived at what is so elementary for us: \( V = \frac{S}{T} \). He considers several cases; for instance, in the case \( S_2 = S_1 \), velocity \( V_2 \) is larger than \( V_1 \) if \( T_2 < T_1 \). To divide a distance by a time was not an acceptable operation, if it was considered at all, while for us it is a common practice to refer to physical quantities of dimensions \( LT^{-1} \) or \( LT^{-2} \), or \( T^{-1} \). [Macagno 1971]. We should be cautious about the limitations of a science that does not define velocity as we do it; such limitations may be less important than we may tend to believe. Experience with computational fluid mechanics using finite differences may show strikingly that one can calculate derivatives (e.g., velocities and accelerations) without the usual operation one learns in calculus courses.

When we examine the basic facets of mathematical kinematics, knowing what the ancient Greek scientist knew about geometry, it seems amazing that they did not create a similar science for the study of motion instead of embarking in seemingly-profound dead-end alleys. However, those interested in the observation and study of the motion of the stars and planets surely accomplished, practically, a study of kinematics of objects reduced to material points. In effect, perhaps the earliest scientific studies of kinematics are those performed by astronomers [see, e.g., Schiaparelli 1874]. We must not forget the astronomical studies of other cultures, especially those of the Mesopotamia, where a numerical rather than a geometrical approach was taken [Neugebauer 1962].
Schiapparelli, Neugebauer, and others have published analyses of early astronomical studies, which are at the same time early examples of kinematics, early examples of \( x_i = x_i(X, t) \). Neugebauer has suggested that the conics may have been discovered thanks to observations of the shadow of the sundial. We must determine what else was done in ancient civilizations concerning studies of motion in other areas than astronomy. The variations of level of seas and rivers, although less regular than the motions of planets are an interesting possibility, as we know that there were important hydraulic works in ancient times, as vital, or perhaps more, than astronomical knowledge. Be as it may, I doubt very much that over periods of millennia we will find much more than tables, or perhaps, graphs, of positions of a particle as a function of time. The notion of velocity remains very primitive when it is found, and that of acceleration; is surely in an even more primitive state, but we should not assume that they did not exist at all [see, e.g., Pedersen 1990].

The history of kinematics, as it can be found in astronomical observations of the remote past, should not be concerned with a number of otherwise very interesting aspects of astronomy, it should focus on the level of theoretical and empirical kinematics used in astronomy through the long history of that science. From the strict point of view of kinematics, we may find more interesting the efforts to describe motion in a geocentric system than in an heliocentric system or in a galactic system. Relative motion may become simpler in some cases, and the skills of the kinematicists less subject to a great challenge. I would say that ellipses are less interesting than hippopedes.
Incidentally, at this point, it should be remembered that Aristarchus of Samos opted for what amounts to a Copernican system. To the rotation of the Earth around its axis he added a revolution around the Sun, and he assumed the Sun and the stars to be fixed. He is supposed to have assumed that all the planets revolve around the Sun. He was even accused of heresy for disturbing the center of universe with his ideas. Clagett [1955, p. 116] thought that Copernicus should be called the Aristarchus of modern times.

In Greek astronomy we find a great use of circular paths, on a plane or on a sphere, or combinations thereof, and our interest resides on the kinematical problem of describing planetary paths including retrogradation, but we must leave to others the study of the dynamical concepts behind such system. Our interest is in how advanced was the study of the motion used in a model, not whether the model was poor or accurate dynamically speaking. We would like, for instance, to know if the notion of velocity was introduced or ignored, and how crude or refined was the way in which it was defined. Or if lack of uniformity in a motion was studied or not, i.e., whether there was some proto-notion of acceleration. Eudoxus and Ptolemy used different models, and rather than comparing them from the point of view of astronomical phenomena, we should consider how well the motion of the model was studied in each case. Another interesting problem is how well did Aristotle understand Eudoxus model in his comments about it. We know that other bodies, those moving as projectiles on the Earth were also studied in Greece, and then there is an interest in comparing the level of kinematics in both fields, rather than conceptual errors and misconceptions in one field or the other. There was
some study of mechanisms or devices to trace some curves. In such a case, it would be interesting to trace the understanding of kinematics if there was any; I mean whether was there a point of view represented by $x_1 = F(t)$, $x_2 = G(t)$? or by $x_2 = f(x_1)$?

There were two important centers of astronomy in the pre-Christian Eurasian world, those two astronomies proceeded in quite different ways. We owe to Neugebauer a good historical analysis of those astronomical sciences, and from the point of view of kinematics perhaps the salient features are the geometric approach of the Greeks and the numerical approach of the Mesopotamians. Kinematics is certainly much related to geometry, but it can also be studied without any geometry, by means of numerical descriptions of functions as the Babylonians seem to have done it.

I believe that the idea that one may opt for a computational kinematics with little reference to geometry needs some lucubrations. Suppose that I have a bad bruise in my leg and it develops into an elliptically shaped ulcer of my skin, and that I am lucky and find a good dermatologist who treats it successfully. Suppose I measure regularly the two axes, $x_1, x_2$, of the ellipse and make a table and feed it to a computer. Then I write a program to find velocity and acceleration for the point of the vector of components $x_1, x_2$. There is no material point moving in the physical space; but is it not this an example of computational kinematics? Who knows the forces acting on my ulcer? Surely not my dermatologist and all his colleagues in the entire world. But, maybe, they can use my view of the kinematics of cicatrization to evaluate different
treatments. Who knows!. May be a few average numbers representing velocity, or acceleration, of cicatrization is all they need, without ever drawing any geometrical diagram. Was this the way the Babylonian astronomers looked at their tables? Could they look at a table and see, with the same ease as we do when looking at a graph, that they had before their eyes a linear function, a polynomial of second or third degree? I tend to believe that all this is plausible.

Of the two important schemes of Eudoxus and Ptolemy, I will choose to call attention to that of Eudoxus because in passing I can refer to the excellent study due to Schiaparelli, a Milanese professor who took a very scholarly approach to this piece of history of science. His was an approach which I have considered as a model for some of my own studies, and that seems worth following by others also. Clagett [955] has praised this brilliant reconstruction from scanty documentation.

The Babylonian approach can be viewed as a precursor of our curve fitting methods which now with the use of electronic computers can be applied with a high degree of refinement. Eudoxus procedure is a precursor of the Fourier analysis of complex motions.

In later Medieval times, Nicole Oresme used a representation in which a segment is taken perpendicular to a line to represent the intensity of a quantity at each point of a subject that affects the intensity. This sounds much like diagrams in use today. Thus longitudo (extension) is represented by a hori-
zontal line drawn in the direction of the subject. The *altitudo or latitudo* (height) of a segment at a given point is proportional to the *intensio* (intensity) of the property at the point. If the figure is a triangle it represents uniformly difform quality (*unifformiter difformis*). A trapezoid is the same only that it does not begin at zero, but at a certain value. A rectangle represents a uniform quality, or we would say, of constant value. *Difformiter difformis* is a quality which is represented by a curve. Oresme made a classification containing 62 different laws of variation. This is superfluous within our methodology governed by the use of advanced mathematics, but it may have been the right choice in Oresme's times.

Oresme extended the study of variation with time to what we would call now two and three variables. When describing velocities, Oresme noted that they should be represented with a double extension, either in time or with respect to the subject. He found the meaning of the area of his diagrams. For instance in uniform notion, it becomes that the area gives the distance traveled. He called this integral, the total quality. He considered a case requiring the sum of a series: he made a diagram consisting of rectangles with base $t/2$, $t/4$, $t/8$ . . . and height $i$, $2i$, $3i$ . . . . He stated that the distance traversed was four times the total quality of the first rectangle. It seems that Oresme did not reach the point of expressing the law so familiar to us: $d = (1/2)gt^2$. According to Dugas [1955 p. 62] some scholastics discussed accelerated motion in theory, others the free fall motion, but they did not connect the two topics. Was this a gap between theoreticians and empiricists?
A very interesting theoretical medieval kinematical study of what is assumed as very simple motion is that of Oresme [Grant 1971] concerning two or more points (supposed to represent planets) in circular motion. Oresme investigated the conjunction of two or more mobiles performing concentric circular motions. He was also concerned with the repetition, or the predictability, of conjunctions, but he assumed that completely accurate positions can be verified to the point that one can in fact distinguish between coordinates that are given by rational or irrational numbers. Nonetheless, this work of Oresme deserves a place in the history of theoretical kinematics.

According to Dugas and Clagett, the developments in Oxford were more advanced, but with no geometric representation. Oresme appears to have referred to certain veteres; and we can presume that they were the logicians of the Oxford School (who else?). Heytesbury [1494] stated the rule for the distance traversed in uniformly difform motion, that was also enunciated by Oresme. He, however, thought that the effective velocity of a rotating body was the maximum velocity. What Heytesbury studied, that in Paris was missed, was the notion of acceleration. Heytesbury used an obscure language but he surely introduced together with that of velocity the notion of acceleration. The description by Heytesbury goes as follows, using modern terminology: If a body starts moving from rest, we can imagine a case in which its velocity increases indefinitely. In the same way we can imagine a more complex case in which the velocity change or acceleration occurs with infinite variations, quick or slow. The relation of acceleration to velocity is analogous to that of velocity to distance traversed. [Dugas 1955, p.67]. Clagett [1959]
says that about one century before, Gerard of Brussels had already considered the notions of Oxford. He considers that Gerard wrote the first treatise on kinematics.

In the study of mathematical kinematics, we must take into account that the history of analysis is surely going to be much easier than that of kinematic synthesis simply because it is a field that has been usually avoided by purely theoretical kinematicists. The methodology for synthesis, which includes necessarily requirements of optimization in one way or another, is much more difficult to put into practice and therefore to study historically. As an example we can consider the use of Chebyshev \([1853]\) polynomials in the optimization of accuracy points in kinematics.

In some cases, of course, dynamics was uncoupled from dynamics because of lack of knowledge. Now, with more knowledge, does it make sense to investigate when kinematics can be uncoupled? All fluid-mechanicists know that irrotational motion of an incompressible fluid can be studied in two steps, first the kinematics and then the dynamics. This is a very interesting modern problem: when is it possible to decouple kinematics from dynamics? For instance, I am thinking of the ballerina who perhaps incidentally discovered that her speed of rotation could be varied by stretching out her arms or by bringing them close to her body. She learned this as an empirical law for which the causes remained unknown. The question, for us, is: can we decouple kinematics from dynamics in this case?
The definition of kinematics seems to make everything simpler, but we must be concerned with the difficult question of the possibility of a meaningful decoupling of kinematics from dynamics. I am aware at least of one notable exception: that of gas dynamics where even thermodynamics must interact with kinematics [Truesdell 1954a]. For many who speak about chaos, I have a question: can geometry and kinematics be decoupled from the dynamics of turbulence? Students of fluid mechanics are sometimes puzzled by the above mentioned analysis of frictionless fluid flow that is carried out without any discussion of the forces involved; forces which are then computed a posteriori. They sometimes ask if there is not an invalidating mistake in ignoring the forces in the first step.
EXPERIMENTAL KINEMATICS

One may become so thrilled with the rational beauty of theoretical kinematics that it seems that the subject does not need observations and experiments. The temptation may be the same that lures many into considering geometry as unrelated to time and to physics. I think there is an analogy between kinematics and its relation with physics and geometry and its relation with kinematics. Geometry cannot be completely separated from experiments, for the simple reason that motion enters in one way or another, especially during periods of creation or revision. In addition, the techniques of the Mathematical Laboratory have been, and still are, necessary to generate new themes in geometry. They, in fact, are more important in that role than in that of teaching. There is however, a difference, because most of kinematics has been, and still is, developed by scientists and engineers who have understood the necessity of observation and experimentation in kinematics, not only as a temporary aid to be discarded eventually, but as the very essence of the subject.

Historically, the great contributions to kinematics of ancient times are the observations of motion of astronomers in Mesopotamia and Greece. It is inconceivable that such kinematics could have been purely mathematical, although the knowledge gained by using with great skill very crude instrumentation was matched only by devising very clever mathematical models of motion and surprisingly accurate calculations. As astronomy progressed, more and more complex phenomena were studied, and the mathematics and
instrumentation required became of higher and higher sophistication, but observation of motion remained for a long time the central object of investigation. At no moment in history can one say that astronomy's kinematics is a purely theoretical or mathematical science.

According to Clagett [1959], Strato the Physicist, a name given to him by Polybius, Cicero and Simplicius (fl. -278), wrote two treatises at least, which may be Mechanics and On Audibles. We know of Strato and his ideas because of the many attributions (which refer to about forty writings) by other authors. Only fragments survive unfortunately. Simplicius quotes from a treatise called On motion [see reproduction in Cohen & Drabkin 1948]. After explaining that it is universally accepted that bodies in natural motion experience acceleration, he says that Strato asserts in his treatise On Motion that a body in natural motion completes the last part of its trajectory in the shortest time. Then he quotes Strato:

In the case of bodies moving through the air this is what happens. For if one observes water pouring down from a roof and falling from a considerable height, the flow at the top is seen to be continuous, but the water at the bottom falls to the ground in discontinuous parts. This would never happen unless the water traversed each successive space more swiftly . .

Strato also used an argument which is clearer than the one above. He called the attention of the reader to the mark left by the same stone when falling from a small height as compared to the mark when falling from a great
height. He emphasized that the stone was the same, the weight and size the same, and that it is not impelled by any different force in each case. It is merely a case of acceleration. Clagett sees in this a kinematic rather than a dynamic analysis. But I would say that, for that, Strato had to try some ways of assessing spaces and times. He infers a different velocity by the use of the complicated phenomenon of forming a dent on the ground. I see Strato very far from Heytesbury, Oresme and Leonardo. In fact, even Aristotle appears more of a kinematicist, if we consider some passages only. But the fact remains that Strato was in these two cases an experimental kinematicist.

The use of experiments in kinematics is as natural and essential as it is in any physical science. The mere fact that both space and time cannot be defined, measured, or investigated, without instrumentation entering the picture in one way or another, makes the investigation of motion an essentially physical science; it is true that it can compete with geometry in mathematical structure and accuracy, but in no way can it be reduced to a theoretical mathematical subject, regardless of the field we consider. At all times, kinematics has received great contributions from engineers and applied scientists. I do not forget that this is a classification of professions of our times, but there were people who under different terminology were playing the role of engineers and applied scientists at all times. What I say of engineering can be said of many other activities in the past. With the ever increasing technological progress since the Renaissance, instruments to observe motion from the stars to the very small particles, and from all kinds of phenomena either natural or
man made, or created in laboratories, experimental kinematics has not ceased to grow during the same period.

The historical development of flow visualization is surely going to be one of the most interesting chapters in any comprehensive history of kinematics. Instances of flow visualization must have happened from the very beginning of an awareness of motion with some intent to understand it and describe it, because the most common fluids, air and water, are in many cases carriers of naturally incorporated tracers. It is enough to think of dust, vapor, and smoke in the case of air, of silt, leaves, and other floating or suspended materials in the case of water. The first engineer-scientist who did a great amount of work on experimental kinematics was Leonardo da Vinci, who studied the motion of particles, of rigid and deformable bodies and of all kinds of fluids [Macagno M. 1987 to 1991]. The method has been extremely useful to all students of motions from very simple to extremely complex ones.

Leonardo has been credited with the invention of flow visualization techniques in water; they may have very well been used before, but what is undoubtedly his merit is an impressive amount of observations of water flows, and he surely can be honored as the pioneer of the modern use of flow visualization [Macagno 1989c, IIHR Monographs]. He made also notable studies of the motion of granular materials induced in different ways (gravity, vibrations, centrifugation, etc.), which in some cases were intended as models for the flow of liquids [Macagno 1982, 1991]. His studies of flow in flames surely are remarkable although we do not know exactly what technique did he
use. It seems that he discovered a primitive optical procedure by projecting the shadow of the flames on a wall in some way. There has been great activity in the field of visualization of movements for more than a century, since the invention of the photographic and the cinematographic cameras. We have very interesting aspects of kinematics from the study of Brownian motion to the flows in nebular stellar dynamics, from traffic flow to forest fires propagation, from laminar to turbulent flow, from sediment transport to blood flow. Every year, there are seminars, meetings and congresses entirely devoted to visualization methods in different fields [See, e.g., Pagendarm et al. 1991 for a seminar with emphasis on computational kinematics. See Voller 1991 for modern video applications]

There are motions of such apparent complexity that, in the first phase of their study, the only possible procedure is the experimental. I have already mentioned the Brownian motion, for which some theory could be developed, but other motions are still more difficult to handle. The studies of animal locomotion by Muybridge [1887] are already classical, but the experimental study of such motions continues to this day [e.g. Alexander R.M. ed. 1977]. Muybridge demonstrated by taking photographs with a battery of cameras, at a time in which the cinematographic camera was not available, that the perception of a running horse by some artists was wrong, and that the horse is never airborne. [Gombrich 1966, Rawlence - Coward 1990]. It is fascinating that, in order to have "living pictures" as Coward says, it was necessary before that there existed enough mechanical engineering kinematics to obtain both the
original and to realize its projection. The "cinematograph" needed a lot of kinematics in addition to optics and chemistry to become efficient and useful.

Another two important lines of study for a history of kinematics are the instrumentation created or adapted for the study of motion and the models that are, and have been used, to investigate motions, existing or to created. We know that models of mechanisms and machines have been used for a long time (Vitruvius already challenged their value, and Leonardo discussed critically his assertions five hundred years ago). Models were also used to demonstrate different kinds of motion, like those which served to illustrate planetary motion. Reuleaux [1876] created a large collection of models in Berlin [see also articles by Kennedy 1876 and Webb 1883]. In rather recent books on engineering kinematics we find authors devoting a chapter or a section to physical kinematic models [see, e.g. Hartenberg 1964]. Although the following is very recent history, nowadays we have the possibility of using computational models [See, e.g., Voller 1991].
Before speaking of applied kinematics, one must explain first what is the fundamental kinematics that is being put into application. I believe that we should not identify fundamental with theoretical mathematical kinematics, but include also the synthesis resulting from observations of and experimentation with motions of all kinds. For instance, if from flow visualizations of turbulent flow we infer that turbulence appears to consist of a whole spectrum of eddies of all kinds, we arrive at a non-theoretical knowledge which can be applied in the efficient generation of turbulence in a given flow, which is being designed to achieve a certain efficiency of mixing of two or more substances. Or, if we have observed very small length and time scale motions in the midst of a liquid, we may apply this knowledge in several areas of physico-chemistry. Or we can use such knowledge for formulation of theories.

In this chapter, as in preceding ones, I will not attempt a systematic survey nor a brief synopsis of the long history of applications of kinematics to many disciplines and activities. I am trying instead to offer a set of representative examples beyond some already mentioned, like astronomy and mechanical engineering, which are classical fields of application. By referring to a certain number of diverse applications, I hope to convey the notion that they are in fact very numerous, and that a historical study of applied kinematics should be mainly the work of a team of scientists and engineers with an interest and also the necessary preparation in history of science. It would be better if some
coordination can be established, but it is important that enough people develop an interest and the will to accomplish such a task, otherwise the work may be done by persons without enough knowledge of the subject.

The fields of applied kinematics are so numerous that one hesitates to attempt the making of a list which everybody will surely find incomplete. However, I believe that I have to offer one. I am including among the applied fields those of science because we have now a body of knowledge which we can classify as an exact science very close to geometry in ancient times. And although one could say that kinematics is part of physics, one can also take the approach of placing geometry and kinematics in the realm of physico-mathematical sciences. At least for the purposes of listing different fields, of application of kinematics, I will take such a position.

For the applications to the sciences, we need only to take into account that it is difficult to find a science in which motion has a negligible role. Even the oldest branch of mechanics, statics, in some of its methods uses motion, and therefore kinematics finds applications. For the applications to engineering, we must take into account that different branches of engineering depend heavily on kinematics: mechanical, aeronautical, hydraulic, environmental, naval, bio-mechanical engineering are good examples. In some branches, the concern is mainly with rigid motion, while in others it is the motion of fluids that takes the fundamental role.
In some cases, kinematics has been recognized as essential from the very beginning, while in others there have been delays. The events vary from one country to another. For instance, Douglas P. Adams [Hain 1967] was of the opinion that in the early 1950's no kinematics beyond the very elementary was taught in the United States to mechanical engineers. I must say that at that time, advanced kinematics was already taught to hydraulic engineers, and surely to aeronautical engineers also, because of the simple reason that the kinematics of fluids is much more complex than that of rigid, or even, elastic bodies. The accomplishments of Euler, Cayley, Chebyschev, and others in the kinematics of rigid bodies were rarely, if ever, mentioned to mechanical engineers [Hain 1967], while hydraulic engineers were not unaware of the rather sophisticated studies of kinematics of vorticity by Truesdell [1954a].

Adams describes the changes and progress that occurred during and after the Second World War in the approach to kinematics of mechanisms and machines in the United States, which appears as having had great German influence. In the field of fluid flow, we see a mixed influence of German and British origins [the schools of Prandtl, von Kármán and Taylor, Rouse 1957] together with a flourishing of an already existing native tradition in the United States. I think there is an interesting line of research, and not only for engineering: that of the history of the reluctance to use fundamental principles and theories in all fields of applications of kinematics. One of the reasons for lack of progress in science and technology one can observe historically in some places is the narrow-mindedness of scientists or engineers in that period. We seem to live in an age in which that does not exist, but it may very well be an
illusion. Anyway, in our times fundamental kinematics is certainly applied to the following fields:

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There is an application that was more important in the past than in our times: the transmission of motion over relatively great distances. The advent of electricity changed this field radically, but the history seems to have never been written. One example of such a transmission existed in the famous Marly pumping installations in France [Decrosse 1991]. There were many also in mining engineering when there was a chance of using the water power of a nearby water stream [Hartenberg 1964].

The general idea of drawing curves by means of mechanisms is old and still used with ever more sophisticated devices. Since mathematicians usually are reluctant to include time as part of mathematical considerations, and do not seem to favor any mathematics done with instruments, I think that we must consider the devices to draw all kinds of curves as mechanisms of interest to engineers and architects in first place, and also to physical scientists and artists. Of course, curves can be drawn also by optical means, not only mechanical devices, and I would say that the corresponding kinematics is just one section of general kinematics. Designing and constructing a device or a machine that performs a certain desired motion is of course a vast endeavor and robotics is part of it.

To mechanical engineers we seem to owe the concepts of analysis and synthesis in kinematics; the analysis is a general study of given motions, while the second is the methodology by which one arrives at certain practical results. I think that what they call synthesis is very much like design. If so, this two notions are applicable to all disciplines in which the production of certain
motions is an objective, and this includes science, engineering and art. In the eighteenth century there are two figures that may be considered as embodiments of analysis and synthesis at that time: Euler and Watt. Euler used mathematics with great skill in his studies of motion in solid and fluid mechanics; I am not aware that he did any work on synthesis as defined above. In contrast, Watt excelled in kinematical synthesis, which is one way of applying the principles of kinematics. Among the interesting contributions of Watt, we have his mechanism for producing (with great approximation) straight-line motion; he considered this as one of his great engineering accomplishments. A warning seems warranted, however: We must guard ourselves from believing that necessarily, the best inventors should be either those who are well armed in the basic science or those who are not.

Many kinematical notions have been transferred to other fields via analogy, even to serve in areas in which motion is not the object of study. Thus we find that the notions of pathlines are introduced in the study of stability of non-linear differential equations [see, e.g., Ross 1964]. The point for which pathlines are thought to exist is a point in the phase plane; it is not a particle or a material point. The "flow" patterns that we can observe in such an application are very similar to those that one sees in books of hydrodynamics. I consider this a very interesting development because in the heart of mathematics, we find that "motion" plays a role, if not for logico-deductive proofs, for an elegant interpretation of behavior of solutions of differential equations.
Kinematics has found and it is finding application in chemistry; perhaps only elementary notions are being used, but the development is surely of interest. Once a reaction starts it continues through some time and therefore one can define a rate of production of a new compound, or a velocity of reaction. Reactions may reach equilibrium after some time. If we want to produce a certain substance, we must remove it so that the reaction continues; but this is another matter. Our interest is here only in that the notion of velocity, and eventually of acceleration, is as present in this field as in kinematics of particles and bodies in mechanics. There are other other processes in which substances move at different speeds under different processes, like electrophoresis and chromatography, for which kinematics appears to have a role.

The motions of the human body have been studied from many different points of view. Rehabilitation professionals need to know much about kinematics of our body, and also seems to be needed by those who want to increase the productivity of all kinds of workers [Rabinbach 1991]. It seems that many who begin working for the latter group end up in the hands of the former group. But there are many other aspects which are related to organs in which motion at different scales is important, as in the case of contractions all along the digestive system [Macagno 1980]. Other applications in biology are the studies of the motions of the heart and its valves, those of the eye, the flows in the circulatory, respiratory, and lymphatic systems.

With the advent of kinetic art, we began to see in Art Museums not only static pieces but many that were in motion or could be set into motion, or
could create the illusion of motion. In many of them there was an application of kinematics involving some mechanism, [see, e.g. Rotative Demisphere by Marcel Duchamp in Kozloff 1969] and thus we saw mechanisms could be used for obtaining not an utilitarian result but an esthetical effect. Of course, fountains did that precisely for millennia. In addition to all this, there is the notion of compositional flow introduced recently [Macagno 1989d], as a counterpoint to the old notion of compositional geometry.
CONCLUSION

When Ampère, in 1834, proposed the development of an independent science under the name of kinematics, there existed already many remarkable studies of motion in a literature pool extending over more than two millennia. The kinematics studies, however, were most of the time immersed in writings about philosophy and physics (in both the old and the new acceptions). Now we have many more documents. The bibliographies by Haine and de Groot alone, have nearly ten thousand entries, and we must take into account that they are practically restricted to one branch of engineering kinematics, in which almost exclusively rigid motion is considered; an immense variety of motions is thus overlooked. There are many thousand more literature sources in the other branches of engineering, in addition to those in several sciences. Moreover, kinematics is not limited to motions in science and engineering, it finds applications in many branches of art and technology, as illustrated in my list in the chapter on applied kinematics. Therefore a comprehensive history of kinematics appears as a monumental task.

For several years I have been working in surveying the entire field of kinematics with a view to formulate an outline for a history of the subject. In this contribution I have given a summary of my conclusions, and of my views on how to proceed. I have also attempted to make a case for the writing of a comprehensive history of kinematics but I did not mean to offer a preview of the work I have undertaken. For the systematic work, I believe that one
should start with the history of theoretical and experimental kinematics, and then proceed to work on the history of empirical kinematics on one side and on that of applied kinematics on the other.

I do not believe that one man alone can do more than draw the general lines of the history of kinematics, and I hope that my initial effort be followed by other historians of science. I would like to see a number of engineers and scientists taking interest in this endeavor, because only they can tackle certain aspects of such a history. As for myself, I will endeavor to present in a few years a synoptic systematic treatment of the four stems that I have decided to introduce and describe in this contribution.
BIBLIOGRAPHICAL REFERENCES

The following references are representative and do not constitute even an attempt at being comprehensive. They are divided in sections that have been convenient in my survey of documents considered useful for my work. From my files, I made this bibliographical selection that I considered useful for further work of my own, and possibly also for the work of others. The sections are under the following headings: General, Ancient, Medieval-Renaissance, Modern, Machines and Mechanisms, Experimental Kinematics, Applied Kinematics, Review Articles and Bibliographies. These sections should be kept in mind when looking for any reference.

GENERAL

This section is not directly related to the history of kinematics; it was useful when preparing myself for work in such a history. Analytic and transformation geometry are included because I found them useful to trace studies of motion of a proto-kinematic type.

AGNESI, Maria Gaetana. 1748. Instituzioni analitiche. Milano.

An analytical geometry, typical of her time, well known in the Continent, and published in English in 1801.


Too brief, but good bibliographic references (Boyer).


See discussion of Decartes's physics, and comparison with Newtonian physics. Bachelard says that most of Decartes' physics is based on geometry and not on algebraic thinking. Bachelard says that there is a tight coupling of mathematics and physics in our times; that one can no longer think physically without thinking mathematically. I believe such coupling to be much older.


It contains a number of discussions of the role of motion in geometry.

Biased in favor of Descartes, as sole inventor of analytic geometry (Boyer).


Excellent summary from Plato to Euler (Boyer).


This is a strange book. The author seems to have been completely unaware of the kinematical aspects of mechanics, or to have chosen to ignore them. However, much valuable information can be found in this book.


EUDEMUS. *History of Geometry*. (See Lanczos).


The author remarks that his understanding of the role of hyperbolic geometry has been growing beyond the importance of the subject as a precursor of relativity theory. He considers that hyperbolic geometry can be grasped as high school geometry (p.2). In Chapter 7, the Poincaré and Beltrami-Klein models are presented to facilitate understanding. About motion, see pp. 72 (see opinion of Helmholtz), 280, 282, 295-298

Important in connection with many mechanisms and machines which are examples of empirical kinematics. It contains also interesting comments on motion, as the ones regarding the inability of ancient Greece to develop a mathematical theory of motion. [See Dingier 1952]. Important was also the reluctance (I suppose of an elite) to engage in practical applications of the theory (See quotations from Plato). See also quotations from Plutarch and Seneca. Plutarch refers to Archimedes shying from writing on technical subjects { true ? }. In a last paragraph of this chapter, Klemm quotes Guiraud [1900] who stated that he was not all that biased and that many others did manual work, and that there were workers in the assembly in Athens. See also quotations from a letter of Pascal explaining the great difficulties in finding skilled mechanics to construct his calculating machine. Klemm refers also to the difficulties found by Guerricke and his assistants. This is important as a hindrance in the accomplishment of experiments.


A reference to Eudemus, a pupil of Aristotle, who wrote a History of Geometry now lost, is given on p. 10. See also comments about motion in chapters on Einstein's theories.


MOREAU, J. 1965. L'espace et le temps selon Aristote. Editrice Antenore, Padova. (No. 4 of Saggi e Testi.)


See the account of the history of elasticity (Chapters VIII, X, XIII.) Compare with the history of fluid mechanics [Tokaty 1971].


Although, to my taste, this book has an overenthusiastic preface, I enjoyed very much reading it, and I strongly recommend it to anybody who wants to write something about non-Euclidian geometry in any of Leonardo's notebooks or similar documents. For a "classic' on the subject, see Klein 1928.

TYLOR 1870, Early History of Mankind. London.

In # 241, see figures for rotary devices to ignite wood. See also KLEMM, Kultur-Wissenschaft, vol. II and III.


An excellent book which can be very useful (together with others mentioned in these references) to anybody interested in the geometry of Leonardo da Vinci.


The chapter on kinematical preliminaries is highly recommended for anybody interested in the science of kinematics, or its history. See, for instance, the beautiful theorem of Rodrigues and Hamilton, or the fundamental theorem that Chasles rediscovered in the years that kinematics was conceived as a new science, or the fascinating connection of rotations of a solid body in space and homographies in the plane (Cayley-Klein).


**ANCIENT**

ARISTOTLE -IV C. *Problems of Mechanics*.


Boyter, p. 32-33. Curves that were described kinematically: spiral of Archimedes, quadratrix of Hippias, cissoid of Diocles (a cubic), conchoid of Nicomedes (a quartic) (These two introduced during earlier part of -2C). Much later Proclus (412-485) applied the kinematic approach to conics showing that any point of a segment with ends sliding on two perpendicular lines describes an ellipse. Perseus (uncertain period) determined solid figures by rotating a circle around an axis in its plane and then intersecting with a plane (spiric curves, oval-shaped curves, and the lemniscate of Bernoulli). The cylindrical helix was known to Geminus, Pappus and Proclus, and perhaps to Apollonius. Eudoxus apparently sought to represent the motion of the planets with the hippopede. A little later the motion of the planets was represented by Apollonius and others as combinations of circular motions; this is tantamount to construction of epicyclic curves but the curves themselves escaped interest, even the cycloid may have escaped attention (although this may be the curve of double motion that according to Iamblichus was constructed by Karpis of Antioch to square the circle.)

See also Medieval history of kinematics.


According to Dingler, there was in ancient Greece an incapacity to develop a theory of motion similar to the treatment of that of statics because of their notions of immutability and immobility of the Idea and the Form. They possessed a static concept of form.
MACHAMER, Peter, 1978, Aristotle on Natural Place and Natural Motion. *Isis*, vol. 69.


Says that geometric algebra of antiquity closely resembles use of coordinates and that Fermat was an almost immediate disciple of Apollonius and Papus. His views here conflict with those of Günther in "Die Anfänge..." (Boyer).

**MEDIEVAL - RENAISSANCE**


On p. 14, Crossby explains the meaning of analogia or proportion. Translation is given on page facing Latin original. Crossby, on p. 48: "the clear distinction which is drawn between the 'qualitative' and the 'quantitative' meanings of the term velocitas, together with a concentration upon the former, provided a most fruitful point of departure for the work of the following generation at Merton College".

The complete title of this book is *Thomae Bradwardini tractatus proportionum seu de proportionibus velocitatum in motibus*. This and other books studied by the school of Wisconsin included in this bibliography are very useful in the study of the history of kinematics in late medieval times.


CLAGETT, M. 1941 *Giovanni Marliani and late Medieval Physics*, New York


Part 1, The origins of science in Antiquity. Part 2, Science in Late Antiquity, Appendices: II. Archimedes and applications of mechanics to geometry. V-VII Epycicles, Ptolemy's descriptions of planetary motions.

Clagett completely ignores deformable bodies and fluid kinematics as part of the history of kinematics, as many others do. Regarding Leonardo da Vinci, Clagett expresses the opinion that he was not familiar with the Merton Rule for uniform acceleration (p.105).

CLAGETT, M. 1968. NICOLE ORESME and the Medieval Geometry of Qualities and Motions. (Tractatus de configurationibus qualitatum et motum) The U. of Wisconsin P., Madison. (QA 32 O 6813)


See, according to Uccelli, vol. III, p. 290 - on origins of kinematics. Uccelli refers to Ms 8680 BN, mentioned by Duhem, without saying much about it


See also The tribus predicamenti and


KARPINSKI, L.C. 1937 Is there progress in mathematical discovery and did the Greeks have analytical Geometry? Isis v. XXVII p. 46-52 (Rejects view that they had it, like Günther and contrary to Coolidge and Heath, and Zeuthen).


See John E. Murdoch and Edith D. Sylla, authors of The Science of Motion, Chapter 7. It is interesting to compare this study with those of Clagett for the same period.

LEONARDO da VINCI. (See the facsimiles of his manuscripts, of which there are publications scattered along the last century. Some new versions are coming out presently.)

LORIA, Gino, 1900. Le ricerche inedite di Evangelista Torricelli sopra la curva logaritmica. Bibliotheca Mathematica (3) I, 75-89

LORIA, Gino, 1902-1903 Sketch of the origin and development of Geometry prior to 1850 (translated by Halsted). Monist v. XIII, p. 80-102, 218-234, for anal. geom. see p. 94, note.
LORIA, Gino, 1923, Da Descartes et Fermat a Monge e Lagrange. Contributo alla storia della
giometria analitica. Reale Acc. dei Lincei Atti. Mem. della classe dio sc. fisiche, mat. e naturali
(5) XIV, p. 777-845.

LORIA, Gino, 1929-33 Storia delle matematiche. Torino, 3 vols.

(Internal Report Biblioteca d'Arte & Istituto d'Idraulica, Milano. In 1988, improved version as
a Report of the Department of Mathematics, University of Iowa).

History of Mathematics, La Crosse, Wisconsin, October 5-6, 1990.

Scheduled for publication in Raccolta Vinciana, Milano.


MARCOLONGO, R. 1932 La meccanica di Leonardo da Vinci, Atti della Reale Accademia di


MURDOCH, John E. 1967. The rise and development of the application of mathematics in

Sect. I. The continuous and the infinite. Discusses the ideas of atomists (which were not like
those of Democritus, Epicurus, Lucretius). The controversy was about mathematical atoms, not
physical atoms.

Sect. II. The quantification of motion. Shows that Brawardine "dynamics" is based on arith­
metic variation for velocities and geometric variation for the ratios of forces.

See also discussion of velocity to be attributed to a body (rigid or deformable) with non-unif­
form velocity distribution (II-B)

Also studied the problem of a long rod falling into planet's diametral tunnel; cannot go through
according to Brawardine's law. (EM: why did they not consider a needle and a magnet ?
which is immediately experimentable ?). Also interesting the use of curvilinear angles.

Velocitas and tarditas are used in medieval physics, always as scalars.

ORESME, Nicole, 1966 De proportionibus proportionum & Ad paucia respicientes. Edited by
E. Grant. The University of Wisconsin Press, Madison and London.

See in this book the different passages and discussions on kinematics. The editor considers
that Oresme has followed a traditional division between kinematics and dynamics (as described
by Clagett 1959, p. 163) At the end contains an extensive Bibliography of sources and an
Index of Latin mathematical terms and expressions. See also article in Isis by Grant 1960.

ORESME, Nicole. 1968. Tractatus de configurationibus qualitatum et motuum. CLAGETT
Nicole Oresme and the Geometry of Qualities and Motions. The U. of Wisconsin P.,
Madison 1968
There is much on kinematics in this book. Regarding Leonardo, Clagett expresses the opinion that he was not familiar with the Merton Rule for uniform acceleration (p.105).


Oresme studied the conditions for two or more bodies in circular uniform motion to conjunct in one or more specific points of one circle or in two or more concentric circles. To him the question of commensurability or incommensurability was crucial, but he left to Apollo to determine the answer. The god never came up with a pronouncement. This seems to reflect the uncertainty of the answer of a problem formulated in purely mathematical terms. The contribution to kinematics as a physical science is really difficult to establish.


Had this been a colloquium concerning a concept in geometry, there would probably have been no need to put it in the context of other sciences. This is revealing of the general notion that kinematics was not pursued for its own sake until rather recent times.


Good analysis of intellectual currents of the time.


The approaches of Galileo, Euler and Atwood are discussed. According to the author, Galileo made possible the development of mathematical kinematics.


Reti describes what he considers the contributions of Leonardo to the systematic study of mechanisms.


Contains interesting Critical Introduction and Study of the Sources. The presentation is somewhat similar to mine in the IIHR Monographs. Many passages belong to kinematics. Uccelli does not want to refer to work by Leonardo as kinematics, which is really silly; perhaps because the term did not exist at the times of Leonardo (see p. XXVII and footnote, p. CI: voler parlar di cinematica nel senso che noi oggi attribuiamo a questo ramo speciale della meccanica, non è forse il caso ). Then Clagett could not refer to Kinematics in Medieval times!


Valuable commentary of the work of the most important precursor of analytic geometry (Boyer).

THOMAS Alvarus, 1509 Liber de triplici motu, Paris.

MODERN


This is part of series edited by Truesdell, whose leadership the author follows in his treating kinematics as a geometric subject for which invariants are sought that exist under change of observers (i.e., frames supplied with a clock).


Excellent book. See Bibl. p. 97: 'the basic material on kinematics goes back to the 17th C. Full references can be found in the works of Truesdell. . ." [Truesdell 1954, 1960].


Chapter 2. Kinematics of the flow field. p. 71-130. See also Ch. 3, where elements of kinematics associated with properties of the fluid can be found. Batchelor says, e.g. (p.131): " As a preliminary piece of kinematics, we consider the changes in size and orientation of material volume, surface and line elements, due to the movement of the fluid." On p. 13, the "rates of change of material integrals are considered". An undefined property is considered; this looks to me as a study of a material geometry in motion; no dynamics is involved yet. I took this approach in my teaching. Look for isolated discussions which are essentially kinematical in the rest of this book.

See also the papers in Z.VDI 1906 on Leonardo da Vinci.


A century before, Descartes said that when a curve C rolls over curve C', the normals to the paths to all points of C pass through the instantaneous points of contact.


Like Levich 1962, this book must be studied carefully to extract the kinematics of the large variety of phenomena considered by the authors. Then it remains to trace the historical roots for each kinematics. Roughly speaking, this can be expected to have strong links with the kinematics of the continuum and of discrete systems of many particles.


"Everything that moves has kinematical aspects". Some fields of application: animal locomotion, art, biomechanics, geology, robots and manipulators, space mechanics, structural chemistry, surgery. Essentially, we are dealing with what mathematicians call transformation geometry. The authors state that when they say displacement, they imply no interest in how a motion actually proceeds: "we consider only the position before and after the motion". *(Sounds like G. Martin !. But soon in the book f(time) is introduced, and so are velocities and accelerations)*


CAUCHY, A. L. 1829. Sur l'équilibre et le mouvement intérieur des corps considérés comme des masses continues. *Exercices de mathématique*, 4. (See also *Oeuvres complètes*, v. 9.).


Three chapters are devoted to kinematics in this book. II. Cinematica del punto. III. Cinematica dei moti rigidi. XIV. Deformazione infinitesime dei sistemi continui.


See also "Random Vibration", a survey article in Applied Mechanics Reviews 12, 11. 1959


EULER, Leonard 1748. Introductio in analysi infinitorum. 2 vols. Lausanne 1748. (Also available in French and German.)


See Chapter I for basic notions of chemical kinetics.


See Ch. 3, Analysis of deformation in a continuum. Well illustrated presentation of Lagrangian and Eulerian descriptions. Strain, rate of strain, Conservation of mass in well balanced discussion.


Sections on kinematics are to be found in some of the papers. Because diffusion is considered by some, there is application of the Lagrangian description of flow in this book.


Among other topics: extension of Euler-Savary formula to 3-D space.


"The full story of the kinematics of mechanisms, doing justice to the many who practiced the art of mechanisms and contributed to the science of kinematics, is yet to be written." (p.22)

A point is always reached at which one must move from the kinematics of a mechanism to its dynamics (forces and torques involved). (p. 46-48).

Cardan joint is called Hooke joint, although none of the two was actually the inventor. Hooke put it to use in 17 C [p.50]

The authors explain that we go from kinematic chain to mechanism by selecting a fixed link. There is no term for the input or driving link, which exists in German for a mechanism in which the driving link has been selected: Getriebe Kette means chain and Mechanismus is the equivalent of mechanism. There is no acceptable translation for Getriebe although drive and train have been used. Getriebe appears as mechanism when used as a noun; when used as adjective as in Getriebelehre (Lehre = theory or science of) is translated as kinematics or mechanisms. (p.54).

See definition of jerk on p. 93

HARTMANN, W. 1890. Geometrie, Mechanik und Kinematik. VDI Z., 34.


Helmholtz after introducing new important kinematical notions, proved very important theorems on vortex flow. After Helmholtz, we can define for any fluid in motion a family of lines, the vortex lines, which accompany the fluid in its motion as a geometry that changes with time and elegantly defines one important aspect of the motion.


See 34. Rotative demisphere by Marcel Duchamp, 1925. Copper disk with metal stand and electric motor.


Characterization of the vicinity of singular points where the velocity is zero in 3-D flows. Useful for interpretation of complex flows.


The author studies a great number of processes in which motion is considered. The kinematics involved is not directly apparent, but it seems possible to extract it after careful examination. This may very well be one of the difficult tasks ahead for the historian of kinematics in a number of fields.


The historical difficulty in introducing kinematic and dynamic physical magnitudes derived from a few fundamental ones is discussed.


In this article, the notion of "rheograms" is introduced to designate symbols used by painters along several millennia. Such rheograms have analogs in the elementary components of complex flows developed only in the last few centuries by scientists.


MACAGNO, Enzo 1989a Unexplored Flow Studies in the Codex Arundel 263. IIHR Monograph No. 106. The University of Iowa, Iowa City, IA, USA.

In Appendix I (43-49) a discussion of the conservation statements is included, which may be useful for the correct interpretation of what is kinematical and what is dynamical in such statements.

MACAGNO, Enzo 1989b Leonardian Fluid mechanics in the Manuscript M IIHR Monograph No.109. The University of Iowa, Iowa City, IA, USA.

MACAGNO, Enzo 1989c Leonardian Fluid mechanics in the Manuscript I IIHR Monograph No.111. The University of Iowa, Iowa City, IA, USA. (See References for papers and other monographs by E. Macagno.)


The paper is divided in three main sections: Geometry, Kinematics, Dynamics. This was, perhaps, facilitated by the easiness in decoupling kinematics from dynamics in the problem considered.


MEYER zur CAPELLEN, W. 1933. Einfache kinematische Probleme in schulmatemathischer Behandlung. Zeitschrift. für den Mathem und Naturwissenschaftliche Unterricht, 64.


There is a great deal of applied mathematics in this history. See also later edition in four volumes. Although in need of some corrections nowadays, it is a valuable source of information.


Sect. I. The continuous and the infinite. Discusses the ideas of atomists (which were not like those of Democritus, Epicurus, Lucretius). The controversy was about mathematical atoms, not physical atoms.

See also discussion of velocity to be attributed to a body (rigid or deformable) with non-uniform velocity distribution (II-B)


As it happens often; the review of the book seems better than the book itself, valuable as it may be to explore an inhuman aspect of kinematics. See TNR July 15 & 22 1991, review by
Jackson Lear, under the title *Man the Machine*. We learn in this review that Hermann von Helmholtz who made outstanding contributions to the kinematics of deformable bodies, also contributed to the "science of work".


See the Preface for a brief history of the adoption of the point of view of Ampère in Paris and its consequences in both research and teaching. Resal attributes to Transon the notion of the rate of change of acceleration, introduced in an article published in 1845. See Transon 1845 in Part II of this Monograph.


See in Chapter 13 the definition of phase plane, and the many "flow" patterns around critical points introduced in the interpretation of the many cases of behavior of solutions of differential equations. Do not be mislead by the circumstances that some equations happen to be from mechanics; this is only incidental.


This author of a well written textbook, emphasizes the importance of learning kinematics before engaging in the study of dynamics, but he defines kinematics as the study of the motion of particles and rigid bodies. This attitude, of overlooking all other kinds of motions, seems to be typical of mechanical engineers; of course, it is not found among aeronautical and hydraulic engineers.


See Ch. IV Kinematics.


The "peculiar and characteristic glory of three-dimensional kinematics" is the subject of this treatise.

On p. 29: Continuum is a region in a Euclidean 3-D space, subject only to the proviso that the region be possessed of a positive volume. By a motion of a continuum we shall mean a one-parameter family of mappings of the continuum onto other continua. The real parameter \( t \) we identify with the time \( -\infty < t < +\infty \), \( t = 0 \) is an arbitrary initial time. At \( t = 0 \), let \( X \) be the coordinates of a typical point (or particle \( X \)). Let the motion be the family of mappings:

\[
x_i = x_i(X, t), \quad i = 1, 2, 3,
\]

or

\[
x = x(X, t)
\]


Truesdell's opinion is that it is impossible to write the history of a science unless it possesses a structure that is clear, explicit and logical. (See Preface, p. vii). Because the theory of heat was in a state of confusion, he adds, we had bad historical essays. Of course, may I add?, bad essays can also result from not having a good knowledge of a sound theory.


This book appears as very valuable for the early history of empirical kinematics. The author points, for instance, that there is information one can obtain, concerning mechanisms and
devices long disappeared, by studying their products. Such is the case of the study of ancient textiles mentioned on pp. 50-51, which reveals much of the looms in which they were woven. It remains to determine, I believe, details of the kinematics of those looms.


The very important kinematical concept of circulation was introduced by Lord Kelvin in this paper.

MACHINES - MECHANISMS


BALL, Robert S. 1876. The Theory of Screws, a Study in the Dynamics of a Rigid Body.


Bresse rediscovered the inflection circle (see de la Hire, L'Hospital).


Data on gears during the period 3000 BC to 100 BC.


He divided the parts of machines into six classes [Reuleaux, p.10-11].


Discussed inflection circle, apparently discovered by L'Hospital 1696.


These authors filled up the details of the master plan of Hachette (1806) based on Monge and Carnot's conception about the separation of mechanisms and machines. Monge had entitled the subject "Elements of Machines" (Remember Elementi machinali of Leonardo), which he intended to be equivalent to the means to change the direction of motion. Monge considered the possible combinations of rectilinear alternative and continuous and circular continuous and alternative motions. In a 2nd ed. 1829 the classes of motions (and I presume of mechanisms) were increased from 10 to 21 by adding other curvilinear motions.


Inflection circle (See de la Hire).


Showed that a coupler-bar point describes a curve of the sixth order. Roberts's theorem states that three different coupler-bar mechanisms describe identical coupler curves. There is another Roberts, Richard, who also made notable contributions, but he is not included in Hain's or in de Groot's bibliographies [see Hartenberg 1964].


TRAENKNER, G. 1953. The significance of kinematics in the design of automatic machines. Maschinenbautechnik. 2,


EXPERIMENTAL KINEMATICS


Together with the *Handbuch der Experimentalphysik* [see Schiller 1932] these two volumes constitute a valuable summary of the status of flow science after the important developments at the beginning of this century. There is a good amount of experimental kinematics in these books.


See Figs. 12-11, 12-13, 12-16 which illustrate several three-dimensional models.


Studies of motions in weaving machines.


Artists anteceded scientists by several millennia in representing elementary components of flow kinematics. The most abundant "rheograms" are those of waves and vortices. For a long time, water was really not depicted, or portrayed "realistically"; the region supposedly occupied by water was covered with one or several "rheograms".


See section on the use of experiments involving motion in transformation geometry.


Totlagen were already considered by Leonardo da Vinci in a truly remarkable page of the Codex Madrid I [Macagno M. 1991].


Among other things, kinematics of data processing.


See chapter on Kinematics, and the numerous illustrations of experimental kinematics throughout the book. Prandtl was a pioneer of flow visualization by modern techniques. In his collected works and books there is much on theoretical and experimental fluid kinematics of the first half of this century.


See comment by D. Coward in TLS, June 1-7 1990, p. 575. "The untold story of the lost inventor of moving pictures". The retina retains the image for a split second (1/10 to 1/4 s) before being stimulated by fresh light signals. In the 1820's, Faraday, Roget, and others noticed that when observed through an aperture of a rotating disk, motion could be accelerated positively or negatively, and even stopped. Some devices were invented to see figures in motion, but 'living pictures' had to wait. Projected in Paris in 1881, Muybridge's sequences of horses in motion proved painters wrong because a galloping horse is never airborne. (See Gombrich's comments in his History of Art.) These pictures came from a series of cameras, rather than a single one.


Models used by Reuleaux are illustrated throughout his book.


Four volumes of this Handbook of Experimental Physics are devoted to fluid flow. [See also Goldstein 1938]

The reader must find here and there the kinematics in this book. See, concerning experimental kinematics, Appendices I and II.


This is a new development in which kinematics plays an important role together with other sciences. According to the editor contributions to this new journal can take the form of computer animations, video recording of flow visualizations among other possibilities.

WEBB, J. B. 1883. Reuleaux's kinematic models. Trans. of the ASME.


WILLIS, ROBERT 1851. A System of Apparatus for the Use of Lecturers in Mechanical Philosophy. Especially in those Branches which are connected with Mechanism. John Weale, London.

REVIEW ARTICLES AND BIBLIOGRAPHIES

The following list is revealing of the state of the art in history of kinematics; almost all the following contributions are in the field of mechanical engineering. There seem to be no interest in the history, or the state, or the future of kinematics in other fields.


This is a good source for material prior to 1930 in the field of mechanisms.

BOTTEMA, O. 1953. Recent work on kinematics. AMR, 6, pp.169-170.


p.287. Whatever history we have in this article begins with A.M. Ampère's definition of kinematics: "the mathematical investigation of the motions that take place in mechanisms and machines and the investigation of the means of creating these motions, namely, of the mechanisms and machines themselves" (see de Jonge, ref. 70), and continues with developments in the 19 C and early 20C. Extensive bibliography. Not much of a history of the subject.
p.289. Advice of Chevyshev to Sylvester: "Take to kinematics, it will repay you; it is more
deciduous than geometry; it adds a fourth dimension to space."

DE GROOT, J. 1970. Bibliography on kinematics. I and II. Eindhoven University of
Technology. Eindhoven, Holland.

Contains about 7000 items. Initially intended for pure kinematics, it was expanded to
include kinematics applied to mechanisms. What was understood by "pure kinematics" is not
clear, but I could not find Euler, Lagrange, Truesdell (to mention just three who made impor­tant contributions) included in this bibliography. With due respect to such great an effort, I
think that the restricted area covered should have been indicated in the title.

DE JONGE, A, E.R. 1942. What is wrong with "kinematics" and "mechanisms", Mechanical
Engineering, 64.


23, 9, p. 127, 200, 202, 204, 206, 208.

FIAT Reports. HAIN, Kurt und W. MEYER zur CAPELLEN 1948. Kinematik in
Naturforschung und Medizin Deutschland 1939-1946 für Deutschland bestimmte Ausgabe der

FREUDENSTEIN, F. und K. HAIN, 1958. Der Stand der Getriebe-Synthese im Schriftum

vol 12, pp. 587-590.

GOODMAN, T.P. 1958. Der Stand der Getriebe-Analyse im Schriftum des Englischen

Purdue University. p. 94-97.

HAIN, Kurt 1959. Ungleichförmige übersetzende Getrieben. Schriftum Übersicht der


Contains bibliography with about 2000 items.


Review paper of research in sixties and seventies. Aspects of the kinematics of the intestinal tube and of the chyle are covered in this review paper. The kinematics of the wall was treated stochastically while that of the chyle was mostly assumed to be deterministic.


Many references and much historical information.


PART II
INCEPTION OF MODERN KINEMATICS

by Enzo Macagno

INTRODUCTION

Kinematics, as an independent science, begins about two centuries ago in France with a gradual change in approach to both teaching and research of Mechanics. Most of the documents in which one can trace this inception emanate from the engineering schools of that time in the French capital, but there are also documents from other institutions like the Academy of Sciences, Collège de France, etc. It is true that in Paris itself, in England, and in Italy there were previous important developments [Clagett 1959, Macagno M. 1987, 1991, and Appendix I in this monograph.] but they did not result in the establishment of a discipline with continuity and life of its own.

Kinematics, in its theoretical form, may be described as time dependent geometry, and like transformation geometry, the methods that are used are not only those of elementary geometry, but those of algebra, group theory, analytic geometry, etc. In addition, experiments are not excluded for rather complex motions, and this makes kinematics part of physics. From the point of view of the physical dimensions, geometry is concerned with length, kinematics with length and time, and dynamics with length, time and mass. One
could perhaps say that kinematics is halfway between geometry and dynamics both in their development and in their teaching. However, one must be careful not to take a simplistic view. Kinematics is not just geometry dependent on a parameter, or geometry in motion, as it may be suggested by some theoretical studies of given displacements of given figures.

In this contribution, I will trace the development of the notion and the first steps in the development of an independent new science lying in between geometry and physics but closer to the latter. It is in fact akin to a geometry in which the figures are parameter dependent, but with the added condition that the parameter is the time, and that we need to connect all the work with the requisites of physics rather than those of mathematics. Thus, although superficially kinematics may appear very close to geometry, it is part of physics without any doubt. Instead, geometry may be viewed as part of mathematics as long as mathematicians do not claim that it is very valuable in solving physical problems, because then it should be submitted to certain tests based on experiments.

Usually, Ampère [1834] is credited with having launched (in 1834) modern kinematics, but in fact he should share this credit with several of his colleagues. The trend towards the separation of kinematics from geometry and dynamics, as an independent science is noticeable at least half a century before. Ampère himself, can be seen in his writings moving in that direction years before the publication of his famous essay on the philosophy of science. I will summarize the results of examining the writings of Ampère and others.
[Bossut, Chasles, Bélanger, Coriolis, Carnot, D'Alembert, Deidier, du Buat, Francoeur, Girault, Hachette, Haton, Lagrange, La Hire, Laplace, Monge, Navier, Parent, Poinsot, Poisson, Prony, Resal, Saint Venant].
KINEMATICS BEFORE AMPÈRE

As stated in Part I of this monograph, the study of motion is very old, and the first documents go back more than two millennia. But whatever kinematics existed until late medieval times and until Leonardo da Vinci, it was always integrated into some other discipline. Even the name, kinematics, only appears a little more then a century and a half ago. In Part I there is enough information on previous developments by the schools of Oxford and Paris, and by Leonardo; there is therefore no need to say more at this point.

In the seventeenth and eighteenth centuries, there were many contributions to mechanics which necessarily contained in some cases important aspects of theoretical kinematics, either because their authors had no alternative than ignoring the forces involved or saw great advantage in beginning by a study of motion in a purely geometric manner, without considering the causes. A very good example of the latter case is found in some of Euler's writings. But neither himself nor colleagues and followers thought of an independent discipline. Euler wrote that

*The investigation of the motion of a rigid body may be conveniently separated into two parts, one geometrical, and the other mechanical. In the first part, the transference of the body from a given position to any other position must be investigated without respect to the causes of motion, and*
must be represented by analytical formulae, which will define the position of each point of the body. This investigation will therefore be referable solely to geometry, or rather to stereotomy.

It is clear that by the separation of this part of the question from the other, which belongs properly to mechanics, the determination of the motion from dynamical principles will be made much easier than if the two parts were undertaken conjointly.

Although not so clearly expressed, D'Alembert shared similar ideas, as also did Kant according to Hartenberg [1964]. D'Alembert, in his *Dynamique*, stated that all that we can see without doubts in the motion of a body is that it traverses a certain distance in a certain time. He thought that one could derive from this simple fact all the principles of dynamics; in this he was less accurate than Ampère who put the pure study of motion on a less ambitious but firmer base. Here is D'alembert statement:

*Tout ce que nous voyons bien distinctement dans le mouvement d'un corps, c'est qu'il parcourt un certain espace, et qu'il emploie un certain temps à le parcourir. C'est donc de cette seule idée qu'on doit tirer tous les principes de la mécanique, quand on veut les démontrer d'une manière nette et précise; ainsi on ne sera pas surpris, qu'en conséquence de cette reflexion, j'aie, pour ainsi-dire, detourné la vue des dessus les causes motrices, pour n'envisager uniquement que le mouvement qu'elles produisent.*
Indeed, D'Alembert can hardly be called a modern kinematicist although there is in him an evident recognition of how powerful can be a deep understanding of motion. D'Alembert's principle for dynamics appears like a conception in which one uses kinematics to reduce dynamics to the classical statics' methodology.

In contrast, if we examine the famous Mécanique Analytique by Lagrange [1788], we find that the approach is the one that gives to the forces the primordial role. Statics, taking about one third of the volume, includes hydro- and aerostatics. The author states, in his Avvertissement, at the beginning of the book, that he deals in the second part with la Dynamique ou la Théorie du Mouvement, but there is no attempt made at separating the study of what we call now kinematics and dynamics. Of course, Lagrange's is not the only book to take this approach which seems to be quite common during the eighteenth century, but the contrast with Euler is quite interesting. Incidentally, it is in this Avvertissement that Lagrange boasted that On ne trouvera point des Figures dans cet Ouvrage.

For a historical study, the Mécanique Analytique constitutes an example of the methodology discussed in Part I, when it becomes necessary to trace the kinematics submerged in a document. As soon as one begins the reading of the first part of this book, one discovers that for Lagrange motion actually plays an important role in Statics. On page 2 of his treatise, he states that there are three principles in statics: 1. that of the equilibrium of the lever; 2. that of the composition of motions; 3. that of the virtual velocities. I have
underlined two key words because they are extremely important to understand the position taken by Ampère. Statics may be about systems that appear to us as static, but the theory of this discipline requires that we have a knowledge about motion.

Most of the teachers and writers of textbooks in the field of mechanics (either theoretical or practical) of the times of D'Alembert were people with experience in research. Thus two genuine sources of innovation were within them. They came to realize that for both teaching and research a separate study of motion and its general independent properties were essential. Ampère himself surely went through such a process, as reflected in his comments concerning kinematics in his *Essai*. His proposal was the result of experience, very well conceived and founded, and it was accepted by a number of contemporaries and by posterity.

Although at that time, mechanics was being taught and developed in several other places than Paris, the inception of modern kinematics appears to be essentially a French contribution, at least in the sense of actually undertaking the writing of essays and books on the subject, and clearly defining the purpose and goals of the new discipline. Of course, much of kinematics already existed incorporated in a large number of memoirs, papers, and books. In Part I, I have already described, however succinctly, the beginnings of mathematical kinematics in the astronomical works of Greece and Mesopotamia, and those of empirical kinematics much before since the dawn of civilization in
different lands. But all that kinematics was not recognized as a separate body of knowledge.

It is interesting that at the very moment when the formulation of physics in general and mechanics in particular was reaching a level that for macrophenomena is still generally valid today, and when certain sections of those sciences could have been very well absorbed into a more general scheme, as simple links of a well set methodological chain, mechanicists began to realize the need for kinematics to become an independent discipline emerging with undeniable force.

One aspect that is common to several authors of the period I am describing is that they grew uncomfortable with the introduction of forces ad-hoc in their studies of motion, and wished they could leave out what appeared to them as metaphysical considerations concerning mysterious causes of motion. After all, even today the teacher of mechanics does not have a way of rationally introducing the gravitational force! And pressure in fluid mechanics is also introduced quickly and empirically (although one could make, at least for gases, a simplified appeal to the kinetic theory).

At the close of the eighteenth and beginning of the nineteenth century, the École Polytechnique saw a band of teachers, who were very innovative, change the approach to the teaching of several subjects, among which theoretical and applied mechanics received, nor without controversy, an extraordinarily progressive reshaping and a revolutionary approach. Referring to the ele-
merits of machines, Monge proposed that the corresponding course dealt with the means by which the direction of any motion are changed. He explained this by saying that teaching should aim at showing how motion along a straight line, rotation around an axis, and back and force motion could be transformed one into the other. He proposed that, since machines embody the result of combinations of certain motions, a complete enumeration of such motions be established. Hachette prepared a chart in which he included illustrations showing, for example, how circular continuous motion could be transformed into rectilinear alternating motion. This chart was presented in 1806, and it was followed by a book in 1811. By this time, Lanz and Bétancourt had already published *Essai sur la composition des machines* [1808]. Borgnis (Italian engineer and professor at the University of Pavia) also proposed a classification system that Coriolis simplified. These were works that are important in the genesis of the notion of kinematics as an independent science, but are still efforts to modernize engineering mechanics rather than launch a new science. Monge and also L.N.M. Carnot had a direct influence in those efforts, especially through their teaching and the shaping of the Ecole Polytechnique.
AMPÈRE's CONTRIBUTION

In the preface of his Essai sur la philoshopie des sciences, or Exposition analytique d'une classification naturelle de toutes les connaissances humaines, Ampère [1834] stated that already in 1829, when preparing his course to be delivered at the Collège de France, he was considering two important questions. The first was a definition of general physics including how to distinguish it from other sciences. The second was about the different (both existent and inexistent) branches of physics.

"En 1829, lorsque je préparais le cours de physique générale et expérimentale dont je suis chargé au Collège de France, il s'offrit d'abord à moi deux questions à résoudre:

1o. Qu'est-ce que la physique générale, et par quel caractère précis est-elle distinguée des autres sciences.

2o. Quelles sont les différentes branches de la physique générale ainsi circonscrite, qu'on peut considérer, à volonté, comme autant des sciences particulières, ou comme les diverses parties de la science plus étendue dont il est ici question?

The book makes interesting reading even today. Ampère, apparently knew his Greek well enough to discuss his choice of new names for sciences still without a name. One of them was kinematics. He started with κίνεμα, meaning motion, and formed the adjective κίνεματικός, from which we
have now kinematics. The reader will excuse me if I do not include the correct accents in these two Greek words. My point is to show a bit of Ampère’s erudition, rather than my own.

A word is necessary to prepare the reader for the hierarchy adopted by Ampère, when he considered sciences of different order. I believe this will be obvious if one of his tables is shown:

<table>
<thead>
<tr>
<th>Sciences de 1ère ordre</th>
<th>Sciences de 2ème ordre</th>
<th>Sciences de 3ème ordre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mécanique. élémentaire</td>
<td>Cinématique</td>
<td></td>
</tr>
<tr>
<td>Statique</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dynamique</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mécanique transcandante</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MÉCANIQUE</td>
<td>Mécanique moléculaire</td>
<td></td>
</tr>
</tbody>
</table>

Ampère also mentions that long time before writing his Essai he had noticed that all books on mechanics omit, at their beginning, general considerations relative to motion. Such considerations should constitute a science of the third order. In partial form something of this kind has been done by authors like Carnot in his writings about motion considered geometrically, and in Lanz and Betancourt’s *Essai sur la composition des machines*. 
Referring to kinematics, Ampère argued that kinematics should comprise all that there is to be said about the different kinds of motion, regardless of the forces that may produce them. He added that it must cover all the considerations about spaces traversed in different motions and about the times employed. It should deal with the calculation of the velocities attained depending on the functions relating space and time in each case. Then the different instruments, or mechanisms, that can change one motion into another should be studied. The definition of machine should be changed to say that they are instruments or devices by means of which one can change the direction and the velocity of a given motion, instead of continue to say that a machine serves to change the direction and intensity of a given force [For as collection of definition of machine, see Reuleaux 1876, Note 7]. Here are the concepts of Ampère:

_Cette science, doit renfermer tout ce qu'il y a dire des différentes sortes de mouvements, independamment des forces qui peuvent les produire. Elle doit d'abord s'occuper de toutes les considérations relatives aux espaces parcourus dans les différents mouvements, aux temps employés à les parcourir, à la determination des vitesses d'après les diverses relations qui peuvent exister entre ces espaces et ces temps. Elle doit ensuite étudier les différents instruments a l'aide desquels om peut changer un mouvement en un autre; en sorte qu'en comprenant, comme c'est l'usage, ces instrumens sous le nom de machines, il faudra définir une machine, non pas comme on le fait ordinairement, un instrument a_
l'aide duquel on peut changer la direction et l'intensité d'une force
donnée, mais bien un instrument à l'aide duquel on peut changer la direc-
tion et la vitesse d'un mouvement donné.

He added that the consideration of forces will only serve as a distraction for whoever tries to understand the mechanisms involved. He gave the example of the mechanism in a watch which determines the same ratio of velocities no matter how it is driven, by the watch motor or by hand. He also said that a treatise on the subject serait d'une extreme utilité dans l'instruction; the reason being that the student would overcome the difficulties in understanding mechanisms without the added hindrance of having to study also the forces involved.

Ampère insisted that kinematics must deal also with the ratios of velocities of different points of a machine, and more generally of any system of material points. This was related to the determination of the virtual velocities. Ampère believed that the teaching of the principle of virtual velocities, usually hard to grasp by the students, would become free of difficulties if they were already familiar with the kinematical aspects. Maybe he was referring to his experiences as a teacher, when he stated: ne leur présentera plus aucune difficulté.

In Ampère's plan, after kinematics, one would continue with the study of statics, where he says that one considers the forces independently from motion. Statics must come only after kinematics because motion is of imme-
diate perception and knowledge, _tandis que nous ne voyons pas les forces que produisent le mouvemens observés._

After studying motion without forces, and forces without motion, one should attack the problem in its totality; il reste à les considérer simultanément, à comparer les forces aux mouvements qu'elles produisent, et à déduire de cette comparaison les lois connues sous le nom de lois générales du mouvement, . . . . Thus, the purpose of dynamics is defined; it must establish general laws, and use them to predict motions given the forces, or serve to find the forces, given the motion

**EMBRANCHEMENTS SOUS-EMBRANCHEMENTS**

**Sciences de 1ère ordre**

Arithmologie

Mathématiques proprement dites

Géometrie

**SCIENCES**

**MATHEMATIQUES**

Mécanique

Physico-mathématiques

Uranologie

He added that Uranology should comprise Uranography, Heliostatics, Astronomy, Celestial mechanics.
AFTER AMPÈRE

The idea of Ampère of creating an independent science dealing exclusively with the study of motion was well received by many; in fact it was already in the making by different people in 1834, when he published his *Essai*. Some did not understand Ampère and his colleagues who never conceived kinematics as not supplemented by whatever other branches of science had to be applied, in the same way that the creators of geometry never thought that their science would alone suffice to deal with problems beyond geometry, like optical perspective, for example, or geodesics, or celestal mechanics. Astronomy with only geometry would never have gone beyond the Ptolemaic approach. Among the critics of the approach we have some French professors and also Franz Reuleaux in Germany. Among those who misunderstood his idea there are also some in other countries. Reuleaux [1876] expressed criticism towards "pure" kinematics, without realizing that by definition it was as "pure" as geometry. He thought that true kinematicists fell into Redtenbacher's nihilism, and cut 'Cinématique pure' from "Cinématique appliquée" (Redtenbacher was Reuleaux's teacher at Karksruhe, where an engineering school had been established taking the École Polytechnique as a model. ) . Reuleaux criticized Resal's *Cinématique pure* as an example of the sublimation of problems of kinematics into those of pure mechanics [Reuleaux 1876, p. 16]. In this passage, obviously, Reuleaux refers to his kind of kinematics (the restricted one of the mechanical engineer) and not the generalized one conceived by Ampère.
Concerning Ampère’s proposal, Reuleaux [1875] considered the year 1830 as one that saw a great change, but I hope I will be able to show that such a change had been in the making for several decades before the *Essai* of Ampère. I believe that Reuleaux never fully understood the long term scope and also the limitations of Ampère’s proposal, and thought that Kinematics should only be concerned with mechanisms and machines and not be an *absolutely isolated science*. In addition to kinematics, dynamics and other disciplines, are essential for Reuleaux’s purposes, which are much more restricted than those of Ampère who was not propounding what we call an engineering science but a new scientific discipline. When Alex B.W. Kennedy [1876] was confronted with the choice of the title to be given to the translation of the famous book by Reuleaux, he opted for *Kinematics of Machinery*, and not for a literal translation of *Theoretische Kinematik*, which would have been really misleading. By 1876, it was already obvious to everybody that, in less than half a century, the idea of Ampère had prevailed without distortion.

After Ampère, kinematics was gradually accepted as a new discipline, and developed to ever higher levels. According to Resal, Poncelet in his lectures at the Faculté des Sciences in Paris, from 1838 to 1840, put into practice Ampère’s plan. Among the important contributions of an early period we must recognize those of Chasles and Poinsot both graduates of the École Polytechnique who studied rigid geometric bodies in motion. Of course, the notion of instantaneous center had already been introduced by Johann Bernoulli [1742]. But there were also important developments in the kinematics of elasticity and in that of fluid mechanics.
In the Avant-Propos of his Cinématique, Bélanger [1864] still finds necessary to make clear what is the role of kinematics. In a comment to Laboulaye's Traité de Cinématique, he stated that one should consider kinematics as a science possessing its own theories, independently from dynamics and from the knowledge of the physical and experimental properties of the moving body. Bélanger, who taught at the École Polytechnique from 1851 to 1860, compares algebra and geometry in an analogy with kinematics and dynamics, arguing that they can be taught separately but they do not need to be imparted at sequential times. He mentions that this is precisely what is done successfully at the École Centrale.

The movement initiated in Paris soon spread over other countries, and soon papers and books appeared there. [Willis 1841 in England and Giulio 1847 in Italy], but this study is limited to the inception period. Anyway, for some time it was in France where most of the activity resided. [Chasles, Poinsot, Girault, Haton, Bélanger, Poncelet, Navier, etc. [See Reuleaux 1876]

In his preface to the English version of Reuleaux's book, Prof. Alexander Kennedy refers to Willis 1[841] as a treatment in which motion is considered merely for its own sake, without reference to force or time. If time is excluded, this should not be considered as a book on kinematics. Kennedy says also that later writers did not carry the analytic process further. I presume he refers to English books and articles only.
CONCLUSION

Even after the preceding narrative, the idea of studying thoroughly kinematics without consideration of the causes of motion, before one studies dynamics, or even statics, may still appear to some readers as putting the cart before the horse. It may seem that to study motion *per se* was unavoidable to the astronomers of ancient times or to the engineers that were then developing different kinds of mechanisms and machines, but why such an approach should be continued after the equations of physics were finally brought to a reliable complete system, at least for macro-scale phenomena ?. However, if one considers problems beyond the elementary ones, the need for studying kinematics first and then dynamics becomes obvious in many cases. It is true that the historian of science, who is not at the same time a well trained physicist or engineer, will have difficulty in discovering this point, but if one is familiar, for instance, with Eulerian and Stokesian fluid mechanics, the point emerges spontaneously with great clarity and force. Euler's equations for hydromechanics (which are equations for inviscid fluids) could be formulated with little general notion about motion and forces, but the forces due to viscosity could not be easily expressed. The complete constitutive equations for viscous fluids are not easy to guess before the kinematics of rates of strain has been developed. It is true that Newton gave a formula for an extremely simple situation, but to do that also required a basic understanding of the kinematics of fluids. The difficulty becomes monumental for fluids that are non-Newtonian in their rheological behavior, and it took a great effort to achieve
general and intrinsically valid formulations for non-Newtonian fluids. In such efforts, kinematics based on tensorial calculus was of paramount importance. There should be no doubt that Ampère was in the right track in propounding kinematics as an autonomous science. I am sure that those who know relativity better than I, would make a point similar to the above one to show how important was a deep knowledge of theoretical kinematics in the formulation of Einstein's theories.

The great historical role of the study of motion in all its aspects has been overlooked perhaps because the notion of kinematics as an independent science has been perceived and emphasized only in relatively recent times thanks to the simple and at the same time great conception of Ampère. We can draw a parallel between the conceptions of Euclid and Ampère who, separated by more than two millennia, tried to separate the mathematical and the physical aspects of two sciences, although one undertook to summarize all the knowledge available while the other did not. But what one perceived about the great value of studying geometry *per se*, it was equally perceived by the other regarding motion *per se*. In the same way that the history of geometry goes back to times much before Euclid, that of kinematics also goes back to much earlier times than those of Ampère. We only need to think that the motion of stars and planets was studied for millennia without regard to the forces involved, to realize how old kinematics really is.

In a future more detailed study, I plan to give more coverage to precursor movements of that of Ampère and his colleagues. I think that one should
include with enough detail at least those of the kinematicists of Oxford and Paris and the kinematical studies of Leonardo da Vinci as foreshadows of modern kinematics. Most of the Medieval kinematicists were truly unconcerned about the causes of motion. Leonardo, at least in part of his studies, clearly dealt with the study of the geometry of motion and not with dynamics, following in fact what is now Ampère's approach.

I feel, because of questions and comments at the occasion of lectures I have given on the subject of this monograph before historians of science, that I must say a few words about the rationale of my study of the history of kinematics and about the role of kinematics itself. Many consider geometry and dynamics as subjects that are well and alive, but regarding kinematics there is a general notion that it is a closed chapter in the development of physics. The truth is that in several aspects it is still advancing and the object of intense research efforts in science, engineering and art. One striking example is the great flurry of activity in flow visualization only matched by computational kinematics. The history of modern kinematics has not been written except for some partial aspects mainly related to mechanical engineering applications. It will not be an easy task to do it in a comprehensive way.
ACKNOWLEDGEMENTS

This contribution (Part II) is mainly based on research carried out during two periods in Paris in the springs of 1990 and 1991. The most helpful institution was the École Polytechnique in whose modern and modernly-run library I had the opportunity to collect a large amount of very useful information in a rather short time. I express here my heartfelt thanks to Mesdames Masson and Billoux and the helpful staff under them.
This list contains more items than those mentioned in the text, because in this way some indication of the repercussions of the Paris movement can be surmised.


This is certainly a contribution to the science of kinematics, a decade before the proposal in the Essai sur la philosophie des sciences.


In the Preface of this sixth edition of his treatise, Bélanger acknowledged the leadership of Ampère. There is a first part on general theoretical kinematics, while the second is devoted to kinematics applied to mechanisms and machines. There are also some notes of works preceding his: Hachette and Lanz & Bétancourt in 1808, Poncelet (2nd. edition) 1845, Willis 1841, Laboulaye 1849.


Whatever history we have in this article, begins with A.M. Ampère's definition of kinematics and continues with developments, in the 19 C and early 20C, in mechanical engineering kinematics. Extensive bibliography. On p.289: Advice of Chevyshev to Sylvester: "Take to kinematics, it will repay you; it is more fecund than geometry; it adds a fourth dimension to space."


In their Preface, the authors state that kinematics has to do with everything that moves. In spite of that, and of listing many areas of application of kinematics, they seem blind to the existence of certain areas in science and technology in which deformation and flow occurs, from meteorology to blood circulation, and from stellar dynamics to Brownian motion.

See historical notes.


We must take into account that a century before, Descartes said that, when a curve C rolls over curve C', the normals to the paths to all points of C pass through the instantaneous points of contact.


The first part of vol. I is on kinematics.


According to C. C. Gillispie, this is an application of the science of motion to the study of the principles governing the operation of machines, and it was written as what we call now an engineering science. Carnot introduced the idea of "geometric motions", a kind of proto-vector analysis.


First edition in 1783, under the title *Essai sur les Machines en général*. In the Preface of the second edition (1803), Carnot discusses his approach by saying that there are two possibilities: either to consider mechanics as the theory of the forces which produce the motion, or as the theory of the motions themselves. He adds that the first approach is followed almost by everybody. We have, in this preface, a recognition of a new approach.

CAUCHY, A.L. 1827 *Exercises de Mathématiques*, II. Paris, (See also *Oeuvres*, (2), VII, p. 94.)


According to Bricard and Whittaker, Chasles rediscovered a theorem already established by Mozzi 1763. (See also Cauchy 1827),


Biased in favor of Descartes, as sole inventor of analytic geometry (Boyer). But regarding kinematics there are very useful passages. He refers frequently to kinematics, mechanical construction of curves, etc.


Clagett deals with the origins of kinematics, but he completely ignores the kinematics of deformable bodies as part of its history, as many others do. Regarding Leonardo da Vinci, Clagett expresses the opinion that he was not familiar with the Merton Rule for uniform acceleration (p.105), but he overlooks most of the kinematical work of Leonardo.


FOURIER, J. B. Leçons d'analyse et de mécanique professés à l' École Polytechnique. Ms. de l'auteur.


Contains facsimiles of unpublished writings on mechanics and calculus, and interesting comments and discussions.


Contains Halphen's theorem on the composition of two general displacements (See also proof by Burnside, Mess. of Math. XIX, p.104, 1889).


Chapter 1: An Outline of Kinematics to 1900 This chapter is excellent but the authors vision is very narrow; they see the history of kinematics as that of the growth of machines and mechanisms and the related mathematical developments. On p. 22: "The full story of the kinematics of mechanisms, doing justice to the many who practice the art of mechanisms and contributed to the science of kinematics, is yet to be written". See also Ch. 3 on kinematic models, which includes some historical data. See also Reuleaux.

Great historical figures listed in Ch. 1


In his Avvertissement for this edition, Lagrange stated: On a déjà plusieurs Traités de Mécanique, mais le plan de celui-ci est entièrement neuf. He could not foresee that not many years afterward there would be still more innovative approaches under the leadership of Ampère. From another statement in this Avvertissement is evident that his innovation was to convert mechanics into algebra rather than geometry: On ne trouvera point des Figures dans cet Ouvrage. Les méthodes que j'y expose demandent ni constructions, ni raisonnements géométriques ou mécaniques, mais seulement des opérations algébriques, ... (underlining added). He ends by claiming that he has made mechanics a branch of (mathematical) Analysis.

They implemented the master plan of Hachette (1806) based on Monge and Carnot's conception about the separation of mechanisms and machines. Monge had entitled the subject "Elements of Machines" (Remember Elementi machinali of Leonardo), which he intended to be equivalent to the means to change the direction of motion. Monge considered the possible combinations of rectilinear alternative and continuous and circular continuous and alternative. In a 2nd ed. of 1829 the classes of motions (and I presume of mechanisms) were increased from 10 to 21 by adding other curvilinear motions.


Nearly a century after Lagrange, Moigno sticks to the Lagrangian approach. In another contribution, more will be included about this book; suffice it to quote one paragraph from the long Preface of this book: On trouvera étrange que le nom de M. Poncelet, le législateur en France de la Mécanique Appliquée, ne soit pas prononcé dans cet ouvrage; cela vient de ce que je dus faire ici de l'analyse, de l'équilibre et du mouvement virtuel, tandis que Poncelet est le chef d'école de la synthèse, du mouvement et du travail.


NAVIER, Louis M. H. 1831-34. Leçons de Mécanique donnés à l'École Polytechnique. 1ère et 2ème Années.

Navier did also research into the fundamental theory of viscous fluids, and there he proved to be a powerful kinematicist when formulating equations which were also established by Cauchy, St. Venant and Stokes; they are now called the Navier-Stokes equations. In spite of this, his approach as a teacher was rather conservative, compared with that of St. Venant.


From Preface: . . . . *je ne me suis assujetti exclusivement, ni à la méthode synthétique, ni à la marche analytique*. . . . Poisson took the same approach in the 1833 edition of this book, i.e. he does not appear to have had early ideas about kinematics as an independent science.


PRONY, R Cours de mécanique à l'École Polytechnique. Ms École des Ponts et Chaussées, Paris.

PRONY, R. 1810-1815. Leçons de Mécanique Analytique données à l'École Impériale Polytechnique, Imprimerie de l'École Impériale des Ponts et Chaussées, Paris.


See the Preface for a brief history of the adoption of the point of view of Ampère in Paris and its consequences in both research and teaching.


Of historical interest are the Introduction pp. 1-25, and Ch. VI, Sketch of the History of Machine Development, pp. 201-246.


See annotations for Navier 1831. The title of Chapter 2, Cinématique, ou étude géométrique du mouvement, is revealing of St. Venant's approach.


TRANSON, 1845. Journal des Mathématiques pures et appliquées. (See Resal 1862).

WILLIS, 1841 Principles of Mechanism. London/
Willis' approach differs from that of Monge. The scheme of Lanz, *notwithstanding its apparent simplicity*, must be considered *a merely popular arrangement*. Reuleaux [1876] is somewhat critical of Willis, especially of his idea of considering only rigid elements and exclude water-wheels, windmills and other fluid machines.

**Fascicule Quinquenal de la Société Amicale des Anciens Elèves de l'Ecole Plovtechnique.**

From the 1973 issue, I have extracted the following list of teachers of Mechanics at the E. P. from 1794 to 1903. I believe that it speaks for itself, and that it may be very useful for the study of developments affecting kinematics during the 19th century. I have not been able yet to compile a list of those who taught courses on mechanisms and machines.

APPENDIX I

LEONARDO DA VINCI's
SCIENTIFIC AND TECHNICAL KINEMATICS
by Matilde Macagno*

Leonardo's studies in geometry as a part of physics have been only superficially explored although there are a good number of investigations published regarding classical aspects[see, e.g. Marinoni1[982]. The most interesting notes and drawings on geometry left by Leonardo are those in which he considers motion as a means of deriving geometrical results, and geometry as means of studying motion itself. There is a full spectrum of notes going from purely geometrical questions [See, e.g., Leonardo's Theorem in Martin 1982], to configurations in art and science [Macagno, M. 1987], and to the analysis of engineering kinematical problems [Macagno, M. 1988]. Leonardian kinematics comprises from the study of motion of a single material point to that of the turbulent flow of fluids, passing through questions of rigid body motion and performance of connected rigid and flexible elements.

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Leonardo's notes on geometry in motion and scientific and technical kinematics are spread in the thousands of folios he left in the hands of his last disciple, Francesco Melzi, when he died in France in 1519. Of those manuscripts only about half are now available. (the originals are in Italy, France, Spain, England and USA). In most of the extant manuscripts one finds an important part devoted to questions of motion. He dealt with such questions from different points of view and with varying methodology, depending on Leonardo himself approaching the topic in question as an artist, or an engineer, or a scientist. For this contribution, I have chosen a number of representative accomplishments in his studies of motion. Presently I will offer only a summary; a paper has been accepted already for publication with a more comprehensive treatment, including the Mathematical Laboratory studies I have performed to gain insight into the work of Leonardo [Macagno M. 1988].

1. Leonardo applied the finite motion of figures and its parts to gain knowledge about the equivalence of areas and volumes of figures of different shape. This became important for science and engineering when he related this result to what we call now the equation of continuity. This is in fact a misnomer and a more adequate name would be the equation for conservation of volume.

2. Although Leonardo did not recognize the existence of the instantaneous center of rotation in plane motions of figures, that was going to be discovered by J. Bernoulli, he was aware of the relation between the velocities of two points in rigid-body motion, as shown by his study of the classical connecting
rod mechanism. This finding was, perhaps, more the result of tactile than visual experimentation, as demonstrated by a laboratory study, in which a mechanical model is operated by the two hands, one driving a nearly uniform circular motion and the other, as passive as possible, following the motion of the other end of the rod.

3. An interesting study of Leonardo comprises the analysis of the motion of string and pulleys in a variety of tackles (systems of pulleys, strings and weights). His motivation was the statics of these systems, but he did it by studying their kinematics, believing that the ratios of forces was related to that of displacements.

4. Another important result is that of the basic elements of what we call now the theory of deformable bodies. Leonardo was really assuming that the transformations were linear (linearization is an assumption or an approach often present in Leonardo's notes). His sketches of deformation of plane figures are strongly reminiscent of the theory developed centuries after by Helmholtz. (See the notable sketches in Folio 72R of Codex Madrid II).

5. I favor the hypothesis that Leonardo owed much of his deep understanding of transformation geometry to his apprenticeship in Verrochio's art shop, and to the work with his hands in his own studio. But there is also the evidence of a profound interest in geometry as a science (which he seems to have had difficulties in learning as handed down by tradition) that he appears to have mastered when approached in his own way, driven by his own motivations. I
think that there is a link between the almost obsessive study of the subdivision of figures of different kinds, as attested, for instance, by a number of sheets in the Codex Atlanticus, and his advanced understanding of the geometry in motion of deformable bodies. I find that is fortunate that one of the sheets assembled by Pompeo Leoni in Codex Atlanticus (CA 602R) should contain an extraordinary depiction of deformation in the inner space of a figure, thus offering a proof of how far Leonardo went in his exploration of the geometria che si prova col moto.

In closing this contribution, I want to share some of the insight I have gained on Leonardo and his studies of motion. I see him in three phases that may have happened all nearly at the same time. I believe that he began with some study of elementary geometry, apparently not very successful, but from there he developed what seems to be an obsessive interest in the figures of equivalent area and volume, and then moved from static ways of doing this work to displacement and motion of figures. Thus he developed a geometry which he called appropriately kinematical geometry. To this way of approaching geometry surely contributed also his ability, as an artist, to take a portion of any material and shape it in any desired form. The frozen notion of fixed form prevalent in his inherited knowledge of geometry was thus replaced by the study of changing form, either by steps or in a continuous process. The second phase is Leonardo's strong interest in the subdivision and aggregation of figures, which one cannot separate from his artistic motivations, but that is intrinsically a geometric procedure and it was for him an important source of ideas and also a tool. We must think that, if one deforms any of
these complex figures, one visualizes the internal changes together with those of the perimeter, and this must have played a great role in his studies of the geometry of deformation. Finally, the engineer in Leonardo was concerned with curves described by moving particles and bodies, by parts of different mechanisms, and by the amazingly complex families of curves that water constructed before his curious eyes. But this was also a source of geometry in motion for him, because, in turn, only geometry could provide a frame to arrive at some understanding of flow phenomena. Thus Leonardo must have learnt from art, nature and technology, a geometry much more interesting than that of Pacioli and, closing the circle, used it to study complex phenomena of all kinds. Leonardian geometry is not only geometry done with motion is also the geometry of motion.
REFERENCES


