LEONARDIAN FLUID MECHANICS
IN THE MANUSCRIPT G
ENZO MACAGNO

"GEOMETRIA CHE SI FA COL MOTO"
IN LEONARDO's Ms G
MATILDE MACAGNO

IIHR Monograph No. 114
Iowa Institute of Hydraulic Research
The University of Iowa
Iowa City, Iowa 52242-1585
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LEONARDIAN FLUID MECHANICS
IN THE MANUSCRIPT G

INTRODUCTION

General Remarks

This is the fourteenth IIHR Monograph I have written on the Leonardian science of flow and transport phenomena. The Manuscript G contains a good number of notes and drawings on fluid mechanics, hydraulics, aerodynamics, and related subjects. Regarding the format and the history of the Ms G, the reader can find an excellent section in the introduction to the transcription of this manuscript published by Prof. A. Marinoni [1989]. Marinoni suggests that the Ms G was written in between 1510 and 1515. He states that, together with the Mss E, F, G, L, and M, and the Volo degli Uccelli, the Ms G belongs to a group of manuscripts we have inherited very much as they were in the hands of Leonardo. In the Ms G, the folios 7, 18 and 21 have been missing for a long time.

When one holds in his hands the manuscripts of Leonardo da Vinci - or what is left of them - the feelings of direct contact and access may be overwhelming. On the one hand, one knows that these documents are free from the pitfalls of transcription, translation, editing, printing, etc. Once Leonardo expressed his feelings about being alone: Quando sarai solo sarai tutto tuo. That is the way we find him in his manuscripts, free from the
interference of any other person. On the other hand, had Leonardo arrived at the point of having some of his writings in print, such books would not be as close to him as his notebooks are, although in a more refined form. Anybody who has published his own work, and studied papers by others, knows very well that much is discarded concerning the inner thinking of any author, either by his own hand or by those of editors and reviewers. Not that the printing in itself would be the cause of this, because Leonardo's writings cannot be compared either with the printed books of his time or with those that existed before in handwritten form; these were also edited carefully and expurgated from trial-and-error steps, doubts, vacillations and, if possible, also from inconsistencies and contradictions.

It is true, however, that in Leonardo's notebooks we find plenty of problems due to the lack of organization of such manuscripts, which could have been avoided in a printed book. He was aware of this hindrance; thus, referring to what we call now the Codex Arundel, he noted that it was a collection without order, made up of many sheets which I have copied here, hoping afterward to arrange them in their proper places according to the topics covered in them. Zubov [1968] has discussed this aspect of the manuscripts and its meaning to the student of Leonardo. Garin [1952] considered the notebooks of Leonardo to be the result of the intensively lived days of an unusual man and never like the fragments of a book or material for a book.

In the case of Leonardo, perhaps due to the difficulties involved in handling the original form of his writings, there has been an interesting
development. We know that some students of the past must deal with an additional cause of remoteness and barriers, when they must rely only on the doxographers; such is the case, for instance, for students of the predecessors of Aristotle [Taylor, 1989]. Paradoxically, many writers have written about Leonardo after reading only those whom we can call the "doxographers" of Leonardo. What I mean is that the writings of Leonardo have been transcribed in either diplomatic or critic ways, or summarized in anthologies. It is not difficult to detect in any paper or book, whether the author has relied on first- or on second-hand sources. One notorious case of trusting in anthologies appears to be the famous essay *Leonardo da Vinci and a Memory of Childhood* by S. Freud [1966], in which Freud appears to have used the translation into German of a passage in the notebooks of Leonardo in which the name of a rather small bird was rendered as "vulture". Freud jumped to a number of conclusions precisely because of believing that the bird was a vulture. Even if the essay contained psychiatric lucubrations that were clever and valid, it seems unbecoming that a man of such international culture, who visited Italy repeatedly, would not care much to quote from the original words, and to make sure that he knew exactly what they meant [See, e.g., the criticism of Peter Gay 1989]. One point that I have not seen made about this mistake of Freud, is that the famous passage could be one of those that Leonardo copied, or summarized, from a reading, and therefore not directly connected with his childhood. This is one of the serious pitfalls to which one is exposed when working with his notebooks. In spite of important investigations about Leonardo's sources, much remains to be established in this area. Perhaps, in fluid mechanics one is relatively safer, because then
so little was known about flow and transport phenomena. It is generally agreed that Leonardo learned some mathematics from Luca Pacioli, and perhaps some anatomy from Marcantonio della Torre, but who could have taught him some fluid mechanics?

Even in these days, one can find more often than not, books and papers on Leonardo which quote from anthologies like those of Richter and McCurdy [see e.g., Maiorino 1992], and not from Leonardo directly. In many cases drawings from the hand of Leonardo are reproduced without any reference to the accompanying text in the corresponding original manuscripts. Prof Marinoni stated once that The majority of the readers of Leonardo have seen only anthologies such as those of Richter, McCurdy, Solmi, Fumagalli and Brizio [Marinoni 1980]. I could add that many of them have used only one or two of such anthologies, although it is already a century that a good number of Leonardo’s manuscripts were reproduced and transcribed, and about half a century that almost all of them have been available in that form. Besides, my own experience has shown that it was not too difficult to consult the original manuscripts if one wanted to do it, and quite easy to work with facsimiles. Therefore, there is little excuse for the exclusive use of anthologies, which are certainly known not to be comprehensive. One of the pitfalls of anthological works is that they overlooked in different degrees the importance of Leonardo’s drawings. I consider highly questionable any rendition of a text without a serious study of the original drawings related to it. In addition, all the anthologies I know are by a single author. How could one person feel that he or she knew enough art, anatomy, aerodynamics, architecture, astronomy,
botany, dynamics, geology, geography, geometry, hydraulics, hydrology, kinematics, mechanics, meteorology, morphology, optics, physics, physiology, technology, transfer and transport phenomena, zoology, etc., etc. to feel comfortable about taking passages from his notebooks on any subject and offering translations of them? How can anyone believe that such anthologies can be reliable primary and sole sources for scholarly work?

To be sure, much more is needed than studying the original writings and drawings, but, in my opinion, the examination of the original form is essential. We know that it is already difficult to read profitably many of the writings of our times; those of past centuries offer an increased difficulty which stems from other forms and ways, not only of the language used but also of the views and of the thinking pertaining to a given topic. This difficulty is compounded when the writings are by somebody who is undergoing an intellectual transition because he discovers that the knowledge he has received needs a revision and a new cast. In the case of Leonardian fluid mechanics, we are bound to find great difficulties because he ventured into many questions of dynamics, from the motion of a single particle to the complexities of turbulent flow. We know now that his efforts were fraught with enormous difficulties not having as a tool a system like Newtonian mechanics. One must study Leonardian fluid mechanics being fully aware of this aspect. Leonardo was able to detect flaws in the received knowledge of physics but not to find an effective substitute, excepting, perhaps, in the field of kinematics. T.S. Kuhn [1977] has given the following advice: "When reading the works of an important
thinker, look first for the apparent absurdities in the text and ask yourself how a sensible person could have written them. When you find an answer, . . . . then you may find that more central passages, ones you previously thought you understood, have changed their meaning."

In examining these monographs, the reader may think that I have yielded to the temptation of taking the approach I criticize. I have reluctantly decided to transfer, using my own criteria, the text of those passages which I found to fall within the scope and goals of this project. Such passages are then given in my own interpretation in the English language. I wish I knew Italian well enough to do all this work in the language of my ancestors, but not being so, my way of telling others what I have found is to express it in English. Moreover, English is so widely used that my chances of getting comments and criticism of my work and, perhaps, persons who may want to use it or, hopefully, continue it, are really maximized. Besides, I do not offer my work as an anthological one, even for fluid mechanics; any scholar who uses it should still take as primary source the manuscripts themselves.

I believe that only fluid-mechanicists can undertake the analytic work reported in these monographs, as well as the synthesis that must complement such analysis. In this sense, I feel on a very firm ground. One of the motivations I had, from the beginning of this project, was a statement by Prof. Ernst Gombrich [1964] concerning the need that art historians and analysts had for a study of all the work on water, in Leonardo’s writings and drawings, done by those who could do it
because of knowing enough hydraulics and hydraulics history. The comment by Gombrich read in part as follows: "...while historians of science have provided us, art historians, with an admirable key to the understanding of Leonardo's anatomical drawings, no historian of hydraulics has yet obliged us with a similarly detailed study of his drawings of water". The same notion, I believe, is applicable to the historians of science and technology. Much ink has been spilled by historians of art, science, and technology offering general views before documents, like those of Leonardo, had been previously studied by those who had enough knowledge to interpret, analyze, and synthesize them. One interesting example is the apparently well known subject of hydrostatics; many have ventured opinions about the history of hydrostatics believing they were familiar enough with Archimedean hydrostatics, and could e.g. speak about Leonardian hydrostatics also. But until Dijsterhuis [1957] published his excellent work on Archimedean hydrostatics, we did not really knew exactly how to interpret and evaluate it. Perhaps, a partial exception to this is the work of Arredi [1942-43 ] who pointed at the lack of connection of Archimedean hydrostatics with dynamics. It is of great importance to see that hydrostatics cannot be fully understood unless one considers it as part of a wider building, that of dynamics. And dynamics, in turn, needs a deep understanding of kinematics, as it was intuitively realized by Leonardo, and afterward, at a much higher level, by Euler, Ampère, D'Alembert and many others about two centuries ago [Macagno 1991].
All along my work on Leonardo's manuscripts, I have studied the papers and books of analysts of the history of science and technology, as reflected by the references in which I include the sources that I considered most valuable. In this monograph, one of those included, is T.S. Kuhn [1970, 1971, 1977]. I believe that his concepts may be very useful to persons doing the kind of work I am doing, even if one is not persuaded by many of his arguments. I have found in Kuhn a very sincere student of the history of science. See, e.g. some of his notes to his 1971 article in *Daedalus*. I believe that the following quotations will whet the appetite of my readers: Footnote 3, for instance, points at a fundamental difference between critics in art and in science. In science, the criticism is usually done by scientists (how could it be otherwise ??). Kuhn notes that - Historians usually rely *exclusively* on these works of "criticism" - (italics and quotation marks by Kuhn). In Footnote 4, Kuhn advises the reader to read a book by C. C. Gillispie [1960] if he is interested in *someone who knows the science and its history*.

What concerns me, is that Kuhn, beyond his doctoral work in physics, does not seem to have ever worked as a productive scientist, and therefore how could he have an accurate vision of what is actually going on when science is created? By contrast, it is amazing how little scientists are able to say about their own creations of new knowledge, but at least when they do it we hear the voice of those who have actually lived through the process [See. e.g., the classic essay by Hadamard 1945]. When reading Kuhn's writings, I had the same feelings I have developed when reading other writings of the same or similar kind. I
think that there are phenomena and processes which have not been considered, as well as approaches that have been overlooked. Few have recognized that much of science began in artists' workshops, in architects' ateliers, in a great variety of engineering activities. In such places, paradigms, philosophical principles and prejudices, metaphysical dogmas, beliefs, etc. may have had little weight. Of course, with received knowledge much of that may come in disguise and as a substratum, and many historical figures may reflect, for instance, Aristotelian, or Platonic, or Medieval, or Modern "paradigms", even if they were not aware of sharing them. Some students of Leonardo have gathered and analyzed those statements in his notes which could, perhaps, be called paradigms [e.g. Grillandi 1975]; it seems difficult to ascertain the importance of their role in Leonardo's studies; he appears to have relied on his own observations and experiments much more than on the tenets and teachings of the past. There are, of course, different views, considering a sense of continuity as that of Niels Bohr [see Toulmin 1972, Krajewski 1977, Shapere 1980, Hallyn 1987].

It is unfortunate that the education of scientists and technologists of our times did not include much attention, if any, to the history of ideas and of their own disciplines, so that distinguished engineers and scientists could easily, as they worked and reached maturity, acquire a wider view than the usually narrow one. I do not believe that the history of science must be the sole work of scientists; historians, with a general humanistic formation, are as essential to arrive at a good historical synthesis as those scientists who once in a while become
historians. An illustrated humanistic component is essential, because science as well as engineering is a socio-cultural phenomenon. This does not mean that all cultures come up with equally efficient and valid results in engineering and science as some seem to proclaim dogmatically. The center, or centers, of excellence has always moved around through the centuries. Take, as an illustration, irrigated agriculture, one of the hydraulic technologies, which is old enough to have been practiced with success (sometimes followed by overuse and failure) in a large number of places in the planet. Its history is multicultural, because its great achievements flow from one culture to another, rather than because of a permanent uniform excellence of all contributors.

Because both science and technology have powerful influences on society, and because many powerful members of society take advantage for good and for evil of the products of scientific and technological research and invention, there is a need for an approach to history in which all participate. One cannot take only into account the opinion of scientists and engineers about the history of science and technology. Thagard [1992], may not be defending exactly this thesis, but he gives a fair enough view of the different opinions, and his book on conceptual revolutions may be profitably read. Personally, I tend to be cautious about the overused term revolution, as I am of the term paradigm, or metaphor. If seems hard to go against the opinion of Cohen [1985] when he argued that there is a revolution when scientists themselves have considered that there was a revolutionary change.
However, I think that we need to have science and technology developments and history examined from different viewpoints, and thus nobody, among philosophers, historians, scientists and engineers, should claim the possession the most profound understanding and the ultimate clear vision.
METHODOLOGY USED IN THIS STUDY

General considerations

In some of the previous IIHR Monographs and in papers containing my work on the synthesis of Leonardian Fluid Mechanics, I have included sections on the methodology of Leonardo da Vinci as it emerges from his writings. This methodology, became quite clear after some years of thorough examination of his manuscripts and was one of the first rewarding pieces of synthesis of his work that I could achieve [Macagno 1974, 1982, 1985b]. At this juncture, I do not have more to say about such question than what can be found in recent monographs. Instead, I feel that, in order to avoid persistent misinterpretations, more must be said about my methodology in studying his notebooks.

I began preparation for this task ever since I found myself with a deep interest in Leonardo's manuscripts, more than three decades ago. The method I have developed grew with the challenges I was finding in my way. Some comments about such period may be in order. I see in retrospective, that I may not have been aware of some aspects of my own way of proceeding, as once I was told by Professor Hunter Rouse, after he inquired about it once, when he was Director of the Iowa Institute of Hydraulic Research, and I could not give a satisfactory answer. I believe that some of my teachers, from elementary
school on, have instilled in me a discipline of work and a number of general procedures along with their specific teachings. They come into use automatically, or almost so, when the need arises. Since then I have done some introspection, and I think I have a better notion about how I proceed in my studies, and to whom I am indebted.

Faithful to my own natural inclination of learning always from those with more experience, I looked for those persons who could continue to instill some basic methodology for my investigation of Leonardo's manuscripts, since the very beginnings of my project. I have listened to the comments of those who were kind enough to discuss with me this project and who were, in a variety of ways, authorities in different fields, from art history to advanced technologies in fluid flow and hydraulic engineering, and from the language of Leonardo to the state of engineering in his and in our times.

I have also learned much from my students of fluid mechanics and transport processes who were, sometimes unknowingly, re-living experiments designed by Leonardo da Vinci. Usually, undergraduate students reflected primitive sources of received knowledge which have survived the scientific "revolution", and it was more rewarding not to reveal to them the source. One example of such survival is the conviction that water is incompressible and that air is highly compressible. Instead, graduate students usually liked to contribute thoughts about their reactions concerning experiments with sources going back half a millennium. I must add that some graduate students could be highly
critical toward a teacher who dare to put qualitative and quantitative experimentation on an equal level of value. Usually, they were the same who could not see that qualitative thinking is at the root of a good dimensional analysis.

Leonardo's manuscripts are a difficult material to handle, and I am well aware of the mistakes one can make. As Keynes did for analogies [Macagno 1986], I believe that it may be useful to regard methodology as containing two components, one positive and one negative; of course in this case one should strive to minimize the negative component, a difficult task if there is one. I believe the most serious negative aspect is to rely on any reading by someone else and not on the original writings, and to do it without a careful reading, at the same time, of the corresponding drawings. Among the positive components, I consider the laboratory method which has required a continued effort to improve it, and has also been most rewarding.
The Laboratory Method

Along more than three decades of use of this method, I have discovered that most of those to whom it is described for the first time often form incorrect ideas about it. This may be true, of course, even when any current laboratory work is reported to those who have not worked in a laboratory. It even happens that an experimental biologist, for instance, would not understand correctly what is reported as laboratory work by a physicist, and vice-versa; and I speak of this after some experience. I believe then, that it is necessary to continue to describe as well as possible, the essentials of the especial methodology I have developed and improved through the years, and that has served me to great advantage for the understanding and interpretation of both the writings and the drawings of Leonardo concerning flow and transport phenomena.

As an example of misunderstanding, I would like to mention the way a TV channel reported my work on Leonardian fluid mechanics in Europe. It included some of my experiments, under the title "Leonardo on the Test Stand". The central idea of my experiments is not that of merely testing those of Leonardo, but that of understanding what he may have done, and if possible arriving at plausible conjectures of what his thoughts may have been. Perhaps, the best way of conveying what is the role of my work in the laboratory is to describe some examples and point at their significance and the way in which they contribute to other purposes than those of a mere test-stand. One problem is that some
people, when they listen about some events, immediately imagine what they are about, instead of trying to acquire some more knowledge before jumping to conclusions. The role of experiments in this project is not what one may quickly imagine without a good deal of information about them.

Even to those who are research scientists in either hydraulic or fluid-mechanical laboratories, one needs to explain that their usual work is quite different from the work of a historian who uses the laboratory in his research. Usual research is intended to generate new knowledge and to expand the present frontiers [Macagno1963], while the historian is trying to relive (not simply repeat) experiments performed in the past, centuries ago, in a different world than ours. Although it is true that my own acquaintance with ordinary research was of some usefulness, for the most part I had to learn to work in a different way and with a quite different approach. This laboratory method should not be confused with an unimaginative way of checking Leonardian fluid mechanics, in a bare confrontation of his notes and drawings against modern experiments. In fact, one should try to produce non-modern experiments, like those that could be performed at the end of the fifteenth or beginning of the sixteenth century. I must say that this is not easy at all. Most probably, many of those experimenters were not members of any university, but engineers and artists.

In the case of Leonardo, usually, there is little information about the way in which the experiments were performed. For each experiment,
one may find at most a few paragraphs, and in many cases, only a few if any sentences, and some drawings in the form of sketches; nothing liked a the refined text and the well finished figures one expects to find in a published book or report. Many conditions that are quite clear in the usual research projects, are not spelled at all. For instance, if Leonardo states that clear water has a different kind of turbulence and erosive power than sediment-laden water as the flow goes around an obstacle implanted in the bed of a river, there are many parameters that are not mentioned, and one must play with them until one can consider that the experiment one is performing is close enough to the one he mentions. It should be kept in mind that, for Leonardo, qualitative experiments were very important, and most of the time were the ones he could and would perform, because of he was exploring a field that was little known. We must not forget, however, that even today, qualitative experiments may be very useful to the fluid mechanicist, both as a teacher and as an experimentalist [Macagno 1953, Vanoni 1990].

In addition to the very particular kind of documents we have inherited from Leonardo, we must take into account that as a man, he defies classification. One should imagine experiments as they can be performed by an artist, and also by an engineer, without excluding the scientist [Macagno 1987a]. His experiments have deep roots in the art workshop, in which he was a brilliant apprentice, and in his own studios, where he was a great artist. [M. Macagno 1987]. But he partook also of the received knowledge of his time (through readings and mostly, perhaps, his verbal exchanges with engineers, mathematicians
and scientists) Hence, at least two traditions must be taken into account, as pointed out by several students of Leonardo [see Ono 1952, Lilley 1953, Zubov 1968, Macagno M. 1987]. In any of the two traditions, experiments were very different five centuries ago from what they are nowadays.

It is not easy to establish how the experiments mentioned by Leonardo were performed, but it seems safe to assume the simplest conditions imaginable; few, if any, measurements; little systematic work and few, if any, calculations. One should remember also that he could perform experiments outdoors in improvised conditions [See, e.g. experiments with flames in Codex Atlanticus 728, or with sand and water waves in the same codex, CA 105a V]. We should not expect sophistication, as we have today in abundance, in data acquisition and processing. Regarding laboratory devices, it is unfortunate, but I do not believe that any of the instruments and apparatuses Leonardo may have used have survived; I have only found one reference to such devices in the memoirs of Giovanni A. Mazzenta [Grammatica 1919], who mentioned that  manuscripts, books and apparatuses  left by Leonardo were seen by him around 1595 in Villa Melzi, near Milano. Regarding mathematical analysis of experimental results, Leonardo had to rely mainly on geometry. We know that he developed his  geometria che si fa col moto  , to the point that could be useful in interpreting flow characteristics [Macagno M. 1987]. The extent of help that he got from his geometry of motion is not easy to evaluate, but it is surely a very interesting goal for future work.
As a first example, I would like to describe the experiments with floating bodies constituted by two cylindrical flat boxes attached one on top of the other. One box is filled with water and the other with air. If flat enough, this body can float in a stable position, with either the air or the water on top. Leonardo repeatedly described this experiment in his notebooks [See CM I 145V, CA 1016R, and Macagno 1987a]. As a first step, I had one such body constructed to be used by my students of the course Experimental Method in Fluid Mechanics, which I developed in the seventies at the University of Iowa. I thought, at the beginning, that Leonardo was studying the stability of floating bodies and I wanted my students to know about that.

As I saw the difficulties of my students in dealing not only theoretically with the two-compartment box but also in considering heuristically the seemingly paradoxical nonuniform floater with two stable positions, I came to think that the experiment was perhaps not viewed by Leonardo as a problem of instability, even if instability had not been foreign to Archimedes. One must take into account that Leonardo appears as having been unaware even of the main principle of Archimedes about the buoyancy force. In the mean time, I had found in other passages that Leonardo was concerned with the old notion that each element "desires" to join its sphere, if it is placed outside it. I believe that my watching of the model and the way in which it stayed put with the water above, in spite of disturbances introduced to reverse it, may have been also his main view of this experience. And that the
experiment was significant to Leonardo, because in it he found a case in which water and air did not seem to desire to return to their own spheres. This is obviously a question in which we are nowadays at a level of knowledge that allows us to determine theoretically (no need for the laboratory!) that Leonardo's floater has two stable positions of equilibrium (given a rather flat double box). There is however an advantage in repeating the experience, in reliving it, because the real situation provides an atmosphere of more fruitful examination of what he may have had in mind. I am confident that those with laboratory experience will understand my point. For Leonardo, this must have been a crucial experiment regarding the inherited physics, and this is, I believe, more exciting than him discovering or expanding the knowledge of hydrostatics stability. Finding experimental evidence that some aspects of Aristotelian physics to be wrong must have been a much more momentous event.

But there are other cases with other different aspects to be considered. One can find a great variety of intellectual excitements when going through the different experiments one finds in Leonardo's manuscripts. When I came across the experiment of two impinging water jets, that according to Leonardo, would not coalesce but bifurcate nicely after osculation [Macagno 1982], I had my doubts, which were shared by a number of my colleagues whom I consulted at that time. In fact, most of them thought that Leonardo was deadly wrong. But, by then, I had already found him quite reliable, and I decided to explore jets of all kinds. And thus I found that laminar jets, within a certain domain of the
parameters involved, did exactly what Leonardo reported in his notes. He also included two jets which would merge, and a jet which would either merge with a nappe or that could go limpidly across it. The latter was an experiment which I repeated because I thought that the nappe would always be split in two under the jet; a circumstance that may or may not happen. I was in this case trying to learn from Leonardo, and not trying to test his reliability [see Macagno 1982, 1988a].

A somewhat different situation occurs when Leonardo includes in his manuscripts notes that represent received knowledge, like the tenet inherited apparently from G. Nemorarius [Marinoni 1989], which asserts that the heavier part of a body moving through air becomes the guide of the motion [Ms G 51]. I thought I could find examples that would prove this statement wrong. A general statement can be proved not valid if one finds just one case in which it fails. Leonardo mentions a pyramid falling down, but according to the Medieval rule it could not fall with the vertex downward. Note that a cut of a pyramid by a plane halfway between base and vertex leaves a portion seven times heavier on one side than in the other. In the experiments, I used pyramids, cones, and other bodies. The pyramid, for instance, repeatedly fell point down when dropped with the vertex down; in no case it turned around to let the heavier half become the guide of the motion. I could not find any indication of Leonardo having challenged this rule, in spite of a rather large number of passages in which falling bodies are considered [See Macagno 1987c and other multichannel tabulations by the same author]. One may suspect that the Medieval tenet was not tested by Leonardo for
a large enough variety of bodies so that he could discover exceptions to the medieval rule. Let me say again, that I did these experiments because I needed to gain insight myself on this question, to better understand how to interpret a given situation. I found out that I did not know enough about falling bodies, and I think that very few know much about all the possibilities.

After performing many experiments as part of my use of the laboratory methodology, I cannot find a case in which I could conclude beyond reasonable doubt that Leonardo could not be relied upon concerning the veracity of his depictions of observed phenomena. In my opinion, he was an honest, although, according to our standards, not very careful, experimenter. Even his mistakes come in support of this judgement. Leonardo's notes present a problem in that he, in many cases, did not specify when he had actually ran an experiment, or whether it was a thought experiment or not.

Thought experiments may cause some anxiety to some persons, but they have been used for a long time as a valid and inspiring method. Of course, a problem arises when the writer does not specify whether the experiment was actually performed or not. That Leonardo may have depicted what he reasoned, through the use of the received knowledge of physics, is illustrated by several examples. The most flagrant is the one in which he said that water must go up in an inverted partially filled flask (mounted like a barometric tube) under the action of heat applied to the top of the flask, when it surely goes down [Codex Hammer 3V]. We
could consider this as either a case in which he did not actually perform the experiment, or one in which something went wrong because of leaving the apparatus unattended. It may happen that, under excessive heating, air bubbles out of the inverted flask through its submerged opening, and then, when it eventually cools off, the water re-enters the flask and will go higher than it was. If one does not watch the experiment all the time, this may appear, a posteriori, as a simple rise in water. Nobody is free from making mistakes like this, but I consider more probable that this passage corresponds to a thought experiment. [See discussion in Macagno 1885b, 1988a].

There are also cases in which one finds experiments which are part of the familiar background or the particular research field of the student of Leonardian fluid mechanics; for example, the determination of the force of a water jet on a flat plate (which was for some time part of my regular instructional tasks at the University of Iowa), or the force and pressure distribution on the wall of a tank, an experiment which would nowadays be considered unnecessary [See Codex Madrid I 149V and Codex Hammer 6R and discussions by Macagno 1982, 1987, 1988a] In such cases, one feels much less compelled to resort to some work in the laboratory, but there are nonetheless aspects that must be analyzed by the experimentalist, as in the case in which Leonardo proposes the replacement of a wall in the water tank by a series of strips with the purpose of sensing the force on each of them. We do not see the need for this nowadays, but how would we ensure a good result for such an
experiment? could Leonardo have performed it successfully? or was it beyond his available experimental techniques and means?

A very interesting episode occurred when I consulted Prof. Vito Vanoni about Leonardo's passage in Ms C 24 V on enhanced erosion of the bank of a canal at the places where diagonal waves are reflected [Macagno 1988b]. His answer was that Leonardo was right, because he was able to see that effect when he himself did his own experiment in the CALTECH laboratory. His exact expression was "the old boy was right" [Vanoni 1990]. I must add that, before writing to Vanoni, I had consulted this point with several experts in fluvial sediment transport, and all of them discarded almost instantly the possibility of such increased erosion taking place. Vanoni, a man with a vast experience, resorted to the laboratory method. I suppose, this is the example I could offer of having appealed - indirectly - to a test of veracity. I must add that Vanoni's test of the increased power of erosion was very clever.

Finally, I would like to say that I consider the laboratory method as a common ground on which historians with quite different backgrounds can meet. Of course, this would require an effort in cooperative work, but I am hopeful that this may happen in view that there is no other way more promising of desired valuable work. Although nothing done by us, human beings, is without some degree of subjectivity, this method provides a degree of objectivity which is higher than that of many other methods, especially in this case, in which we are in the field of physics a science known to stand on much firmer
ground, relatively speaking, than others. Because in this work we are also in the field of history, in which a well publicized negativism has been introduced in recent times, it seems important to adhere to conventions that do not seem in need of being discarded. I agree very much with an opinion expressed recently by G. S. Wood about those conventions *scrupulously developed in the western world and with respectable justifications*. He adds that we should not throw away such conventions without first examining critically the modern skepticism or the narrative experiments [Wood 1992]. I have come to see the method I have developed as a bulwark against purely subjective irresponsible narratives.

I hope that the above discussion, which is by no means complete, makes clear what the laboratory method meant, and still means, in my studies of Leonardian fluid mechanics. In addition to that, the reader can consult the other monographs and papers I have written, in which more cases are included.
Analysis of the Manuscripts

In the analysis of Leonardo's manuscripts, it is important to identify not only the individual passages of interest but to cluster together those in which a topic, or a procedure, or a method are found to be included. Having devised from the beginning an ad-hoc type of multichannel tabulation, it is easy to put together such clusters. I consider very useful for my work of synthesis to have at hand tabulations of paradoxes, analogies, experiments and experimental situations, basic concepts and notions, etc. In the monographs I have included tables of analogies and experiments corresponding to the particular document studied in each monograph.

In this monograph, the reader will find the arrangement in pairs of pages used in previous IIHR Monographs for the survey of the work of Leonardo. It should be said that much more than a bare survey is offered. The page on the left-hand side contains selected texts from the Ms G in Leonardian Italian with my own separation of words, my version in English of such passages, and my comments. These comments are both critical and suggestive of work that is needed. On the right-hand side page there is a succinct "map" of the corresponding page in the Ms G, showing the approximate location of texts (T) and drawings (D); beside this map, I give the profiles for the topical blocks I have identified in that page. The way in which such profiles are generated has been described in papers and monographs; they are the result of a detailed study [Macagno 1987c]. Also included are the
drawings I have considered necessary. I have drawn the sketches myself, because I want to offer them with my own interpretation and emphasis as a fluid-mechanicist. These drawings also reflect a detailed study of the question involved.

I have adopted an eclectic approach to translation, because each passage (text and drawings) requires to be rendered as a unique piece. In some cases, I have considered better to be as literal as possible, to convey what Leonardo was actually saying; in other cases, I thought that he was obviously not careful with his writing, but his intention was transparent, and I treated those passages as something to be explained, as a matter of exegetical translation. I have had no hesitation to show my doubts whenever they existed. It is perhaps paradoxical, but it seems that the more one knows about a given field, the more difficult is to be sure of what Leonardo says concerning specific topics in such a field. Only generalists seem to make confident translators and not to be disturbed by doubts. In documents, as those left by Leonardo, there are many ambiguities and many obscure passages. It is quite probable that some points may remain for ever in doubt. There is still much work to be done before we can offer for the general public a truly coherent synthesis of all the fluid mechanical work of Leonardo. However, consistent views are emerging in some areas, and I would like to refer the reader to the synthesis papers I have already published [Macagno 1982, 1984/5, 1985, 1986, 1987, 1988, 1989, 1991, 1992].
ANALYSIS OF THE MANUSCRIPT G

Analogies

As I have already done for other manuscripts, the analogies I have identified in the Ms G are presented in tabular form (Table I). For each of the analogies listed, I have included a summary description and a simplified sketch' either when there was a figure in the Ms G, or when it would help in understanding the text. The interested reader will find some comments about these analogies that I have included in the corresponding pages of the monograph. Suffice here to state the basic notion that analogies have, according to Keynes [1921] two components: the positive and the negative analogy. This was already repeatedly recognized by Leonardo. One of those cases is in the Codex Atlanticus, when he stated, in connection with an analogy regarding the transport of sand and water, that he should

Make the sand flat when there is wind and see how the wind generates its waves, and not how much slower the sand moves than the air. Do a similar test with water and make notes on the differences between water and sand. (CA 105a V).

In 1921, Keynes, after discussing his general concepts about analogy, stated

I hold then, that our object is always to increase the Negative Analogy, or, which is the same thing, to diminish the
characteristics common to all the examined instances and not yet taken into account by our generalization.

Any reader interested in the study of the differences may find useful to consult, besides Keynes, other authors [for example, Stegmüller 1969, Fielder 1978).

In this analysis of the Ms G, I was able to identify nine analogies one of whose terms at least is fluid-mechanical. There are three analogies in which three terms were included by Leonardo. In two of them the three terms are fluid-mechanical (See entries 1 and 3 in Table I).

I have presented already my ideas about the evaluation and classification of analogies [Macagno 1982, 1986, 1989b]; I have found appropriate to divide the analogies in internal and external. Internal are those analogies which are totally within mechanics or fluid mechanics, while external are those in which at least one term belongs to some other science, even if it implies some flow. For instance, in an analogy between the flow of water in canals and that of blood in the arteries or veins of a human being, I see an external analogy because one of the terms lies in the field of physiology, even if flow is the main concern in both terms. According to this criterion, the nine analogies can be divided into SIX internal and three external.
Experiments

In my tables of *Experiments and Experimental Situations*, I include any passage in Leonardo’s manuscripts which may be useful in a survey of those questions which he tried to approach experimentally. Some of the items in my tables may be discarded in a future re-evaluation, but I prefer to sin by over-inclusion than to do it by omission. To warn the reader I have added the words "experimental situations". I believe I have demonstrated that Leonardo performed experiments [Macagno 1982, 1987a, 1989a, 1991b] but I have also voiced a warning about many cases in which he indicated *experimentato* or *experimentata* and there is no evidence of him having run such experiment. I must also mention that there is not an exact correspondence between profiles of key words in the text of the monographs and the tables of *Experiments and Experimental Situations*. Entries which belong to Geometry, Kinematics, Mechanics, etc. have been included in all those cases in which I have considered that the passage could be of some value for the study of Leonardian Fluid Mechanics.

My investigation has led to the identification of 22 groups of experiments and experimental situations in the Ms G, which are presented in tabular form (Table II). To avoid misunderstandings, I must say once more that tables like this do not mean that I believe that Leonardo performed all the experiments listed. I am surveying documents and I report what I find in them that appears as experimental
in one sense or another. By using the laboratory methodology I have
developed for the study of Leonardo's scientific writings, we have a
rather objective procedure to examine and evaluate the work he did,
but I believe that in many cases we will remain in doubt about what he
actually performed in the area of experiments [Macagno 1989a].

Experiments and experimental situations in the Ms G belong to
several areas: general mechanics, deformable bodies, hydromechanics,
hydraulics, aerodynamics, bio-fluid-mechanics, ballistics, fluvial
hydraulics, visualization of motion.

The sketches in Table II (many of which are simplified) are either
based on Leonardo's drawings, or introduced when there was no
drawing by Leonardo. In the latter case, I have indicated my intrusion
by inscribing my initials in the lower right-hand-side corner of the
corresponding box. I have summarily described each experiment or
experimental situation; an analysis of these experiments is part of my
ongoing investigation, consisting of a systematic analysis of experiments
and experimental situations in all of Leonardo's extant notebooks, as a
necessary step for the synthesis of his science of flow. Some
preliminary critical comments are to be found together with my
renditions of the fluid-mechanical passages in the Ms G, in the second
part of this monograph.
COMMENTS ON AREAS OF INTEREST IN THE Ms G

The following comments cover salient questions of geometry, kinematics, dynamics, fluid mechanics and transport phenomena. They refer to specific topics included by Leonardo in the Ms G, but are, at the same time, inclusive of general considerations about the methodology that has been considered appropriate, an approach that has been used by the author in all his publications on synthesis of Leonardian fluid mechanics.

**Geometry**

In his manuscripts, Leonardo included many notes on geometry. Folios of the Ms G containing geometrical notes and drawings are the following: 1, 13, 17, 29, 32, 37, 38, 39, 40, 41, 42, 43, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 66, 67, 68, 69, 75, 96. In addition to this, there are many geometrical constructions he used when dealing with different topics, like shadow and light in folio 3, perspective in folio 29, or optics in folio 75. Some of the above mentioned notes are about received knowledge, while others contain new notions. It seems well established that Luca Pacioli was an important source of learning for Leonardo [Kemp 1981, Marinoni 1986]. He had also some knowledge of arithmetic and very rudimentary notions of algebra; Prof. Kemp [1981, p. 248] stated that Leonardo's natural abilities were far better suited to geometry. However, Leonardo could play with sequences of numbers to express some new notions as in the case of functional relationships [In the Ms G, see 85R].
would say that he could come up with some original concepts whenever there was a situation leading to them or necessitating them. Instead, I am rather skeptical about Leonardo's foresight concerning infinitesimal calculus [Marinoni 1989].

The most important progress made by Leonardo in mathematics is the conception and development of geometry as a subject associated with motion and not as one concerned only with rigid static figures. Without geometry in motion, it is inconceivable that he could have gained so much new insight on a number of difficult kinematical questions. Even a distinguished mathematician as R. Marcolongo [1932, 1934] failed to see what was so obvious to Hermann Weyl [see Appendix], and, much more recently, to other students of Leonardo's geometrical notes [Macagno M. and Macagno E. 1987, Macagno M. 1987, 1992]. The geometry in motion in the Ms G has been examined by Prof. Matilde Macagno who kindly agreed to write an appendix on the subject for this monograph. Therefore, in this section only a few more general remarks will be included.

After wondering why some original aspects of Leonardo's work were overlooked in many studies, I came to recognize that most publications about Leonardo reveal blind spots in the author's investigative vision. One tends to see only what one is familiar with. I suppose I also have blind spots, and some day another student of Leonardo, with more advanced knowledge of flow phenomena than mine, is going to describe them. However, much wasted time could be avoided would all scholars proceed according to the best of their abilities.
exempting themselves from engaging in areas without having some justified claims of expertise and experience, and not just on the basis of a general training. Otherwise, we are bound to find misconceptions that may persist for a long time. Because Leonardo used motion to develop geometry, a scholar who believes that geometry must be a subject dealing only with rigid and static figures is ill prepared indeed for an investigation of Leonardian geometry.

A few passages in the Ms G refer to geometric properties as we move towards a point following a given pattern. Leonardo, by having made statements that seem inspired in philosophical views about the infinitely small and the nothingness, elicits attraction for metaphysical considerations, but I believe that it is more appropriate to look at this topic within the modern mathematical scene. Thus, in a study of Leonardian geometry, we should take into account that his notes have almost always practical sense. It must be kept in mind that he was after the solution of problems of quadrature and cubature, and of subdivision of figures (with an eye towards problems of statics, for instance). Concerning the possibility of Leonardo having a foresight of infinitesimal calculus [Marinoni 1989], I believe that we should take into account that one of the main applications of finding a limit mathematically is the determination of differential quotients, or derivatives, or gradients. Therefore it is not the metaphysical meaning of a "zero" or another, but the ratio of two "zeros" that may constitute the crux of the matter, and this does not appear to have been grasped by Leonardo.
Concerning the discussion of Leonardo in Ms G 1 about what happens to the top of a triangular pyramid as its size is made smaller and smaller and it becomes a point, I think that we can again use our understanding of limits that are related to applications in physics. Remember, for instance, Cauchy's tetrahedron in elasticity and fluid mechanics. Or the analysis of stresses around a point, leading to the tensor of stresses at a point. Through that analysis, such a point is not anymore an ordinary point of Euclidean geometry, it is a point belonging to three planes on which there exist stress components, in the number of three on each. Or consider a singularity in Hydrodynamics like a source or a sink; infinite streamlines issue from there or concur to that point. It is, I would say, similar to a radiation of rays in projective geometry. Such points contain much more than the "nulla" Marinoni[1989] wants to associate to a point as an "indivisible entity". Of course, it would be unwise to attribute to Leonardo a foresight about Cauchy's tetrahedron, or about tensors, but knowing about them, one is better armed to try to conjecture what Leonardo could or could not be adumbrating in his analysis of the vanishingly small top of a pyramid.
Kinematics

To study Leonardo's notes on flow and transport phenomena without considering at the same time his notes on geometry and particularly those on kinematics would be a great mistake. Greek physics remained in a primitive state because geometry was not extended to the study of motion with the same genius evidenced in the books of Euclid. We have now much evidence of the important role of kinematics in different sciences and in engineering [Macagno 1991a]. Particularly, in the case of fluid mechanics, one glaring example is the essential role attributed to kinematics in the studies of turbulence, one of the most difficult problems still without a satisfactory solution; in recent years important contributions have been and are being made in the development of a topology and kinematics for three-dimensional turbulent flow [Macagno 1991a]. Leonardo's notes on kinematics, although very rudimentary compared with our present knowledge, should appear as remarkable to any reader who is familiar with the present status of that science. Leonardo entered entirely new areas of the science of motion and was able to gain a good understanding about figures and bodies undergoing deformation. Without this, the kinematics of flow phenomena is an impossible task.

The most interesting passages on kinematics in the Ms G are those describing the rolling of a curve or a surface on another (G 38V, 39R-V, 42V, 58R, 67V, 68R). By rolling a circumference over a straight line, its length can be determined mechanically. Once this is done, calculation of the
area of the circle is also possible. This may appear to some analysts of Leonardo's writings as a crude way of working, more fit to mechanical engineers than to mathematicians, but conceptually it contains a model for the mathematical integral formula for the rectification of curves. In the same way, the rolling of a body of revolution on a plane surface, contains a model for the determination of the area of curved surfaces by double integrals and also a model for mapping of curved surfaces on the plane as explained in detail in the Appendix. Engineers have always considered inseparable mechanical theory and mathematics, either in mechanisms or in instruments. Marinoni [1989] says that this is relying on "un metodo non propriamente matematico, ma empirico e meccanico"; but this smacks of a very narrow view of what mathematics is, both historically and in its present state.

If one would paraphrase the above sentence to say "a method non truly mathematical, but empirical and electronical" it could be applied negatively to the use of modern computers which are known to be so useful to both applied and theoretical mathematics. The fact is that modern computers are instruments of much more value to this science than either the compass or the straight edge, the instruments that were not proscribed but widely used by Euclidean geometry along the centuries and are still the only acceptable by some persons. Should not we say that a circumference drawn with a compass is a mechanical construction which does not belong in mathematics? Circumferences and straight lines were always constructed by moving a pencil manually either attached to a compass or sliding along a straight edge, and these are surely mechanical devices. What is so
different in Leonardo rolling a wheel or a disk over a straight line, or what is wrong with the body of revolution which he rolls repeatedly (along different lines) on the plane? His *geometria che si fa col moto* has been developed to a high degree of refinement by the science of kinematics developed centuries after Leonardo.

The originality of Leonardo's approach in kinematics cannot be assessed properly without a solid understanding of the discipline at an advanced level; otherwise, how can one judge the difference between Leonardo's treatment and that of some of his sources? Let us take Ms G 54R, for instance; in T1, Leonardo expresses clearly the results of his application of the notion of relative motion to an arrow shot from a ship in motion. In T2-3, Leonardo levels critical remarks to Vitruvius and Alberti. After examining these notes, a thorough study of what they say is in order to tell clearly whether the criticism is justified or not. Concerning Leonardo's method, which is not described by him in folio 54, one must search in the rest of the documents to find it and then analyze it. History must use the light of the present as well as that of the past. To look at a study of Leonardo only under the light of his predecessors and not also under that of his successors is surely a wrong approach.

Another, although somewhat more difficult, problem for the analyst of Leonardo's work is offered by the verso of folio 54, concerning the view of a stream of particles coming out from the orifice of a moving vessel; it should be discussed using kinematics of relative motion, and assessed as to whether it is all wrong or if it has some elements of truth.
[See Marinoni 1989 for historical aspects]. Leonardo may not have originated the idea of this phenomenon as a device for velocity measurements, but his analysis of the phenomenon in other manuscripts is very interesting. In them, he presents a clearer picture and description of the jet coming out from a moving vessel. I have analyzed this problem and it is quite probable that Leonardo gave it a final formulation in terms of a vessel being accelerated rather than in uniform motion [Macagno 1992]; if this is true, his conclusions appear to be generally correct. As for the specific statements in 54V about jets, they seem to be generally wrong.

Concerning the motion of a projectile launched upward at some point on the Earth and its eventual return, I believe that anybody who cares to discuss it should be familiar with motion relative to a rotating coordinate system. We know about Coriolis acceleration only since the kinematic studies of that French scholar in the past century. In addition, one should know what simplifying assumptions can be introduced concerning air resistance and instability in motion through a fluid. Leonardian fluid mechanics should be taken into account also to evaluate his notes on this topic. I believe that, in spite of his very rudimentary knowledge compared to ours, he was well ahead in this field relative to any of his predecessors.

I would say that Leonardo's drawings in G 54 V are correct if they are considered simply as geometric constructions for the absolute motion of a point radially while the circle is in uniform rotation, the motion relative to the circle being by definition rectilinear. Considered as the motion of body falling towards a gravitational center it is flawed with several errors. I obviously find this passage about the spiral path of a body falling over a
very long distance in a rotating planet very interesting because it offers no doubt that Leonardo considers rotation of our planet as a given fact. In folio 54V we have another passage that requires a knowledge of the kinematics of relative motion as it was developed by Coriolis.
Dynamics

Leonardo obviously did not have, as we do, a body of received knowledge, or one developed by himself, comprising a working theory that could be applied to questions of dynamics. Such a theory was developed much later by Newton, allowing physicists to deal qualitatively and quantitatively with mechanical phenomena on a solid basis, achieving results that sometimes possess an amazing accuracy. We only need to remember the prediction of the existence of a planet through accurate perturbation calculations. For the problems that Leonardo considered, Newtonian mechanics is surely appropriate, and none of the refinements and corrections introduced to it in this century is really needed.

We must keep in mind that Newtonian mechanics came more than two centuries after the death of Leonardo, and realize that he had a great chance of going astray in the dynamical analysis of any problem with subtle questions about different kinds of motion. He could predict that water, set completely free in the air sphere, would move down to the water sphere, but he could not find any way of predicting that water could remain in the air sphere, in the case of the double-compartment floater discussed above (see section on experiments), in spite of being apparently completely free to do so. The same difficulty appears in another experiment in which water is elevated peripherally due to the rotation of a tronco-conical container while air moves down centrally [Macagno 1989a]. Although lacking an explanation based on the notions of his time, Leonardo used this effect to
design (probably not ever construct) an early version of a centrifugal pump [G. de Toni 1989]. Of course, although we use the term "centrifugal pump" we have in Newtonian-mechanics a more accurate explanation for this phenomenon without appealing to a questionable reference to centrifugal forces. But to Leonardo, this was surely a puzzling experience, and he could not find any consistently clarifying explanation for it within the physical theories of his time.

Regarding the Ms G, I have included some questions which are evidently of dynamics, such as the balance with two kinds of dynamic loads (See Ms G 13V and 17V) and the two-body projectiles connected by a cord, discussed in Ms G 86V and 87R. It may be asked what have those questions to do with Leonardian fluid mechanics? The first reason is that it is remarkable that the balance, an instrument for a long time stereotype of statics, is applied by Leonardo to gain a sense of dynamic effects. The second is that, although I do not think those experiments are directly related to a specific topic of fluid mechanics, I believe that a mind capable of dealing with those experiments with the right intuition - I would not dare to say scientific knowledge - can also be powerful when dealing with dynamic effects of fluid flow. I would say that perhaps Leonardo connected, via analogy, the balance with a system of two weights mounted on a pulley on one side of a balance with some of his experiments on impact of jets on a plate mounted on one side of a balance [Codex Hammer 23 V]. I have searched in his manuscripts for a closer analogy, that of a container hanging on one side of a balance and spilling some water through an orifice in the bottom, but I could not find it.
I do not believe that Leonardo could not engage in problems of dynamics with some chances of success, even if in our view he was so poorly armed to deal with them. I think that Kuhn understood how much progress one can make if one uses the "paradigms" of his own time in his or her research. I believe that Leonardo and others may have used Aristotelian or Medieval physics to some advantage. In addition to that, his observations and experiments, although they did not crystalize in a manifest body of theory, were part of his inquisitive mind and provided a firm basis for understanding the physical world. We must remember that he was not only an engineer and a scientist; he was also an artist, and I think that he could arrive at knowledge in different ways. Access to knowledge may occur through methods that do not seem rigorous and according to modern scientific methodology, and thus sometimes results could be obtained like ours, who are scientifically much better armed.

Leonardo's discussion of a projectile moving up and down in Ms G 54V can be expected to be purely kinematical rather than a true dynamical analysis. This may be something that dynamically is incorrect, but that, reduced to a study of a kinematical situation, is admissible. One must look at those passages within the notions of motion relative to an observer within and outside the system. Even the notion of what is for Leonardo the vertical line should be examined carefully. Assuming that a truly cylindrical tower would be built with great precision, would a long plumb line coincide with a line on its side? Would any of those two lines
(whether different or not) coincide with a line of the force field under consideration?. One should think of the equilibrium position of a weight hanging from a long string and try to determine whether it is a perfect straight line in a rotating planet.

If statics on the surface of our planet is not simple, much less is dynamics. In a rotating planet, a body, released in free fall, will not follow the line of the static force field. Marinoni [1989] has discussed the passages in the Ms G concerning rotatory motion of the elements [Moto rotatorio degli elementi in the introduction to his transcription of the Ms G], giving very interesting historical information, but concerning the mechanical aspects, I believe that he could have consulted, e.g., the Meccanica Razionale by Pietro Burgatti [1921] among many books by Italian physicists who discussed motion of a body relative to the rotating Earth. The body (due to the Coriolis "force") may hit the wall of the above mentioned perfect tower, even assuming that it falls in vacuum; because a falling body experiences a deviation towards the east in the Northern Hemisphere. The fall with an initial horizontal velocity does not follow really a parabola but a three-dimensional curve; the difference must be important for such long trajectories as those considered by Leonardo.
Fluid mechanics and transport phenomena

There is less fluid mechanics in the Ms G than in other notebooks and codices; certainly much less than in the Codex Hammer or in the Ms F. Some notes, however, are remarkable, like the ones on blood flow in the heart and to aorta through the cuspidal valve. Leonardo's notes in this case, are essentially in the very revealing drawings rather than in the words. [See Giaquinta 1970, Giaquinta and Macagno 1992].

I found very interesting the notes on the fall of cards, boards and similar objects (See entries 13 and 14, in Table II), having done myself many experiments of this kind before and after studying Leonardo's writings. For those interested in inventions and their history, these notes should certainly be related to the parachute, because of the case in which a man is hanging from a very large plane surface and trying to maneuver it. However, more exciting to the researcher in fluid mechanics is the difficult question of flow mechanics that are implied in the fall of bodies who enter into complex paths rather than staying close to the vertical line.

The notes on siphons in other manuscripts, specially in the Codex Atlanticus are much more informative than those in the Ms G, where siphons are only introduced as parts of some gadgets, or in analogies in which some other phenomenon is considered as the central one. There are much more interesting passages on siphons in other manuscripts;
especial mention is deserved by those in which siphons are examined under the light of some paradoxes [Macagno 1989a].

The brief discussions of two important areas of fluid mechanics in Ms G 50V, i.e., the flow around different kinds of bodies and the erosion at the foot of a water fall are valuable, but there is much more on these two subjects in other manuscripts [Macagno 1987a, 1988a]. One important point to be taken into account is that, in comparing boats, birds and fish in flow mechanics, there are interesting analogies, but there are also important differences.

A number of pages of the Ms G deal with questions of flight [see in the facsimile of Ms G, folios 41, 42, 49, 63, 64, 65]. In addition to the flight of birds, Leonardo discussed that of bats and of insects. In view of the excellent work by Giacomelli [1936] concerning aerodynamics in the manuscripts of Leonardo, I have considered that only when such notes bear on fundamental aspects of fluid mechanics would I include them in these monographs.

It may appear odd that I do not say much about fluid mechanics in this section; perhaps it is because I know much more about these subjects than I know about geometry and kinematics. In fact, it is because I have included much already in the constructive and critical comments accompanying most of the passages I have selected for this monograph. Those are notes for myself as I continue work on the synthesis of Leonardian fluid mechanics, and also for those who in the
future may undertake further studies on this subject. What I have
to say in conclusion about the core of my project on Leonardian fluid
mechanics goes, and will go, into papers in which the synthesis I am
trying to establish is being offered area by area.
FUTURE WORK

I have found my thoughts about the future very much in the same status as a year ago, when I was completing my study of the Ms F; therefore, this section will be very much a repetition of the section included at that time in IIHR Monograph No. 113.

Much work remains still to be done to arrive at a satisfactory complete synthesis of Leonardo's science of flow and transport phenomena. I have attempted to formulate syntheses for limited areas, as those of kinematics, hydrostatics, basic fluid mechanics, analogies, paradoxes, experimentation, but many others remain to be thoroughly investigated and synthesized. I believe that Leonardo's methodology for his studies of fluid flow phenomena is clearly emerging from my studies, and I have written a synthesis of such methodology already. Of course, until all the documents are examined thoroughly, a final total synthesis must be postponed. Too many times, Leonardists have rushed to write conclusively without a serious study, and I am as cautious as possible not to fall in the same error. I very much hope that all the analytical work that lies ahead can be accomplished in the years I have left, but most probably, this monumental work will have to be completed by others. My hopes of attracting co-workers for an appropriate and effective transfer are always there, but I may be overoptimistic as usually.
An area in which work is very much needed is that of other disciplines important to the historical development of fluid mechanics. I have tried to establish some criteria for the study of Leonardian fluid mechanics, which is my central interest, but I have come to realize that more than I initially envisioned is necessary. Without an analysis of a trilogy encompassing geometrical, kinematical and dynamical developments in the notes of Leonardo it is difficult to analyze thoroughly his work on flow phenomena. I have encouraged already the study of that part of his geometry that is very original and innovative, and I am glad to report that a collaborator has been found in this area [Macagno M. 1990, 1992]. I have made some inroads myself into Leonardian kinematics and the history of kinematics in general [Macagno 1991]. I hope very much that interest in his dynamics entices some student or scholar of both science and history, because this trilogy of disciplines is very important in the investigation of the science of flow and transport phenomena in Leonardo's writings.

Iowa Institute of Hydraulic Research

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Along a number of years, my research work on Leonardian Fluid Mechanics received financial support from several sources. I was awarded a joint grant from the National Science Foundation and the National Endowment for the Humanities and then a second grant from NEH. I am also grateful for a Fulbright Award for the academic year 1986-87, during which I was able to work with Professor Augusto Marinoni, of the Commissione Vinciana of Italy, at the Biblioteca d'Arte of the city of Milano. The staff of the Biblioteca d'Arte at the Castello Sforzesco was extremely kind and helpful. I am greatly indebted to Professor Marinoni for his advice and help, always granted with great generosity and efficiency, and also with sincere criticism. I am also grateful to the Politecnico of Milano, in which Istituto di Idraulica I was given a spacious office and generous and efficient secretarial help. Before that, I enjoyed for several years support from the Alexander von Humboldt Foundation and the Volkswagen Foundation from Germany, and from the Universities of Karlsruhe and Paris. In Karlsruhe much work was done both on documents and in the excellent laboratory facilities that were made available to me.

Many persons have helped me by agreeing to discuss aspects of my work, or by lending a hand in libraries, laboratories and class rooms, or by kindly and efficiently answering my letters full of questions, or by doing searches for me. In this respect, I extend my
gratitude especially to Professors A. Marinoni, V. Vanoni, J. F. Kennedy and R. Ettema. Many of my students have helped me unknowingly by answering quizzes tailored to discover primitive notions or their reactions to questions that were considered by Leonardo. A number of graduate students performed experiments which were in some ways repetitions of those of Leonardo.

My wife has helped me in all possible ways, including the undertaking of her own research into the geometry of Leonardo. In this monograph, I am happy to include her study of geometry in the Ms G. But the most important contribution from her has been her constructive criticism blended with an unshakable faith in this work.

Mr. Mark Wilson, of the Iowa Institute of Hydraulic Research, has assisted me very efficiently in the production of the manuscript master copy. His cordiality at all times is very much appreciated.
REFERENCES


The general introduction, under the same title as the book, makes for very good reading. Among other things, it refers to the effects of the philosophical discussions by Greek scholars after they became familiar with the achievements of Babylonian mathematics and astronomy. The Greek thinkers became familiar with the great difficulties inherent in the concepts of continuity, motion, infinity and measurement. The challenge was met and the result was the theory of the geometrical continuum at the time of Eudoxus (see p. XV). The final sentences of this introduction deserve quotation in full: Fortunately creative minds forget dogmatic philosophical beliefs whenever adherence to them would impede constructive achievements. For scholars and laymen alike it is not philosophy but active experience in mathematics itself that alone can answer the question: What is mathematics?.


It is doubtful that Leonardo could ever have seen this kind of centrifugal pump elevating water at any significative height, had he constructed one. Boldetti used an electrical motor to run his model, and probably the speed used in the model was far superior to what Leonardo could have produced. As for Leonardo not providing blades for this "invention" (see p. 46), I would examine thoroughly all his manuscripts, were I interested in machines and gadgets, rather than in general scientific principles.


Strachey discusses the mistranslation of nimbio. It is not clear if it was due to Freud directly, or to some source he consulted; the fact is that instead of Milan, Geier was used. Strachey warns about following the impulse to dismiss the entire work because of this mistake (p. 6).


The use of the wrong version of a passage mentioned in the text of this monograph, continues to this day; see an article by C. E. Schorske, Freud's Egyptian Dig, in The New York Review of Books, May 27, 1993.


Unknown to Gombrich were, perhaps, the publications of Arredi and Giacomelli, to whom he could have referred as the initial steps of the body of work he was requesting from the historians of hydraulics. Unfortunately, historians of science have usually stayed away from events in the field of flow and transport phenomena. Arredi and Giacomelli were both engineers, and it seems that only engineers (or applied scientists) can work with some success in the history of fluid mechanics. Can this be due to the fact that little fluid mechanics is taught in physics courses for non-physicists or non-engineers?

See p. xxiii, where Gray states that Freud used a mistranslation of *nibbio*, a serious flaw by which the structure of Freud’s speculation collapses.

See, e.g., on p. 24: Ogni azione fatta dalla natura non si può fare con più breve modo co’ medesimi mezzi. The notion of minimization of some action, property, or function has been attractive for a long time. In the Ms G 75R we have a discussion of falling bodies in which Leonardo makes an application of a principle of minimal length of a path.
The author remarks that Until now the work of artisans was in the blind spot of the historian of science, and ends his contribution with the following statement: If we do not open our eyes (to see) what was going on among practicians, we are in danger, at any time, of overlooking facts only because they are outside the academic world, but are so important as demanding that we rewrite the fundamental point of the history of science. Perhaps Ono saw Leonardo as a man who could develop science starting from his own experience as an apprentice in an art workshop.


In this book, Keynes included a chapter on The Nature of Argument by Analogy, which is surely worth studying, especially by those who emphasize the positive analogy to the detriment of the negative analogy.


Niels Bohr introduced the name of correspondence principle for a thesis of continuity in science or progress.


Kuhn begins his Preface, with the sentence: "The essay that follows is the first published report on a project originally conceived almost fifteen years ago." At that time he was a graduate student of theoretical physics working on his doctoral dissertation thesis. In those years, Kuhn certainly gained much insight into what was going on in science. Even if one may not agree with some of Kuhn's conclusions, it is clear that not many researchers have at the end of their lives, a vision like the one he gained early in his life. For a different notion about the role of paradigms in science see Courant [1943].


This text of this article was included in Kuhn 1977. There are, however, very interesting footnotes in the Daedalus issue. Footnote 3, for instance, points at a very interesting
difference between critics in art and in science. In science, the criticism is usually done by scientists (how could it be otherwise ??). Kuhn notes that Historians usually rely exclusively on these works of "criticism" (italics and quotation marks by Kuhn). In Footnote 4, Kuhn advises the reader to read a book by C.C Gillispie [1960] if he is interested in someone who knows the science and its history.


The Preface of this book makes very interesting reading. In part, Kuhn responds to criticism of his approach and ideas, and in part he narrates his experience as he left the field of physics to become a historian. He also gives an introduction to papers included in this book.


Lilley stated that two traditions must be taken into account when we consider the experimental method: the university tradition and the craftsman tradition.


This paper was originated by an innovation in the teaching of fluid mechanics to engineering students, using observations in nature, man made hydraulic systems, and the laboratory. Inspiration to do so came from a course by Prof. R.W. Pohl [*Einführung in die Physik*, Göttingen] Qualitative analysis was a requirement in my course, before the learning of dimensional analysis and theory of models.


See References to Leonardo's writings on page 9 of the Introduction. See also Modeo 1993 (pp. 15-18) for additional indirect references to Leonardo's writings.


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MODEO, Sandro 1993. La mente per il tutto. La Rivista dei Libri, Luglio/Agosto 1993, Milano. [See also Maiorino 1992].


See Section 3 for an account of Freud's deep involvement with Italian culture.


One of the interesting points in this paper is the distinction between experience and experiment. Reading this paper may help to understand Leonardo's writings regarding such and other points.

VANONI, V. 1990. Private communication.

A paper is being written in which his contribution will be reported in detail.


See comments on p. 21 about the role of historiographic conventions. They are important to me, as I adhere to them believing that otherwise I would do a poor job without feeling constantly that I must be as objective as possible, and that, trying to give a synthesis of Leonardian fluid mechanics, I cannot allow myself to make of it a mere narration.


Zubov believes that Leonardo antedated Galileo in the fusion of the craftsman and the university tradition; he, of course, grants that they proceeded with a great difference of sophistication [see also Lilley 1953].
<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
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<tbody>
<tr>
<td>ANALOGIES IN THE MANUSCRIPT G</td>
</tr>
</tbody>
</table>

| 1 | 10R T1-2 | Bodies floating in a river. Clouds floating in the air. Lump of wax flowing between the fingers squeezing it. |
| 2 | 38R T3-4 | Analogy between the behavior of fresh and salt water at sea-earth interface and in a piece of cloth. |
| 3 | 50V T1 D1-2 | Analogy among the flows around boats, fish, and birds. |
| 4 | 69R T1-2 D1 | Some analogies and differences between the flow of air and the flow of water. |

<table>
<thead>
<tr>
<th>ARIA</th>
<th>ACQUA</th>
<th>VAPOR</th>
<th>FLOW</th>
<th>FLUV</th>
<th>WIND</th>
<th>BOMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACQUA</td>
<td>TERRA</td>
<td>FL P SU TE</td>
<td>PERCOL</td>
<td>SEA</td>
<td>EXPER</td>
<td></td>
</tr>
<tr>
<td>ACQUA</td>
<td>FLOW</td>
<td>BOMO</td>
<td>BO FLO</td>
<td>BOAT</td>
<td>ANIMAL</td>
<td></td>
</tr>
<tr>
<td>ARIA</td>
<td>ACQUA</td>
<td>COMPR</td>
<td>CANAL</td>
<td>IMPACT</td>
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<p>| | | | |</p>
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</thead>
<tbody>
<tr>
<td>5</td>
<td>The capillary flow in a layer of soil is compared to the flow in a strip of fabric of similar shape.</td>
<td>ACQUA TERRA FL P SU TE SIPHON</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>The persistence of an image in the retina is compared to a lasting sound in air and to the conservation of motion of a body.</td>
<td>BOMO OPTIC ACOU</td>
<td></td>
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<tr>
<td>7</td>
<td>Analogy between the &quot;domino effect&quot; in bricks and the human affairs.</td>
<td>BOMO BO FA EXPER LIV SYS</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Analogy between the flow of heat and light, the intensity of impact between solid bodies, and the degree of concentration of substances dissolved in water</td>
<td>ACQUA THERM OPTIC MIX BOMO IMPACT</td>
<td></td>
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<tr>
<td>9</td>
<td>91V</td>
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<tr>
<td>T1-3</td>
<td>D1-3</td>
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<tr>
<td></td>
<td><img src="image" alt="Diagram" /></td>
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</table>

Similarity between a cloud rotated by an air current and the wheel of a water mill.

<table>
<thead>
<tr>
<th>10</th>
<th>ACQUA</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAPOR</td>
<td>ROTAT</td>
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<tr>
<td>WIND</td>
<td>W WHEEL</td>
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<th>11</th>
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<th>12</th>
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<td></td>
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</tbody>
</table>
| 1 | Two different weights attached to the ends of a string mounted around a pulley hanging from a balance. Presumably, Leonardo experimented with two degrees of freedom of this system, as he did in Ms G 76V, 77R. | 13V | T2 D1
    |                                                                           | 17V | T3-4 D2|
| 2 | A balance is subjected on one side to a static load while the other side supports a dynamic load consisting of a pendulum. Several cases are considered: simple pendulum, conical pendulum, etc. | 76V | T4-5 D2
    |                                                                           | 77R | T2-3 D2-5|
| 3 | Body whose driving motor suddenly stops. It seems that Leonardo assumed in this case a rigid link between the motor and the body. | 86R | T1,3-4|
| 4 | "Chain reaction" of several falling bodies. Leonardo saw an analogy between the mechanical effect in case and human behavior. | 89R | T3 D3|
## TABLE II
### EXPERIMENTS AND EXPERIMENTAL SITUATIONS IN THE MANUSCRIPT G

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BALLISTICS</strong></td>
<td>Observation of the path of a projectile. The initial portion of the path is assumed to be horizontal. Was he influenced by an Aristotelian received knowledge? A three-piece approximation to the path of a projectile was popular for a long time.</td>
<td>77R T1 D1</td>
</tr>
<tr>
<td><strong>BALLISTICS</strong></td>
<td>Projectile formed by two bodies attached to the ends of a cord and launched by means of different devices.</td>
<td>86V T2 D1-4 87R T3-5 D1-2</td>
</tr>
<tr>
<td><strong>MECHANICS OF FLUIDS</strong></td>
<td>The capillary rise and the subsequent evaporation in a strip of cloth, are used to determine the concentration of a given substance in water. One must wait until the strip is dry</td>
<td>37V T2</td>
</tr>
<tr>
<td><strong>MECHANICS OF FLUIDS</strong></td>
<td>Comparison of the capillary rises in two strips of cloth to demonstrate the difference in concentration of two water solutions.</td>
<td>38R T3-4</td>
</tr>
<tr>
<td>Experiments and Experimental Situations in the Manuscript G</td>
<td></td>
<td></td>
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<tr>
<td>-----------------------------------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>MECHANICS OF FLUIDS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emptying of a vessel by a hidden automatic siphon. The physical principle of the &quot;man-made&quot; Tantalus cup.</td>
<td>40 V T1-3 D1-3</td>
<td></td>
</tr>
<tr>
<td><strong>MECHANICS OF FLUIDS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Siphon with a minute rate of flow of mercury. Apparently, this was a clock-device intended to cause a fire after a long time (!!).</td>
<td>44 V T1 D1 48 R T2-3 D2</td>
<td></td>
</tr>
<tr>
<td><strong>MECHANICS OF FLUIDS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A jet comes out from an orifice in the bottom of vessel which is being moved horizontally. [See Macagno 1992].</td>
<td>54 V T1</td>
<td></td>
</tr>
<tr>
<td><strong>MECHANICS OF FLUIDS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flow around bodies of different shapes. Several bodies are included; some of them, streamlined. One looks like a fat Joukowsky two-dimensional hydrofoil</td>
<td>50 V T1 D1-2</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td><strong>MECHANICS OF FLUIDS</strong></td>
<td></td>
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<tr>
<td></td>
<td>Cards falling through air. Leonardo indicates that he used a slightly curved card. (See next entry for similar experiments.)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>14</th>
<th><strong>MECHANICS OF FLUIDS</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fall of boards through air, either by themselves or with a man hanging from them and maneuvering them. (Is this related to the studies of parachutes ?)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>15</th>
<th><strong>VISUALIZATION OF MOTION</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Visualization of motion by retinal-image retention or persistence. This method is applied to several rapidly moving sources of light.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>16</th>
<th><strong>FLOW VISUALIZATION</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flow visualization by addition of die in the case of two streams impinging on each other.</td>
</tr>
<tr>
<td></td>
<td>Experiment Description</td>
</tr>
<tr>
<td>---</td>
<td>----------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>17</td>
<td>Flow visualization by means of neutrally buoyant solid tracers floating amidst flowing water.</td>
</tr>
<tr>
<td>18</td>
<td>Erosion at the foot of a waterfall. The study included the vortex system involved in the scouring.</td>
</tr>
<tr>
<td>19</td>
<td>Possible effect of a difference in temperature on the equilibrium of a balance from which cold and warm water vessels are suspended.</td>
</tr>
<tr>
<td>20</td>
<td>Effect of heating on the initiation of flow in a layer of varnish applied to a plane surface.</td>
</tr>
<tr>
<td>Table II: Experiments and Experimental Situations in the Manuscript G</td>
<td></td>
</tr>
<tr>
<td>---------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td><strong>Flow of blood in the heart, its valves, and the aorta.</strong></td>
<td></td>
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<tr>
<td><strong>BIOFLUIDMECHANICS</strong></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>1V</td>
</tr>
<tr>
<td><strong>Respiratory flow of air through the nose and the mouth.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>BIOFLUIDMECHANICS</strong></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>96V</td>
</tr>
<tr>
<td><strong>Other experimental situations may be included.</strong></td>
<td></td>
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<tr>
<td>23</td>
<td></td>
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<tr>
<td>24</td>
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</tbody>
</table>
LEONARDIAN FLUID MECHANICS
IN THE MANUSCRIPT G

by
ENZO MACAGNO

Iowa City, 13 August, 1993
The feet of trees have globular surface which is caused by the roots sending nutriment to such a tree. Those surface globbosities produce rare crevices in the bark. The bark spaces in between have the concavity with fissures because they receive lesser amounts of nutriment.

This passage should be related to others in which Leonardo referred to the flow of sap in trees. See, e.g., Ms I 12R-V and also Ms G 17R, 34 V.
The heart is a main force muscle and it is very powerful relative to the other muscles.

Write about the location of the muscles that go from the base to the point of the heart and of the muscles that originate at the point and go to the top.

The auricles of the heart are its vestibules. They receive, from the beginning to the end of its contraction, the blood from the heart when it flows out of its ventricle. This is so because, without such partial outflow, the heart could not possibly contract.

Here it is asked whether the subdivision of a triangle can destroy its figure when such subdivision is unending. If the figure of the triangle is destroyed, without doubt the three sides of the triangle (?) concur in one point which is not true (?)

Never can the three sides of a pyramid, with a triangular base, concur to a point. Here is the proof: If they would concur, such point would be divisible, which is against the definition of a point in geometry. If one of its sides concurs to a point to which the three sides do not concur, without destruction of its triangular figure . . . . .

The last sentence in T6 seems to have been left unfinished. In T5-6, I believe that Leonardo is reflecting received knowledge. In my opinion one should not attribute to this passage a significance out of proportion as some writers have done. From the viewpoint of this investigation, however, any notions Leonardo may have developed, or acquired, concerning mathematical limits are certainly of interest, even if they are rudimentary for our times. What would be of great interest is to see whether Leonardo ever had any intuition about the limit of the ratio of two quantities, as in the case of studying the notions of force, area, and pressure [Macagno 1987].
Figuration of the deluge

The air was dark due to the thick rain falling obliquely because of the thrust of the wind coming across it. The rain came in waves through the air not differently from those of dust, although it differed in that such flood was crisscrossed by the lines drawn by the water drops as they fell. Their color was tinged by the fire generated by the thunderbolts that splitted and disintegrated the clouds. The flashes of lightning impacted and opened the "pelaghi" of the great filled valleys; such openings showed in their depths the bent tops of the trees. Neptune could be seen with his trident in the midst of the waters while Æolus with his winds turned over the eradicated trees floating in the immense waves. The horizon and the hemisphere were misty and fiery with the lightning of the unending thunderbolts.

Men and birds were seen crowding the great trees still uncovered by the great waves embracing the hills surrounding the great chasms.

Another, longer, passage on the deluge can be found in Windsor 158R-V. One should examine the several drawings at Windsor describing catastrophic events. Of course, from the point of view of fluid mechanics a special approach will be needed for an analysis of passages like the above one. For instance, the waves in the air seem to relate to some structure of turbulent flow visualized by either water drops, mist or dust. Leonardo, in his notes, refers more than once to the visualization of turbulence by the dust suspended in the air.
Ms G 6V

T1

ACQUA
ARIA
FOCO
GRAN
DROP
METEO
WAVE
WIND
ART
The wind. The air flows like a river and carries in it the clouds in the same way flowing water transports those things that are sustained by it. This is proved by considering that, were the wind penetrating the air and pushing the clouds, these clouds would be condensed between the air and the motor, and they would acquire lateral 'inpeto' like the wax enclosed between the fingers (of our hand).

Of the motion of the air
The air flows when it is pulled to fill the vacuum or driven by the rarefaction of the humor of the clouds.

In T1, note the two analogies: one between water flow and air flow, and the other between fluid flow and the flow of a plastic material as it is subjected to the movement of the walls containing it.

In T2, I found difficult to determine the meaning because it seemed to me that both vacuum and rarefaction at some place should produce the same effect, of flow towards that place.
<table>
<thead>
<tr>
<th>T1</th>
<th>T1-2</th>
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<tbody>
<tr>
<td></td>
<td>ACQUA</td>
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<td></td>
<td>ARIA</td>
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<td>FLOW</td>
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<td>FLUV</td>
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<td>T2</td>
<td>WIND</td>
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<td>METEO</td>
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<td></td>
<td>VAC</td>
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<td>D1</td>
<td>INPETO</td>
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<td>T3</td>
<td>ANAL</td>
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</tbody>
</table>
As it runs in water, the horse that is deeper in water will produce less foam. The one that is less submerged makes more foam. This is so because the legs that are less submerged are less hindered and become speedier and thus, having more speed, they push more the water than the knee and the thigh.

The flow and spray due to a body moving through water, or the similar case in which the body is fixed and water flows around, has been analyzed for a variety of Froude numbers, and there seem to be enough work done already to serve as a guide for the analysis of the above passage.

Of the science of weights.

The 'grave' that descends freely does not give by itself weight to a given support. Here is the proof: Let $a$ be one and $b$ be 2; it follows that $m$ sustains only 2 because the excess of 2 $b$ with respect to one is one, and such one not having a support in $a$ falls freely. Hence it is without support, and not having it nothing prevents it from moving. Hence $m$, end of the balance, does not sense such an excess, because it is falling and it is not supported.
Diagonal (flow).
The water flowing in straight rivers is always of great or small inclination depending on having great or small velocity.

Water.
The water flowing down a straight river has always a diagonal motion from the middle to the bank facing it and from the side facing it to the middle of the river. This is proved by the ninth of this (book ?) where it says: the water in straight rivers is always higher in the middle of their width and on the sides than in between the middle of their width and those sides. This was proved in the seventh where it says: the water of straight rivers never flows along a straight line because it is speedier the farther is from the banks, (which are) its hindrance. And this was proved there where it says: where its flow is impeded, there the reflected motion is generated. According to the tenth of this (book ?): the minimal level of the water in the width of the river is always beween the incident motion and the reflected motion. And according to the eleventh: after the last elevation of the reflected water the beginning of the incident motion is generated. And by the twelfth: the incident motion of the water does not change into the reflected motion without impacting either the bottom or the side of the river; where the water impacts either the bottom or the side of the river, earth is eroded from the bottom or the side of the river; under the incident motion there is always erosion of the bottom of the river, while under reflected motion the bottom of the river recovers its elevation.

The flow to which Leonardo refers here in T1 and T2 seems to be supercritical flow in a straight channel (much like the ones represented with great detail in the Ms C [see Macagno 1988]. At this instance, he seems to confuse the configuration on the surface due to diagonal waves with the water flow; however after some time, he was able to realize the difference between the kinematics of waves and that of the water flow. There is a remarkable drawing in the Codex Arundel (see Folio 23V) showing this distinction. There are several questions which deserve further study in this page. One is the discussion about the elevation of the water which may be correct in spite of the confusion between wave kinematics and water kinematics. At the point where the wave is incident on the wall and it is reflected there should exist an increased elevation.
Of proportions.
If from two similar (figures?) we remove similar parts the ratio of part to part will be the same as the ratio of total to total.
It follows that, being these two circles one double the other, the portion that occupies one fourth of the larger is twice the portion occupied by a quarter of the smaller.
Such proportion is valid for remainder to remainder as it is for total to total. And also for portion to portion as for remainder to remainder.

The two circles having four points in common with the same square are one twice the other.
The two squares with four points in common with the same circle are also one twice the other.

Between one ramification and the following one, lacking especial branches, the cross-sectional area remains the same. This is so because the total amount of humor that nourishes the base of such branch nourishes also the following branching. Equal nutrients produce equal effects.

In T1, I have rendered simili as similar after examining the figure D1 together with the text. In some cases Leonardo means equal when he uses the word simile; I believe this not true in this case.
In T2, Leonardo means that the circle circumscribing a square is twice in area the one inscribed in the same square.
T3 should be analyzed in relation to other passages on the flow of sap in trees (See MsG 1R, MsI 12R-V and other notebooks). In T3, Leonardo states the same principle he stated in the Ms I, i.e., assuming conservation of flow rate in sap, there should be conservation of cross-sectional area along the tree.
Ms G 17R

T1
D1
T2
D2
T3

D1

TI-2 D1
GEOM
CIRC
SIMIL

T3 D2
LIQ
FLOW
CONSER
PLANT
Geometry.
The circle through the three angles (vertices) of the equilateral triangle is thrice the triangle tangent to three sides of the same triangle.
The diameter of the largest circle on the triangle is two thirds of the axis (height) of the same triangle.

Experiment for which there is a question below.

Question about falling weights.
It is asked whether the weights, falling from the pulleys, produce from themselves more or less weight on the axes of those pulleys when they fall than when they are fixed.

In the second part of T1, Leonardo says that the diameters of the two circles in figure D1 are one twice the other, while in the first part he states that the corresponding areas are one thrice the other. It is easy to find errors in elementary geometry in his notes; he was better at developing new geometric notions [Macagno M. 1992]. Errors in the arithmetical calculations of Leonardo have been well documented by Marinoni [1982].

T3 is a note under the figure D2. In T4, we find a question, for which an answer is provided in Ms G 13V.
<table>
<thead>
<tr>
<th>T1</th>
<th>D1</th>
<th>D1</th>
<th>TI D1</th>
<th>T3-4 D2</th>
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<td>T2</td>
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<td>GEOM</td>
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</table>
Pico gave those opinions.

Light-beam from Moon with image of the Sun

If the Sun $f$, mirrored in a water surface $nm$, seems to be in $d$ (that is, seems to be as much beneath water as it is above) and, to the eye $b$, seems to have the dimension $a$, and, moving the eye from $b$ to $c$, the image $a$ doubles (in size?), how much would the image increase if the eye were to move from $c$ to the Moon? Use the rule of three and you will see that the light that the Moon has in plenilune cannot ever be the light received being spherical. Hence it is necessary that the Moon be water. n

Marinoni [1989] suggests that Leonardo may have meant Pico della Mirandola, while Ravaisson-Mollien considered T1 illegible. My conjecture is that, if Marinoni is right, perhaps somebody could find the notions in T2 in some of the writings of della Mirandola.

In the second part of T2, quintadecima means the fifteenth day after the novilune (new moon). Perhaps, this was an opinion of della Mirandola.
Ms G 20R

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<th>T1</th>
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<td>SOURCE?</td>
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</table>
Of smoke in cities.
The smoke is seen better and more neatly in the oriental than in the occidental part when the Sun is at the orient. This is due to two causes. The first is that the Sun rays are translucent in the particles of such smoke thus making them bright and evident. The second is that the roofs of the houses seen at orient at such time are in shadow because of their inclination which prevents the Sun from illuminating them. The same happens in the dust, and both are moreuminous when they are more dense, and more dense towards the middle.

Of smoke and dust.
When the Sun is at the orient, the smoke of the city will not be seen at the Occident, because it is not seen when the light rays penetrate it. It is not seen either when it is in a dark place because the roofs of the houses show to the eye that same part that it is exposed to the Sun and in such bright place such smoke is hardly seen.

But the dust in similar situation appears as darker than the smoke because it is of heavier material than the smoke which is more humid.

This passage extends over two pages of the Ms G, but with such a unity that it has been represented by a single profile. To be sure, it belongs more to the field of painting than to that of fluid mechanics, but some aspects are of interest because some optical physical properties of a fluid with suspended material in its midst are discussed.
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<td>LIGHT</td>
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</table>
Leaves always turn their direct side, with all their surface towards the sky, so that they better receive, with all their surfaces, the dew that slowly comes down from the air. Such leaves are distributed all over the tree in such a way that they do overlap the least possible in the same way that one sees in ivy-covered walls. Such a distribution satisfies two conditions, i.e., (one is) to leave spaces between them through which the air and the sunlight can penetrate, and the second is that the drops falling from the first leave may fall over the fourth and the sixth of the other trees (?)

I have rendered the end of T2 as I can understand it, but it does make little sense to me. The logical way of ending this text could have been, in my opinion: "...the second is that the drops falling from the first leave may fall over the other leaves of the tree".
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<td>DISTRIB</td>
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</table>
Of the difference between force and weight but first, of the force.

Between the spring and the counterweight with equal 'potentie', the spring counts more because its 'potentia' is pyramidal. Its maximal 'potentia' is at the beginning of its motion. But the counterweight has composite 'potentia', one columnar and the other pyramidal. The columnar means that the weight is constant and pulls with equal 'potentia' at the beginning as well as at the end of the motion. But the pyramidal begins as instant and point and then, in each degree of the motion acquires intensity and velocity; its motion is free and fast. But in the slow motion of the 'grave', the pyramidal 'potentia' ceases and only the columnar 'potentia' remains; this, as already said, has the same value at the beginning, in the middle, at the end, or in any other point of its motion, etc.

Note that Leonardo uses here the terms pyramidal and columnar to describe variations of a quantity in terms of another. I believe this is one of the influences, perhaps indirect, of Medieval geometers and kinematicists on the notes of Leonardo.

The main topic is one of mechanics, and it seems easy to interpret if we use the notions that within certain small region the weight of a body is a constant, while the force of a spring varies strongly as a function of its length. This is one of the passages included in this series of monographs because of its importance as a basic notion in relation to fluid mechanics.
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In the tree branches, the lower small branches grow more than the upper ones because the sap that nourishes them is affected by gravity and has easier movement downwards than upwards, and also because the lower ones are farther from the shadow which is near the center of the tree.

This statement should be compared with Ms I 12R-V where Leonardo appears to consider that the sum of the cross sections of all branches at different levels is the same.
The semblances made by things in motion, i.e., over their field, so that they, in their motion, appear as they are not.

The water droplets in the rain, the spools and rotating wheels, the stones under flowing water, and the firebrand turned around in a circle appear as continuous in what they are discontinuous.

Cicatrices in the trees.
The cicatrices in the trees become thicker than required by the sap provided for their nutriment.

In T1-2, Leonardo describes one of the oldest visualizations of motion due to the time the image of a given point remains in our retina. One must be careful with his statement about continuity and discontinuity. If instead of using the eye, we register the motion with a multi-exposure, very fast-speed camera we see something discontinuous if the interval of exposures is long enough. I believe this may help to understand T2.

In T3, I have followed Ravaisson-Mollien in rendering "margini" as "cicatrices". This choice seems to be supported by the text of Ms G 1R T4.
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<td>FLOW</td>
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</table>
To test the light waters.
One can determine the density (?) of waters by suspending with equal levels the opposite ends of a clean old linen cloth strip. The strip must penetrate to the bottom two vessels filled to capacity by the two waters under experimentation. Then those waters will climb up in the cloth and little by little the cloth will dry up; thus more water will rise up, until the vessels become empty. If you refill the vessels, all the water will go up very slowly and at the same time it will evaporate as said before. In this way the strip will become filled with the residue of the evaporated water. In this way, by determining the increase in weight, you will be able to find out which water has more earth (salt?).

The method here described by Leonardo seems very clumsy, because evaporation of the water in order to separate it from the dissolved or suspended solids could be obtained by evaporation due to heat in a more efficient manner. Incidentally, I have been unable so far to find any reference in his notes to hydrometers, which were in use already many centuries before Leonardo.
Ms G 37V

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<td>ACQUA FL P SU TE VAPOR MEAS EXPER</td>
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</table>
How is it that the sea does not penetrate into the Earth. That the sea does not penetrate into the Earth is shown by the many different veins of fresh water that, in different places at the sea, penetrate it from the bottom to the surface. The same is also shown by wells, dug at beyond the distance of one mile from the sea, which become filled by fresh water. This happens because fresh water is lighter than salt water, and hence is more penetrating.

Which one weighs more: the frozen water or the one that is not frozen.

Fresh water penetrates more against the salt water than the salt water against the fresh one.

That the fresh water penetrates more against salt water, than the salt water against the fresh water, is shown by a light old dry cloth hanging with equal level of the two ends inside the two different waters, provided the water surfaces have the same level. Then the fresh water will rise in the strip so much more than the salt water the lighter is the fresh water than the salt water.

Note that Leonardo believes that his experiment with a strip of cloth gives the clue for the encroaching of salt water on fresh groundwater close to the sea (See Ms G 37V). Note that in both cases he deals with porous media; is that enough to make the analogy justifiable?
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<td>ACQUA</td>
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<td>FL P DENS</td>
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<td>TERRA</td>
<td>THERM</td>
<td>FL P DENS</td>
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<td>ANAL</td>
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</table>
On a plane, let us make the straight line \( rS \) with the motion of the fourth of the great circle of the sphere; undoubtedly, the true linear length will occupy \( rt \), similarly to the \( rS \), at right angle in \( r \). Let us make the curve \( Sxt \). Then divide \( rS \) in as many equal parts as you wish and mark the indefinite circumferential portions \( cy, hz, Li \) going over the line \( rt \). Then take paper and cut it and move the circumference of the cut circle upon the marked circumference \( cy \) and go beyond the line \( rt \) indefinitely, as long as there is contact of the moving circumference with the fixed circumference, and mark such end. Do the same with the following circumference \( hz \), and also with all the other circumferences. Mark the ends of the motion for each circumference. Along such end points draw the curve \( rvt \).

If you would make such parallel motions along the straight lines of the third or fourth figure it would be a better approach, because the wheel (made of paper) rolls better over straight lines than over curves, given that the straight parallels be of the same length as the curved parallels. Then you will make with motion the quadrature of the portion of the circle with the said motion; i.e. the portion \( aci \) taken from the triangle \( abc \), and after the quadrature add it to the triangle \( abc \) and the sum will be \( 1/8 \) of the surface of the sphere, etc.

I found necessary to examine critically the discussion of this page by Marinoni [1989], because it seems to have escaped to him that in T1-2 above there are three procedures for the determination of the area of a sphere which lead to different results. (See in the Introduction of this monograph the section on geometry).
Ms G 39R  T4 D3-4

Quadrature of the hemisphere.

Let us rectify by means of motion one half of the periphery (circumference) \( pm \) in the straight segment \( nm \). And let us rectify along the (one-fourth of) great circle \( qm \) along the straight segment \( mo \), at right angle with the first segment \( nm \). Let then rectify the second circle \( ab \), which gives the segment \( bc \) parallel to the line \( mo \), a let us do the same with all the circles which divide the hemisphere \( qpm \).

semi-diameters (radii?) to be rectified.

\( \Phi \)ra del semisperio

Sia dirizzato la meza periferia \( pn \) nella retta \( nm \) mediante il moto e sia asteso in lungho il magior cierchio \( qm \) nella recta \( mo \) in congiuntione rettanghula cholla \( pa \) \( nm \) di poi sia asteso il \( 2^0 \) \( O \) \( ab \) in equale e lla linja parallella \( bc \) cquidistanz alla linja \( mo \) e chosi seghuj in tuttj li cierchi in che djuijdi il semisperio \( qpm \)

e semjdiamjtrj de cierchi che ssanno ad dirizzare.(T4)

T4 D3-4 shows one additional procedure with respect to Ms G 38V. The same commentscare also valid for this passage. The last sentence in T4 refers to figure D4.
Ms F 39R

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T4 D3-4
GEOM
SUBDIV
MAP
INTEG
BO MO
KINEM
The circles with a common center will be one twice the other, if the square placed between them makes contact with each of them.

The squares with common center, and with a common circumference in between that touches the two of them, will be one twice the other.

Proof: of the eight triangles forming the large square four are contained in the small.

Such ratio holds from circle to circle that holds from square to square which are made through multiplication of their diameters.

Property of the right angle that rules the proportions.

Of all the portions of circles in contact with a right angle always the larger equals (?) all the smaller; and of all the parallels that contain such portions, always the larger contains and is equal (?) to all the smaller within such right angle abc.

Note that T1 refers to D1 and T2 to D2. In the case of T3, D3 satisfies his statement, but so do other pair of circles and squares not related in the same way as as the ones in D3.

T4 is not entirely clear to me, but if Leonardo meant what I understand fir D4, then he was wrong. (See Introduction).

There pages devoted to geometry in the Ms G which I have not included and will not include because I only work on those which may be of interest from the point of view of their application to kinematics, mechanics, and fluid flow sciences. (See section on Geometry in the introduction to this monograph.)
<table>
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<th>T1</th>
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<th>T1-4 D1-4</th>
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<td>T2</td>
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This device is siphon-like.

In these layers (?) of the drinking glass, the illusion is made of wine which in the rest of the glass flows out from the bottom.

Either water or wine flows in through the orifice $a$, and goes up through the channel $ab$, and comes down through the second channel $bc$; all the wine above the orifice $a$ discharges through $bc$. This channel can be shown to be needed to discharge all the wine by the fourth of the conduits called siphons.

Even if waters come from the same source, one will be heavier than the other if it acquires more heat than the other. The waters that acquire cold become lighter.

The heavier the water the noisier its impact will be. The warmer the water the heavier will be.

When I was in highschool, we were taught about Greek mythology; I remember very well, the Tantalus cup, or was it a pool in which he was placed; anyway, when he thought that he was ready to drink, the water, or wine, would recede. I do not remember if this was due to a hydro-mechanical device or due to the will of the gods who punished the unfortunate king.

In T4-5, I do not believe Leonardo was right. We know now that the density of water is not a simple function of the temperature; the density has a maximum at 4 °C, and liquid water at 0 °C has the same density as water at approximately 8 °C. It is very doubtful that Leonardo had any knowledge of this, which makes untrue the statement: "when we heat water its density decreases". It also makes untrue the opposite statement. It is true for only temperatures above 4 °C. Most probably, Leonardo was referring to this range, and then T4 is wrong.
<table>
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<th>T1</th>
<th>D1</th>
<th>T1-3</th>
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- **D1**: Diagram of a siphon-like device.
- **D2**: Diagram of a flow device.
- **D3**: Diagram of a thermometric instrument.
- **D4**: Diagram of an experiment setup.
Oil lamp, whose wick goes up as much as the oil goes down. This happens because the wheel that raises the device floats on the oil. The wheel descends as much as the oil diminishes. The wheel turns due to a string wound around its axle and the teeth of the wheel drive the tube with teeth that contains the wick.

Lucerna che quanto chala lolio tanto si inalza lo stoppino suo e cquessto nasscie perché la rota che inalza il disegno si sosstiene sopra lolio e cquanto e oljo diminuisscie tanto la rota dissciende ed dissciendendo gira in se medesimo mediante il filo che ssi suolgie dintorno al suo polo e lli denti della rota spincie la channa dentata che ricieve lo sstoppino. (T1)

Yet, the same will do the axle $a$ of the wheel that does not descend while only the floater $b$ sitting on the oil descends together with the level of such oil. The floater drives the wheel which with its teeth drives slowly the aforesaid toothed tube, etc.

anchora fara il medesimo sel polo $a$ della rota non dissciendera ma sol disscienda la leuja $b$ che ssta a ghalla sopra lolio la qual levita dissciende insieme cholla superficie desso olio e fa voltare la rota la qual cholla sua dentatura spinfie in alto con lente moto la predetta channa dentata, etc. (T2)

The pyramid divided at half its height, is divided in seven eigths of its quantity (volume?). The proof: the entire pyramid is divided in 12 pyramids, of which there are 8 which form the body with eight faces which is equal to 4 of the large pyramids.

La piramjde divisa per altezza chon taglio equallymente disstante alla sua basa e diujsa ne sette ottavi della sua quantita provasi neche il tutto si diujsed in 12 piramjde delle qualj vene 8 che conponghano il corpo dotto base che vale 4 piramjde delle magiori. (T3)

In fact, the description of these lamps, while interesting as gadgets contributes very little to fluid mechanics, and I have included T1-2 D1-3 only for the sake of completeness. As in other cases concerning machines and devices, Leonardo may have been describing existing mechanisms.

In T3, Leonardo clarifies a question that stems from a number of other notes about the subdivision of the pyramid. In this case he may be describing received knowledge, because the quantitative question about this subdivision is old (and I believe trivial also) while the qualitative question of partition in all congruent pyramids in analogy to the subdivision of a triangle is the subtle question [Macagno E. ?? and Macagno M. 1987, 1990].
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**Diagram**:

- **D1**: Abstract image with a circular shape and multiple elements.
- **D2**: Abstract image with a complex structure and various components.
- **D3**: Abstract image with a geometric shape and intricate details.
- **D4**: Abstract image with a geometric structure and multiple facets.
When the wind flows against the course of the birds, they always fly low. This shows us that the wind is more intense above than below. The adversary says that the wind hitting the Earth, suddenly acquires more density than before, and thus becomes more powerful and heavier.

When the bird is pushed by the wind, it lets itself go down along an inclined motion; when it wants to recuperate the initial height, it turns against with the velocity of the acquired 'inpeto'; this is consumed against the wind which acts as a wedge and raises it up even higher than the initial height. There, the bird goes down obliquely again, and then does the same as said above, and thus continually acquiring degrees of elevation, it arrives finally at the desired altitude.

The problems of flight of either machines or birds in the notes of Leonardo was well treated by Giacomelli [1936]; I only include, concerning aeronautics, some passages which I consider of interest in the context of the study of Leonardian fluid mechanics and transport phenomenology. Giacomelli's monograph contains some comments on the fluid mechanics related to flight, and is surely worth consulting.
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<th>T1</th>
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<td>D1</td>
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<td>KINEM</td>
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D1
Quadrature of the surface of a sphere.

Let us rectify the curves $ab$ and $ac$, which meet at right angle in the point $a$. Then let us draw the quarter of circumference using the segments $ab$ or $ac$ as semi-diameter. Beyond this, divide such semi-diameters in as many parts as you like and draw the curves $no$, $mp$, $Lq$. These curves, or curvilinear parallels, will exceed that quarter of circumference by all the portion $acS$; this originates as shown by the figure below with half the sphere $hiLKm$ and the cone $hiK$. The semi-sphere is made up of curvilinear pyramids, while the cone is made up of rectilinear pyramids. The difference between one and the other is shown by the portion $acS$ of the circle (figure ?). Quadrature of this portion must be done with motion of the curved side. The same you will do with the curved side of the quarter of circle $bc$. Add the two quadratures in a single square with the rule of the next to the last of Pythagoras.

Once you have performed the quadrature of this surface, divide it in six squares and construct the cube and you will obtain the cubature of the solid sphere.

In T1, in fact, a quarter of the circumference of a great circle needs only a rotation of a quarter of a turn to describe $1/8$ of the sphere. The distinction between circle and circumference is clear to Leonardo, and I have assumed that he wanted to make it in this case. Note also that $ab$ and $ac$ may indicate a curve or a segment of straight line of the same length. Once more, I tend to believe that he may have learned this from somebody (Luca Pacioli ?), and made a note from memory, including several mistakes. A detailed study is needed (see comments in my Introduction to this monograph).
Of the siphon.

The prepared mercury, when flowing through a siphon made of copper tubing which is extremely fine over all its length, where the liquid rises and descends, will be like a powder (sand?) clock. This is the slowest and finest fall (flow) possible; so much that in one hour less than one grain (idrop?) would flow from one vessel to the other.

And the surface of its bath is of sensible surface by means of the opacity of the mercury. The skin of that mercury should descend insensibly relative to the descent of the mercury pouring out though the siphon. Thus you will be able to generate by means of a percussion, within a year or even more. Until the fire starts this device is noiseless.

This is drawn on the margin of the fourth folio from this which shows how to mount such vessel so that it will finally do its work as expected.

A much more detailed drawing for the siphon device, in Ms G 44V D1, can be found in 48R D2, as indicated by Leonardo’s phrase: "E cuesto e disegnato in margine della 4a carta da piedi...". In the first part of T1 we find that the siphon seems part of a clock, while in the second we are informed that this device can be used for aggressive purposes. I am somewhat disconcerted by what seems to be a device to start a fire after one year, or more; I cannot see that this would not be a very evil action. Of course, from the point of view of fluid mechanics, I have performed some experiments with a thin capillary tube, whose results will be reported in another publication.
Ms G 46R T1 D1-2

Of 'potentia' of he voice.

In the proportion of the voice (area?) $ab$ to the voice (area?) $fn$, the ear at $cd$ is more powerful; it receives more voice (sound?) from $ab$ than from $fn$. This is so because $ab$ is almost infinitely larger than $fn$; hence the voice impacting the ear at $cd$ is infinitely larger than it would be if it were impacted by $fn$.

De potentia della vocie

Tanto quanto la vocie $ab$ riceve in se la uocie $fn$ tanto lorechio $cd$ he ppìu potente riceve piu vocie dal $ab$ che dal $fn$ e perche in acto $ab$ e quasi infinitamente maggiore che $fn$ la vocie perchotendo lorechio $cd$ si fa infinitamente maggiore che sella fussi perchossa dal detto $fn$. (T1)

At the beginning of T1, taking D1 into account, I believe that Leonardo meant a ratio of areas; hence an alternative is offered. I am old enough to have seen a device like the one depicted in D1-2 when I was a child.
Water.
Of the motion (propagation?) in a river of a sudden discharge into its dry bed.

The flow of the water from the drained lake to the dry river is slower or faster depending on the river being locally larger or narrower, flatter or deeper.

According to the above, the flux and reflux of the sea that enters the Mediterranean sea, and of the corresponding rivers, is of more or less excursion depending on that sea being more or less narrow.

Of the quicksilver siphon to ignite a fire.

When the water decreases in a vessel, the free surface is going down; hence the siphon discharges less as the water surface descends. But if the siphon would descend together with the water surface onto which it floats, there is no doubt that the discharge through the siphon would remain the same all the time. Hence, to satisfy this invariance, we will make the vessel \( n \) as a vessel that floats on the pool of quicksilver \( m \). The vessel \( n \) is like a boat for the siphon; through the bottom of such boat, the siphon coming from the air penetrates into the quicksilver. This quicksilver flows through such siphon \( n \) into the vessel \( f \). The descent of the quicksilver and that of the boat and the siphon are the same. The siphon is made of very fine polished copper tubing and feeds a vessel which, upon acquiring weight, will fall and, on impact, start a fire.

A negligible difference in level is needed from the quicksilver surface to the end of the siphon.

This is a detailed description of the device briefly mentioned on Ms G 44V.
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Why the water is salty.

Pliny in his second book chapter 103 says that the sea water is salty because the heat of the Sun burns and dries the humidity and absorbs it; all this gives to a large sea the salty taste. But this is not acceptable, because if the heat of the Sun would make the sea salty, so it would, undoubtedly, make lakes, reservoirs and marshes; they should be even saltier in the proportion that their waters are less mobile and of less depth. Experience shows the contrary: the waters of such reservoirs are entirely devoid of salt. In the same chapter, Pliny adds that such salinity could be due to the heat easily taking away the sweet and light component and leaving the rougher and coarser part; because of this the water at the surface sweeter than near the bottom. To this one can level objections based on the same reasons as above; i.e., the same should occur to the marshes and other waters that dried up because of the heat. It was also said that the salinity of the sea is due to exudations from the Earth. To this one can object that then all the water veins of the Earth should be salty. It is concluded that the salinity of the sea is due to the many water veins, that as . . .

...they penetrate the Earth, come upon the salt mines, dissolve part of the salt and carry it with them to the Ocean and to other seas where never go up clouds that generate the rivers. The sea should be saltier in our times than in any other (before). If the adversary would say that an infinite time would have dried up into salt the sea, one can answer that such salt returns to the earth as the earth goes up with such acquired salt and the rivers take it back to the submerged land.
Salt is extracted from places where the pigs urinate. The sea winds are salty.

To say it better, if the entire world is eternal, it is necessary that its population be eternal, hence the human species has been and will be an eternal consumer of salt. If the entire Earth would be salt it would not be enough for the human meals. Then, we have to admit that the salt as a substance be eternal with the rest of the world, o that it die and be reborn together with the humans that consume it. But experience shows that it does not die as shown by the fire being unable to consume it and by the water that becomes saltier and saltier the more salt it dissolves. Evaporating the water, salt appears in the original quantity. The salt brought into a city each year, is not destroyed by the passage through human bodies, because it reappears in urine, sweat, or other secretions. We will say that the rains that penetrate the earth, be those under the foundations of cities and villages or through the conduits of the earth return the salt taken from the sea. The mutations of the sea once over the mountains resulted in the salt mines found in those mountains, etc.

Ma a dire meglio essendo dato il mondo eterno e gli e neciessario che lli sua popoli sieno anchora loro eternj onde etternalmente fu e ssarebbe la sspetie vmana consumatrice del sale e sse ttutta la massa della terra fussi sale non bassterebbe alli cibi vmanj per la qual cosa ci bisognia chonfessare o chella spetie del sale sia eterna insieme chol mondo o che e quella mora e rinasca insieme cho gli omnj dessa divotori Ma sselia essperienza cinsegnja chel non avere morte come per il focho si manfesta il qual no llo consuma e per lacqua che di tanto si sala di quanto ella in se rerisolue evaporando lacqua seppe il sale ressta nella prima quantita ne vale passare per li corpori vmanj che in orina o in sudore o altre superfluita fia ritrovato e eqato e il sale che onnj anno si porta alle citta adunque direno che llo acque che pe pioggie penetratrici della terra sie quella che sotto allj fondamenti delle cita e popolj sie quella che per li meati della terra renda la salsedine leuata dal mare e chella mutation del mare stato sopra tutti li monti lo lasci per le mjniere ritrovate n essi monti, ecc. (49R T4)
Of the variations (?) of the earth

The groundwater courses, as well as those between air and ground, continually erode and deepen their own beds.

The earth picked up by the rivers settles down in the last part of their courses; i.e., the earth picked up in the high part of the courses of the rivers is deposited in the last low parts of their flow.

Where the fresh waters well up at the surface of the sea the formation of an island is a known phenomenon. The island will appear sooner or later depending on the amount of water coming up being large or small. Such an island is generated by the earth or the erosion of stones due to the groundwater course through the places where it passes.

The title of T1 was either meant for some other topic or Leonardo had another notion about vibration della terra. Ravaisson-Mollein translated this phrase into vibration de la terre; I believe that Leonardo may have misused the word vibration while actually meaning Changes of the Earth. Marinoni also has a comment on this inconsistency: "La pagina parla piu di erosione che di vibrazione".
Ms G 50V T1 D1-2, T2-3 D3-5

Of the motion of boats.

These three boats, of the same width, length and depth, moved with equal 'potentia' will have different velocities, because the boat with the wider portion ahead is faster and is similar to the shape of birds and of the fish called mullets. This boat opens sideways and in front great quantity of water, which in its revolutions (vortices?) closes on the ship over two thirds in the back. The opposite does the boat $dc$; the boat $df$ is of middle speed between the two aforementioned.

Of the erosion by water in its falls.

The fall of waters over the banks erodes always the foot of such banks and damages their foundations. Proof: let $ac$ be the height of the bank from which the water $an$ falls, impacting and eroding the impacted place. The middle (area?) of the impact is $mnc$; there the flow bifurcates into the reflected motions $nmo$ and $ncb$. They, in all aspects (directions?) erode the bank (boundaries?) by friction due to their vortical motions. Hence, as the foundations of the bank are eroded, it collapses at the places where their support is lacking.

The water falling from $ab$ to $nm$ will gradually erode the bed, all along from $ab$ to $cd$, at the level where it falls.

Note that in the drawing $ab$ in $D1$, there are some indications of the flow around the bodies. If one looks at these two sketches in the context of many others in Leonardo's manuscripts, it seems obvious that he is, in a sort of short-hand picture, reminding himself that there are waves and vortices in the near region of the flow. $T1$ contains some errors because ship hydrodynamics (occurring as it does at an interface) is different from fish hydrodynamics and bird aerodynamics.

$T2-3$ $D3-5$ must be studied in connection with many other similar situations to be found in Leonardo's notes [See IIHR monographs, Macagno 1986-92]. I found $T2-3$ quite well explained, especially the continual recession of the erosion in a water fall.
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<th>D1</th>
<th>T1  D1-2</th>
<th>T2-3 D3-5</th>
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Of the body with nonuniform weight that moves through air or water.

In moving bodies of uniform matter and nonuniform weight the heavier part guides the motion. The pyramidal body of crossection uniformly disform, which is launched point ahead with a bow, will immediately turn its base towards the direction to which the whole body moves.

Marinoni [1989] notes - without further comment - that "pars gravior priora occupabit' is a rule from *De ratione ponderis* of G. Nemorarius, but one only has to perform some simple experiments to show that the rule is not always valid. Among the bodies I used was my paper-weight, which is an hexagonal pyramid. I dropped it on a piece of paper with a carbon and a cushion under it; in all cases the point left a clear mark after falling several feet. So far, I have not found that Leonardo investigated this problem to the point of finding the medieval rule wrong. I find this strange, because this is an area in which he took especial interest.
Ms G  51V  T1 D1,  T2 D2

Of the excavation of ports.

Construct compartments, and when one compartment is empty of water and earth, discharge the water of the second compartment into the first to be emptied; once it is dry extract the pile planks of the compartment full of water and with the same wood built the following compartment, etc.

Different ways of discharging water into the sea.

In T1, since both earth and water are removed it seemed to me preferable to use excavation in the title. D1 corroborates this interpretation.
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<th>T1</th>
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<th>T1 D1</th>
<th>ACQUA</th>
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D1

D2
Of the moving body.

The arrow shot from the bow of the boat in direction opposite to that of the boat motion will not leave the place where it is shot, if the motion of the boat is equal to that of the said arrow.

But, if the arrow from that boat is thrown with the above said velocity, towards the place that the boat is going, the arrow will separate from the boat at twice the velocity of such arrow.

Of knowing how much a boat sails per hour.

Our ancestors of Antiquity used several devices to determine the distance traversed by a boat during one hour. Vitruvius describes one in his book on architecture, but it is defective as well as others. This consists of a water wheel that touches the sea waves at their crests; the sum of its complete turns gives a straight line which is related to the circumferential periphery of the wheel when rectified. But this device is not applicable, except in the case of plane and quiet surface in lakes; note that if the water moves together with the boat with the same motion (velocity), the wheel remains immobile. If the wheel has velocity greater or smaller than that of the boat, then the wheel does not have a motion equal to that of the boat. Therefore such a device is of scant utility.

There is another way based on the known distance from one island to another. In this, a light board is used that, under the wind, becomes more or less inclined depending on the impacting wind being of more or less velocity. This is in Battista Alberti ('s book).
The method of Battista Alberti, based on knowing the distance from one island to the other, is valid only for a boat similar to the one in which the calibration was made. It requires also that it be with the same load, and the same sails arranged the same way, and with waves of the same size. Instead, my method is useful always for any boat using oars or sails, large or small, short (?) or long, high or low.

In the first part of T1, we would say that the velocities must be equal rather than the motions. I could have rendered *moto* - in this case - as velocity, but I felt that it could be misleading for some readers. More serious of course, is the danger of reading T1 without taking into account the theory of relative motion.

In connection with T2, I feel that a comment about the value of tradition, which is both respected and subjected to examination, is in order. I found remarkable that he, who is going to write a note about the Vitruvius method of measuring a distance, refers to "li nostrij antichj"; I am glad to see that I share with him a veneration for the same distinguished ancestors; of course, that does not exclude sound criticism when warranted. In addition, I would like to comment that the distance from one island to another had to be known by the use of an independent procedure; today we have many ways of doing this, but even at the times of Vitruvius there were several methods, especially for given islands.

What is not clear at all to me in this page, is the method proposed by Leonardo. Could it be that Leonardo was thinking here of using two measurements of relative motion to determine the speed of the ship? In T1, at least, he did not explain how that could be done.
Of the motion of the 'mobile'.

Of the motion of the 'mobile' pouring continually on a moving body or when the pouring (liquid) is itself moving.

The motion of the liquid which pours out from the bottom of a moving vessel will follow an inclined straight line. Such inclination will be greater or smaller depending on the velocity of the vessel, producing it, being greater or smaller.

Of the motion of the body receiving the thing pouring out from the vessel.

It is the same to receive on the moving body the thing pouring out of the moving vessel than to move the pouring vessel over the still body. However, if the motion of the pouring vessel is the same as that of the body receiving the spilling thing, then the motion of the thing that comes down will be straight, as shown above.

The 'grave' that descends through the circumvolutory elements does it always along that straight line going from the initial point to the center of the world.

The 8 lines with the 8 divisions into which they are divided are to illustrate (?) a single line which is straight. The weight falling through the circumvolutory elements passes through each of the 8 subdivisions of the line and finally returns to the same location from which it departed. The motion of the 'grave' has two descriptions (?); i.e., spiral curve and straight line.
In T1, the title appears to have needed some clarification on the part of Leonardo (see long subtitle), probably because he was going to refer to several cases. It is interesting that in English one of the acceptions (in the Webster Dictionary) for mobile is that of a fluid. I could not confirm the same to be valid for Leonardo in Italian; perhaps it was. If so, he was using mobile in different ways. Ravaisson-Mollien understood this passage quite well; his handling of it was valuable for my work.

Without relating T1 D1 with others passages especially those in the Codex Arundel, it is not easy to interpret it correctly [See Macagno 1992, Raccolta Vinciana].
Of the moving body.

Of the 'grave' falling through the air, taking into account that the elements rotate with a complete revolution in 24 hours.

The body descending from the top of the sphere of fire will move straight down to earth, even if the elements would be in a continual circumvolution around the center of the world. Proof: let $b$ be the 'grave' descending through the elements. $b$ starts at $a$ and descends to $m$, the center of the world. I say that even if such 'grave' descends in curvilinear motion along a spiral, in fact, never departs from the rectilinear descent. This is a continual process between the point at which it departed and the center of the world, because if it departed from $a$ and fell down to $b$, during the time it descended to $b$ and was carried to $d$, the place of $a$ has changed to $c$; thus the moving body finds itself in the straight line going from $c$ to the center of the world. If the body descends from $d$ to $f$, which is the beginning of the motion, the same time it moves from $c$ to $f$, if $f$ descends to $h$, it passes through $g$. In this way, in 24 hours, the body falls down to earth in the location where it departed initially; such a motion is composite.

If the body falls from in 24 hours from the highest level of the elements to the lowest, its motion is a resultant of rectilinear and curvilinear motions. I say straight because it will never deviate from the briefest path going from the initial point to the center of the elements. It will stop at the lowest end of such straight line which always passes through the zenith of the place where the body departed. And such motion is curvilinear in all parts of it, and consequently over the entire line; from this follows that the stone thrown from the tower does not ever hit the side of such tower before hitting the Earth.

$T2$ has been a rather difficult paragraph to deal with. I believe that a careful analysis of text and figures may lead to the conclusion that Leonardo constructed correctly the path of the body in space for a special case in which a body would fall with uniform velocity from the outer layer of fire to the interface air-water in 24 hours. Was he referring to the motion relative to the planet, when he said that in fact the path was straight? I found that $T3$ may be helpful in clarifying the meaning of $T2$. 

sel mobile disciende dalla supplema all infima parte delli elementi in 24 ore il moto suo fia chonposso di retto e di curvo Retto dicho perchè mai si suiera della linja breuissima che saastende dal locho donde si diuise al ciento dellalementi e ssi fermera nello stremo infimo di tal rettitudine la qua senpre sta per zenit sotto il locho donde tal mobile si diuise E tal moto in se e churvo chon tutte le quanti le parte della linja e per conseghuentia e al fine curvo chon tutta la linja e di cuj nasscie che il sasso gittato della torre no perchote nel lato dessa torre prima che in terra. (T3)
Ms G  55R

T1  
D1
T2  
T3

T1-3 D1
FOCO
ARIA
BO FA
PLANET
ROTAT
KINEM
REL MO

D1

35
With the curved motion made by the two sectors of a given circle with the angles (vertices ?) fixed at the center of such circle, triangles of sinuous sides will be obtained. They will be equal (areawise) to the triangles with rectilinear sides as can be shown by shearing (?) the sides \( a b c \) in continual fashion; thus the said triangle will form oppositely a triangle with sinuous sides equal to the rectilinear triangle \( a b c \).

Straight base for one surface and the other.

These two figures identified with \( ab \) are of the nature of the figures above; the rectilinear triangle \( amo \) is equal to the triangle of curvilinear sides \( pnQ \).

There are many examples of circular shearing motion in Leonardo's notebooks [See Macagno M. 1992].
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Geometry.

Let be given a bisangular area equal to a square which has irregular sides with nonuniform curvature all with its concavity turned toward the same center. The question here is about the spiraling bisangle \( adbc \) which satisfies the above said conditions. This (figure) is converted, by means of motion, into a rectangular surface; it is equal to a quarter of the circle, as it is shown in the top figure, first divided into four equal bisangles, and also divided into four triangles of the area (?) of the given bisangle. One of the four spirals is \( becnd \) and one of the four triangles is \( bdf \). Note that the deformation (?) of its curvilinear side results into a straight line equal (or equivalent ?) to the curvilinear side.

This figure is called irregular lunula.

This invention about lunulas of irregular sides are infinite; i.e., of infinite varieties of curvature.

In these 3d and 4th I mean to have the same as it was written above.

In D1-4 we have some drawings that even without an accompanying text would represent very well a remarkable example of the use Leonardo made of motion in the study of geometry. In this case he uses circular shear to great advantage. Note that he performs finite deformations of sectors of a circle. This passage is, in fact, part of his kinematics. I believe that drawings like these may have attracted the attention of Hermann Weyl when he examined Leonardo's manuscripts and concluded that he understood dihedral and cyclic symmetries.

Ms G 58R contains another example of the use of motion in geometry, but its significance in connection with kinematics is minor compared with that of Ms G 57V.

I could not find the word bisangle in English dictionaries. I believe that there are cases in which we must coin new terms if necessary. A plane geometric figure with two sides and two angles is possible (within Euclidean geometry) if one of the sides at least is curvilinear. The circle in D1 is therefore divided into four bisangles.
<table>
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<td>T2</td>
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<td>T4</td>
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![Diagram](image-url)
Here the hammer moves upward against the nail that must be driven into the wood.

Of the percussion.

Among the accidental 'potentie' in nature, the percussion exceeds greatly any of the others which are effected by the motors of weights in the same time, same motion, weight and force. Percussion can be divided in simple and composite. Simple is the one in which the motor and the impacted (?) moved body are conjoined at the place of percussion. Composite is the one in which the percussor body does not end its motion at the place of its impact; e.g.: the percussion hammer of the die used to stamp coins. The composite percussion is quite weaker than the simple percussion, because if the face of the hammer would have been engraved with one face of the coin, would carry the coin to be stamped and would impact it over the engraving of the other face of the coin, the coinage would be more expedite and neat on the side impacted by the simple motion than on the side of the composite percussion, as it happens to the coin impacted by the die, where the hammer impacts it and the percussion is reflected and reverberates against the face of the hammer.

Percussive forces are often a subject that is found in notebooks. Regarding fluid mechanics, Leonardo relates his knowledge about impact to considerations about jets, or about bodies falling into a liquid or moving through water and air. For instance, in Ms G 63V to 65V in which he considers matters relating to the flight of birds, bats, and insects, he introduces the effect of percussion of wings onto the air. Anybody familiar with Bergeron's book on impact phenomena [Bergeron 1949] will surely understand why any considerations about impact in the notebooks may be of interest in studying Leonardian fluid mechanics.
The plane surface \( abc \) is equal to the surface of the shield-like body \( ab \); the straight segments including the plane, parallel (strips) are (of the same length as ) the curvilinear parallels around of such body.

When you rectify the irregular periphery \( abc \) by its motion, it is equal to the segment \( de \). Having divided the shield-like body in five parallel (strips) of equal width in themselves and among themselves, I will divide also in five equal parallels the line \( de \) together with the surface \( def \). Such parallels will be of indefinite length; then, with circular motion I will cover each indefinite line with the corresponding circle two the extent of the length of each circle.

When you rectify the irregular periphery \( abc \) by its motion, and you mark the subdivision along the curved line and along the indefinite straight line \( de \), before separating the curved line from the straight one, something you will do with all parallels successively, do not care about being all of the same width. Make sure though that each curve becomes rectified on its straight line and that you perform entire circular motions.

Quando tu dirizzi la curva \( abc \) choll suo moto e ttu ssegnja le patitionj nella linja curva e nella retta indifinjta \( d e \) e avanti che ttu seperi la curva dalla retta e chosi farai in tutti i paralellj di mano in mano e non fa chaso che sieno in fra lloro di largheza equali pur che ciasscuno de curvi si dirizi sopra il suo retto etc e ssi termjnaj nello intero suo moto ciechulare. (T2)

The first paragraph of T1 is related to D1 and the second to D2. T2 and the second paragraph of T1 seem to be saying the same thing, but there are some important differences. To clarify this passage it seems appropriate to use our integration formulas for the area of bodies of revolution. It does not seem easy to determine whether Leonardo had a good grasp of this question.
The complete revolution of the largest circle of the sphere gives the straight segment $ab$. The rolling of one fourth of the largest circle gives the straight segment $ad$; i.e., the curve $da$ gives $ac$ as a rectified line. Now divide the periphery in four equal parts, and divide in four parts the straight segment $ac$, and draw the four parallels included between the three lines $ab$, $bc$, $ca$, and the four parallels should be given with their true lengths, with the complete revolutions of the four anuli surrounding the hemisphere $adf$.

The straight segment $ab$ is equal to the curvilinear $ac$. All the triangle $abc$ is equal to all the surface of the oval half figure divided in five anular parallels.

Lanterna revolutione fata dal maggiore cl della spera fa la retta $a b$ e la 4a revolutione deesso maggiore cl della spera fara la recta $ad$ coe la churua $d a$ dirizzata fa a c linja retta diujiadunque in 4 equali parte la periferia $a d e$ in 4 diuiderai la recta $a c e$ ffalli 4 paralelli inclusi in fra lle tre linje $ab$ $bc$ ca e lli 4 paralelli saran dati cholle lor vere lungheze cholle intere revolutione delli 4 anulj che uesstano il semj spericho $a d f$.

(T1)

a b rettilinjo vale a c churvilinjo e ttutto il trianghola $a b c$ val tutto la sspoglia della meza fighura ovale diujsa in cinque paralelli anulari. (T2)

The procedures given in T1 tiofund the area of the sphere and the one in T2 for an elongated body of revolution are of a kinematical nature, and their interpretations in terms of finite elements corresponds to the modern expressions that one could use for the numerical calculation of such areas.
The reflected wind, the one running against its own flow, defeats the incident wind up to the point that the reflected one becomes weak. Afterward, it picks up force, when it joins the incident motion. The corresponding potentia comes from its compression at the place where it impacted. Such compression always penetrates into the incident wind, up to the point in which it disgregates and has the velocity of motion diminished.

Water does the same, not by compressibility, but because it jumps into the air and acquires weight.

*In most of his notes, Leonardo is quite consistent regarding the consequences of his notions about the high compressibility of air and the incompressibility of water. One interesting investigation within his fluid mechanics, would be to gather all his notes on this topic and interpret them through a rigorous analysis and a creative synthesis. It is not enough to say that he overestimated both the incompressibility of water and the compressibility of air. Knowing that many students who begin the study of fluid mechanics do the same, there is here - in my opinion - a very interesting line to pursue. We must answer questions like this: Is one mislead by an implicit extension to dynamics of what is in fact a knowledge for quasi-equilibrium conditions? I mean that we are all imbued of notions of thermo-statics rather than thermo-dynamics. When compressibility plays an important role in fluid mechanics and when not is not so simple a question as many believe. It is not simply a question of the value of the Mach number. For instance, not many fluid mechanicists are familiar with water-hammer phenomena in which the Mach number can be very low. There is an acceleration number to be taken into account also.*

Il uento reffesso che ssi rivolta contro al suo avenimento vincie il uento incidente insino a tantro che esso reffesso indebolisscie e poi ripiglia forza quando si congivgnie chol moto incidente e ttal potentia nasscie per la sua condensatione acqujstata nell 8 doue perchosse la qual condensatione senpre penetra nel uento incidente insino a tantro che ssi disgregha e dimjnuisscie la velocita del moto. (T1)

Lacqua fa il medesimo non gia per chondensatone ma perche salza in fra laria e acqujsta peso. (T2)
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<td>T5</td>
<td>D4</td>
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<td>T1-2 D1</td>
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ARIA
ACQUA
COMPR
WEIGHT
FLOW
IMPACT
ANAL

![Graphical diagram](image-url)
Whether the water can rise from the sea to the top of the mountains

The Ocean cannot penetrate from the roots to the summits of the mountains adjacent to the sea; it only raises to the height warranted by the dryness of the mountain. And if, for the *aversario*, the rain - infiltrating the mountain from the summit down to its roots which are adjacent to the sea - descends and softens the opposite foot of the same mountain, and if such rain acts continually in the way of a siphon which flows along its longer side; i.e. (the rain) would be the one which pulls up the sea water - as if *Sn* would be the sea surface while the rain descends from the summit of the mountain from *a* to *n* on one side and from *a* to *m* on the other side - this would, without doubt, be (like) the distillation with felt, or be like the tube of the above said siphon. Thus, the water of the great rain, which has softened the side of the mountain and descends on both sides of the mountain, would pull for ever with it on the longer side; hence, also the rain *an* together with the sea water, if the side *am* of the mountain would be longer than the side *an*. But this is impossible, because no part of the Earth not covered by the sea, is lower than the said Ocean, etc. 

I agree with Marinoni that T1 is *alquanto confuso*, but I do not agree with his comments in his footnote No. 2 for Ms G 70R [Marinoni 1989]. It seems that also Ravaisson Mollien had problems with this passage; his interpretation went wrong from the very beginning because he misread the phrase *la seccita del monte* as *la sécheresse du monde* (de la atmosphère).

*T1 cannot be clarified without an understanding of how a common siphon works, how the inner pressure in the liquid varies along the tube. Since Leonardo refers to the felt-strip siphon, this has also to be understood taking surface tension into account. One should be familiar with some experiments with siphons which visualize its limitations; a very illustrative experiment can be run by siphoning mercury from one container to another using a transparent flexible tube. This was a routine experiment in my course of Fluid Mechanics aimed at instilling the notion that if the top of the siphon is elevated too much relative to the reservoir level the siphon stops working. What I mean is that if one wants to interpret Leonardo's writings on fluid mechanics, all the knowledge of this science one can muster is necessary; and may not be enough in some cases. In the context of Leonardo's apparent belief that a siphon would work for any height, his text is quite clear. Of course, he did not know about the role of the atmospheric pressure on a siphon.*
Ms G 70R

DI

T1

D1

T1D1
ACQUA
TERRA
POR MED
FL P SU
TE
SEA
SIPHON
METEO
ANAL

DI

S

Of the simple *inpeto*.

Simple *inpeto* is the one that drives the arrow or the dart. Composite *inpeto* is the one that drives the stone coming out of the sling. This kind of *inpeto* does not have long duration because, as it is obvious, the moving body finds resistance in its penetration of the air. This is illustrated by the noise produced by the circular motion of the body.

This distinction between the *inpeto* imparted by a motor that describes a straight line as compared to another one that drives the projectile along a circumference, before launching it, is certainly interesting, even if Leonardo may have grasped only the kinematics rather than the dynamics involved.

In this case, I have modified somewhat the construction of this paragraph because I am sure that Leonardo did not mean that the noise was the cause of a shorter duration rather the air resistance. The noise was for him what revealed the air resistance.
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<tr>
<td>T2</td>
<td>ARIA</td>
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<tr>
<td>D1</td>
<td>BO MO</td>
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<td>ROTAT</td>
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<td>T3</td>
<td>RESIS</td>
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<td>D2</td>
<td>INPETO</td>
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<td>ACOUS</td>
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<td>T4</td>
<td>CONCEP</td>
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Ms G 72V
About *inpeto*.

*Inpeto* is impression of motion transferred from the motor to the moving body. *Inpeto* is a *potentia* that is an impression of the motor into the moving body. Any impression tends to be permanent or, in fact, desires permanence. It is shown by the impression produced by the Sun in the eye observing it and by the sound made in a bell by the impact of a hammer hitting it. Any impression desires permanence as shown by the image of the Sun made in the eye and the image of the motor in the moving body, etc.

The transfer of *inpeto* from one body to another is likened here to the image of a bright light which remains for some time in our retina or by the sound elicited from a bell. In the second case, I assume that Leonardo considers the long period of time of vibration of the bell as being in analogy with the duration of *inpeto* in the moving body.
Ms G 73R

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<td>T6</td>
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<td>T2</td>
<td>T1</td>
<td>D1</td>
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</table>

T1
- BO MO
- IMPACT
- POT
- INPETO
- OPTIC
- ACOUS
- CONCEP
- ANAL
Of percussion.

The air, which is being condensed under the moving body which descends through it in an oblique position, flows always more along the upper part than along the lower part of it.

The contiguous (?) portions of air are condensed as much on one side as they are rarefied in the other. The rarified air is less resistive while the condensed air is more resistive. Hence, the part behind the moving body will descend with more impeto than the part in front of it. This will be the cause for the frontal part to go up at the end of the reflected motion, etc.

It is desired to make an experiment to see whether the non-mist (?), varnish liquified by fire, flows away from the oblique places where it is not very thick. This varnish (once liquified) must be spread with a perpetual (?) brush.

According to Marinoni Tl-2 is the end of the text on Ms G 74 R. It seems that Leonardo not only wrote from right to left but he also began his notebooks on what for us would be the last page of the book.

In Tl and T2, I have preferred condensed to compressed because in this context, I believe ha Leonardo was thinking of the change in density produced by compression, while in other places he seems to use condensata to refer to the compression of the air. Note that compression (as a decrease in volume) and increase in density go together.

When reading notes like Tl and T2, we must always remember that for Leonardo the air was highly compressible, even for low velocities and accelerations. For those who understand the role of the parameter called Mach number it may seem that his was a rough error, but in fact it is not so easy to establish the knowledge that may seem almost obvious to the fluid-mechanicists of today.
<table>
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<th>T1-2</th>
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<td>D1</td>
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**ARIA**

**COMPR**

**FLOW**

**BO FA**

**FLOW**

**THERM**

**EXPER**

---

**Ms G 73V**

---

**T1**

**D1**

**D2**

**T2**

**T3**

**D3**
Man can descend as it is shown below.

Of percussion.

Of the things falling through the air from the same height, the one following the longest path will produce the least impact. It follows that the one falling along the shortest path will produce the maximum impact.

This first moving body, which is a piece of paper somewhat curved, begins its fall with the front at \( b \), and moves from \( a \) to \( c \). In this motion \( a \) descends more than \( b \). Hence, at the end of the reflected motion, \( a \) is in \( c \) and \( b \) goes up to \( d \). This is proven by the ninth of this (subject ?) which says: The thing that impacts the air with the larger part of itself is the one that penetrates less the air. And also by the tenth: A thing penetrates the air with maximum velocity when it falls through it with its minimum dimensions. And by the eleventh: The heavier part of a body moving through the air becomes the guide of the motion of such body.

This one will move to the right if he bends the right arm and extends the left arm, and then he will move from the right to the left if he changes the movements of the arms.

The proof: let us consider the grave ab, which front, although being of uniform dimensions and weight, weighs more because of its oblique orientation, even if the width is the same all over. Due to this, the front becomes guide of such fall. According to the twelfth: The air resistance to the (falling) body is more the more the air is compressed. Hence, that side weighs less that has its (sur)face in contact with the compressed air. According to the thirteenth: The air with higher velocity moves more. If follows that . . . .

According to Marinoni, this page is continued on Ms G 73V; note that T5 ends with an incomplete sentence. Besides, Leonardo joined this last line of T5 with the beginning of T1 in Ms G 73V.
In T3, Leonardo quotes from a source, or sources, I have not been able to determine, except for the last statement for which the source has been identified already [Marinoni 1989]. See also my comments to Ms G 51R in this monograph.

I tend to believe that Leonardo, in what may be a lost notebook, kept a record of propositions, some of which may very well be his own. They must have been numbered, and it was easy for him to refer to them by the corresponding numbers.

I have experimented with falling bodies of many different shapes and I am sure that there is much more to be learned before one can analyze with some sound base passages like the above one. The reader can arm himself with a pair of scissors and pieces of paper of different thickness and drop pieces of paper into quiet air. An interesting experiment is to let drop a card with letters written on both sides and observe how it falls when it is as straight as possible; use different initial orientations. Then try with a slightly curved card. Then cut the long sides just a bit to change the rectangle into a trapezoid with slight difference of its two bases. Experiment also with rectangles of different aspect ratios. This will produce, I believe, a rather cautious attitude towards final conclusions regarding this folio. There are already too many comments that are wrong in the literature that also support the notion of being more serious about the analysis of Leonardo’s writings.
Gravity.

Any inclined motion of the *grave* in the air divides the gravity of the moving body into two aspects (directions?) one responds to the place where it moves and the other to the cause that impedes it.

_I tend to agree with Ravaisson-Mollien that aspetti here may indicate the directions of two components._
Of the fall of a grave.

Any natural action is effected along the minimal way. Because of this, the free fall of the grave is towards the center of the world, following the minimal distance between the moving body and the ultimate lowness of the universe.

The uniform grave that falls obliquely divides its weight in two different directions. Proof: Suppose \(ab\) is the moving body oriented along the inclined line \(abc\). I say that the weight of the grave \(ab\) acts along two directions; i.e., along \(bc\) and along \(nm\). The reason why the weight can be more on one side than the other and for which inclination both will be equal will be said in the book on weight.

This board has rotatory motion; the inpeto of such rotation prevents the direct (?) fall.

\(T2\) offered some difficulties, among other things because the verb is missing in the second sentence. I believe that Leonardo meant that rotation of the body will set it into a different path, as it falls, than the straight one.
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D1

48
There seem to be no notes pertaining to this two drawings in the rest of the Ms G. My conjecture is that Leonardo meant to write some comments and for some reason never did it. D1 is interesting in the sense that there is a pier which is quite close to the water fall, a situation I do not remember having seen in the manuscripts I have examined.

I am not completely sure that D2 is an independent flow with respect to the water fall, but to facilitate further study of this page I have preferred to describe this page as containing two drawings.

Of the oscillating weight.

The weight oscillating from right to left and from left to right becomes so more grave to its appendage (?), or to its sustentaculum of the appendage, as the appendage is less oblique.

When its pendulum is less oblique.

I believe that appendicholo in T4 was used in the sense given in the Webster for append (ception I). This seems to be clear from observing D2. Note that in T2 I have rendered appendicholo as pendulum; I think this is really what Leonardo meant. This is a pendulum attaches to a balance.

In D2, a balance is shown in which a pendulum has been installed on one of the sides. Presumably, equilibrium was obtained, and then the pendulum was set in motion; maybe this was due to the wind as suggested by Ravaison Mollien, when he dealt with this folio. If we do not know the answer to this problem, we can find it - in a somewhat simplified form - in a textbook on dynamics [e.g., Synge and Griffith 1949], and see whether Leonardo was right or wrong about the force applied on the sustentaculum.
Ms G 77R  T1 D1,  T2-3  D2-5

Of weight.

Any grave moving along a horizontal line does not weigh but along the line of its motion. This is proven by the first part of the path of the ball of a bombard, being such motion along a horizontal line.

But the body oscillating in any direction at one end (?) of the arm of a balance will have more or less gravezza, depending on the angle between the pendulum and the arm of the balance being more (or less) close to the right angle.

The circumferential motion of a weight around its fixed axis will give to such weight more weight the faster such circumferential motion will be.

In T1, Leonardo appears to be accepting the notion that the path of a projectile can be described by two straight lines with a curve in between. This notion was accepted in ballistics for a long time, and it is seen for jets in some paintings [Macagno M. 1993].

Note that in D2-5 we have a variety of circular motions around the point of suspension of the pendulum. Unfortunately, Leonardo does not give any indication of how these experiments might have been performed.

del peso

On grave che ssi move per il sito della equalita non pesa se non per la linja del suo moto provasi nella prima parte che ffa il moto della pallotta della bombarda il quale moto e nel sito della equalita. (T1)

Ma il grave ventilante per qualunche asspetto intorno alla fronte che a llasste della bilancia ara tanto piu o men gravezza quanto e fia-piv vicino o fremoto alla linja cientrale La congiuntione che a llapendicholio del peso chol br della bilancia sara piu vicina all angholo recto. (T2)

Il moto circhunuolubile fatto con velocita dal peso intorno al sito al fermnamento del suo polo sara a esso peso di tanto magior peso quanto esso moto circhunvolubile fia di piu velocie moto. (T3)
Of the nature of heat.

If the base of four braccia over a distance of one braccio transfers the potentia, the heat from the base increases by a factor of four. If such base (cross-section ?) is reduced to one fourth of braccio, such potentia acquires sixty four degrees over such base. Those diminishments of base and increments of potentie are written here below.

As the base decreases, the potentia of the pyramid increases inversely. If the base increases, the potentia of the pyramid decreases, etc.

If you would reduce the base of four braccia of diameter to the size of a vecchia, the potentia you will have will be 4,194,304. Thus, if ones keeps quadruplicating, the base increases and the potentia decreases.

I have interpreted T1 as a description of the stationary flow of heat generated at constant rate at the base of a of pyramidal "conduit", as it flows through different cross-sections towards the vertex. The cross sections are not at equal distance one from the other. In T1, Leonardo seems to use basa as base and as cross-section.

T2-4 are numerical calculations which give the powers of 4 up to the eleventh power (4194304). T2 has been crossed over by Leonardo, but it looks very much like the other two. I have considered necessary to reproduce here only the upper part of T3 and T4. It is true that Leonardo lacked a notion of functional relationship as ours; but the fact is that these two columns represent a functional relationship as many tables do even today, and did in Mesopotamian astronomy (Neugebauer 1952). T3 gives the size of the base as it begins with 1 and it decreases to 1/4, 1/16, etc. and T4 gives the corresponding values of the potentia as it increases from 1 to 4, to 16, etc. T4 shows, in fact, a calculation of the powers of 4. In fact, Leonardo gives area of "conduit" and rate of flow as a function of distance, only that the distance is not varied by constant increments. Note that $1 \times 1 = 4 \times (1/4) = 16 (1/16)$, etc. We apparently have here a statement of conservation in a flow of heat instead of water.
What is *inpeto*.

The *inpeto*, also called derivative motion, is born from the primitive motion which existed when the moving body was joined together with its motor. Never will we find in the derivative motion any velocity equal to that of the primitive motion. Proof: In each degree of the motion of the cord of the bow there is a loss of the *potentia* transferred to it by its motor. Because any effect shares with its cause, the derivative motion of the arrow diminishes by degrees in *potentia*. Thus, it participates in the *potentia* of the bow, which, as it was created by degrees, it is destroyed by degrees also, etc.

*The inpeto* communicated by the motor to the moving body is infused in all the coherent parts of such moving body. This is shown by all parts of the body, in the interior or at the surface, having the same velocity, except in the rotatory motion, because in it the part with more *inpeto* turns around the part with less *inpeto*, i.e., those which are closer to the center of the moving body. And that part which moved first remains always more distant from the beginning of its motion if it is not impeded; this happens because that part is the more *potente* in such rotation. If the *avversario* would say that the *inpeto* driving the moving body is in the air around it from the middle to the rear, one must negate this, because the air that follows the body is pulled by such body in order to fill the vacuum it leaves behind. In addition, the air compressed ahead of the moving body flows backward in opposite sense.

Here continues what is missing below.

And if the air turns around in the rear, it is evident that it must collide with the one that the body pulls towards itself in the rear. When two things impact there is for each of them a reflected motion, which are both transformed into opposite vortical motions and carried by the air that fills the vacuum which is left behind by the body. It is impossible that the motion of the moving body be increased by its own motion at the same time, because the motor is always of more *potentia* than the moving body, etc.
This page is very important in the study of Leonardian fluid mechanics. The rejection of old wrong ideas about the flow around bodies is very clear. But to see that Leonardo could find the old physics in error should not be the most exciting event for historians as it has been; what is of utmost significance is to discover the extent and depth of his new ideas concerning this question, to study them critically because they are most likely bound to contain flaws together with original correct thinking and findings. Some of those ideas are sketched on this page, but they are spread over many others; one must examine not only the writings but also the drawings representing flow around bodies [see, e.g., Ms H 16R or CH 25V, discussed in Macagno 1987]. Leonardo's notions about resistance due to surrounding fluids are also to be considered in this context, as well as his conclusions about dissipation of energy.
Of the motion of a body.

Which one will send farther the same body? a large potentia with a short motion? or a small potentia with a long motion? That derivative motion of the same body will be longer which will have longer derivative motion of the same motor.

Proof: The fifth of this says that, for the same body, the proportions of the different lengths of the primitive motion will be found to exist also in the different lengths of the derivative motion. This is because, if a given motor moves the body away the distance of one dito in an harmonic time, the same potentia will move away the body two of the same dita in two harmonic times. And this is so because the ratio between derivative and primitive motions is constant.

The inpeto of the body is not always generated because not always the motor possesses an inpetuoso motion. As it is shown by a light cart pulled by oxen in a level place: as soon as the oxen stop the cart motion is terminated.

As I have done with other passages, I resorted to the study of a simple theoretical model in order to gain insight into what Leonardo may have meant when writing this page. Regardless of the above being received or original thinking, it should reveal the level of his understanding of the dynamical problem of setting into motion a given body by means of some device, some "motor". I think that some readers may like to have a brief description of the model I used, a full discussion of which will be part of a paper now under preparation. I considered as motor a linear-spring gun ejecting a spherical projectile in several situations. Most useful to me were the case of vertical upward shots in vacuum under a uniform gravitational force and the case of horizontal shots into a linearly resistive medium and no gravity force. Both provide a finite reach distance and seem quite appropriate to shed light into the above page, as it becomes easy to compare different reach distances with different loading distances of the spring.
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<tbody>
<tr>
<td>T1</td>
<td>T1,3-4</td>
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<tr>
<td>T2</td>
<td>BO MO POT</td>
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<tr>
<td>T3</td>
<td>INPETO KINEM MECH</td>
</tr>
<tr>
<td>T4</td>
<td>EXPER CONCEP</td>
</tr>
</tbody>
</table>
Of the five directions of motions.

The different local motions are five. The first is upward, the second, downward, the third is horizontal, the fourth is inclined upward, the fifth and last is inclined downward.

delli cinque asspecti de moti
cinque sono le uarieta de motj lochalj d equali il primo e in su il sechondo in giu terzo e nel sito della equaljta 4° e obbliquo in su e l qujnto e ultimo e obbliquo in giv. (T1)

How the inpeto of the moving bodies joined by a cord is transferred from one body to the other.

The inpeto generated by moving bodies joined together by a cord, of which alternatively one becomes the motor of the other, will separate the two bodies from the original motor over a small distance.

When the two moving bodies attached to the ends of a given cord are different one from the other, the sum of their motions will be less than if they were equal.

When the larger of the two bodies joined by a cord leads the motion, the motion of the two bodies joined together will be larger than if the beginning of the motion were effected by the smaller body.

When the two bodies joined by the cord are equal . . . .

come linpetto delli mobili congiunti per corda si trassmjeta dal un mobile all altro

Linpetto gienarato dallj mobili con corda chongiuncti delli qualj scanbievolmente lun si fa motore dell altro removera con picholo spatio li due mobili dal lor p° motore

Quando li 2 mobili per li due oppositi stremj a vna medesima corda congiunti saran mjnorj lun che llaltro allora la somma del loro moto sara mjnore che sse tali mobilj fussino in fra lloro equali

Quando el maggiore de due pesi a corda congiuntj sara il primo nel moto allora il moto delli dua congiunti fia maggiore che sse il principio del moto fuzzi chomjnciato dal mobile mjnore

E see llj due mobili da chorda congiunti sara con parj . . . . . . (T2)

T2 in this page can be studied with the same model described in the comments to Ms G 86R
<table>
<thead>
<tr>
<th>T1</th>
<th>T1</th>
<th>T2 D1-4</th>
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<tr>
<td>BO MO</td>
<td>BO MO</td>
<td>WEIGHT</td>
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<td>D1 GEOM</td>
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*Diagram of connections between T1 and T2.*
Ms G  87R  T1,  T2,  T3-5  D1-2

Of the two moving bodies attached to the ends of a cord and thrown through the air.

What is primitive motion?
Primitive motion is that which is made while the moving body is attached to its motor.

What is derivative motion.
The derivative motion is that made in the air by the moving body once it separates from its motor.
The derivative motion derives from the primitive motion, and never has velocity or potentia equal to the velocity or potentia of the primitive.

The path of the moving body will conform to the rectilinearity of the path of its motor when all the parts of such moving body would be of motion equal to the primitive motion of its motor.

If all the parts of the motion effected by the parts of a whole are of equal motion, then such body will not be in rotation, and it will receive the entire potentia of its motor and it will accomplish the expected length due to its motion, the weight of the moving body being proportionate to the potentia of its motor.

Find the proportion of the weight required by the moving body so that it is proportionate to the potentia of its motor.

The problem of two bodies connected by a cord moving through air may seem foreign to fluid mechanics, but it surely presents a serious challenge to a modern fluid-mechanicist. I have been studying fluid mechanics for six decades and I remember to have seen only a simple description or formulation of this problem; I believe that Prof. E. Loedel Palumbo included in one of his books a discussion of the problem for the case of the two bodies falling down vertically (in vacuum ?). If the two-body system is launched as shown by Leonardo, the problem seems very difficult to study even with the present knowledge of fluid mechanics. If this problem is ever studied, I would advice to study first two disconnected bodies dropped into air at a short vertical distance one from the other. Some idea of the complexity of the air resistance can be surmised from examining Table III in the Elementary Fluid Mechanics by Hunter Rouse (see the data for Tandem disks). Perhaps, the laboratory methodology is what is in order in this as in a number of other cases [Macagno, 1982].
These bricks show life and human affairs. One falls down the other.

I always thought that the "domino theory" was invented by those who wanted to justify warring all the time. I tend to believe that the idea is as old as the invention of bricks!! In a more serious note, I suppose that one could use this phenomenon in an analogy with some fluid-mechanical phenomena in instability theory.
Of potentia.

A given virtu is so much potente the less space it occupies. This is meant for heat, percussion, force, and many other things.

We will say first that the heat of the Sun which is incident on the concave mirror and, from it, is reflected in the form of a pyramid that the more it converges, the more it acquires proportionally potentia. I.e., if the pyramid impacts the object at half its length it reduces its potentia to one half of its magnitude at its base. If it impacts at ninety nine hundredths of its length it reduces to ninety nine hundreds of its base and reaches ninety nine hundredths of the heat received by the base of the heat from the Sun or from fire.

Similarly, the impact of an iron pyramid will go deeper in the penetrable material the more pointed its vertex is. Similarly, the grave with less area and greater weight, because less quantity of air offers a resistance to it. Of motion and force we will tell somewhere else. Also other things like sweet, bitter, acid, tart, strong, do the same as said above. The illustrative example is the mixing of some of these in increasing quantity with snow or water, which by themselves do not give or remove flavor but the whole looses potentia.

I have consistently resisted the temptation to give words in English (or any other language) for some terms used by Leonardo. This is due to my early discovery that terms like potentia, inpeto, etc. could mean many different things in Leonardo’s writings. This always raises the thorny question of who should study scholarly Leonardo’s writings. I truly believe that it should be done in Italian until a synthesis that is acceptable is reached, but the reality of life indicates that most of the people who engage in this kind of study are not Leonardo’s countrymen.

As Leonardo was fond of analogies I offer for T1 the additional analogies of a metal or air spring, which the more compressed they are the more potente they become.
<table>
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<tr>
<th>T1</th>
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<td>D1</td>
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Ms G 89V

Diagram with cone-shaped structure labeled D1.
Of motion.

Whether the rotatory motion of the wheel around its axis makes the wheel more or less grave upon the axis. The same I ask about the wheel rolling over a plane.

We may think that this are trivial questions, but five centuries ago they were by no means easy to answer. The distinction between a fixed axis of rotation (which is also an axis of symmetry of the wheel!) and an instantaneous axis on the periphery of wheel is, in my view, very clever. So far, I have not found an answer to the above questions in other passages of Leonardo's writings.
Ms G 90R

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<td>CONCEP</td>
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</table>
This vortices must be made of died, turbid (?) water in clear water.

Marinoni [1989] in a footnote, suggests either turba or torba and also orba, as possible renditions for orba in T1. I believe that Leonardo made a note here on how to visualize the eddy motion in the flows sketched in D1-2, and therefore I chose turbid for the English rendition. Perhaps, Leonardo was explaining the method followed by him in obtaining this and other sketches.
Wind

How the wind impacting a cloud on its side can turn it around.

The wind that impacts a cloud only on one side while the opposite side of the cloud is in quiescent air, will push that cloud ahead and rotate it, producing a roto-translatory (?) motion as the water turns around the wheel of a water mill.

What is the cause for the clouds to grow upward.

When the flow of two opposite winds takes two clouds to a mutual impact, such clouds coalesce one into the other and not being able to dilate or to go down due to the wind passing below them, such clouds elongate along the space where the passage is less hindered, i.e., upward.

When two clouds meet each other along the same path.

When within the same wind two clouds meet each other, because it has the more powerful wind, the larger envelopes the smaller and they condense one into the other along the common contact, and rain is made.

In Ms G 91R there is a brief note on the northern winds which I found unrelated to fluid mechanics. Instead in the verso we have a full page of remarks on the fluid mechanics of the interaction that Leonardo assumed between winds and clouds. In T1, was Leonardo considering a small free-jet stream or a shear flow? My initial reaction to this and other similar comments on the interaction between wind and clouds has been that perhaps Leonardo considered the clouds as independent objects more like boats in a river (remember prophets and saints standing on clouds or angels sitting here and there on them, so profusely depicted during long time). But on second thoughts, he may have been able to gain a more advanced view. The analogy with the water wheel is not very good because the cloud does not have a fixed axis of rotation.

In T2, Leonardo uses a purely mechanical explanation for what must have been in fact a thermal effect if he was referring to summer clouds. A similar comment should be valid for T3, in which the notions of separate bodies in a fluid seems to emerge.
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<td>T2</td>
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<td>VAPOR</td>
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<td>WIND</td>
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<td>T3</td>
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<td>ANAL</td>
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![Diagram with symbols and labels D1, D2, D3]
The underneath wings are more oblique than the ones above both longitudinally and laterally.

The fly, in its standing still on its wings in the air, beats such wings with great velocity and sound, pulls up from the place of equilibrium (?) and goes up as much as the wings are long. In going up, puts it forward obliquely in such a way that almost impacts the air cutting it. As it lowers it, it impacts directly the air. It would go a bit upward were not because of the opposing weight of the animal with its oblique (component?). Let us take the inclination of the standing fly along the line $ef$ and the inclination of the wings motion between up and down along the lines $ab$, $cd$, which intersect with the line of descent $ef$ under right angles in such a way that the potentia of the descent along the inclination $ef$ is equal to the potentia of the ascent along the inclination of the wings motion along the inclination(s) $abca$. The hind legs act as a rudder, and when it wants to escape it lowers the wings.

With the advent of photography and cinematography it became possible to have reliable visual data of the details of wings of birds and insects during flight. For instance, Nachtigall [1966, 1974] made a movie of the wing motions of a fly with a time of $1/3200$ sec between each picture. What are we supposed to think of Leonardo's description in T2 of the motions of the fly wings? Maybe, this passage will, some day, be studied critically, by one of those few fluid-mechanics who have taken an interest in the flight of insects. Perhaps, Leonardo was conjecturing those motions through his knowledge of the flight of larger animals, assuming an analogy of behavior at very different scales. Hovering flight has been studied, among others, by Greenwalt [1960], Bennett [1966], Weis-Fogh [1975, 1977], Norberg [1975], Lighthill [1975], Ellington [1978,1980].
Wind.

Whether the wind is generated by excess or defect. The southern parts which disaggregate the humid reaching them become condensed and not being able to receive the increase they reject it back to where they are attracted by the vacuum created in the cold region where such humid collects in the formation of clouds, or in the northern parts where other clouds are formed.

Of the velocity of clouds.

The cloud in its path is of less velocity than its shadow moving upon the Earth. Proof: Let \( e \) be the body of the Sun, \( a \) the cloud, and \( c \) its shadow. Hence, while the cloud moves from \( a \) to \( b \), the shadow will move from \( c \) to \( d \). The shadows that go from the clouds to the Earth do it along straight lines all concurrent to the Sun, therefore, according to the fourth of this what is proposed is true. The said fourth says: of the equidistant (parallel ?) cuts of the angle between two concurrent lines the closer to the point of concurrence is smaller. Hence, being the clouds closer to the Sun than their shadows, there is no doubt that, in the same time, the shadow on the Earth will traverse a long distance than its cloud in the air.

Once I worked on T1, I compared my rendition with that of Ravaisson-Mollien into French, and I found both quite similar. In spite of that, I feel that I have not been able to capture the sense of this passage. For instance, I cannot answer the question: Was Leonardo referring to a small or a vast region of the planet?

Instead, T2-D1 has been very easy to handle. I would not dismiss this passage as a lack of sense of proportion because the difference of velocities between the cloud and its shadow is so small, Leonardo could be very bad in arithmetical calculations [Marinoni 1982], but he had a sense for the rigor one must show when dealing with general principles. What he is saying is that the velocity of a cloud can never become equal to that of its shadow, not even if the Sun were a million times the distance it actually is.
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<td>ARIA</td>
<td>VAPOR</td>
<td>WIND</td>
<td>BO MO</td>
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<tr>
<td>METEO</td>
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<td>GEOM</td>
<td>KINEM</td>
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Water.

Flowing water contains infinite motions, some faster and some slower than its main course. This is demonstrated by the things that float between "two waters", which are of the same weight (density?) as water, and show very well in clear waters the true motion of the water carrying them. Sometimes, the fall of the wave towards the bottom carries them (the things?) to impact the bottom and should bounce with them to the surface, were the floating body spherical. But oftentimes does not do this because the body may be longer or narrower one way than the other; its nonuniformity is impacted on the larger side by another wave that takes the body rotating and moving and transported in a motion which is at times fast, at times slow, now to the right and then to the left, now upward and then downward, tumbling and rotating now in one sense and then in another; all this in response to all its motors. In the struggle of such motors it is always the prey of the winner.

The waves are created by . . . .

The first sentence of T1 contains a good statement of the qualitative behavior of all flows, except the ideal case of a completely uniform flow. Leonardo relies here on the visualization of flow that is naturally produced by neutrally buoyant particles and bodies carried by water. He first states that they reveal truly the water flow, but then seems to discuss the reliability of this method when the "tracer" is not spherical.

acqua

Lacqua corrente a in se infiniti moti magori e minori che l suo corso principale quessto si prova per le cose che ssi sostengano in fra le 2 acque le qualj son di peso ecqale all acqa e mostra bene nell acque chiare il uero moto dell acqua che le conduce perche alcuna volta la caduta dell onda in verso il fondo le porta con seclo alla percussione di tale fondo e reffetterebbe con seco alla superficie dell acqa sel corpo notante fassi sperico ma isspesse volte no riporta perche e sarai piu largo o piu stretto per vn uerso che per laltr e lla sua inuniformjta e percossa dal magore lato da vna altra onda refresa la qual ua rivolgendo tal mobile il quale tanto si move quanto ell e portato il qual moto e quando veloce e quando tardo e quan si volta a desstra e quando a sinistra ora in su e ora in giu rivoltandose e girando in se medesino or per un verso e ora per laltr obidendo a tuttj i sua motori e nelle battaglie fatte da tal motori senpre ne va per preda del uincitore. (T1)

dell onde sono create dal . . . (T2)
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Ms G 95R T3

There will not be flux and reflux where several rivers do not flow into the same pool. non sara frusso e reflux dove piu fiumi non versan nun medesimo pelagho. (T3)

The language of T3 seems to be easy to translate, but its deep meaning escapes me completely. Was Leonardo trying to say that if we have a single river running into a given reservoir there will be flux and reflux? while several rivers will not produce flux and reflux?

Ms G 96R T1

Of the quadrature of the circle and the first who found it by chance. de 'ra del cl e chi fu il primo che lla trovo a chaso

Vitruvius, while measuring miles with many entire revolutions of the wheels on which carts move, extended over the *stadi* many circumferential lengths of the circle of such wheels. In fact, he learned that from the animals pulling such carts, but he did not know that that was the way of finding the square equal to a circle. The Syracusan Archimedes was the first to find that the multiplication of the semi-diameter of the circle times the one half of its circumference gave a rectilinear quadrilateral equal to the circle. Vetruvio mjsurando le mjglia cholle molte intere revolutionj delle rote che movano i charri distese nelli sue stadi molte linje circhunferentialj del cl di tali rote Ma llui le inparo dalli anjmalj motori di talj charri Ma non chonobbe quello essere il mezo a dare il ° equale a vn cl il quale pà per archimede siraghusano fu trovato che lla multiplitachatione del semjdiamjtro dun cl cholla meta della sua circhunferentja facieva vn quadrilatero rettilinjo equale al cl. (T1)

I suppose that Leonardo meant a rectangle when he wrote un quadrilatero rettilineo; perhaps he visualized in his mind a quadrilateral with all right angles.
### Ms G 95R

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### Ms G 96R

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64
One cannot breathe through the nose and the mouth at the same time. This is shown by somebody breathing in through his mouth and exhaling through the nose; one always hears the noise at the opening close to the uvula as it opens and closes.

I have some evidence contrary to T4. As soon as I read the first sentence of T4 I remembered a friend in my university-student days in France, who could inhale cigarette smoke let it come out slowly through nose and mouth and then take it back through both openings.
"GEOMETRIA CHE SI FA COL MOTO"

IN LEONARDO's Ms G

by Matilde Macagno
INTRODUCTION

In Leonardo's manuscripts there are a large number of notes on geometry, one part of which has been the subject of many papers. However, there have been almost no publications on the other part, the one that contains novel aspects of Leonardo's work on geometry. One notable exception was Hermann Weyl [1952], who detected the observations of Leonardo regarding symmetry, and considered them so remarkable that he named after Leonardo a theorem in the theory of symmetry [Martin 1987]. There are, however, other novel aspects beyond dihedral and cyclic symmetry in the geometrical notes. Those aspects received practically no attention, probably because they were not fully understood by previous students of Leonardo's geometry. In the Manuscript G, there are several such notes; remarkable are those on ways of mapping on a plane the surfaces of bodies of revolution. Instead, there is little on geometry of bodies undergoing deformation [Macagno M. 1987a-b, 1992], an area that is one of the most original among Leonardo's studies, unless we are willing to consider the mapping of surfaces of revolution on a plane as a form of deforming a system of lines into another shape. It is well known that such mapping is not as straightforward as it is in the cases of the cylinder and the cone. Leonardo handled this problem in a way that deserves detailed study. Several other questions, in which motion is given a role, i.e., belonging to the geometria che si fa col moto, are included in the Manuscript G and comments on them will constitute the first part of this contribution leaving the mapping of surfaces for the second part.
MOTION IN GEOMETRY

In the introduction to his transcription of the Ms G, Prof. A. Marinoni included a section with comments on geometry in this and other manuscripts [Marinoni 1989, pp. 15-21]. That section is very useful for the valuable historical background it offers; there we learn, for example, about the lively interaction between Luca Pacioli and Leonardo da Vinci. Other possible sources for Leonardo’s ideas on geometry are also mentioned, from Greek and Medieval authors to Francesco de Giorgio Martini. Also very useful are Marinoni’s notes on terminology. It is of interest to learn, for instance, that at the time of Leonardo the prism was called *colonna laterata quadrangolare*. Having studied in the last decade the notes on geometry by Leonardo and published several papers reporting my conclusions regarding novel aspects [Macagno M. 1987, 1991, 1992], I find it both interesting and challenging to analyze as a mathematician the same material that Marinoni considered as a historian and linguist.

The role of motion is perhaps more important in the generation of ideas in geometry than in the exposition of results and in the teaching of the subject. Such a role is illustrated by a number of circumstances that can be identified both in the history of geometry and in the terminology that has been adopted. In transformation geometry, for instance, one finds revealing terms like translation, halfturn, rotation, screw, glide, dilation, dilatation, shear, etc. However, many authors of books on geometry explain that transformations in geometry are about correspondences of points of two separate figures; for instance, Prof. Martin in his book *Transformation*
Geometry, says that in transformations there is no physical motion being described. According to Martin, we are describing only the initial and the end positions of physical motion; he considers that the description of physical motion belongs to differential geometry [Martin 1987]. In fact, in this and in his language throughout his book, he seems to adopt a more open position than other mathematicians.

Although Leonardo wrote about transformations and the *geometria che si fa col moto* several centuries ago, only recently do we see motion incorporated in the teaching of geometry in high-schools [e.g., Serra 1993]. For a long time there has been little reference to motion in the presentation of Euclidean geometry [see, e.g. a paper by Tucker 1959], perhaps because of being considered as a distraction from purely mathematical considerations, or perhaps because of more profound reasons, like the disturbing influence of the philosophical paradoxes of Zeno of Elea [Courant 1943, Lanczos 1970]. It must be understood that motion in geometry does not need to satisfy the stringent conditions we impose in engineering kinematics. For instance, in his book on geometry, M.J. Greenberg [1980], explains the use of motion as follows: *The term motion as we used it here does not mean continuous movement of a physical body as in common usage...* Therefore, even reflection in a line or in a plane can be conceived as points which are given a motion we would not tolerate in discussions of fluid flow. For an example of this kind of motion, see Courant [1943], where the arbitrary motion of points is considered within a circle in connection with Brower's theorem that states that there must always be a point that will remain where it was initially. In the same book, we
can find a discussion of quite arbitrary ergodic motions with a purely geometric treatment.

It is interesting to examine also books by historians of science, where motion in geometry and kinematics are usually overlooked or given little attention. Discussions of certain developments in geometry are widely discussed, while others seem to have never existed. For instance in Cohen [1985] there are thirty nine entries on geometry, but none refers to motion of figures; moreover, there is no entry on kinematics.

To investigate the geometrical notes of Leonardo armed only with the help of static geometry is surely a mistake [Courant 1943]. Marinoni, in his notes for the recent publication of the facsimile of the Ms G, mentions a sentence from the *Trattato della Pittura* in which painting, and even philosophy, are considered as disciplines concerned with motion [Marinoni 1989]. However, in his examination of the geometrical works of Leonardo, he does not remark the extent and the depth of those notes and drawings concerned with motion applied to geometry or those aspects in which geometry is applied to the study of motion. The fact is that Leonardo wrote very clearly about the transformation of figures passing, by means of different motions, from one shape to another that may be quite different and "irregular"[see Ms G 56R]. Among other things, we must distinguish between the use of motion to arrive at some geometrical results, which is nowadays typical of the Math Laboratory, and the use of geometry to describe the complexities of the kinematics of rigid and deformable bodies.
QUALITATIVE THALES' THEOREM

In Ms G 92V, Leonardo considered the length of the displacement of a cloud and that of its shadow. In support of his statement, Leonardo enunciates a qualitative (instead of quantitative) form of the so-called Thales theorem about the segments of parallel lines which intersect the two sides of an angle. It is one thing to use proportions to state that the four segments involved satisfy $CD:AB = SC:SA$, and another that if $SC > SA$, then $CD > AB$ (Fig. 1). In view of the great preference for quantitative geometry that existed for a long time, if Leonardo learned some form of the Thales theorem, it was most probably in the form of proportions and not of inequalities.

In the qualitative form, as expressed by Leonardo: *gli tagli equidistanti dall'angolo delle due linee concorrenti saran tanto minori, quanto e'fieno piu vicini all loco del concorso* [Ms G 92V], it seems plausible that this form of the theorem was adopted and not learned by him. His analysis does not take into account that the difference between the distance from the Sun to the cloud is almost equal to that of such a source of light to the shadow. It is only in terms of qualitative analysis that his statement is not trivial. An appreciation for qualitative knowledge is a mark of many passages in Leonardo's mathematics. I believe this is a rather modern characteristic.

Another comment of interest, according to several historians [Tannery 1887, Boyer 1950, Heath 1981], is that Thales may have proceeded very much in an empirical way in his works on geometry, at least in his own periods of acquiring knowledge. Leonardo himself
proceeded in a similar way. According to Tannery, as quoted by Boyer (1950), Thales may have appealed in his demonstrations to the evidence of the senses: in fact his theorems were those whose truth one would recognize by the execution of some practical construction. Boyer (1985), in another book, summarizes the arguments of several historians who consider that the mathematics of the sixth and fifth centuries B.C. was too primitive to permit the introduction of deductive methodology by Thales.

PARALLEL LINES AND LIMITING POINTS

In Ms G 59R, Leonardo tried an extension of the concept of parallelism from straight to curved lines (Fig. 2). In order to realize that such extensions are possible, we only need to remember that in geographical coordinates we speak of parallels for those lines with constant values of latitude that do not intersect each other. Moreover, there exists a family of lines orthogonal to the parallels along which the distance between any pair of parallels is constant. Leonardo introduces four kinds of parallel lines: the common Euclidean parallels, a pair of circular arcs of the same radius (one results from a translation of the other), two concentric circular arcs, and a circumference and its center.

I will discuss first the last category of parallels. It is, in my view, a limiting case of the third kind of parallels; the inner circumference tends to one of vanishingly small radius, but remains a circumference through the entire process. We cannot know how Leonardo would have
explained this case, but I do not believe it would have been as I see it, five centuries later. No doubt he had something specific in mind, because more than once he wondered what happens as a geometric configuration diminishes indefinitely in scale. In Ms G 1V, Leonardo considered the top portion of a triangular pyramid assuming that its height is made smaller and smaller. At the very top, there is a point, but in the limiting process the trihedron is always there; therefore, the limiting point and the point at the limit are two geometric entities with different properties. Leonardo's explanation is in the form of a mixed question/answer: *Equi si dimanda se mai la division del triangolo destrugge la figura d'esso triangolo, ancora se tal division sia in verso l'infinito; e se tal triangulo si destrugge dessa figura, senza dubbio li tre lati della piramide concorre in un punto. (Il) che non e vero*. I would say - in spite of the passage not being very clear - that Leonardo shows at least some intuition of the limiting process he describes.

The second kind of parallels (Ms G 59R D2) does not resist analysis, and must be considered as a misconception. The two circular arcs are of the same radius, and one can be obtained from the other by means of a translation. It is true that this produces pairs of corresponding points separated by the same distance, but it is also easy to see that the supposed parallel lines, if prolonged, will intersect each other (Fig. 3). In addition to this, there is no family of lines orthogonal to the putative parallel curved lines such that along them we find the same distance.
Although motion is less significant in the subdivision of figures, in the complementary operation of aggregation of figures motion is required at a larger scale [Martin 1987, Serra 1993]. In his manuscripts, Leonardo offers many examples of both subdivision and aggregation of figures [Macagno M. and Macagno E. 1987]. In the Ms G, the salient note on this topic is that of the subdivision of the pyramid [Ms G 41R, 50R]. Leonardo dealt with this problem extensively; this aspect of his work has been analyzed in another publication [Macagno M. and Macagno E. 1987]. What is novel in this question is that Leonardo was much less concerned with the ratios of volumes in a quantitative form than he was with a quality to be preserved. His subdivision of figures has been related to some applications, as the determination of centers of gravity, while a basic mathematical aspect of his work was overlooked. The essential point is that Leonardo was trying to subdivide figures into smaller figures congruent among themselves and similar to the given figure. In this endeavor, he was guided by the following attractive analogy.

One can subdivide a rectangle, or a triangle, into congruent smaller rectangles or triangles similar to the corresponding given figure (CM II 68V). It is easy to see that one can subdivide a right-angle parallelepiped in the same way. If one would reason that a rectangle is geometrically analogous to a parallelepiped and a triangle to a tetrahedron, it is plausible to assume that a tetrahedron should be susceptible to subdivision in the same way. I have studied this question and the conclusion is that the analogy fails to materialize (Fig. 4). The
most striking way of verifying this failure is to construct eight congruent tetrahedra and try to put them together to form a larger similar tetrahedron [Macagno M. and Macagno E. 1987]. Did Leonardo discover that his analogy does not hold? I believe that he did, even if there is no clear statement of this in his manuscripts. But the main point I want to make is that one misses entirely the picture, if one discusses this question in terms of ratios of volumes of figures that are not congruent. The note in the Ms G is a comment of Leonardo about ratios of volumes, and it is much less interesting than similar notes in the Codex Madrid, where he worked in great detail with quadrangular pyramids; note that in such a case, the base as well as all sides can be subdivided according to his criterion, but no so the pyramid itself.

It seems obvious that lack of familiarity with modern transformation geometry may lead any student to misunderstand what Leonardo was actually doing. In his work on the subdivision of the pyramid, he was pursuing a subtle point which eluded Marinoni. This question has been discussed in several publications [see, e.g., Macagno E. 1985, Macagno M. and Macagno E. 1987]. What Leonardo was trying to do was to investigate an analogy that attracted him very much. The problem is not of a quantitative but of a qualitative nature. It is trivial to show that a pyramid (or a tetrahedron) has a volume eight times larger than the smaller pyramid determined by a cut at half its height (this was known to Euclid, and probably to others before him). What Leonardo was after was the possibility of actually cutting eight such pyramids out of the given pyramid in a manner similar to the subdivision of a triangle in four similar congruent triangles. Conversely, we could look at this problem as the question of putting
together eight small congruent pyramids to form a pyramid of twice the dimensions of the given ones. This proves to be impossible in a more striking form than the subdivision approach. The sketches in Ms G 41R, 51R may have been a phase of the search for the desired partition, and at the same time a verification that it was hopeless. The main sketch is the most revealing and deserves more detailed study than it has received.

In Ms G 40R D1-4 there are four drawings which illustrate never-ending geometrical nesting processes. In D3, for instance, Leonardo begins with a circumference into which a square is inscribed; within the square he inscribes then a circle, and so on. All the four drawings in Ms G 40R can be continued indefinitely and have as limits vanishingly small figures. I have considered already a similar process in previous publications [Macagno M. and Macagno E. 1987, Macagno M. 1990], but here I want to emphasize the dynamic kind of geometry Leonardo used, as opposed to the static one prevailing before him. In all these drawings one can trace a piecewise continuous trajectory of a point moving toward a limiting position; the singularities of such a path are related to the diminishing size of the figures in the sequence. Taking into account that motion is always present in Leonardo's considerations, I am led to conjecture that a similar, but surely not identical idea, may have been in Leonardo's mind.

Another subdivision proposed by Leonardo pertains to the surface of a sphere. In Ms G 67R, he sketched a subdivision of the surface of the sphere into supposedly congruent triangles (Fig.5). Such subdivision, as depicted, is geometrically impossible. Only five regular
polyhedra, the Platonic bodies, can be constructed in Euclidean geometry. This is in contrast to the regular polygons, which can be infinite in number. Three of the Platonic bodies have equilateral triangles as faces, and the one with the maximum possible number is the icosahedron, in which five triangles concur around a vertex. If we try to construct a body with six such triangles a figure in the plane results and, if we continue the construction, a tiling of the plane with triangles (or with hexagons) results. This is a tiling that cannot be done on a sphere, or more accurately, inscribed into a sphere, or even circumscribed if preferred. Hence the nearest approximation to the surface of a sphere feasible in this way (with congruent triangles) would be an inscribed and a circumscribed icosahedron. Perhaps, Leonardo could have proceeded with semi-regular polyhedra, but nothing of this kind has been found in his notes.

In fact, the subdivision of the surface of the sphere discussed above is part of a passage in which he proposes a way of calculating the volume of the sphere; in such a context, it will be considered again in the last section of this contribution.

FLEXIBLES LINES

Leonardo considered systems of pulleys in several of his manuscripts. I have examined all of his available notes on this subject and found that the ability to raise heavy weights with a small force is only part of his concern. He was also very interested in the convoluted flexible line formed by the cable, rope or string as it moves through the
device considered in each case. I have already discussed what we can call the kinematics of the flow of strings through a mechanism in other publications [Macagno M. 1987, 1992]. This facet of Leonardo’s studies of motion is very revealing of his interest in fundamental questions, and was completely disregarded by many students of his work, who focused on more practical topics.

Regarding the Ms G, a number of notes on systems of pulleys can be found in 17V, 47V, 78R, 80R-V, 81R-V, 82R, 87V, 95V. However, the notes in other manuscripts are more explicit in the study of the changing shape of the string as the force is exerted. In 17V there is a clear statement about the velocity with which the string moves, that it is different at different places along the length: "in the string moving through the pulleys, the last (part) has more velocity than the others" (Fig. 6) In Ms G 82R T2, Leonardo stated more clearly that the string winding around the pulleys is less and less for those pulleys farther and farther away from the point of pull. In a qualitative statement, he said: Tanto e ppiu tardo delle corde che ppenetranno infra le taglie qunto esse son piu remote dal suo motore. In Ms G 87V T3, the statement is quantitative in terms of a functional relationship which is described as "uniformly disform": Epiramjdale he il moto che han le corde desse taglie perch e vnjformemente disforme dalla prima corda insino all ultima si va ritardando. Without doubt, the analysis of the motion of the cord is more accurately discussed by Leonardo in other manuscripts [see Codices Atlanticus, Arundel, Madrid and Forster, Ms E, discussed in Macagno M. 1992].
CIRCULAR SHEAR

By parallel linear shear, a triangle can be transformed into another triangle with the same base and altitude, thus preserving the area. There is a proposition of Euclid about this, but there is a chasm between his approach and Leonardo's use of motion in this geometric transformation. This is illustrated by Leonardo in many passages of his notebooks [Macagno M. 1987a]. He went considerably further than the above by imposing a shear motion that was a function of y in a plane in which the triangle had the base on the x-axis (Fig. 7). He understood very well that the sides of the transformed figure could be of any shape. Moreover, he showed that a similar transformation was possible for circular shear (see Codex Arundel). In this case, there have been some doubts about this process conserving area, but they can be discarded when modern kinematics is used to examine the problem [Macagno M. 1987a]. The relation between Leonardo's approach and the so-called Cavalieri's principle [Boyer 1959] has been discussed in a previous publication [Macagno M. 1990]. I should add to those comments that Leonardo's approach in the Ms G is closer to our finite-difference calculations of areas than to Cavalieri's principle.

In Ms G 56R, Leonardo illustrates transformations of two circular sectors by circular shear: col moto curvo fatto delli due settori . . . Of course, the base of the two figures must be of the same length (Fig. 8). He deforms very arbitrarily one side (one radius), then the motion supposedly takes this sinuous line in a rotation that places its points on the circumference at the required distance; thus we can generate two figures of different shape but of the same area. See Fig. 8, which is a
composite of Ms G 56 R D2 and D3. I have illustrated in a recent paper [Macagno M, 1992] that there is a counterpart to this transformation in the Couette flow of an incompressible fluid, both theoretically and experimentally.

Marinoni's analysis of Ms G 56R shows a lack of understanding of the geometry in motion that he himself considers as an important step forward. If we allow motion, we must not reject that a rectilinear triangle be transformed into a curvilinear one, the curvilinear sides do not need to be solely arcs of some supposedly "perfect" figure, they can be sinusoidal or as arbitrary as the mathematician, or even nature, wants to make them, relative to some ideal conventional model. I believe that most of what is considered geometry would be unacceptable to Marinoni; see e.g. all the geometry contained in Fearful Symmetry by Stewart and Golubitsky [1992]. For those who can only conceive of a static and frozen oversimplified geometry, this book may be of interest. Here is a sentence from the Preface: "... how can deterministic mathematical models produce random behavior?" Although Leonardo does not seem to have introduced randomness in geometry, he studied the kinematics of turbulence, one of the most fascinating phenomena in which determinism and randomness come together. Even discontinuities should be allowed; geometry is not only about nicely behaved classic curves. (Fractal geometry would not be geometry according to the "perfectionist's" criterion.) One wonders what comment would be elicited by Peano's path inside a rectangular area in the proof that all its points can be covered by it; perhaps another statement like: "Ma qui siamo fuori della matematica".
One cannot accuse Leonardo, who in four decades produced enough writings to occupy scholars for centuries, of being "sbrigativo", without thinking that so were Fermat, Abel, and many others, before and after him, who left for us conjectures that have been so puzzling. His D1 in Ms G 56R is an interesting subdivision of a quarter of a circle into several either equivalent areas, or one twice another.

In Ms G 57V, Leonardo combines the notions of symmetry and motion in a way that, one can conjecture, attracted Hermann Weyl in his study of the manuscripts at the middle of this century (Fig. 9). To reproduce 57V D1 in Fig. 9 we can proceed as follows: suppose we draw one radius and then rotate it by steps of 90° each time. This is equivalent to dividing the circle into four quadrants, obviously of the same area. Suppose we distort the radius into a spiral curve and then apply the same rotations, the circle is divided again into four equal areas. Why does the distortion have to be a traditionally nice curve? It certainly can have any shape we want to give to it. As if to hammer down his point, Leonardo drew another set of drawings (Ms 57V D3) in which the sectors are eight instead of four. The spirals in Ms G 57V D1 and D3 are drawn as Archimedean spirals. We should realize that the transformation in Ms G 56R need not be supported by cyclic symmetry; the true sequence of these two pages may very well be from 57V to 56R. This reverse order is found often in Leonardo's writings as a consequence of his being a truly left-handed person.
Nobody would claim that one can determine with high accuracy the value of \( \pi \) by rolling a wheel on the pavement or by staining the periphery of a disk and rolling it on a piece of paper to transform mechanically a circumference into a straight line segment (See Ms G 58R T5). However, this simple experiment, if performed carefully, can lead to the practical determination of rather useful values of \( \pi \). This is an example of empirical geometry. It would seem that if experiments and laboratory methods are appropriate for any branch of physics, one should not avoid such methodology when dealing with geometry. After all, what is the sense, for instance, of using \( \pi \) accurate to the tenth significant figure when the radius is obtainable only up to the second or third figure? On the other hand, let us not forget the geodesic operations of Gauss trying to verify by measurements the theorem about the sum of the angles of a triangle. Leonardo appears to have recognized the place of theory in geometry, but not even mathematics was out of bounds for him when laboratory methods appeared necessary or useful. Besides, there is no other way of ascertaining whether a geometrical theory applies to the physical world than approaching geometry as a part of physics; in this opinion I feel supported by Gauss' geodesic work. Of course, I am not claiming that Leonardo knew all of this as we know it today, only that he was eclectic in his pursuit of knowledge. We use numerical computational methods today to increase our knowledge; does a computer, for instance, need to recognize whether \( \pi \) is rational or irrational? We need this knowledge when working in certain highly theoretical areas of mathematics. It does not seem fair to Leonardo to say that the problem of the irrationality of
π was "tranquillamente eluso" (Marinoni 1989), when in fact it does not play a role in what we may call math-lab methods.

In Ms G 96R, we find a passage that reveals the roots of Leonardo's empirical approach to estimating the length of the circumference and the areas of the circle and its segments, as well as of bodies of revolution. In other manuscripts, Leonardo has also shown his awareness of more theoretical approaches. I will quote the beginning of the passage that describes Vitruvius' method for measuring distances: "Vetruvio misurando le miglia colle molte intere revoluzioni delle rote che movano i carri, distese nelli sue stadi molte linie circunferenziali del cerchio di tali rote..." At the end of the same passage, we find a reference to the use of the length of the circumference to find the area of the circle according to Archimedes: "Prima per Archimede siragusano fu trovato che la multiplicazione del semidiamitro d'un cerchio colla metà della sua circunferenza faceva un quadrilatero rettilineo eguale al cerchio."

**AREAS OF SECTORS AND SEGMENTS**

Before I discuss the notes on calculation of areas in the Ms G, I would like to describe a passage in Leonardo's Anatomical Drawings 121R. It consists of two sketches which I have used to draw Fig. 10. The text consists only of the words quadratura darchimede. One figure shows a circle which has been divided into a number of congruent sectors, which in turn have been subdivided each into a triangle and a circular segment (Fig. 10-B). In Fig. 10-A we see a rearrangement of
the figures into which the circle was initially decomposed. The idea is that a circle has been transformed into a figure that would tend to a rectangle if the subdivision is pursued toward larger and larger number of elements. This approach is entirely different from the one we find in the Ms G.

In the Ms G, Leonardo deals with the quadrature of the full circle (39V D6), the half circle (58V), and segments of a circle (39V D4, 68R D2). Marinoni [1989] has described very well the procedure for the case of a circle. The circle is first divided in an even number of relatively small sectors, then one rolls the circle on a line in the plane, marks the half length of its circumference and constructs triangles of the same height and base as the sectors (Fig. 11A-B). A similar, comblike figure corresponds to the other half of the circle and can be placed on top of the first one so that a full figure very close to a rectangle is obtained. The area of such a rectangle is equal, in the limit, to that of the circle. By this method we could also obtain the area of any given sector by noticing what fraction of the rectangle corresponds to the given sector. The general idea is always to rectify the arc of circumference involved, or part of it, and then construct on it either a triangle or a rectangle.

Another way of finding the area of the circle, or that of a semicircle, is in Ms G 58V (Fig. 12). In this case the small triangles corresponding to the small sectors are in the same sequence but they are deformed so that they end up having a common vertex, their height being as before equal to the radius of the circle. One by one they are
equal to the equal sectors that originated them. Hence Leonardo procedure in this case is correct.

In Ms G 39V, Leonardo dealt also with a segment of a circle under the title: "Quadratura d'una porzione mediante il moto che dirizza la periferia della portione". In Fig. 13, I have reproduced his sketches in 39V D3. It seems that Leonardo expected that the upper vertices of the small differently shaped sectors into which he divided the segment of circle would fall along a straight-line path, but this is not true, and so the figure is not a trapezoid with all rectilinear sides. The upper side of it is actually a segment of an hypocycloid. Besides, the empty spaces are not an exact match for the other half of the figure as in the case of a circle. Leonardo does not seem to have realized that a simpler way to solve this problem was to find the radius of the given segment, find the area of the corresponding sector and subtract the triangle formed by the two limiting radii and the chord of the segment. It is true that he tries in Ms G to find the center of an arc (39V D5), which will give the desired radius, but there seem to be no explicit connection between the two problems. Anyway, for Leonardo, the rolling on a plane of a curvilinear segment could have been a useful practical means for finding its area when the curvilinear side is not circular, or elliptical, but part of some other curve such as the one we will consider in Fig. 19.

AREAS OF BODIES OF REVOLUTION

In the Ms G several folios (38, 39, 61, 62, 67, 68) contain procedures based on rolling a body over the plane, in order to
accomplish a mapping of curved surfaces onto a plane surface. This is generally done by Leonardo in order to find the area of the body. I consider these passages as the highlight of the geometrical studies in the Ms G. They are in several ways a preview of distant future (relative to Leonardo) developments in the use of coordinates on curved and plane surfaces, methods of mapping curved surfaces, and procedures for grapho-mechanical calculation of areas. Such procedures are structured in a way that foreshadow the future approach that centuries later will appear in the integral calculus. I believe that we should consider Leonardo as a forerunner of those developments in a strict sense. Although some ideas and notions were clearly in an embryonic state in his mind, he did find rather effective ways of putting them into practice within the limits of available techniques. This seems an unavoidable conclusion if we carefully follow his considerations and descriptions and try to repeat what he may have actually done to arrive at the results that are summarized in the above mentioned folios of the Ms G. In this section, I will begin by describing what Leonardo did. I will then describe my own repetition of his constructions using geometry in motion and examine them critically in order to evaluate his efforts.

For 38V and 61V, Marinoni [1989, p. 162] correctly describes these passages as *sviluppo su una superficie piana*; in English, we would say *mapping on a plane surface*. Leonardo considers cylinders (Ms G 59V, 61R-V), cones (Ms G 42V, 59V, 61R-V, 68V), spheres (Ms G 38V, 39R, 42V, 68R-V) and bodies of revolution other than the sphere (Ms G 67V, 68R). I will summarize some interesting aspects of Leonardo's notes on cylinders and cones (Fig. 14) and consider in full detail bodies of
revolution, since in the latter one can find the more novel aspects of Leonardo's method and at the same time see what difficulties he experienced in these attempts.

To understand better the following analysis, we should keep in mind that for cylinders and cones, a point by point or line by line correspondence with a portion of a plane can be made without any distortion. In modern terminology, a conveniently chosen double family of lines on the cylinder would map into a Cartesian orthogonal system of coordinates, while those on the cone would leave an imprint on the plane like a polar system of coordinates (See below the analysis of Ms G 42V).

The many figures in the Ms G showing the circular sector resulting from rolling out a cone on its generatrices on a plane should be examined taking into account a simple geometric ratio, i.e., the length of the circumference of the base, \(2\pi r\), over the length \(R\) of the generatrix. This ratio, \(2\pi (r/R)\), which gives the angle of the above sector, must be less than \(2\pi\). Therefore, in the cases in which Leonardo gives the sector as half a circle, the generatrix must be equal to twice the radius of the base. This is useful in interpreting the use of the cone in relation with the area of the sphere in Ms G 42V.

In Ms G 38V, under the title *De quadrature della superficie sferica*, Leonardo included two procedures to obtain the area of one eighth of the sphere. He also considered the quadrature of the sphere in Ms G 39V, 42V, 68R-V. Here one should be concerned with Leonardo's attempts to develop on a plane a surface which is not amenable to such
an operation without losing certain of its properties. An illustration of this is given by the impossible converse operation of trying to wrap a piece of paper around a sphere without wrinkles. It is because of this impossibility that there is such variety of geographic coordinates for the maps of the surface of the Earth. Although Leonardo does not seem to be interested in maps, he is actually performing mappings when he tries to find the area of a sphere in this manner.

Continuing the description of Ms G 38V, we find in D1 a representation of a sphere in which two views of parallels have been drawn. We see an elevation above and a plan view below of those parallels (Fig. 15), much in the way that Monge introduced them in his descriptive geometry centuries after Leonardo. In Figs. 16A-B, I have included 38V D2 and 39 R D3. There are two similar drawings, 38V D4 and 39V D3, but I have selected the second because it carries more information about what Leonardo actually did. Regarding 38V D2, Leonardo obtained it by rolling successively on a plane the parallels drawn on one eighth of the sphere. We can assume that such parallels were freshly painted and so they left an imprint on a piece of paper. According to him, a quadrant of a circle plus a segment would be covered (Fig. 16A). Leonardo gave a detailed description of his grapho-mechanical construction of Fig. 16A. In 38V T1, he began by saying: chol moto de el 4° del maggior cierchio della spera si fara la linja retta rS i locho piano . . . ; i.e., "by the motion on a plane of one fourth of the maximum circle of the sphere (meridian), one obtains the straight-line segment rS". In the rest of this paragraph Leonardo explains that rt is perpendicular to rS and of the same length; then rS is divided in equal parts and the arcs of circumference Sx, cy, hx, Li
are drawn going somewhat beyond the line \( rt \). At this point, one should cut disks of paper with radii presumably equal to the radii of the corresponding parallels on the surface of the sphere. Leonardo is not explicit about this, but there seem to be no other choice for such radii if one takes into consideration the rest of 38V. Such disks should be rolled over the corresponding arcs over one fourth of their periphery; Leonardo says that this would give points beyond the line \( rt \) thus determining the curve \( rvit \). The area enclosed by \( rSxtv \) should be equal to the area of the eighth of the sphere.

In the following paragraph (38V T2), Leonardo begins by saying, regarding a construction similar to that in Fig. 16B: *he ssettv volessi fare tal motj paralelli sopra le rette della 3\(^{a}\) o 4\(^{a}\) fighura e sarebe migliore isspediente perche meglio si move la parte della rota fatta di carta sopra linie rette che sopra linje churve purche li paralellj rettj sien di larchezza equale alli paralellj churvilinj*, meaning that it is better to draw straight parallel lines perpendicular to the base line rather than the arcs of circumference in 38V D2. He then considers that the area \( moed \) can be decomposed into a triangle and a circular segment. The area of the latter should be determined by geometry in motion (i.e. by the procedure given in Ms G 38V and 68R).

A drawing similar to 38V D2 is in 42V, but in this case Leonardo included a more detailed mapping, which I have shown in Fig. 17A. I have also reproduced another drawing (Fig. 17B) in which Leonardo sketched roughly and side by side a system of orthogonal lines on the cone and on the sphere. This sketch (42V D3) is suggestive of an analogy between the mapping on the plane of the two surfaces of
revolution, the cone and the sphere. An error of Leonardo in 42V D2, to
which I will refer later on, may be caused by relying on such an
analogy. Regarding the mapping of two families of lines, I would like to
mention that I had discussed already a similar mapping in the Codex
Atlanticus (CA 602R), where Leonardo maps the family of orthogonal
lines formed by the radii and circumferences within a circle into those
(not orthogonal) within an ellipse that results from the semicircle by
means of an affine transformation [Macagno M. 1992].

In Fig. 18, I have reproduced 67V D2 and 68R D1, two mappings of
bodies of revolution for which Leonardo used the method he described
as preferable in Ms G 38 T2. I consider this a Cartesian coordinate
choice instead of the polar coordinate choice we can see in Fig. 17A. As
an incidental remark, I would like to say here that the logical sequence
in the way Leonardo progresses through the topic of mapping surfaces
of revolution in Folios 38, 39, 42, 67 and 68 seems to point in a direction
opposite to that of the book having been written from the pages with
high numbers toward the pages with low numbers (See remarks about
chronology in Macagno E. 1985, 1988, 1989 a,b,c].

To analyze the work by Leonardo that I have described above, I
will use mathematical tools that are much more advanced than his. I
believe that this provides an objective basis to establish whether he
was proposing a kinematic operation that was feasible and that would
lead to correct results or not. I know the dangers pointed out by Zukov
[1969] of either viewing Leonardo only from the past or only from the
future. On the one hand, I have tried to see what he could have
acquired as received knowledge, and on the other hand to detect what
was his own invention. It would be a mistake to renounce the use of all the mathematical knowledge I have acquired, in an entire life devoted to mathematics, in trying to understand what he did. When he was creating new ways in geometry, it is much better to consider his work in the light of modern mathematics rather than in that of the geometry he could have inherited.

I will consider some additional figures constructed according to Leonardo's descriptions, including the rolling of disks on paper as he did. My analysis begins with what we may call the Cartesian-coordinates diagram (See Fig. 19). My figure is the result of grapho-mechanical constructions, except for the theoretical curve denoted with open circles, (o); the abcissae of such points are proportional to \( \cos \phi \), where \( \phi \) is the latitude if we refer to geographical coordinates on the sphere. As prescribed by Leonardo, I actually cut thick-paper disks and rolled them on the parallel lines trying to obtain the same points. I found, after doing it several times, that one could come quite close to the theoretical points. I did not plot my results here, considering more important the abcissae indicated in Fig. 19 with +, x, A, which I obtained by directly measuring distances on the free-hand sketches of Leonardo in the Ms G. I decided to take such measurements when I felt that his sketches looked quite correct, qualitatively speaking. After obtaining the results illustrated in Fig. 19, I believe that it is quite plausible that Leonardo experimented with this question in the way we work in the Math Lab today, and that thus his sketches had a sound basis. I think that an artist like him could convey easily the information contained in carefully drawn figures when he was describing his findings and making the sketches in this notebook. In


Fig. 19, I have shown, as he did in Ms G 38V D4, the cord of the full-line curve, thus showing a triangle and a segment; the curve is obviously not an arc of a circumference, but I used such an arc as an approximation to calculate the area, and found a reasonably good result. I also drew an average curve based on my measurements on three of Leonardo's sketches, which gave an excess of 5% relative to the accurate value. This is in fact remarkably good if one looks at the size and quality of Leonardo's sketches.

In Figs. 20 and 21, I have summarized my analysis of Ms G 38V D2 and 42V D2. In this analysis, I had to make some assumptions, because Leonardo seems to have been less careful with these figures, in which he used what we call today polar coordinates. For instance, I drew the quadrants for the two figures as given by $\pi R/2$ and $R$ respectively, although this is implicit rather than explicit in the Ms G. I found that the additional segment of area should not be beyond line $rt$ in drawing 38V D2. In 42V D2, the radial lines should be curves that are not orthogonal to the other family of lines (See Fig. 21). One can obtain conformal mapping regarding area but not preserve the angles at the same time. This procedure may yield correct values for the area of the sphere if carefully carried out, but, as Leonardo himself realized, the procedure illustrated in 38V D4 and 39V D3 is better.

In addition to my above-described study, which is as empirical as the procedures of Leonardo that I am examining, I found that a theoretical analysis was warranted. Except for some quantitative mistakes, I found that what he was doing is closely paralleled by the contemporary treatment in integral calculus of the areas of the sphere
and other bodies of revolution. If we take into account that to draw the figure 39R D3 (Fig. 16B), Leonardo used what amounts to a system of Cartesian coordinates in which \( yR\cos\phi \) is represented on the abscissae and \( R\phi \) on the ordinates, where \( R \) is the radius of the sphere, \( \phi \) is the latitude and \( y \) is the longitude, then the element of area will be given by \( R^2\cos\phi \, df \, dy \). The corresponding integral gives \( 2\pi R^2 \) as the area of the figure \( mbdco \). Hence Leonardo is correct in stating that this mapping leads to the area of the hemisphere.

In considering the other procedure, in which Leonardo uses a polar diagram (see Figs. 16A and 17A), I have found that his idea is fundamentally correct but his sketches contain some errors. It seems to me, in fact, that they do not follow the descriptions of the procedures given in 38V and 42V. Leonardo says that \( rs = rt \) (Fig. 16A) should be equal to one fourth of the maximum circumference, i.e., \( \pi R/2 \), but in assuming that \( sx = \pi R/2 \) he runs into an inconsistency. Hence, the figure \( rhsxtv \) is wrong in shape and in area. What seems to have happened is that he in fact made \( rs = rt \) equal to the radius \( R \). To illustrate this point I have drawn Fig. 21. Utilizing integral calculus, it can be very easily proved that the construction - according to his written instructions - should give the correct area. Leonardo's grapho-mechanical construction would give an area \( \pi/2 \) times smaller than the correct area. In figure 42V D2, one can see another error, because the radial lines are not line segments but curves as I have shown in Fig. 21.

When Leonardo recognized that the method shown in Fig. 16B and 18 is more accurate and easier to apply, the other method may have lost importance for him. Even so, I believe that it is useful for us that he
did not discard it from his notes, because it is very revealing of his versatile creativity, even if it suffers from some errors of geometric construction.

VOLUMES

Leonardo used motion much more in mapping the surface of a body than in the determination of the corresponding volume. Nevertheless it is worth discussing the ways of finding volumes that Leonardo considered in the Ms G as part of his notes on geometry of transformations. In Ms G 39V T2 D3, we find the essence of his procedure (Fig. 22). Once the surface of the sphere has been mapped onto the plane and transformed into a two dimensional figure of equal area, this area is subdivided in equal-area elements and, using them as bases, pyramids are constructed. In his own words: Quadrato che aj la superficie della sfera con parti esso quadrato in tanti quadrieti quanto atti piacie pur che sieno infra lloro equali e ffa che ciasscuno quadretto sia basa duna piramide della quale lassi sia semjdiamjtro della spera che voi chubare e sien tutti equali.

Assuming that Leonardo did the mapping of the area of the sphere conformally (preserving areas instead of angles or lengths) his drawing with small pyramids with height equal to the radius of the sphere appears adequate (Ms G 39V D3). In this method, I believe that Leonardo proceeded by analogy to an extension of the procedure he described for finding the area of the circle (Fig. 11). In this analogy, as I conceive it, the corresponding elements would be, first,
mappings onto a plane of the length of a circumference and of the surface of a sphere, and, second, the placing of small triangles on that segment and of small pyramids on the area equivalent to that of the sphere. Leonardo's method of determining the volume of the sphere is easy to justify by using elementary integral calculus. We only need to write the integral for the volume of a sphere in polar spherical coordinates and compare it with the expression for the area in the same coordinates. The only operation that is new is the introduction of a factor of $\frac{R}{3}$ as the common height of all the elementary pyramids into which the sphere is decomposed (Fig. 22).

CONCLUSION

I am aware of the criticism that is sometimes leveled at papers that discuss just one of Leonardo's manuscripts. I also know that there are different stages in investigating the works of Leonardo, who left only what can be considered as the working notes of a researcher and a student of many topics. However, a first stage must necessarily comprise analyzing the documents as they have come to us. It is in this context that I have written this appendix as an analysis of the Manuscript G focused on the innovative studies of geometry by Leonardo. The work of synthesis belongs to another of the above mentioned stages.

In a passage in the Anatomical Manuscripts, Leonardo wrote the following thought about the role of the mathematics he was using in his studies: "non mi leggha chi non e mathematico nell'ni principij
"[Folio 116R TV]. I take this as a warning not to interpret his work only in the light of classic geometry and unaware of the new geometry. Kenneth D. Keele, in his comments to the Anatomical Manuscripts 116R TV, emphasized Leonardo's use of geometry in physiology and concluded with a quotation from 153R which asserts that nature cannot generate motion in animals without a mechanical system. It seems obvious that Leonardo could not have successfully studied motion without his "geometria che si fa col moto".

To avoid confusion, I want to explain also that one thing is geometria che si fa col moto, and another is the study of motion per se, i.e., what Ampere called kinematics. There is much that constitutes the theoretical basis for Leonardo's remarkable work on kinematics, an area in which he went far beyond any researcher before him. The fact that he did remarkable studies of kinematics should not obscure the view of what was for him, actually, a new kind of geometry, the geometry that an artist like him could create, going far beyond what others had done and were doing in the field of scientific perspective. Leonardo developed a wide and deep view of transformation geometry, of which perspective is only a small area. Perspective was, and remains, within the field of a geometry without motion, while Leonardo's conception of geometry is deeply and inventively associated with motion. In this he appears to me as very much ahead of most geometry that followed him.
REFERENCES


Boyer traces the failure to include motion in geometry to the Greek approach. See, e.g., on p. 25: *Only the statics aspects of optics, mechanics, and astronomy found a place in Greek mathematics, and it remained for the Scholastics and early modern scientists to establish a quantitative dynamics. Concerning Cavalieri's principle, see p. 22, 118. Cavalieri's books were published more than a century after Leonardo's death.*


In the general introduction, we find an interesting comment concerning the consequences of adhering to pure axiomatic geometry by the Greeks and followers: *For almost two thousand years the weight of Greek geometrical tradition retarded the inevitable evolution of the number concept and of algebraic manipulation, which later formed the basis of modern science (see p, XVI).*


See section on theorems attributed to Thales. It seems that what we call now Thales' theorem is based on particular cases of measurement of distances by means of similar triangles by Thales. This would not be the first case of attribution, or the naming, of a theorem on the basis of practical applications, rather than theoretical achievements. In such category, we have Leonardo's theorem in the theory of symmetry, for instance, which was named by Hermann Weyl after Leonardo [Macagno M. 1990].

According to Lanczos, Zeno of Elea put forward his famous arguments to show that the concept of motion leads to absurdities (see p. 20).


Martin's position seems to be that motion is useful but not essentially needed for his book, although he uses motion whenever it helps in his discussions and his problems. Moreover, he agrees that physical motion is described in differential geometry (p. 18). Leonardo's theorem can be found on p. 66.

From Chapter 7: "Geometry is not only the study of figures, it can also include the movement of those figures. If you move all the points of a geometric figure to set rules you can create a new geometric figure. . . . This is called a transformation". From the Preface, Prof. Prof. J.M. Shaughnessy: "The spirit of this text is remarkably consistent with recent research on the development of geometric thinking in adolescents . . . ."

TANNERY, Paul. 1887 La géométrie grecque. Paris


FIG. 1. In a qualitative form of Thales theorem, Leonardo related the inequality between the distances traversed by an object and its shadow, and the distances of object and shadow from the source of light. (Sketch based on Ms G 92V D1).

FIG. 2. Attempt by Leonardo to extend the notion of parallelism between straight lines to curved lines. (From Ms G 59 R).

FIG. 3. In case 2, Fig. 2, the lines considered by Leonardo to be parallel intersect when they are extended.
FIG. 4. The analogy conceived by Leonardo, regarding the subdivision of figures into smaller similar figures (A : B :: C : D), failed to be true; the pyramid cannot be subdivided into eight similar pyramids. Conversely, eight congruent small pyramids cannot be put together to construct a larger similar one.

FIG. 5. A subdivision of the surface of a sphere depicted in Ms G 67R D1 is impossible, because it would imply the existence of a Platonic body with more than twenty congruent equilateral faces. A plan view of Leonardo's subdivision in 60 triangles has been drawn to make clear that not all triangles are congruent.
FIG. 6. The flow of the cord in a system of pulleys was studied by Leonardo, who found that different portions of the cord have different velocities.

FIG. 7. Change of shape by rectilinear shear. It is well within geometry of motion that the displacements \( d \) be given by a rather general function \( d = f(y) \) as shown.

FIG. 8. Counterpart of Fig. 7. By circular shear, a given common circular sector can be transformed into a sector of as sinuous sides as desired. Area is conserved during this geometric process. (From Ms G 56R)

FIG. 9. The notions of circular shear and cyclic symmetry lead Leonardo to the subdivision of a circle into congruent figures he called *elijke bisanghule*. (Ms G 57V). As in Fig. 7, the shearing function can be quite arbitrary.
FIG. 10. In the Anatomical Drawings 121R D6-7, Leonardo made a sketch under the title *quadratura darchimede*. This is geometry without motion, while in Ms G, Leonardo included different procedures involving motion; they could be called *quadrature di Leonardo*.

FIG. 11. After rectification of the circumference, by rolling it over a straight line on a plane, the area of the circle can be mapped with the desired accuracy, by application on the plane of a large number of sectors of the circle. Note that deformation is present (\( \alpha \neq \alpha' \)).

FIG. 12. Another way of mapping the sectors; in this case, for a semicircle. The first step is the rolling of the semicircumference on a line in the plane. This rolling of the circumference, or part of it, or of a wheel, is a common feature in the geometry in motion in the Ms G.
FIG. 13. Leonardo's attempt at determining the area of a curvilinear segment by an extension of the procedure illustrated in Figs. 11 and 12. A third drawing has been added with a different mapping than that sketched by Leonardo in the Ms G (see 39V and 68R).

FIG. 14. The Ms G contains several procedures for the mapping of the surface of circular cones and cylinders on the plane.
FIG. 15. For his mappings of the surface of the sphere, Leonardo introduced projections of parallels drawn on the sphere both in elevation and in plan views. (From Ms G 38V D1).

FIG. 16. On Ms G 38V and 39R, two procedures for the mapping of one eighth of the surface the sphere on a plane are described. According to our notions they can be classified as being one in polar coordinates and the other in Cartesian coordinates. (The drawings in this figure are based on 38V D2 and 39R D3).
FIG. 17. In Ms G 42V, Leonardo introduced a system of orthogonal lines drawn on the surface of the cone and that of the sphere, and tried to relate the mappings of the two surfaces on the plane.

FIG. 18. Mapping of parallels drawn on the surface of bodies of revolution onto the plane. As in other cases, the guiding rule was conservation of area. (Based on Ms G. 67V D2 and 68R D3. (68R D1 is similar to 39R D3.)
FIG. 19. Analysis of Leonardo's procedure for the calculation of the area of the sphere by mechano-graphical mapping of its surface on a plane. In this case the length of parallels, or parts thereof, are rectified along parallel lines. This gives a curve, the area under which should be the area of the sphere, or of a fraction of it. The theoretical curve has been denoted with 0. The symbols +, x, Δ, have been used to transfer to this drawing values read out from Leonardo's sketches, 38V D4, 39R D3, 68R D1, respectively. The area under Leonardo's average curve (see dashed line) is in excess from the correct value by about 5%. Both qualitatively and quantitatively, those sketches seem to be remarkably good.
FIG. 20. Analysis of Leonardo's mapping of the sphere on a plane in Ms G 38 V D2. In this case, his mechano-graphical procedure was followed to obtain the points indicated by +. Theoretical points are denoted with o.

FIG. 21. Analysis of Leonardo's mapping of the sphere in Ms. G 42V D2. Leonardo's idea of rolling disks to effect the mapping was used (see +). The theoretically calculated points are indicated by o. In this case two families of lines were considered by Leonardo; i.e. meridians and parallels. Note important differences with Leonardo's drawing (see Fig. 17).
FIG. 22. Once the surface of a sphere is mapped onto the plane, its interior can be mapped above that plane. Leonardo did it by means of pyramids of height equal to the radius of the sphere as a means of calculating the volume of the sphere as described in Ms G 39V D2.
PUBLICATIONS ON HISTORY OF SCIENCE AND TECHNOLOGY


These Internal Reports contain descriptions of the work at the fluids laboratory of the Karlsruhe Hydromechanics Institute, most of which has been included in publications listed below.


The development of the laboratory methodology is first described in this publication.


The notion of rheogram is introduced and illustrated in two tables for waves and vortices.


In this paper, a number of experiments are reported which were performed as an application of the laboratory methodology in the analysis of documents, developed by the author from 1965 to 1980, at the Iowa Institute of Hydraulic Research first, and then at the Institut für Hydromechanik Universität Karlsruhe.


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Analogies are studied historically, bringing them under the light of the Keynesian concepts of positive and negative analogies, before dealing with Leonardian analogies.


Written as a challenge to the owners of the codex to support a scholarly study of the document.


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MACAGNO, E. 1989d Leonardian Fluid Mechanics in the Manuscript L. IIHR Monograph No. 108. The University of Iowa, Iowa City, IA.
The priority in developing the two main views of fluid flow has been debated (H. Lamb and C. Truesdell) because it was developed during the same years by both Euler and Lagrange, but indeed, both descriptions were already used by Leonardo. This, of course, he did with much simpler mathematical tools.


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