MEASUREMENT OF MICROPRESSURES IN LIQUIDS

by

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INTRODUCTION

The measurement of very small static pressures (of the order of 0.001 inch of water) can be accomplished with instruments that are essentially uncomplicated. The measurement of pressures of such magnitude at low frequencies, i.e., from a fraction of a cycle per second to the order of 10 to 20 cycles per second, is, by comparison, a formidable problem. In this frequency range, it is generally not possible to take advantage of piezoelectric or magnetostrictive effects, for example, and the problem reduces to one of detecting the (very small) motion of a mechanical system deforming in response to the applied pressure.

For measurements in the interior of liquids, diaphragms of some type must usually be employed in a leak-proof container. The design of the instrument then resolves itself into consideration of essentially three elements: the diaphragm and its behavior as affected by the transducer system; the response of the fluid system in the container or housing and the feedback to the diaphragm; and, the transducer and recording system.

Early in World War II, the David Taylor Model Basin was called upon to make such measurements in support of the development of countermeasures against pressure mines. The pressure mine is actuated by the change in ambient pressure associated with the flow field about a ship as it passes in close proximity to the mine. Combined with magnetic and acoustic influences, the pressure signal required can be coded into the mine mechanism to make a particularly vicious and selective weapon which can single out for attack a ship of certain size. Some of the history of this type of mine warfare and examples of ship and mine characteristics are given in reference [1]. A typical example of the pressure "signature" recorded on the bottom of a channel for a ship model moving in shallow water is shown in Fig. 1.

Model studies were initiated in 1940 with two objectives: the determination of the characteristic pressure signatures of Naval and merchant vessels in order to establish safe speeds for transit

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* Office of Naval Research.
** David Taylor Model Basin.
of minefields sown with mines of given firing rules; and, concurrently, the development of specific devices designed to produce pressure fields suitable for sweeping.

In addition to the need for measurements for the above application, data of this type were also required in the investigation of the forces associated with the movement of ships in shallow water and restricted channels. Thus, while the early work during World War II was carried out with temporary equipment, there existed a need of a sufficiently long range nature to lead to the installation early in 1944 of an essentially permanent pressure-measuring range.

It is the purpose of this paper to describe the equipment which comprises the present DTMB micropressure range and in this context discuss the treatment of the various problems mentioned above as encountered in developing this installation. Hence, not all possible methods of accomplishing the purpose will be discussed here but only those considered as practicable for the DTMB problem.

**The Required Instrument Characteristics**

In the installation described here, measurements of pressure changes were desired over an operating range of 2 inches of water with a resolution of 0.001 inch of water under ambient water heads of \( \frac{1}{2} \) to 10 feet. It was required that the response of the system be insensitive to the absolute magnitude of this ambient head, be essentially linear throughout the operating range of pressures, and be reasonably flat over a range of 0 to 20 cycles per second.

In addition to the requirements specific to the DTMB installation,
other general requirements peculiar to systems placed under water for appreciable lengths of time are the following. The system must be insensitive to temperature changes; in the case of the DTMB installation, in which a closed gas system is used, particular care must be taken to eliminate such changes. Provision must be made for electrical stability of the transducer and recording system, and the gages must be designed to eliminate difficulties due to the presence of moisture. Watertight integrity must be maintained and, for systems exposed for long periods, noncorrodible materials are essential. Finally, for permanent installation of a range containing a large number of gages in a basin in which the water level is often changed for various experimental purposes, the system should be self-monitoring and self-adjusting to balance out the ambient head. This is required in order to protect diaphragms from damage resulting from excessive pressures and to provide an easy means of establishing the reference pressure level.* Consequently, such an installation requires a remote means of calibration.

Aside from the instrument and recording problems outlined in the foregoing, the measurement of such small pressures requires that special attention be given to the experimental environment, i.e., the facility in which the experiment is conducted and the method of gage installation. This will be discussed briefly before proceeding to the description of the DTMB micropressure range, itself.

Possible Errors Associated with the Experimental Environment

The facilities employed at DTMB for micropressure measurements are two rectangular towing basins. The larger one, in which the permanent range is installed, the "Shallow Basin," is 260 feet long by 52 feet wide with a water depth which may be varied from 0 to 10 feet; the smaller of the two, which is used with portable equipment, is 140 feet long by 10 feet wide with a water depth variable between 0 and 6 feet.

The possible sources of error associated with the characteristics of these facilities are: the modifications to the pressure field associated with flow about the gage housings, the limited distance available for the establishment of steady-state conditions, and modification of the flow field associated with wall proximity.

The difficulties resulting from flow about the housing may be

*As will be shown, thin diaphragms are required for large sensitivity, so that sufficient strength to withstand the maximum possible head may be incompatible with sensitivity requirements. Furthermore, the linear range of the system may be asymmetrical with respect to the zero position depending upon the operating head.
Fig. 2. Installation of Transducers in the David Taylor Model Basin Shallow Basin

Fig. 3. Layout of the Micropressure Range in the DTMB Shallow Basin
eliminated by flush mounting where possible. In the DTMB installation, this is accomplished by placing the gages in channels cut into the basin floor. These gages are covered and held in place with heavy brass cover plates as shown in Fig. 2. The cover plates are of sufficient weight to prevent their displacement under the forces acting during the passage of a model. For use in channels in which it is not possible to arrange flush mounting, a number of gages have been designed with faired contours to minimize errors due to the local flow conditions.

While the model basins at the David Taylor Model Basin have hard (concrete) bottoms, and the water level is varied to obtain different depths, it may sometimes be necessary to use a false bottom to obtain depth variation, as was sometimes done in the earliest experiments relating to the mine problem. In this case, extreme care must be exercised in assuring rigidity of the bottom and in sealing the gaps between the false bottom and the basin walls. Under the loading produced by the pressure field of a model, deflections large enough to significantly alter the desired pressures may be produced if adequate stiffening is not provided. The presence of even very small gaps allows flow which again may alter the pressure field significantly.

The transducers are arranged on the basin floor in the manner shown in Fig. 3. Since three or more transducers are located on the basin centerline along the path of the models, comparison of the records at successive stations discloses whether steady-state conditions have been attained during a given run.

It is, of course, well known that wall corrections must often be applied to data taken in facilities of the type considered here, and it is not the intention to discuss such corrections in detail. However, it is well to point out that in measurements of the magnitudes considered here such corrections assume a much greater importance than in the types of measurements ordinarily made in model basins. For methods used in correcting pressure field data, reference may be made to a report by Borden [2] and references therein.

**THE TRANSDUCER PROBLEM; TYPES SELECTED FOR THE DTMB MICROPRESSURE RANGE**

When applied to an ideal pressure transducer, the pressure signal created by the model and the influences of the facility will be converted to an electrical signal having an output proportional to the applied pressure. Since appreciable distortion, attenuation
and phase lag may result in liquid-filled instrument lines, it is desir-able that the sensing element be located at the point of measure-
ment. Hence, a diaphragm or bellows whose deflections are detected by electrical means was determined to be the most practical method for satisfying this requirement and that of flush mounting without excessive deflections. The design of the diaphragm then becomes a problem of finding a suitable compromise between the requirements for deflections that are easily detectable and high frequency re-
response. While the latter is compatible with the need for small diameters (to obtain measurements representative of conditions at a point, i.e., diameters very small compared with the distance over which significant changes in pressure may occur), the requirement for large deflections is not. Although the actual design is in-
fluenced by the method used for detecting the deflections, these points may be illustrated by examining the diaphragm alone.

The deflection \( w_c \) at the center of a thin circular diaphragm or plate clamped rigidly around the circumference and loaded uniformly by an excess pressure \( \Delta p \) is given by \([3]\)

\[
w_c = \frac{3}{16} \frac{\Delta p}{E} \left(1 - \nu^2\right) \frac{a^4}{h^3}
\]

where \( E \) is the modulus of elasticity and \( \nu \) is Poisson’s ratio for the material, \( a \) is the radius and \( h \) the thickness of the diaphragm.

The fundamental frequency \( \omega_o \) of such a diaphragm is, approxi-
mately \([4]\),

\[
\omega_o = 10.21 \frac{h}{a^2} \sqrt{\frac{E}{12 \rho_m \left(1 - \nu^2\right)}}
\]

where \( \rho_m \) is the mass density of the material plus the added mass associated with the motion of the ambient fluid. Thus, where high sensitivity (large deflection per unit of pressure change) is ob-
tained by making \( a \) large and \( h \) small, just the opposite relation is required for high frequencies, i.e., \( h \) large and \( a \) small.

The first transducer system employed at the Taylor Model Basin was based on the change in the natural frequency associated with a variation of the inductance in an inductance-capacitance circuit. In particular, this system was designed to detect the motion of a circular copper diaphragm placed near an inductive coil \([5]\). The motion of the diaphragm under changing pressures induced a change in an oscillating signal which was measured as the deviation from the tuned frequency. Development of a suitable diaphragm and mounting method was a very difficult process. The diaphragm finally developed was of 0.004 inch rolled copper, one inch in di-
ameter, soldered to a steel ring. However, only one diaphragm in four was satisfactory in spite of careful preparation and stress relieving; buckling due to thermal stresses and hardspots due to the rolling process or non-uniform stress relieving accounted for most of the failures. Although this system was used essentially as described in reference [5] throughout the war years of the program, the problems of diaphragm fabrication and limitations in linearity and stability of the circuits led in 1946 to examination of other systems.

Condenser type transducers were considered as an alternate solution; such a system had already been used successfully by the Admiralty Experiment Works in England [1, 6]. Variation in capacitance due to displacement of the diaphragm acting as one plate of a condenser may be used for frequency or amplitude modulation of a sharply tuned signal, and very high sensitivity may be obtained. However, it was feared that difficulties would arise as a result of moisture accumulation in the gap between plates over long periods of immersion, non-linearity of output signal over the required pressure range, temperature drift, and the problem of the capacitance of long cables leading from the gage housing to the recording equipment. Since much experience had been gained at the Taylor Model Basin with resistance-wire strain gages by this time and because of the anticipated difficulties with condenser-type gages, the latter were abandoned for the present application. Subsequently, however, condenser-type gages were developed for other applications [7].

Attention was then turned to the application of resistance-wire strain gages for measurement of micropressures. This was motivated by the previous successful use of such gages in dynamometer and balance equipment and by the availability of a highly sensitive and stable carrier-type strain indicator [8]. The first attempts centered on the use of gages of the SR-4 type bonded to metal strips in various configurations to sense the movement of a diaphragm. It was soon apparent, however, that the restraints introduced by such methods were too large for attainment of the sensitivities required. Development was then undertaken of methods for mounting unbonded strain wires in suitable configurations. Shortly after the latter program began, the Statham Laboratories of Los Angeles, California, announced the development of a suc-

*Reference 6 is of additional interest since it describes what is evidently one of the first successful applications of the hot-wire for practical measurements of velocities in water.
cessful system employing unbonded wires, and early in 1948 it was decided to use Statham gages fabricated to TMB specifications.

In addition to the specifications for the gage housing required to satisfy requirements for long periods of immersion and for the diaphragm to attain the needed frequency response, the following specifications were laid down to insure the attainment of required sensitivity when used with the DTMB strain indicators:

"1. Each micropressure gage should have two active strain sensitive resistance elements arranged so that they form two adjacent arms of a Wheatstone bridge. The remaining two arms of the bridge are in the recording system and are inactive. If more than two active resistance elements are used, they should be wired in parallel so as to form effectively the two adjacent arms of a bridge. This will permit compensation for temperature variations.

"2. The resistance of one arm of the Wheatstone bridge consisting of one element or set of elements should be as near 120 ohms as is practicable.

"3. The strain sensitive resistance elements should be made from an alloy having a low temperature coefficient such as Ad-

![Fig. 4. Top View of Statham Laboratories P-18 Pressure Transducer Fabricated for the DTMB Micropressure Range](image-url)
vance or Constantan and should have a very stable strain-resistance coefficient.

"4. A change of resistance of not less than 50 micro-ohms per ohm for a pressure change of plus or minus 0.009 pounds per square inch is required for each of the two active strain-resistive resistance elements.

"5. The variations of the mechanical system due to the hysteresis, vibration, and thermal variations must not cause a variation in output voltage greater than 2\% of the output voltage at a pressure of 0.009 pounds per square inch."

The Resistance-Wire Strain-Sensitive Transducers and the Amplifying and Recording Equipment

The transducers fabricated to the specifications outlined in the foregoing are illustrated in Figures 2 and 4. The corrugated diaphragm has an active diameter of 2 inches and is formed of tinned brass having a thickness of 0.0015 inch. The unbonded strain wires may be seen in Fig. 5 which is a photograph of the bottom of the
transducer with the watertight cover plate removed. These strain wires are 0.003 inch in diameter with an electrical resistance of approximately 120 ohms per arm wired in a two-arm bridge circuit and activated through a mechanical linkage by movement of the diaphragm. The natural frequency of the diaphragm-linkage combination in water is approximately 23\* cycles per second. However, the true resonant frequency of the entire system depends not only on the mechanical characteristics of the transducer itself but also on the response characteristics of the internal gas system. This problem and the measures which must be taken to insure the necessary behavior of the gas system are discussed in the appendix.

![Diagram of the amplifying and recording system](image)

**Fig. 6. Block Diagram of the DTMB Type 1-A Strain Indicator Used with the Micropressure Transducers**

Since the characteristics and complete circuits of the DTMB strain indicator used with these transducers have been described elsewhere \[8\], only a brief description of the components will be given here. A block diagram of the amplifying and recording system is shown in Fig. 6. The carrier frequency is generated by a sinusoidal oscillator which drives the buffer amplifier; the latter furnishes power to excite the bridge and also delivers a voltage through the isolator, to the mixer circuit. The output or signal voltage from the bridge is passed by the preamplifier to the attenuator. The output voltage of the attenuator is further amplified by a fixed-gain amplifier. The amplified signal voltage is impressed on the null detector and also passes through the driver to the mixer circuit, which is a polarity discriminating demodulator. The composite output voltage from the mixer circuit drives two independent power amplifiers, which in turn operate into full-wave rectifiers and thence

* Employed more recently is Statham Model P18-A which has a diaphragm with an active diameter of about 2\(\frac{3}{4}\)". The natural frequency of this model is 35 c.p.s.—the difference between models being due primarily to differences in the linkage systems. The characteristics of Model P18-A are used in the computations in the appendix.
through a high-pass filter and milliammeter to a multi-channel oscillograph.

A typical calibration of the pressure transducers at a single gain is shown in Fig. 7. Although the nominal operating range is ± 2 inches, the output is actually usable over a much wider range —being linear up to ± 8 inches of water. This is also the operating limit—stops being provided to prevent further movement of the diaphragm under greater pressures and, thus, to prevent damage to the diaphragm should there be malfunctioning of the internal gas system.

![Typical Calibration of a Pressure Transducer](image.png)

**Fig. 7. Typical Calibration of a Pressure Transducer**

**The Internal Gas Reference, Monitoring, and Calibration System**

All of the transducers in the installation described are pressurized initially to a pressure equivalent to the ambient, undisturbed water head so that only the deviations (pressure changes) are detected. The internal gas system must accomplish a three-fold purpose: provide the static reference level to which the pressure changes are referred; be capable of monitoring the internal pressure to insure that the ambient head is always balanced; and, be usable for in-place calibration. Before the design of the multi-channel micropressure range described here, the early inductance-type gages were pressurized with air which was simultaneously fed to a submerged bell. The appearance of air bubbles escaping from under this bell indicated when the ambient head was balanced.
Fig. 8. Schematic Diagram of the Gas Pressure Reference, Monitoring, and Calibration System
Since the bell was open at the bottom, it was able also to compensate for small changes in head. Calibration was accomplished by changing pressure internally by changing the volume of the system with a small bellows. It was necessary, of course, to remove this system from the basin whenever it was not in use.

For a permanent multi-gage installation to be operated under large changes of ambient head and to be left unattended over long periods of time, a closed, gas-tight system was required. A schematic diagram of the gas reference, monitoring, and calibrating system designed for simultaneous operation of ten transducers is shown in Fig. 8. The control console from which this system is operated is shown in Fig. 9. The control loops for each of the required functions may be identified from the following description.

The control console embodies three major sections: the supply and exhaust, the control and calibration, and the transducer feed. Each of these sections can be isolated from the others and vented to the atmosphere separately, if necessary (thus, making it possible to repair or remove any single component without disturbing the system). Due to the high humidity prevailing in the basin building and the long immersion of the transducers, oil-pumped nitrogen is used as the gas in the system rather than air compressed at the test site. As added protection for the transducers, the gas is passed through a series of silica gel dehydrators. The latter are
required because of the presence of a water surface in the calibrating manometer.

The hydrostatic head on the basin is compensated by means of the automatic balancing supply and exhaust system. Control is maintained by a balancing chamber, Fig. 10, submerged in the basin. Through a mechanical linkage, Fig. 11, motion of the bellows actuates the relays (seen on the backboard of the control console in Fig. 9) which in turn activate the appropriate inlet or exhaust solenoid valves. Thus, when there is an increase in water level, the bellows of the balancing chamber is compressed, the inlet solenoid is actuated, and gas is fed into the system until equilibrium is restored. Conversely, a falling water level allows the bellows to expand as a result of the excess internal pressure, the exhaust solenoid is actuated, and the excess nitrogen is exhausted to the atmosphere. The nitrogen reservoir pressure is maintained at a pressure equivalent to a few feet greater than the maximum water depth in the Shallow Basin.

The electrical contacts in the balancing chamber are set so that the circuits are made by changes of $\pm 0.07$ inch of water head, and the pressure lag is just sufficient to return the floating contact to
a central position between the contacts for the two circuits. Although smaller limits of control may be obtained, the value of 0.07 inch was chosen as compatible with the average roughness of the concrete basin floor. Stability of the gas circuit for the balancing chamber was attained by the use of needle valves near the inlet and exhaust valves and by proper choice of gas lead from the console to the chamber as selected by test. The balancing chamber is protected against overpressure by internal and external stops incorporated into the watertight base and into the protective covering for the bellows. During calibration and operation of the range, this circuit is isolated from the internal pressure system so that activation (caused by waves, for example) does not occur during a test. Since this balancing chamber is inaccessible for quick, emergency repair, a less sensitive but easily accessible standby balancing chamber has been added. The latter utilizes opposed bellows, one carrying internally the full head on the basin by direct communication with the water in the basin, the other operated in the same manner as that of the system described above.

The control console is equipped for in-place static calibration of the transducers. Incremental changes in pressure are obtained by changing the volume of a bank of small Hydron bellows (shown to the right in Fig. 9). The change in pressure is monitored on the slant-tube manometer which is connected to the calibrating bellows and transducer manifold on one side and to a gas reservoir or reference volume on the other. Thus, the response of the transducers as recorded on an oscillograph record is obtained directly in terms of a pressure change.

![Fig. 11. Interior of Balancing Chamber Showing Mechanical Linkages and Electrical Contacts](image-url)
Errors Associated with the Internal Gas System

Since the gas system is completely closed and limited in overall size, errors will be introduced if there are temperature fluctuations and are inherent as a result of internal volume changes not associated with temperature changes. Since the resolution of the system is to be of the order of 0.001 inch of water, the maximum temperature fluctuation, within the range of temperatures and total pressures encountered at DTMB, must be less than 0.001°F during the course of a test run. To accomplish this, the entire gas supply and control console are encased in an enclosure of 3/4-inch plywood and 1/2-inch Cellotex (not shown in Fig 9). The console, together with the amplifiers and recording instruments, are further housed in an insulated room in which the temperature fluctuation is limited to plus or minus 1°F.

Changes in internal volume not associated with temperature fluctuations are associated with the displacement of liquid in the slant-tube manometer during calibration. The gas-reference volume tends to minimize the error caused by the volumetric change in this manometer. In this installation, the reference volume is 3 cubic feet so that a change of pressure equivalent to 0.1 inch of water produces a change in volume of 0.049 cubic inches when the manometer is set for a slope of 1/10. This will cause a change in the reference tank of approximately 5 percent of the differential pressure at the maximum head of 10 feet in the basin, and all calibrations must be corrected for this effect. For smaller total heads and smaller manometer magnifications, the error is correspondingly less, being negligible for a total head of about 4 feet and a magnification of 1/4, for example.

Errors in frequency response associated with the gas system are discussed in detail in the appendix.

Concluding Remarks

The DTMB micropressure range essentially as described here has been in operation for the past twelve years with only one major overhaul and such inactivation as needed to incorporate improvements. In spite of the adverse conditions under which many of the components operate, the reliability has been most satisfactory.

A portable micropressure range based on this system has also been constructed for use primarily in the small basin mentioned previously, but suitable for use in any of the DTMB facilities.
Acknowledgments

The early inductance gages were developed by E. Plesset and C. Starr and the first gas system by A. Kalinske and J. M. Robertson. The present multi-transducer micropressure range was designed by P. Eisenberg; improvements were added by J. P. Craven and J. A. Luistro who also designed the portable range.

Discussion

Mr. Robertson, who had been mentioned by Mr. Eisenberg for his early work on the micro-pressure system reminisced that when he had last heard about the project, in 1942, the system had consisted of one unit instead of ten. He re-emphasized the point that in a system where small, transient pressures are being measured it is necessary to consider the frequency responses of all parts of the system, electrical, electronic, and mechanical. If the frequency response is poor, the measurements may be deceptive. When he had worked on the problem he had made a brief study of other systems that had been used to study transient pressures. One of the earliest ones, he found, was made by Capt. Geeler and reported in his book on Why Wave Action, in which he described pressure measurements taken with a diaphragm type pickup. Because of the possibility that his transients could have been in tune with the natural frequencies of the system, Mr. Robertson questioned the reliability of Capt. Geeler’s work. A system where a large volume at the diaphragm is not available and a limited volume with a long tube is used for calibration may lead to difficulty.

Mr. Yih wondered whether fish could actuate a pressure sensitive mine, but Mr. Eisenberg had never heard of a fish being blown up by one.

Mr. Calehuff then raised the question of hysteresis in the diaphragm and asked how it had been avoided. Mr. Eisenberg agreed that there had been a great deal of difficulty in the early days of the project because the gages were made by soldering two steel rings. A man at the Taylor Model Basin had developed the skill and technique of applying a process of rolling, soldering, and annealing for stress relief which on the average yielded a successful diaphragm in one of three attempts. With the Statham gage, however, this difficulty does not seem to exist, probably because a corrugated diaphragm, screwed onto the gage, is used.

Dr. Craven remarked, at this point, that a corrugated diaphragm is difficult to machine.
Mr. Ingram then raised the question as to whether the operation of the gages with a common back pressure induced any errors; whether there was interaction between the diaphragms. Mr. Eisenberg was of the opinion that in a reach as long as in this system the interaction is negligible.

Finally, at the request of Mr. Robertson, Mr. Eisenberg sketched and described a corrugated diaphragm.

REFERENCES


APPENDIX

The frequency response of the transducer is a function of the mechanical impedance of the gage diaphragm and strain-wire system and of the "acoustic impedance" of the instrument lines and reference volume. The gage system shown schematically in Fig. 12 may be divided into four components, the mechanical elements of the transducer, the instrument volume, the instrument gas line, and the reference volume.

The diaphragm, its attached strain wires and the mounts for the wire form a complex mechanical-spring system such that an
approximation based on diaphragm properties alone is not applicable. Practically, the problem may be greatly simplified by considering the system as a simple spring whose constants are determined empirically. The equation for such a simple spring system is

\[(\Delta p_0 - \Delta p_1)A_0 = kx_{\text{max}} \sqrt{\frac{c_0^2}{k^2} + \left(1 - \frac{m}{k} \omega_0^2\right)^2} \epsilon^{i(\omega t - \phi_1)}\]

(1)

Here \(\Delta p_0\) is the applied external pressure increment, \(\Delta p_1\) is the resulting internal pressure increment, \(A_0\) is the effective area of the diaphragm, \(k\) is the spring constant, \(c\) is the damping coefficient, \(m\) is the effective mass of the spring, \(\phi_1\) is the phase angle equal to \(\tan^{-1}\left[\frac{c_0/k}{1 - m\omega_0^2/k}\right]\), and \(x\) is the deflection of the center of the diaphragm.

It is not necessary to determine all of the constants in this expression since the apparent or recorded pressure \(\Delta p_r\) may be determined by a static calibration:

\[\Delta p_r = \frac{k}{A_0} x = \frac{k}{A_0} x_{\text{max}} \rho_1\omega t\]

(2)

Therefore

\[\frac{\Delta p_0 - \Delta p_1}{\Delta p_r} = \sqrt{\frac{c_0^2}{k^2} + \left(1 - \frac{m}{k} \omega_0^2\right)^2} e^{-i\phi_1}\]

(3)

The magnitude of the internal pressure increment \(\Delta p_1\) differs from zero as a result of the finite geometry of the instrument and
instrument line and the compressibility and viscosity of the gas. Since the surface area to volume ratio of the instrument and line is large the process may be considered isothermal and the ideal gas law applied. Thus, for the instrument volume

$$\frac{\Delta p_1}{p_0} = -\frac{\Delta V}{V_o} + \int_{-\infty}^{t} \frac{Q dt}{V_a}$$  \hspace{1cm} (4)

where $\Delta V$ is the change in instrument volume
$V_o$ is the instrument volume, and
$Q$ is the volume rate of gas flow from the instrument volume into the instrument line.

As the recorded pressure is simply a measure of diaphragm deflection, we may write

$$\frac{\Delta V}{V_o} = k_2 \frac{\Delta p_r}{p_o}$$  \hspace{1cm} (5)

Here $k_2 V_o/p_o$ is an experimentally determined constant. If we consider the pressure increments to vary in a simple harmonic manner we may differentiate equation (4) with respect to time and with equation (5) determine

$$\frac{\Delta p_1 + \frac{ip_2 Q}{\omega V_o}}{\Delta p_r} = -k_2$$  \hspace{1cm} (6)

The flow $Q$ through the instrument lines has been determined by Iberall (9). He assumed that the flow follows Poiseuille's law of viscous resistance,

$$\frac{\partial p}{\partial x} = -\frac{128}{\pi D^4} \mu Q$$  \hspace{1cm} (7)

where $\frac{\partial p}{\partial x}$ is the pressure gradient in the instrument line,
$D$ is the diameter of the line, and
$\mu$ is the dynamic viscosity of the gas.

If it is further assumed that the reference volume $V_R$ is very large, then it can be determined from Iberall's analysis that
\[
\frac{\Delta p_1}{128 \mu Q} = \frac{L}{\pi D^4} \frac{(tanh^2 A' + tan^2 A')^{1/2}}{A' \left(1 + tanh A' \tan A'\right)} e^{i\phi_2}
\]

where

\[
\phi_2 = tan^{-1} \left[ \frac{\tan A' - tanh A'}{\tan A' + tanh A'} \right]
\]

\[
A' = \frac{4}{D} \left( \frac{\omega \mu}{p_0} \right)^{1/2}
\]

and \(L\) is the length of the instrument line.

Equations (3), (6) and (8) may be solved simultaneously to determine the response characteristics of the system.

\[
\frac{\Delta p_0}{\Delta p_r} = z_1 e^{i\phi_1} - k_2 \left[ 1 + \frac{p_0}{V_0 \omega} z_2 e^{i(\pi/2 - \phi_2)} \right]^{-1}
\]

where

\[
z_1 = \sqrt{\left( \frac{c_0 \omega}{k} \right)^2 + \left[ 1 - \frac{m}{k} \omega^2 \right]^2}
\]

and

\[
z_2 = \frac{\pi D^3}{32} \left( \frac{\omega}{p_0 \mu} \right)^{1/2} \frac{1 + tanh A' tan A'}{(tanh^2 A' + tan^2 A')^{1/2}}
\]

The response curves for the TMB range are shown in Fig. 13. Three conditions are shown

(a) the frequency response of the gage without instrument lines,
(b) the frequency response of the system as originally installed, and
(c) the frequency response of the system if modified by reducing the length of piping.

The spring constants were determined experimentally by striking the gage in water and measuring the resulting natural frequency and exponential decay time. The volume coefficient was determined by a static calibration with the reference volume sealed. The system characteristics employed are listed in Table I. At the present time the instrument range has been modified by the addition of a volume reservoir at the instrument. This has the effect of reducing the length of the instrument line to zero.
Fig. 13. Response Characteristics of DTMB Micropressure Range

<table>
<thead>
<tr>
<th>TABLE I</th>
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<tbody>
<tr>
<td>RESPONSE CHARACTERISTICS OF DTMB MICROPRESSURE RANGE FOR</td>
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<tr>
<td>$p_o = 19$ p.s.i.a.</td>
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$k/m = 4.8 \times 10^4$
$c/m = 3.22$
$L = 100$ ft.
$V_R = 3$ ft.$^3$
$V_o$ (before modification) = 1.37 in.$^3$
$k_2$ (before modification) = 0.121
$V_o$ (after modification) = 29.6 in.$^3$
$k_2$ (after modification) = 0.0056