Investigation of Three-Axis Accelerometer Calibration Techniques

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Investigation of Three-Axis Accelerometer Calibration Techniques

by

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A thesis submitted in partial fulfillment of the requirement for graduation with Honors in the Department of Electrical and Computer Engineering

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Abstract

Accelerometers are used in a variety of applications ranging from navigation systems to stand-alone tests. In each application, the accuracy of the accelerometer is crucial. There are various calibration techniques that range in precision, yet each apparatus devised to calibrate the accelerometer is quite complex. A calibration method of less complexity is desired to calibrate stand-alone accelerometers that require moderate accuracy. One stand-alone accelerometer, the X16-1D, was used to investigate calibration techniques. Six calibration techniques were investigated to determine an efficient method to calibrate a stand-alone accelerometer. Each method provided a different acceleration component that was modeled by theoretical equations. For all six methods, test variables were established to determine the accelerometers ability to track acceleration. Each test was compared to the theoretical acceleration model to determine the effectiveness of the calibration technique. The overall errors of each method and their significance on the calibration are modeled to determine the characteristic offsets inherent to the accelerometer. The overall consistency in error and accelerometer offset between tests proves that the accelerometer is inherently well calibrated.
Introduction

The use of accelerometers in everyday applications and experiments has been rising. Accelerometers can be found in cell phones, cars, airplanes, and many other devices, however, accelerometers can also be used in stand-alone applications. Stand-alone accelerometers are used to test research or experiment specific tests that can vary from using one to one thousand accelerometers at the same time. Most stand-alone accelerometers are calibrated within a certain tolerance range to meet company regulations thus most accelerometers will not have the same calibration. This variation possesses problems for users that want to obtain very accurate data across a plentiful number of accelerometers. Users of such accelerometers must either sacrifice time to calibrate each accelerometer or risk having inaccurate readings.

Therefore, in order to reduce errors in accelerometer readings, each accelerometer has to be calibrated by hand using multiple calibration methods. These hand calibrations can be inaccurate if not performed properly. If a number of accelerometers need to be calibrated at a single time, it could take the researcher hours to calibrate each accelerometer resulting in down time for the research. The lack of a consistent and accurate calibration technique for these accelerometers has led to countless hours and money being spent on calibrating accelerometers where the aim of the research to measure and acquire accurate data from the accelerometers. Therefore, an accurate calibration technique is desired in order to improve reliability of accelerometers in the field.

The accelerometer used in testing these new calibration techniques is the X16-1D. The X16-1D is a three-axis accelerometer produced by Gulf Coast Data Concepts (GCDC). The X16-1Ds compactness and ability to store data for further evaluation makes it easy to evaluate and analyze the acceleration read by the accelerometer.

![Gulf Coast Data Concepts X16-1D Accelerometer](image)

Figure 1: Gulf Coast Data Concepts X16-1D Accelerometer

The X16-1D accelerometer stores acceleration using inertia forces. The device uses a microelectromechanical system to detect the acceleration of each axis. The microelectromechanical system (MEMS) uses sensors to measure the forces seen in each
direction. When a force is applied to an axis, the accelerometer takes the value of force and then converts that force into acceleration. This force is converted into acceleration by using Newton’s Second Law (F=ma). Because the mass of the object in the MEMS is known, the system can easier convert the force reading into an acceleration.

This inertia force reading technique means that the accelerometer will always read an acceleration even when the system is stationary. When the accelerometer is stationary, the accelerometer will read the acceleration due to gravity. This means that if one axis is pointed parallel to the direction of gravity, the acceleration read on that axis will be equal to 9.81 m/s² because that axis feels the force of gravity. Therefore, the accelerometer bases its’ acceleration off the acceleration due to gravity which is 9.81 m/s² because the system will allows feel this acceleration thus its’ acceleration values are normalized off of this value. If the accelerometer outputs a value of .5g, then the actually acceleration is .5*9.81m/s². In the context of this paper, all accelerations measured are with respect to g’s or all accelerations are a fraction or multiple of g.

Each axis of the accelerometer is orthogonal to each other. The axis orientation of each axis is shown in figure 2. If an acceleration is going in the same direction as the axis then the accelerometer will output a positive acceleration, and it will output a negative acceleration if the acceleration is in the opposite direction of the axis. In the context of this paper, all accelerations denoted as x, y, or z axis accelerations are with respect to the axis orientation of the accelerometer as seen in figure 2.

**Literature Review**

Accelerometers are used in the medical field, general applications, elevators, and precise navigation systems. Each application has its own requirements for accuracy, precision, and reliability. In the medical field, the precision and accuracy of accelerometers is not as stringent as ones used in the fields of aviation and navigation systems. In the application of calibration techniques described in this paper, they lean toward the less precise calibration much like in the medical field because of the application of the accelerometers. The accelerometers calibrated in this paper are used for measuring the acceleration of tree movement.

In the medical field, accelerometers are used to monitor patient movement. The accelerometers used in the medical field are susceptible to offsets from long term use, an in-field calibration scheme is desired. Using electronic circuitry to estimate the direction of error based off when the user moves randomly, the calibration can self-correct and calculate within 3% the error of 1 g (Lötters, Schippe, Veltink, Olthuis, & Bergveld, 1998). Other methods include using Kalman filters to measure inclination activity of an individual which yields a 2° RMS inclination error (Luinge & Veltink, 2004).

Accelerometers can also be calibrated on their own without having a predetermined application. The actual calibration of a stand-alone accelerometer can be determined using a simply physical layout. Using a level table and cube with the accelerometer screwed in, the cube can be rotated in all six positions and the offset and scale factor can be determined with calculations (Gulf Coast Data Concepts, 2017).

The calibration of accelerometers used for elevators requires higher accuracy and more consistent calibration due to the elevators constant movement. In this field, it is hard to set downtime to calibrate the accelerometer therefore programmable algorithms are derived to continuously update and ensure the accelerometers are reading data correctly (Wu, Wang, & Ge, 2002).

The aviation industry requires navigational systems to have very accurate readings. The three main errors that are present in accelerometers are scale factor, offset, and non-orthogonality (Šipoš, Pačes, Roháč, & Nováček, 2012). Most commonly scale factor and offset, are calibrated assuming that the non-orthogonality of the device is very small to negligible. Current navigational calibrations systems rely on other components like gyroscopes or GPS to calibrate (Lötters et al., 1998). In the multi-position calibration technique, the calibration deviates from the common six-position rate and static tests because it neglects non-orthogonality (Syed, Aggarwal, Goodall, Niu, & El-Sheimy, 2007). The multi-position approach allows the use of one-axis turntable to calibrate the MEMS accelerometer and can be used in the field (Syed et al., 2007). However, this approach has some flaws as it heavily relies on the earth’s rotation rate which is very small in nature (Li, Niu, Zhang, Zhang, & Shi, 2012). The need for external equipment in the multi-position calibration creates a burden on the user. The Kalman filter
algorithm paired with the hand rotation algorithm scheme allows the user to quickly calibrate the device without external equipment (Syed et al., 2007).

Each of the calibration methods utilized in each industry has requirements in precision, accuracy, and application. The calibration requirements of the medical field or general calibration will be used because the accelerometers used in this study are used to measure acceleration of tree in which high precision accuracy is beyond the scope of the application.
Methods

Drop Calibration

When an object is dropped from a predetermined height, the only acceleration the object sees is the acceleration due to gravity, therefore if an object is dropped from a variation of heights the acceleration seen on the axis that is parallel with gravity should be 9.8 m/s². Using this fundamental relationship, the accelerometer can be dropped from a variation of heights and the acceleration in the direction of gravity should remain 9.8 m/s². Using this knowledge, the accelerometer was dropped from five total heights; 2ft, 3ft, 4ft, 5ft, and 6ft. At each height, the accelerometer was dropped five times with the z-axis pointed in the opposite direction of gravity, thus the gravity seen in the -z-axis should be -1 g or -9.8m/s².

Figure 3: Drop Calibration Configuration

Pendulum

One of the most fundamental approaches to calculating the acceleration of an object is to use pendulum motion. The acceleration of an object can be easily determined with the relationships that hold for object in pendulum motion. Using the simple pendulum equation, the period of a system can be calculated using the acceleration due to gravity and the length of the pendulum.
\[ T = 2\pi \sqrt{\frac{L}{g}} \]

Equation 1: Periodicity for an ideal pendulum

The simple pendulum equation only holds true when the length of a pendulum is very large because it reduces slack in the line that will result in irregular oscillations. Another interesting characteristic is the mass has no effect on the period of oscillation, thus a larger mass can be used to reduce slack in the pendulum arm. Using this relationship, the acceleration seen on the accelerometer can be modeled as a sinusoid with period (T), therefore as the accelerometer swings in motion, its’ period should remain constant as the pendulum swings. The acceleration amplitude should remain constant as well because of the sinusoidal nature of the pendulum.

As the length of the pendulum is increased the acceleration amplitude should remain constant, but the period should decrease thus determining the consistency of the accelerometer. Also, as the drop angle of the accelerometer increases, the acceleration should increase in order to maintain the same period. Investigating these two characteristics, helps distinguish the accelerometer consistency and accuracy in all three axes’ as a function of pendulum length and drop angle of the pendulum.

**Turntable – Gear Driven**

A turntable is a device much like a record player. In the context of this paper, a turntable is a table driven in a uniform circular motion by a motor. The motor turns the table by using gears to connect the table to the motor. The corresponding gear ratio drives the table at constant speeds. The motor encoder takes a DC voltage and translates the voltage level to the corresponding speed. The motor encoder takes voltages in the range of 6V to 12V with 12V being the fastest speed. The turntable rotates at a constant frequency which allows for the ability to calibrate an accelerometer accurately. The turntable itself lying flat on a surface with the accelerometer flush with the table can only view two distinct accelerations, one from centripetal acceleration and the other from gravity. However, when the turntable is set an angle, the accelerometer can be calibrated in all three axes. The figure below demonstrates the setup of the accelerometer in the three-axis calibration.
As mentioned above, there are only two accelerations involved in the rotation of the turntable; one is the centripetal acceleration and the other is acceleration due to gravity. Because the turntable is moving at a constant speed in a uniform circular motion, there exists an acceleration inward on the accelerometer due to classical physics. The centripetal acceleration acts on an object that is radius \( r \) away from the center of the device. As the object rotates around the center of the device with period \( T \), there is a constant acceleration \( a_c \) pointed directly from the object to the center of the device. This acceleration can be modeled using the equation below.

\[
a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}
\]

Equation 2: Centripetal Acceleration

As seen in figure 4, the accelerometer is set at an angle of 45 degrees. The accelerometer is set at such an angle to create a distinct acceleration in each of the three axes. The turntable is set at angle as well in order to distribute the acceleration due to gravity among the three axes. When the turntable is set to an angle \( \Theta \) the acceleration is split between the z-axis and the combination of the x-axis and y-axis by \( \Theta \). Using trigonometry, it can be seen that the acceleration due to gravity for each axis can be described below.

\[
a_{g,z} = gcos(\Theta)
\]

Equation 3: Accelerometer Z-Axis Acceleration due to Gravity

\[
a_{g,x} = a_{g,y} = gsin(\Theta)
\]

Equation 4: Accelerometer X-Axis and Y-Axis Acceleration due to Gravity
The acceleration due to gravity seen by the x and y axis varies as the device rotates with a constant angular frequency. As the device rotates through one period, the acceleration due to the gravity changes from the x-axis and y-axis and will vary from the maximum value to zero. Therefore, the complete acceleration due to gravity seen by the device can be expressed using the equation below.

\[
a_{gx} = g \sin(\theta) \cos(\omega t) \quad a_{gy} = g \sin(\theta) \sin \omega t
\]

Equation 5: Accelerometer X-Axis and Y-Axis Acceleration due to Gravity per Time

Centripetal acceleration is the other component that affects the acceleration seen on the accelerometer. The z-axis doesn’t see any centripetal acceleration because it is orthogonal to the direction of the acceleration, therefore only the x-axis and y-axis feel the effects of the centripetal acceleration. Because the accelerometer is set to an angle of 45° to the direction of centripetal acceleration, the acceleration is equal in both the x and y direction.

\[
a_{c,x} = a_{c,y} = a_{c} \sin(45°)
\]

Equation 6: Accelerometer X-Axis and Y-Axis Acceleration due to Centripetal Acceleration

After combining the two accelerations seen by the accelerometer, the overall acceleration for each axis can be characterized as below.

\[
a_x = a_{c} \sin(45°) + g \sin(\theta) \cos(\omega t)
\]

\[
a_y = a_{c} \sin(45°) + g \sin(\theta) \cos(\omega t)
\]

\[
a_z = g \cos(\theta)
\]

Equation 7: Turntable Total Acceleration for Each Axis

**Linear Actuator**

A linear actuator is a device that extends and retracts in a linear fashion. In most cases the linear motion is produced by a motor that rotates to push the arm forward or back. The rotations of the motor are translated into linear motion. The linear motion produced creates a constant speed in a given direction. The linear actuator used in this paper is driven by a voltage. A negative voltage extends the linear actuator arm while a positive voltage retracts the linear actuator arm. The calibration setup can be seen below.
The accelerometer and linear actuator are each set to an angle because of the linear actuators inability to move in more than one axis. The 45° offset of the accelerometer allows the interaction of two-axis acceleration and the angle of the linear actuator allows for the third axis to be calibrated. In this setup, there are only two forms of acceleration; acceleration due to gravity and linear acceleration. However, since the linear actuator moves at a constant speed, the linear acceleration seen on the accelerometer is zero, therefore the only acceleration acting on the accelerometer is gravity.

The acceleration due to gravity remains constant in the system and its value per axis is given by the angle of the accelerometer and the angle the linear actuator forms with the horizontal. The angle the linear actuator forms with the horizontal is denoted as Θ. Due to the fact the accelerometer is offset by an angle of 45°, the x-axis and y-axis should see the same value of acceleration due to gravity.

\[ a_z = g \sin(\theta) \]

Equation 8: Accelerometer Z-Axis Acceleration due to Gravity

\[ a_x = a_y = \frac{g \cos(\theta)}{2 \cos(45^\circ)} \]

Equation 9: Accelerometer X-Axis and Y-Axis Acceleration due to Gravity
As the linear actuator moves at a constant speed back and forth, the only acceleration is the acceleration due to gravity that is described in the previous equations. The acceleration should remain constant throughout the process of linear motion no matter the speed of the linear actuator.

**Linear Stage**

The linear stage is a device that uses a screw drive to move a slide back and forth according to the frequency and voltage applied to the motor drive. These variations affect the speed and linear distance the slide moves. Varying the frequency and voltage changes the acceleration in each axis of the accelerometer. The variation in frequency to the motor driver, changes the how fast the slide will move back and forth thus increasing angular frequency. The variation in voltage to the motor driver changes the distance the slide moves ($X_m$), but it doesn’t increase the angular frequency. The calibration configuration can be seen below.

![Figure 6: Linear Stage Calibration Configuration](image)

The two variables; frequency and voltage, affect the acceleration that is caused by the linear motion of the linear stage. This acceleration denoted as ($a_l$), is caused by the linear actuator movement pattern. The linear stage moves back and forth in a periodic fashion. This periodic fashion makes the distance traveled by the slide to vary sinusoidally. Since acceleration is the second derivative of distance, the acceleration will vary sinusoidally as well. Due to the orientation of the linear stage, the accelerometer will only feel the effects of this linear acceleration in the x-axis and y-axis. The 45° angle the accelerometer makes with the direction
of the linear acceleration means that the linear acceleration in the x-axis and y-axis components of the accelerometer will be the same because the three axes are orthogonal. The linear acceleration of the linear stage can be denoted as below.

\[ a_l = (2\pi f)^2 X_m \cos \omega t \]

**Equation 10: Linear Acceleration of Linear Stage**

The orthogonality of the x-axis and y-axis and 45° angle with the linear acceleration means that the x-axis and y-axis accelerometer acceleration will be equal.

\[ a_{l,x} = a_{l,y} = \frac{a_l}{2 \cos(45^\circ)} \]

**Equation 11: X-Axis and Y-Axis Linear Acceleration**

The other acceleration acting on the accelerometer is the acceleration due to gravity. When the linear stage is set at an angle with the horizontal, much like the configuration figure, the acceleration due to gravity will split between all three axes. In this method, the angle the linear stage makes with the horizontal is defined as 21.876°. Much like the linear acceleration, the acceleration due to gravity component will split equally between the x-axis and y-axis of the accelerometer due to the 45° angle. The acceleration due to gravity for each axis is described as follows:

\[ a_z = g \cos(21.876^\circ) \]

**Equation 12: Z-Axis Acceleration due to Gravity**

\[ a_{g,x} = a_{g,y} = \frac{g \sin(21.876^\circ)}{2 \cos(45^\circ)} \]

**Equation 13: X-Axis and Y-Axis Acceleration due to Gravity**

The total acceleration for each axis is the combination of the acceleration due to gravity and the linear acceleration. The total acceleration in the z-axis for the accelerometer is equal to the acceleration due to gravity as it is the only acceleration acting on it.

\[ a_x = a_y = a_l + g \sin(21.876^\circ) \]

**Equation 14: Linear Stage Total Acceleration for X-Axis and Y-Axis**

Using these equations for the acceleration of each axis of the accelerometer, the varying effects of the frequency and voltage can be seen in the linear acceleration which is translated to the x-axis and y-axis acceleration of the accelerometer.
Robot Arm

The robot arm calibration method utilized a robot kit that involves the use of stepper motors. The stepper motors are powered using an Arduino which uses its built-in servo motor functions to drive the stepper motors to desired positions. The first servo motor moves the robot arm around the base of the system and the accelerometer as setup in the figure below doesn’t move as the system is rotated around the center. The second servo motor rotates the stand in which the accelerometer is resting on, thus as it moves the system will rotate the accelerometer in a circular rotation about the top gear. The accelerometer is firmly attached to the stand and doesn’t fall off when the top motor rotates the stand to the state in which the accelerometer is upside down. The overall setup of the configuration can be seen below.

Figure 7: Robot Arm Calibration Configuration

The robot motors are driven by two factors: one the amount of rotation and the speed in which it arrives at the desired positions. The amount of rotation determines how far the system rotates until reaching its final position and then returns to the other bound. The motor has a range of positions, spanning from 0° to 180°. If the positions are set from 30° to 180°, then the system will complete a near full rotation about the axis of rotation. If the positions are set from
90° to 180° then the system will only complete a half rotation about the axis of rotation. The other factor, speed, effects the time it takes to reach the desired position thus decreasing the time to get from position a to position b. Both these factors affect the characteristics of the acceleration that the accelerometer will experience.

The two accelerations acting on the accelerometer are the acceleration due to gravity and centripetal acceleration. The centripetal acceleration the accelerometer from the first axis of rotation (the servo motor connected to the bottom gear) is driven to zero. This acceleration is set to zero because the accelerometer is set at the center of rotation of the bottom axis of rotation associated with the servo connected to the bottom gear system. The other form of centripetal acceleration is associated with the top gear system. This acceleration is not equal to zero because the accelerometer is set at a radius (r) from the axis of rotation. This acceleration, however, is also neglected because of the negligible acceleration it will cause on the accelerometer. This is due to the fact that the radius from the center of rotation to the accelerometer is so small that the centripetal acceleration is almost zero.

With the centripetal acceleration neglected in this experiment, the only acceleration acting on the accelerometer is the acceleration due to gravity. The acceleration due to gravity will split between all three axes. The acceleration splits between each axis because of the rotation of the stand in which the accelerometer rests on. When the z-axis is pointed up as shown in the robot arm calibration configuration figure, all acceleration due to gravity is felt by the z-axis. When the top servo motor rotates the stand to the point in which the z-axis is now perpendicular to gravity, the acceleration due to gravity is split between the x-axis and y-axis. It is split evenly because the accelerometer is set at a 45° degrees to the center of the rotation of the top gear system. This distribution of acceleration points to the fact that the acceleration due to gravity will vary sinusoidally with time because the accelerometer position varies with respect to time, thus the acceleration due to gravity will switch between the axes. Therefore, the acceleration for each axis of the accelerometer can be described as below.

\[
\begin{align*}
    a_x &= a_y = \frac{g \sin(\omega t)}{2 \cos(45^\circ)} \\
    a_z &= g \cos \omega t
\end{align*}
\]

Equation 15: Robot Arm Total Acceleration for Each Axis

As the position and speed of the robot arm motors varies, the overall peak acceleration for each axis should not vary, but the sinusoidal function should change to match the oscillation pattern of the servo motors.
Results

Drop Calibration

The average acceleration in the z-axis for each height can be seen in the table below.

<table>
<thead>
<tr>
<th>Height (ft)</th>
<th>Trial 1</th>
<th>Trial 2</th>
<th>Trial 3</th>
<th>Trial 4</th>
<th>Trial 5</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.94541</td>
<td>-0.95513</td>
<td>-0.94804</td>
<td>-0.9443</td>
<td>-0.95123</td>
<td>-0.94882</td>
</tr>
<tr>
<td>3</td>
<td>-0.95276</td>
<td>-0.94783</td>
<td>-0.95036</td>
<td>-0.94965</td>
<td>-0.9489</td>
<td>-0.9499</td>
</tr>
<tr>
<td>4</td>
<td>-0.9391</td>
<td>-0.9504</td>
<td>-0.95158</td>
<td>-0.95078</td>
<td>-0.95147</td>
<td>-0.94867</td>
</tr>
<tr>
<td>5</td>
<td>-0.95306</td>
<td>-0.95367</td>
<td>-0.95173</td>
<td>-0.95467</td>
<td>-0.9475</td>
<td>-0.95213</td>
</tr>
<tr>
<td>6</td>
<td>-0.95076</td>
<td>-0.94988</td>
<td>-0.94203</td>
<td>-0.9476</td>
<td>-0.94514</td>
<td>-0.94708</td>
</tr>
</tbody>
</table>

Table 1: Average z-axis acceleration for various heights

After reviewing the acceleration of the accelerometer in the direction of gravity, the acceleration is not exactly 9.8 m/s² or 1g. The average error between the experimental and the theoretical value is roughly .05068g which equates to .497 m/s². This may be caused by precision error from the manufacturing process which cannot be controlled in this scenario, but what is even more obscure is the variation in acceleration as it is falling because in theory the acceleration should remain constant. There are a few reasons for this fluctuation of acceleration as the accelerometer falls; the accelerometer axis of measurement was not perfectly parallel to gravity and the drag force was not accounted for in the testing. First, diving into the drag force calculation, there is a slight drag force seen on the object as it falls, but due to small drop heights, the drag force is negligible.

\[ D = \frac{(C_d \rho v^2 A)}{2} \]

Equation 16: Drag Force

Next, the accelerometer axis alignment inaccuracy can be attributed from two things, drag force and human error. If the accelerometer is dropped at a slight angle the accelerometer axis that is aligned with the axis of gravity will be offset which can lead to inaccurate readings of acceleration. If there is a slight angle, the drag force can also play a role in shifting the way the object falls because of the redistribution of mass. Again, the effects of the drag force are very low, but with the human error, the errors accumulate and led to a significant error that can skew the data.

After investigating the errors that have contributed to the inconsistently in data, dropping an accelerometer with one axis aligned with the direction of gravity is not a plausible approach to calibrating an accelerometer. The human error alone makes it difficult to replicate consistent data. Therefore, this calibration technique can only be plausible if a very precise machine is designed that can drop the accelerometer with precision.
**Pendulum**

The length of the pendulum arm and the initial drop angle of the pendulum were varied to see the effects on the periodicity of the accelerometer. The angle at which the pendulum is originally released should have no effect on the period of the acceleration due to ideal pendulum motion. However, as the length of the pendulum arm is increased the period of oscillation will increase as well.

![Period vs. Length of Pendulum](image)

**Figure 8: Turntable period with respect to pendulum arm length**

![Period vs. Pendulum Angle](image)

**Figure 9: Turntable period with respect to initial drop angle**

As expected, the periodic acceleration seen by the accelerometer is almost identical to the theoretical period that one should expect. As the length of the pendulum increased, the period increased. On the other hand, as the drop angle increases the period should not. As the initial
drop angle increased, the period of oscillation varied and wasn’t consistent. The reason for this inconsistency is the inability to fully remove any slack from the line. When the pendulum is dropped, there may be a slight slack from the release point which causes oscillations. This causes the pendulum to stray from ideal characteristics and the pendulum period is then dependent on the angle at which it is dropped. In addition, the pendulum had a tendency to rotate about its connection to the mass which led to the inconsistent periods when the system's angle increased.

Another interesting discovery came from observing the amplitude of the acceleration. For an ideal pendulum, the amplitude should remain constant as the system continues to oscillate, but this is not the case in observed trials. This is due to the fact that the pendulum rotated about its connection point as well as wind resistance which slowed the oscillation down. In due time, the apparatus would eventually come to a stop due to the inability to force these undesired characteristics to zero which was seen on the observed data.

The pendulums inability to output consistent waveforms makes it an implausible approach to calibrating the accelerometer. The apparatus needed to create an ideal pendulum would require too much space and equipment to make it a practical technique to use.

**Turntable – Gear Driven**

The two parameters varied in testing the accuracy of the turntable were the speed and the angle the turntable makes with the horizontal. The speed of the turntable varies with the angular frequency of rotation. The varying speed changes the centripetal acceleration, whereas the angle of inclination changes the acceleration due to gravity seen on the accelerometer.

![Figure 10: Turntable centripetal acceleration with respect to the angle of inclination](image)

**Figure 10: Turntable centripetal acceleration with respect to the angle of inclination**
Figure 11: Turntable centripetal acceleration with respect to the period of oscillation

As the angle of the turntable is increased, the centripetal acceleration in the x-axis and y-axis should remain constant. However, the centripetal acceleration of each axis varies by around .005g to .010g which is not a very significant amount. The x-axis and y-axis acceleration increase in the same nature as the theoretical model for acceleration with respect to period. The accelerometer values fluctuates, but in general increases with respect to the period of the turntable. The x-axis and y-axis values should be identical, but they are not because the accelerometer isn’t set to an exact angle of 45°. This variance and other factors like the exact measurements of the radius and period may lead to a theoretical period that is not perfect.

Figure 12: Turntable acceleration due to gravity with respect to angle of inclination
The acceleration due to gravity on the x-axis and y-axis should increase as the angle increases which are shown in the figure above. The experimental acceleration follows the theoretical with little to no error. As the period of oscillation increases, the acceleration due to gravity shouldn’t change, but it varies. The x-axis and y-axis differ by the same amount thus the accelerometer is not set to the ideal 45° or there is a tolerance error between the two axes. However, the error between the ideal and theoretical hovers in the range .005g to .01g which means that the acceleration is quite accurate.

The figure above gives a graphical representation of the acceleration in all three axes. This graph shows the acceleration for when the turntable is set at 18.43° and a voltage of 12V is applied which leads to a period of 2.32s. The z-axis acceleration according to calculations should be around -.95g. The x-axis and y-axis waveforms are identical, but they are offset by 90°
because of the effect of the gravitational acceleration component on each axis. As the accelerometer rotates, the axis that is in line with the gravitational acceleration changes as it moves around in a circular motion. Since the two axes are orthogonal, their values should be offset by 90°. At this angle and speed, the centripetal acceleration which cause the waveform to be slightly offset to -.04g. The amplitude of the acceleration due to gravity is around .316g which is the max value the accelerometer should reach with the sinusoidally nature of the gravitational acceleration component.

The turntable provides a good calibration scheme that is easy to model. The overall apparatus is simple and produces very few errors. The accelerometer mapped the sinusoidal nature of acceleration caused by the turntable and even the orthogonality of the waveforms appeared as expected. Overall, the turntable provides very accurate calibration reading that can be used to calibrate multiple accelerometers at once.

**Linear Actuator**

The applied voltage and the angle at which the linear actuator makes with the horizontal were the two variables tested in the linear actuator calibration. The voltage applied to the linear actuator changes the speed in which the arm moves back and forth, but the arm still moves at a constant speed, so it is expected that there is no acceleration caused from the motion of the arm.

Testing this theory, as the voltage given to the actuator increased from 8 V to 12 V the average acceleration on each axis did not change which is expected. Therefore, the fact that the linear actuator arm movement is not accelerating denotes that its speed should not affect the accelerometer readings on each axis as the voltage varied.

The second variable, the angle, should adjust the amplitude of acceleration seen on the accelerometer. As the angle at which the linear actuator makes with the horizontal increases, the acceleration due to gravity seen by each axis varies. After varying the angle from 16° to 23°, the effects on the acceleration for each axis is observed and plotted in the graphs below.
Looking at the results, the magnitude of acceleration increases in the z-axis while the magnitude of acceleration for the x-axis and y-axis decrease because of the shifting of gravity on the accelerometer as the angle is increased. In all three axes, the acceleration decreases or increases linearly with respect to the angle. This is expected, as the acceleration of each axis is dependent on the angle at which the linear actuator makes with the horizontal. In all three axes, the experimental acceleration and theoretical acceleration are offset by nearly the same error. This consistent error means that the accelerometer is mapping the correct acceleration with a constant offset that could be adjusted. The offset can also be attributed to the fact that the z-axis is not fully parallel with the arm of the linear actuator. In addition, the x-axis and y-axis 45° offset to the arm may not be perfectly 45°, thus the acceleration is not evenly distributed between the two axes.
Overall, the linear actuator provides a simplistic approach to calibrating the accelerometer. However, due to the inability to produce more than constant linear acceleration, the accelerometers would have to be calibrated with multiple orientations. In addition, the attachment method used to connect the accelerometer to the linear actuator makes it difficult to set the accelerometer in the desired orientation to calculate precisely and reduce apparatus induced errors. Therefore, the linear actuator does not provide an adequate method to calibrate the accelerometers.

**Linear Stage**

The two variables tested in this method were the voltage and frequency applied to the motor that drove the linear stage. The voltage variation effected the distance the slide traveled whereas the frequency varied the frequency of oscillation. Both variables varied the acceleration caused by the linear stage. Both variables did not affect the acceleration due to gravity as it is independent of the two test variables. This means that the acceleration due to gravity should ideally remain constant for each trial.

The acceleration due to gravity was tested against the oscillation amplitude and the frequency of the linear stage. In both cases, the acceleration due to gravity for each axis remained relatively constant as predicted by the theoretical calculation. The x-axis and y-axis values were offset by around .03g, but both were very close in value which means that the angle at which the linear stage was set at was slightly off from the angle used in the theoretical angle or the x-axis and y-axis have tolerances that are offset by roughly .03g.

![Linear Acceleration Component](image)

**Figure 18:** Linear stage x-axis and y-axis linear acceleration amplitude with respect to frequency
The linear acceleration of the x-axis and y-axis components varied linearly with respect to the oscillation amplitude and the frequency of oscillation. As described in the method section, the linear acceleration amplitude component varies linearly with the oscillation amplitude and varies by the square of frequency. In both cases, as the variables increased the overall amplitude increased. The x-axis and y-axis acceleration follows the theoretical curve in both cases with a maximum error of .015g.

The figure above gives a visual representation of the acceleration in each axis of the accelerometer. The x-axis and y-axis vary sinusoidally much like the theoretical equation. The z-
axis acceleration which is unaffected by the linear acceleration of the linear stage, sits at a steady value of -0.93g. This value is only off by 0.01g compared to the theoretical acceleration. The sinusoidal functions of the x-axis and y-axis are offset by around 0.23g and the theoretical acceleration due to gravity that causes the offset is around 0.26g. The amplitude of the sinusoidal is dependent on the voltage applied and the frequency. The amplitude of the sinusoidal function should be around 0.04g, whereas the theoretical acceleration is around 0.049g.

In all of the trials, the x-axis and y-axis are offset from each other, but there is consistent gap between them that doesn’t vary substantially. Overall, each axis was able to track the theoretical acceleration as each variable was changed. There were small, but not substantial deviations that occurred between the theoretical and experimental.

The linear stage provides an adequate method to calibrating the accelerometers. The overall accuracy of the linear stage movement allowed for an easy comparison between the experimental and theoretical acceleration. The linear stage adds another acceleration which improves the robustness of calibration. On the other hand, the complexity of the apparatus leads to the possibility of equipment errors that could be impossible to account for in the overall calibration process.

**Robot Arm**

The two parameters varied in the investigation of the robot arm were the speed of the servo motors and the distance the servo motors moved back in forth. In both cases, the speed and the position effect the period of rotation of the servo motors. However, the only acceleration affecting the accelerometers is the acceleration due to gravity. The period of the servo motor rotation should not affect the amplitude of the acceleration due to gravity.

![Figure 21: Robot arm gravitational acceleration in the x-axis and y-axis](image)
Looking at the acceleration in the z-axis of the accelerometer, the amplitude should always be at -1g, but the graph shows that the z-axis accelerations hovers around -.993g which is a deviation of .007g. The x-axis and y-axis acceleration should register .707g each because of the angle at which the accelerometer is placed on the robot arm. The x-axis acceleration is lower than the desired value and the y-axis values are higher. As the period increases, the x-axis and y-axis acceleration values stay the same. They share a .09g offset between them which could be due to the fact that the accelerometer is not set at the desired angle of 45° thus more acceleration may have been disturbed toward the y-axis than the x-axis.

Overall, the robot arm design possesses the ability to calibrate each axis at the same time, but there are many errors that arise from using the gear system and the two servo motors. The code used to run the servos is not the most reliable in moving the servo motors at the exact speed or position desired to calibrate the accelerometer.
Conclusions

After investigating all six calibration techniques, each method has its advantages and disadvantages. The drop calibration, pendulum, and linear actuator are the least reliable in providing consistent data due to the various errors that are introduced by human error and the inability to design a perfect model. On the other hand, the linear stage, turntable, and robot arm had more consistent data as the test variables were adjusted. These techniques required more moving parts to model the acceleration that was desired. In theory, this is ideal because they provide the best model to compare the experimental to theoretical acceleration. However, as more equipment is added to create a better theoretical model, the more error is introduced in the system. In the use of the linear stage, the driver motor was needed to perform the desired acceleration of the slide thus there is possible error induced by the driver motor. On both spectrums, there is a fine line between complexity of equipment and the possible errors induced. The drop calibration and pendulum require very few components which minimizes the equipment errors or tolerances, but they lack calibration accuracy. On the other hand, the linear stage and robot arm are complex calibration techniques that have many moving parts that each induce errors, but they accurately calibrate the accelerometers. In the application and accuracy of the calibration for the scope of this investigation, the turntable provides the perfect medium. The turntable uses very few parts to operate and produce an accurate calibration of accelerometers. The turntable provides consistent replicability that is vital to calibrating accelerometers. This method provides the best possible calibration method and the fewest errors to account for in the equipment itself for the accuracy needed. Overall, each method showed that the accelerometers can follow various acceleration calibration techniques. In each case they had small enough errors that they could be considered negligible. In the end, the X16-1D accelerometer tolerances are negligible, thus only minor calibration is needed to ensure the accelerometer accurately records acceleration.
References


