1917

Variation of angular displacement in binaural localization with relative intensity

Eugene Manasseh Berry

State University of Iowa

This work has been identified with a Creative Commons Public Domain Mark 1.0. Material in the public domain. No restrictions on use.

This thesis is available at Iowa Research Online: https://ir.uiowa.edu/etd/4009

Recommended Citation
Berry, Eugene Manasseh. "Variation of angular displacement in binaural localization with relative intensity." MS (Master of Science) thesis, State University of Iowa, 1917.
https://doi.org/10.17077/etd.cd5lh1ld.

Follow this and additional works at: https://ir.uiowa.edu/etd
Variation of angular displacement in binaural localization with relative intensity.

by Eugene M. Berry.

A Thesis

Submitted to the Faculty of the Graduate College
of the
State University of Iowa

in partial fulfillment of the requirements for
the degree of

Master of Science

June 13, 1917
The general problem that we are working on is sound
the effect of intensity ratio on localization. The specific problems considered in this thesis are as follows.

1. The relation between the angular displacement and the intensity ratio in the case of a pure tone;
   (a) with no difference of phase,
   (b) with a large difference of phase.

2. The relation between the angular displacement of a complex tone of two components and the intensity ratio of one of these components, the intensity ratio of the other being kept constant.

On this problem, C. C. Bunch, a graduate student in psychology and myself a graduate student in physics have worked together this year.
Historical. The influence of intensity in binaural localization has been considered to be an important factor and by many the most important factor. Very few accurate measurements have been made but many experiments have been made which could easily be interpreted to favor the intensity theory as it is often called. Practically all the work bearing on the subject of intensity and localization is referred to in one or more of the historical accounts of A.H. Pierce, P. Rostosky, M. Matsumoto, and C. E. Perree and R. Collins. Much of this work has little connection with our work and is of little value.

Steinhauser comes to the following conclusions. For direct hearing, i.e. when the sound can directly reach both ears, let $\theta$ be the angle between the plane of the pinna and the line of sight, and $\alpha$ the angle which the line of sight makes with the direction of the sound, let $i_1$ and $i_2$ represent the relative intensities with which the sound is heard in the two ears, then,

$$\tan \alpha : \tan \beta :: i_1 - i_2 : i_1 + i_2$$

1. A.H. Pierce, Studies in Space Perception
5. Steinhäuser, The theory of binaural audition, Phil Mag., 1879 (5) VII 181, 261. (This is a translation of Steinhäuser, Die Theorie der binauralen Hörens, Wein 1877.)
For the case where the sound will be reflected from a wall to the ear, he also calculates the effect of the reflection. He devised an instrument which he called homophone to test his theory and claimed to have proved his theory, but he apparently assumes that the head casts a perfect sound shadow, which is incorrect.

Thompson\(^1\) and Urbantschitsch\(^2\) found that when one ear was fatigued, the sound was shifted toward the opposite ear. C.E.Ferree and Ruth Collins\(^3\) found that subjects having a natural difference in sensitivity of the ears tend to shift the sound toward the axis of the stronger ear. Similar displacements could be made by plugging one ear; differences in sensitivity so produced had a greater effect than natural differences in sensitivity. To correct for the natural difference in sensitivity, they plugged the stronger ear but not enough to make it equal to the weaker ear. In this way they could obtain no displacement. They used a galton whistle of 20,000 vibrations per sec. and a tuning fork of 480 vibrations per sec. with essentially the same results.

J.R. Angel\(^1\) in 1903 and D. Starch\(^2\) in 1905 experimenting with forks in the open, conclude that intensity ratio is an important factor.

Lord Rayleigh\(^3\) is perhaps the first one to suggest that difference of intensity or intensity ratio cannot account for our ability to localize low pitched tones. Later\(^4\) he concludes that phase differences account for the localization of tones of low frequencies, but intensity is the most important factor for tones of high frequencies.

C.S. Myers and H.A. Wilson\(^5\), L.T. More and H.S. Fry\(^6\), L.T. More\(^7\), and T.J. Bowlker\(^8\) performed similar experiments. They conducted the sound to both ears by tubes, the length of which could be easily changed so they could get any phase difference desired. These experiments tend to show that phase differences are appreciated by the ears, but C.S. Myers and H.A. Wilson advance a

theory to explain their work. The theory is that by conduction from one ear to the other and a change of phase in conduction thru the head, the phase differences cause a difference of intensity at the ears themselves, and so try to explain it fundamentally by intensity. Lord Rayleigh criticizes their theory, saying that according to the theory a small external difference of intensity would cause quite a difference in the localization, but actually this is not true for low frequencies. Ferree and Collins also make the point that in these experiments with unequally long tubes there is a difference of intensity at the ears.

The fact that differences of intensities will cause a shift of the sound from the median plane was observed by S.P. Thompson in 1878, by Tarchanoff in 1878, and by Kessel in 1882.

Matsumoto was the first to make a careful study of the effect of intensity ratios on localization. He observed that sounds so placed that there was equal intensity at the ears were always located in the median.

4. Tarchanoff, St. Petersburg med. Wochenschrift, 1878, No. 43.
5. Kessel, Ueber die function der Ohrmuschel bei den Raumwahrnehmungen, Arch.f. Ohrenheilk., 1882 XVIII 120.
plane. He placed two telephone receivers, each run by a 256 d.v. fork and so arranged that he could vary the intensity in either telephone. By a sufficient relative intensity he could cause the sound to swing from 90° right to 90° left. Most observers localized the sounds in front but one always in back. Front and back were often confused. This work showed that there was some relation between the angle and the intensity ratio but he makes no attempt to establish a numerical relation between the angle and the intensity ratio. He says, "We must be satisfied if the general dependence of the latter on the former is proved."

Münsterberg tested the least noticeable change in the direction of a sound. He found the change least in front and greatest in back with a comparatively regular increase as the angle from the front increased. He assumes the localization is based upon sensations of movement and accomplished either by actual head movements or the sensation of strain when the head is not moved. This sensation of strain must increase as the angle from the front increases. He says that these experiments conform exactly to Weber's law.

Bloch repeated Münsterberg's experiments. He found a minimum sensitivity at 90° right or 90° left and a

maximum at 0° and at 180°; the one at 180° being less than the one at 0°.

Matsumoto found that when the relative intensities at the ears were in such a relation that the perceived sound was located at 90° right or 90° left, a small change in the relative intensities was not generally perceived and did not change the localization of the sound; but when the intensities were such that the sound was located just in front, a small change of relative intensity was usually perceived as a change of direction.

He confirmed this by observing the least noticeable change of distance of one of the phones for different relative intensities, which he says should, according to Weber's law, be expected to depend upon the initial difference of intensity of the two sounds. His experiments show this, for the increment distance was smallest when the sound was initially located in front or behind and gradually increased as the initial difference of intensity increased.

1. Loc. cit.
In the Summer of 1916 G.W. Stewart and O. Hovda made some rather careful measurements on the relation between the intensity ratio and the localization. The sound from an electrically driven tuning fork of 256 d.v. is conducted separately to each ear. One of the tubes is kept at a fixed distance from the fork, while the distance of the other tube from the fork can be varied. By using a Rayleigh disk the relative intensities of the sound at the binaurals, i.e. at the ears, can be found for any given position of the "movable" tube. In this way known intensity ratios at the ears can be produced and their localization observed.

They found that if they plotted the angular displacement from the front, as the ordinate and the logarithm of the intensity ratio at the ears, as the abscissa, the points were grouped in general about a straight line; or if \( \theta \) is the angle from the front, \( I_r \) the intensity at the right ear, and \( I_l \) the intensity at the left ear,

\[ \theta = k \log \left( \frac{I_r}{I_l} \right). \]

This relation they verified for angles up to 45°.

This they show to be an extension of Weber's law in the following manner. Let \( \frac{I_r}{I_l} = R \) be the intensity ratio at the two ears, and let \( \theta \) be the angle corresponding to \( R \); then let \( \Delta R \) be the change of intensity ratio which will cause the least noticeable change of direction, \( \Delta \theta \);

1. These results will soon be published in the Physical Review.
then if we assume that,
\[ \Delta \theta = k(\Delta R/R) \]
or \[ d\theta = k(dR/R) \]
and integrate we get,
\[ \theta = k \log R \pm \text{a constant of integration}, \]
but for equality of phase \( \theta = 0^\circ \) when \( R = 1 \)
\[ \therefore \theta = k \log R = k \log(I/L) \].

The assumption that \( \Delta \theta = k(\Delta R/R) \) would be what Weber's law gives if we suppose that the ears interpret intensity ratio as angle. Thus it appears that this law is an extension of Weber's law. Weber's law has been verified for many cases in the differential form, but this is probably the first verification in the integrated form.

The first part of our work is an extension of this law for angles up to \( 90^\circ \) from the front, with essentially the same apparatus as Stewart and Hovda used.
Apparatus:—Fig. 1 is a general diagram of the apparatus. As we wished to get as pure a tone as possible we used two 256 d.v. forks in series, i.e. the same current runs through the magnet coils of both forks. Fork B has the contact D to make and break the circuit. A condenser C is put in parallel with the contact to cut down the sparking. Fork A is the source of the sound and in this way the noise of the sparking does not enter the tubes leading to the ears. It is very difficult to get the contact so adjusted that the forks will run with reasonable steadiness.

To measure the amplitude of the fork A we had a microscope with a micrometer eye-piece focused on a bright spot near the end of one of the prongs of the fork A. This bright spot was the end of a short piece of wire fastened on with wax. This was illuminated by focusing on it the light from one of the lamps on the ceiling. When the fork was vibrating the spot of light widened out into a line, the length of which gave us the amplitude. There were fifty-five divisions per mm. in the microscope, so we used one of these divisions as our unit of amplitude. We could get any amplitude we pleased up to about one mm. by properly loading the forks and adjusting the resistance in the circuit. When the forks were exactly in tune the amplitude was very large; then if one of the forks was loaded the amplitude decreased. The reason for this is as
Fig. 1.
follows. When a fork is loaded its natural frequency is changed. When the fork A is exactly in tune with the fork B there is resonance and the fork A has a large amplitude. When the fork A is loaded the resonance is more or less destroyed and the vibrations of the fork A are forced vibrations and the amplitude of vibration is more or less diminished for the same amount of energy used to run the forks. At first we loaded the fork A with wax then
later we used wire riders which had enough spring in them to stay when put on the fork.

Close to the fork A (Fig. 1.), on one side were two glass tubes about 7 mm. internal diameter; one marked F we called the "fixed" tube because it was kept at a fixed distance from the fork; the other tube marked M we called the "movable" tube, because it was fastened to a piece of tin so that the whole thing could be slid back and forth while keeping the tube perpendicular to the fork and so varying the distance from that tube to the fork. A mm. scale, fastened on a board which was fixed in reference to the fork, enabled us to set this "movable" tube at any desired position. A part of the tin slider marked the relative position of the tube and when at the zero of the scale the "movable" tube was about 1 mm. from the fork.

As the ears are quite easily fatigued by any tone for that particular tone, some device was necessary to shut off the sound when desired. G. W. Stewart and O. Hovda in their work on this simply inserted a piece of cardboard between the fork and the tubes. We could not use this method because we had our "fixed" tube too close to the fork (about 2.5 mm. while they had theirs about 10 mm.

First we used water traps, $T_f$ and $T_m$, Fig. 1. for that purpose; simply a glass U tube with another tube

1. loc. cit.
joined on to the base of the $U$, to which a rubber tube about 80 cm. was attached. The trap is opened or closed by lowering or raising the end of the rubber tube. In order to avoid being bothered by bubbles we used for the water traps, tubes about 13 mm. internal diameter. When closed these let thru a very little sound and even when open cut the intensity down to about 15% of its original intensity.

For some of our experiments we could not get sufficient intensity thru these water traps so we used mercury traps, Fig. 2. These consisted of the same kind of $U$ tubes as before, except the internal diameter was much smaller, about 6 mm., and we used mercury instead of water. This time we had the short glass tube at the base of the $U$ dip in a cup of mercury. This cup was fastened on a board and normally held up by a spring and so kept closed and could be opened by pressing on the board.

We used the same cup of mercury for both traps, but the sound passing from one trap to the other thru the mercury was almost subliminal, certainly too small to affect the results any. The mercury traps were more effect-
tive than the water traps.

The sound was lead from the fork to the traps and from the traps to the ears or the Rayleigh disk by heavy rubber tubing about 4.5 mm. internal diameter. The length of path from the ears to the fork was about three meters. We made these tubes as near the same length as possible so as to have no phase difference, but our results seem to indicate that there was a small constant difference of phase. In fact in some of our work we changed the length of one of the tubes to correct this.

To measure the intensity of the sound we used a fairly sensitive Rayleigh disk. The Rayleigh disk and the observer were in a separate room from the forks and the traps. The tubes go thru two small holes in the wall; in this way the sound from the forks are scarcely audible to the observer except thru the tubes. We also arranged a couple of electric lights so we could signal from one room to the other. The Rayleigh disk gives deflections proportional to the intensity. In fact we found that when other things were constant the deflections of the disk were proportional to the square of the amplitude of vibration of the fork, i.e. proportional to the intensity.

Fig. 3. shows the Rayleigh disk drawn to scale. A glass tube B is fixed in the open end of the Rayleigh disk. At the end a short rubber tube was fixed so the binaurals could be easily inserted at A one at a time.

![Fig. 3.](image)

The binaurals used here were the ordinary stethoscope binaurals.

We first had to get an intensity curve, i.e. the curve showing the relation between the intensity ratio and the distance or the position on the scale, Fig. 1. What we actually used was a curve having position on the scale and the logarithm of the intensity ratio as the coordinates.

Our method was for one of us to keep the amplitude of the fork constant, as near as possible, while the other observed the deflections of the disk for the "fixed" tube and for several positions of the "movable" tube; then dividing one by the other we obtained the ratio of intensities. Table 1. is a sample of our data for this. The first column gives the position of the "movable" tube on the scale; the second, the deflections of the disk; the third, the ratio of the deflections of the disk to the average of the four deflections for the "fixed" tube. The average ratio for the
Table 1.
Amplitude of the fork 23

<table>
<thead>
<tr>
<th>Position on scale</th>
<th>Def. of the disk</th>
<th>Ratio to fix. tube</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td>14.8</td>
<td>1.040</td>
</tr>
<tr>
<td>Moveable</td>
<td>0</td>
<td>1.910</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1.250</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.647</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.352</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.232</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.120</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>0.092</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.056</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.028</td>
</tr>
<tr>
<td>Fixed</td>
<td>13.8</td>
<td>0.972</td>
</tr>
<tr>
<td>Moveable</td>
<td>7</td>
<td>0.227</td>
</tr>
<tr>
<td>Fixed</td>
<td>14.1</td>
<td>0.993</td>
</tr>
<tr>
<td>Moveable</td>
<td>7</td>
<td>0.232</td>
</tr>
<tr>
<td>Fixed</td>
<td>14.2</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Amplitude of the fork 50

<table>
<thead>
<tr>
<th>Position on scale</th>
<th>Def. of the disk</th>
<th>Ratio to fix. tube</th>
<th>Ratio to mov. at 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moveable</td>
<td>7</td>
<td>0.226</td>
<td>0.983</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>0.039</td>
<td>0.387</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.054</td>
<td>0.235</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>0.037</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.023</td>
<td>0.122</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.233</td>
<td>1.015</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.230</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>0.092</td>
<td>0.400</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.058</td>
<td>0.252</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>0.037</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.023</td>
<td>0.122</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.232</td>
<td>1.016</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.013</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.008</td>
<td>0.035</td>
</tr>
</tbody>
</table>
"movable" tube at 7 is 0.230; this is taken from the first part of the table where the amplitude is 23. This is used as the standard for the second part of the table where the amplitude is 50, for with this amplitude the deflection for the fixed tube is too great to measure, i.e. of the deflections of the disk to the average goes off the scale. Column four gives the ratio of deflections of the "movable" tube at 7, then from this the third column in the second part is obtained by making the average of the four ratios for the "movable" tube at 7 to be 0.230.

It is difficult to make accurate measurements of the intensity ratio with the Rayleigh disk; one reason is that noises around the building occasionally bothered, but the chief trouble is that it is almost impossible to keep the amplitude of the fork constant to within 2%, and as the intensity varies as the square of the amplitude this makes quite a variation. However by taking three or four sets similar to table 1. and averaging the results, we obtained a curve far more accurate than our localization and consequently sufficiently accurate for our purpose.

As we couldn't get sufficient intensity thru the water traps for our intensity curve, we replaced the traps by short glass tubes to get our intensity curve. Then we took four positions of the "movable" tube and measured their ratio to the "fixed" tube, both with and without the traps. We did this to see if the fraction of
the intensity which got thru the traps was a constant for all intensities and also to compare the traps. We found that the traps cut down the intensity to a constant fraction of the original intensity regardless of what that intensity is; this fraction is approximately 1/7. We found that one trap cut the intensity 3% more than the other trap; we do not need to correct for this as this is too small a difference to cause an appreciable error in our results, in fact the only effect it could have on our localization curves as we plotted them would be to shift the zero a trifle.

Table 2. gives the data for the curve showing the relation between the log of the intensity ratio and the position of the "movable" tube on the scale. As we always had the "fixed" tube to the right ear and the "movable" tube to the left ear we used $I_r$ for the intensity at the left ear or from the "movable" tube and $I_l$ for the right ear or the "fixed" tube. As we wished to verify the law $\theta = k \log(I_r/I_l)$, it was necessary to plot our results so the curves would be a straight line if that law held, i.e. we must use as our coordinates $\theta$ and $\log(I_r/I_l)$. This is the reason for getting a curve using the log of the intensity ratio instead of the ratio itself.

Fig. 4. shows the curve connecting the log of the intensity ratio and the distance. From this we made our scale, shown also in Fig. 4., which we used to plot our results. The upper part which has divisions running from
### Table 2.

<table>
<thead>
<tr>
<th>Dist.</th>
<th>$I_r/I_c$</th>
<th>$I_r/I_l$</th>
<th>Log($I_r/I_c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.910</td>
<td>0.523</td>
<td>-0.281</td>
</tr>
<tr>
<td>.5</td>
<td>1.590</td>
<td>0.629</td>
<td>0.201</td>
</tr>
<tr>
<td>1</td>
<td>1.224</td>
<td>0.817</td>
<td>0.088</td>
</tr>
<tr>
<td>3</td>
<td>0.663</td>
<td>1.510</td>
<td>0.179</td>
</tr>
<tr>
<td>5</td>
<td>0.369</td>
<td>2.710</td>
<td>0.433</td>
</tr>
<tr>
<td>7</td>
<td>0.237</td>
<td>4.220</td>
<td>0.625</td>
</tr>
<tr>
<td>10</td>
<td>0.133</td>
<td>7.520</td>
<td>0.876</td>
</tr>
<tr>
<td>12</td>
<td>0.093</td>
<td>10.75</td>
<td>1.032</td>
</tr>
<tr>
<td>15</td>
<td>0.059</td>
<td>16.95</td>
<td>1.229</td>
</tr>
<tr>
<td>18</td>
<td>0.036</td>
<td>27.80</td>
<td>1.444</td>
</tr>
<tr>
<td>20</td>
<td>0.028</td>
<td>36.70</td>
<td>1.552</td>
</tr>
<tr>
<td>30</td>
<td>0.012</td>
<td>83.40</td>
<td>1.921</td>
</tr>
<tr>
<td>40</td>
<td>0.007</td>
<td>143.0</td>
<td>2.155</td>
</tr>
</tbody>
</table>
0 to 40 and unequally spaced are distances on the scale (the distances on this scale plus a certain constant which we did not determine are the distances of the "movable" tube from the fork) while the lower part on which the divisions are equally spaced gives the logarithm of the intensity ratio for the corresponding distance given on the upper part of the scale.

We used two kinds of binaurals; one the ordinary stethoscope binaurals which we called "closed" binaurals, because they close the external meatus of the ear; the other kind did not touch the ears, hence we called them "open" binaurals. Fig. 5. is a diagram of them. Each consisted simply of a glass tube passing thru a cork which was fitted into a hole in a triangular board. This board had three legs which rested on the head and kept the glass tube from touching the ears.

The end of the tube was adjusted as close as possible to the ears without touching them. We also fastened them together by a strip of metal which served as a spring to hold them against the head.

We did practically all our work with the closed binaurals first and used the water traps, but when we started to use the "open" binaurals we found that the
water traps would not allow enough sound to pass thru, so we used the mercury traps which did not greatly diminish the intensity of the sound. We had to use much as ten times the intensity when we used the "open" binaurals as when we used the "closed" binaurals.

The reason the water traps cut the intensity of the sound so greatly was because the diameter of the tube in the traps was several times the diameter of the hole in the rubber tubing; for any bulged tube in place of the traps also cut down the intensity greatly. The tubing of which the mercury traps were made had an internal diameter only a little greater than the rubber tubing we used.

We tried to use these small traps first with water but couldn't on account of bubbles, then we discarded them and used the larger traps with water until we came to use the "open" binaurals, then we fixed the small traps up with mercury and called them mercury traps. These can be used for the "closed" binaurals just as the water traps in fact they are preferable to the other.

Method of observation. We took fifty different settings of the "movable" tube. These were so taken that in respect to the log of the intensity ratio they would be uniformly distributed. It was necessary to take a large number for the localization of these sounds were rather inaccurate, frequently varying 15° to 20° for the same setting, i.e. the same intensity ratio. We then took these fifty distances and mixed them up so they were in
random order except we tried not to have two settings in succession which would be so close to each other that they would sound the same.

The observer sat in the center of a semi-circle of about 70cm. radius on which were marked the angles to every five degrees from 90° right to 90° left.

One person adjusted the position of the "movable" tube. He would set the tube at a certain position and then flash the light as a signal to the other, then open the trap for five seconds then close it for about three seconds then open it for five seconds, repeating this until the observer flashed back to indicate that he had localized the sound. Then he sets the "movable" tube at the next distance and so on till the set is completed. The observer recorded the localizations, numbering them from 1 to 50. In some of the sets the one at the fork would keep the trap open until the observer had formed his judgment and flashed the light, then he closed the trap and opened it again for the observer to confirm his judgment, then when the observer flashed, went on to the next one in the set.

At every fifth one, the one at the fork flashed five times, otherwise we occasionally get confused.

After the set we then combined the distances and the angles.

In practically every case the observer did not know the order the distances were to be given in. Part of the time we would only take half of the fifty at one sitting, and
would take a rest in the middle, for it is tiresome to take fifty at one sitting. Some of the time one would take half a set, then the other a half a set, then the first one a half set, then the other finish his set.

As already stated we plotted our results with angles as ordinates and the log of the intensity ratio as abscissa. If the points are grouped about a straight line, then the extension of Weber's law,

$$\theta = k \log \left( \frac{I}{L} \right)$$

is verified. Whether we use the natural or the common logarithms is immaterial for one is a constant times the other.

Results with the "open" and "closed" binaurals, with nearly equal phase. The curves are in general straight lines. They are practically the same both in straightness and in slope, whether the "open" or "closed" binaurals are used. Fig. 6. shows four curves with the same observer; curves (1) and (2) are with the "closed" binaurals, while (3) and (4) are with the "open" binaurals. Curves (1) and (3) are as near alike as any two sets ever are, yet (1) was taken with the "closed" binaurals on March 8 while (3) was taken with the "open" binaurals on April 23. Curves (1), (3), and (4) are probably straight lines from about 30° left to 90° right. In all these the intensity at the right ear was kept constant while the intensity at the left ear was varied, by the means described above, from twice
Fig. 6
the intensity at the right ear to about \( \frac{1}{35} \) the intensity at the right ear. In some of our first experiments we used ratios as small as 1 to 140, but as the angle reaches 90° when the ratio is about 1 to 25 and the angle does not go beyond 90°, we thought it to be of no advantage to use ratios smaller than the ratios that the distances from 1 to 20 of the "movable" tube would give us, which is about 1 to 35.

Curve (2) is not a single straight line, but appears to be a line with two distinct slopes, i.e. there is a break in the curve at about 15° right. Curves (1) and (3) were taken only three days apart under similar conditions, as far as we could get them. Occasionally there was a second break in the neighborhood of 90°, probably these would occur in every case if we had carried the intensity ratios far enough, for in every case where we used the distances from 0 to 40 we found the break at 90°.

As to the frequency of the broken curves as compared with the straight ones the following will give some idea. Out of six curves for the "closed" binaurals of one observer, one was straight, one had possibly a slight break, and four had a decided break in each curve. For another observer the breaks were not so marked, i.e. the difference of slope was not so great; out of six for the "closed" binaurals he had two straight ones. These breaks occur with the "open" as well as with the "closed" binaurals.
Fig. 7. shows a curve with a break at about $15^\circ$ and one close to $90^\circ$.

We do not know what causes these breaks, but we believe that up to about $90^\circ$ the straight line curves are the normal type of curves and that the breaks are due to some abnormal physical or mental condition of the observer. G.W. Stewart and O. Hovda\textsuperscript{1} found in their work similar breaks. In one particular case they thought that a slight cold affected one ear and so caused the break.

\textsuperscript{1} G.W. Stewart and O. Hovda, The intensity factor in binaural localization, Phys. Rev.
**Discussion of errors.** The intensity curve is somewhat difficult to get accurate, but for each point on the intensity curve (see table 2. and Fig. 4.), except for the distances 30 and 40, the intensity ratio used is the mean of five or more observations. The deviation of any observation from the mean is not more than 5%, and the probable error less than 1%. In Fig. 4. the points lie very closely on the smooth curve.

When our localization curves are plotted as in Fig. 6. an error of 3% of the intensity ratio of any point would displace the point an amount equal to its diameter; hence a 3% difference in the water traps can safely be neglected, especially since the variation in the localization for the same relative intensities is frequently 20° or more.

There is the danger of the fork moving in respect to the "fixed" tube and the scale. As we have these things held in place by clamps the vibration of the fork may jar something loose. After we had taken an intensity curve and a few observations, the fork came discarded loose, then we took a new intensity curve and all the we had taken observations which before. The intensity ratios were very much changed as can be seen from table 3., yet if plot the same set of observations with the two different intensity curves, we get curves almost identical in shape. The slope is considerably changed and the zero a little displaced. However I am quite confident
Table 3.

Data for intensity curves used in plotting curves (1) and (2) above.

<table>
<thead>
<tr>
<th>Dist.</th>
<th>I/I₀ for curve (1)</th>
<th>I/I₀ for curve (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.9100</td>
<td>1.770</td>
</tr>
<tr>
<td>1</td>
<td>1.224</td>
<td>0.940</td>
</tr>
<tr>
<td>3</td>
<td>0.663</td>
<td>0.460</td>
</tr>
<tr>
<td>5</td>
<td>0.369</td>
<td>0.224</td>
</tr>
<tr>
<td>7</td>
<td>0.237</td>
<td>0.140</td>
</tr>
<tr>
<td>10</td>
<td>0.133</td>
<td>0.068</td>
</tr>
<tr>
<td>12</td>
<td>0.093</td>
<td>0.036</td>
</tr>
<tr>
<td>15</td>
<td>0.059</td>
<td>0.016</td>
</tr>
<tr>
<td>20</td>
<td>0.028</td>
<td>0.008</td>
</tr>
</tbody>
</table>
that for all the results used the fork had not moved after obtaining the intensity curve enough to change any intensity ratio by more than ten percent. Fig. 8. shows the result of plotting the same set of observations by the two intensity curves. Curve (1) is plotted correctly while (2) is plotted with the wrong intensity ratios.

Another possible error is that by moving the "movable" tube, a small difference of phase is introduced. In most of our observations we only moved this tube 2 cm. For a 256 d.v. fork the wave-length would be about 134 cm., then the phase difference introduced would be \( \frac{2}{134} \times 360^\circ = 5.5^\circ \). G.W. Stewart\(^1\) has experimented quite carefully on the effect of phase differences and has found that the displacements due to phase differences are very nearly equal to the phase differences themselves. About the only effect this could have on our curves would be to increase slightly the slope of the curves.

With the open binaurals it is almost impossible always to adjust them alike so that they will be in exactly the same position for the two ears. The effect of not having the ends of the glass tubes at the same position in the ears would be to change the intensity ratio of every setting of the "movable" tube by a constant factor. As we plot our curves with log of the intensity ratio as abscissa, this would amount to a translation of the coordinate axes,

\(^1\) G.W. Stewart, Binaural beats, Phys. Rev.
i.e., would shift the curve without changing its shape or slope. The shape of the ears is such that the actual position when very close to the external meatus probably would not make a great difference in the intensity. For this reason it would appear that the position of the zero with the "open" binaurals might not be very constant. As a matter of fact the curves do not cross the the line of equal intensity always at the same point even with the "closed" binaurals. There seems to be considerable variation in these curves from one time to the next.

The effect of phase differences. The fact that phase differences can cause angular displacements about equal to the phase differences has already been referred to. In our first work we tried to get the paths to the ears equal in order to have equality of phase. But most of our curves cross the line of equal intensity at anywhere from 7° to 20°, i.e. the localization for equal intensity was on the average from 7° to 20° left. The probable explanation of this is that a constant difference (perhaps due to differences in the diameter of the tubing) of phase displaces the localizations, in these cases, toward the left.

To test the effect of phase differences, we replaced a short glass tube by a longer glass tube in the tube to one ear thus changing the length of path and so introducing a constant phase difference. This also changed the intensity ratio for any given position of the "movable" tube by a constant factor. Hence we need only measure the
fraction of the intensity cut down by the additional tube; or, measure the ratio for any one setting of the "movable" tube. Then as we use the log of the intensity ratio, we have simply to transform the coordinates by translation of the axes, after having plotted the localizations by the original intensity curve.

Fig. 9 shows four curves which are the results of observations taken to show the effect of phase differences. The curves are very similar to the ones shown before which are from observations taken with practically no phase difference. Curves (1), (2), and (3) were from observations with the "open" binaurals, (4) was with the "closed" binaurals.

For curve (1) the length of path to the right ear was made 15.5 cm longer than usual. For curve (2) the length of path to the left ear was 15.5 cm longer than with the usual arrangement. This made a total difference between the two cases of 31 cm. This would mean that the change of phase differences between the two cases would be about 90°. Actually curve (1) crosses the line of equal intensity ratio i.e. the zero of the abscissa at 30° left, curve (2) at 3° left which is a difference of 27° in the two curves.

For curves (3) and (4), as the length of path to the right ear was 39 cm longer than that which had existed in the usual arrangement, the change in the phase difference from what it was with the usual length of tubes should be about 110°, but the curves cross the line of equal intensity at about 40° left which is about 30° further left than curves
with the usual length of tubes.

From curves (1) and (2) it would appear that a large phase difference changes the slope of the curve. Two curves for another observer show similar differences of slope. The "closed" binaurals do not show any marked differences of slope for large phase differences.
Fig. 9.
The slopes of the curves. These are very changeable. The slope of the curve gives the value of $k$ in the equation,

$$\theta = k \log \left( \frac{I_r}{I_l} \right).$$

If we express $\theta$ in degrees and use the common logarithms, i.e. with the base ten, then $k$ in our work varies, for the "closed" binaurals, from 45 to 90, and for the "open" binaurals from 54 to 118. When there is practically no phase difference, for the "open" binaurals the range is representing the experiments in which there was from 54 to 73, but for the three curves the extra tube to the right ear (which shifts the localization toward the left) the values of $k$ are 102, 114, and 118, while when the extra tube is to the left ear, $k$ has the values 56 and 73. 70 seems to be about the normal value for equality of phase with either set of binaurals.

The values of $k$ in the work of Stewart and Hovda on the intensity ratio in binaural localization ranged from 31 to 51. They found that $k$ for the "open" binaurals was less than for the "closed" binaurals. Our work does not seem to bear this out. Our work seems to show that $k$ is more variable for the "open" than the "closed" binaurals. They found that it required an intensity ratio of about 10 to 1 to cause a displacement of 45°, hence to cause a displacement of 90° would require an intensity ratio of 100 to 1. For, according to the law,

$$\theta = k \log \left( \frac{I_r}{I_l} \right) = k \log R$$

1. loc. cit.
if \[ \theta_i = k \log R_i \]
then \[ 2\theta_i = 2k \log R_i \]
i.e. \[ 2\theta_i = k \log R_i^2 \]
if \( R = 10 \), when \( \theta \) in degrees is 45
then \[ 45 = k \log 10 = k \]
or \[ k = 45 \]

In our work we found that for the best curves (Fig. 6. three of shows our best curves) with practically equality of phase, the curve crosses 45° when the log of the intensity ratio is about 0.70 or 0.75 i.e. for a ratio of about 5 or 6 to 1, and the curves reach 90° when the log of the intensity ratio is about 1.4 i.e., for a ratio of about 25 to 1. But as the zero was 10° to 15° off, the intensity ratio necessary to produce a displacement of 90° with exact equality of phase would be less, probably about 18 to 1, which is the value that substitution in the equation,

\[ \theta = k \log \left( \frac{I_i}{I_r} \right) \]

would give \( k = 70 \)

G.W. Stewart and O. Hovda show that the intensity ratios which they found are necessary to cause a displacement of 45° are many times greater than the intensity ratios experienced in actual localization. The same would hold true for our work.

1. loc. cit.
A double localization. Occasionally it appeared that the sound could be localized in two directions, frequently as far apart as 90°. When this was noticed, with the "closed" binaurals, there seemed to be two sounds; one a smooth ringing sound which did not change much as the intensity ratios were changed, but was decidedly shifted when a large phase difference was introduced by using a long glass tube instead of a short tube; the other was a sort of a buzzing sound which shifted its position as the intensity ratio was changed. This was most noticeable when there was a large phase difference, say 50° to 70°, and an intensity ratio sufficient to produce a displacement of normally 20° to 40° in the opposite direction.
Then the two sounds were sometimes so separate and distinct as to be very confusing. For the "closed" binaurals, whenever I tried to notice this these were generally clearly distinguishable except when both were located very close together or when the ratio of the intensities was such as to cause localization close to 90°. In general I tried not to notice this or pay any attention to it.

With the "open" binaurals this was not generally noticeable and when it was, was not so distinct as with the "closed" binaurals.

This effect was observed only by Bunch and myself out of four observers, however only one of the other two took any observations with a large phase difference and that only once. We noticed it only after very many observations.

S.P. Thompson in describing the location of the acoustic image says that when there is a phase difference for a pure tone the sound is located partly in the ears and partly in the back of the head. For our double localization, one was decidedly in the front of the head, as to the other I would not positively state that it did not sound in the ears, but it did not seem that way to me.

Varying one component of a complex tone, which is composed of two simple tones. To do this we took another electrically driven fork of a different frequency than 256 d.v. We then placed the end of a glass tube close to each prong of the fork and by means of two T tubes connected these in to the other tubes, one to each tube. Then the sound in the right ear was made up of two components of constant intensity. The sound in the left ear was also of two components, one of constant intensity, the other, of variable intensity. The variable component was of the frequency of 256 d.v. The sound coming to one ear was entirely separate from that coming to the other ear.

We first tried as our second fork one of 435 d.v. In this case each tone was distinctly heard and localized separately, irrespective of the other.

We then tried as our second fork one of 128 d.v. With the "closed" binaurals there two localizations, one nearly the same as with the 256 d.v. fork alone and the other nearly constant in position. Curves (1) and (2) Fig. 10. show localizations plotted separately. The log of the intensity ratio of the variable component is the abscissa. In this the assumption is made that the relative intensities for this component are the same as they are without the tubes from the 128 d.v. fork connected in. As we took some time to localize the sound and the phase of the forks relative to each other was continually changing and could still get definite localizations, it would seem that the phase of the forks relative to each other had no effect on our localizations.
Fig. 10.
With the "open" binaurals the sound did not in general seem to be double, hence we could get a single localization curve. Curves (3) and (4) fig. 10 show two such sets from two different observers. The slopes of these two curves are decidedly less than any obtained with the single fork. In these no attempt was made to keep the amplitude of the 256 d.v. fork constant. The two forks were simply adjusted so both could be heard easily; but with the two for the "open" binaurals, the 128 d.v. fork was relatively fainter than with the "closed" binaurals.

On account of varying the intensity at one ear only, the intensity sum varied in the ratio of about 2 to 1. The question might be asked whether this could make any difference in the results of the localizations and whether the absolute value of the intensity sum made any difference. It does probably not in the case of a pure tone, for we did not always have the same amplitude of the fork and sometimes the amplitude was not constant during a set of observations. Possibly the actual intensity might have some effect on the slope of the curves.

Where one component of a complex tone was varied this could easily cause a difference, for if the sound from the 256 d.v. fork is equal in intensity to that of the 128 d.v. fork, when the "movable" tube is close to the fork, it would only have about half the intensity when the "movable" tube is at quite a distance from the fork.
Summary. Similar results are in general obtained with the "open" and the "closed" binaurals. Our experiments show that the extension of Weber's law, in the integrated form, to localization,

$$\theta = k \log(I_r/I)$$

where $\theta$ is the angle of localization from the front, and $I_r/I$ is the ratio of the intensities at the ears, holds in general up to $\theta = 90^\circ$. We plotted our curves with $\theta$ as the ordinate and $\log(I_r/I)$ as the abscissa.

There are frequently breaks in the curves which we cannot explain, except that we think that the breaks are due to some abnormal condition of the observer.

A large constant difference of phase displaces the curves somewhat but not by an amount equal to the phase difference and possibly increases the slope of the curves.

The relation between the intensity ratio of one component of a complex tone, (made up of two tones) and the localization is also expressed by the same law,

$$\theta = k \log(I_r/I)$$

i.e. when the sounds fuse. The value of $k$ the slope of the curve is very much smaller when the sounds do not fuse the component sounds are localized separately with little interference. This has not been tried out very carefully, and only a very few curves for this were taken.
Conclusion. These results tend to show that while with sufficient intensity ratios the sound may be shifted as far as 90° from the front, yet the intensity ratios necessary to cause a given angular displacement is so much greater than that found generally in actual localization that intensity ratio in itself can hardly be a very prominent factor in the actual localization of a pure tone of this frequency.

There remains very much work to be done on the question of the intensity factor in binaural localization. Forks of higher and also of lower frequency should be tried to see if the frequency has a large effect on the importance of the intensity ratio in binaural localization.

I wish to express my appreciation of the valuable assistance and suggestions of Professor G.W. Stewart, under whom this work was carried on. Also I wish to acknowledge my indebtedness to Mr. C.C. Bunch, who worked with me on this problem, and to Mr. Victor Hoersch and Mr. William Schriever for their assistance as observers.