Investigation of collective phenomena in dusty plasmas

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INVESTIGATION OF COLLECTIVE PHENOMENA IN DUSTY PLASMAS

by

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A thesis submitted in partial fulfillment of the requirements for the Doctor of Philosophy degree in Physics in the Graduate College of The University of Iowa

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ABSTRACT

I study dusty plasma produced by electrostatically confining melamine formaldehyde microparticles in a radio-frequency glow discharge plasma. Dusty plasma is a mixture of particles of solid matter (dust), electrons, ions, and neutral gas atoms. The dust particles have a very high charge and a mass compared to the electrons and ions in the ambient plasma. As a consequence, a dusty plasma exhibits collective phenomena such as dust acoustic waves, crystallization, and melting. The discrete nature of dust particles gives rise to compressibility.

In this thesis I report findings of four tasks that were performed to investigate dust acoustic waves, compressibility, and melting. First, the nonlinear phenomenon of synchronization was characterized experimentally for the dust acoustic wave propagating in a dust cloud with many layers. I find four synchronized states, with frequencies that are multiples of 1, 2, 3, and 1/2 of the driving frequency. Comparing to phenomena that are typical of the van der Pol paradigm, I find that synchronization of the dust acoustic wave exhibits the signature of the suppression mechanism but not that of the phaselocking mechanism. Additionally, I find that the synchronization of the dust acoustic wave exhibits three characteristics that differ from the van der Pol paradigm: a threshold amplitude that can be seen in the Arnold tongue diagram, a branching of the 1:1 harmonic tongue at its lower extremity, and a nonharmonic state.

Second, to assess which physical processes are important for a dust acoustic instability, I derived dispersion relations that encompass more physical processes than commonly done. I investigated how various physical processes affect a dust acoustic wave by solving these dispersion relations using parameters from a typical dust acoustic wave experiment. I find that the growth rate diminishes for large ion
currents. I also find that the compressibility, a measure of the coupling between the dust particles, have a strong effect on the wave propagation. Comparing the kinetic vs. hydrodynamic descriptions for ions, I find that under typical laboratory conditions the inverse Landau damping and the ion-neutral collisions contribute about equally to the dust acoustic instability.

Third, I performed dust acoustic wave experiments to resolve a previously unremarked discrepancy in the literature regarding the sign of the compressibility of a strongly-coupled dust component in a dusty plasma. According to theories compressibility is negative, whereas experiments suggest that it is positive. I find that the compressibility is positive. This conclusion was reached after allowing for a wide range of experimental uncertainties and model dependent systematic errors.

Finally, the polygon construction method of Glaser and Clark was used to characterize crystallization and melting in a single-layer dusty plasma. Using particle positions measured in a previous dusty plasma experiment, I identified geometrical defects, which are polygons with four or more sides. These geometrical defects are found to proliferate during melting. I also identify a possibility of latent heat involvement in melting and crystallization processes of a dusty plasma.
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CHAPTER 1
INTRODUCTION

A plasma is an ionized gas consisting of electrons, ions, and neutral gas atoms. Naturally occurring plasmas are rare on earth, however more than ninety percent of the matter in the universe exists as a plasma. Numerous examples of plasma can be found in our own solar system. The planets including our earth is bathed in a constant stream of plasma emanating from the sun called the solar wind. Terrestrially, plasmas can be found inside common household items like the florescent bulbs and plasmas are also used in many industrial applications such as production of integrated circuits and treatment of aircraft construction material.

A dusty plasma contains particle of solid matter in addition to electrons, ions, and neutral gas atoms [1, 2, 3, 4]. These particles are usually dielectric and they range in size from nanometers to micrometers. They are also the heaviest component in a dusty plasma. Dusty plasmas are found in nature, for example, in planetary rings, comet tails, interstellar clouds, and Earth’s ionosphere [5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. In laboratory experiments, that I discuss in this thesis, I prepare a dusty plasma by introducing melamine formaldehyde polymer particles, Fig. 1.1, to an argon plasma. In this thesis, I will refer the particles of solid matter in a dusty plasma as dust particles.

Dusty plasmas are of interest in astronomy, space exploration, nuclear fusion, and industry. Astronomers were among the first to study dusty plasma ever since its discovery in the rings of Saturn by the Voyager spacecraft [7]. This was soon followed by Gary Selwyn’s observation of dust particle levitation and confinement in a plasma processing reactor used for semiconductor manufacturing. The latter discovery motivated experimenters to study dusty plasmas in the laboratories.

Dust particles charge by collecting electrons and ions from the background
Figure 1.1: Melamine formaldehyde polymer particles imaged by scanning electron microscopy. I prepare a dusty plasma by introducing these particles to an argon plasma. The polymer particles do not tend to stick to one another and this is one of the advantages of using these. The particles I use for my dust acoustic wave experiments are smaller than the ones shown in the image and they have a diameter of 4.83 $\mu$m. Image courtesy of Karl-Heinz Lerche.

plasma [1, 15]. In a typical laboratory condition, this charge is usually negative because the electron thermal velocity is higher than that of the ions. A dust particle of few microns in size can have a charge of several thousands of electronic charge. In astrophysical conditions, photoionization and secondary electron emission become dominant processes and the dust particles can become positively charged.

Dust particles in a dusty plasma show collective behavior for example crystallization, melting, and waves. The high dust particle charge enables them to interact with one another as well as with the electrons and ions in the background plasma. When the dust particle interaction potential energy is larger than their kinetic energy they are said to be strongly coupled and a collection of dust particles can exhibit properties of solids or liquids [7, 16]. Thus, a dusty plasma can be used as a model system for a study of crystallization and melting. Dusty plasmas also support a variety of waves, for example, dust acoustic waves.

Glow discharge plasmas are typically used for an electrostatic confinement of dust particle in experiments [1, 2]. Glow discharge plasmas can be produced
using direct current (dc) or radio-frequency (rf) sources. I use a rf glow discharge plasma in my experiments. For an ignition of an rf glow discharge plasma, a vacuum chamber is filled with a noble gas like argon and a radio frequency high voltage is applied to a metal disc, an electrode, inside the chamber, Fig. 1.2. The rf voltage helps to accelerate electrons which in-turn ionize argon atoms by electron-impact ionization.

An ambipolar diffusion in glow discharge plasmas gives rise to dc electric fields that helps confine dust particles. A neutral gas atom ionizes creating a pair of an electron and a positive ion. The less massive electron is swept away leaving the more massive ion near its place of birth. After many such births of electron and ion pairs, a positive space charge is developed in the bulk plasma due to an accumulation of positive ions. In order to maintain a steady state, a dc electric field develops which expels the ions at the same rate as the electrons.

Figure 1.2: Photographs of outside and inside of a vacuum chamber. (a) The vacuum chamber is used for an ignition of a radio-frequency (rf) glow discharge plasma. The vacuum chamber is filled with a noble gas like argon. A 13 MHz rf high voltage is applied to an electrode inside the chamber to ionize the gas. The outside chamber wall is grounded and it is the second electrode. (b) Inside the vacuum chamber with arrows indicating the dc ambipolar electric fields in the bulk plasma. These electric fields naturally arises due to the ambipolar diffusion of electrons and ions and help confine and levitate dust particles when introduced into the plasma.
Experimenters typically use dust suspensions with a single or many layers. Under gravity, the dust particles usually sediment into few layers. By carefully controlling the number of dust particles introduced into the plasma, a single layer can be formed, Fig. 1.3(a). A single layer of dust particles is suitable for a study of two-dimensional melting. In microgravity conditions, the dust particles usually fill a volume creating a 3D dust cloud. Such 3D dust clouds can also be confined in laboratory conditions by enhancing the dc ambipolar electric fields. This can be done by introducing a glass box, Fig. 1.3(b). A 3D dust cloud is suitable for an observation of the dust acoustic wave, which is spontaneously excited by a flow of ions in a glow discharge plasma.

Figure 1.3: Images for single and multi-layer dust suspensions. (a) A single layer of dust particles. The negatively charged dust particles levitate by balancing the upward electric force, $QE$, with the downward gravitational force, $mg$. Image courtesy of Yan Feng. (b) A 3D dust cloud that fills a volume consisting of many layers. In addition to the vertical dc electric fields that help levitate the dust particles, there are also much weaker horizontal dc electric fields in a glow discharge plasma. By placing a glass box, the horizontal electric fields can be enhanced and the dust particles can be confined in a 3D cloud. Image courtesy of Tim Flanagan.

In addition to the electric and gravitational forces, Lorentz force, radiation pressure, and various drag forces act generally on dust particles [1]. Lorentz force acts on dust particles if there is a strong magnetic field. When powerful laser beams are used, they can impart forces to dust particles via momentum exchange. There
are two varieties of drag forces, ion and neutral drag, which result from the collisions of dust particles with ions and neutral gas atoms, respectively. Neutral drag usually is the source of energy dissipation in a dusty plasma and is the only force that is significant in my experiments next to electric and gravitational forces.

The dust particles are large enough that they can be observed with a naked eye or a camera and this is one of the biggest advantage of studying dusty plasmas in the laboratory. The dynamics of the particles are slow enough due to their high inertia and particle motions can be tracked by recording movies. The waves in dusty plasmas have frequencies of few tens of Hz and can be recorded with high speed cameras.

The organization of this thesis is as follows. In Ch. 2, I report results from an experiment which was designed to investigate the nonlinear phenomenon of synchronization of a dust acoustic wave. In Ch. 3, I report results from an analytical modeling task. In this task, I derived a dispersion relation for a dust acoustic wave including more physical processes than commonly done and assessed how those physical processes affect the wave propagation. In Ch. 4, I report results from an experiment that was conducted to determine the compressibility of a strongly-coupled dusty plasma. Finally in Ch. 5, I report results from a data analysis task that was performed to investigate melting in a strongly-coupled dusty plasma.
CHAPTER 2
SYNCHRONIZATION OF THE DUST ACOUSTIC WAVE

2.1 Introduction

The dust acoustic wave (DAW) is a compressional wave analogous to an ion acoustic wave in a plasma or a sound wave in air and was first predicted by Rao et al. [17]. The dust particles participate in the wave dynamics leading to a periodic modulation of their number density. Due to the large mass of the dust particles, the wave has a low frequency, which in laboratory experiments is typically 10 – 100 Hz [18]. The low frequency enables observation of this wave by video imaging [19]. DAW can appear spontaneously, without any external excitation, as has been observed in the laboratory [20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52] and under microgravity conditions [53, 54, 55, 56].

The DAW appears spontaneously because it is self-excited by an instability that is driven by streaming ions, similar to a Buneman-type instability in plasma [57, 58, 59, 60]. The wave usually propagates parallel to the ion streaming direction. Such a streaming of ions is common in glow discharge plasmas, where it is driven by an ambipolar electric field. The ion-driven instability must compete with wave damping due to frictional drag of the dust particles as they move through the neutral gas [60]. At a sufficiently low gas density, damping is so weak that it cannot overcome the instability, and the self-excited DAW can grow to large amplitudes and become nonlinear [50].

Several nonlinear phenomena for the DAW have been observed experimentally, including harmonic generation [50], shocks [44], wave breaking [43], frequency clustering [56], and synchronization [37, 45, 46, 55, 61].
The nonlinear phenomenon that I investigate in this chapter is synchronization, in which a self-excited oscillation or wave interacts with a driving force, resulting in an adjustment of the oscillation or wave frequency [62, 63]. Synchronization was observed by Huygens for two pendulum clocks that were mechanically coupled [62]. Since then, synchronization has been observed in biological, chemical, electrical, and mechanical oscillatory systems [62, 63].

Plasma physics experimenters have observed several kinds of waves and oscillations that exhibit synchronization. These include ion sound waves [64, 65, 66], ionization waves [67, 68], drift waves [69, 70, 71, 72, 73, 74, 75], ion cyclotron waves [76], plasma relaxation oscillations [77, 78, 79, 80, 81], and beam plasma oscillations [82, 83, 84].

A common mathematical model that exhibits synchronization is a periodically forced van der Pol oscillator [85],

\[ \ddot{x} - (\alpha - \beta x^2)\dot{x} + \omega_0^2 x = A_{dr}\cos(2\pi f_{dr}t) \]  

(2.1)

which describes the position \( x \) of a harmonic oscillator with a natural frequency \( \omega_0 = 2\pi f_0 \), with terms for a nonlinear damping \( \beta x^2 \dot{x} \), a source of energy for self-excitation \( \alpha \dot{x} \), and a periodic driving at a frequency \( f_{dr} \). This van der Pol oscillator can exhibit synchronization not only at \( f_{dr}/f_0 \approx 1 \) which is called harmonic synchronization, but at ratios that are rational numbers. If \( f_{dr}/f_0 > 1 \), the synchronization is said to be “superharmonic,” whereas if \( f_{dr}/f_0 < 1 \) it is “subharmonic.”

For plasma waves, the van der Pol oscillator has been used both as a quantitative model and as a qualitative reference for the characteristic properties of synchronization. As a quantitative model, Eq. (2.1) has a limited usefulness because its nonlinear term does not exactly correspond to the nonlinearities in most kinds of plasma waves. Moreover, unlike a wave equation, Eq. (2.1) is a differential equation only in time, and not also in position. For these reasons, in many cases
the van der Pol oscillator is often best suited only as a qualitative model. In other cases, where the nonlinearity happens to take the same form as in Eq. (2.1), it can be used quantitatively, as has been suggested theoretically for ion sound waves that are self-excited by ionization [64, 65, 66], ionization waves self-excited by fluctuations in electron temperature [68], and oscillations in a beam plasma system [82].

The study of synchronization that I discuss in this chapter is motivated by previous experiments of self-excited DAW, in the laboratory [37, 45, 46, 61] and in microgravity conditions [55]. Using a sinusoidal driving voltage, Trottenberg et al. [37] observed harmonic synchronization and a narrower frequency spectrum, and later Pilch et al. [45, 61] also observed subharmonic and superharmonic synchronization. Even without an external sinusoidal driving, some qualities of synchronization have been observed in microgravity experiments [55]. Synchronization played a role in earlier experiments with this wave as well; for example, in measurements of dispersion relations, the wave’s frequency was determined by an externally applied modulation [24, 39, 40, 48].

To characterize the synchronization of self-excited DAW, I carry out two experiments with a sinusoidal voltage applied to an electrode that is separate from the electrodes that sustain the plasma. I vary the driving frequency and driving amplitude, allowing a determination of the conditions that result in synchronization and to identify the mechanism for synchronization. I also observe a nonsynchronized oscillation, which I term as a nonharmonic state, at a frequency that is well below both the natural oscillation frequency and the driving frequency.

2.2 Experimental design

I performed two experiments under nearly identical conditions. I used the first experiment to plot an Arnold tongue diagram which shows where synchronization occurs in a parameter space of driving amplitude and driving frequency. I used the
second experiment to identify the synchronization mechanism for the DAW. The only difference between these two experiments is the method used for scanning the driving frequency and driving amplitudes as discussed in Sec. 2.2.3.

2.2.1 Dusty plasma preparation

I prepared a plasma that has a dc electric field that can levitate dust particles against the downward force of gravity in a plasma sheath. The plasma was formed in a vacuum chamber, Fig. 2.1, with argon gas at 120 mTorr pressure. A low power radio-frequency (rf) voltage, with a 57 V peak-to-peak amplitude was applied between two electrodes, one which was a horizontal lower electrode and the other which was the grounded wall of the vacuum chamber. This rf voltage produced an rf electric field that sustained the plasma by accelerating electrons that can partially ionize the gas. The 13.56 MHz frequency is so high that only electrons respond to it. In addition to the rf electric fields, there is also a dc electric field that arises naturally due to ambipolar transport of electrons and ions. In the sheath, immediately above the lower electrode, this dc electric field is mostly downward with a smaller horizontal component.

I used monodisperse melamine formaldehyde microspheres of diameter 4.83 μm as the dust particles and they were introduced into the plasma with a dust dispenser that is similar to a salt shaker with a single hole. Once they were immersed in the plasma, the dust particles acquired a negative charge. In the presence of the downward dc electric field, they experienced an upward electric force that balanced the downward force of gravity, so that the dust particles were levitated, Fig. 2.2. To confine the dust particles in the horizontal direction, I enhanced the horizontal component of the dc electric field by placing a glass box on the lower electrode. Oliver Arp introduced the use of a glass box to confine dust particles in a three dimensional cloud and this method was later used by Tim Flanagan in his DAW.
Figure 2.1: Schematic of the vacuum chamber, shown without flanges (a) and its electrical and mechanical configuration (b). The high-speed camera was used for imaging the dust cloud. The lower electrode was inserted into the chamber during experiments. The 13 MHz rf oscillator was used to generate the plasma. By shaking a dispenser, I introduced dust particles so that they fall through the plasma. They collected a negative charge and became levitated and confined in a 3D cloud inside a glass box resting on the lower electrode. The plasma ion density was modulated by a ring-shaped driving electrode powered by a sinusoidal voltage, with an amplitude $A_{dr}$ and a frequency $f_{dr}$. A Langmuir probe at a location outside the dust cloud, as marked $\ast$ was used to measure the ion density fluctuation due to driving. This probe was removed prior to adding dust. Note that the rf voltage and the driving voltage values are specified in peak-to-peak volts ($V_{pp}$).

The box had an open top, and its dimensions are $3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$. A three-dimensional dust cloud, with a size of about 1 cm, was confined in the center of this box.

The electric field that provides dust levitation also drives an ion beam which is the energy source for the excitation of the DAW. The wave propagates in the direction of the ion flow. The wave also grows in amplitude as it propagates because the energy gain from the ion-driven instability exceeds the dissipation due to
frictional drag on the neutral gas atoms. I chose a gas pressure of 120 mTorr so that the frictional drag would be small enough to allow the wave to grow to nonlinear amplitudes after traversing only one-sixth of the cloud’s height.

The natural frequency of the DAW, i.e., the frequency in the absence of any periodic driving, was $f_0 = 22$ and 21 Hz in the first and second experiments, respectively. I determined these frequencies not only by viewing the video, but also using the spectral analysis methods that I describe in Sec. 2.3.1.

2.2.2 Illumination and image capture

I used a vertical sheet of 577-nm laser light for illuminating the dust cloud. The dust cloud was imaged from the side with a digital camera, Fig. 2.1(a). I used an illumination power of 0.9 W which was low enough that it did not visibly perturb the dust cloud.

A digital camera was used for recording images at a rate of 256 frames per second, which I chose to provide an adequate temporal resolution for the wave and its
harmonics. The camera, with a 105-mm focal length lens and a doubler, provided a spatial resolution of 78 pixels/mm. At this resolution, a typical wavelength of 2.5 mm was resolved with about 200 pixels. The camera’s field of view included the entire cross-section of the dust cloud that was illuminated, as shown in Fig. 2.2(b). In the image, a single particle spreads over about 25 pixels.

2.2.3 External driving

The self-excited DAW was synchronized by applying an external sinusoidal disturbance at a chosen frequency $f_{dr}$ and amplitude $A_{dr}$. To do this, I disturbed the overall ion density in the plasma by applying sinusoidal voltages to a negatively biased electrode located 2 cm above the top of the glass box, Fig. 2.1(b). This “driving electrode” was a flat copper ring with a surface area of 15 cm$^2$.

The external driving modulates the overall ion density in the chamber. I verified in a test that the sinusoidal ion density modulation varies linearly with the amplitude of the sinusoidal driving voltage at the same frequency. This linearity, shown in Fig. 2.3, extends over the entire range of driving amplitudes used in the experiments, including the lowest driving amplitudes, so that I am confident that the wave observations I will interpret as signatures of synchronization are not instead artifacts of a nonlinear response of the plasma to the driving. This linearity test was performed using lock-in-amplifier detection of current in a Langmuir probe biased for ion saturation.

Because harmonics can play an important role in nonlinear synchronization, I verified that there is no detectable harmonic content in the ion density modulation. This test, which was performed using a Langmuir probe, gave me confidence in my observations of harmonic content in the DAW. These observations, that I present in Sec. 2.3.1, are not merely due to any harmonics present outside the dust cloud, but instead due to the more interesting physics of nonlinearity in the dust cloud.
The ion density modulation has the effect of modulating the Debye length and therefore the thickness of the sheath above the lower electrode [88]. Since the dust cloud was levitated by the dc electric field in this sheath, the small modulation of the ion density leads to a vertical shaking of the entire dust cloud, with a maximum displacement of \( \approx 1 \text{ mm} \), as measured at \( f_{dr} \ll 20 \text{ Hz} \) and \( A_{dr} = 46 \text{ V}_{pp} \). Thus, the external modulation can affect the dust cloud in more than one way: a modulated ion density, a modulated dc electric field, and a resulting vertical shaking. I cannot identify which of these is responsible for the synchronization that I observed.

I varied the driving frequency from 2 to 55 Hz. The upper limit of this range was chosen because I found in a test no signatures of synchronization at higher frequencies. The two experiments differ in the procedures used for scanning the frequency \( f_{dr} \) and amplitude \( A_{dr} \). In the first experiment, I slowly swept the frequency, while in the second I held the frequency constant. In both experiments the driving amplitude was held steady during the recording of a movie, and movies were repeated for various amplitudes starting with 2 \( \text{ V}_{pp} \).

In the first experiment I swept the frequency which allowed measurement of wave conditions over many values of the driving parameters, \( A_{dr} \) and \( f_{dr} \). These two parameters will be the axes of the Arnold tongue diagram that I discuss in Sec. 2.3.3, so that it is necessary to record wave data for many values of these two parameters in order to populate the diagram. Those data must be recorded for many values \( A_{dr} \) and \( f_{dr} \) rapidly enough so that the number of particles within the dust cloud does not vary significantly. If I operated over a longer time, I would encounter the difficulty that the number of dust particles would diminish with time, especially when large-amplitude driving was applied. These requirements led me to use the method of Brandt [89]: sweeping the frequency \( f_{dr} \) at a constant amplitude, for a succession of amplitudes. I recorded data for the highest amplitudes at the
end, so that the low-amplitude data are not affected by a loss of dust particles. The frequency was swept at a steady rate, from 2 to 55 Hz, over 145 s. One movie was recorded during this sweep, with its first frame triggered at the start of the sweep.

In the second experiment I held the frequency constant, so that the system was in a steady state during the recording of a time series. I rely on this second experiment for two purposes: to verify the validity of the results from the first experiment, and to identify the synchronization mechanism. For the mechanism identification, I used the method of Balanov [63], who held the frequency \( f_{dr} \) constant and then repeated for a series of driving amplitudes \( A_{dr} \). Since this method of holding the frequency constant does not allow me to make observations over a wide a range of parameters as does the sweeping method in the first experiment, in this second experiment I recorded data primarily for lower values of driving amplitudes, as is needed to identify the synchronization mechanism.

2.2.4 Analysis

Here the analysis centers on determining the dust number density \( n_d \). I can characterize \( n_d \) using images of the dust cloud because the camera’s sensor has a linear response to incident light, and very little scattered laser light is absorbed within the dust cloud. To obtain the spectra of the fluctuations of \( n_d \), I Fourier transformed the time series of the brightness. In particular, I used the brightness averaged over a thin rectangular sample region in the image, indicated by a box in Fig. 2.2(b). This sample region was chosen to be only 0.25 mm in the \( z \) direction, i.e., about one tenth of a wavelength in the direction of wave propagation, but a much wider 2.5 mm range in the \( x \) direction. Averaging over such a wide range of \( x \) is possible because the wavefronts were nearly planar in the center of the dust cloud.

The wave spectra of dust number density fluctuations were used in two ways.
Figure 2.3: Test of linearity in the response of the plasma to the driving. I measured the ion saturation current drawn by a negatively-biased Langmuir probe, as a proxy for the ion density, at a location shown in Fig. 1. While applying a sinusoidal waveform at $f_{dr} = 20$ Hz to the driving electrode, I measured the fluctuation in ion density, at that same frequency, using a lock-in amplifier. The inset is a magnification for the lowest driving amplitudes. This test demonstrates a linear plasma response to the driving over the entire range of driving amplitudes used in the experiments.

First, I determined the wave conditions over many values of the driving parameters, $A_{dr}$ and $f_{dr}$. These two parameters will be the axes of the Arnold tongue diagram. Second, I visually inspected the spectra for various driving amplitudes. I compared the features of the spectra to the van der Pol paradigm to find the synchronization mechanism of the DAW.
2.3 Results and discussion

2.3.1 Time series and spectra of $n_d$

Examples of the time series of dust number density $n_d$ are shown in Fig. 2.4(a) and (b), for two different driving frequencies $f_{dr}$, but the same large driving amplitude $A_{dr} = 46 \text{ V}_{pp}$ as obtained from the first experiment. In Fig. 2.4(a) and (b) the fluctuation of dust number density is large, with a peak-to-peak fluctuation of about 50%, as compared to the time-averaged dust number density.

I calculated a spectrum of dust number density fluctuations from an interval within the time series. This is done by subtracting the time average and using a fast Fourier transform (FFT). I denote the frequency variable for this FFT, i.e., the spectral frequency, as $f_{sp}$, to avoid confusion with the driving frequency, $f_{dr}$. The time interval for computing an FFT has a duration of 2 s, consisting of 512 frames. This duration was chosen to be short enough so that $f_{dr}$ can be considered as a constant during that interval, but long enough so that the corresponding frequency resolution of 0.5 Hz allows to distinguish the various synchronized states. Example spectra are shown in Fig. 2.4(c) and (d), computed for the time series data when the slowly swept $f_{dr}$ was about 20 Hz and 40 Hz, respectively.

I combined spectra for various driving frequencies, yielding Fig. 2.5(a). Such a graph of power as a function of the two frequencies, $f_{dr}$ and $f_{sp}$, is useful for identifying synchronized states. The data in Fig. 2.5, as in Fig. 2.4, are for the highest driving amplitude, $A_{dr} = 46 \text{ V}_{pp}$.

2.3.2 Features in the spectra

The spectra in Fig. 2.5(a) have a richness of detail, including three kinds of strong features that I will discuss, as well as some weaker features that I will not attempt to explain. In other words, I will only remark upon the features that have
a good signal to noise ratio. The strong features are labeled in Fig. 2.5(b) according to my interpretation: the natural oscillation at \( f_0 \), synchronized states, and a non-harmonic state. I will discuss the natural oscillations and the synchronized states next. The nonharmonic state is a feature that is neither at \( f_0 \) nor has signatures of synchronized states, and I discuss it in Sec. 2.3.5

2.3.2.1 Natural frequency

The natural frequency \( f_0 \) can be identified as the dominant frequency in the spectrum *in the absence of driving*, i.e., at \( A_{dr} = 0 \text{ V}_{pp} \). Using data recorded without any driving, I was able to determine \( f_0 = 22 \text{ Hz} \). This same natural frequency can also be present without, and even with, high-amplitude driving, provided that the driving frequency is either very low or high, as marked in Fig. 2.5(b).

2.3.2.2 Synchronized states

At least four synchronized states are seen in Fig. 2.5. These appear as straight lines with an upward slope. The various synchronized states are distinguished by their slopes in Fig. 2.5. For example, the line marked 1:2 corresponds to oscillations at a frequency \( f_{sp} \) that is one half of the driving frequency, \( f_{dr} \). This feature can be seen in the upper portion of Fig. 2.5. The 3:1, 2:1, 1:1, and 1:2 synchronized states are easily seen in Fig. 2.5. Synchronized states of 4:1 or higher may also be present, but their signatures at the bottom of Fig. 2.5 are too weak for me to identify them conclusively.

Now I discuss the various synchronized states that I identify in Fig. 2.5. The synchronized state labeled 1:1 is termed the *harmonic* synchronized state. In this state, the external driving frequency \( f_{dr} \) is near, but not necessarily the same, as the natural frequency \( f_0 \), and the wave oscillates at the driving frequency \( f_{dr} \). The synchronized states at an integer multiple of the driving frequency, labeled 2:1
and 3:1, are termed subharmonic synchronized states. In these states, the wave oscillates at a harmonic of the driving frequency. Finally, in the state 1:2, which is a superharmonic synchronized state, the wave oscillates at one half the frequency of the external driving.

All of these synchronized states are visible in Fig. 2.5 as a main spectral peak as well as peaks at integer multiples, i.e., at harmonics of the main peak. I have marked the second harmonics in Fig. 2.5(b) as dashed lines. Weaker features for third and higher harmonics can also be detected in Fig. 2.5(a). All these harmonics are present because the waves are non-sinusoidal, due to nonlinearities when the wave amplitudes are large [50]. I consider the peaks at the second and higher-order harmonics as essentially part of the same synchronized state as the main peak, even though they can be distinguished in Fig. 2.5. For example, I consider the solid and dashed lines labeled 2:1 as both belonging to the same feature, which in this case is a subharmonic synchronized state.

Usually there is only one main peak for a given driving frequency, $f_{dr}$. The reader can confirm this by drawing a horizontal line across Fig. 2.5(a) and noting that this horizontal line intercepts only one peak with significant power, along with harmonics of that peak. For example at $f_{dr} = 20$ Hz, almost all the wave power is concentrated at $f_{sp} = 20$ Hz, i.e., the 1:1 synchronized state.

2.3.3 Arnold tongues

2.3.3.1 Calculation method

I will next explore how the synchronization depends on the driving amplitude. This effort includes preparing a plot, known as an Arnold tongue diagram, which is a traditional characterization of synchronization. I will use data for the power as a function of frequency $f_{sp}$ and driving frequency $f_{dr}$, as shown in Fig. 2.5(a).
Figure 2.4: Brightness time series and spectra. Representative time series (a) and (b) of brightness, which is a measure of the dust number density $n_d$. The corresponding power spectra (c) and (d), calculated by fast Fourier transforming the fluctuations of $n_d$. The 2 s time interval used for the FFT is short enough that the slowly swept driving frequency $f_{dr}$ can be considered to be constant. Two different synchronized states are shown: in (c) the observed wave at 20 Hz is synchronized to the 20 Hz driving frequency and a small harmonic is also present, while in (d) the wave at 20 Hz is synchronized to one half the 40 Hz driving frequency. These data from the first experiment are for $A_{dr} = 46$ V$_{pp}$. 
Figure 2.5: Power spectra results. (a) Power spectra, as in Fig. 2.4, plotted with color representing power, for various driving frequencies. Data shown are from my first experiment for the highest driving amplitude, $A_{dr} = 46 \ V_{pp}$. Synchronized states, when they are present, are revealed by a line that has a constant slope, which is later referred to as the winding number. (b) Guide for interpreting the spectra in (a). At extremely low and high driving frequencies, oscillations are observed at the same natural frequency $f_0$ as without any driving. Features of interest include: subharmonic 3:1 and 2:1 synchronization for $5 < f_{dr} < 12$ Hz, harmonic 1:1 synchronization for $12 < f_{dr} < 25$ Hz, and superharmonic 1:2 synchronization for $35 < f_{dr} < 47$ Hz. The strongest spectral features are usually accompanied by their second harmonic, which is indicated by dashed lines in (b). Additionally I observe a state marked $f_\ast$, termed the nonharmonic state in the text; it is neither a synchronized state nor an oscillation at the natural frequency, $f_0$. The presence of a lower sideband at a frequency $f_{ls} \equiv f_{dr} - f_\ast$, marked by a dotted line, and the presence of a second harmonic $2f_\ast$ indicate that this state is nonlinear. There are additional features (with low signal-to-noise ratio), beyond those that I choose to label, that may represent additional states that are physically significant.
While the spectrum in Fig. 2.5(a) is prepared for only one driving amplitude, I also prepared similar spectra over a range of amplitudes. Altogether, these spectra represent power $P$ recorded in a three dimensional parameter space, with 120 values of spectral frequency $f_{sp}$, 105 values of driving frequency $f_{dr}$, and 29 values of driving amplitude $A_{dr}$.

An Arnold tongue is an indication, in the two-dimensional parameter space of driving amplitude vs. driving frequency, of whether synchronization occurs. In my case, the parameter space for the Arnold tongue consists of 105 values of $f_{dr}$ multiplied by 29 values of $A_{dr}$, for a total of 3045 elements. For each of these elements, I must make a binary decision whether the synchronization occurs, and if it does occur I darken that element in the parameter space. In this manner I drew an Arnold tongue.

One step in this process of drawing an Arnold tongue is often not described by practitioners: the manner of making the binary decision of whether synchronization occurs, for a given element of parameter space. I chose a specific procedure, with a quantifiable criterion for making the decision, which I now describe. First, using spectra data as in Fig. 2.5(a), I plotted Devil’s staircases, Fig. 2.6. The vertical axis in a Devil’s staircase is a so-called winding number [63], which is the ratio of $f_{dr}$ to the peak frequency, while the horizontal axis is the driving frequency, $f_{dr}$. To understand what I mean by the peak frequency, consider a horizontal profile of Fig. 2.5(a); this will be a line graph similar to the spectrum shown in Fig. 2.4(c). This spectrum has a distinctive peak, and it is the frequency of this peak that is used in calculating the winding number. To gain greater precision in determining the peak frequency, instead of merely selecting the spectral frequency with the highest power, the peak frequency is calculated [63] as a weighted mean $\bar{f}$,

$$\bar{f} = \frac{\sum f_{sp}P}{\sum P}$$  \hspace{1cm} (2.2)
Here, $P$ is the spectral power as a function of $f_{sp}$, as in Fig. 2.4(c) and (d). In using Eq. (2.2), I used values of the spectral frequency in the range $0.5 \leq f_{sp} \leq 30$ Hz. The higher limit of this frequency range is selected in order to minimize the contribution to the above summation from the second harmonic component of the wave. By inspecting Fig. 2.5 the reader can note that the second harmonic of the wave lies above $f_{sp} = 30$ Hz for the range of driving frequencies used.

I inspected the resulting Devil’s staircase for a specific value of the driving amplitude, as in Fig. 2.6, and identified flat spots as a signature of synchronization. A quantifiable criterion was needed to decide whether a data point in the Devil’s staircase is part of a flat spot. I made this decision by requiring that the winding number for the data point must be within 2% of the value of the data point to its left as well as within 2% of the data point to its right.

I generated a graph of the Arnold tongues by repeating this decision for all 105 values of $f_{dr}$ and all 29 values of $A_{dr}$, for a total of 3045 elements of the graph. Darkening an element when it was determined that it belonged to a flat spot in the Devil’s staircase, I obtained the desired graph of Arnold tongues, Fig. 2.7. I used this method of generating Arnold tongues because the parameter space for a tongue is marked using only quantitative criteria, without any subjective interpretation to draw a tongue’s boundary.

### 2.3.3.2 Discussion of Arnold tongues

Here for the first time in the study of synchronization of the DAW, I am able to show how it varies with both the driving amplitude and the driving frequency. Now I discuss and interpret the interesting features that can be observed in the Arnold tongue diagram, Fig. 2.7.
Figure 2.6: Devil’s staircases, for measuring the range of driving frequencies for the synchronized states. (a) Data shown are from my first experiment for $A_{dr} = 46 \text{ V}_{pp}$. This staircase plot was prepared using spectra as in Fig. 2.4 to calculate the frequency $\bar{f}$ of the strongest peak, as in Eq. (2.2). The four flat spots indicated here correspond to harmonic 1:1, superharmonic 1:2, and subharmonic 3:1 and 2:1 synchronized states. Using a Devil’s staircase like this one, I can measure the range of driving frequencies for each synchronized state; for example the 1:1 synchronized state at this driving amplitude occurs for $14.5 \leq f_{dr} \leq 24 \text{ Hz}$. (b) Data shown for $A_{dr} = 9 \text{ V}_{pp}$. The two separate flat spots marked 1:1 indicate a branching of the 1:1 synchronized state, which will be shown in greater detail in Fig. 2.7.
Figure 2.7: Arnold tongue diagram, i.e., indications of the driving conditions that result in synchronized states. Here I can identify four tongues for the harmonic 1:1, superharmonic 1:2, and subharmonic 3:1 and 2:1 synchronized states. To prepare this diagram, I marked the elements in the parameter space that correspond to a flat spot in the Devil’s staircase in Fig. 2.6. Unlike Arnold tongues in the van der Pol paradigm, here the tongues do not narrow to a point at $A_{dr} = 0$ V_{pp}. Instead, I find a threshold driving amplitude for exciting synchronized states. Data shown are for the first experiment.
Threshold

The Arnold tongues in Fig. 2.7 reveal a threshold for the driving amplitude required for synchronization. Unlike the Arnold tongues for the van der Pol oscillator, mine do not have a sharp tip at zero driving amplitude, but instead vanish at a small but finite amplitude. For example, there is no synchronization observed for $A_{dr}$ below a threshold of 6 V$_{pp}$ for the 1:2 superharmonic synchronized state.

In considering the origin of this threshold, I must determine whether it is the result of nonlinearities in the dust cloud that I wish to study, or whether it is instead an uninteresting consequence of the coupling of the driving electrode to the dust-free plasma located between the electrode and the dust. I can dismiss the latter possibility by examining Fig. 2.3, which demonstrates a linear response of the ion density to the driving amplitude. This response exhibits no threshold or other peculiarities, even at the lowest driving amplitudes. Thus, I assume that the origin of the threshold must lie somewhere within the dust cloud.

Branching

The 1:1 Arnold tongue also has a distinctive branching at low forcing amplitudes. These branches can be seen in Fig. 2.7. The two branches are at spectral frequencies of about 16 and 22 Hz.

I consider two possible explanations for this branching feature: the branches are either all part of the same 1:1 synchronized state or they indicate a merging of two different synchronized states. To test these explanations requires a way of distinguishing synchronized states, and for this purpose the Devil’s staircase is a better tool than the Arnold tongue diagram, which is prepared from the Devil’s staircase. By examining the rich detail in the Devil’s staircase in Fig. 2.6(b), I find a signature of 1:1 synchronization feature with two separate flat spots at the same $A_{dr}$ as the two branches in the Arnold tongue diagram. This indicates that both
branches are in fact part of the same 1:1 synchronized state. A feature in Arnold
tongue diagrams similar to branching feature that I observe can be seen in the results
of van der Pol simulations, which is not all belonging to the 1:1 synchronized state
but instead a merging of two distinct nearby states such as 3:2 and 1:1 [90]. I can
exclude the latter possibility by noting that the Devil’s staircase lacks any signature
of 3:2 synchronization, which would appear at a winding number of 0.6 if it were
present in my experiment. Thus, I conclude that the branches I observed are all
part of the same 1:1 synchronized state.

Similar branching of an Arnold tongue can be seen in previously reported
experimental data for other physical systems, including the 1:1 synchronization of
ionization waves in a non-dusty-plasma experiment [68] and the 1:1 synchronization
of ruby laser output [91]. I use the term “branching” and not “splitting” as in
[91] because only the tip of the 1:1 Arnold tongue is divided, not the entire tongue
as in some of the results of [91]. Although this 1:1 branching is visible in these
previously reported Arnold tongues [68, 91] I have not found any explanation of
this phenomenon in the literature.

2.3.4 Phaselocking vs. suppression mechanisms

I will next determine which of the two synchronization mechanisms, in the
context of the van der Pol paradigm, is responsible for the synchronization that
I have observed. I will use the terms “phaselocking” and “suppression” to distin-
guish these two mechanisms, as defined by Balanov et al. [63]. To avoid confusion,
I should mention that “phaselocking” and “suppression” also have other meanings
in the synchronization literature. Phaselocking is used by some authors as a syn-
onym for synchronization itself, regardless of its underlying mechanism [81]. The
term “suppression” is used sometimes as a synonym for “oscillation death” in the
literature for the nonlinear dynamics of biological systems [62].
Figure 2.8: Testing for signatures of suppression vs. phase locking mechanism, for 1:1 synchronization. I examine these spectra to identify how the peak for the natural frequency $f_0$ behaves with respect to that of the driving, $f_{dr}$. (a) At the lowest driving amplitude, there is only a peak at the natural frequency $f_0$. (b)-(e) At a slightly higher driving amplitude, nonlinear coupling occurs, as indicated by the presence of sidebands, although the amplitude is not sufficient for synchronization. A transition to synchronization develops as the driving amplitude is increased (b)-(e), until the oscillation is fully synchronized with the driving in (f). In the transition to synchronization, I observe two indications of suppression: the two peaks do not merge, and the peak at $f_0$ diminishes in height. Similar spectra are shown in the right column (g)-(l) and they exhibit the same signatures. These data are for the second experiment.
To distinguish suppression and phaselocking, I follow the prescription in Sec. 3.9 of Balanov et al. [63]. They provided a comprehensive review of the fundamental theories and they presented a discussion of the signatures of suppression and phaselocking that I will use. These prescriptions were developed for use with the van der Pol oscillator, and I will use them even though that oscillator is essentially a single point and not an extended system that sustains a propagating wave. Previous investigators of synchronization of plasma waves [67, 83] relied upon the van der Pol paradigm for other purposes; here I rely upon it for distinguishing the two mechanisms that are possible for the van der Pol oscillator, phaselocking and suppression. In the van der Pol paradigm, suppression occurs over a wide range of driving amplitudes except for the lowest amplitudes, where phaselocking can occur [63].

I now summarize the prescription of Balanov et al. [63] for determining whether the suppression and phaselocking mechanisms are present. The data used are power spectra. In particular, one examines two peaks in the spectra, at \( f_{dr} \) and \( f_0 \). A signature of phaselocking is a merging of the two peaks as the driving amplitude increases, while a signature of suppression is that the two peaks remain separate [63]. Another signature of the suppression is a significant reduction of the height of the peak at \( f_0 \), as the driving amplitude is increased. It is particularly useful to inspect the power spectra at lower driving amplitudes, corresponding to the lower portion of an Arnold tongue, because it is for these conditions that phaselocking, if it occurs at all, should be identifiable.

I next examine my spectra for the signatures of suppression and phaselocking. The spectral data I use for this purpose, Fig. 2.8, are from my second experiment. In particular, the conditions I consider are low driving amplitudes, \( A_{dr} \leq 12 \text{ V}_{pp} \), at \( f_{dr} = 16 \) and 18 Hz, which are the same conditions as at the lowest extremity of
the 1:1 Arnold tongue. I have marked the peaks at $f_{dr}$ and $f_0$ with heavy arrows at the top of each panel of Fig. 2.8. I will give close attention to the separation between these peaks, as indicated by the separation between the heavy arrows.

I can exclude the possibility of the phaselocking mechanism by observing that the peaks at $f_{dr}$ and $f_0$ never merge. Examining a column in Fig. 2.8, for example the left column, I see that the separation between the peaks remains nearly constant, and does not merge, as the driving amplitude increases from the top panel to the bottom. Thus, the signature of phaselocking (a merging of the two peaks) is absent. This result is the same for both cases, for $f_{dr} = 16$ and 18 Hz.

I observe the signature for the suppression mechanism, as indicated not only by the absence of merging, but also a significant reduction in the height of the peak at $f_0$ as the driving amplitude increases. This reduction can be seen by scanning the eye downward in the left column of Fig. 2.8, focusing on the peak at $f_0$. For example, the power is reduced by about a factor of three between panels (b) and (d), and by about a factor of five between panels (d) and (e). This result for the driving frequency of $f_{dr} = 16$ Hz in the left column is also confirmed in the right column for $f_{dr} = 18$ Hz. Thus, as one of my chief results, I find that the 1:1 synchronization occurs through the mechanism of suppression and not phaselocking.

Additionally, I note that the spectra in Fig. 2.8 exhibit sidebands. These sidebands are peaks at frequencies that are linear combinations of $f_{dr}$ and $f_0$ and their harmonics, for example $f_{dr} + f_0$, $2f_0 - f_{dr}$, and $3f_0 - 2f_{dr}$, as marked by thin arrows in Fig. 2.8. In general, the presence of sidebands is interpreted as an indication that nonlinear coupling between the waves and the external driving is present, but too weak to result in synchronization [63].
2.3.5 Nonharmonic state

In the power spectra of Fig. 2.5, I noted three kinds of strong features, which I interpret as: the natural oscillations at $f_0$, the synchronized states and a nonharmonic state that is neither synchronized nor at $f_0$. I now discuss this nonharmonic state, which is marked $f_*$ in Fig. 2.5.

The feature $f_*$ in Fig. 2.5 appears as almost a vertical line, meaning that the spectral frequency varies only a little while the driving frequency is varied over a wider range. In Fig. 2.5, the spectral frequency for this feature lies in the narrow range $15 < f_{sp} < 17$ Hz while the driving frequency has a wider range, $25 < f_{dr} < 35$ Hz. I have verified that this feature is present in this same spectral frequency range for all driving amplitudes $> 2$ $V_{pp}$ that I tested.

I term this feature as a “nonharmonic state” since it appears to be different from both the natural oscillation at $f_0$ and the synchronized states. The reason it appears not to be associated with the natural oscillation at $f_0$ is that its spectral frequency is quite different, with $f_*$ well below $f_0$. The reasons that I consider it not to be a synchronized state is that its frequencies do not match a subharmonic or superharmonic of the driving frequency $f_{dr}$, and it appears almost as a vertical line in the Fig. 2.5. This vertical line is unlike the synchronization features, which have constant slopes and pass through the origin.

I also find indications of nonlinearities for this nonharmonic state. These indications of nonlinearity are a harmonic at $2f_*$, as indicated by a dashed line in Fig. 2.5(b), and a lower sideband frequency, $f_{ls} \equiv f_{dr} - f_*$, as indicated by a dotted line in Fig. 2.5(b). The presence of this sideband indicates a nonlinear coupling of the external driving and the DAW. Thus, while I am unable to fully explain this feature, I can conclude that it is a nonlinear oscillation that is different from both the natural oscillation at $f_0$ and the synchronized states. This nonharmonic state
seems to occur above a threshold driving amplitude, since I do not observe it at the lowest driving amplitude of 2 $V_{pp}$.

### 2.4 Conclusions

I have characterized synchronization of the self-excited dust acoustic wave in a dust cloud in a laboratory plasma. In the absence of driving, the wave propagates at a natural frequency $f_0$. To provide driving, I applied a sinusoidal voltage with an adjustable driving frequency $f_{dr}$ and driving amplitude $A_{dr}$ to an electrode located above the dust cloud, causing the ion density throughout the plasma to be modulated sinusoidally at $f_{dr}$. As in the experiment of [50], the wave grows in amplitude as it propagates downward, attaining nonlinear amplitudes. I determined spectra for fluctuations in the brightness in video images, since the brightness is proportional to the dust number density. I examined how these spectra depend on the driving amplitude and frequency.

I find at least four distinct synchronized states 3:1, 2:1, 1:1, and 1:2. In the harmonic synchronized state, 1:1, the wave oscillates at the external driving frequency. For the subharmonic synchronized states, 3:1 and 2:1, the wave oscillates at a harmonic of $f_{dr}$. I detect only one superharmonic synchronized state, 1:2, in which the wave oscillations are at one-half the driving frequency.

Examining the spectra for the common signatures in the van der Pol paradigm, I find that synchronization of the wave has the signatures of the suppression mechanism, but not the signature of the phaselocking mechanism.

The synchronization I observe differs in at least two additional ways from the van der Pol paradigm. First, there is a threshold that the driving amplitude must exceed for synchronization to occur, as can be seen in the Arnold tongue diagram, Fig. 2.7. Second, for 1:1 synchronization, the Arnold tongue does not have a single pointed tip, but instead has a branched tip.
I find that sidebands appear in the spectra at frequencies that are the sum or difference of $f_0$ and $f_{dr}$ or their harmonics. These sidebands are indications of nonlinear coupling. These are seen for driving conditions $f_{dr}$ and $A_{dr}$ that are slightly outside the Arnold tongues, i.e., for conditions that do not quite allow synchronization. These sidebands vanish for the synchronized states, as also occurs in the van der Pol paradigm.

I observe a feature in the spectra that I term a “nonharmonic state,” which appears to be a nonlinear oscillation. This feature appears in Fig. 2.5 at a spectral frequency between 15 to 17 Hz, which is neither the natural frequency nor a sub-harmonic or superharmonic of $f_{dr}$, as would be expected for a synchronized state. This nonharmonic state is not a familiar feature of the van der Pol paradigm.
CHAPTER 3
DISPERSION RELATION OF THE DUST ACOUSTIC WAVE
UNDER EXPERIMENTAL CONDITIONS

3.1 Introduction

A variety of physical processes can affect the dust acoustic wave (DAW). Inertia for the DAW is provided by the heaviest species, the dust. The restoring force for the DAW, on the other hand, is mainly the electric force arising from charge separation of all three charged species, dust, electrons, and ions, as they are compressed and rarefacted. In this wave, the dust particles behave differently from electrons and ions in several ways: they have a much larger inertia, a large cross section for collisions with gas atoms, a charge that fluctuates [92], and a charge that is large enough to have strong-coupling effects [93, 94, 95].

The first goal of this chapter is to assess which physical processes are important for a DAW under experimental conditions. I will rank these processes based on their importance. The second goal is to weigh the comparative advantages of a kinetic vs. hydrodynamic description of ions. This is needed because most authors use only one of these descriptions, without justifying the choice based on an assessment of its accuracy. I provide that assessment in this chapter. The third goal is to quantify how much the dispersion relation is affected by ion collection current onto dust particles. The final goal is to formulate a way to incorporate strong-coupling effects using a compressibility parameter for dust, as a measure of the equation of state, and to determine how the sign and magnitude of this parameter affect the dispersion relation.

To meet these goals, I include more physical processes than is commonly done when deriving the dispersion relation of the DAW. The processes included are:

- charge separation
• dust inertia
• ion drift
• ion-neutral collisions
• dust-neutral collisions
• dust compressibility
• finite temperature effects for electron and ions
• ion collection (depletion) onto dust
• dust-charge fluctuations
• inverse Landau damping for ions.

I devise a method of testing and ranking the importance of these processes. I calculate exponents for the percentage variations of the wave frequency $\omega_r$ and growth rate $\omega_i$ with respect to a parameter that quantifies these processes; a larger exponent indicates a greater effect of this process on the dispersion relation.

I review the previous DAW literature, emphasizing the methodology of derivations and the expressions for susceptibilities in Secs. 3.1.1 and 3.2, respectively. I derive new susceptibilities for ions, dust, and the dust charge fluctuation in Sec. 3.2, that incorporate more physical processes. I use these expressions in Sec. 3.3 to derive three new dispersion relations. In Sec. 3.4, I present results corresponding to my four goals: ranking the various physical processes according to how much they affect the dispersion relation, assessing the advantage of a kinetic vs. hydrodynamic description of ions, quantifying the change in the dispersion relation due to ion collection by the dust particles, and describing how the sign and magnitude of the compressibility affect the dispersion relation. For the calculations in Sec. 3.3 and Sec. 3.4, I assume typical experimental conditions, in particular, the parameters of the DAW experiment of Flanagan and Goree [49] (denoted henceforth as FG).
3.1.1 Approaches for including physical processes in a dispersion relation

Two approaches for deriving dispersion relations in a dusty plasma are the lattice wave and hydrodynamic approaches. In the lattice wave approach, all the physics of electrons and ions are incorporated in a screening length for the interaction of point-like dust particles, and the equation of motion of the dust particles is solved to determine the mode frequencies \cite{96, 97, 98, 99}. This lattice wave approach is useful mainly for strongly-coupled dusty plasmas in a crystalline state. It predicts longitudinal and transverse waves, but neither of these waves is the same as a DAW; a DAW involves charge separation among the charged species, and the reduced treatment of electrons and ions in the lattice approach does not allow an accounting for this charge separation.

In the hydrodynamic approach, which I use, the fluctuating densities are treated separately for three components: electrons, ions, and dust. The linearized fluctuating densities \( \tilde{n}_j \) for each species \( j \) are related to the linearized wave potential fluctuations \( \tilde{\phi} \) by the susceptibility

\[
\chi_j = -\frac{Q_j \tilde{n}_j}{\varepsilon_0 k^2 \tilde{\phi}}. \tag{3.1}
\]

Since the charge on a dust particle can fluctuate, \cite{1, 92} one can define a susceptibility \( \chi_{qd} \), which relates the linearized fluctuating dust charge \( \tilde{Q}_d \) to the linearized wave potential fluctuation \( \tilde{\phi} \). This susceptibility can be written as

\[
\chi_{qd} = -\frac{\tilde{Q}_d \tilde{n}_d}{\varepsilon_0 k^2 \tilde{\phi}}. \tag{3.2}
\]

To derive a dispersion relation, Eqs. (3.1) and (3.2) are combined in the linearized and Fourier transformed Poisson equation, \( \varepsilon_0 k^2 \tilde{\phi} = \tilde{n}_i Q_s + \tilde{n}_e Q_e + \tilde{n}_d Q_d + \tilde{n}_d \tilde{Q}_d, \) yielding

\[
\varepsilon(k, \omega) = 1 + \chi_e + \chi_i + \chi_d + \chi_{qd}. \tag{3.3}
\]
Here, $k$ is the wave number, while $\chi_e$, $\chi_i$, and $\chi_d$ are the linear susceptibilities for electrons, ions, and dust.

The first DAW dispersion relation was derived by Rao et al. [17]. Essentially they solved Eq. (3.3), omitting $\chi_{qd}$, using separate hydrodynamic descriptions of electrons, ions, and a continuum description of cold dust. The dispersion relation of Rao et al. [17] is not suitable for laboratory experiments because it does not take into account effects that are generally present in experiments: drifting ions, frictional gas drag on ions and dust, strong-coupling effects for the dust, and depletion of electrons and ions onto the dust. Some of these effects were included in varying combinations by other authors [57, 59, 60, 92, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114]. Among them, D’Angelo et al. [100] added drifting ions and gas friction acting upon ions and dust, but they did not include dust-charge fluctuation. Melandsø et al. [92] added dust-charge fluctuation to the Rao derivation, but they did not include ion drift or gas friction acting on ions and dust.

Depletion of electrons and ions from the plasma [14, 15, 115], due to collection onto the dust, can be a significant factor in experimental plasmas if the cloud of dust particles fills a three-dimensional volume. In a wave, this depletion is enhanced when the cloud is locally compressed. Melandsø et al. [92] took this process into account in a theory intended for planetary rings, but without other processes that are significant in experimental conditions such as dust-neutral collisions.

Strong-coupling effects arise due to the discreteness of particles. Discrete particles cause microscopic variations in the electric field. These microscopic effects are most severe if a particle’s charge is large, which is the case for dust particles. For this reason, dust particles under experimental conditions tend to be strongly coupled, so that they act collectively like atoms in a liquid or solid as described in
Ch. 1. Electrons and ions, on the other hand, remain weakly coupled, so that they have collective properties more like those of a gas.

There are three approaches that have been taken in the literature to account for strong coupling of dust. Rosenberg and Kalman [103] used a quasi-localized charge approximation for dust to derive the longitudinal component of a dynamical matrix, and they included this in $\chi_d$. Kaw and Sen [104] approximated $\chi_d$ as that of a one-component plasma (OCP), as derived by Ichimaru et al. [116]. Murillo [105] derived the dust susceptibility taking into account strong-coupling effects by using a static local field correction parameter.

One of the goals in this chapter is to formulate a hydrodynamic description of strong coupling in terms of a compressibility $\beta$. In general, the compressibility of a substance is a measure of the fractional change in its volume divided by the change in pressure. I add a compressibility term, which can be adjusted to account for either strong or weak coupling among dust particles, in the dust equation of motion when I derive a dust susceptibility.

All of the dispersion relations that I discussed above are for waves with small amplitude, i.e., linear waves. The wave amplitude in DAW experiments can be linear [49, 117] or nonlinear, [50, 55, 56, 117] although experimenters have often found that even under nonlinear conditions the wavelength is predicted reasonably by a linearized dispersion relation [29].

3.2 Susceptibility derivation

Here, I present expressions for ion, electron, and dust susceptibilities that include various combinations of physical processes. By combining the various susceptibilities in different combinations one can include the physical processes that are deemed to be important.
3.2.1 Ion susceptibility

I present five expressions for $\chi_i$, each with a different combination of ion processes. The first four expressions were first reported by other authors, while the last one is developed here. The ion processes that I consider are ion drift, ion-neutral collisions, finite ion-temperature effects, and ion losses by collection onto dust particles.

Either a Vlasov or a hydrodynamic description can be used to describe ions. Other authors have generally chosen one or the other, without justification. An advantage of the Vlasov description is that it is kinetic, so that it includes two effects that are important for the instability: inverse Landau damping (ILD) and ion-neutral collisions. Only the latter is included in the hydrodynamic description. However, the hydrodynamic description has its own advantage: it can easily be adapted to account for ion currents collected on the dust particles. One of the goals of this chapter is to weigh the comparative advantages of these two descriptions, for typical experimental conditions; I do this in Sec. 3.4.

3.2.1.1 Vlasov kinetic description

A kinetic Vlasov description for ions is [102]

$$\chi_i = \frac{1}{k^2 \lambda_D^2} \frac{1 + \xi_i Z(\xi_i)}{1 + (i \nu_{in}/\sqrt{2}kV_{Ti})Z(\xi_i)}. \quad (3.4)$$

This assumes an ion-neutral collision rate $\nu_{in}$ and a Maxwellian ion distribution centered at an ion drift speed $U_0$. Here, $\xi_i = (\omega - kU_0 + i \nu_{in})/\sqrt{2kV_{Ti}}$, $V_{Ti} = \sqrt{k_B T_i/m_i}$, and $Z(\xi_i)$ is the plasma dispersion function [118]. The imaginary part of Eq. (3.4) would correspond to unstable wave growth.
3.2.1.2 Hydrodynamic descriptions

Here I present four hydrodynamic descriptions with different physical processes.

The first and simplest hydrodynamic description of ions assumes a Boltzmann response \( n_i = n_{i0} \exp(-e\phi/k_BT_i) \). The susceptibility is [17]

\[
\chi_i = \frac{1}{k^2 \lambda_{Di}^2},
\]

(3.5)

where \( \lambda_{Di} \) is the ion Debye length. Equation (3.5) assumes that the ions are inertialess, and it neglects many processes for ions important for laboratory experiments. I will next add several processes, one at a time.

In my second hydrodynamic description, I add ion drift, which is one of two required elements for destabilization of the wave in a hydrodynamic approach. Assuming a drift speed \( U_0 \), the ion fluid velocity is written as \( u_i = U_0 + \tilde{u}_i \), which I use in the ion fluid equations

\[
\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial z} n_i u_i = 0
\]

(3.6)

and

\[
\frac{d}{dt} u_i = -\frac{e}{m_i} \frac{\partial}{\partial z} \phi - \frac{k_B T_i}{n_i m_i} \frac{\partial n_i}{\partial z}.
\]

(3.7)

After linearizing, Fourier transforming, and combining Eqs. (3.6) and (3.7) into Eq. (3.1), I obtain

\[
\chi_i = \frac{\omega_{pi}^2}{V_{Ti}^2 k^2 - (kU_0 - \omega)^2}.
\]

(3.8)

Here, \( n_i \) is the ion number density, while \( m_i, T_i \), and \( \omega_{pi} \) are the ion mass, ion kinetic temperature, and ion plasma frequency. The ion momentum equation, Eq. (3.7), has terms on its right hand side for the macroscopic electric force (due to charge separation) and the ion pressure. Since the ions are weakly coupled, an ideal gas equation of state is used for the ion pressure.

My third description adds the other required element for destabilization of
the wave in a hydrodynamic approach: ion-neutral collisions. I rewrite the ion
momentum equation, Eq. (3.7), as
\[
\frac{d}{dt} u_i = - \frac{e}{m_i} \frac{\partial}{\partial z} \phi - \frac{k_B T_i}{n_i m_i} \frac{\partial n_i}{\partial z} - \nu_{in} u_i. \tag{3.9}
\]
Combining this with Eq. (3.6) yields
\[
\chi_i = \frac{\omega_{ps}^2}{V_{Ti}^2 k^2 - (k U_0 - \omega)^2 + i \nu_{in} (k U_0 - \omega)}. \tag{3.10}
\]
The three hydrodynamic descriptions listed above, Eqs. (3.5), (3.8), and (3.10),
have previously appeared in the literature. Equation (3.10), in particular, was used
earlier by FG [49] for the limit \(U_0 \gg \omega/k\).

My fourth hydrodynamic description adds one more process: ion losses due
to the collection of ions on the dust particles. To do this, I add a term to the ion
continuity equation,
\[
\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial z} n_i u_i = - n_d I_i / e. \tag{3.11}
\]
The last term is the ion loss rate onto the dust, which is proportional to both the
dust number density \(n_d\) and the ion current onto dust \(I_i\). In a wave the ion current
\(I_i\) will be modulated and this will modify the modulation of \(n_i\) and thereby affect
\(\chi_i\). One can in principal use any model for this ion current. Here, I will use two
such models. First, the orbital-motion limited (OML) ion current is [119]
\[
I_i = \pi a^2 n_i u_i e \left(1 - \frac{2e \phi_s}{m_i u_i^2} \right), \tag{3.12}
\]
and this takes into account ion drift, but not ion-neutral collisions. Here, \(\phi_s = Q_d / 4\pi \varepsilon_0 a\) is the dust particle surface potential for a particle of radius \(a\). Second,
the Lampe ion current [120], which accounts for ion-neutral collisions but not ion
drift, is
\[
I_i = \sqrt{8\pi a^2 n_i V_{Ti} \left[1 + z\tau + (R_0^3 / a^2 l_i) \right]}. \tag{3.13}
\]
The symbols inside the square brackets of Eq. (3.13) are defined in Khrapak et al.
[121].
For the experimental conditions of FG [49], the Lampe ion current (without ion drift) is approximately double the OML ion current (without ion-neutral collisions). Ideally, a third ion current model is needed that accounts for both ion drift and ion-neutral collisions, but I am not aware of any analytical model that does this.

After the usual linearization and Fourier transformation, I combine Eqs. (3.9), (3.11), and (3.12) into Eq. (3.1). This yields

\[ \chi_i = \frac{\Omega_{\phi}}{k^2 \lambda_D^2 (\Omega_n - i\omega)}. \]  

(3.14)

Here, \( \Omega_n \) and \( \Omega_{\phi} \) are quantities which have dimensions of inverse time, and they depend only on equilibrium parameters such as \( n_i \) and \( U_0 \). Expressions for \( \Omega_n \) and \( \Omega_{\phi} \) are rather lengthy; they are presented in Appendix A along with an adjustable parameter \( \gamma \) that allows their use with the Lampe model or any other model of ion current.

3.2.2 Electron susceptibility

Electrons in a dusty plasma, like ions, are generally weakly coupled, so that they can be described either hydrodynamically, or with a kinetic Vlasov description.

Neglecting inertia, electrons are described by the Boltzmann response, \( n_e = n_{e0} \exp(e\phi/k_B T_e) \), as in Rao et al. [17]. The susceptibility is then

\[ \chi_e = \frac{1}{k^2 \lambda_{De}^2}, \]  

(3.15)

where \( \lambda_{De} \) is the electron Debye length.

Alternatively, the Vlasov description of electrons [102] including electron-neutral collisions is

\[ \chi_e = \frac{1}{k^2 \lambda_{De}^2} \left[ 1 + \xi_e Z (\xi_e) \right]. \]  

(3.16)

Such a kinetic description retains the effects of electron Landau damping, but this is usually unnecessary as the phase velocity of the DAW is usually very slow compared
to the electron thermal velocity. For this reason, I only use Eq. (3.15) in the derivation of DAW dispersion relations.

3.2.3 Dust susceptibility

The dust particles experience many processes that affect their motion, and these enter into the dust susceptibility $\chi_d$. Most significantly, the dust particles provide inertia to the wave and they participate in the charge separation that is responsible for wave’s electric field. Additionally, dust-neutral collisions introduce wave damping. As the dust is compressed and rarefied, the dust equation of state comes into play, and this is described by the compressibility, which must be chosen differently according to whether the dust is weakly or strongly coupled.

I do not use a Vlasov kinetic description for the dust particles in a dusty plasma. Such a description is not appropriate when collisional nearest neighbor interactions of dust particles are strong, as is often the case under experimental conditions. Thus, I choose a hydrodynamic description for dust, which I can adapt to include strong-coupling effects.

I generalize the hydrodynamic approach of FG [49] by including the dust compressibility $\beta$ to the dust fluid equations. To derive the dust susceptibility, the dust fluid equations

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial z} n_d u_d = 0 \quad (3.17)$$

and

$$\frac{d}{dt} u_d = - \frac{Q_d}{m_d} \frac{\partial}{\partial z} \phi - \frac{1}{\beta n_d^2 m_d} \frac{\partial n_d}{\partial z} \quad (3.18)$$

are linearized, Fourier transformed, and combined into Eq. (3.1) to yield

$$\chi_d = \frac{\omega_{pd}^2}{k^2 (\beta n_d m_d)^{-1} - \omega^2}. \quad (3.19)$$

Here, $m_d$ and $u_d$ are the dust particle mass and dust fluid speed. The two terms on the right hand side of the dust momentum equation, Eq. (3.18), are the wave’s
macroscopic electric force (due to charge separation) and the dust pressure gradient terms.

I can further generalize the dust susceptibility to incorporate dust-neutral collisions. Accordingly, I introduce the dust-neutral damping term with a rate $\nu_{dn}$ to the right hand side of Eq. (3.18), yielding

$$\frac{d}{dt} u_d = -\frac{Q_d}{m_d} \frac{\partial}{\partial z} \phi - \frac{1}{\beta n_d^2 m_d} \frac{\partial n_d}{\partial z} - \nu_{dn} u_d. \quad (3.20)$$

The dust susceptibility is then

$$\chi_d = \frac{\omega^2_{pd}}{k^2 (\beta n_d m_d)^{-1} - \omega (\omega + i \nu_{dn})}. \quad (3.21)$$

The compressibility $\beta$ deserves discussion because this parameter is the only one in my dispersion relation that can account for strong-coupling effects for the dust. In the case of weak coupling, with no microscopic electric forces between individual dust particles, I would have an ideal gas equation of state for the dust. The compressibility then has the positive value $\beta = 1/n_d k_B T_d$, and Eq. (3.21) becomes

$$\chi_d = \frac{\omega^2_{pd}}{k^2 V^2_{Td} - \omega (\omega + i \nu_{dn})}, \quad (3.22)$$

where $V_{Td} = \sqrt{k_B T_d / m_d}$. For strong coupling, the compressibility arises from microscopic variations in the electric field due to the discreteness of particles. The compressibility can be a complex number due to viscoelastic effects, and my expressions allow for this possibility, although later I will assume $\beta$ is real and does not vary with $\omega$ or $k$. The effects of $\beta$ on the DAW dispersion relation will be quantified in Sec. 3.4.4.

3.2.4 Susceptibility due to dust-charge fluctuation

Dust particles charge by collecting electrons and ions. These collection currents can fluctuate at the wave’s frequency so that the dust charge $Q_d$ will fluctuate as well [92]. Due to this dust-charge fluctuation, some authors have included a
fourth susceptibility, Eq. (3.2), into their derivation of the DAW dispersion relations. All the other susceptibilities I have considered are based on Eq. (3.1) due to fluctuations of a number density.

Here I derive an expression for $\chi_{qd}$ by generalizing the hydrodynamic approach of Melandsø et al. [92] by including four more processes: ion drift, ion-neutral collisions, dust-neutral collisions, and strong-coupling effects for dust. I start with the dust charging equation

$$\frac{\partial Q_d}{\partial t} = I_e + I_i,$$

where $I_e$ and $I_i$ are the electron and ion currents, respectively. In principal, one can use any model for these currents. For consistency, I again use the OML current model, [119] Eq. (3.12) for ions and

$$I_e = -4\pi a^2 n_e e \sqrt{\frac{k_B T_e}{2\pi m_e}} \exp \left( \frac{e\phi_s}{k_B T_e} \right)$$

for electrons.

To obtain an expression for this susceptibility, I linearize, Fourier transform, and combine Eqs. (3.12), (3.23), and (3.24) into Eq. (3.2). This yields

$$\chi_{qd} = -\frac{4\pi a n_d \Omega_{V0}}{k^2 (\Omega_{\phi_s} - i\omega)}.$$ 

Here, $\Omega_{\phi_s}$ and $\Omega_{V0}$ are quantities having the dimensions of inverse time and they are functions of equilibrium quantities like $n_{i0}$ and $n_{e0}$. Expressions for $\Omega_{\phi_s}$ and $\Omega_{V0}$ are rather lengthy and they are presented in Appendix A. For use with other ion current models, these expressions contain an adjustable parameter $\gamma$.

### 3.3 Models for DAW dispersion relations

I now derive three new dispersion relations that include a combination of physical effects from the list in Sec. 3.1. I do this by combining various susceptibilities from Sec. 3.2 into Eq. (3.3). The sensitivity of the dispersion relations to these
effects is quantified in Sec. 3.4.

3.3.1 Baseline hydrodynamic model

I start with a baseline dispersion relation that includes minimal effects appropriate for experimental plasmas: charge separation, dust inertia, ion drift, ion-neutral collisions, dust-neutral collisions, an adjustable compressibility, and finite temperature effects for electrons and ions. These are the first seven effects from the list in Sec. 3.1. My other two dispersion relations will be generalizations of this baseline dispersion relation, which in turn is a generalization of the dispersion relation of FG [49].

I use a dust susceptibility that includes a compressibility. The compressibility can be selected either for a weak-coupling case using \( \beta = 1/n_d k_B T_d \) or a strong-coupling case using another value.

I combine Eqs. (3.21), (3.15), and (3.10) for \( \chi_d \), \( \chi_e \), and \( \chi_i \), respectively, in Eq. (3.3). For now, I omit \( \chi_{qd} \), i.e., I neglect the dust-charge fluctuation at the wave’s frequency. The resulting baseline dispersion relation is

\[
\varepsilon(k, \omega) = \begin{cases} 
1 + \frac{1}{k^2 \lambda_{De}^2} + \frac{\omega_p^2}{k^2 (k U_0 - \omega)^2 + i \nu_{in} (k U_0 - \omega)} \\
+ \frac{\omega_{pd}^2}{k^2 (\beta n_d m_d)^{-1} \omega (\omega + i \nu_{dn})}
\end{cases} = 0. \tag{3.26}
\]

It is typically the case that \( U_0 / (\omega/k) >> 1 \); for example in the experiment of FG [49] this ratio is 9500. In this limit, Eq. (3.26) has an analytical solution:

\[
\omega(k) = \omega_{pd} \left[ k \lambda_{De} \sqrt{\frac{1}{1 + k^2 \lambda_{De}^2 \alpha} + \frac{1}{\beta n_d m_d \omega_{pd}^2 \lambda_{De}^2} - \frac{\nu_{dn}^2}{4 \omega_{pd}^2 k^2 \lambda_{De}^2} - i \frac{\nu_{dn}}{2 \omega_{pd}}} \right]. \tag{3.27}
\]

In Eq. (3.27), \( \alpha \equiv 1 + \omega_{pd}^2 / [(V_{Ti}^2 - U_0^2) k^2 + i \nu_{in} U_0 k] \) is a dimensionless complex quantity which has a value of about 0.8—1.7i for the typical experimental conditions of FG [49].
The wave frequency and growth rate are the real and imaginary parts of Eq. (3.27), respectively. I plot them in Fig. 3.1, assuming real $k$, using the plasma parameters of FG [49] (see Appendix B for a list of these plasma parameters).

In this baseline model, ion-neutral collisions destabilize the wave while the dust-neutral collisions provide wave damping. This can be seen in Fig. 3.2, which is a plot of the growth rate $\omega_i$ vs. ion-neutral collision rate $\nu_{in}$ based on Eq. (3.27). The instability occurs only for $\nu_{in} > 2.5 \times 10^6$ s$^{-1}$ for the conditions of FG [49].

3.3.2 Hydrodynamic model with more processes

Next, I add two additional processes to my baseline model: ion collection onto dust and dust-charge fluctuations.

Combining Eqs. (3.25), (3.21), (3.15), and (3.14) for $\chi_{qd}$, $\chi_d$, $\chi_e$, and $\chi_i$, respectively, in Eq. (3.3) yields

$$
\varepsilon(k, \omega) = \left\{ 1 + \frac{1}{k^2 \lambda_D^2} + \frac{\Omega_n}{k^2 (\Omega_n - i \omega)} + \frac{\omega_p^2}{k^2 (\beta n m_d)^{-1} - \omega (\omega + i \nu_{dn})} \right\} = 0. \tag{3.28}
$$

In this dispersion relation, the quantities $\Omega_\phi$, $\Omega_n$, $\Omega_{\phi s}$, and $\Omega V_0$ all have dimensions of inverse time as presented in Appendix A. Typical values of these four quantities for the experimental parameters of FG [49] are: $\Omega_\phi = (7.1 - 3.2 i) \times 10^5$ s$^{-1}$, $\Omega_n = (0.7 + 4.2 i) \times 10^6$ s$^{-1}$, $\Omega_{\phi s} = 4.7 \times 10^5$ s$^{-1}$, and $\Omega V_0 = -(0.3 + 2.7 i) \times 10^6$ s$^{-1}$. These values are for the OML ion current model, which I also assume in the remainder of this chapter except in Sec. 3.4.3, where I examine the sensitivity of the dispersion relations to the choice of ion current model.

In this hydrodynamic model, as with my baseline model, the mechanism that destabilizes the DAW is ion-neutral collisionality. However, unlike the baseline model, Eqs. (3.26) and (3.27), in Eq. (3.28) wave damping is provided not only by dust-neutral collisions but also by dust-charge fluctuations. Equation (3.28) requires
Figure 3.1: Real frequency $\omega_r$ (a) and imaginary frequency $\omega_i$ (b) as a function of wavenumber $k$ for three dispersion relation models derived in Sec. 3.3. These models are: the baseline hydrodynamic model Eq. (3.27), hydrodynamic model with more processes Eq. (3.28), and hybrid hydrodynamic-kinetic model Eq. (3.29), respectively. I use the experimental conditions of FG [49] as input parameters for these models. All the three models yield a maximum growth rate near 4 mm$^{-1}$ which is the experimentally observed wave number in FG [49].
Figure 3.2: Sensitivity of the growth rate $\omega_i$ to the ion-neutral collision rate $\nu_{in}$. The curve is obtained by solving Eq. (3.27), the dispersion relation for the baseline model, for the experimental conditions of FG [49] except that I allow $\nu_{in}$ to vary. The wave number is $k = 4 \text{ mm}^{-1}$. A tangent is drawn at the estimate of $\nu_{in}$ in FG.

a numerical solution. I plot this solution in Fig. 3.1.

Comparing the curves in Fig. 3.1, I find that including the two additional processes, ion collection on dust particles and dust-charge fluctuation, has an effect up to 40% in the dispersion relation. This is seen in Fig. 3.1(a) and (b), where I compare the model that includes these processes to the baseline model, which does not. In particular, both $\omega_r$ and $\omega_i$ are changed by as much as 40% over a wide range of $k$.

3.3.3 Hybrid hydrodynamic-kinetic model

Next, I include the same effects as in my baseline model, except now for ions I use a kinetic description, which retains the effects of inverse Landau damping.

I use the kinetic susceptibility Eq. (3.4) for ions with the hydrodynamic susceptibilities Eqs. (3.21) and (3.15) for dust and electrons. These are combined in
Here, I omit the susceptibility due to dust-charge fluctuation, i.e., $\chi_{qd}$. The resulting hybrid hydrodynamic-kinetic dispersion relation is

$$
\varepsilon(k, \omega) = \left\{ \begin{array}{c}
1 + \frac{1}{k^2 \lambda_{De}^2} + \frac{1}{k^2 \lambda_{Di}^2} \left[ 1 + \left( \frac{\nu_{id}}{\sqrt{2} k V_{Ti}} \right) \frac{1 + \xi_i Z(\xi_i)}{1 + \xi_i Z(\xi_i)} \right]
+ \frac{\omega_{pd}^2}{k^2 (\nu_{id} m_d)^{-1} - \omega(\omega + \nu_{id})}
\end{array} \right\} = 0,
$$

which also requires a numerical solution. This solution is plotted in Fig. 3.1 for the experimental parameters of FG. [49]

As it was for the baseline model, the wave damping is provided by dust-neutral collisions. However, unlike the baseline model, here there are two wave destabilization sources: ion inverse Landau damping and ion-neutral collisions. In Sec. 3.4.2, I will compare these wave destabilization sources and determine their relative contributions to the instability for typical experimental conditions.

I find that the dispersion relation for this hybrid model yields about the same $\omega_r$ and $\omega_i$ as for my baseline model. As can be seen in Fig. 3.1, the percentage difference for $\omega_r$ between the baseline and the hybrid models are less than 2% over a wide range of $k$. However, the difference for $\omega_i$ is much larger, with a percentage difference of up to 8%.

### 3.4 Results and discussion

I present results organized into four subsections that correspond to the four goals. First, I perform a sensitivity test to quantify how much the real and imaginary parts of the wave frequency change, for a given change in an input parameter (such as the ion-neutral collision rate). Second, I determine whether it is important to use a kinetic treatment for ions (which takes into account inverse Landau damping), or whether a simpler hydrodynamic model for the ions is adequate. Third, I quantify how much the dispersion relation depends on the choice of ion current models (OML vs. Lampe). Finally, I use my formalism of $\chi_d$ including a compressibility (which
can account for strong coupling) to learn how the dispersion relation depends on the sign and magnitude of the compressibility.

3.4.1 Sensitivity to parameters

To determine the sensitivity of the dispersion relation to the physical parameters, I calculate exponents

\[ \delta_r = \frac{(\Delta \omega_r/\omega_r)}{(\Delta F/F)} \]  \hspace{1cm} (3.30)

and

\[ \delta_i = \frac{(\Delta \omega_i/\omega_i)}{(\Delta F/F)} , \]  \hspace{1cm} (3.31)

where \( F \) is a parameter such as the ion-drift speed \( U_0 \). In this test, I make a 1% change in \( F \) and I determine the fractional change in \( \omega_r \) and \( \omega_i \) to compute \( \delta_r \) and \( \delta_i \), respectively. As an example, if \( \omega_r \) is proportional to \( F \), the exponent will be unity and if it is proportional to \( \sqrt{F} \) the exponent will be 0.5. When calculating these exponents, I assume the experimentally observed wave number \( k = 4 \text{ mm}^{-1} \) of FG [49].

The results for the exponents of \( \omega_r \) and \( \omega_i \) are summarized in Table 3.1 for my three dispersion relation models derived in Sec. 3.3. In Table 3.1, I highlight the exponents \( > 0.4 \) because these indicate a particularly significant sensitivity. Note that an entry of 0.00* in this table indicates that the magnitude of the exponent is less than 0.01.

In general, I find that \( \omega_i \) is more sensitive to changes in various parameters than is \( \omega_r \). For example, in Table 3.1, \( |\delta_i| \) can be as large as four, whereas \( |\delta_r| \) is never large as unity.

I now rank the parameters that cause the largest changes in the growth rate \( \omega_i \). When they are increased, the parameters that cause the largest positive change in \( \omega_i \) are the dust plasma frequency followed by the ion-drift speed. For these
Table 3.1: Exponents for the dispersion relation models.

| $F$ | Hydrodynamic models | | | Hybrid | |  
| | Baseline model | with more processes | | |  
| | $\delta_r$ | $\delta_i$ | $\delta_r$ | $\delta_i$ | $\delta_r$ | $\delta_i$ |  
| $\beta$ | −0.11 | +0.44 | −0.09 | +0.34 | −0.11 | +0.43 |  
| $\nu_{dn}$ | −0.12 | −2.24 | −0.09 | −2.39 | −0.12 | −2.28 |  
| $\nu_{in}$ | +0.47 | −0.16 | +0.42 | −0.61 | +0.45 | −0.17 |  
| $U_0$ | +0.48 | +0.87 | +0.29 | +1.49 | +0.48 | +0.80 |  
| $V_{Te}$ | −0.00* | +0.13 | +0.12 | +0.04 | −0.00* | +0.14 |  
| $V_{Ti}$ | −0.00* | −0.42 | −0.00* | −0.43 | +0.01 | −0.44 |  
| $\omega_{pe}$ | +0.00* | −0.13 | −0.25 | +0.08 | +0.00* | −0.14 |  
| $\omega_{pi}$ | −0.95 | −0.35 | −0.73 | +0.15 | −0.94 | −0.26 |  
| $\omega_{pd}$ | +0.88 | +4.13 | +0.87 | +4.39 | +0.88 | +4.16 |  
| $I_i$ | +0.14 | −0.59 |  
| $I_e$ | −0.12 | +0.15 |  

parameters $\delta_i > 0.4$ in Table 3.1. On the other hand, the parameters that cause the largest negative change in $\omega_i$ when they are increased are the dust-neutral collision rate followed by the ion-neutral collision rate, ion current, and ion thermal speed. All of these have $\delta_i < -0.4$ in Table 3.1.

I also rank the parameters that cause the largest changes in the wave’s frequency $\omega_r$. Only two parameters, the dust-plasma frequency followed by the ion neutral collision rate, have a significant positive sensitivity with $\delta_r > 0.4$. Only one parameter, the ion plasma frequency, has a significant negative sensitivity with $\delta_r < -0.4$. This list is shorter than for $\omega_i$ because, in general, $\omega_i$ is much more
sensitive than $\omega_r$ to a change in a parameter's value.

An experimenter additionally might wish to know the sensitivity of $\omega_r$ and $\omega_i$ to experimental parameters such as the macroscopic dc electric field $E_{z0}$ or the ambient gas pressure $P$. I perform a sensitivity analysis for seven such experimental parameters in Appendix C.

I note a limitation of these rankings based on exponents: they are valid only within a narrow range of parameters that brackets the conditions I assumed. While the conditions of FG [49] that I assumed are representative of many experiments, it would be necessary to recompute these exponents if the conditions differ significantly from those of FG. To illustrate this, I show in Fig. 3.2 results for growth rate $\omega_i$ as a function of the dust neutral collision rate $\nu_{in}$. In Fig. 3.2, a tangent is drawn at the conditions of FG. The exponent is proportional to the slope of the tangent which varies with $\nu_{in}$.

3.4.2 Comparison of the sources of the DAW instability

In Sec. 3.3, I derived three dispersion relation models for the DAW, and among them only the hybrid hydrodynamic-kinetic model has two sources for the DAW instability: ion-neutral collisions and inverse Landau damping (ILD). Here, I compare the contributions to the instability from these two sources.

To distinguish the instability contributions from the ion-neutral collisions and ILD, I compute the imaginary frequency by numerically solving one of my dispersion relations, Eq. (3.29), two ways: with an ion-neutral collision rate that has a realistic value for experiments and a zero rate, $\nu_{in} = 0$. I subtract the imaginary frequencies calculated using these two ways to find the contribution due to ion-neutral collisions.

The results in Fig. 3.3 are presented with different hatching patterns for the two contributions to $\omega_i$. I calculate these for the experimental conditions of FG [49].
Figure 3.3: Contributions of inverse Landau damping (ILD) and ion-neutral collisions to the DAW instability for the conditions of FG [49]. The total $\omega_i$ (heavy line) and the contributions from ILD (crosshatch pattern) were obtained from Eq (3.29) using experimental estimate of $\nu_{in}$ in FG [49] and $\nu_{in} = 0$, respectively. The difference between the total and ILD contributions is attributed to ion-neutral collisions (single hatch pattern). I find that ILD and ion-neutral collisions contribute about equally to the DAW for these typical experimental conditions.
I find that the contributions to the instability from ILD and ion-neutral collisions are of the same order of magnitude. In Fig. 3.3, both contributions are displayed as positive quantities that offset the negative contribution due to gas damping $-0.26\omega_{pd}$. The gas damping contribution is shown as a solid line at the bottom of the Fig. 3.3. Although the contributions from ILD and ion-neutral collisions generally vary with $k$ in Fig. 3.3, as depicted by bars of varying heights, they are generally of the same order of magnitude. For example, at a typical experimental value of $k = 4 \text{ mm}^{-1}$ ($k\lambda_{Di} = 0.2$ in dimensionless units) as observed in FG [49], the contribution of ILD is approximately 51% of ion-neutral collisions. I cannot yet, however, make a clear recommendation to use the kinetic model because it neglects ion currents collected by the dust particles, which I will evaluate next.

It is intriguing that while my baseline hydrodynamic and hybrid hydrodynamic-kinetic models have different sources for the instability, these two models yield roughly the same value for the imaginary part of the frequency, as can be seen in Fig. 3.1(b). I believe that this equality arises purely by chance for the conditions used. For example, at the much lower gas pressure conditions of an anodic plasma an instability requires kinetic effects [37].

3.4.3 Sensitivity to ion current model

I find here that $\omega_i$ is quite sensitive to the effects of ion current onto the dust particles. The growth rate is significantly suppressed by this ion collection, and this trend has not been noted in the literature, to the best of my knowledge.

In Fig. 3.4, I compare one of my dispersion relations, Eq. (3.28), for three cases: $I_i$ is either zero, the OML current, or the Lampe current. The Lampe current is double the OML current for the experimental conditions I consider.

I find that the effect of ions collecting on the dust particles is a reduced growth rate as seen in Fig. 3.4(b). At $k = 4 \text{ mm}^{-1}$, for example, $\omega_i$ is reduced by 32% for
the OML current as compared to the case for $I_i = 0$. It is reduced even more, by 82%, for the Lampe current. The real part of the dispersion relation is also affected, but to a lesser degree than $\omega_i$.

As I mentioned in Sec. 3.2.1.2, the OML ion current, Eq. (3.12), neglects ion-neutral collisions while the Lampe current, Eq. (3.13), neglects ion drift. These are both significant processes under experimental conditions. However, I cannot determine which is more suitable because each model neglects a significant process.

3.4.4 Strong coupling effects

The compressibility of a strongly coupled plasma arises from microscopic variations in the electric fields. This compressibility does not generally provide the wave’s restoring force; that is done by the macroscopic electric fields arising from charge separation. The role of the compressibility is to somewhat alter that restoring force. In general, a small compressibility would have a large effect on the wave, while a large compressibility would have a little effect; this is because a small compressibility would indicate a significant force for a given change in volume.

In the limit of weak coupling, the compressibility has the ideal gas value $\beta = 1/n_i k_B T_d$. This has such a large value that it will result in little effect on the restoring force. For strong coupling, however, $|\beta|$ can have a much smaller value, thereby have a larger role in altering the wave’s restoring force. However, the value of $\beta$ for a strongly-coupled dusty plasma has not been well established, as I will discuss in the next chapter. For now, I will adopt a general view, allowing the sign of $\beta$ to be either positive or negative for strong coupling, when I evaluate the dispersion relation.

I can illustrate how the real and imaginary frequencies are affected by the compressibility. To do this, I solve Eq. (3.27) for three cases for $\beta$: a positive value of $1.83 \times 10^5$ Pa$^{-1}$ which I base on an empirical estimate in FG [49], a negative value
Figure 3.4: Real frequency $\omega_r$ (a) and imaginary frequency $\omega_i$ (b), from Eq. (3.28), for three different ion currents. For the Lampe current, I use the expression in Appendix A with a parameter $\gamma$ chosen as the ratio of the ion currents in Eqs. (3.13) and (3.12). In particular, for the condition of FG [49] the Lampe ion current is twice the OML ion current, i.e., $\gamma = 2$. The imaginary frequency is affected significantly by the change in the ion current while the real frequency is affected to a lesser degree. These results reveal that the instability is significantly suppressed for enhanced ion currents.
of $-1.83 \times 10^6 \text{ Pa}^{-1}$, and a weak-coupling value of $2.08 \times 10^9 \text{ Pa}^{-1}$. The latter was evaluated as $\beta = 1/n_d k_B T_d$ assuming $n_d = 1.2 \times 10^{11} \text{ m}^{-3}$ from FG [49] and an estimated value $T_d = 1/40 \text{ eV}$, which is room temperature. The room-temperature assumption for dust is appropriate only for low-amplitude waves, when the dust’s random motion is nearly in thermal equilibrium with the neutral gas.

Results plotted in Fig. 3.5 reveal that compressibility can have a significant effect on the DAW dispersion relation. This is especially so as the wave number $k$ becomes larger, i.e., as the wavelength becomes smaller. The imaginary part is affected more than the real part.

Assuming that strong coupling leads to a negative compressibility, I find the trends reported by Rosenberg et al. [122]. These trends are a smaller $\omega_r/\omega_{pd}$ and a larger $\omega_i/\omega_{pd}$ as strong-coupling effects are increased. In other words, if all other things including $\omega_{pd}$ are held constant, the instability can be enhanced by strong coupling if $\beta < 0$. Alternatively, assuming strong coupling leads to a positive compressibility, the trends are reversed: for strongly coupling, the real frequency would be enhanced and the imaginary part would be diminished.

### 3.5 Conclusions

Including more physical processes than are typically accounted for, I derived new susceptibility expressions for the ions, dust, and dust charge fluctuation. I have also reviewed other susceptibilities that were previously used in the literature. Demonstrating the use of these susceptibilities, I combined them differently to derive three new dispersion relations which I plotted using the experimental parameters from Flanagan and Goree [49].

I find that, in general, varying the experimental parameters or selecting different physical processes results in a larger effect on the imaginary part of the
Figure 3.5: Real frequency $\omega_r$ (a) and imaginary frequency $\omega_i$ (b) as a function of wavenumber $k$ from Eq. (3.27), for three different compressibility values. The three curves in each figure are for a positive compressibility equal to an empirical estimate in FG [49] (solid line), a negative value with the same magnitude as the positive value (dot-dashed line) and a weak coupling limit value for compressibility (dashed line). I find that the instability is enhanced with a larger $\omega_i$ if the compressibility is negative.
dispersion relation than on the real part. This means that for a study of the instability, it is crucial to include the appropriate physical processes. I performed tests to determine which processes are important in typical laboratory experiments.

For my first goal, I quantify how sensitive the dispersion relation is to a physical parameter, and the corresponding physical process, by calculating exponents $\delta_r$ and $\delta_i$, defined in Eq. (3.30) and Eq. (3.31). I judge a parameter to be significant if it alters the frequency enough as judged by $|\delta_r| > 0.4$ or $|\delta_i| > 0.4$. I find that six theoretical parameters affect $\omega_i$ substantially; ranking them starting with the largest $|\delta_i|$, for the experimental conditions of FG [49]. These six parameters are:

- dust plasma frequency
- dust-neutral collision rate
- ion-drift speed
- ion-neutral collision rate
- ion current
- ion thermal speed.

I also find three experimental parameters that affect $\omega_r$ substantially. Starting with the largest $|\delta_r|$, they are:

- dust plasma frequency
- ion-plasma frequency
- ion-neutral collision rate.

I find that the DAW instability is significantly suppressed if any physical process leads to a greater collection of ion currents on the dust particles. The real frequency, on the other hand, has little dependence on the ion current. These ion-current effects for the DAW instability will be most important at high dust number densities that are typical of laboratory experiments. This importance of ion currents
to laboratory conditions has not been remarked upon in the literature to the best of my knowledge.

I find that the instability contributions from inverse Landau damping and ion-neutral collisions are of the same order. All the other things being the same, it is best to use a kinetic descriptions for ions. However, I am faced with the problem that ion currents, which I also deemed to be important just above, are not easily accounted for in a kinetic model. Thus, there remains a need to derive a more complete expression for $\chi_i$ that includes both ILD and ion collection onto dust.

For my final goal, I find that strong coupling can change the growth rate substantially, and it can also make a measurable change in the real frequency as well. I determined this by comparing three cases for the compressibility. This finding that the dispersion relation is sensitive to the compressibility indicates a need for further studies of compressibility of strongly-coupled plasmas to determine its magnitude and sign.
CHAPTER 4
COMPRESSIBILITY MEASUREMENT IN A DUSTY PLASMA

4.1 Introduction

As mentioned in Ch. 3, the compressibility of a strongly-coupled dusty plasma has not been well established. In particular, there is a discrepancy between theory and experiments regarding the sign of the compressibility $\beta$ of the strongly-coupled dust component in a dusty plasma. In theories $\beta$ is negative but it is positive according to experimental results, for compressions provided by the dust-acoustic wave (DAW) in a dusty plasma. This discrepancy has not been remarked upon previously, to the best of my knowledge. In both the experiments and theories, a single strongly-coupled component (dust) is mixed with weakly coupled components (electrons and ions). The question, whether compressibility is positive or negative, should apply not only to dusty plasmas, but also to any other multicomponent plasma having one strongly-coupled component.

To help resolve this discrepancy, I report in this chapter new dusty plasma experiments to determine the sign of $\beta$. I use a DAW, measuring its frequency $\omega$, and wavenumber $k = k_r + ik_i$ along with other plasma parameters. I compare these experimental inputs to a dispersion relation $D = D_r + iD_i = 0$ derived in Eq. (3.28) of the previous chapter. In doing so, I consider $\beta$ for the dust to be a free parameter which could be positive or negative. I find solutions to $D = 0$ requiring that $k_i$ be near its maximum and I find several possible solutions to the dispersion relation that match the experimental measurements for positive values of $\beta$, but none for negative values of $\beta$. This finding leads me to conclude that $\beta > 0$.

A material’s compressibility, i.e., the inverse of its bulk modulus, is

$$\beta \equiv -(1/V)\partial V/\partial P = (1/n)\partial n/\partial P,$$  \hspace{1cm} (4.1)
where $P$, $V$, and $n$ are the pressure, volume, and number density. Compressibility determines a material’s sound speed, and it is used in fields such as engineering [123] and earth science [124]. In any common material (of solid, liquid, or gaseous form), compressibility is positive. This means that an object resists being longitudinally compressed. When squeezed more ($\Delta P > 0$), the object becomes smaller ($\Delta V < 0$). A negative compressibility would correspond to an explosive instability. While some exotic substances expand transversely when squeezed [125, 126], they do not expand longitudinally along the axis of compression, as would be the case for a negative compressibility. Unlike other matter, however, plasmas have macroscopic electric fields arising from charge separation among the different plasma components, and these fields provide a restoring force which is stabilizing. One could entertain the theoretical idea of a negative compressibility for just one component in a plasma, because its explosive tendency could be overwhelmed by the stabilizing effect of the macroscopic electric field.

Compressibility in a plasma arises because of either thermal effects or Coulomb collisions, according to whether the plasma component is weakly or strongly coupled. In a weakly-coupled component, collisions are seldom and an ideal gas law $P = nk_B T$ is obeyed, so that the compressibility is

$$\beta = 1/(nk_B T),$$  \hspace{1cm} (4.2)

where $T$ is the plasma component’s temperature. In a strongly-coupled component, on the other hand, collisions are frequent, and particle kinetic energies are less than the interparticle potential energies. Plasmas are strongly coupled if charge or number density are very large, or the temperature is very low. Like a liquid or a solid, a strongly-coupled component does not satisfy an ideal gas law [49]. It is possible for only one component to be strongly coupled in a multicomponent plasma; this is often the case for dusty plasmas.
Compressibility has been studied experimentally in nonideal plasmas, which have a coupling that is between strong and weak [127]. In those experiments, shocks and explosives compress all components of the plasma together [128, 129, 130]. The dusty plasma experiments that I perform differ in several ways: compression is provided more gently by an oscillatory wave, the various plasma components compress differently from one another, with a self-consistent electric field and one component is strongly coupled while the others are weakly coupled.

A key assumption for compressibility is that the dust component can be described as a continuum that obeys hydrodynamic equations including the dust equation of motion:

\[
\frac{d}{dt}u_d = -\frac{Q_d}{m_d} \frac{\partial \phi}{\partial z} - \frac{1}{\beta n_d^2 m_d} \frac{\partial n_d}{\partial z} - \nu_{dn} u_d,
\]

where \(u_d, m_d, \) and \(n_d\) are the dust velocity, mass, and number density. On the right side, the first term has a macroscopic electric field arising from charge separation while the second term is from the equation of state, as described by \(\beta\). The only way that strong vs. weak coupling effects are distinguished in this approach is the value of \(\beta\). The last term in Eq. (4.3) is for a dust-neutral friction with a friction rate \(\nu_{dn}\).

A compressional wave requires a restoring force. For sound waves in an ordinary material, the compressibility term represents the only restoring force. For a multi-component plasma, however, the term \(\partial \phi/\partial z\) in Eq. (4.3) represents an additional restoring force due to charge separation among the components.

To determine \(\beta\) for the dust component, the most direct method would be measurements of \(P\) and \(n_d\), exploiting the position-dependent compression provided by the wave. While video imaging in my experiments can yield a measurement of \(n_d\), I am unable to measure \(P\). I therefore need another method: a fitting of experimental and theoretical dispersion relations.
The dispersion relation $D = 1 + \sum \chi_j = 0$ can be written in several forms, depending on the physics that is included in the susceptibilities $\chi_j$ for various species $j$ as described in Ch. 3. For my experimental data, I will use the dispersion relation Eq. (3.28) because it takes into account almost all the significant processes that occur in the experiment. For the limited purpose of assessing existing theories, however, I will first examine a simpler dispersion relation, Eq. (3.27) from Ch. 3, 

$$\frac{\omega(\omega + i\nu_{dn})}{\omega_{pd}^2} = \frac{1}{1 + \chi_e + \chi_i} + (\beta n_d m_d)^{-1} \left( \frac{k}{\omega_{pd}} \right)^2,$$

which neglects ion and electron collection onto dust, and dust charge fluctuations as well as ion kinetic effects. For this limited purpose, for $\chi_e$ and $\chi_i$, I use Eqs. (3.15) and (3.14).

In the literature, I have identified three theories for the DAW with strong-coupling effects which all include a negative value for the dust compressibility [105, 122, 104]. However, to describe the dust these papers used parameters that were named and defined differently from the traditional compressibility of Eq. (4.1).

- Murillo [105] used a static local field correction parameter $G_{ocp}(k)$ in his Eq. (28), which is a dispersion relation. Comparing term-by-term to the simple dispersion relation in Eq. (4.4), I find that $G_{ocp} \propto 1/\beta$, but with the opposite sign as $\beta$. Murillo [105] states that $G_{ocp}$ is positive, meaning that in his theory $\beta$ is negative. In the other two theories, I focus on expressions for $\chi_d$ that resemble my expression for dust susceptibility from Ch. 3

$$\chi_d = \frac{\omega_{pd}^2}{k^2 (\beta n_d m_d)^{-1} - \omega (\omega + i\nu_{dn})},$$

Equation (5) for $\chi_d$ in Rosenberg et al. [122] has a parameter $D_L(k)$, called a longitudinal projection of a dynamical matrix, which is $\propto 1/\beta$ and with the same sign, based on a comparison to Eq. (4.5). Rosenberg et al. [122] report $D_L(k)$ as having a negative value, meaning that $\beta$ is negative.
Similarly, Eq. (12) of Kaw and Sen [104] has a parameter $\mu_d$ which is negative, and comparing to my Eq. (4.5) I find that that $\mu_d \propto \beta$ with the same sign. Thus, their theory is also for a negative $\beta$.

In contrast to these three theories, the literature also includes three experiments that are consistent with a positive $\beta$. I inspect their equations to identify which parameter is analogous to $\beta$. Flanagan and Goree [49] obtained agreement with a dispersion relation like Eq. (4.4), except they used a differently named parameter, which is positive. Comparing their Eq. (5) to Eq. (4.5), I find that their parameter is $\propto 1/\beta$, with the same sign. Thus, $\beta$ is positive. In two other experimental papers, Williams et al. [40] and Yaroshenko et al. [112] used a dust kinetic temperature parameter $T_d$. They modeled the dust as an ideal gas, with an ad-hoc increase in $T_d$ to achieve agreement with experiments. Since an ideal gas has a positive compressibility, Eq. (4.2), these papers are consistent with a positive $\beta$.

4.2 Experimental design

After identifying this discrepancy in theory and experiments, I determined the sign of $\beta$ by performing two experiments, with and without synchronization. In the experiments, a partially ionized argon plasma was sustained by applying a 13 MHz, 80 V peak-to-peak voltage to a lower electrode in a vacuum chamber, Fig. 4.1(a). Before introducing dust particles, I used a Langmuir probe (LP), to measure the ion number density, $n_i$, electron temperature, $T_e$, and the local time-average electric plasma potential $\phi_0$. By subtracting $\phi_0$ in two locations, I estimated the local dc electric field strength, $E_{z0}$, that arises due to an ambipolar transport. These parameters for a dust-free plasma are listed in the last column of Table 4.1, and as an approximation I will assume they are the same when dust is introduced.

I injected monodisperse melamine formaldehyde polymer microspheres of radius $a = 2.4 \, \mu m$ into the plasma, where they gained a negative charge. They were
levitated by a downward electric field $E_{z0}$ above a horizontal lower electrode that had a dc self bias of -20 V with respect to the grounded electrode, which consisted of the rest of the vacuum chamber. The dust cloud was shaped, as in [87, 49, 50], using a cubical glass box of length 4 cm with open top and bottom. The dust particles were strongly-coupled as seen in Fig. 4.1(b). A DAW propagated downward in the dust cloud due to an ion streaming instability. The dust cloud was illuminated with a 532-nm laser sheet and imaged with a digital camera at 256 frames/sec.

The two experiments were done at nearly identical conditions. The amplitude for the radiofrequency power, the same size polymer (MF) dust particles, and the same side-view imaging setup. Three differences in the experiments were the use of synchronization and slightly different values of $P_{\text{gas}}$ and $n_{d0}$. In the main experiment, the wave’s frequency $\omega/2\pi = 17$ Hz and wavenumber $k_r = 1.7$ mm$^{-1}$ were determined naturally by the ion-driven instability, without any external synchronization. In the synchronized experiment, I controlled the wave’s frequency by applying a low-frequency sinusoidal modulation to an upper driving electrode, Fig. 4.1(a). Synchronization requires nonlinearity, so that this was done farther from the instability’s threshold to yield a higher wave amplitude, $\tilde{n}_d/n_{d0} = 63\%$ as compared to 23% for the main experiment. These amplitudes were controlled by choosing gas pressures $P_{\text{gas}} = 200$ mTorr and 210 mTorr, respectively. Additionally, $n_{d0}$ was 15% smaller in the synchronized experiment.

4.3 Analysis

In my experimental determination of $\beta$, I give attention to the accuracy of the experimental inputs for the calculation of susceptibilities in the dispersion relation. Three groups of the measurements pose no problem at all because they only have uncertainties of few percents. These accurately known values are $\omega$, $k_r$, and $k_i$ obtained from imaging; $m_d$, $a_d$ as specified by the manufacturer; $P_{\text{gas}}$ as measured by
a precise capacitance manometer, and $T_{\text{gas}}$ based on room temperature. The latter two values are required for determining $\nu_{dn}$. The remaining four input parameters, however, had a lower accuracy. The measurement of $n_{d0}$ from images (IM) has a significant uncertainty due to the soft edge of the Gaussian laser sheet used for imaging. Three other parameters, based on Langmuir probe measurements, also have significant uncertainties; these are $n_{i0}$, $T_e$, and $E_{z0}$. My method assumes that these three parameters remain the same when dust is added; this assumption introduces an additional systematic error. I estimate that $n_{d0}$, $n_{i0}$, $T_e$, and $E_{z0}$ have a factor of two uncertainty, and to be extremely conservative, I will allow for a factor of six uncertainty in my analysis.

In addition to these experimental uncertainties, there are model-dependent systematic errors, and the most prominent of these is the one for $\chi_i$. For $\chi_i$, I use Eq. (3.14) which includes: ion drift, ion-neutral collisions, finite temperature effects for ions, and ion collection onto dust. Limitations of the ion models force me to
choose between neglecting ion collection on dust particles or ion kinetic effects; I chose to neglect the latter, which are shown in Ch. 3 to cause systematic errors as large as a factor of two. I expect that due to ion-neutral collisions, the actual ion current collected by a dust particle will actually be larger than predicted by OML, perhaps by a factor of two or three as described in Ch. 3. Thus, I will allow the ion current to be a variable quantity in my analysis. In addition to $\chi_i$, the expression I use for $\chi_e$ has a model-dependent bias: it assumes a Maxwellian electron distribution. If the true distribution is non-Maxwellian, then the true value of $\chi_e$ will be either reduced or enhanced; I allow for this simply by allowing $T_e$ to vary.

I determined $\beta$ from the experimental data using a four-step procedure. The first two steps found possible solutions of the dispersion relation $|D| = 0$ that agree with the observed $\omega$, $k_r$, and $k_i$ that were experimentally determined with high precision in the main experiment. In the third step, I narrowed these solutions by requiring that their imaginary dispersion relation have a maximum $k_i$ (as a function of $k_r$) near the experimentally measured $k_i$. I find that this third step yields the best solutions for $\beta > 0$, so that my procedure could end after this step. Nevertheless, I performed a fourth step to test the remaining solutions by requiring that their real dispersion relation qualitatively agree with the $\omega$ vs. $k_r$ as measured in the synchronized experiment. I next provide details of these four steps, which include an exhaustive search in a six dimensional parameter space of $\beta$, $n_{i0}$, $n_{e0}$, $T_e$, $E_{z0}$ and $I_i$. This parameter space is extended over a very wide range in every direction to provide very conservative allowances for experimental errors and systematic errors in the dispersion relation model.

In the first step there were $800 \times 35^5 \approx 4 \times 10^{10}$ grid points. In this first step, the compressibility $\beta$ was varied over a wide range from $-1 \times 10^6$ to $+1 \times 10^6$ with
800 linearly spaced values, while the other five parameters \((n_{d0}, n_{i0}, T_e, E_{z0} \text{ and } I_i)\) were varied logarithmically over a 36-fold range with 35 grid values each. These 36-fold ranges were centered on the measured values in the last column of Table 4.1, except for \(\gamma\) which had a range centered on unity. Among these \(4 \times 10^{10}\) grid points, I then selected those that satisfied \(|D| < 0.5\) at the experimentally measured \(\omega, k_r, \text{ and } k_i\). This first step ended with 165,000 surviving gridpoints, which I used as starting points for the second step.

In the second step, I used the downhill simplex method [131] to finish the fit of the dispersion relation to the experimentally measured \(\omega, k_r, \text{ and } k_i\). Requiring that \(|D|\) be minimized to \(< 10^{-11}\) yielded 19 solutions.

The third step is motivated by the principle that the observed \(k_r\), in the absence of external synchronization, will correspond to the maximum growth rate \(k_i\) of the instability [49]. Starting with the 19 solutions of the second step, I calculated their imaginary dispersion relations, \(k_i(k_r)\). I then imposed a slope requirement that \(|(d k_i/d k_r)| < 0.1\), which narrowed the survivors to only four solutions, which are labeled \(A, B, C, \text{ and } D\) in Table 4.1. All four of these have \(\beta > 0\). This outcome already signals that \(\beta\) is likely to be positive. However, before dismissing the possibility of a good solution with \(\beta < 0\), I relaxed the slope requirement to \(|(d k_i/d k_r)| < 0.6\), to yield the most reasonable of all the negative \(\beta\) solutions; it is labeled \(E\) in Table 4.1.

In the fourth step, in Fig. 4.2 I test the real dispersion relations of the five surviving solutions. In this test, I compare these five solutions to the theoretical dispersion relation, shown as curves, to the data points from the synchronized experiment. I seek only a qualitative agreement, because the solutions were for the conditions of the main (unsynchronized) experiment, which differed a few percent from those of the synchronized experiment. Examining Fig. 4.2 (a), I find a good
Table 4.1: Comparison of parameters for the best solutions of the fit.

<table>
<thead>
<tr>
<th>parameter</th>
<th>solution</th>
<th>measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>with $\beta &gt; 0$</td>
<td>with $\beta &lt; 0$</td>
</tr>
<tr>
<td>fit parameters</td>
<td>$A$</td>
<td>$B$</td>
</tr>
<tr>
<td>$\beta$ ($10^4$Pa$^{-1}$)</td>
<td>+9.6</td>
<td>+8.5</td>
</tr>
<tr>
<td>$T_c$ (eV)</td>
<td>3.1</td>
<td>4.6</td>
</tr>
<tr>
<td>$E_{z0}$ (V/cm)</td>
<td>19.8</td>
<td>17.9</td>
</tr>
<tr>
<td>$n_{i0}$ ($10^{14}$m$^{-3}$)</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>$n_{d0}$ ($10^{10}$m$^{-3}$)</td>
<td>8.7</td>
<td>7.8</td>
</tr>
<tr>
<td>$\gamma = I_i/I_{OML}$</td>
<td>4.9</td>
<td>3.7</td>
</tr>
<tr>
<td>computed parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M = U_0/\sqrt{k_B T_c/m_i}$</td>
<td>0.27</td>
<td>0.21</td>
</tr>
<tr>
<td>$10^{-3}Q_{d0}/e$</td>
<td>−2.7</td>
<td>−2.9</td>
</tr>
<tr>
<td>$\tilde{n}<em>i/n</em>{d0}$ (%)</td>
<td>13.6</td>
<td>13.7</td>
</tr>
<tr>
<td>$\widehat{\phi}$ (mV)</td>
<td>171.1</td>
<td>152.9</td>
</tr>
</tbody>
</table>
qualitative agreement with solutions A and B for positive $\beta$. In Fig. 4.2(c), however, I find a striking disagreement for solution E, which has a negative $\beta$.

Figure 4.2: Dispersion relation data. Symbols are from the two experiments while curves are for the theoretical dispersion relation, Eq. (3.28). The theoretical curves were fit to the experimental $\omega$ and $k_i$ for the main experiment. The curves are shown for five solutions listed in Table 4.1; their agreement with experiment is tested in steps two and four of my analysis.
4.4 Results and discussion

I find that for the wide range of $\beta$ that I explored, only a positive value is possible for my experimental conditions. The value of $\beta$ is in the range $+2.9 \times 10^4 \text{ Pa}^{-1}$ to $+9.6 \times 10^4 \text{ Pa}^{-1}$, corresponding to the four best solutions, Table I. The most likely value is about $+9 \times 10^4 \text{ Pa}^{-1}$, which is the midpoint of solutions A and B. In coming to this conclusion, I made very conservative allowances for experimental errors and systematic errors in the dispersion relation model.

Now, I compare this result for the compressibility to two cases. First, I compare it with a weakly-coupled dusty plasma, which in the collisionless limit obeys an ideal gas law so that $\beta = 1/(n_d k_B T_d)$. This has a value of $+3.0 \times 10^9 \text{ Pa}^{-1}$, for the mean value of $n_d$ for solutions A and B and assuming that the dust fluid kinetic temperature is $T_d = 1/40 \text{ eV}$ (which approximates room temperature conditions without the collisional heating associated with finite amplitude waves). Comparing with the experimental result, I find that the compressibility of the dusty plasma is four orders of magnitude smaller than that of a weakly-coupled dusty plasma.

Second, I compare it with an estimate of the compressibility using dimensional analysis noting that $\beta$ has the dimensions of inverse pressure, i.e., volume divided by energy. The potential energy of a dust-particle pair is of order $U = Q_d^2 / 4\pi\varepsilon_0 n_d^{-1/3}$. Thus, I estimate that $\beta$ is roughly $\beta_0 = 1/n_d U = 4\pi\varepsilon_0 Q_d^2 n_d^{4/3}$ for a strongly coupled plasma. I find that the compressibility, $\beta$, of the dusty plasma is $0.06\beta_0$ in dimensionless units.

4.5 Conclusions

I performed a DAW experiment to resolve a discrepancy in the literature regarding the sign of the compressibility of a strongly-coupled dust component in a dusty plasma. I find that $\beta > 0$ which in disagreement with the three theories, which all have $\beta < 0$. One must ask whether this discrepancy is due to shortcomings
in the experiment or in the theories. For the experiment, a possible shortcoming is my assumption that the collection of dust particles can be described as a continuum. The observed wavelength corresponds to about sixteen interparticle spacings, i.e.,

\[ 2\pi n_{d0}^{1/3} / k_r \approx 16. \]

In other dusty plasma experiments, a continuum description was found to be useful for even shorter scale lengths, for hydrodynamic parameters such as viscosity [132] but I have not assessed whether this is true also for compressibility. I believe that a more likely problem lies in the three theories. They all have two treatments of electrons and ions that may not be consistent. In one treatment, they use a \( \chi_d \) derived using from one-component-plasma treatment of the dust, which describes electrons and ions as being fixed, in a non-self-consistent way. In the other treatment, in \( \chi_e \) and \( \chi_i \) electrons and ions are described again a second, more self-consistent way, by solving their separate equations of motion to yield \( n_e \) and \( n_i \), for use in Poisson’s equation to obtain the dispersion relation.
CHAPTER 5
POLYGON CONSTRUCTION TO INVESTIGATE MELTING IN A
2D STRONGLY COUPLED DUSTY PLASMA

5.1 Introduction

As I described in Ch. 1, a strongly-coupled plasma is a collection of charged particles that have neighboring interaction potential energies larger than their kinetic energies. The dust particles in a dusty plasma comprise a strongly-coupled plasma due to their large charge which is acquired by collecting electrons and ions from the background plasma.

In general, unlike more common weakly-coupled plasmas, strongly-coupled plasmas can exhibit the properties of liquids or solids [133]. The melting transition between solid-like and liquid-like phases of strongly-coupled plasmas has been studied for many years. Many of the earliest melting theories and simulations were carried out assuming a $1/r$ potential, i.e., the one-component plasma (OCP) model [116, 134, 135, 136, 137, 138]. Experiments appeared later, when apparatus to cool and confine pure-ion plasmas was developed [139, 140]. Later, simulations of melting in plasma were carried out assuming a Yukawa (Debye-Hückel) potential [141, 142, 143, 144, 145].

Model systems are often used in the field of condensed matter physics for the study of crystallization and melting. These systems usually allow direct observation of the positions and motions of the constituent particles. A colloidal suspension is an example of a model system that allows imaging of the positions of particles, which is ideal for an experimental study of crystallization and melting at an atomistic scale [146, 147].

Dusty plasmas can serve as another model system and provide a practical
way to perform laboratory experiments of crystallization and melting in a strongly-coupled plasma, Fig. 5.1. A Dusty plasma allows imaging of particles and their motion with video microscopy [93, 148, 149]. It also allows manipulation of dust particles, for example by applying radiation pressure forces from laser beams to increase the kinetic temperature [150, 151].

Figure 5.1: Video microscopy images of a single layer dusty plasma in a solid-like state (a) and a liquid-like state (b). The solid is highly-ordered with long linear chains whereas the liquid is a disordered state. Image courtesy of Yan Feng.

One approach to study melting theoretically and experimentally is with two-dimensional (2D) systems. Melting experiments have been reported with many 2D or quasi-2D physical systems, including colloidal suspensions [152, 153, 154], electrons on liquid helium surface [155], and gases adsorbed on graphite surfaces [156]. Two-dimensional melting experiments have also been performed in dusty plasmas with a single layer of dust particles suspended in a glow discharge plasma [150, 157, 158, 159, 160, 161]. Quasi-2D melting experiments have also been performed in dusty plasmas, with multiple layers [162, 163, 164, 165, 166].

A defect is a deviation of structure from an ideal ordering and they play a central role in several descriptions of 2D melting [167, 168, 169, 170, 171, 172, 173].
A common way of identifying defects in 2D systems is an analysis of coordination number, that is, the number of nearest neighbors. For example, in a perfect 2D triangular lattice every particle has a coordination number of six. A defect is present if there is a particle with a coordination number that is not equal to six.

Voronoi analysis is commonly used in 2D simulations and in 2D experiments to identify defects. This analysis divides a 2D plane into a polygon network in which the center of each polygon coincides with the location of a particle. The number of sides of a polygon is equal to the coordination number of the central particle.

Voronoi analysis starts with measurement of particle positions of individual particles, Fig. 5.2(a). Next, a map of bonds between particles is calculated by a Delaunay triangulation, Fig. 5.2(b). This map consists of triangles that fills the plane with a vertex located at the position of each particle. By drawing perpendicular bisectors to these bonds, a map of polygons can be obtained which is called a Voronoi diagram, Fig. 5.2(c). A perfect 2D lattice consists of triangles with hexagonal symmetry, and in such a lattice all the Voronoi polygons are six-sided. When the lattice is not perfect, the defects are identified as non-six sided polygons, with five or seven sides being the most common, as can be seen in Fig. 5.2(c). These defects identified by the Voronoi analysis are called topological defects.

Polygon construction [171, 172] is a different way of characterizing defects in a 2D structure with polygons. Like Voronoi polygons, these polygons form a network that covers the entire plane, Fig. 5.2(d). Unlike Voronoi polygons, however, the sides of these polygons coincide with bonds identified by triangulation, and there are usually fewer sides in the polygon and are called geometrical defects. Few examples of geometrical defects are the quadrilaterals, pentagons, and hexagons.

The polygon construction method has the advantage of providing a gradation in the severity of geometrical defects. Quadrilaterals are the least severe, while
pentagons, hexagons, etc. are progressively more severe. In contrast, defect identification using Voronoi analysis is usually a binary measure when used in dusty plasma experiments, i.e., at the location of a particle, there is either a topological defect, or there is not [164, 174]. The gradation of defects in polygon construction allows a greater sensitivity in identifying and classifying disorder.

![Figure 5.2](image)

Figure 5.2: A particle position map (a) produced by analyzing an experimental image. This is used to construct a triangulation map (b), which indicates bonds. The triangulation map can be used to compute a Voronoi diagram (c), which depicts topological defects, which are non-six-sided cells shown shaded. By removing bonds that are opposite large angles from the triangulation map, marked by bold lines in (b), I obtain the polygon construction (d). Non-three-sided polygons in (d) indicate geometrical defects, which do not have a one-to-one correspondence with the topological defects in (c).

I characterized melting in a 2D strongly-coupled dusty plasma using the more sensitive polygon construction. For this, I analyzed data from a 2D dusty plasma experiment conducted by Feng et al. [174]. This experiment was previously reported, along with Voronoi analysis [174].

I am one of the first to apply polygon construction to analyze experimental
data and in fact I only find two previous reports of its use in the literature. It was introduced with simulations to study melting in a two-dimensional system of particles interacting with a WCA potential [171, 172]. Later it was used for analyzing scanning tunneling microscope (STM) images from an experiment to study melting in a Cs overlayer adsorbed on a Si-Ag surface [175].

I used polygon construction to compare the 2D structure in the solid and liquid states, and also studied the development of structure during rapid heating. The rapid heating that was employed in the experiment of Feng et al. [174] yielded a short-lived superheated solid, i.e., a structure with few defects as in a solid but a temperature above the melting point. I further investigated the superheated solid, exploiting the sensitivity of polygon construction.

The order of melting in a 2D system in general has been a long debated issue. One of the difficulties in settling this issue is that there is no practical way of measuring the interparticle potential energy in a 2D experiment. I counted the abundance of polygons and used it as an indirect measure of the potential landscape to see whether there was any signature of latent heat which is an indication of a first-order melting transition.

5.2 Method

I will briefly summarize the experimental conditions and procedures used by Feng et al. [174] here. Argon gas at 7 mTorr pressure was partially ionized by applying a low power RF voltage at 13.56 MHz with a 97 V peak-to-peak amplitude. Monodisperse melamine-formaldehyde dust particles of diameter 4.83 μm were introduced into the plasma. The mutual repulsion of these dust particles in the experiment can be modeled by a Yukawa potential [176], with $-4360e$ for the electrical charge and 0.65 mm for the screening length. The dust particles were levitated in an electric sheath above the lower electrode. By limiting the number
of dust particles introduced into the plasma, a suspension was prepared with only a single layer and a lattice constant $b = 0.86 \text{ mm}$. In the absence of manipulation, the dust particles self-organized in a solid triangular lattice with six-fold symmetry.

In the experiment, Feng et al. [174] used video microscopy to track particle motion. A digital camera operated at 55 frames per second imaged a $34.2 \times 25.6 \text{ mm}^2$ region that contained about 1000 dust particles. The particle positions, which are the starting point for my polygon construction, were measured using an image analysis procedure [177] for each particle in each video frame.

In the experiment, Feng et al. [174] used laser manipulation to increase the kinetic temperature to cause melting. This laser manipulation scheme was previously developed by Nosenko et al. [151]. In the experiment the laser manipulation, which applied nearly random kicks to dust particles was sustained for 55 s. Afterwards, laser heating was abruptly turned off to allow a cooling due to gas friction. The resulting time series for kinetic temperature is shown in Fig. 5.3(a).

The kinetic temperature in the context of this thesis describes the kinetic energy of the random motion of the dust particles. This kinetic temperature is not a thermodynamic temperature, because the surrounding gas and plasma had different temperatures. The polymer material inside the dust particles probably had yet another temperature that is never hot enough to melt them. The melting that I study in this chapter is not for the polymer material itself, but rather for the spatial configuration of dust particles that are suspended in space.

5.3 Analysis

For polygon construction, I used the measurements of individual particle positions from the video images, Fig. 5.2(a). Next, a map of bonds between particles was calculated using a Delaunay triangulation, Fig. 5.2(b). Then, I removed unusually long bonds as shown in Fig. 5.2(b), yielding a polygon construction, Fig. 5.2(d).
Figure 5.3: Results from an analysis of the sudden-heating experiment of [174]. Starting with a low-temperature solid structure, laser heating was suddenly applied and then later turned off. (a) Time series of dust particle kinetic temperature reported in [174]. Times marked b-g corresponds to the panels below. (b)-(g) Polygon construction: triangles are non-defects, while quadrilaterals, pentagons, hexagons, etc. are geometrical defects. Starting with a solid (b), after laser heating is turned on a proliferation of geometrical defects is seen during melting, (c) and (d). Diminishment of geometrical defects is seen during cooling, (e)-(g), as crystallization gradually progresses.
In polygon construction it is necessary to define a somewhat arbitrary threshold when identifying unusually long bonds or unusually large angles for bond removal. I used the bond-angle approach, with the same 75° threshold as in [171, 172].

I used this polygon construction in three ways. First, to determine how geometrical defects proliferate and cluster as melting progresses, I inspected maps visually. Second, to characterize disorder during the formation of a superheated solid and subsequent melting, I counted types of polygons, for example triangles and quadrilaterals. Third, to quantify how defects tend to cluster, I classified each vertex according to the sequence of polygons around the vertex.

5.4 Results and discussion

5.4.1 Comparison of solid and liquid structures

The geometrical defects proliferate when sudden laser heating is applied, and then diminish after laser heating is stopped, as seen in Fig. 5.3(b)-(g). Initially, before applying laser heating, the collection of particles in the suspension has the structure of a solid. The polygon construction for this solid consists mainly of triangles, Fig. 5.3(b). Next, during the application of laser heating, particles move and non-triangular polygons proliferate in Fig. 5.3(c) and (d). During the cooling immediately after the laser heating stopped, non-triangular polygons diminish in Fig. 5.3(e) and (f). Finally, after the kinetic temperature drops to its original value, I find that the geometrical defects in the structure slowly become less numerous, Fig. 5.3(g).

To identify which polygon types proliferate the most during heating, I quantified them with order parameters $P_3$, $P_4$, $P_5$, and $P_6$. These order parameters, defined by Glaser and Clark [172], are a measure of the abundance of a particular polygon type. For example, $P_3$ and $P_4$ are the respective number of triangles and
quadrilaterals, normalized by $2N$. Here, $N$ is the number of analyzed particles. Time series results for these order parameters are presented in Fig. 5.4.

Figure 5.4: Time series for order parameters $P_3$ to $P_6$ and the kinetic temperature (normalized by its maximum). The abundance of geometrical defects, as indicated by $P_4$ to $P_6$, is greater in a liquid than in the initial solid. Quadrilaterals, indicated by $P_4$, are the most frequent geometrical defect (in both a liquid and a solid), while pentagons and hexagons are the next most frequent.

The time series of order parameter reveal that when comparing a liquid to a solid, quadrilaterals are the most abundant geometrical defect, followed by pentagons and hexagons. This ordering is as expected, since quadrilaterals are the least severe type of geometrical defect.

Geometrical defects in strongly-coupled dusty plasma tend to cluster, so that a polygon tends to be adjacent to another polygon of the same type. For example, quadrilaterals tend to adjoin other quadrilaterals, as was noticed previously by Glaser and Clark in their WCA simulations [171, 172, 173]. This tendency of quadrilaterals to cluster is noticeable in both the solid, Fig. 5.5(a), and the liquid,
Figure 5.5: Enlargements (a) of Fig. 2(b) for the initial solid and (b) Fig. 2(d) for a liquid. Quadrilaterals tend to cluster with other quadrilaterals, often forming ladder-like structures like the examples encircled here. Likewise, pentagons tend to cluster with other pentagons, as seen in (b).

Fig. 5.5(b). Additionally, in the liquid I observe that pentagons tends to cluster with other pentagons, as can be seen in Fig. 5.5(b).

When quadrilaterals cluster with one another, the result is often a ladder-like structure. Examples for the experiment are circled in Fig. 5.5(a) for a solid and Fig. 5.5(b) in a liquid. These ladder-like structures are often linear, but sometimes they bend or branch. Similar structures were observed in a WCA liquid simulation [171, 172, 173].

In order to better understand how the polygons adjoin, I used the vertex classification scheme of Glaser and Clark [172]. They identified 25 common configurations of polygons arranged around a vertex, and labeled them with the letters $A - Y$. Some of the vertex types that occur most frequently are sketched in Fig. 5.6.

In a perfect crystal, one would observe only vertex type $A$, where six triangles join.

To quantify the abundance of vertex types, I calculated the vertex fraction, which is the ratio of vertices of a certain type to the total number of vertices in the polygon construction [172]. For example, in Fig. 5.3(b), I counted 62 occurrences of vertex type $B$, and 1066 total vertices, so that I calculated the vertex fraction for type $B$ as $\frac{62}{1066} = 0.058$. 
Figure 5.6: The first 12 vertex types, among the 25 classified in [172]. Vertices are at the locations of particles. In a perfect 2D crystal, only type A would be found.

I find that in the dusty plasma, the vertex fractions are quite different in the solid and the liquid, Fig. 5.7(a). Vertex type A is the most common in the solid, but not in the liquid. The most common vertices in the liquid are B, C, E, F, and G, in that order. All those vertex types include a geometrical defect, and they are all more common than A in the liquid.

5.4.2 Hysteresis of structural disorder

Hysteresis is the delay in response of a system property to an external stimulus and it is observed in fields like natural sciences, engineering, and mathematics. For example ferromagnetic materials exhibit hysteresis by retaining magnetization even after the external magnetic field is removed.

A hysteresis diagram is typically prepared by combining time series measurements for two quantities, a system property and a measure of the external stimulus. For an example in a ferromagnetic material, measurements of magnetization M and applied field H are combined into a single graph with M as the vertical axis and H as the horizontal axis.

In this chapter, hysteresis I study is the delay in response of a structure to
Figure 5.7: Dusty plasma vertex fraction, calculated as the number of observations of a particular vertex type such as $A$ or $B$, divided by the total number of vertices observed. Error bars based on counting statistics are too small to plot. Many video frames were used in the calculation: 1000 frames for the solid, 2000 for the liquid, and 500 for the solid undergoing crystallization. The latter is for the time interval $92.2 - 100$ s.

a change in temperature. Hysteresis has been observed in previous melting experiments for example melting of lead implanted into aluminum [178], Lithium subjected to high pressure [179], Ar condensates in a porous Vycor glass [180], and Ga droplets confined in epoxy rasins [181]. In analyzing the hysteresis in these experiment, hysteresis diagrams were drawn by combining measurements of temperature and a measure of structure. Since these experimenters were unable to directly track particle motion, as in the dusty plasma experiments, they use an indirect measures of structure for example x-ray diffraction, resistance, or optical transmission.

Hysteresis occurred in the sudden heating experiment of Feng et al. due to a delayed response as in the case of rapid heating and a gradual response as in the case of rapid cooling [174]. Feng et al. [174] plotted a hysteresis diagram by
Figure 5.8: Hysteresis diagrams from combining a time series for kinetic temperature and a time series of a measure of disorder. Each data point corresponds to one video frame. (a) Hysteresis, as it was computed from Voronoi diagrams in [174], reveals a superheated solid when rapid heating commences, as indicated by the broken circle. (The defect fraction was calculated as the ratio of the area filled by five and seven-sided Voronoi polygons to the total area.) (b) Hysteresis, as computed from polygon construction diagrams, reveals additional features. Here, the time series for the disorder parameter $P_4 + P_5 + P_6$ was combined with the time series for kinetic temperature. I note two previously unremarked features marked $H$ and $C$. Observing these unremarked features, which are less prominent in (a), was made possible by the greater sensitivity of the polygon construction method.
combining two time series, the kinetic temperature and defect fraction computed by counting defects in a Voronoi diagram which I have replotted in Fig. 5.8(a).

I prepared a hysteresis diagram by combining a time series of kinetic temperature and disorder parameter calculated based on Polygon construction. I calculated the disorder parameter as $P_4 + P_5 + P_6$. Results for this hysteresis diagrams are shown in Fig. 5.8(b).

At the start of the experiment, the structure is solid, in the lower left corner of the hysteresis diagram, Fig. 5.8. When laser heating was applied, the kinetic temperature increases, ultimately leading to a liquid in the upper right corner. Later, during cooling, the temperature diminishes and the structure gradually crystallizes, eventually returning to the solid condition in the lower left corner.

I identify two previously unremarked features in the polygon-construction hysteresis diagram, Fig. 5.8(b). The first feature is an upward row of data points marked $\mathbb{H}$ that is observed in the sudden heating process. The second feature is a downward row of data points marked $\mathbb{C}$ that is observed during the cooling process. Both of these features appear near the melting point, where there is a significant change of disorder parameter without much change in kinetic temperature.

Features $\mathbb{H}$ and $\mathbb{C}$ are visible with a more profound signature when the hysteresis diagram is prepared using a measure of disorder on polygon construction Fig. 5.8(b). I attribute the clarity in detecting these features to the sensitivity of polygon construction method due to the gradation of geometrical defects as compared to the more binary measure of the presence of topological defect. These features may be indicative of latent heat and I describe this next.

5.4.3 Latent heat

Latent heat in a first-order transition corresponds to a change in interparticle potential energy without a change in kinetic temperature. The disorder parameter
in the polygon construction hysteresis diagram serves as an indirect measure of a change in the potential landscape. Thus, the hysteresis diagram shows how the potential landscape change with the kinetic temperature.

Latent heat is possibly involved in the melting and rapid cooling process of a 2D strongly coupled plasma, which would be an indication of a first-order melting transition. The features $\mathbb{H}$ and $\mathbb{C}$ in the polygon construction hysteresis diagram appear as vertical rows of data points in Fig. 5.8(b), suggesting that the potential landscape changes in these instances without a change in the kinetic temperature, and this is the signature of latent heat. To make a definitive conclusion of whether these features indicate a first-order transition would require measurements of the internal potential energy of the system.

5.4.4 Solid superheating

A superheated solid has the structure of a solid at a temperature above the melting point. Superheating can be achieved by two methods: suppressing surface melting, or transferring a large amount of energy to a bulk solid in a brief time.

The rapid heating that was employed in the sudden heating experiment of Feng et al. [174] yielded a short-lived superheated solid, that was identifiable in their hysteresis diagram Fig. 5.8(a). The signature of a superheated solid is a horizontal row of data points after the temperature exceeds the melting point, as indicated by a broken line. The signature of superheated solid can also be seen in the hysteresis diagram that I constructed by using polygon construction which is also marked with a broken line Fig. 5.8(b).

Polygon construction allows me to expand what is known about the structure of a superheated solid. Despite having more defects than a solid, a superheated solid has quadrilateral and pentagon defects with proportions that are like those for a solid, not a liquid. This is shown in the time series of the quadrilateral to
pentagon ratio, Fig. 5.9. I expect this ratio to have a smaller value as disorder increases, due to an increased proportion of pentagons as compared to the less severe quadrilaterals. In Fig. 5.9 this ratio is indeed much smaller for a liquid than for a solid. What is interesting is that during the time interval of a superheated solid, this ratio is almost the same as for a solid, not a liquid. Thus, I conclude that the defect structure of a superheated solid more closely resembles that of a solid than a liquid.

![Figure 5.9: Samples of the time series for $P_4/P_5$. This ratio diminishes as pentagons (which are a more severe geometrical defect) become more abundant as compared to quadrilaterals. Each data point here corresponds to one video frame. As judged by this measure of defect severity, a superheated solid more closely resembles a solid than a liquid.](image)

5.5 Conclusions

The polygon construction method of identifying geometrical defects in 2D structures is used to analyze data from a strongly coupled dusty plasma experiment
in several conditions: a solid, a superheated solid during rapid heating, a liquid, and a solid undergoing crystallization. I exploit the advantage of polygon construction that it distinguishes defects according to severity.

Comparing solids and liquids under steady conditions, I find that while both quadrilaterals and pentagons become more numerous in a liquid, their ratio does not remain constant. I find that the quadrilaterals tend to cluster with other quadrilaterals in ladder-like structures in both the solid and liquid. I also find that the pentagons tend to cluster with other pentagons in the liquid. In order to classify the arrangement of polygons, I measure the vertex fractions. I find that almost all vertex types, except for type A which is typical for a hexagonal crystal, become more numerous in a liquid as compared to a solid.

The hysteresis diagram I prepared from the polygon construction suggests that sudden heating has three steps. First, at a temperature near the melting point, there is an increase in disorder without much change in temperature. Second, a superheated solid appears at temperatures above the melting point. Third, the superheated solid melts, yielding a liquid. The first of these three steps was not remarked upon previously, when a hysteresis diagram was prepared using Voronoi analysis. Further experiments or simulations are needed to determine whether this step commonly occurs during sudden heating.

Finally, polygon construction is used to characterize the nature of geometrical defects present during the transient conditions of solid superheating and crystallization. Geometrical defects in a superheated solid are more numerous than in a solid, and less numerous than in a liquid. I find that the nature of the geometrical defects in the superheated solid more closely resembles that of a solid than a liquid. This conclusion was made possible by polygon construction, which allows a calculation of the ratio of quadrilaterals to pentagons.
Here I list the dimensionless parameters introduced in Section 3.2.

$$\Omega_n = \omega_{pi} \left\{ \frac{i k U_0}{\omega_{pi}} + D_0 D_3 \frac{U_0}{V_{Ti}} + k^2 \lambda_{Di}^2 D_2 - i D_1 D_2 D_3 k \lambda_{Di} \right\}$$  \hspace{1cm} (A.1)

$$\Omega_{\phi} = \omega_{pi} \left\{ k^2 \lambda_{Di}^2 D_2 - D_0 D_3 D_4 \frac{U_0}{V_{Ti}} \frac{m_a}{m_d} Z_d k^2 \lambda_{Di}^2 D_2 D_3 k \lambda_{Di} \right\}$$  \hspace{1cm} (A.2)

$$\Omega_{\phi s} = \omega_{pe} \left\{ \frac{1}{\sqrt{2\pi}} \frac{a}{\lambda_{De}} e^{\eta} + \gamma \frac{a}{2 \lambda_{Di} \omega_{pe}} \frac{V_{Ti}}{U_0} \right\}$$  \hspace{1cm} (A.3)

$$\Omega_{V0} = \omega_{pe} \left\{ \frac{1}{\sqrt{2\pi}} \frac{a}{\lambda_{De}} e^{\eta} + D_0 \frac{a U_0}{3 \lambda_{De} \omega_{pe}} \frac{\Omega_{\phi}}{U_{in} - i (\omega - U_0 k)} \right\}$$  \hspace{1cm} (A.4)

These expressions use the dimensionless parameters: $\eta = e \phi_s/k_B T_e$, $D_0 = (1 - 2 e \phi_s/m_i U_0^2) \gamma$, $D_1 = (1 + 2 e \phi_s/m_i U_0^2) \gamma$, $D_2 = \omega_{pi}/[\nu_{in} - i(\omega - U_0 k)]$, $D_3 = \pi a^2 \lambda_{Di} n_{i0}$, and $D_4 = \omega_{pi}^2 / [\omega^2 - k^2/\beta n_d m_d + i \nu_{dn} \omega]$. Here, $\gamma = I_i/I_{OML}$ is an adjustable dimensionless parameter that allows the use of any ion current model. For the OML ion current model, $\gamma = 1$. 
APPENDIX B
PLASMA PARAMETERS

Here I list the plasma parameters used for solving the dispersion relation models discussed in Sec. 3.3. These parameters are based on the dust acoustic wave experiment by FG [49]. The value for $\beta$ is based on an empirical estimate of a sound speed in FG [49].

Table B.1: Input parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$n_{i0}$</td>
<td>$6.0 \times 10^{14}$ m$^{-3}$</td>
</tr>
<tr>
<td>$n_{e0}$</td>
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<td>$\nu_{dn}$</td>
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<tr>
<td>$\beta$</td>
<td>$1.83 \times 10^5$ Pa$^{-1}$</td>
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APPENDIX C
RESULTS FOR SENSITIVITY TO EXPERIMENTAL PARAMETERS

Here I report a sensitivity analysis of the dispersion relation solutions to experimental parameters, such as ambient gas pressure, which do not explicitly appear in my dispersion relations. I compute the exponents for $\delta_r$ and $\delta_i$ as described in Sec. 3.4.1 of Ch. 3. Results are listed in Table C.1 where the first column is the parameter $F$ in Eq. (3.30). Note that an entry of 0.00* in this table indicates that the magnitude of the exponent is less than 0.01.

Among the experimental parameters listed in the first column of Table C.1, most of them only affect one theoretical parameter in the dispersion relations. For example, the ion density $n_i$ only affects the ion plasma frequency $\omega_{pi}$. There are altogether five such parameters: the ion density, electron density, dust density, electron temperature, and ion temperature. The microscopic dc electric field $E_z$ and the ambient gas pressure $P$, however, affect more than one theoretical parameter in the dispersion relations. In particular, a change in $E_z$ (proportional to $U_0$ and inversely proportional to $Q_d$) affects the ion-drift speed and the dust charge due to the levitation requirement, whereas a change in $P$ (proportional to $\nu_{in}$, $\nu_{dn}$ and inversely proportional to $U_0$) affects the ion-neutral collision rate, dust-neutral collision rate, and the ion-drift speed.

As can be seen in the results in Table III, four of the seven parameters have large exponents, i.e., large $|\delta_r|$ or $|\delta_i|$. These four are: ion density, dust density, dc electric field, and gas pressure. Hence, errors in measurements of these parameters can greatly affect theoretical calculation of $\omega_r$ and $\omega_i$ using the dispersion relations.
Table C.1: Sensitivity to experimental parameters.

<table>
<thead>
<tr>
<th>$F$</th>
<th>Hydrodynamic models</th>
<th>Hybrid hydrodynamic-kinetic model</th>
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<tbody>
<tr>
<td></td>
<td>Baseline model with more processes</td>
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<tr>
<td>$\delta_r$</td>
<td>$\delta_i$</td>
<td>$\delta_r$</td>
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</tr>
<tr>
<td>$n_{d0}$</td>
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<td>$+2.06$</td>
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<tr>
<td>$T_e$</td>
<td>$-0.00^*$</td>
<td>$+0.06$</td>
</tr>
<tr>
<td>$T_i$</td>
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</tr>
<tr>
<td>$E_{z0}$</td>
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</tr>
<tr>
<td>$P$</td>
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<td>$-3.27$</td>
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APPENDIX D
PAPERS BASED ON THESIS


REFERENCES


