A study of magnetic fields in HII regions using Faraday rotation

Allison Hainline Costa

University of Iowa

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A STUDY OF MAGNETIC FIELDS IN H\textsubscript{II} REGIONS USING FARADAY ROTATION

by

Allison Hainline Costa

A thesis submitted in partial fulfillment of the requirements for the Doctor of Philosophy degree in Physics in the Graduate College of The University of Iowa

May 2018

Thesis Supervisor: Professor Steven Spangler
This is to certify that the Ph.D. thesis of

Allison Hainline Costa

has been approved by the Examining Committee for the thesis requirement for the Doctor of Philosophy degree in Physics at the May 2018 graduation.

Thesis Committee:

Steven Spangler, Thesis Supervisor

Cornelia Lang

Shea Brown

Kenneth Gayley

Charles Kerton

Robert Mutel
To my husband, Tim, who has given me endless support over the past 10 years.

For my mother, who has always believed in me and encouraged me.

For my brother, who has challenged me as only an older brother can.

For my late father, who took me stargazing.
ACKNOWLEDGMENTS

I would like to thank my advisor, Steven Spangler, for his guidance throughout my career. With his careful mentoring, I have matured as a researcher and writer. I enjoyed my time as his student immensely.
Massive young stars dynamically modify their surroundings, altering their stellar nurseries and the gas that exists between stars. With my research, I assess the modification of the Galactic magnetic field within H\textsc{ii} regions and stellar bubbles associated with OB stars. Because H\textsc{ii} regions are plasmas, magnetic fields should be important to the dynamics of the region. Understanding how the magnetic field is modified in these structures is critical for inputs to simulations and for assessing stellar feedback. To obtain information on the properties of the magnetic field, I measure the Faraday rotation of linearly polarized radio waves that pass through the plasma of the H\textsc{ii} region.

In this thesis, I present results of Faraday rotation studies of two Galactic H\textsc{ii} regions. The first is the Rosette Nebula (l = 206°, b = -1.2°), and the second is IC 1805 (l = 135°, b = 0.9°), which is associated with the W4 Superbubble. I measure positive rotation measure (RM) values in excess of +40 to +1200 rad m$^{-2}$ due to the shell of the Rosette nebula and a background RM of +147 rad m$^{-2}$ due to the general interstellar medium in this area of the Galactic plane. In the area of IC 1805, I measure negative RM values between +600 and –800 rad m$^{-2}$ due to the H\textsc{ii} region. The sign of the RM across each H\textsc{ii} region is consistent with the expected polarity of the large-scale Galactic magnetic field that follows the Perseus spiral arm in the clockwise direction, as suggested by Van Eck et al. (2011, ApJ, 728, 14).

I find that the Rosette Nebula and IC 1805 constitute a “Faraday rotation anomaly”, or a region of increased RM relative to the general Galactic background value. Although the RM observed on lines of sight through the region vary substantially, the |RM| due to the nebula is commonly 100 – 1000 rad m$^{-2}$. In spite of this,
the observed RMs are not as large as simple, analytic models of magnetic field amplification in H\textsc{ii} regions (such as by magnetic flux conservation in a swept-up shell) might indicate. This suggests that the Galactic field is not increased by a substantial factor within the ionized gas in an H\textsc{ii} region. Finally, these results show intriguing indications that some of the largest values of $|RM|$ occur for lines of sight that pass outside the fully ionized shell of the IC 1805 H\textsc{ii} region, but pass through the Photodissociation Region (PDR) associated with IC 1805.
PUBLIC ABSTRACT

Massive young stars dynamically modify their surroundings, altering their stellar nurseries and the gas that exists between stars. These massive stars create regions of ions and free electrons (plasmas) in which the magnetic field is “frozen”, so magnetic fields should be important to the dynamics of these regions. Magnetic fields may provide support against gravity, which would suppress future star formation in the region, or magnetic fields may suppress turbulence, which would encourage star formation.

With my research, I investigate how the Galactic magnetic field is modified in these regions with respect to the ambient (external) medium. In this work, I present results of a study of two regions in the Milky Way where massive stars have modified their surrounding. The stars are relatively young and nearby in the Perseus Arm of the galaxy. From my analyses, it appears that the Galactic magnetic field is not sensitive to the presence of these regions because the magnitude and the direction of the magnetic field is not modified due to the plasma in these two regions.
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CHAPTER 1
INTRODUCTION TO THESIS RESEARCH

1.1 Introduction

OB associations energize the interstellar medium (ISM) via their stellar winds and by photoionizing the surrounding gas. In doing so, they modify the material in the ISM and interact with the pervasive Galactic magnetic field, which is locked into the ISM plasma. While shocks from stellar winds and ionization fronts may quench or trigger star formation in the region, it is necessary to also include magnetic fields in stellar feedback models. Models that include magnetic field information suggest that magnetic pressure may provide support against gravitational collapse (Passot et al., 1995; Körtgen & Banerjee, 2015); however, recent simulations by Zamora-Avilés et al. (2018) suggest that magnetic fields can suppress turbulence, which then promotes star formation. Observations of magnetic field properties in regions of recent star formation provide constraints on stellar feedback simulations and on models to understand magnetic fields in star forming environments.

Magnetic fields should have a direct impact on the growth of stellar bubbles and H\textsc{ii} regions, as well as the future star formation in the region. Magnetic fields are frozen into the plasma of the H\textsc{ii} region, and the stellar winds of the star can transport magnetic field lines out into the ISM. The simple case of radial expansion of the stellar wind and H\textsc{ii} region is modified by the presence of the swept up magnetic field lines. The shells of magnetized bubbles can be elongated in the direction of the field lines as well as thicken the shells of the bubbles (Ferrière et al., 1991).

With my research, I assess how the Galactic magnetic field is modified near OB associations by probing the magnetic field within the shells of H\textsc{ii} regions, wind-blown bubbles, and peripheries of the H\textsc{ii} regions. By determining the orientation
and magnitude of the magnetic field in these dynamic structures, I investigate links between the properties of the magnetic field and the environment, such as star formation activity, structure of the H\foreignlanguage{en}{\textsc{ii}} region, and the wind luminosity of the stars responsible for the H\foreignlanguage{en}{\textsc{ii}} region.

As part of this work, I am developing techniques to identify and interpret magnetic field information along complex lines of sight that may, for example, contain multiple media with turbulent magnetic fields. This work is important to studies of magnetic fields in other environments such as the solar corona, and the analyses and discussions in this work are applicable to magnetic fields in extragalactic environments as well.

With my research, I attempt to answer the following questions. 1) How is the Galactic magnetic field altered, in either magnitude or direction, due to the presence of wind-blown bubbles and H\foreignlanguage{en}{\textsc{ii}} regions? 2) To what extent is the modification dependent on properties of the exciting star cluster (e.g., stellar content, wind luminosity, extent of the bubble)? To answer these questions, I have undertaken observing projects of two Galactic H\foreignlanguage{en}{\textsc{ii}} regions with the NSF’s Karl G. Jansky Very Large Array (VLA).

1.2 Faraday Rotation as a Measurement of Magnetic Fields

Faraday rotation is a wavelength dependent rotation of the plane of polarization of a wave as it propagates through a magnetized thermal plasma, which is described by

\[
\chi = \chi_0 + \left[ \frac{e^3}{2\pi m_e^2 c^4} \int_\text{source}^{\text{observer}} n_e \mathbf{B} \cdot ds \right] \lambda^2,
\]

where \(\chi\) is the polarization position angle of a radio wave after propagating through a plasma, \(\chi_0\) is the intrinsic polarization position angle of the radio wave before propagating through a plasma, \(e, m_e,\) and \(c\) are the usual fundamental physical constants, \(n_e\) is the electron density of the plasma, \(\mathbf{B}\) is the vector magnetic field, \(ds\)
is a vector increment of the path length from the source to the observer, and \( \lambda \) is the wavelength.

The quantity that contains information on the magnetic field and the thermal electron density of the rotating medium is the Faraday depth, \( \phi \), which can be written in convenient units as

\[
\phi = 0.81 \int n_e (\text{cm}^{-3}) B (\mu \text{G}) \cdot ds (\text{pc}) \text{ rad m}^{-2},
\]

For a radio wave that propagates through a uniform magneto-ionic plasma, then \( \chi = \chi_0 + RM\lambda^2 \), where RM is equal to \( \phi \).

To probe the plasma structure of \( \text{H II} \) regions and stellar bubbles, I perform polarimetric observations of extragalactic radio sources with lines of sight through or near to an \( \text{H II} \) region. The sources of choice are distant galaxies behind the nebula, and they act as point-like probes of the plasma shell of the \( \text{H II} \) region. By observing many lines of sight through a number of \( \text{H II} \) regions, I can assess the modification of the magnetic field in different plasma environments.

### 1.3 Previous Results of Importance

Three previous works are of primary interest to this line of research. The first is the study of the Cygnus OB1 by Whiting et al. (2009), in which they probed the Cygnus OB1 association with Faraday rotation to confirm a “Faraday Rotation Anomaly” for this region. They confirmed the anomaly, which is a large change in the RM over a small distance in the sky, and Whiting et al. (2009) attributed this anomaly to the plasma bubble associated with Cygnus OB1. The main aspect of the Whiting et al. (2009) paper that is of interest is the simple shell model that was developed to model the change in magnitude and sign of the RM in the Cygnus region.
The second work is that of Harvey-Smith et al. (2011). In this study, Harvey-Smith et al. (2011) measured the electron density and the line of sight magnetic fields in five Galactic H\textsc{ii} regions using Faraday rotation and H\textalpha measurements. They found that each H\textsc{ii} region has a coherent magnetic field, and in contrast to the Whiting et al. (2009) study, they concluded that there was not an amplification of the magnetic field in the shells of these H\textsc{ii} regions (Harvey-Smith et al., 2011).

In Savage et al. (2013), I performed polarimetric observations of 23 lines of sight that pass through or near to the shell of the Rosette Nebula H\textsc{ii} region. I obtained RM measurements for these lines of sight and compare the observations to simple shell models to determine if there is an amplification of the magnetic field due to the shell of the Rosette Nebula H\textsc{ii} region. This work comprised the majority of my Master of Science thesis, and the following chapters in this document expand greatly on this initial work.

1.4 Outline of Thesis

I have studied two wind-blown bubbles and H\textsc{ii} regions: the Rosette Nebula and IC 1805. The OB associations associated with these H\textsc{ii} regions are NGC 2244 and OCl 352, respectively. Since the completion of the upgraded VLA, the new observations of background sources through the Rosette Nebula and IC 1805 have a spectral coverage of 2 GHz, so it is possible to employ RM Synthesis to determine RM values. Additionally, I employ a Bayesian statistical analysis to compare the two forms of the simple shell model, which I utilize to reproduce the spatial distribution of the |RM| from the center of the nebula.

In Chapter 2, I discuss follow up observations of 11 lines of sight near the Rosette H\textsc{ii} region and a new conclusion for the amplification of the magnetic field in the shell. The results of IC 1805 are presented in Chapter 3. Within this chapter, I also discuss a larger picture of the magnetic field within the shells of the Rosette
and IC 1805 H\textsc{ii} regions.

In Chapter 4, I discuss supporting work that is not formerly presented in Costa et al. (2016) and Costa & Spangler (2018). This includes an analysis of spectral indices of radio sources in Section 4.1, the results of a Faraday complexity study in Section 4.2, a discussion of photodisassociation regions near the Rosette and IC 1805 in Section 4.3, and a review of Faraday rotation studies of H\textsc{ii} regions from the literature in Section 4.4. Finally, I summarize and provide concluding remarks in Chapter 5, and I discuss possible future work in Chapter 6.
CHAPTER 2
DENSER SAMPLING OF THE ROSETTE NEBULA WITH
FARADAY ROTATION MEASUREMENTS: IMPROVED ESTIMATES
OF MAGNETIC FIELDS IN H II REGIONS

This chapter is taken directly from Costa et al. (2016), which was published in
the Astrophysical Journal.

2.1 Introduction

Throughout the main sequence lifetimes of O and B stars, these massive stars
modify the surrounding matter by photoionizing it, creating an H II region, and
through their stellar winds. The stellar wind expands out into the interstellar medium
(ISM), sweeping up material, and inflating a bubble of hot, ionized gas. The Weaver
et al. (1977) solution for a stellar bubble inflated by the stellar wind of a single star
consists of four regions. These regions are (see Figure 1 of Weaver et al. 1977) (a)
the inner region closest to the star with the hypersonic stellar wind, (b) a bubble of
hot, low density gas, (c) an annular shell of swept up shocked ISM gas, which may
constitute part or all of the observed H II region, and (d) the ambient ISM exterior to
the bubble. In Figure 1 of Weaver et al. (1977), region (c) constitutes an H II region.
A further diagram of an H II region is shown in Figure 3 of Weaver et al. (1977). H II
regions are plasmas, and magnetic fields affect the dynamics of H II regions and stellar
bubbles through magnetic pressure and magnetic tension. In magnetohydrodynamic
(MHD) simulations, magnetic fields can elongate the shells of stellar bubbles prefer-
entially in the direction of the field lines and may thicken the shell perpendicular to
the magnetic field, altering the shape of the stellar bubble (Ferrière et al., 1991; Stil
et al., 2009). Possible observations of the elongation of young bubbles with respect
to magnetic fields are discussed in Pavel & Clemens (2012). However, magnetic field
properties are difficult to measure in \( \text{H}_\text{II} \) regions.

The goal of this research is to understand how the general interstellar magnetic field, \( B_{\text{ISM}} \), is modified in the interior of the \( \text{H}_\text{II} \) region. The technique we employ to investigate the role of magnetic fields in stellar bubbles is Faraday rotation. Faraday rotation is the rotation of the plane of polarization of a radio wave as it passes through a plasma that contains a magnetic field and is described by

\[
\chi = \chi_0 + \left[ \left( \frac{e^3}{2\pi m_e^2 c^4} \right) \int_{\text{source}}^{\text{observer}} n_e \, B \cdot ds \right] \lambda^2, \tag{2.1}
\]

where \( \chi \) is the polarization position angle of a radio wave after propagating through a plasma, \( \chi_0 \) is the intrinsic polarization position angle of the radio wave before propagating through a plasma, \( e, m_e, \) and \( c \) are the usual fundamental physical constants, \( n_e \) is the electron density of the plasma, \( B \) is the vector magnetic field, \( ds \) is a vector increment of the path length from the source to the observer, and \( \lambda \) is the wavelength. The quantity in the square brackets in Equation (2.1) is termed the rotation measure (RM) and can be written in convenient units as

\[
\text{RM} = 0.81 \int n_e \, (\text{cm}^{-3}) \, B \, (\mu \text{G}) \cdot ds \, (\text{pc}) \, \text{rad} \, \text{m}^{-2} \tag{2.2}
\]

(Minter & Spangler, 1996), where the term in the parentheses in Equation (2.1) equals 0.81 in these units. To obtain information on the magnetic field, the electron density needs to be independently determined since the integrand in Equation (2.2) is the product of \( n_e \) and \( B \). Such independent measurements are provided by a number of techniques such as thermal radio emission (used in this paper), intensity of radio recombination lines, or pulsar dispersion.

The Rosette Nebula is a good candidate for this line of research, as it is an \( \text{H}_\text{II} \) region with an obvious shell structure and central cavity. Menon (1962) determined
that the Rosette Nebula is ionization bounded from radio continuum observations, which was later confirmed by Celnik (1983, 1985, 1986). Celnik (1985) determined the inner and outer radii of the shell and the electron density from radio continuum observations at 1410 and 4750 MHz with the 100 m telescope of the Max Planck Institut für Radioastronomie at Effelsberg.

The star cluster responsible for the H\textsc{ii} region is the OB stellar association NGC 2244. The nominal center that we adopt for the center of the H\textsc{ii} region is the center of NGC 2244, R.A.(J2000) = 06\textdegree31\textquoteright55\textsec, Dec.(J2000) = 04\textdegree56\textquoteright34\textsec (l =206.5, b = −2.1) (Berghöfer & Christian, 2002). The nebula is located 1600 parsecs away (Román-Zúñiga & Lada, 2008). The age of NGC 2244 is less than 4 Myr old (Perez et al., 1989), and there are 7 O type stars within the association (Park & Sung, 2002; Román-Zúñiga & Lada, 2008; Wang et al., 2008). The stellar winds of these stars are believed to have inflated a bubble of hot ionized gas around the star cluster, which provides the environment for our Faraday rotation study. The mass loss rates of these O stars have been estimated to be of order \( \dot{M} \sim 10^{-6} \ M_\odot \ yr^{-1} \) (Howarth & Prinja, 1989), terminal wind velocities of order \( v_{\text{term}} \sim 3000 \ km \ s^{-1} \) (Chlebowski & Garmany, 1991), and wind luminosities \( L_w \sim 10^{36} \ ergs \ s^{-1} \), where \( L_w = \frac{1}{2} \dot{M} v_{\text{term}}^2 \). However, recent studies by Bouret et al. (2005) and Mokiem et al. (2007) have shown that the mass loss rates of stars may be over estimated by a factor of 3–5 due to clumping in the winds. These lower mass loss rates and wind luminosities would modify the expected evolution of stellar bubbles in general, including the Rosette.

2.1.1 Previous Results on Faraday Rotation through H\textsc{ii} Regions

In Savage et al. (2013), we investigated the role of magnetic fields in H\textsc{ii} regions with polarimetric observations of extra galactic radio sources whose lines of sight pass through or close to the Rosette Nebula. We made observations of 23 background radio sources using the Karl G. Jansky Very Large Array. The background radio sources
were selected from the National Radio Astronomy Observatory VLA Sky Survey (NVSS) (Condon et al., 1998). Twelve sources had lines of sight within 1° of the nominal center of the nebula, and the remaining 11 sources had lines of sight that passed through an annulus of 1°–2° of the center.

Savage et al. (2013) measured a background RM due to the Galactic plane in this region of the sky of +147 rad m⁻² and an excess RM of +50 to +750 rad m⁻² due to the shell of the Rosette Nebula H II region (Table 3 in Savage et al. (2013)). We employed a physically motivated shell model developed by Whiting et al. (2009) to reproduce the magnitude and sign of the RM observed in the shell as a function of distance from the center of the nebula (Section 4.1 of Savage et al. (2013)). The simple shell model assumes spherical symmetry and that the electron density, \(n_e\), in the shell is an independently determined quantity. The parameters for the inner and outer radii of the shell and the electron density were adopted from Model I of Celnik (1985), where it is assumed that the electron density is uniform within an annulus with outer radius of \(R_0\) and inner radius of \(R_1\). For a strong shock, the shell model predicts that the largest value of the RM should be near the outer radius of the shell. This “rotation measure limb brightening” is due to the MHD shock that is conjectured to define the outer radius, \(R_0\), of the bubble. This shock strengthens the interstellar magnetic field and “refracts” it into the shock plane (see discussion in Section 2.5.1 below).

An assumption of the Whiting et al. (2009) model, employed in Savage et al. (2013), is that the entire Rosette Nebula H II region consists of a Weaver-style bubble, with the annular shell thickened by a process such as that suggested by Breitschwerdt & Kahn (1988). This model may not be strictly correct. However, we will employ it in this paper as a simple representation of a class of models in which the magnetic field is amplified at the interface with the HII region.

Harvey-Smith et al. (2011) conducted a Faraday rotation study of five Galactic
$\text{H}\,\text{II}$ regions in which they estimated the electron density and the line of sight component of the magnetic field and did not find an amplification of the general interstellar magnetic field due to the shell. In the report of Harvey-Smith et al. (2011), the increase in the RM is attributed entirely to an increase in the density of the shell. Savage et al. (2013) employed two simple models for an interstellar shell, one due to a wind-driven stellar bubble as well as one in which there is no change in the interstellar magnetic field, to investigate which physical situation better describes the magnetic field within the shell of the Rosette Nebula. Savage et al. (2013) argued that the case with an amplified magnetic field in the shell better described the observed magnitude and spatial distribution of the RM from the center of the Rosette Nebula, and suggested that “RM limb brightening” was present in the data. In this case, the $|\text{RM}|$ values are largest near the edge of the shell but then decrease abruptly outside the outer shock front as well as downstream of the contact discontinuity. However, this statement was more a suggestion than a conclusion due to the relatively small number of lines of sight probed within the shell. Savage et al. (2013) concluded that either of these models was viable as a simplified description of the shell structure.

Recently, Purcell et al. (2015) performed a Faraday rotation and radio polarization study of the Gum nebula. They suggest that the Gum nebula is not a supernova remnant but is an $\text{H}\,\text{II}$ region surrounding a wind-blown bubble. The ionized shell model they employ has similarities to the one employed by Savage et al. (2013). The model of Purcell et al. (2015) reduces to Savage et al. (2013) when the nebula subtends a small angle. Purcell et al. (2015) perform a multi-parameter analysis to simultaneously determine the electron density, shell thickness, filling factor, and the magnetic field instead of assuming fixed parameters, as is done in this paper and Savage et al. (2013).

In this paper, we report and discuss additional data taken to clarify the structure of the Rosette Nebula bubble. We performed polarimetric observations of 11
additional sources observed through the shell of the Rosette Nebula. These sources produce values of RM along 15 lines of sight that are added to those reported in Savage et al. (2013). The combined and enlarged data set permits a more detailed study of the magnetic field within the Rosette Nebula bubble. The outline of this paper is as follows. Section 2.2 outlines the instrumental configuration and the observations performed. Section 2.3 describes the data reduction process, and the methods for determining the RM values. In Section 2.4, we report the results of the RM analysis and Section 2.4.1 compares the two methods of determining the RM. Section 2.5 describes the shell models, and in Section 2.5.2, we discuss the Bayesian statistical analysis employed to compare the two shell models. In Section 2.5.3, we modify our shell models to accommodate the inhomogeneity of the nebula. Finally, Section 2.6 contains our conclusions and summary.

2.2 Observations

We observed our target sources with the Jansky Very Large Array (VLA)\textsuperscript{1} radio telescope in C configuration on the 5th of February 2012. Table 2.1 lists the details of the observation. We selected 11 extragalactic radio sources with lines of sight within 1° of the nominal center of the Rosette Nebula to probe radii within the shell of the nebula. The sources are listed in Table 2.2 with their right ascension (\(\alpha\)), declination (\(\delta\)), Galactic longitude (\(l\)), Galactic latitude (\(b\)), the impact parameter of the line of sight from the center of the nebula (\(\xi\)), and the peak values of Stokes I intensity at 4.3 GHz. Figure 2.1 shows the positions of the sources from this work as well as those of Savage et al. (2013), plotted over a mosaic of the Rosette Nebula compiled from the Second Palomar Observatory Sky Survey\textsuperscript{2}.

\textsuperscript{1}The Very Large Array is an instrument of the National Radio Astronomy Observatory. The NRAO is a facility of the National Science Foundation, operated under cooperative agreement with Associated Universities, Inc.

\textsuperscript{2}The Second Palomar Observatory Sky Survey (POSS-II) was made by the California Institute of Technology with funds from the National Science Foundation, the National Geographic Society,
In addition to the 11 program sources in Table 2.2, we observed J0632+1022, J0643+0857, and 3C138 as calibrators. The flux density and polarization position angle calibrator for these observations is 3C138. The primary calibrator, J0632+1022, was used to determine the complex gain of the antennas as a function of time, as well as the instrumental polarization parameters (D factors). Similarly, we observed J0643+0857 as a secondary gain calibrator to independently determine the instrumental polarization and confirm the solutions for the D factors. All sources were observed in a single eight-hour observing run. Five-minute observations of the targets were interleaved with observations of J0632+1022 and J0643+0857, which guarantees that the calibrators have sufficient parallactic angle coverage to solve for the D terms.

2.3 Data Reduction

The data were reduced and imaged using the NRAO Common Astronomy Software Applications (CASA). The data were reduced following a similar procedure as in Savage et al. (2013). The procedure we implemented is as follows:

1. We initially flagged data using systematic flagging procedures (e.g., “Quack”) before visually inspecting the data to manually remove data corrupted by radio frequency interference (RFI). In the frequency ranges we observed, there was minimal RFI except in two subbands, 7378–7506 MHz and 7506–7634 MHz, which were flagged completely.

2. Calibration of the data included determining complex gains, instrumental polarization parameters, and the R–L phase difference. After applying the solutions to the data, we ran a CASA systematic flagging mode in FLAGMANAGER, “Rflag”. Rflag performs a second pass through a calibrated data set, reading the data in chunks of time and accumulating statistics to identify outlying data.
Table 2.1: Log of Observations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date of Observations</td>
<td>February 5, 2012</td>
</tr>
<tr>
<td>Duration of Observing Session (h)</td>
<td>7.6</td>
</tr>
<tr>
<td>Frequencies of Observation (GHz)</td>
<td>4.720; 7.250</td>
</tr>
<tr>
<td>Number of Frequency Channels per IF</td>
<td>512</td>
</tr>
<tr>
<td>Channel Width (MHz)</td>
<td>2</td>
</tr>
<tr>
<td>VLA array</td>
<td>C</td>
</tr>
<tr>
<td>Restoring Beam (diameter)</td>
<td>5.53</td>
</tr>
<tr>
<td>Total Integrated Time per Source</td>
<td>35 minutes</td>
</tr>
<tr>
<td>RMS Noise in Q and U Maps (mJy/beam)</td>
<td>0.029b</td>
</tr>
<tr>
<td>RMS Noise in RM Synthesis Maps (µJy/beam)</td>
<td>7.0c</td>
</tr>
</tbody>
</table>

- a The observations had 1.024 GHz wide intermediate frequency bands (IFs) centered on the frequencies listed.
- b This number represents the average RMS noise level for all the Q and U maps.
- c Polarized sensitivity of the combined RM Synthesis maps.

Table 2.2: List of Sources Observed

<table>
<thead>
<tr>
<th>Source Name</th>
<th>α(J2000)</th>
<th>δ(J2000)</th>
<th>l (°)</th>
<th>b (°)</th>
<th>ξ (arcmin)</th>
<th>I (mJy beam⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I9</td>
<td>06 32 08.15</td>
<td>04 14 26.8</td>
<td>206.95</td>
<td>-2.35</td>
<td>42.3</td>
<td>11.4</td>
</tr>
<tr>
<td>I11</td>
<td>06 32 47.83</td>
<td>04 38 22.3</td>
<td>206.68</td>
<td>-2.02</td>
<td>43.8</td>
<td>11.0</td>
</tr>
<tr>
<td>I13</td>
<td>06 33 28.81</td>
<td>04 08 37.4</td>
<td>206.19</td>
<td>-2.09</td>
<td>53.3</td>
<td>1.9</td>
</tr>
<tr>
<td>I17</td>
<td>06 34 38.88</td>
<td>05 36 42.5</td>
<td>206.02</td>
<td>-1.16</td>
<td>57.2</td>
<td>14.0</td>
</tr>
<tr>
<td>N4</td>
<td>06 31 45.53</td>
<td>05 23 49.9</td>
<td>205.88</td>
<td>-1.90</td>
<td>27.4</td>
<td>18.0</td>
</tr>
<tr>
<td>N5</td>
<td>06 32 52.66</td>
<td>05 21 45.9</td>
<td>206.04</td>
<td>-1.67</td>
<td>29.0</td>
<td>3.8</td>
</tr>
<tr>
<td>N6</td>
<td>06 30 32.34</td>
<td>04 56 11.0</td>
<td>206.15</td>
<td>-2.38</td>
<td>20.6</td>
<td>—</td>
</tr>
<tr>
<td>N7</td>
<td>06 30 36.62</td>
<td>04 55 28.2</td>
<td>206.17</td>
<td>-2.37</td>
<td>19.6</td>
<td>7.6</td>
</tr>
<tr>
<td>N8</td>
<td>06 30 41.82</td>
<td>04 55 14.2</td>
<td>206.18</td>
<td>-2.35</td>
<td>18.3</td>
<td>—</td>
</tr>
<tr>
<td>N9</td>
<td>06 30 28.13</td>
<td>04 41 58.6</td>
<td>206.35</td>
<td>-2.50</td>
<td>26.1</td>
<td>2.2</td>
</tr>
<tr>
<td>N10</td>
<td>06 33 39.67</td>
<td>05 09 40.9</td>
<td>206.31</td>
<td>-1.59</td>
<td>29.2</td>
<td>5.0</td>
</tr>
</tbody>
</table>

- a Angular distance between the line of sight and a line of sight through the center of the nebula.
- b Stokes I intensity at 4.3 GHz. Note, value is for the (a) component when lines of sight has more than one.
Figure 2.1: Palomar Sky Survey Mosaic of the Rosette Nebula with lines of sight probed by the present study and Savage et al. (2013). Measurements are represented by circles scaled to $\sqrt{|RM|}$. The red circles represent the new lines of sight observed in this work, and the gray circles are from our previous work (see Savage et al. (2013), Figure 1). The ‘X’s are depolarized sources, and the open circles are sources with negative RM values. All other RM values obtained are positive.

points. It then implements thresholds to auto flag these outlying data points. After running Rflag, a new calibration of the data was performed.

3. The calibrated visibility data were then imaged using the CASA task CLEAN to produce images in Stokes I, Q, U, and V. CLEAN is a Fourier transform task which forms the “dirty map” and “dirty beam” of the data, implements the CLEAN deconvolution algorithm, and then restores the image by convolving the CLEAN components with the restoring beam. The restoring beam was set to 5'53 for all the sources, and natural weighting was used in order to obtain the highest signal-to-noise-ratio (SNR). The images of the Stokes parameters I,
Q, U, and V were made in two ways. These two sets of images were the input
data for the two methods of determining the RM, described further in Sections
2.3.1.1 and 2.3.1.2.

(a) In the first approach, a single image of each Stokes parameter was made
in 128 MHz-wide subbands within each 1.024 GHz intermediate frequency
band (IFs). The actual spectrum utilized was slightly less than 128 MHz
because of discarded edge channels as well as channels lost to RFI. Since the
bandwidth of 128 MHz is non-negligible compared to the center frequency,
particularly for the lower frequency subbands, we used the “mfs” (multi-
frequency synthesis) mode in the task CLEAN. This algorithm takes into
account the different \((u, v)\) tracks corresponding to different frequencies.
The image resulting from mfs CLEAN corresponds to a frequency at the
center of the 128 MHz-wide subband. These images were the inputs to the
\(\chi(\lambda^2)\) analysis (Section 2.3.1.1).

(b) The second approach was to make I, Q, U, and V images for each 4 MHz-
wide piece of spectrum in our observations using the mode “channel” in
CASA, which averages two adjacent 2 MHz channels. The resulting set of
maps were used as input to the RM synthesis analysis (Section 2.3.1.2).

4. Finally, phase-only self-calibration was performed on all sources. For the sources
with sufficient SNR (i.e., SNR > 20)\(^3\), we performed two iterations of phase-
only self-calibration. The sources with SNR < 20 did not improve with phase-
only self-calibration.

\(^{3}\text{NRAO Data Reduction Workshop 2012}\)
2.3.1 Determination of Rotation Measures Using Two Techniques

For all 11 sources, we employ two methods to determine the RM for each source or source component. The first method (Section 2.3.1.1) was implemented in Savage et al. (2013) and consists of a least-squares linear fit of $\chi(\lambda^2)$. This is a technique that has traditionally been used to measure Faraday rotation from radio astronomical polarization measurements. The second method (Section 2.3.1.2) is Rotation Measure Synthesis (RM Synthesis) (Brentjens & de Bruyn, 2005). RM Synthesis exploits the large, multi-channel data sets generated by modern interferometers like the VLA and avoids some of the shortcomings of the $\chi(\lambda^2)$ fit. The following sections detail the imaging process for the two methods.

2.3.1.1 Rotation Measures from Least-Squares Fit of $\chi(\lambda^2)$

The output of the CASA task CLEAN is a set of images in Stokes I, Q, U, and V. With these images, we use the CASA task IMMATH to generate maps of linear polarized intensity $P$,

$$P = \sqrt{Q^2 + U^2},$$

and the polarization position angle, $\chi$,

$$\chi = \frac{1}{2} \tan^{-1} \left( \frac{U}{Q} \right).$$

Figures 2.2a and 2.2b are examples of maps for two sources, where the vectors are $\chi$, the gray scale is $P$, and the contours are the Stokes I intensity. Though some of the sources observed in this project are point sources to the VLA in C array, other sources like N10 (Figure 2.2b) have resolvable structure. For each source, we produce maps in each of the fourteen, 128 MHz-wide subbands. To obtain measurements of
χ, we select the pixel of the highest value of P of the source in the 4338 MHz map and measure χ at that location in each 128 MHz-wide subband.

Figure 2.2: Map of (a) N4 and (b) N10 at 4.85 GHz. The gray scale is the linear polarized intensity, P, the vectors show the polarization position angle, χ, and the contours are the Stokes I intensity with levels of -2, -1, 2, 10, 20, 40, 60, and 80% of the peak intensity, 24.5 mJy beam⁻¹ and 4.87 mJy beam⁻¹ for N4 and N10, respectively. The circle in the lower left is the restoring beam. In image (b), there are four resolved components of N10, and we call the northern component “a” in Table 2.3 and the following three components are marked “b”–“d” in a clockwise direction from “a”.

Figure 2.3: Plot of the polarization position angle as a function of the square of the wavelength, χ(λ²), for the source N4, RM= +1383 ± 18 rad m⁻². The reduced χ² is 1.5. Each plotted point results from a measurement in a single 128 MHz-wide subband.
Figure 2.4: Graph of the percent polarization, \( m \), as a function of \( \lambda^2 \) for sources (a) I9 and (b) I17. These sources show a decrease in \( m \) with increasing \( \lambda^2 \), which is characteristic of depolarization. The dashed line shows the median value of the percent polarization.

We obtain RM measurements by carrying out a least-squares linear fit of \( \chi(\lambda^2) \), where RM is the slope of the line. Figure 2.3 shows an example of the least-squares fit for the source N4. The errors in \( \chi \) are \( \sigma_\chi = \frac{\sigma_Q}{P} \) (Everett & Weisberg 2001, Equation (12)), where \( \sigma_Q = \sigma_U \) is the rms noise in the Q map. This method assumes that there is only one source along the line of sight, unaffected by beam depolarization and without internal Faraday rotation. Depolarization manifests itself as a change in \( m=P/I \) with frequency. Depolarization may occur for a number of reasons, such as external Faraday dispersion, multiple interfering RM components, differential Faraday rotation, or internal Faraday rotation (Burn, 1966; Sokoloff et al., 1998; O’Sullivan et al., 2012).

For all sources, we inspected plots of \( m(\lambda^2) \) to determine if there were depolarization effects. Figures 2.4a and 2.4b show the graphs for the two sources, I9 and I17, respectively, which show depolarization. Despite the detection of depolarization for these two sources, we do not believe our resultant RM values to be biased or in error. The reduced \( \chi^2 \) of the fit of \( \chi(\lambda^2) \) is satisfactory and suggests no departure from the \( \chi \) vs \( \lambda^2 \) dependence (Table 2.3).
2.3.1.2 RM Measurements via the Technique of RM Synthesis

The basic mathematics and physics of RM Synthesis was first discussed by Burn (1966). However, the implementation of the fundamental idea with data from modern, wide bandwidth interferometers is due to Brentjens & de Bruyn (2005). With the upgraded continuous spectral coverage of 1–2 GHz of the VLA, it is possible to implement RM Synthesis for VLA polarimetric studies.

The Faraday dispersion function, $F(\phi)$, is a function of Faraday depth, $\phi$, and is related to the observed quantity, the complex polarized flux, $\tilde{P}(\lambda^2)$, where $P = Q + iU$ (Sokoloff et al., 1998). This function is defined as $\tilde{P}(\lambda^2) = P(\lambda^2)W(\lambda^2)$, and $W(\lambda^2)$ is a weighting function that is zero for $\lambda^2 < 0$ (Brentjens & de Bruyn, 2005; Heald, 2009). It is also zero for wavelengths at which observations do not exist, including wavelengths excised for RFI, and for wavelengths at which measurements exist, it is weighted by $1/\sigma^2$, where $\sigma^2$ is the variance. Other possible weighting schemes include uniform weighting, where $W(\lambda^2)$ is unity for $\lambda^2 > 0$ and at the wavelengths at which measurements were made.

RM Synthesis utilizes a Fourier transform relationship to convolve $F(\phi)$ with the rotation measure spread function (RMSF). Mathematically, RMSF, or $R(\phi)$, is

$$R(\phi) = K \int_{-\infty}^{\infty} W(\lambda^2) e^{-2i\phi\lambda^2} d\lambda^2,$$

(2.3)

where $\lambda$ is the wavelength, and the output of the convolution is the reconstructed Faraday dispersion function, $\tilde{F}(\phi)$,

$$\tilde{F}(\phi) = F(\phi) * R(\phi) = K \int_{-\infty}^{\infty} \tilde{P}(\lambda^2) e^{-2i\phi\lambda^2} d\lambda^2,$$

(2.4)
where $K$ is a normalization function given by

$$K = \left( \int_{-\infty}^{\infty} W(\lambda^2) d\lambda^2 \right)^{-1},$$

(see Brentjens & de Bruyn (2005) for the full derivation). To recover $F(\phi)$, $\tilde{F}(\phi)$ is deconvolved via a CLEAN algorithm such as the ones discussed in Heald (2009) and Bell & Enßlin (2012). In the case of a single point source behind a Faraday screen, $F(\phi)$ is a delta function at a Faraday depth equal to the RM through the screen. “Faraday Complexity” (Anderson et al., 2015) will lead to a broadening of the Faraday dispersion function, $F(\phi)$, and $Q$ and $U$ will have non-sinusoidal behavior. This broadening can also occur in the case of depolarization of a single component.

In practice, we utilize an IDL implementation of an RM Synthesis and CLEAN code. The inputs to the IDL code are FITS files of the CLEANed images of Stokes $Q$ and $U$ exported from CASA. The $Q$ and $U$ images are a function of frequency, $Q(\nu)$ and $U(\nu)$, and are composed of fourteen subbands containing twenty-four 4 MHz-wide channels. Ideally, an RFI free data set would permit the use of all sixteen

Figure 2.5: KVIS output map from the RM Synthesis analysis where the gray scale is the polarized intensity for the source N4 at $\phi=+1408$ rad m$^{-2}$. 

20
Figure 2.6: Top: Plot of the clean Faraday dispersion function, $F(\phi)$, for N4 at the position corresponding to the peak value of the linear polarized intensity determined in the $\chi(\lambda^2)$ analysis. The curve peaks at $+1408 \pm 16$ rad m$^{-2}$. The solid line is the amplitude, the dash-dot line is the real part, and the dashed line is the imaginary part. The red dotted line marks the $3\sigma_Q$ threshold. Bottom: Plot of RMSF.

128 MHz-wide subbands containing thirty-two 4 MHz channels. In practice, the edge channels within each subband are typically flagged and thus not included in the final analysis. The data set input to the RM Synthesis code is therefore a set of 336 images in Q and 336 images in U. The frequency spacing between images is 4 MHz and is approximately the effective bandwidth in each image, except for gaps due to discarded edge channels between subbands. There are also gaps due to discarded or heavily flagged subbands. The output from the IDL code is an image in Faraday depth space, which retains the astrometric headers (e.g., R.A. and Dec.). As CASA cannot yet read images in Faraday depth space, we use the Karma package (Gooch, 1995) tool KVIS to read the images. In KVIS, we extract the polarized flux from a single pixel at the location of peak linear polarized intensity in each Faraday depth plane to acquire the Faraday dispersion function spectrum. We then fit a Gaussian to the Faraday dispersion function to recover the RM at $F(\phi_{\text{max}})$. Multiple RM components may be present in the Faraday dispersion function and can be identified by comparing
the data to the RMSF. The Faraday dispersion function was measured at the same spatial location as used for the $\chi(\lambda^2)$ analysis, for each source or source component. This was done so that we can compare the two measurements.

For each source, we implemented a search range of $\phi = \pm 10000$ rad m$^{-2}$ to identify possible peaks at large values of $\phi$. We are sensitive up to $\phi_{\text{max}} \sim 3.0 \times 10^5$ rad m$^{-2}$, the full width at half-maximum (FWHM) of the RMSF is $\phi_{\text{FWHM}} = 1067$ rad m$^{-2}$, and the largest detectable scale in Faraday depth space (max-scale) is 2085 rad m$^{-2}$ (see Equations (61)–(63) in Brentjens & de Bruyn 2005). In Table 2.3, column 5 lists the effective RM values derived from the peak of the Faraday dispersion function from the RM Synthesis analysis.

Figure 2.5 shows an example of a KVIS map for source N4 at $\phi = 1408$ rad m$^{-2}$. The top panel of Figure 2.6 is the Faraday dispersion function corresponding to one pixel from Figure 2.5, and the bottom panel plots the RMSF. Using the spatial location of the peak linear polarized intensity from the $\chi(\lambda^2)$ analysis, we select the same spatial location on the RM Synthesis map so that we may compare the two measurements. In general, this method samples the Faraday dispersion function at $F(\phi_{\text{peak}})$ and at $P_{\text{max}}$ in the RM Synthesis data. The exception to this is N9(b). The spatial location of $F(\phi_{\text{max}})$ in the RM Synthesis map does not coincide with the location in the $\chi(\lambda^2)$ maps, which is most likely due to the values of $P$ not exceeding the $5\sigma_Q$ threshold that is implemented in the imaging process in CASA (see Section 2.4.1) in the $\chi(\lambda^2)$ analysis. The reported RM value derived from the RM Synthesis analysis is spatially coincident with the $\chi(\lambda^2)$ value to maintain consistency.

### 2.4 Observational Results

For the 11 sources observed through the shell of the Rosette Nebula, we measured RM values for fifteen lines of sight, including secondary components. Table 2.3 lists the RM values and associated errors for the least-squares method and from
the RM Synthesis analysis. We have three lines of sight that do not pass through the shell of the nebula, I9, I13, and I17. Including the secondary components, we measure an average background RM due to the general ISM of $+146 \pm 37 \text{ rad m}^{-2}$. The background RM is in perfect agreement with the value from Savage et al. (2013). We see no evidence for a gradient in the measured background RM values over the $4^\circ$ diameter region centered on the nebula. We measure an excess RM ranging from $+40$ to $+1200 \text{ rad m}^{-2}$ due to the shell of the nebula. Figure 2.1 shows the new sources in red in combination with the results from Savage et al. (2013), where the symbols are scaled to $\sqrt{|RM|}$. Figure 2.7 plots the observed RM measurements as a function of distance from in the center of the Rosette Nebula in parsecs, using a distance of 1600 pc to the nebula (Román-Zúñiga & Lada, 2008).

Double or multiple sources with large changes in RM values can probe small scale structure in the Rosette Nebula. We use the variable $\Delta RM$ to indicate differences in the RM between components or parts of a source with angular separations larger than the restoring beam of $5''.5$. Of the 8 sources with RMs, 5 sources have more than one component that yield RM values. N10 has a maximum angular separation of $53''$ between the northern and the southern component and $13''$ between the eastern and western components. For I13 and I17, the angular separations are $30''$ and $10''$, respectively. In these cases, the $\Delta RM$ values are relatively small, nearly within the errors in the case of I17. The $\Delta RM$ values for N5 and N9, however, appear to be significant, with $\Delta RM \sim 350$ and $260 \text{ rad m}^{-2}$ for N5 and N9, respectively. The angular separation between the components for N5 is $18''$, and N9 has a separation of $50''$. The linear separations between the components for these sources are 0.14 and 0.39 pc, respectively. Our measurements of $\Delta RM$ may indicate small scale gradients in the electron density or line-of-sight component of the magnetic field in the Rosette Nebula. These variations could be due to inhomogeneities in the shell, or turbulent fluctuations in $n_e$ and $B_{\parallel}$ on spatial scales of 0.1 to 1 parsec.
Table 2.3: New Faraday Rotation Measurements through the Rosette Nebula

<table>
<thead>
<tr>
<th>Source</th>
<th>Component</th>
<th>RM$^a$ (rad m$^{-2}$)</th>
<th>Reduced $\chi^2$$^b$</th>
<th>RM$^c$ (rad m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I9</td>
<td>a</td>
<td>+318±24</td>
<td>1.2</td>
<td>+276±26</td>
</tr>
<tr>
<td>I11</td>
<td>a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I13</td>
<td>a</td>
<td>+39±33</td>
<td>3.2</td>
<td>+90±15</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>+130±24</td>
<td>0.6</td>
<td>+139±19</td>
</tr>
<tr>
<td>I17</td>
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<td>+81 ±4</td>
<td>2.1</td>
<td>+83±3</td>
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<tr>
<td></td>
<td>b</td>
<td>+116±8</td>
<td>0.8</td>
<td>+87±10</td>
</tr>
<tr>
<td>N4</td>
<td>a</td>
<td>+1383±18</td>
<td>1.5</td>
<td>+1408±16</td>
</tr>
<tr>
<td>N5</td>
<td>a</td>
<td>+1062±21</td>
<td>1.8</td>
<td>+1074±14</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>+1332±74</td>
<td>2.1</td>
<td>+1426±52</td>
</tr>
<tr>
<td>N6</td>
<td>a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N7</td>
<td>a</td>
<td>+697±17</td>
<td>0.8</td>
<td>+719±19</td>
</tr>
<tr>
<td>N8</td>
<td>a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N9</td>
<td>a</td>
<td>+175±20</td>
<td>1.9</td>
<td>+165±9</td>
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<td></td>
<td>b</td>
<td>+571±98</td>
<td>1.3</td>
<td>+421±51</td>
</tr>
<tr>
<td>N10</td>
<td>a</td>
<td>+619±42</td>
<td>1.3</td>
<td>+569±29</td>
</tr>
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<td></td>
<td>b</td>
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<td>8.4</td>
<td>+556±14</td>
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<tr>
<td></td>
<td>c</td>
<td>+507±20</td>
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<tr>
<td></td>
<td>d</td>
<td>+664±40</td>
<td>9.2</td>
<td>+603±26</td>
</tr>
</tbody>
</table>

$^a$ RM value obtained from a least-squares linear fit to $\chi(\lambda^2)$.

$^b$ Reduced $\chi^2$ for the $\chi(\lambda^2)$ fit.

$^c$ Effective RM derived from RM Synthesis

We did not obtain RM measurements for three of the sources, I11, N6, and N8. The sources N6, N7, and N8 were closely-spaced sources taken from the NVSS survey, with a maximum angular separation of 1.4 arcmin. We imposed a cutoff in the uv plane, which excluded data with uv distances $<$ 5000 wavelengths in the CLEAN task for these sources. For the VLA in C array at C-band, this (u, v) cutoff should eliminate any foreground emission due to the nebula and not emission from the extragalactic sources. N6 and N8 were completely filtered out by this process. Source I11 is very weakly polarized or unpolarized at 4.3–7.7 GHz and did not improve with phase-only self-calibration. The RM synthesis technique allows us to determine
that I11 is unpolarized (no significant peak in the $F(\phi)$ plot), and we can exclude the possibility that it possesses an extremely large RM that would result in depolarization over the 128 MHz-wide subband.

Figure 2.7: Plot of observed RM as a function of distance in parsecs from the center of the Rosette Nebula. The lines of sight observed in this paper are denoted by red circles, and the plotted RM values are derived from the peak of the Faraday dispersion function. The results from Savage et al. (2013) are the black circles. All of the sources have error bars; in many cases, the size of the measurement error is smaller than the plotted point. Secondary source components are represented.

2.4.1 Comparison of Techniques for RM Measurement

In this section, we discuss the results of the traditional $\chi(\lambda^2)$ method of obtaining RM values and RM Synthesis. For each of the sources and source components in Table 2.3, we have two measurements of RM, one from each of the two techniques. Figure 2.8 plots the $\chi(\lambda^2)$ RM values against the RM measurements obtained through RM Synthesis. The line shows the case of perfect agreement between the two methods. There is very good agreement between the two measurements. The RM Synthesis technique has higher precision, evident in the smaller errors, since it uses the entire 2 GHz bandwidth to calculate the RM. We feel the results of Figure 2.8 lend confidence to the RM values we report.

The only source that deviates strongly is component (b) of N9. Component
Figure 2.8: Plot comparing the RM measurements (rad m$^{-2}$) obtained from RM Synthesis and the traditional least-squares fitting method of $\chi(\lambda^2)$. The line represents perfect agreement between the two measurements, and errors for both methods are represented.

(b) of N9 is a weak source. The Stokes I intensity at 4.3GHz $\sim$1.2 mJy beam$^{-1}$. N9(b) did not improve with phase-only self-calibration, and the linear polarized intensity only exceeds the $5\sigma_Q$ threshold in one subband in the $\chi(\lambda^2)$ analysis. N9(b) is present, however, in the RM Synthesis analysis. RM Synthesis has the advantage over the $\chi(\lambda^2)$ method of being less sensitive to low SNR levels than the traditional $\chi(\lambda^2)$ method. In spite of the large errors associated with the components of N9, the difference in $\Delta$RM is large enough to be significant. In our analysis in the following sections, we use the RM values derived from the RM Synthesis technique for all of the sources.

2.5 Diagnostics of Magnetic Fields in H II Regions Utilizing Plasma Shell Models

2.5.1 Empirical Models for Stellar Bubbles

Savage et al. (2013) employed a physically motivated shell model to represent
the magnitude and spatial distribution of the observed RM in the shell of the Rosette Nebula. The equation describing this shell model is

$$RM(\xi) = \frac{Cn_e L(\xi)}{2} [B_{zI} + B_{zE}], \quad (2.5)$$

where $B_{zI}$ and $B_{zE}$ are the line of sight components of the magnetic field in the shell at ingress and egress, respectively, and $C$ is a set of constants equal to 0.81 defined by Equation (2.2) when $L(\xi)$, $B$, and $n_e$ are measured in parsecs, $\mu$Gauss, and cm$^{-3}$, respectively (see Equation (10) in Whiting et al. 2009). $B_{zI}$ and $B_{zE}$ are defined to be in the shell, and downstream from the hypothesized outer shock that defines the outer limit of the bubble. Relations between $B_{zI}$ and $B_{zE}$ and the upstream, undisturbed ISM magnetic field are given in Equations (6)–(9) of Whiting et al. (2009). $L(\xi)$ is the chord length through the shell, and it has two domains given by

$$L(\xi) = 2R_0 \sqrt{1 - \left(\frac{\xi}{R_0}\right)^2}, \; \text{if} \; \xi \geq R_1, \; \text{and} \quad (2.6)$$

$$L(\xi) = 2R_0 \left[ \sqrt{1 - \left(\frac{\xi}{R_0}\right)^2} - \left(\frac{R_1}{R_0}\right) \sqrt{1 - \left(\frac{\xi}{R_1}\right)^2} \right], \; \text{if} \; \xi \leq R_1,$$

where $R_0$ and $R_1$ are the outer and inner radii of the shell, respectively, and $\xi$ is the distance between the line of sight and the center of the shell (see Figure 6 of Whiting et al. 2009). Outside the outer radius of the H II region ($R_0 < \xi$), the model predicts $RM=0$, so the background RM due to the Galactic plane in this region is determined empirically through observations in the vicinity of, but outside the Rosette Nebula. The external general interstellar magnetic field, $B_0$, can be treated as a constant, meaning we assume $B_0$ does not vary significantly over the extent of the Rosette Nebula, i.e., outside the nebula, $B_0$ is constant in magnitude and direction. At the
shock front located at \( R_0 \), \( \mathbf{B}_0 \) can be decomposed into components perpendicular (\( B_\perp \)) and parallel (\( B_\parallel \)) to the shock normal. The perpendicular component is amplified by the density compression ratio (\( X \)) in the shell. Use of all these considerations then gives (Savage et al. 2013, Equation (8))

\[
RM = C n_e L(\xi) B_{0z} \left( 1 + (X - 1) \left( \frac{\xi}{R_0} \right)^2 \right).
\]  

(2.7)

\( B_{0z} \) is the z-component of \( \mathbf{B}_0 \), such that

\[
B_{0z} = B_0 \cos \theta,
\]

(2.8)

where \( B_0 \) is the magnitude of the ISM magnetic field and \( \theta \), the angle between the direction to Earth and \( \mathbf{B}_0 \), becomes the only free parameter in the model. We assume \( B_0 \) to be a constant and known. K. Ferrière (private communication) has pointed out that in reality, there will be fluctuations in \( |B_0|\) as well as \( \theta \) in the interstellar medium. As a result, it can be argued that \( B_{0z} \) should be considered the true independent parameter in Equation (2.7). In the limit of a purely adiabatic strong shock, i.e., \( X=4 \), Equation (2.7) predicts that the maximum value of the RM occurs near the outer radius of the shell for \( \xi < R_0 \). As discussed in Section 2.1.1, Harvey-Smith et al. (2011) consider a case in which the increase in RM is due solely to an increase in density and not the magnetic field. In this limit where \( X=1 \), Equation (2.7) models the expected RM in the shell without an enhancement in the magnetic field,

\[
RM(\xi) = C n_e L(\xi) B_{0z}
\]

(2.9)

and it predicts the maximum value of the RM is at the contact discontinuity, \( R_1 \).

Equations (2.7) and (2.9) represent two different models for the structure of the bubble, albeit highly simplified and approximate. The model described by Equation
(2.7) posits an increase in the interstellar magnetic field in the shell as well as an increase in the plasma density that is incorporated in the empirical parameter $n_e$. A comparison between the models of Equations (2.7) and (2.9) and the data is shown in Figure 2.9. Figure 2.9a represents the model with shock-enhanced magnetic field (Equation (2.7)), and Figure 2.9b is the model with no modification of the interstellar field (Equation (2.9)). Both models incorporate shell parameters ($R_0$, $R_1$, and $n_e$) from Celnik’s Model I (Celnik, 1985).

The models plotted here differ slightly from those discussed in Savage et al. (2013). In the present paper and Savage et al. (2013), we adopt a distance for the Rosette Nebula of 1600 parsecs, whereas Celnik (1985) assumed a distance of 1420 pc. In the present paper, we have therefore scaled up $R_0$ and $R_1$ by 13% and correspondingly lowered the value of $n_e$. The new values are given in Table 2.4. This correction was noted by Planck Collaboration et al. (2015). Given the rough approximation of the Celnik models to the structure of the Rosette Nebula, the model used in Savage et al. (2013) is still useful as one approximation of the nebular structure. The values of $\theta$ in Table 2.4 and used in Figure 2.9 are slightly modified from Savage et al. (2013). These new values of $\theta$ are the result of the Bayesian analysis discussed in Section 2.5.2.

![Graph similar to Figure 2.7 with models superimposed for (a) Equation (2.7) with $\theta = 73^\circ$ and (b) Equation (2.9) with $\theta = 47^\circ$.]
Table 2.4: Input Parameters for Shell Models

<table>
<thead>
<tr>
<th>$R_0$</th>
<th>$R_1$</th>
<th>$n_e$</th>
<th>$B_0$</th>
<th>$X$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(pc)</td>
<td>(pc)</td>
<td>(cm$^{-3}$)</td>
<td>((\mu)G)</td>
<td>(deg)</td>
<td></td>
</tr>
<tr>
<td>19.0</td>
<td>7.0</td>
<td>12.2</td>
<td>4</td>
<td>4</td>
<td>73</td>
</tr>
<tr>
<td>19.0</td>
<td>7.0</td>
<td>12.2</td>
<td>4</td>
<td>1</td>
<td>47</td>
</tr>
</tbody>
</table>

\[ a \] A value of 4 is for a purely adiabatic shock, and 1 is for no modification of magnetic field.

\[ b \] Values of $\theta$ derived from the Bayesian analysis in Section 2.5.2.

2.5.1.1 Physical Context of Bubble Models

The physical content of the two models discussed here is described in Section 2.5.1 above, as well as Savage et al. (2013) and Whiting et al. (2009). Both are subject to criticisms as regards physical content, for reasons we briefly consider here.

The analysis of Harvey-Smith et al. (2011) may be incomplete because it is difficult to see how the general interstellar magnetic field would not be modified by advection and compression due to the highly conducting plasma shell of the nebula. However, the model of Whiting et al. (2009) is subject to criticism for reasons brought to our attention by K. Ferrière (private communication). The main physical ingredient of the Whiting et al. (2009) expression is the modification of the interstellar field $B_0$ by the strong outer shock wave assumed to be present at $R_0$. Given the ionized or partially ionized state of this part of the interstellar medium, this will be an MHD shock. The Whiting et al. (2009) model applies the magnetic field jump conditions that occur at an MHD shock (e.g., Gurnett & Bhattacharjee (2005)).

Aside from the question as to whether a strong shock exists in this part of the Rosette Nebula, the point raised by Dr. Ferrière addresses the extent of the amplified field throughout the shell. In the Whiting et al. (2009) expression, it is assumed
that the amplified field applies to a large portion of the shell (see fuller discussion in Whiting et al., 2009). Dr. Ferrière’s point is that while this simplification might be valid in the case of a thin shell, it becomes questionable in the case of an object like the Rosette Nebula, in which the shell between \( R_1 \) and \( R_0 \) occupies most of the volume of the nebula.

The points raised by Dr. Ferrière are included in Planck Collaboration et al. (2015) (Section 5 and Appendix A). That paper also includes a model of the nebular field based on the assumption of conservation of magnetic flux. While recognizing the points raised above, we will continue to use the expressions contained in Equations (2.7) and (2.9) as part of the discussion in the present paper. These expressions represent limiting behavior of the magnetic field within a bubble shell. For example, Equation (2.7) may be considered a proxy for any mechanism that would enhance the interstellar field within the nebular bubble, and rotate it into a plane perpendicular to the radial direction. It is perhaps worth pointing out that the RM(\( \xi \)) curves presented by Planck Collaboration et al. (2015) in their Figure 5 also show “rotation measure limb brightening” that is characteristic of the Whiting model. In Figure 5(a) of Planck Collaboration et al. (2015), the peak of the curve is located at \( \xi \sim 12 – 13 \) pc for the blue and red curves, respectively. This peak is only slightly interior to where the peak of Equation (2.7) is located at \( \xi \sim 14.2 \) pc. In both Equations (2.7) and (2.9), the discontinuity in the derivative of RM at \( R_1 \) results from the assumption that a contact discontinuity occurs here. Furthermore, both models assume the region interior to \( R_1 \) is a vacuum. In any case, the comparison of measurements to a set of simplified descriptions of magnetic field behavior should further the goal of these investigations, which is to better specify the magnetic field within these shells, and how they are related to the general interstellar magnetic field.
2.5.2 A Bayesian Statistical Approach to Compare Shell Models and Determine the Orientation of the Interstellar Magnetic Field

In this section, we carry out a Bayesian statistical analysis to determine which of the models described in Section 5.1 better reproduces the observed RM measurements, and therefore offers a better representation of the magnetic field in the Rosette Nebula. The Bayesian analysis is suitable for our purposes because the two models may be considered “hypotheses” that are being tested (Gregory, 2005). Our approach and terminology follows that presented in Gregory (2005). We adopt from Celnik (1985) the parameters for the shell such as the inner and outer radius, $R_1$ and $R_0$, and the electron density, $n_e$. These parameters have been scaled to the distance to the Rosette Nebula of 1600 pc. As in Planck Collaboration et al. (2015), we adopt $R_1 = 7.0$ pc, $R_0 = 19.0$ pc, and $n_e = 12.2$ cm$^{-3}$. We retain the value of $4 \mu$G for the magnitude of the external magnetic field, $B_0$ (Ferrière, 2011). Given the measured and adopted values of the parameters, the RM through the bubble is determined by the angle $\theta$. In Savage et al. (2013), a value of $\theta = 72^\circ$ for a shell with an amplified magnetic field was obtained by visually inspecting the fit of the model to the data. For the model without the amplification of the magnetic field (Equation (2.9)), $\theta = 54^\circ$. The Bayesian analysis to be presented below automatically yields the optimum value of $\theta$ for each model. Although the primary reason for carrying out the Bayesian analysis is to determine which model better represents the observations, irrespective of the value of $\theta$, this quantity is nonetheless of astronomical interest.

Bayes’ Theorem states that the probability of a hypothesis being true given some statement of prior information, is

$$p(M_i|D,I) = \frac{p(M_i|I)p(D|M_i,I)}{p(D|I)},$$

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where \( M_i \) is a model we are testing, \( I \) is a statement representing our prior information, and \( D \) is the data set. The denominator \( p(D|I) \) is referred to as the “global likelihood” (Gregory, 2005). The likelihood function is

\[
p(D|M, I) \equiv \int_0^{\pi/2} d\theta \ p(\theta|M, I) p(D|M, \theta, I), \tag{2.10}
\]

and \( p(\theta|M, I) \) is the prior. Each hypothesis is a function of the unknown “nuisance parameter”, \( \theta \), which must be marginalized to compare two different hypothesis (shell models in our case) (Gregory, 2005). The value of \( \theta \) can range from 0 to \( \frac{\pi}{2} \) on the surface on a hemisphere. From this treatment, we will know the likelihood function, regardless of the value of \( \theta \). We can, however, obtain estimates of \( \theta \) as well. Our choice of \( p(\theta|M, I) \) allows \( \theta \) to range on the surface of a hemisphere,

\[
p(\theta|M, I) = \sin \theta.
\]

The data consist of \( N \) elements \( d_i \), where \( N \) is the number of RM measurements (19 in the present case of \( \xi \leq R_0 \)). We may write \( d_i = \text{RM}_i + e_i \), where \( \text{RM}_i \) is the RM predicted by the model (Equation (2.7) or Equation (2.9)), and \( e_i \) is simply the difference between the model and the measured value. The probability of a model being true can be represented as a product of the independent probabilities for each measurement, and if the \( e_i \) are independent (see Section 4.8 in Gregory 2005), then

\[
p(D|M, \theta, I) = \prod_i p(d_i|M, \theta, I) = \prod_i \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{e_i^2}{2\sigma^2} \right), \tag{2.11}
\]

where \( \sigma^2 \) is the variance. The likelihood function for a model \( M \) is then

\[
p(D|M, I) = (2\pi)^{-N/2} \int_0^{\pi/2} d\theta \ \sin \theta \ \prod_i \sigma_i^{-N} \ \exp \left( -\frac{(d_i - \text{RM}_i)^2}{2\sigma_i^2} \right). \tag{2.12}
\]
In practice, we perform an explicit summation over $\theta$ to obtain the likelihood functions, $p(D|M_1,I)$ and $p(D|M_2,I)$.

We can compare the two models by computing the odds ratio (Gregory, 2005),

$$O_{12} = \frac{p(M_1|D,I)}{p(M_2|D,I)}.$$  

Assuming the prior probabilities are equal, $p(M_1|I)=p(M_2|I)$, the odds ratio reduces to the Bayes factor and is then a ratio of the likelihood functions

$$B_{12} = \frac{p(D|M_1,I)}{p(D|M_2,I)}.$$  \hspace{1cm} (2.13)

To select the preferred model, we use the ‘Jeffreys’ scale’ (Table 1 in Trotta 2008), which categorizes the evidence as “inconclusive”, “weak”, “moderate”, or “strong” based on the value of the Bayes factor. Gordon & Trotta (2007) use the variable $B$ to denote the Bayes factor such that on the Jeffreys’ scale, $|\ln B| < 1$ is inconclusive, $1 \leq |\ln B| \leq 2.5$ is weak, $2.5 \leq |\ln B| \leq 5$ is moderate, and $5 \leq |\ln B|$ is strong evidence.

We adopt the following prescription for $\sigma$ in the Bayesian analysis. In Equation (2.11), the values of $\sigma_i$ recognize the fact that the two models described by Equations (2.7) and (2.9), while physically motivated, are approximations at best. With this viewpoint, the measurement errors on RM, determined by radiometer noise, are irrelevant. The main factor governing the departure of the measurements from the model is the crudeness of the model. We allow the model to deviate from the observed RMs by a scale factor, $\alpha$, such that $\sigma_i = \alpha R M_i$, where $R M_i$ is the calculated model value of the RM. There is no rigorous way of specifying the parameter $\alpha$. We determine $\alpha$ as follows:
1. We perform the Bayesian analysis with an arbitrary value of $\alpha = 0.2$ to determine $\theta_{\text{peak}}$, which is the value of $\theta$ at maximum likelihood, $\mathcal{L}_{\text{max}}$, where

$$\mathcal{L}_{\text{max}} = \left[ p(D|M, \theta, I) \right]_{\text{max}},$$

for both models.

2. Using the values of $\theta_{\text{peak}}$, we calculate RM values for each line of sight with Equations (2.7) and (2.9) and find the distribution of

$$x_i = \frac{d_i - \text{RM}_i}{\text{RM}_i}.$$

3. We then calculate the standard deviation, $\sigma_{\text{SD}}$, of the distribution of $x_i$, and we then perform another iteration of the Bayesian analysis with $\alpha = \sigma_{\text{SD}}$. From our analysis, we determined that $\alpha$ equals 0.41 and 0.43 for the homogeneous and inhomogeneous (to be defined in Section 2.5.3 below) models, respectively.

The results of the Bayesian analysis are listed in Table 2.5. The first column lists the type of model, homogeneous or inhomogeneous, used in the Bayesian analysis. Column two lists the value of $\alpha$, and column three gives the value of density compression ratio, $X$. The likelihood function, $p(D|M_i, I)$, is reported in column four, and the corresponding value of $\theta_{\text{peak}}$ is in column five. Columns six and seven give $B_{12}$ and $(B_{12})^{-1}$, respectively. Finally, column eight lists $|\ln B|$ and the corresponding strength of evidence on the Jeffreys’ scale is given in column nine. Figures 2.10a and 2.10b plot the integrand of Equation (2.12) for the models with fixed values of $R_1$, $R_0$, and $n_e$ and where $\alpha = 0.41$. For the model with an amplification of the magnetic field in the shell (Equation (2.7)), $p(D|M_1, I) = 4.1 \times 10^{-54}$ with $\theta_{\text{peak}} = 75^\circ$. For the model without an amplification (Equation (2.9)), $p(D|M_2, I) = 2.2 \times 10^{-53}$ with $\theta_{\text{peak}} = 54^\circ$. 

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The Bayes factor is $B_{12} = 0.19$ or $B_{12}^{-1} = 5.4$. Thus, $|\ln B| = 1.7$, and this is considered “weak” evidence in favor of the model without an amplification of the magnetic field in the shell, i.e., that considered by Harvey-Smith et al. (2011), on the Jeffreys’ scale. In this analysis, we have restricted the shell radii and the electron density to be global parameters for the shell. The Bayesian analysis does not conclusively favor either model. There is a weak tendency to favor the model without modification of the general interstellar field.
Table 2.5: Results of Bayesian Analysis

| Shell          | $\alpha$ | $X$  | $p(D|M_{t,I})$ | $\theta_{peak}\, (\text{deg})$ | $B_{12}$ | $(B_{12})^{-1}$ | $|\ln|B||$ | Jeffreys’ Scale |
|----------------|----------|------|----------------|---------------------------------|----------|-----------------|----------|-----------------|
| Homogeneous$^a$| 0.20     | 1    | $1.9 \times 10^{-69}$ | 47                              | 0.06     | 16.0            | 2.8      | moderate        |
|                |          | 4    | $9.9 \times 10^{-71}$ | 73                              |          |                 |          |                 |
| Homogeneous$^a$| 0.41     | 1    | $2.2 \times 10^{-53}$ | 54                              | 0.19     | 5.4             | 1.7      | weak            |
|                |          | 4    | $4.1 \times 10^{-54}$ | 75                              |          |                 |          |                 |
| Inhomogeneous$^b$| 0.20    | 1    | $5.4 \times 10^{-61}$ | 17                              | 11       | 0.1             | 2.4      | weak            |
|                |          | 4    | $6.1 \times 10^{-60}$ | 68                              |          |                 |          |                 |
| Inhomogeneous$^b$| 0.43    | 1    | $2.6 \times 10^{-45}$ | 33                              | 0.37     | 2.7             | 1.0      | weak            |
|                |          | 4    | $9.2 \times 10^{-46}$ | 71                              |          |                 |          |                 |

$^a$ For global shell values of $R_0 = 19.0$ pc, $R_1 = 7.0$ pc, and $n_e = 12.2$ cm$^{-3}$.

$^b$ Localized shell parameters as described in Section 2.5.3.
Figure 2.10: Plots of $p(\theta|M,I)p(D|M,\theta,I)$ from Equation (2.12) as a function of $\theta$ in increments of $d\theta = 0.25^\circ$. The values of the shell model parameters are listed in Table 2.4 for (a) the shell model with an amplification of the magnetic field in the shell (Equation (2.7)) and (b) the shell model without an amplification of the magnetic field in the shell (Equation (2.9)). The values for the inhomogeneous shell model parameters are listed in Table 2.6 (see Section 2.5.3) for (c) the model with an amplification of the magnetic field and (d) the model without an amplification.

2.5.3 Simplified Inhomogeneous Shell Models
Table 2.6: Local Parameters for Lines of Sight Through the Shell

<table>
<thead>
<tr>
<th>Source</th>
<th>$T_B$ (K)</th>
<th>$\xi^a$ (pc)</th>
<th>$R_0$ (pc)</th>
<th>$R_1$ (pc)</th>
<th>$n_e$ (cm$^{-3}$)</th>
<th>RM$^b$ (rad m$^{-2}$)</th>
<th>RM$^c$ (rad m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I7$^{d,e}$</td>
<td>0.49±0.13</td>
<td>4.60</td>
<td>15.1±0.3</td>
<td>9.2±0.1</td>
<td>17.6±3.0</td>
<td>+454±73</td>
<td>+764±176</td>
</tr>
<tr>
<td>I8$^{d,f}$</td>
<td>0.46±0.04</td>
<td>16.0</td>
<td>19.1±0.4</td>
<td>8.40±0.3</td>
<td>13.4±0.7</td>
<td>+1063±59</td>
<td>+909±62</td>
</tr>
<tr>
<td>I10$^{d,g}$</td>
<td>0.30±0.03</td>
<td>16.4</td>
<td>18.8±0.2</td>
<td>7.4±0.1</td>
<td>11.5±0.5</td>
<td>+881±38</td>
<td>+724±33</td>
</tr>
<tr>
<td>I12$^{d,h}$</td>
<td>0.59±0.09</td>
<td>9.60</td>
<td>18.2±0.4</td>
<td>11.3±0.1</td>
<td>16.0±1.6</td>
<td>+726±65</td>
<td>+961±141</td>
</tr>
<tr>
<td>N4</td>
<td>0.51±0.04</td>
<td>12.7</td>
<td>17.7±0.4</td>
<td>7.4±0.1</td>
<td>13.0±0.6</td>
<td>+1005±41</td>
<td>+1014±53</td>
</tr>
<tr>
<td>N5</td>
<td>0.44±0.05</td>
<td>13.5</td>
<td>16.5±0.5</td>
<td>7.4±0.1</td>
<td>13.7±1.0</td>
<td>+973±75</td>
<td>+855±82</td>
</tr>
<tr>
<td>N7</td>
<td>0.75±0.07</td>
<td>9.10</td>
<td>18.2±0.5</td>
<td>7.9±0.1</td>
<td>13.9±0.7</td>
<td>+955±42</td>
<td>+1338±75</td>
</tr>
<tr>
<td>N9</td>
<td>0.70±0.08</td>
<td>12.1</td>
<td>16.5±0.5</td>
<td>9.8±0.4</td>
<td>15.8±1.1</td>
<td>+1127±72</td>
<td>+1110±89</td>
</tr>
<tr>
<td>N10</td>
<td>0.12±0.05</td>
<td>13.6</td>
<td>14.1±0.4</td>
<td>5.2±0.1</td>
<td>11.4±3.4</td>
<td>+500±162</td>
<td>+388±120</td>
</tr>
</tbody>
</table>

$^a$ Distance between the line of sight and the line of sight through the center of the nebula at a distance of 1.6kpc.

$^b$ RM value calculated by Equation (2.7) using $\theta = 71^\circ$ and $\alpha = 0.43$.

$^c$ RM value calculated by Equation (2.9) using $\theta = 33^\circ$ and $\alpha = 0.43$.

$^d$ Source from Savage et al. (2013)

$^e$ Component (a) RM = $+749 \pm 27$ rad m$^{-2}$ and component (b) RM = $+704 \pm 33$ rad m$^{-2}$.

$^f$ Component (a) RM = $+461 \pm 4$ rad m$^{-2}$ and component (b) RM = $+238 \pm 73$ rad m$^{-2}$.

$^g$ RM = $+841 \pm 117$ rad m$^{-2}$.

$^h$ RM = $+835 \pm 17$ rad m$^{-2}$.
In Savage et al. (2013), we assumed that the shell is spherically symmetric and the density is homogeneous as described by Model I of Celnik (1985). However, the real structure of the Rosette Nebula has variation in both the inner and outer radii $R_1$ and $R_0$ (and thus the shell thickness $(R_0-R_1)$) as well as the density $n_e$. The simple shell model implemented in Section 2.5 is heavily dependent on these three parameters. In Equation (2.6), the location of the contact discontinuity ($R_1$) determines whether the line of sight resides in the region of shocked stellar wind closest to the star ($\xi < R_1$), within the shell of shocked ISM gas ($R_1 \leq \xi \leq R_0$), or upstream of the shock in the ambient ISM ($\xi > R_0$).

In reality, the Rosette Nebula is not a spherically-symmetric object, although it is a better approximation to this ideal than most H II regions. The lack of spherical symmetry, visible on optical or radio images, is doubtlessly due to the inhomogeneous nature of the medium into which the bubble expands. As a result, the local effective values of $R_0$, $R_1$, and $n_e$ will differ from one source line of sight to another. This would obviously affect our analysis of which shell model better represents the observed RM.

![Figure 2.11: Plot of the measured brightness temperature from the 4.95 GHz radio continuum map as a function of $\xi$ from the center of the Rosette Nebula in parsecs to the source N4. The black line is the data, the blue line is the fit to the data, and the shaded gray region represents the range in measured $T_B$ for the sources in Table 2.6. The fitted parameters are the inner and outer radii and the electron density.](image)
In this section, we discuss our method of obtaining the local values for the inner and outer shell radii and the electron density. By doing so, we can marginally relax the assumptions of spherical symmetry and homogeneous density within the shell. In essence, we are fitting a homogeneous spherical shell to a slice of the thermal radio emission passing through each source. The inhomogeneity of the Rosette Nebula is then accommodated by allowing these parameters to vary for different slices through the nebula. The result is that we obtain a set of $R_0$, $R_1$, and $n_e$ for each source. Consequently, we are using the brightness temperature profile through each source to the nebular edge to estimate the line-of-sight path length.

To determine the local shell parameters, we used the archival 4.95 GHz radio continuum map of the Rosette Nebula from Celnik (1985), made at the Max Planck Institut für Radioastronomie, and generously furnished to us by Dr. Wolfgang Reich. We measured the brightness temperature, $T_B$, as a function of $\xi$ along slices through the nebula at different position angles $\psi$. The data consist of measurements of brightness temperature that we then convert to density by first calculating the emission measure, $EM$,

$$EM = 282 \ T_8^{0.35} \ T_{GHz}^{2.1} \ T_T \ cm^{-6} \ pc$$  \hspace{1cm} (2.14)

(Equation (19) of Spangler & Cordes (1998)), where $T_8$ is the electron temperature in units of 8000 K, $\nu_{GHz}$ is the frequency of the observation in units of GHz, and $T_T$ is the thermal component of the brightness temperature in units K. In the case of the Rosette Nebula, we make the assumption that all of the radio emission above the background level is thermal, $T_T=T_B$. The emission measure is related to the electron density by

$$EM = \int n_e^2 dz = fn_e^2L,$$  \hspace{1cm} (2.15)
where \( n_e \) is the electron density in cm\(^{-3}\), \( f \) is the filling factor, and \( L \) is the length of the path through the shell in pc, given by Equation (2.6). For present analysis we assume a filling factor \( f = 1 \); further discussion of the role of the filling factor is given in Section 2.5.3.1. We then carry out a least-squares fit of the model to the observed brightness temperature along the slice. To determine shell parameters \( n_e \), \( R_0 \), and \( R_1 \) for a given slice, we use Equations (2.14) and (2.15) and Equation (2.6) for the chord length \( L(\xi) \). For example, Figure 2.11 plots the measured brightness temperature as a function of \( \xi \) for one source, N4, as well as the range in brightness temperature profiles for the interior sources listed in Table 2.6. We fit for the inner and outer radii, and then calculate the electron density for each line of sight.

Since we obtained estimates of the electron density and shell radii as a function of the angle \( \psi \), we calculate the expected RM on a specific line of sight using the local values for the radii and electron density and with values of \( \theta \) determined from the Bayesian analysis (see Section 2.5.3.2.) The results of this analysis are listed in Table 2.6 for sources with \( \xi \leq R_0 \) from both this work and Savage et al. (2013). A disadvantage of this approach is that three source components, I6a, I6b, and I15 were excluded from the inhomogeneous shell model analysis. In these three cases, the source was beyond the model outer extent of the shell, defined by the fit value of \( R_0 \). As such, a meaningful model estimate for RM was not available. Examination of the 4.95 GHz Effelsburg map showed that there was indeed plasma along the line of sight to each source, as indicated by thermal radio emission, but that brighter emission closer to the nebular center dominated the fit for the local shell model. These sources do not therefore pose any problem in our understanding, but this situation did mean that the number of source/source components was reduced from 19 in the case of the homogeneous shell analysis to 16 for the inhomogeneous shell model.

The first column of Table 2.6 lists the source defining the line of sight, and column 2 lists the measured brightness temperature at the position of the source.
The value of $\xi$ for the source is listed in column 3. Columns 4, 5, and 6 list the estimates of $R_0$, $R_1$, and $n_e$, respectively. Finally, columns 7 and 8 give the calculated RM values from Equations (2.7) and (2.9), respectively.

Figures 2.12a and 2.12b plot the observed RM vs the modeled RM values calculated with Equations (2.7) and (2.9), respectively, for sources with $\xi \leq R_0$. The line in these figures represents the case of perfect agreement between the observed RM and calculated RM values. The errors for the modeled RM values are propagated from the fits of $T_B$, $R_0$, and $R_1$. For both cases, Figure 2.12a and Figure 2.12b, there is rough agreement between observations and the model when the model predicts intermediate RM, i.e., $+400 \leq \text{RM} \leq +1000 \text{ rad m}^{-2}$. When the models predict high RM, however, the results are mixed. For some lines of sight, the observed RM are in reasonable agreement with the model values. However, in other cases, the observed RM is considerably lower.

It is unclear physically why this is the case. It is possible that some of these lines of sight pass through regions in which $\mathbf{B}$ is strongly modified from the simplified forms assumed in our analysis. In such cases, reversal of the shell magnetic field for some portion of the line of sight could lead to a drop in the total RM. Alternatively, it is possible that lines of sight characterized by low observed RM relative to model predictions are probing plasma cavities within the shell. This possibility is rendered less likely by the fact that the results of Figure 2.12 have taken into account the emission measure along a slice at the same position angle $\psi$ as the source. It has been suggested to us (D. Schnitzeler, private communication) that these low RMs for some lines of sight could indicate a low value of the filling factor $f$. This is more-or-less equivalent to the above suggestion that the anomalously low RMs are due to cavities in the nebula.

It is possible to have very high values of the RM that could potentially wrap within the bandwidth, which may cause depolarization. However, we do not believe
this to be the case for the RM values we report. There are two reasons as to why we do not expect this: 1) the reduced $\chi^2$ analysis showed no deviations of $\chi$ from $\chi^2$, and 2) in the RM Synthesis analysis, we are sensitive to up to $|\phi_{\text{max}}| \sim 3.0 \times 10^5 \text{ rad m}^{-2}$, so we would be able to detect large RM values. In general, we do not believe that the observed RMs are the source of disagreement with the calculated values of the RM.

Figure 2.12: Plots comparing the observed RM values (rad m$^{-2}$) with calculated RM values using the local values of the shell radii and electron density for (a) Equation (2.7) with $\theta = 71^\circ$ and (b) Equation (2.9) with $\theta = 33^\circ$. These are the peak values of $\theta$ from the Bayesian analysis (Section 2.5.3.2). The line shows the case of perfect agreement between the observed RM and the model RM values. Only the sources with $\xi \leq R_0$ are plotted (Table 2.6).

2.5.3.1 Effect of Possible Filling Factor on Inferred Shell Parameters

The analysis above interprets our observations in terms of highly simplified shell models. In all the models, the density is assumed uniform along the line of sight, and in the homogeneous models of Section 5.1, the density is constant at all points within the shell (illustrated in Figure 6 of Whiting et al. 2009). If $X = 1$, the magnetic field is also uniform along the line of sight. In the alternative case we have considered, $X = 4$, the field is constant with one value along half the line of sight (one half of the chord through the nebula), and constant with another value on the other half.
This homogeneity has been assumed in recognition of the patent oversimplification of these models in describing the true H II region associated with the Rosette Nebula.

Another parameter which frequently appears in the literature in discussions of H II regions is the filling factor f, conventionally defined as the fraction of the H II region volume that is occupied by ionized gas, as opposed to a vacuum which allegedly occupies the remainder of the volume. Harvey-Smith et al. (2011) adopt a value of f = 0.1 on the basis of numbers reported in the literature, and this value is incorporated in their estimates of $B_\parallel$. Purcell et al. (2015) adopted a value of f = 0.3 for the Gum Nebula, but noted that it was poorly constrained, and could be significantly larger. Planck Collaboration (2015), in their study of the Rosette Nebula, assumed a value of $f = 1$, apparently in view of the absence of data indicating a smaller number.

There are two ways of estimating the filling factor for an H II region.

1. In the first investigation of this matter, Osterbrock & Flather (1959) measured the density in the central part of the Orion Nebula from the density-sensitive line ratio OII 3729/3726. They showed that the inferred density, if taken as uniform through the nebula, would result in an EM much larger than that obtained by measurement of the radio thermal continuum emission. The resolution, illustrated in Figure 7 of Osterbrock & Flather (1959), was to posit that the ionized gas was contained in clouds occupying a small fraction of the nebular volume. This same approach, using density-sensitive infrared lines of SIII was used by Herter et al. (1982) to reach similar conclusions about the H II regions G75.84+0.4 and W3 IRS 1. We were unable to find suitable measurements of the OII 3729/3726 doublet in the Rosette Nebula, so an estimate of f in this way could not be done.

2. A second method for determining the filling factor was presented in Kassim et al.
(1989). It relies solely on radio measurements and could be termed a radiometric method of determining $f$. The estimate of $f$ from Kassim et al. (1989) results from requiring consistency between the optically thin radio flux of a nebula and estimates of EM obtained from the low frequency radio spectrum, at and below frequencies at which the nebula becomes optically thick (see Equations (6) and (7) of Kassim et al. 1989). The filling factor is then defined as

$$f \equiv \frac{\Omega}{\Omega_{\text{beam}}}$$

(2.16)

where $\Omega$ is the inferred solid angle subtended by regions of emission, and $\Omega_{\text{beam}}$ is the beam of the radio telescope, i.e. the solid angle over which all emission is averaged.

An apparently equivalent (i.e. using the same physical concepts) formulation of the Kassim et al. (1989) method compares the measured brightness temperature $T_B$ of the H$\!\!\iota$ region in the optically thick part of the spectrum (more precisely, the antenna temperature $T_A$ attributable to the thermal emission of the nebula) to the independently-determined electron temperature $T_e$,

$$f = \frac{\Omega}{\Omega_{\text{beam}}} = \frac{T_B}{T_e}.$$  
(2.17)

Equation (2.17), applied to the data of Kassim et al. (1989) on H$\!\!\iota$ regions associated with the S53 complex, reproduces the filling factors in Table 5 of Kassim et al. (1989). It should be noted that the filling factor calculated in the method of Kassim et al. (1989) will not usually equal that defined by Osterbrock & Flather (1959), but should indicate if the latter is substantially less than unity, and may be used as a rough estimate in the absence of other information.

An estimate of $f$ using the method of Kassim et al. (1989) is possible for the
Rosette Nebula. Graham et al. (1982) assembled radio continuum measurements of the Rosette Nebula over a wide range of frequencies, and showed that the nebula becomes optically thick at a frequency of about 400 MHz. From a fit to the lower frequency part of the spectrum, they obtain an optically-thick brightness (antenna) temperature of $T_B = 4100^{+700}_{-500}$ K. Celnik (1986) reports an independent measurement of the electron temperature $T_e$, based on radio recombination line observations. Celnik measures $T_e = 4700\pm600$ K if LTE level populations are assumed, and $T_e = 5800\pm700$ K on the basis of a non-LTE calculation. Adopting the latter value for $T_e$, a value of $f = 0.71 \pm 0.13$ results. Given the uncertainty in this whole procedure, this value should probably not be considered strongly inconsistent with $f = 1$.

We now briefly discuss the consequences of a filling factor $f \sim 0.7$ on the analysis presented in this paper. If $f$ (now taken to be the fraction of line of sight occupied by clumps containing plasma) is less than unity, the true density in those clumps will be larger by a factor of $1/\sqrt{f}$ than deduced for a uniform shell. Likewise, the value of $B_{0z}$ in the clumps will be larger by the same factor. Since our modeling of RM observations constrains the quantity $B_{0z} = B_0 \cos \theta$ (Equation (2.8)), the consequences of $f < 1$ are given in the following expression,

$$\frac{B_0^1 \cos \theta^1}{\sqrt{f}} = B_0^f \cos \theta^f$$

(2.18)

with the variables defined as follows. The quantities $B_0^1$ and $\theta^1$ are the magnetic field strength and inclination angle in the case $f = 1$ (homogeneous shell). These are the quantities presented in the previous sections. The variables $B_0^f$ and $\theta^f$ represent the corresponding quantities when $f \leq 1$. If we assume that $B_0^1 = B_0^f = B_0$ is known, Equation (2.18) becomes an identity relating $\theta^1$ and $\theta^f$.

Table 2.7 reports our results concerning the effects of a value of $f$ less than unity. This table presents values of $\theta^f$ corresponding to different values of $f$. Column
Table 2.7: Results of Filling Factor

<table>
<thead>
<tr>
<th>Model</th>
<th>$\theta^f=1$</th>
<th>$\theta^f=0.71$</th>
<th>$\theta^f=0.50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X = 4$</td>
<td>73°</td>
<td>70°</td>
<td>66°</td>
</tr>
<tr>
<td>$X = 1$</td>
<td>47°</td>
<td>36°</td>
<td>15°</td>
</tr>
</tbody>
</table>

1 gives the value of the compression factor $X$, since this determines $\theta$. Columns 2, 3, and 4 give the corresponding values of $\theta^f$ for $f = 1$, 0.71, and 0.50. The value $\theta^f=1$ gives the reference case for a nominally uniform shell. The case $\theta^f=0.71$ corresponds to the estimate of $f$ based on a comparison between the brightness temperature and electron temperature for the Rosette. Finally, $\theta^f=0.50$ corresponds to a more strongly clumped case, intermediate between more extreme estimates of $f \leq 0.1$ obtained by Osterbrock & Flather (1959), Herter et al. (1982), and Kassim et al. (1989), and the unclumped limit of $f = 1$.

The obvious effect of $f < 1$ is to require that the interstellar magnetic field $\vec{B}_0$ be closer to the line of sight. This tendency is clearest for the $X = 1$, $f = 0.50$ case. To the extent that a larger value of $\theta$ indicates a more realistic model, given that the Rosette Nebula is roughly in the anticenter direction (Savage et al. 2013, Section 4.1), smaller values of $f$ are less preferred.

This thinking can be pursued to obtain a highly model-dependent constraint on $f$. Obviously $\cos \theta^f \leq 1$, which is satisfied for the $X = 1$ case for $f \geq 0.46$. A smaller value of $f$ can be accommodated within the $X = 4$ model. However, in both cases the assumption of a filling factor significantly less than unity implies that the ISM magnetic field $\vec{B}_0$ at the location of the Rosette Nebula would be significantly rotated from its mean value for an azimuthal magnetic field. A final remark is that less highly inclined fields (larger $\theta$) can be accommodated if the true value of $B_0$, the magnitude of the interstellar magnetic field at the location of the Rosette Nebula, is larger than our assumed value of 4 $\mu$G.
2.5.3.2 Bayesian Analysis for Inhomogeneous Shell Models

In Section 2.5.3, we marginally relaxed the homogeneous shell model to account for inhomogeneity in the shell of the Rosette Nebula. We employ the Bayesian analysis to determine which shell model better describes the observed RM values and whether the interstellar magnetic field is alternatively strongly modified or unmodified in the shell. We perform the Bayesian analysis in the same manner described in Section 2.5.2. Rows three and four of Table 2.5 lists the results of the first iteration of the Bayesian analysis with \( \alpha = 0.2 \) and the second iteration where \( \alpha = \sigma_{\text{SD}} = 0.43 \).

In Figures 2.10c and 2.10d, the integrand of Equation (2.12), \( p(\theta|M,I)p(D|M,\theta,I) \), is plotted as a function of \( \theta \) for the model with amplification of the magnetic field and that without, respectively. The calculated values of the RM for the inhomogeneous models are listed in columns 7 and 8 in Table 2.6 using the values of \( \theta_{\text{peak}} \) from the second iteration of the Bayesian analysis.

Using the localized parameters for the shell and electron density, \( \alpha = 0.43 \), and \( N = 16 \), \( p(D|M_1,I) = 9.2 \times 10^{-46} \) with \( \theta_{\text{peak}} = 71^\circ \), and \( p(D|M_2,I) = 2.6 \times 10^{-45} \) with \( \theta_{\text{peak}} = 33^\circ \). The Bayes factor is \( B_{12} = 0.37 \) \( (B_{12}^{-1} = 2.7) \) and \( |\ln B| = 1.0 \), which is “weak at best” (Gordon & Trotta, 2007) evidence on the Jeffreys’ scale in favor of the model of Harvey-Smith et al. (2011) model. The conclusions of the Bayesian analysis for the inhomogeneous case verges on neither model being favored. While incorporating inhomogeneity by determining the electron density and inner and outer radii for each line of sight yields better and more accurate models, this procedure does not eliminate large residuals between the models and the measurements. These large residuals apparently prevent the Bayesian analysis from convincingly selecting one of the models as the preferred one.
2.5.4 Summary of Bayesian Test of Models

The Bayesian analysis was undertaken to determine which of the simplified models discussed in Section 2.5.1 is a better representation of the Rosette Nebula Faraday rotation data. The results of the Bayesian analysis are listed in Table 2.5. If a global shell model (i.e., the same values of \( R_0 \), \( R_1 \), and \( n_e \) for all lines of sight) is applied to the data, the Bayes factor is \( B_{12} = 0.19 \) (\( B_{-12} = 5.4 \)) and \( |\ln B| = 1.7 \), indicating that the picture of Harvey-Smith et al. (2011), i.e., a model without modification of \( B_0 \), is weakly favored in its ability to reproduce the observations (Section 2.5.2).

If an attempt is made to accommodate the inhomogeneity of the nebula by having the effective values of \( R_0 \), \( R_1 \), and \( n_e \) change for different slices through the nebula, the Bayes factor is \( B_{12} = 0.37 \) (\( B_{-12} = 2.7 \)) and \( |\ln B| = 1.0 \), which is “weak at best” (Gordon & Trotta, 2007) in favor of the Harvey-Smith et al. (2011) model.

The results of the Bayesian analysis depend on the choice of \( \alpha \). This probably is a consequence of the fact that as \( \alpha \) increases, the role of outliers in the likelihood function decreases. In any case, as the value of \( \alpha \) is increased from 0.20 to 0.43, the preferred inhomogeneous model changes from weakly in favor of the \( X=4 \) model to weakly in favor of the \( X=1 \) (unmodified) model.

Our analysis is also sensitive to our choice of prior, i.e., allowing \( \theta \) to range on a surface of a hemisphere. This choice of prior can bias the results at higher values of \( \theta \). We initially performed the Bayesian analysis with a uniform prior such that \( p(\theta|M, I) = \frac{1}{\Delta \theta} \), where \( \Delta \theta = \frac{\pi}{2} \). However, the choice of prior does not change which model is favored in the Bayesian analysis.

The conclusions of the Bayesian analysis are not strong, and the inability of this analysis to identify a clear preferred model is apparently due to the oversimplified nature of all models considered. Inhomogeneities or other structures in the real nebula
partially mask the average properties of the nebula. However, a positive point is that
either model can reproduce the magnitude and extent of enhanced RMs over the right
angular extent on the sky, for plausible values of the magnitude and orientation of
the interstellar magnetic field. For the global models, which predict that the RM
should depend only on the impact parameter $\xi$, the plot to consult is Figure 2.9.
Both models account for the abrupt onset of the RM at the edge of the nebula, and
both reproduce, with plausible values of the independent parameter $\theta$, the values of
RM actually observed. However, an equally striking aspect of Figure 2.9 is the large
dispersion of measured RMs about the model curves. Apparently, variations in the
plasma structure of the stellar bubble, describable by variations in $n_e$, $|B|$, and $\theta$ are
as important as the description of the mean shell characteristics, and in fact obscure
the signature of this mean structure.

For the case of the “inhomogeneous models”, the relevant plot is Figure 2.12. In
the inhomogeneous case, the predicted RM does not depend only on $\xi$, and we make
a comparison between the model and observable values for all lines of sight. Here,
again, the models meet with some success in that they can reproduce the magnitude
of the RMs observed, and are also able to partially account for the variation of RM
from one line of sight to another. That is, the models seem to successfully predict
the upper envelope of the observed RMs. However, the models do not account for
the fact that some lines of sight, expected to have large RMs, have intermediate to
low values. Again we must appeal to spatial variations within the nebula, potentially
describable as large-scale turbulence, that cause variations in the values of RM.

As a final comment, it is worthwhile to know that plausible variations in one
parameter, $\theta$, allow two models that significantly differ in a physical sense to rea-
onably account for the data. We suspect that this statement would extend to the
model presented by Planck Collaboration et al. (2015). Their physical description of
the nebular shell is very different and probably more realistic than those considered
in this paper. Their result for the prediction of RM(ξ), presented in Figure 6 of their paper, is similar in form to that shown in Figure 2.9 of our paper, with different but plausible values of |B₀| and θ. However, it is not capable of accounting for substantial dispersion in RM for lines of sight with the same value of ξ.

2.6 Summary and Conclusions

1. We performed polarimetric observations using the VLA of 11 new radio sources observed through the shell of the Rosette Nebula. This significantly increases the number of lines of sight probed relative to that of Savage et al. (2013).

2. We obtained RM measurements for 15 lines of sight (including secondary components), using two techniques: a least-squares fit to χ(λ²) and RM Synthesis. The measured RM values are in excellent agreement between the two techniques. We measure an excess RM of +40 to +1200 rad m⁻² due to the shell of the Rosette Nebula and confirm the background RM of +147 rad m⁻² from Savage et al. (2013). The large range in values of RM through the shell is an indicator of inhomogeneity in the shell, which is beyond the scope of the present paper.

3. We employ a Bayesian analysis to determine which of two models better represents the observed dependence of RM with distance from the center of the nebula, a model with a shock-enhanced magnetic field in the shell or one without. We first treat the Rosette Nebula as a spherically-symmetric shell having global parameters for the inner and outer radii of the shell and the electron density. With this calculation, the model without modification of the interstellar field is favored. The Bayes factor is 5.4 (B₁₂⁻¹) and on the Jeffreys’ scale, the analysis of Harvey-Smith et al. (2011) is weakly favored.

4. We perform a second Bayesian analysis using localized parameters determined from radio continuum data to account for the inhomogeneity of the shell. In
this analysis, we model the inner and outer shell radii and calculate the electron
density to each line of sight. We then employ these localized parameters in the
Bayesian analysis and find the model without modification of the magnetic field
in the shell is weakly favored. The Bayes factor is 2.7, which on the Jeffreys’
scale ($|\ln B| = 1.0$) is considered “weak at best” (Gordon & Trotta, 2007).

5. The results of the Bayesian analysis are dependent on the value of $\alpha$, which
specifies the expected departure of the measurements from the model. For a
value of $\alpha \simeq 0.4$, neither model is significantly superior to the other in a Bayesian
sense. We attribute this to the fact that neither model can account for large
departures of the observed RMs from the model predictions. This situation is
unlikely to change, even for analytic and numerical models that include more
sophisticated physics than the empirical expressions we have employed.

6. Regardless of the specific choice of model, the magnitude and spatial extent
of the rotation measure “anomaly” is completely consistent with independent
information on the plasma density in the nebula, and the interstellar magnetic
field.

7. To further investigate the role of magnetic fields in H II regions, one would want
more lines of sight through the shell of the Rosette H II region and the stellar
bubble to obtain a large sample of RM values. In many cases though, such
a sample is limited by the number of strong, linearly polarized, extragalactic
sources behind the nebula. Simulations of H II regions and stellar bubbles with
magnetic fields, such as those by Stil et al. (2009), van Marle et al. (2015),
Ferrière et al. (1991), and Walch et al. (2015), are vital to understanding how
the magnetic field and the density contribute to the large RM values within the
shell of the nebula. It would be of considerable interest to compare calculated
RMs on lines of sight through these simulations with the measured values. Such
comparison would indicate if those simulations are capable of reproducing the average features of Faraday rotation through these “Faraday Rotation Anomalies” (Whiting et al., 2009), as well as the large fluctuations in the observed RMs with respect to the expected values. Finally, similar analyses with more H II regions and stellar bubbles would further illuminate the role of the magnetic field and would allow one to consider differences in stellar composition, wind luminosities, and ages of the nebulae. Presently, we are undertaking such an analysis as presented in this paper for W4/IC 1805 and IC 1396.
CHAPTER 3
A FARADAY ROTATION STUDY OF THE STELLAR BUBBLE AND \textit{H} \textsc{ii} REGION ASSOCIATED WITH THE W4 COMPLEX

This chapter is taken directly from Costa & Spangler (2018), which has been submitted to the Astrophysical Journal.

3.1 Introduction

Young massive stars in OB associations photoionize the surrounding gas, creating an \textit{H} \textsc{ii} region, and their powerful stellar winds can inflate a bubble around the star cluster. Magnetic fields are important to the dynamics of these structures (Tomisaka, 1990; Ferrière et al., 1991; Vallée, 1993; Tomisaka, 1998; Haverkorn et al., 2004; Sun et al., 2008; Stil et al., 2009), and they can elongate the cavity preferentially in the direction of the magnetic field and thicken the shell perpendicular to the field (Ferrière et al., 1991; de Avillez & Breitschwerdt, 2005; Stil et al., 2009), causing deviations from the classical structure of the Weaver et al. (1977) wind-blown bubble. Knowledge of the magnitude and direction of the magnetic field within stellar bubbles and \textit{H} \textsc{ii} regions is important for simulations and for understanding how the magnetic field interacts with and modifies these structures.

In previous work (i.e., Savage et al. 2013 and Costa et al. 2016), we investigated whether the Galactic magnetic field is amplified in the shell of the Rosette Nebula, an \textit{H} \textsc{ii} region and stellar bubble associated with NGC 2244 ($\ell = 206.5^\circ$, $b = -2.1^\circ$). In this work, we continue our investigation of how \textit{H} \textsc{ii} regions and stellar bubbles modify the ambient Galactic magnetic field by considering another example of a young star cluster and an \textit{H} \textsc{ii} region that appears to be formed into a shell by the effect of stellar winds.
3.1.1 Techniques for Measuring Magnetic Fields in the Interstellar Medium

Faraday rotation measurements probe the line of sight (LOS) component of the magnetic field in ionized parts of the interstellar medium (ISM), provided there is an independent estimate of the electron density. Faraday rotation is the rotation in the plane of polarization of a wave as it passes through magnetized plasma and is described by the equation

\[
\chi = \chi_0 + \left[ \left( \frac{e^3}{2\pi m_e^2 c^4} \right) \int n_e \cdot B \cdot ds \right] \lambda^2, \tag{3.1}
\]

where \( \chi \) is the polarization position angle, \( \chi_0 \) is the intrinsic polarization position angle, the quantities in the parentheses are the usual standard physical constants in cgs units, \( n_e \) is the electron density, \( B \) is the vector magnetic field, \( ds \) is the incremental path length interval along the LOS, and \( \lambda \) is the wavelength. We define the terms in the square bracket as the rotation measure, RM, and we can express the RM in mixed but convenient interstellar units as

\[
RM = 0.81 \int n_e \ (cm^{-3}) \ B \ (\mu G) \cdot ds \ (pc) \ rad \ m^{-2}. \tag{3.2}
\]

3.1.2 The H II Region and Stellar Bubble Associated with the W4 Complex

The H II region and stellar bubble of interest for the present study is IC 1805, which is located in the Perseus Arm. The star cluster responsible for the H II region and stellar bubble is OCl 352, which is a young cluster (1–3 Myr) (Basu et al., 1999). OCl 352 has 60 OB stars (Shi & Hu, 1999). Three of these are the O stars HD 15570, HD 15558, and HD 15629, and they have mass loss rates between \( 10^{-6} \) and \( 10^{-5} \) \( M_\odot \) yr\(^{-1} \) (Massey et al., 1995) and terminal wind velocities of 2200 – 3000 km
s$^{-1}$ (Garmany, 1988; Groenewegen et al., 1989; Bouret et al., 2012). We adopt the nominal center of the star cluster to be R.A.(J2000) = 02$^h$ 23$^m$ 42$^s$, decl.(J2000) = +61$^\circ$27$'$ 0$''$ ($\ell = 134.73$, $b = +0.92$) (Guetter & Vrba, 1989) and a distance of 2.2 kpc to IC 1805 to conform with previous studies of the region (e.g., Normandeau et al. 1996; Dennison et al. 1997; Reynolds et al. 2001; Terebey et al. 2003; Gao et al. 2015). In the literature, other distance values include: 2.35 kpc (Massey et al., 1995; Basu et al., 1999; West et al., 2007; Lagrois et al., 2012), 2 kpc (Dickel, 1980), 2.04 kpc (Feigelson et al., 2013; Townsley et al., 2014), and 2.4 ± 0.1 kpc (Guetter & Vrba, 1989).

We refer to the H II region between −0.2$^\circ$ < $b$ < 2$^\circ$ as IC 1805. This structure is also known as the Heart Nebula for its appearance at optical wavelengths. We differentiate this region from the northern latitudes that constitute the W4 Superbubble (Normandeau et al., 1996; West et al., 2007; Gao et al., 2015), and we use the nomenclature of W4 to describe the entire region, which includes IC 1805 and the W4 Superbubble. Below we summarize the structure of IC 1805 and Figure 3.1 is a cartoon diagram of the structure described here.

- **South.** On the southern portion of IC 1805, there is a loop structure of ionized material at 134$^\circ$ < $\ell$ < 136$^\circ$, $b$ < 1$^\circ$, which we call the southern loop. Terebey et al. (2003) find that at far infrared and radio wavelengths, the shell structure is well defined and ionization bounded, since the ionized gas lies interior to the dust shell. However, they also find that there is warm dust that extends past the southern loop and a faint ionized halo (see their Figure 6). Terebey et al. (2003) argue that the shell is patchy and inhomogeneous in density, which allows ionizing photons to escape. Gray et al. (1999) discuss extended emission surrounding IC 1805 and suggest that it may be evidence of an extended H II region (Anantharamaiah, 1985). Also surrounding IC 1805 are patchy regions of H I (Braunsfurth, 1983; Hasegawa et al., 1983; Sato, 1990) and CO (Heyer &

Terebey et al. (2003) model the structure of the southern loop using radio continuum data. They assume a spherical shell and place OCl 352 at the top edge of the bubble instead of at the center to accommodate spherical symmetry (see their Figures 4 and 5). The center of their shell model is at \((\ell, b) = (135.02^\circ, 0.42^\circ)\). They find an inner radius of 30 arcmin (19 pc) and a shell thickness of 10 arcmin (6 pc) and 2.5 arcmin (2 pc) for a thick and thin shell model, respectively. Terebey et al. (2003) report electron densities of \(10 \text{ cm}^{-3}\) and \(20 \text{ cm}^{-3}\) for the thick and thin shell models, respectively (see Section 3.5 and Table 3 of Terebey et al. 2003). While we utilize and discuss these models in the following sections, the center position of the shell in Terebey et al. (2003) was selected to fit the ionized shell, and as such, the shell parameters should only be used to describe the bottom of IC 1805. For latitudes near the star cluster, the model fails, as the star cluster is at the top edge of the bubble instead of at the center.

- **East.** On the eastern edge of IC 1805 \((\ell > 134.6^\circ, b < 0.9^\circ)\), Terebey et al. (2003) find that warm dust extends outside the loop boundary and suggest that if the warm dust is associated with the ionized gas, then the bubble has blown out on the eastern side of IC 1805. At the Galactic latitude equal to the star cluster, the ionized gas appears to be pinched (Basu et al., 1999), which is usually caused by higher densities. There is a clump of CO emission in the vicinity of the eastern pinch at \((\ell, b) = (135.2^\circ, 1.0^\circ)\) (Lagrois & Joncas, 2009a), and there is HI emission on the eastern edge at \((\ell, b) = (136^\circ, 0.5^\circ)\) (see Figure 1 of Sato 1990).

- **West.** On the western edge of IC 1805 is the W3 molecular cloud and the W3 complex, which hosts a number of compact H II regions and young stellar
objects (see Bik et al. 2012 and their Figure 1). Dickel (1980) modeled the structure of W3, which is thought to be slightly in front of W4, and they argue that the advancement of the IC 1805 ionization front and shock front into the W3 molecular cloud may have triggered star formation. Moore et al. (2007) similarly conclude that the W3 molecular cloud has been compressed on one side by the expansion of IC 1805. While infrared sources nestled between the western edge of IC 1805 and eastern edge of the W3 molecular cloud are thought to be the product of this interaction, W3 Main, W3 (OH), and W3 North are thought to be sites of triggered star formation from IC 1795, which is part of W3 as well and not from the expansions of the ionization front (Nakano et al., 2017; Jose et al., 2016; Kiminki et al., 2015). There is therefore uncertainty regarding a physical connection between W3 and IC 1805.

– North. North of OCl 352, the bubble opens up into what is called the W4 Superbubble (Normandeau et al., 1997; Dennison et al., 1997; West et al., 2007; Gao et al., 2015), which is a sealed “egg-shaped” structure that extends up to $b \sim 7^\circ$ (Dennison et al., 1997; West et al., 2007). At the latitude of the star cluster, Lagrois & Joncas (2009a) estimate the distance between the eastern and western shell to be $\sim 1.2^\circ$ (46 pc) that increases in size up to $1.6^\circ$ (61 pc) at $b = 1.8^\circ$ (see Figure 11 of Lagrois & Joncas 2009a). At higher latitudes, Dennison et al. (1997) model the thickness of the shell to be between 10–20 pc (16 – 31 arcminutes) from H$\alpha$ observations.

The “v”-shaped feature seen in Figure 3.2 at $(\ell, b) \sim (134.8^\circ, 1.35^\circ)$ is prominent in the ionized emission, and Heyer et al. (1996) report a cometary-shaped molecular cloud near $(\ell, b) \sim (134.8^\circ, 1.35^\circ)$. The alignment of the cometary cloud, as it is pointed towards IC 1805, suggests that the UV photons from the star cluster are responsible for the “v” shaped feature in the ionized emission.
on the side closest to the star cluster (Dennison et al., 1997; Taylor et al., 1999). Lagrois & Joncas (2009a) argue, from radial velocity measurements, that the cloud is located on the far side of the bubble wall, and while it may appear to be a cap to the bubble connecting to the southern loop, it is simply a projection effect. As such, the ridge of ionized material directly north of OCl 352 is not the outer radius of the shell but is part of the rear bubble wall.

– **PDR.** The H\textsc{i} and molecular emission near the southern (\(\ell < 0.9^\circ\)) portions of IC 1805 suggest that a photodissociation region has formed exterior to the H\textsc{ii} region. PDRs are the transition layer between the fully ionized H\textsc{ii} region and molecular material, where far UV photons can propagate out and photodissociation molecules. We discuss the importance and observational evidence of a PDR in Section 3.5.3.

There is an extensive literature on the W4 region and its relationship to W3, dealing with the morphology (Dickel, 1980; Dickel et al., 1980; Braunsfurth, 1983; Normandeau et al., 1996; Dennison et al., 1997; Heyer & Terebey, 1998; Taylor et al., 1999; Basu et al., 1999; Terebey et al., 2003; Lagrois & Joncas, 2009a,b; Stil et al., 2009) and star formation history (Carpenter et al., 2000; Oey et al., 2005). In the following paragraphs, we summarize those results from the literature that are most relevant to our polarimetric study and inferences on magnetic fields in this region.

Measurements of the total intensity and polarization of the Galactic nonthermal emission in the vicinity of H\textsc{ii} regions is of interest because the H\textsc{ii} regions and environs act as a Faraday-rotating screen inserted between the Galactic emission behind the H\textsc{ii} region and that in front. Few radio polarimetric studies exist in the literature to date of the IC 1805 stellar bubble. Gray et al. (1999) present their polarimetric results of the W3/W4 region at 1420 MHz with the Dominion Radio Astrophysical Observatory (DRAO) Synthesis Telescope. They find zones of strong
depolarization near the H II regions, particularly in the south, where there is a halo of extended emission around IC 1805. They conclude that RM values on order $10^3$ rad m$^{-2}$ and spatial RM gradients must exist to explain the depolarization near the H II region. More recently, Hill et al. (2017) present results of their polarimetric study of the Fan region ($\ell \sim 130^\circ$, $-5^\circ \leq b \leq +10^\circ$), which is a large structure in the Perseus arm that includes W3/W4. While the focus of their study was not on W4 specifically, they find similar results to Gray et al. (1999) in that there is sufficient Faraday rotation to cause beam depolarization in the regions of extended emission.

In the W4 Superbubble, West et al. (2007) determined the LOS magnetic field strength by estimating depolarization effects along adjacent lines of sight. Using estimates of the shell thickness and the electron density from Dennison et al. (1997),
West et al. (2007) estimate $B_{\text{LOS}} \sim 3.4 - 9.1 \, \mu\text{G}$ for lines of sight at $b > 5^\circ$. Gao et al. (2015) also report $B_{\text{LOS}}$ estimates in the W4 Superbubble by assuming a passive Faraday screen model (Sun et al., 2007) and measuring the polarization angle for lines of sight interior and exterior to the screen. For the western shell ($\ell \sim 132.5^\circ$, $4^\circ < b < 6^\circ$) and the eastern shell ($\ell \sim 136^\circ$, $6^\circ < b < 7.5^\circ$) in the superbubble, Gao et al. (2015) report negative RMs between $-70$ and $-300$ rad m$^{-2}$ in the western shell and positive RMs on order $+55$ rad m$^{-2}$ in the eastern shell. Gao et al. (2015) conclude that the sign reversal is expected in the case of the Galactic magnetic field being lifted out of the plane by the expanding bubble. With H$\alpha$ estimates from Dennison et al. (1997) for the electron density and geometric arguments for the shell radii of the W4 Superbubble, Gao et al. (2015) estimate $|B_{\text{LOS}}| \sim 5 \, \mu\text{G}$.

Stil et al. (2009) compare their magnetohydrodynamic simulations of superbubbles to the W4 Superbubble. In general, they find that the largest Faraday rotation occurs in a thin region around the cavity, and inside the cavity, it would be smaller. They also present two limiting cases for the orientation of the Galactic magnetic field with respect to the line of sight, and the consequences for the RMs through the shell. If the Galactic magnetic field is perpendicular to the observer’s line of sight, then the contributions to the RM from the front and rear bubble wall would be of equal but opposite magnitude, except for small asymmetries which would lead to low RMs ($\sim 20$ rad m$^{-2}$) through the cavity. This requires the magnetic field to be bent by the bubble to have a non-zero line of sight component. If the Galactic magnetic field is parallel to the line of sight, then the RMs through the front and rear bubble wall reinforce each other, and there are high RMs for lines of sight through the shell. In this case, there are higher RMs ($\sim 3 \times 10^3$ rad m$^{-2}$) everywhere.

There are also studies of the magnetic field for W3. From H$\text{I}$ Zeeman observations, van der Werf & Goss (1990) conclude that the $B_{\text{LOS}}$ has small-scale structures that can vary on order of 50 $\mu\text{G}$ over $\sim 9$ arcsec scales. Roberts et al. (1993) report
values of the LOS magnetic field from H\textsuperscript{i} Zeeman observations towards three resolved components of W3. The three components are near ($\ell$, $b$) $\sim (133.7^\circ, 1.21^\circ)$, with a maximum separation of 1.5 arcmin, and the LOS magnetic field is between $-50 \mu G$ and $+100 \mu G$. Balser et al. (2016) observed carbon radio recombination line (RRL) widths to estimate the total magnetic field strength in the photodissociation region (see Roshi 2007 for details). They report $B_{\text{tot}} = 140 – 320 \mu G$ near W3A ($133.72^\circ$, $1.22^\circ$) and argue that for a random magnetic field, $B_{\text{tot}} = 2 |B_{\text{LOS}}|$, which would then be consistent with the Roberts et al. (1993) estimates of the $B_{\text{LOS}}$. It should be noted that these magnetic field strengths are substantially larger than those inferred for the W4 Superbubble on the basis of polarimetry of the Galactic background (see text above).

In this paper, we present new Faraday rotation results for IC 1805 to investigate the role of the magnetic field in the H\textsuperscript{ii} region and stellar bubble. As in Savage et al. (2013) and Costa et al. (2016), we utilize an arguably simpler and more direct method of inferring the LOS component of the magnetic field in H\textsuperscript{ii} regions. This is the measurement of the Faraday rotation of nonthermal background sources (usually extragalactic radio sources) whose lines of sight pass through the H\textsuperscript{ii} region and its vicinity. In Section 3.2, we describe the instrumental configuration and observations, including source selection. Section 3.3 details the data reduction process, including the methods used to determine RM values. In Section 3.4, we report the results of the RM analysis and discuss Faraday rotation through the W4 complex in Section 3.5. We present models for the RM within the H\textsuperscript{ii} region and stellar bubble in Section 3.6. We discuss our observational results and their significance for the nature of IC 1805 in Section 3.7 and compare the results of this study with our previous study of the Rosette nebula in Section 3.8. We discuss future research in Section 3.9, and present our conclusions and summary in Section 3.10.
3.2 Observations

3.2.1 Source Selection

Figure 3.2: Mosaic of IC 1805 from the Canadian Galactic Plane Survey at 1.42 GHz, with Galactic longitude and latitude axes. The lines of sight listed in Table 3.1 are the red and blue symbols, where positive RMs are blue and negative RMs are red. The green and purple symbols are RM values from Taylor et al. (2009) or Brown et al. (2003), where positive RMs are green and negative RMs are purple. We utilize the naming scheme from Table 3.1 for the RM values from the literature for ease of reference, but we omit the “W4-” prefix in this image for clarity. The size of the plotted symbols is proportional to the $|\text{RM}|$ value.
Our criteria for source selection were identical to Savage et al. (2013) and Costa et al. (2016) in that we searched the National Radio Astronomy Observatory Very Large Array Sky Survey (NVSS, Condon et al. 1998) database for point sources within 1° of OCl 352 (the “T” sources) with a minimum flux density of 20 mJy. We also searched in an annulus centered on the star cluster with inner and outer radii of 1° and 2° for outer sources (“O”) to measure the background RM due to the general ISM. We identified 31 inner sources and 26 outer sources in the region. We then inspected the NVSS postage stamps to ensure that they were point sources at the resolution of the NVSS (∼ 45 arcseconds). We discarded sources that showed extended structure similar to Galactic sources. We selected 24 inner sources and 8 outer sources from this final list.

The sources are listed in Table 3.1, where the first column lists the source name in our nomenclature. The second and third columns list the right ascension (\(\alpha\)) and declination (\(\delta\)) of the observed sources. The positions are determined with the imfit task in CASA, which fits a 2D Gaussian to the intensity distribution at 4.33 GHz. Columns four and five give the Galactic longitude (\(\ell\)), Galactic latitude (\(b\)), which is converted from \(\alpha\) and \(\delta\) using the Python Astropy package, and the angular separation from the center of the nebula (\(\xi\)) is given in column six. Column seven lists the flux density at 4.33 GHz calculated with imfit, and column eight gives the mean percent linear polarization (\(m = P/I\)) as measured across the eight 128 MHz maps and assuming a Faraday simple source. Figure 3.2 is a radio continuum mosaic from the Canadian Galactic Plane Survey (CGPS) (Taylor et al., 2003; Landecker et al., 2010) with the location of the sources, along with the names, indicated with filled circles.
<table>
<thead>
<tr>
<th>Source</th>
<th>α(J2000)</th>
<th>δ(J2000)</th>
<th>l</th>
<th>b</th>
<th>ξ^a</th>
<th>S_{4.33GHz}^b</th>
<th>m</th>
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<td>02 30 16.2</td>
<td>+62 09 37.9</td>
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</tr>
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<td>+60 58 19.6</td>
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<td>0.51</td>
<td>88.0</td>
<td>64 3</td>
<td></td>
</tr>
</tbody>
</table>

^a Angular distance between the line of sight and a line of sight through the center of the star cluster.

^b NVSS position. No source detected in the Stokes I map in any frequency bin.

^c High Mass X-Ray Binary LSI +61°303.
Table 3.2: Log of Observations

<table>
<thead>
<tr>
<th>VLA Project Code</th>
<th>13A-035</th>
</tr>
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<tbody>
<tr>
<td>Date of Observations</td>
<td>2013 July 10, 13, 16, and 17</td>
</tr>
<tr>
<td>Number of Scheduling Blocks</td>
<td>4</td>
</tr>
<tr>
<td>Duration of Scheduling Blocks (h)</td>
<td>4</td>
</tr>
<tr>
<td>Frequencies of Observation(^a) (GHz)</td>
<td>4.850; 7.250</td>
</tr>
<tr>
<td>Number of Frequency Channels per IF</td>
<td>512</td>
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<tr>
<td>Channel Width (MHz)</td>
<td>2</td>
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<tr>
<td>VLA array</td>
<td>C</td>
</tr>
<tr>
<td>Restoring Beam (diameter)</td>
<td>4(')81</td>
</tr>
<tr>
<td>Total Integration Time per Source</td>
<td>18–25 minutes(^b)</td>
</tr>
<tr>
<td>RMS Noise in Q and U Maps ((\mu Jy/\text{beam}))</td>
<td>39(^c)</td>
</tr>
<tr>
<td>RMS Noise in RM Synthesis Maps ((\mu Jy/\text{beam}))</td>
<td>23(^d)</td>
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</table>

\(^a\) The observations had 1.024 GHz wide intermediate frequency bands (IFs) centered on the frequencies listed, each composed of eight 128 MHz wide subbands.

\(^b\) The “O” sources (see Table 3.1) averaged 18 minutes, and the “I” sources, being weaker, were between 22–25 minutes.

\(^c\) This number represents the average rms noise level for all the Q and U maps.

\(^d\) Polarized sensitivity of the combined RM Synthesis maps.

3.2.2 VLA Observations

We observed 32 radio sources with the NSF’s Karl G. Jansky Very Large Array (VLA)\(^1\) whose lines of sight pass through or near to the shell of the IC 1805 stellar bubble. Table 3.2 lists details of the observations. Traditionally, polarization observations require observing a polarization calibrator source frequently over the course of an observation to acquire at least 60° of parallactic angle coverage. This is done to determine the instrumental polarization (D-factors, leakage solutions). Since the completion of the upgraded VLA, shorter scheduling blocks, typically less than

\(^1\)The Karl G. Jansky Very Large Array is an instrument of the National Radio Astronomy Observatory (NRAO). The NRAO is a facility of the National Science Foundation, operated under cooperative agreement with Associated Universities, Inc.
4 hours in duration, have become a common mode of observation. It is difficult, if not impossible, with very short scheduling blocks to acquire enough parallactic angle coverage to measure the instrumental calibration with a polarized source. Another method of determining the instrumental polarization is to observe a single scan of an unpolarized source. This technique can be used with shorter scheduling blocks.

In this project we calibrated the instrumental polarization using both techniques. We used the source J0228+6721, observed over a wide range of parallactic angle, and also made a single scan of the unpolarized source 3C84. Use of the CASA task POLCAL on the J0228+6721 data solved for the instrumental polarization, determined by the antenna-specific D factors (Bignell, 1982), which are complex, as well as the source polarization (Q and U fluxes). In the case of 3C84, POLCAL solves only for the D factors. Figures 3.3a and 3.3b show the amplitude of the leakage solutions for one spectral window centered at 4.720 GHz for J0228+6721 and 3C84, respectively. We find no significant deviations between these two calibration methods, indicating accurate values for the instrumental polarization parameters.

3C138 and 3C48 functioned as both flux density and polarization position angle calibrators. J0228+6721 was used to determine the complex gain of the antennas as a function of time as well to as serve as a check, as described above, for the D-factors. We observed the program sources for 5 minute intervals and interleaved the observations of J0228+6721. There was one observation of 3C138, 3C48, and 3C84 each. For our final data products, we utilized 3C84 as the primary leakage calibrator and 3C138 as the flux density and polarization position angle calibrator.
Figure 3.3: Plots of leakage terms solutions (D-factors) from CASA polarization calibration task POLCAL when the calibrator for the instrumental polarization is (a) J0228+6721, a polarized source observed over 60° of parallactic angle, or (b) 3C84, an unpolarized source observed once in each scheduling block. The plots are the solutions at 4.720 GHz, and the plotted symbols are the solutions for each antenna in the R and L polarizations at each 2 MHz wide channel.

3.3 Data Reduction

The data were reduced and imaged using the NRAO Common Astronomy Software Applications (CASA)\(^2\) version 4.5. The procedure for the data reduction as described in Section 3 of Costa et al. (2016) is identical to the procedure we employed in this study. The only difference for the current data set is that in the CASA task CLEAN, we utilized Briggs weighting with the “robust” parameter set to 0.5, which adjusts the weighting to be slightly more natural than uniform. Natural weighting has the best signal/noise ratio at the expense of resolution, while uniform is the opposite. Briggs weighting allows for intermediate options. As in our previous work, we also implemented a cutoff in the \((u, v)\) plane for distances < 5000 wavelengths to remove foreground nebular emission.

\(^2\)For further reference on data reduction, see the NRAO Jan- sky VLA tutorial “EVLA Continuum Tutorial 3C391” (http: //casaguides.nrao.edu/index.php?title=EVLA_Continuum_ Tutorial_3C391)
Similar to Costa et al. (2016), we had two sets of data products after calibration and imaging. The first set of images consisted of radio maps (see Figures 3.4a and 3.4b) of each Stokes parameter, formed over a 128 MHz wide subband for each source. These images were inputs to the $\chi(\lambda^2)$ analysis (Section 3.3.1), and there were typically 14 individual maps for each source per Stokes parameter.

The second set of images consisted of maps of $I$, $Q$, and $U$ in 4 MHz wide steps across the entire bandwidth using the clean mode “channel”, which averages two adjacent 2 MHz channels. To correct for the frequency dependence that can affect the orthogonal Stokes parameters, we apply a correction to the $Q$ and $U$ images. We first determine the spectral index, $\alpha$, of each source from a least-squares fit to the log of the flux density, $S_\nu$, and the log of the frequency, $\nu$. We use the center frequency, $\nu_c$, of the band and the measured value of $Q$ and $U$ at each frequency, $\nu$, to find $Q_o$ and $U_o$ using the relationship

$$Q = Q_o \left(\frac{\nu}{\nu_c}\right)^{-\alpha} \quad \text{and} \quad U = U_o \left(\frac{\nu}{\nu_c}\right)^{-\alpha}.$$

The final images consisted of approximately 336 maps per source, per Stokes parameter, as inputs for the RM Synthesis analysis (Section 3.3.2).

3.3.1 Rotation Measure Analysis via a Least-Squares Fit to $\chi$ vs $\lambda^2$

The output of the CASA task clean produces images in Stokes $I$, $Q$, $U$, and $V$. From these images, we generated maps with the task immath of the linear polarized intensity $P$,

$$P = \sqrt{Q^2 + U^2}.$$
Figure 3.4: Map of (a) W4-I118 and (b) W4-I24 at 4913 MHz. The circle in the lower left is the restoring beam. The gray scale is the linear polarized intensity, $P$, the vectors show the polarization position angle, $\chi$, and the contours are the Stokes $I$ intensity with levels of -2, -1, 2, 10, 20, 40, 60, and 80% of the peak intensity, 21.5 mJy beam$^{-1}$ and 11.7 mJy beam$^{-1}$ for W4-I18 and W4-I24a, respectively.

and the polarization position angle $\chi$,

$$\chi = \frac{1}{2} \tan^{-1} \left( \frac{U}{Q} \right)$$

for each source over a 128 MHz subband. Data that are below the threshold of $5\sigma_Q$ are masked in the $P$ and $\chi$ maps, where $\sigma_Q = \sigma_U$ is the rms noise in the $Q$ data. This threshold prevents noise in the $Q$ and $U$ data from generating false structure in the $P$ and $\chi$ maps. Examples of images are shown in Figure 3.4, which displays the total intensity, polarized intensity, and polarization position angle for sources W4-I18 and W4-I24. W4-I18 is an example of a point source, or slightly resolved source. Twelve of the sources in Table 3.1 were of this type and unresolved to the VLA in C array. Eight sources were like W4-I24, showing extended structure in the observations and potentially yielding RM values on more than one line of sight.

In the case of a single foreground magnetic-ionic medium responsible for the rotation of an incoming radio wave, the relation between $\chi$ and $\lambda^2$ is linear, and we
Figure 3.5: Plot of the polarization position angle as a function of the square of the wavelength, \( \chi(\lambda^2) \), for the source (a) W4-I18, RM = +514 \pm 12 \text{ rad m}^{-2}, \) and (b) W4-I24a, RM = -658 \pm 5 \text{ rad m}^{-2}. Each plotted point results from a measurement in a single 128 MHz-wide subband.

calculate the RM through a least-squares fit of \( \chi(\lambda^2) \). To measure \( \chi \), we select the pixel that corresponds to the highest value of \( P \) on the source in the 4338 MHz map, and we then measure \( \chi \) at that location in each subsequent 128 MHz wide subband. Figure 3.5 shows two examples of the least-squares fit to \( \chi(\lambda^2) \). The \( \chi \) errors are \( \sigma_\chi = \frac{\sigma_Q}{2P} \) (Everett & Weisberg 2001, Equation 12).

3.3.2 Rotation Measure Synthesis

In addition to the least-squares fit to \( \chi(\lambda^2) \), we performed Rotation Measure Synthesis (Brentjens & de Bruyn, 2005). The inputs to RM Synthesis are images in Stokes \( I, Q, \) and \( U \) across the entire observed spectrum in 4 MHz spectral intervals.
We refer the reader to Section 3.1.2 of Costa et al. (2016) for a detailed account of our procedure, which follows the implementation of RM Synthesis as developed by Brentjens & de Bruyn (2005). The goal of RM synthesis is to recover the Faraday dispersion function \( F(\phi) \). Here \( \phi \), the Faraday depth, is a variable which is Fourier-conjugate to \( \lambda^2 \) (see Costa et al. 2016, Equations 3 and 4), and has units of rad m\(^{-2}\). We also refer to \( F(\phi) \) as the “Faraday spectrum”.

<table>
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<td>max-scale</td>
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<td>Sensitivity to extended Faraday structures; Eq (62) Brentjens &amp; de Bruyn (2005).</td>
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<tr>
<td>(</td>
<td>\phi_{\text{max}}</td>
<td>)</td>
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</tbody>
</table>

\(^a\) This bandwidth includes the frequencies not observed that are between the two IFs. They are set to 0 via the weighting function, \( W(\lambda^2) \).

\(^b\) Since flagging for RFI and bad antennas were done individually for each scheduling block, the FWHM of the RMSF can vary slightly from source to source. However, these slight variations are not significant in our interpretation of the RM values report in this paper.

\( F(\phi) \) is recovered via an rmclean algorithm (Heald, 2009; Bell & Enßlin, 2012), and we applied a 7\( \sigma \) cutoff, which is above the amplitude at which peaks due to noise are likely to arise (Brentjens & de Bruyn, 2005; Macquart et al., 2012; Anderson
et al., 2015). The **RMSYNTHESIS** algorithm initially searched for peaks in the Faraday spectrum using a range of $\phi \pm 10,000$ rad m$^{-2}$ at a resolution of 40 rad m$^{-2}$ to determine if there were significant peaks at large values of $|\phi|$. Then, we performed a finer search at $\phi = \pm 3000$ rad m$^{-2}$ at a resolution of 10 rad m$^{-2}$. The RM Synthesis parameters, such as the full-width-at-half-maximum (FWHM) of the rotation measure spread function (RMSF) and the maximum detectable Faraday depth, are given in Table 3.3.

![Figure 3.6](image)

Figure 3.6: (a) Linear polarization map and (b) RM map of W4-I24 from the RM Synthesis analysis. In both images, the data cube was flattened over the Faraday depth axes, and a threshold of $7\sigma$ was applied. W4-I24 has extended structure, so there are two peaks, which are also present in the CASA maps (Figure 3.4b).
As in Costa et al. (2016), we utilized an IDL code for the \textit{rmsynthesis} and \textit{rmclean} algorithms. The output of the IDL code is a data cube in Faraday depth space that is equal in range to the range of $\phi$ that was searched over in the \textit{rmsynthesis} algorithm. The data cube contains, for example, 500 maps of the polarized intensity as a function of spatial coordinates and $\phi$, which ranges between $\pm$ 10,000 rad m$^{-2}$ at intervals of 40 rad m$^{-2}$. Initially, we generated these maps for a 1024 $\times$ 1024 pixel image. We then used the Karma package (Gooch, 1995) tool \textit{kvis} to review the maps to search for sources or source components away from the phase center that, while being too weak to detect in the 128 MHz maps, may be detectable in the RM Synthesis technique since it uses the entire bandwidth to determine the Faraday spectrum\textsuperscript{3}. However, no such sources were identified above the cutoff. From the 1024 $\times$ 1024 maps of the Faraday spectrum, we identified the $P_{\text{max}}$ for the observed sources and extracted the Faraday spectrum at that location. Figure 3.6a shows an example of a $P_{\text{max}}$ map that has been flattened along the $\phi$ axis, i.e., the gray scale in the image represents the full range of $\phi$. From this map, it is easy to identify the spatial location of $P_{\text{max}}$ for the source, which agrees with the location of the peak linear polarized intensity in the $\chi(\lambda^2)$ analysis. We obtained this same result in Costa et al. (2016) for the Rosette Nebula.

To determine the RM, we fit a 2 degree polynomial to the Faraday spectrum at each pixel in the 1024 $\times$ 1024 image above the 7$\sigma$ cutoff. The gray scale in Figure 3.6b shows the RM value from the fit to each pixel. The image is zoomed and centered on the source. While we can mathematically determine the RM at each pixel, the sources are not resolved, so we only select the RM at the spatial location of $P_{\text{max}}$. Figure 3.7a plots the Faraday spectrum and \textit{rmclean} components for W4-I18, and Figure 3.7b shows the RMSF.

\textsuperscript{3}private communication, L. Rudnick
Anderson et al. (2015) describe two cases for the behavior of the Faraday spectrum. A source is considered *Faraday simple* when $F(\phi)$ is non-zero at only one value of $\phi$, $Q$ and $U$ as a function of $\lambda^2$ vary sinusoidally with equal amplitude, and $P(\lambda^2)$ is constant. The *Faraday simple* case has the physical meaning of a uniform Faraday screen in the foreground that is responsible for the Faraday rotation, and $\chi$ is linearly dependent on $\lambda^2$. If a source is *Faraday simple*, then $F(\phi)$ is a delta function at a Faraday depth equal to the RM. The second behavior Anderson et al. (2015) describe for the Faraday spectrum is a *Faraday complex* source, which is any spectrum that deviates from the criteria set for the *Faraday simple* case. A *Faraday complex* spectrum can be the result of depolarization in form of beam depolarization, internal Faraday dispersion, multiple interfering Faraday rotating components, etc. (Sokoloff et al., 1998).

### 3.4 Observational Results

#### 3.4.1 Measurements of Radio Sources Viewed Through the W4 Complex

We measured 27 RM values for 20 lines of sight, including secondary components, through or near to IC 1805. In Table 3.4, the first column lists the source name using our naming scheme, and column two gives the component, if the source had multiple resolved components for which we could determine a RM value. Columns three and four list the RM value from the least-squares method and the reduced chi-squared value, respectively, and column five lists the RM value determined from the RM Synthesis technique.

Figure 3.8 shows the agreement between the two techniques for determining the RM. As in Costa et al. (2016), we find good agreement between the two techniques, and the good agreement between the results using the two techniques gives us confidence in our RM measurements.
Table 3.4: Faraday Rotation Measurement Values through the W4 Complex

<table>
<thead>
<tr>
<th>Source</th>
<th>Component</th>
<th>RM&lt;sup&gt;a&lt;/sup&gt; (rad m&lt;sup&gt;-2&lt;/sup&gt;)</th>
<th>Reduced χ&lt;sup&gt;b&lt;/sup&gt;</th>
<th>RM&lt;sup&gt;c&lt;/sup&gt; (rad m&lt;sup&gt;-2&lt;/sup&gt;)</th>
<th>ξ&lt;sup&gt;d&lt;/sup&gt;</th>
<th>ξ&lt;sup&gt;e&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>W4-I1</td>
<td>a</td>
<td>-277 ± 1</td>
<td>29</td>
<td>-258 ± 3</td>
<td>29</td>
<td>53</td>
</tr>
<tr>
<td>W4-I2</td>
<td>a</td>
<td>-1042 ± 7</td>
<td>1.5</td>
<td>-930 ± 30</td>
<td>30</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>-935 ± 6</td>
<td>1.5</td>
<td>-954 ± 11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W4-I3</td>
<td>a</td>
<td>-876 ± 2</td>
<td>1.9</td>
<td>-878 ± 8</td>
<td>27</td>
<td>16</td>
</tr>
<tr>
<td>W4-I4</td>
<td>a</td>
<td>-139 ± 3</td>
<td>1.9</td>
<td>-153 ± 12</td>
<td>38</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>-91 ± 6</td>
<td>2</td>
<td>-68 ± 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W4-I6</td>
<td>a</td>
<td>-990 ± 8</td>
<td>1.3</td>
<td>-961 ± 23</td>
<td>29</td>
<td>25</td>
</tr>
<tr>
<td>W4-I8</td>
<td>a</td>
<td>-276 ± 2</td>
<td>4.4</td>
<td>-337 ± 8</td>
<td>37</td>
<td>54</td>
</tr>
<tr>
<td>W4-I11</td>
<td>a</td>
<td>-377 ± 8</td>
<td>12</td>
<td>-141 ± 18</td>
<td>35</td>
<td>41</td>
</tr>
<tr>
<td>W4-I12</td>
<td>a</td>
<td>-315 ± 4</td>
<td>2.8</td>
<td>-306 ± 10</td>
<td>34</td>
<td>54</td>
</tr>
<tr>
<td>W4-I13</td>
<td>a</td>
<td>-777 ± 8</td>
<td>1.2</td>
<td>-801 ± 24</td>
<td>28</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>-701 ± 28</td>
<td>0.5</td>
<td>-772 ± 66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W4-I14</td>
<td>a</td>
<td>-678 ± 27</td>
<td>0.6</td>
<td>-666 ± 66</td>
<td>20</td>
<td>36</td>
</tr>
<tr>
<td>W4-I15</td>
<td>a</td>
<td>-157 ± 9</td>
<td>0.8</td>
<td>-124 ± 14</td>
<td>32</td>
<td>10</td>
</tr>
<tr>
<td>W4-I17</td>
<td>a</td>
<td>-492 ± 8</td>
<td>1.6</td>
<td>-440 ± 40</td>
<td>11</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>-509 ± 15</td>
<td>0.6</td>
<td>-464 ± 40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W4-I18</td>
<td>a</td>
<td>+514 ± 12</td>
<td>1.1</td>
<td>+501 ± 33</td>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>W4-I19</td>
<td>a</td>
<td>-407 ± 14</td>
<td>0.3</td>
<td>-431 ± 36</td>
<td>37</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>-53 ± 26</td>
<td>1.4</td>
<td>-167 ± 67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W4-I21</td>
<td>b</td>
<td>-98 ± 28</td>
<td>0.5</td>
<td>-79 ± 62</td>
<td>17</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>-173 ± 34</td>
<td>0.5</td>
<td>-232 ± 70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W4-I24</td>
<td>a</td>
<td>-658 ± 5</td>
<td>1.4</td>
<td>-678 ± 14</td>
<td>39</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>-675 ± 12</td>
<td>0.4</td>
<td>-716 ± 30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W4-O4</td>
<td>a</td>
<td>-31 ± 12</td>
<td>24</td>
<td>-178 ± 18</td>
<td>68</td>
<td>81</td>
</tr>
<tr>
<td>W4-O6</td>
<td>a</td>
<td>-95 ± 1</td>
<td>3.6</td>
<td>-96 ± 4</td>
<td>54</td>
<td>76</td>
</tr>
<tr>
<td>W4-O7</td>
<td>a</td>
<td>-175 ± 24</td>
<td>0.8</td>
<td>-256 ± 56</td>
<td>53</td>
<td>62</td>
</tr>
<tr>
<td>W4-O10</td>
<td>a</td>
<td>-379 ± 5</td>
<td>1.8</td>
<td>-343 ± 16</td>
<td>56</td>
<td>66</td>
</tr>
</tbody>
</table>

<sup>a</sup> RM value obtained from a least-squares linear fit to χ(λ<sup>2</sup>). The errors are 1σ.

<sup>b</sup> Reduced χ<sup>2</sup> for the χ(λ<sup>2</sup>) fit.

<sup>c</sup> Effective RM derived from RM Synthesis.

<sup>d</sup> Distance from center of OCl 352.

<sup>e</sup> Distance from Terebey et al. (2003) center.
3.4.2 Report on Faraday Complexity and Unpolarized Lines of Sight

In the last paragraph of Section 3.3.2, we discuss Faraday complexity. If a source is Faraday simple, then the RM is equal to a delta function in $F(\phi)$ at the Faraday depth. If a source is Faraday complex, then the interpretation of the RM is not as straightforward. There is extensive literature (e.g. Farnsworth et al. 2011, O’Sullivan et al. 2012, Anderson et al. 2015, Sun et al. 2015, Purcell et al. 2015) to understand Faraday complexity.

One indicator of a Faraday complex source is a decreasing fractional polarization, $p = P/I$, as a function of $\lambda^2$. The ways in which this can arise are discussed at
the end of Section 3.3.2. Although depolarization does not necessarily lead to a net rotation of the source $\chi$, its presence indicates the potential for a $\chi$ rotation independent of the plasma medium through which the radio waves subsequently propagate. This could result in an error in our deduced RMs. Nine of the sources, W4-I1, -I3, -I11, -I15, I21b, -O4, -O6, -O7, and -O10 show a decreasing $p$ with increasing $\lambda^2$.

A rough estimate of the potential position angle rotation associated with depolarization may be obtained using the analysis in Cioffi & Jones (1980). These calculations assume that depolarization arises from Faraday rotation within the synchrotron radiation source, and we can estimate the effect of internal depolarization from the changes in fraction polarization. Given the fractional polarization at the shortest and longest wavelength, we obtain the corresponding polarization angle change from Figure 1 of Cioffi & Jones (1980) and then calculate a RM due to internal depolarization. If the calculated RM due to depolarization ($\text{RM}_{\text{depol}}$) is larger than the observed RM, then the RM is potentially affected by depolarization. W4-I1, -O6, -O7, and -O10
show \(RM_{depot} \sim RM_{obs}\), which indicates that internal depolarization could affect the observed RM. The observed RM of W4-I3 is \(\sim 3\) times larger than \(RM_{depot}\), so it is not affected by internal depolarization.

Depolarization due to internal Faraday rotation makes predictions for the form of \(\chi(\lambda^2)\) which would not have \(\chi \propto \lambda^2\) (Cioffi & Jones, 1980). For all of the sources mentioned above, we compared the observed behavior of \(\chi(\lambda^2)\) to the predicted behavior (Equation 4b of Cioffi & Jones 1980). Within the errors, only W4-O7 is consistent with the non-linear behavior of a RM affected by internal depolarization. We interpret this result as meaning that our deduced RM values for most of the sources are not significantly in error due to internal depolarization, and we consider the measurement of depolarization as providing a cautionary flag.

We also considered whether our measurements could have been affected by bandwidth depolarization or beam depolarization. Bandwidth depolarization occurs when the polarization angle varies over frequency averaged bins, \(\Delta \nu\). For example in this study, we use values of \(\chi\) in 128 MHz wide bins (Section 3.3.1) and 4 MHz (Section 3.3.2). For a center frequency of the lowest frequency bin, we use \(\nu_c = 4466\) MHz, and the relationship between the change in polarization position angle, \(\Delta \chi\), is

\[
|\Delta \chi| = 2|RM| \frac{\Delta \nu}{\nu_c^3},
\]

where \(c\) is the speed of light. This formula shows that even for \(|RM| = 10^4\) rad m\(^{-2}\), which is far larger than any RMs we measure, the Faraday rotation across the band is 0.41 radians. This is insufficient to causes substantial bandwidth depolarization. Beam depolarization occurs when there are small scale variations of the electron density or the magnetic field within a beam. It is unlikely that the RMs are affected by beam depolarization as the beam at 6 cm for the VLA in C array is \(\sim 5\) arcseconds. We interpret these RMs as a characteristic value due to the plasma medium (primarily
the Galactic ISM) between the source and the observer. In the analysis which follows, we choose the RM values from the RM Synthesis method.

When the data were mapped and inspected, we found that a few sources that had passed our criteria for flux density and compactness to the VLA D array at L band (1.42 GHz) were completely unpolarized. W4-I16 and W4-O8 are not polarized at any frequency, and the RM Synthesis technique does not show significant (>7σ) peaks at any φ. Three of the lines of sight, W4-I5, W4-I10, and W4-I22, have no source in the field. Despite appearing to be point sources in the NVSS postage stamps (see Section 3.2), we do not observe a source at these locations, and they may have been clumpy foreground nebular emission that was filtered out during the imaging process.

Subsequent investigations determined that some of the selected sources were previously cataloged ultra compact HII regions associated with the W3 star formation region. These sources are W4-I7 (W3(OH)-C), W4-I9 (AFGL 333), and W4-I20 (W3(OH)-A) (Feigelson & Townsley, 2008; Navarete et al., 2011; Román-Zúñiga et al., 2015). The W4-I7 field has no source at the observed α and δ, despite it being identified as W3(OH)-C. We do not observe a source at this location in any frequency bin. W3(OH)-A, however, is observed and is a point source in our maps at all frequencies. Similarly, W4-I9 is detected in each frequency bin and is an extended source. These sources are unpolarized and do not feature in our subsequent analysis.

3.4.3 A Unique Line of Sight Through the W4 Region: LSI +61°303

W4-I19 has a spectrum which is inconsistent with an optically-thin extragalactic radio source. It is linearly polarized, and we measure RM = -431 ± 36 rad m⁻². Investigation of this source during the data analysis phase revealed that it is not an extragalactic source, although it passed our selection criteria for flux and compactness. W4-I19 is the high mass X-ray binary (HMXB) LSI +61°303 (Gregory et al., 1979; Bignami et al., 1981), which is notable for being one of five known gamma ray binary
systems (Frail & Hjellming, 1991). This system has been extensively studied, and
as a result, much is known about the nature of the compact object (Massi et al.,
2004; Massi, 2004; Dubus, 2006; Paredes et al., 2007; Massi et al., 2017), the stellar
companion (Casares et al., 2005; Dubus, 2006; Paredes et al., 2007), orbital period
(Gregory & Neish, 2002), radio structure (Albert et al., 2008), and radial velocity
(Gregory et al., 1979; Lestrade et al., 1999).

The spatial location of LSI +61°303 is important to understanding the RM we
determined for this source. Frail & Hjellming (1991) argue that since signatures of the
Perseus arm shock are present in the absorption spectrum to LSI +61°303 but not the
post-shock gas from the Perseus arm, LSI +61°303 must lie between the two features
at a distance of 2.0 ± 0.2 kpc. They also report that they do not see absorption
features due to the IC 1805 ionization front and shock front. The estimated distance
to LSI +61°303 is consistent with distance estimates to OCl 352. The position relative
to the nebula has consequences for the interpretation of the RM that we measure.
The possibilities are:

1. LSI +61°303 is in front of the stellar bubble and H II region, so it is exterior to
   a region modified by OCl 352. The RM is then an estimate of the foreground
   ISM between us and the nebula.

2. If LSI +61°303 is at the same distance as IC 1805 or slightly behind (greater
distance), then the RM is unique among our sources in that it is not affected by
   Faraday rotation from material in the outer Galaxy. The RM is then probing
   at least a part of the Faraday rotating material due to the nebula.

To further determine the position of LSI +61°303 with respect to IC 1805, we
review the current state of knowledge on the subject from the literature. Dhawan et al.
(2006) observed LSI +61°303 with the Very Long Baseline Array (VLBA) and report
a proper motion of \((\mu_\alpha, \mu_\delta) = (-0.30 \pm 0.07, -0.26 \pm 0.05)\) mas yr\(^{-1}\). Aragona et al.
(2009) report a radial velocity for LSI +61°303 of $V_{rad} = -41.4 \pm 0.6$ km s$^{-1}$, which agrees with previous estimates by Casares et al. (2005). For OCl 352, Dambis et al. (2001) estimate the radial velocity to be $-41 \pm 3$ km s$^{-1}$, and more recent estimates by Kharchenko et al. (2005) ($V_{rad} = -47 \pm 18$ km s$^{-1}$) agree within the errors. Both LSI +61°303 and OCl 352 have similar radial velocities, and the proper motion estimates by Dhawan et al. (2006) indicate that LSI +61°303 is moving similarly on the plane of the sky to OCl 352, which has a proper motion of $(\mu_\alpha, \mu_\delta) = (-1.0 \pm 0.4, -0.9 \pm 0.4)$ mas yr$^{-1}$ (Dambis et al., 2001).

From proper motion and radial velocity estimates, LSI +61°303 appears to be moving in relatively the same direction and speed as OCl 352. Using a distance of 2 kpc to LSI +61°303 and 2.2 kpc to OCl 352, the transverse velocities are $\sim 3$ km s$^{-1}$ and $\sim 14$ km s$^{-1}$, respectively. If LSI +61°303 originally belonged to OCl 352, then it is unlikely that it is in front of IC 1805, given that both are moving at the same radial velocity. While LSI +61°303 appears to be outside the obvious shell structure of IC 1805, it is more likely that it is probing material modified by OCl 352. We discuss this possibility further in Section 3.5.3.

If LSI +61°303 did not originate in OCl 352, then it is possible to still be in front of the nebula, despite the similar velocities. In such a case, the RM we obtained for this line of sight is due to the ISM between us and IC 1805. The RM value we find for LSI +61°303 is nearly 3 times larger than the background RM, which we discuss in Section 3.5.2. This would require a magneto-ionic medium between the observer and the nebula capable of producing $\sim 400$ rad m$^{-2}$ along this line of sight. As may be seen from Table 3.4 and Figure 3.2, other lines of sight near IC 1805, but exterior to the shell, do not have as large of RM values (e.g. W4-O26, -O19, -O7, -I11). It therefore seems most probable that the RM for W-I19 (LSI +61°303) is dominated by plasma in W4.

In summary, there is evidence in the literature that suggests LSI +61°303 may
lie within a region modified by OCl 352, particularly if LSI +61°303 did indeed once belong to OCl 352. If this is the case, then the RM we find is unaffected by the ISM in the outer galaxy and is due to the material near IC 1805.

### 3.5 Results on Faraday Rotation Through the W4 Complex

#### 3.5.1 The Rotation Measure Sky in the Direction of W4

![Plot of RM vs distance from the center of OCl 352 for (a) lines of sight that pass through the W4 Superbubble and (b) lines of sight near and close to the southern loop. The solid line represents the estimate of the background RM using sources in this study and in the literature, and the dashed line is the predicted background RM from the Van Eck et al. (2011) model of the Galactic magnetic field.](image)

Figure 3.9: Plot of RM vs distance from the center of OCl 352 for (a) lines of sight that pass through the W4 Superbubble and (b) lines of sight near and close to the southern loop. The solid line represents the estimate of the background RM using sources in this study and in the literature, and the dashed line is the predicted background RM from the Van Eck et al. (2011) model of the Galactic magnetic field.

Whiting et al. (2009), Savage et al. (2013), and Costa et al. (2016) compared observations to a model of the ionized shell in which the RM depended only on $\xi$,
the impact parameter, or closest approach of a line of sight to the center of the shell.

In anticipation of a similar analysis in this study, we show Figures 3.9a and 3.9b, which plot the RM versus distance from the center of star cluster for the lines of sight through the W4 Superbubble (W4-I1, -I4, -I8, -I12, -I14, -I17, -O4, -O6, and -O7) and the ones through or close to the southern loop (W4-I2, -I3, -I6, -I11, -I13, -I15, -I18, -I19, -I21, -I24, -O10).

In Section 3.1.2, we discussed the morphology of the region around IC 1805 and made the distinction between the southern latitudes and the northern latitudes, so in the following sections, we address each region near IC 1805 separately.

3.5.2 The Galactic Background RM in the Direction of W4

In Savage et al. (2013), we determined the background RM in the vicinity of the Rosette Nebula ($\ell \sim 206^\circ$) by finding the median value of the RM for sources outside the obvious shell structure of the Rosette. Determining the background RM near IC 1805 is difficult, however, due to proximity of W3, the W3 molecular cloud, and the W4 Superbubble. Given the morphological difference between the northern and southern parts of IC 1805, we assume that sources south of OCl 352 ($b < 0.9^\circ$) should be modeled independently of the northern sources, since the W4 Superbubble extends up to $b \sim 7^\circ$ (West et al., 2007). The lines of sight north of the star cluster are intersecting the W4 Superbubble and are not probing the RM due to the general ISM independent of IC 1805. Therefore, the only lines of sight that are potentially probing the RM in the vicinity of IC 1805 are those exterior to the shell structure of the southern loop.

If we apply the thick shell model from Terebey et al. (2003) (see Section 3.1.2 for details), then the lines of sight with RM values exterior to the shell are W4-I2, -I11, and -O10. For the thin shell case, W4-I6, -I13 and -I24 are also exterior sources. The mean RM value for the background using these sources is $-554 \text{ rad m}^{-2}$ and $-670$
Table 3.5: List of Sources with RM values from Catalogs

<table>
<thead>
<tr>
<th>Source Name</th>
<th>α(J2000)</th>
<th>δ(J2000)</th>
<th>RM(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W4-O3</td>
<td>02 35 43.0</td>
<td>+63 22 33.0</td>
<td>−138±18</td>
</tr>
<tr>
<td>W4-O19</td>
<td>02 46 23.9</td>
<td>+61 33 19.9</td>
<td>−157±15</td>
</tr>
<tr>
<td>W4-O26</td>
<td>02 42 32.3</td>
<td>+60 02 31.0</td>
<td>+61 ± 41</td>
</tr>
<tr>
<td>W4-O27</td>
<td>02 25 48.7</td>
<td>+59 53 52.0</td>
<td>−145 ± 22</td>
</tr>
</tbody>
</table>

\(^a\) RM values from Brown et al. (2003) unless otherwise noted.

\(^b\) Taylor et al. (2009) give −75 ± 9 rad m\(^-2\) for this line of sight.

\(^c\) RM value from Taylor et al. (2009).

rad m\(^-2\) for the thick or thin shell, respectively. In Table 3.5, we list RM values from the literature for lines of sight near IC 1805 that we include in our estimate of the background RM. The mean RM value for these sources (excluding W4-O3 for being in the superbubble) is −80 rad m\(^-2\). The sources W4-I2, -I6, -I13, and -I24 are seemingly outside the obvious ionized shell structure; however, they are also the lines of sight for which we measure some of the highest RM values. This is a surprising result, and one we did not observe in the case of the Rosette Nebula. It strongly suggests that the lines of sight to W4-I2, -I6, -I13, and -I24 have RMs that are dominated by the W4 complex, despite the fact that they are outside the obvious ionized shell of IC 1805. We discuss this further in the next section.

For the present discussion, we exclude these sources from the estimate of the background. Using W4-I11, -O10, -O19, -O26, and -O27, we find a mean value for the background RM due to the ISM of −145 rad m\(^-2\). While this value is similar in magnitude to the value of the background RM we found in our studies on the Rosette Nebula, we have significantly fewer lines of sight, and only two of the lines of sight were observed in this study.

Due to a low number of lines of sight exterior to IC 1805, we utilize the model of a Galactic magnetic field by Van Eck et al. (2011) to estimate the background RM
due to the ISM. From their Figure 6, they find the Galactic magnetic field is best modeled by an almost purely azimuthal, clockwise field. Van Eck et al. (2011) use their model to predict the RM values in the Galaxy, and in the vicinity of IC 1805, their model predicts RMs of order $-100$ rad m$^{-2}$. Using this as an estimate of the background RM, we find an excess RM due to IC 1805 of $+600$ to $-860$ rad m$^{-2}$.

3.5.3 High Faraday Rotation Through Photodissociation Regions

The lines of sight with the highest RM values, W4-I2, -I6, and -I24, appear to be outside the obvious shell of the southern loop. These sources are very near to the bright ionized shell. Terebey et al. (2003) and Gray et al. (1999) discuss a halo of ionized gas that surrounds IC 1805, which may be causing the high RM values. Gray et al. (1999) speculate that the diffuse extended structure is an extended HII envelope as suggested by Anantharamaiah (1985). Another possibility is that these high RMs arise in the PDR surrounding the IC 1805 HII region.

PDRs are the regions between ionized gas, which is fully ionized by photons with $h\nu > 13.6$ eV, and neutral or molecular material. PDRs can be partially ionized and heated by far-ultraviolet photons ($6 \text{ eV} < h\nu < 13.6 \text{ eV}$) (Tielens & Hollenbach, 1985; Hollenbach & Tielens, 1999). Typically, the PDR consists of neutral hydrogen, ionized carbon, and neutral oxygen nearest to the ionization front, and with increasing distance, molecular species (e.g., CO, H$_2$, and O$_2$) dominate the chemical composition of a PDR (Hollenbach & Tielens, 1999). One tracer of PDRs is polycyclic aromatic hydrocarbon (PAH) emission at infrared (IR) wavelengths. Churchwell et al. (2006) identify more than 300 bubbles at IR wavelengths in the Galactic Legacy Infrared Mid-Plane Survey Extraordinaire (GLIMPSE), and 25% of these bubble coincide with known HII regions. Watson et al. (2008) examine three bubbles from the Churchwell et al. (2006) catalog with the Spitzer Infrared Array Camera (IRAC) bands 4.5, 5.8, and 8.0 µm and the 24 µm band from the Spitzer Multiband Imaging Photometer.
(MIPS) to determine the extent of the PDR around three young H II regions. One of their main results is that the 8 \( \mu m \) emission, which is due to PAHs, encloses the 24 \( \mu m \) emission, which traces hot dust. Watson et al. (2008) use ratios between the 4.5, 5.8, and 8.0 \( \mu m \) bands to determine the extent of the PDRs, as the 4.5 \( \mu m \) emission does not include PAHs but the 5.8 and 8.0 \( \mu m \) bands do (see their Section 1 for details).

To determine the presence and extend of a potential PDR around IC 1805, we analyze Wide-field Infrared Survey Explorer (WISE) data from the IPAC All-Sky Data Release\(^4\) at 3.6, 4.6, 12, and 22 \( \mu m \). The 4.6 \( \mu m \) WISE bands is similar in bandwidth and center frequency to the IRAC 4.5 \( \mu m \) band, and the WISE 22 \( \mu m \) band is also similar to the MIPS 24 \( \mu m \) band (Anderson et al., 2014). The 12 \( \mu m \) WISE band does not overlap with the 8.0 \( \mu m \) band of IRAC, but the WISE band traces PAH emission at 11.2 and 12.7 \( \mu m \). Anderson et al. (2012) note, however, that the 12 \( \mu m \) flux is on average lower than the 8.0 \( \mu m \) IRAC band, which is most likely due to the WISE band sampling different wavelengths of PAH emission instead of the 7.7 and 8.6 \( \mu m \) PAH emission in the IRAC band.

Figure 3.10 is a RGB image of the southern loop of IC 1805 at 4.6 \( \mu m \) (blue), 12 \( \mu m \) (green), and 22 \( \mu m \) (red). The 1.42 GHz radio continuum emission is shown in the white contours at 8.5, 9.5, and 10 K, and the lines of sight that intersect this region are labeled as well. Similar to the results of Watson et al. (2008), the majority of the 22 \( \mu m \) emission is located inside the bubble. The radio contours trace the ionized shell of the H II region, which show a patchy ionized shell. Outside of the radio contours, there is a shell of 12 \( \mu m \) (green) PAH emission that encloses the 22 \( \mu m \) emission as well. In the northeastern portion of the image, there is extended 22 \( \mu m \) (hot dust) emission, which is spatially coincident with a CO clump (Lagrois & Joncas, 2009a).

\(^4\)http://wise2.ipac.caltech.edu/docs/release/allsky/
The PDR model predicts the presence of neutral hydrogen and molecular CO (see Figure 3 of Hollenbach & Tielens 1999) at increasing distance from the exciting star cluster. Figure 1 of Sato (1990) and Figure 2 of Hasegawa et al. (1983) show H I contours in the vicinity of IC 1805, and the H I emission appears to completely enclose the southern loop except near $135.5^\circ \leq \ell \leq 136^\circ$, $0.2^\circ \leq b \leq 0.9^\circ$. Braunsfurth (1983) report H I emission near IC 1805, and he notes that the hole could be due to cold H I gas or the lack of gas if the winds have sufficiently swept the material away or ionized it. Figure 6 of Digel et al. (1996) shows the CO emission, with the W3 molecular cloud on the western side of IC 1805, CO emission along the southern loop of IC 1805, and the molecular material associated with the W5 ($\ell = 137.1$, $b = +0.89$) H II region on the eastern side of IC 1805. We interpret the WISE data, the radio contours, and the CO and H I maps as a patchy ionized shell surrounded by a PDR.

If there is a PDR surrounding IC 1805, then the highest RM values from our data set, $\text{RM} = -954$ rad m$^{-2}$ and $-961$ rad m$^{-2}$ for W4-I2 and -I6, respectively, lie outside the ionized shell of the H II region and in the PDR. Similarly, the sources W4-I19 and -I24 are also outside the radio continuum contours but appear to be within the 12 $\mu$m (green) emission. This is a surprising result compared with our results from the Rosette Nebula, where we found the highest RM values for lines of sight that pass through the ionized shell. Gray et al. (1999) note zones of depolarization near the southern portion of IC 1805, which require RMs on order $10^3$ rad m$^{-2}$, and spatial RM gradients. The RMs for W4-I2 and -I6 are on this order, but those for W4-I24 and -I19 are not, and we do not find that these lines of sight are affect by depolarization. W4-I24 has two components for which we measure RMs, and the components are separated by $\sim 18$ arcseconds. The $\Delta$RM, which is the difference in RM between the two components is $38$ rad m$^{-2}$, which is not a large change in the RM and is consistent within the errors.

The presence of the PDR is complicated, however, by the extended diffusion
ionized emission reported by Terebey et al. (2003) and Gray et al. (1999). At lower contours, the high RM sources do lie within the radio continuum emission. To fully understand the presence and extent of a PDR or an extended H\textsc{ii} envelope, observations of radio recombination lines on the eastern side of IC 1805 would clarify the structure as well as observations of other tracers of PDRs (e.g., fine structure lines of C and C\textsuperscript{+}, H\textsubscript{2}, and CO). It may be the case that the ionized shell is patchy along the shell wall, which allows photons $>13.6$ eV to escape the shell at places, but the shell is sufficiently ionization-bounded at other places such that a PDR can form.

Figure 3.10: Inset from Figure 3.2. A RGB image of archive WISE data at 4.6 $\mu$m (blue), 12 $\mu$m (green), and 22 $\mu$m (red) with CGPS contours at 8.5, 9.5, and 10 K in white. The lines of sight from the present study are shown with circles and are labeled according to Table 3.1.
3.5.4 Faraday Rotation Through the Cavity and Shell of the Stellar Bubble

There are four lines of sight are through the cavity of the stellar bubble, assuming an inner radius from the Terebey et al. (2003) model. The sources W4-I3, -I15, -I18, and -I21 are through the cavity, and including multiple components, we find 6 RM values. W4-I3 has a high RM ($-878 \pm 18 \text{ rad m}^{-2}$), and W4-I15 and -I21 have comparatively low RM values ($-79$ to $-232 \text{ rad m}^{-2}$). Examination of Figures 3.2 and 3.10 does not reveal enhanced emission near W4-I3 in comparison to W4-I21. W4-I15, however, is in a region of relatively low emission, which may explain why W4-I15 has a RM value at least 4 times smaller than W4-I3.

W4-I13 is outside the shell, assuming a shell radius from either Terebey et al. (2003) model. From Figure 3.2, it does appear to be outside the ionized shell. However W4-I13 is within a 8.5 K contour on the 1.42 GHz radio continuum map, which may indicate that it is probing the ionized shell. We find a high RM for both components of this source, which is similar to the RM values for W4-I2 and -I6.

Across IC 1805, we observe negative RM values for all lines of sight except one: W4-I18, which is 5.6 arcmin (4 pc) from the center of the star cluster. The absolute value of the RM for W4-I18 is also large ($+501 \pm 33 \text{ rad m}^{-2}$), indicating a large change in RM along this line of sight relative to other lines of sight in this part of the sky. This line of sight is probing the space close to the massive O and B stars responsible for IC 1805. In the Weaver et al. (1977) model for a stellar bubble, the hypersonic stellar wind dominates the region between the star responsible for the bubble and the inner termination shock. Equation (12) of Weaver et al. (1977) states that the distance of the inner shock, $R_t$ is

$$R_t = 0.90 \alpha^{3/2} \left( \frac{1}{\rho_0} \frac{dM_w}{dt} \right)^{3/10} V_w^{1/10} t^{2/5},$$

(3.4)

91
where $\alpha$ is a constant equal to 0.88, $\rho_0$ is the mass density in the external ISM, $dM_w/dt$ is the mass loss rate, $V_w$ is the terminal wind speed, and $t$ is time. For a rough estimate of the inner shock distance, we utilize general stellar parameters for OCl 352 of $dM_w/dt = 10^{-5}$ M$_\odot$ yr$^{-1}$, $t = 10^6$ yr, and $V_w = 2200$ km s$^{-1}$ (see Section 3.1.2 or Table 3.7). From the discussion in Section 3.6.2, we adopt $n_0 = 4.5$ cm$^{-3}$ for $\rho_0 = n_0 m_p$, where $m_p$ is the mass of a proton. With these values in the appropriate SI units, $R_t \sim 6$ pc. It is possible that the line of sight to W4-I18 passes inside the inner shock, and the large, positive RM is due to material modified by the hypersonic stellar wind and not the shocked interstellar material. With a positive value of the RM for W4-I18, the line of sight component of the field points towards us while the remaining lines of sight in the cavity are negative, meaning $B_{\text{LOS}}$ points away.

3.5.5 Low Rotation Measure Values Through the W4 Superbubble

North of IC 1805 is the W4 Superbubble, which is an extended “egg-shaped” structure closed at $b \sim 7^\circ$ (West et al., 2007). Basu et al. (1999) utilize an H$\alpha$ map to define the shape, which would include the southern loop ($134^\circ < \ell < 136^\circ$, $b < 0.5^\circ$); Normandeau et al. (1996) examine the H$\text{I}$ distribution, however, and place the base of the structure at OCl 352. Similarly, West et al. (2007) place an offset bottom of the “egg” at OCl 352. The southern loop of IC 1805 is seemingly sufficiently different from the northern latitudes, as it is often not included in the discussion of the W4 Superbubble in spite of the fact that OCl 352 is thought to be responsible for the formation of both structures (Terebey et al., 2003; West et al., 2007).

Nine lines of sight in the present study are north of OCl 352 in the W4 Superbubble. These sources are W4-I1, -I4, -I8, -I12, -I14, -I17, -O4, -O6, and -O7, and they have a mean RM of $-293$ rad m$^{-2}$ and a standard deviation of 178 rad m$^{-2}$. Of these sources, W4-I14 and -I17 have the largest RM values, $-666$ rad m$^{-2}$ and $-460$ rad m$^{-2}$, respectively, and they are close to OCl 352, with distances of 31 arcminutes.
(20 pc) and 17 arcminutes (11 pc), respectively. As discussed in Section 3.1.2, Lagrois & Joncas (2009a) argue that the ionized “v” structure north of OCl 352 is part of the bubble wall and not a cap to southern loop structure, but examination of Figure 3.2 suggests that the bubble walls are denser, or thicker, at latitudes $< 1.5^\circ$ than higher latitudes, which may explain the larger RM associated with W4-I14 and -I17. The remaining lines of sight, however, in the W4 Superbubble have some of the lowest RM values in the data set and are consistently lower RM values than the lines of sight through the PDR.

At higher latitudes, Gao et al. (2015) modeled the polarized emission and applied a Faraday screen model to the W4 Superbubble. They report RMs on the western side of W4 ($\ell \sim 132^\circ$, $b \sim 4.8^\circ$) between $-70$ and $-300$ rad m$^{-2}$ and $\sim +55$ rad m$^{-2}$ for the eastern shell ($\ell \sim 136^\circ$, $b \sim 7^\circ$). Gao et al. (2015) argue that since W4 is tilted at an angle towards the observer (Normandeau et al., 1997), a change in the sign of the RM is consistent with a scenario in which the superbubble lifts up a clockwise running Galactic magnetic field (Han et al., 2006) out of the Galactic plane. The magnetic field would go up the eastern side of the superbubble and then down the western side, resulting in the field being pointed towards the observer in the east and away from the observer in the west. While the lines of sight reported in this paper are at $b < 2^\circ$, we find a similar range of RM values as reported by Gao et al. (2015) for the western side. However, we measure RM values 3 – 4.5 times higher on the eastern side, and we do not observe a sign reversal on the eastern side as suggested by Gao et al. (2015).

West et al. (2007) report positive values of the magnetic field for the western side from a change in polarization angle of $\sim 60^\circ$ at 21 cm, which gives a RM value on order of 20 rad m$^{-2}$. We do not observe RM values this low for any of our lines of sight through the northern latitudes. Our lines of sight, however, do not probe the same regions as the West et al. (2007) and Gao et al. (2015) studies.
The line of sight W4-I4 is arguably within the W4 Superbubble; however, it is also $\sim 8$ arcmin (5 pc) on the sky from W3-North (G133.8 +1.4), which is a star forming region within W3. W4-I4 has two components, separated by 15 arcsec (0.2 pc), and a difference in RM values between the two components of $\Delta RM = 85 \text{ rad m}^{-2}$. The RM values for both components are low ($-153 \text{ rad m}^{-2}$ and $-68 \text{ rad m}^{-2}$) despite being in the superbubble and near to W3, which may have variable but potentially large magnetic fields (van der Werf & Goss, 1990; Roberts et al., 1993) (see Section 3.1.2).

3.6 Models for the Structure of the HII region and Stellar Bubble

3.6.1 Whiting et al. (2009) Model of the Rotation Measure in the Shell of a Magnetized Bubble

Whiting et al. (2009) developed a simple analytical shell model intended to represent the Faraday rotation due to a Weaver et al. (1977) solution for a wind-blown bubble. We employed this model in (Savage et al., 2013) and Costa et al. (2016) to model the magnitude of the RM in the shell of the Rosette Nebula as a function of distance from the exciting star cluster. Figure 6 of Whiting et al. (2009) and their Section 5.1 give the details of the model, and Sections 4.1 of Savage et al. (2013) and 5 of Costa et al. (2016) describe the application of the model to the Rosette Nebula. This model takes as inputs the general interstellar magnetic field ($B$) in $\mu$G, the inner ($R_1$) and outer ($R_0$) radii of the shell in parsecs, and the electron density in the shell, $n_e$ ($\text{cm}^{-3}$). $R_0$ represents the shock between the ambient ISM and the shocked, compressed ISM, and $R_1$ separates the shocked ISM from the hot, diffuse stellar wind in the cavity. Only the component of the ambient interstellar magnetic field that is perpendicular to the shock normal is amplified by the density compression.
ratio, $X$. The resulting expression for the RM through the shell is

$$RM = C \, n_e \, L(\xi) \, B_{0z} \left( 1 + (X - 1) \left( \frac{\xi}{R_0} \right)^2 \right),$$

(3.5)

where $L(\xi)$ is the cord length through the shell in parsecs (see Equation 10 in Whiting et al. 2009 or Equation 6 in Costa et al. 2016), and $B_{0z}$ is the z-component of $B_0$, the magnetic field in the ISM. If $n_e$ has units of cm$^{-3}$, $B_{0z}$ is in $\mu$G, and $L$ is in parsecs, $C = 0.81$ (see Equation 3.2). $B_{0z}$ is at an angle $\theta$ with respect to the LOS and is written as

$$B_{0z} = B_0 \cos \theta.$$

(3.6)

In our previous work, we presented two cases for the behavior of the magnetic field in the shell. The first is that the magnetic field is amplified by a factor of 4 in the shell. The second case, in which there is not an amplification of the magnetic field in the shell, sets $X = 1$. Equation 3.5 then simplifies to

$$RM(\xi) = 0.81 \, n_e \, L(\xi) \, B_{0z}.$$

(3.7)

In Costa et al. (2016), we employed a Bayesian analysis to determine which of the two models better reproduces the observed dependence of the RM as a function of distance. We found that neither model was strongly favored in the case of the Rosette. The model given in Equation (3.5) is subject to the criticism that it applies shock jump conditions for $\mathbf{B}$ over a large volume of a shell, and that the outer radius of an observed H II region need not be the outer shock of a Weaver bubble (see remarks in Section 5.1.1 of Costa et al. 2016). It is worth including this model, however, in our analysis of IC 1805 for completeness and in order to compare our results to those of
the Rosette Nebula.

In Section 3.1.2, we discussed the structure of IC 1805, and we present evidence from the literature that north of OCl 352 is part of the W4 Superbubble. Thus, lines of sight north of OCl 352 may have different model parameters for the shell radii and electron density than the southern loop. For the southern latitudes, we utilize the Terebey et al. (2003) thin and thick shell values for the shell radii and electron density. The remaining parameters in Equation (7) are $B_0$ and $\theta$. As in Savage et al. (2013) and Costa et al. (2016), we adopt $B_0 = 4 \mu G$ for the general Galactic field in front of the H II region. The angle $\theta$ is calculated as follows. Assuming a distance of 8.5 kpc to the Galactic center, a distance to OCl 352 of 2.2 kpc, and given a Galactic longitude of 135°, the angle between the line of sight and an azimuthal magnetic field is $\theta = 55°$. We discuss our comparison of this model with the data in Section 3.7.

<table>
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<th>Center$^a$</th>
<th>$R_i$</th>
<th>$R_o$</th>
<th>$n_e$</th>
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<td>10</td>
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<td>21</td>
<td>20</td>
<td>1, 4</td>
<td>55</td>
<td>3.11b</td>
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<th>$R_s$</th>
<th>$n_0$</th>
<th>$\epsilon$</th>
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<td>25</td>
<td>7.6$^c$</td>
<td>0.62$^d$</td>
<td>55</td>
<td>3.13</td>
</tr>
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</table>

$^a$ Position of model center in Galactic coordinates in the format of ($\ell$, $b$).

$^b$ The model uses either $X = 1$ or $X = 4$.

$^c$ Calculated with Equation (3.12) and $n_s = 10$ cm$^{-3}$, $\Delta R$, and $R_s$ from Terebey et al. (2003).

$^d$ Calculated given $\Delta R$, $R_s$, and $\Theta$ with Equation (3.9).
Figure 3.11: Plots of LOS versus distance with the Whiting et al. (2009) model for \( X = 4 \) (solid) and \( X = 1 \) (dashed) for \( b < +0.9^\circ \) using (a) the thick shell and (b) the thin shell parameters from Terebey et al. (2003). The background \( RM = -100\) rad m\(^{-2}\) from the Van Eck et al. (2011) model. See Table 3.6 for model parameters.

3.6.2 Analytical Approximation to Magnetized Bubbles of Ferrière et al. (1991)

Ferrière et al. (1991) presented a semi-analytic discussion of the evolution of a stellar bubble in a magnetized interstellar medium. The theoretical object discussed by Ferrière et al. (1991) could describe a shock wave produced by a supernova explosion or energy input due to a stellar wind. The main features of the model were an outer boundary (e.g. outer shock) which was the first interface between the undisturbed ISM and the bubble, and an inner contact discontinuity between ISM material, albeit modified by the bubble, and matter that originated from the central star or star cluster.
The main feature of the model is that plasma passing through the outer boundary is concentrated in a region between the outer boundary and the contact discontinuity. In what follows, we will refer to this region as the shell of the bubble. The equation of continuity then indicates that there will be higher plasma density in the shell, and the law of magnetic flux conservation indicates that there will be an increase in the strength of the magnetic field in the shell relative to the general ISM field. Ferrière et al. (1991) were interested in the structure of the bubble, and their results have been corroborated by the fully numerical studies of Stil et al. (2009). However, Ferrière et al. (1991) did not calculate the Faraday rotation measure through their model for diagnostic purposes. Stil et al. (2009) explicitly considered the model RMs from their calculations, but only for a couple of cases and for two values of bubble orientation. It is our goal in this section to use a simplified, fully analytic approximation to the results of Ferrière et al. (1991), that permit RM profiles $\text{RM}(\xi)$ for a wide range of bubble parameters and orientation with respect to the LOS.

The geometry of the bubble is shown in Figure 3.12, which is an adaptation of Figure 1 and Figure 4 from Ferrière et al. (1991). An important feature of Figure 3.12, not present in Ferrière et al. (1991), is the orientation of the line of sight at an angle $\Theta$ with respect to the ISM magnetic field at the position of the bubble, and the impact parameter $\xi$ indicating the separation of the LOS from the center of the bubble. The region interior to the contact discontinuity is referred to as the cavity, and for the purposes of our discussion will be considered a vacuum. A major simplification that we adopt, based on an approximation to the results of Ferriere et al (1991), is that the magnetic field in the shell ($B_s$) is entirely in the azimuthal direction, and that we ignore radial variations within the shell, i.e.

\[
B_s(r, \theta) \equiv \pm B_s(\theta)\hat{e}_\theta
\]  

(3.8)
where $\hat{e}_\theta$ is a unit vector in the azimuthal direction, and the ± is selected by the polarity of the interstellar field at the bubble.

To calculate the Faraday rotation through the shell of the bubble, we need $n_s$ (the electron density in the shell) and $B_s$ as a function of the angular coordinate $\theta$, as well as the geometry of the line of sight. We also need expressions for the outer radius, $R_s(\theta)$, and thickness of the shell. This information is given by three equations in Ferrière et al. (1991). The shell thickness $\Delta R$ is given by Equation (46) of Ferrière et al. (1991),

$$ \left( \frac{\Delta R}{R_s} \right) = \frac{1}{2} \epsilon \sin \theta \quad (3.9) $$

This equation introduces the important dimensionless parameter $\epsilon$, a measure of the relative thickness of the shell. In the analysis that follows, we consider it a parameter that can be adjusted to the observations. Ferrière et al. (1991) stress that
this expression breaks down near the “poles” of the bubble. Ferrière et al. (1991) give the conditions necessary for its validity, and it applies through most of the bubble.

The condition of magnetic flux conservation in the plasma swept up into the shell gives the following expression for the strength of the magnetic field,

\[
B_s(\theta) = \frac{1}{2} \left( \frac{R_s}{\Delta R} \right) B_0 \sin \theta
\]

(3.10)

This is Equation (40) of Ferrière et al. (1991), and \( B_0 \) is the magnitude of the magnetic field in the external ISM. Strictly speaking, the RHS of our Equation (3.10) gives the \( \theta \) component of \( B_s \) in Equation (40) of Ferrière et al. (1991), but given the approximation of Equation (3.8), our Equation (3.10) is self-consistent.

Our final equation for the plasma properties in the shell gives the mass density, which accounts for the sweeping up and accumulation of mass in the shell,

\[
\rho_s = \frac{1}{3} \left( \frac{R_s}{\Delta R} \right) \rho_0
\]

(3.11)

where \( \rho_0 \) is the mass density in the unperturbed ISM. This is Equation (38) of Ferrière et al. (1991). Approximating the plasma by fully ionized hydrogen, the shell and ISM electron densities \( n_s \) and \( n_0 \) are given by

\[
n_s = \frac{1}{3} \left( \frac{R_s}{\Delta R} \right) n_0
\]

(3.12)

3.6.2.1 RM Calculation for Lines of Sight Through Walls of Shell

In evaluating the integral Equation (3.1) or Equation (3.2) through the model shell shown in Figure 3.12, we break the calculation into two parts, each with its own approximations. The first calculation is for lines of sight that pass through a portion of the shell, emerge into the cavity, and then reenter the shell on the opposite side
before exiting the shell entirely. This is the case illustrated in Figure 3.12. We adopt the rough approximation

\[ C \int_{1}^{n} n_s(s) B_s(s) \cdot ds \simeq \pm C n_s(\theta_1) B_s(\theta_1) \Delta s(\theta_1) [\hat{e}_\theta \cdot \hat{e}_s] \]  

(3.13)

In what follows, we omit the ± in front of the RHS, with the understanding that the polarity of the field in the shell is determined by the sign of the measured RM. On the left hand side of the approximation, we introduce the variable \( s \) as a coordinate along the line of sight; \( ds \) is an incremental vector along the line of sight from the source to the observer, and \( \hat{e}_s \) is the corresponding unit vector. The constant \( C \) is the same as introduced in Equation (3.5). On the right side, we express the electron density and magnetic field strength as functions only of the polar angle \( \theta \). \( \Delta s \) is the thickness of the shell along the line of sight, shown in Figure 3.12 with the gray solid line segments, and the subscript “1” indicates the first section of the shell that is transited by the line of sight.

Straightforward algebraic manipulations can be used to evaluate this expression, given relations (3.8) - (3.12), and we obtain the following result for the RM through one of the segments through the shell,

\[ RM_1 = \frac{C n_0 B_0 R_s}{3\epsilon} \frac{\xi}{\sqrt{R_s^2 - \xi^2}} \]  

(3.14)

\[ RM_1 = \frac{C n_0 B_0 R_s}{3\epsilon} \frac{x}{\sqrt{1 - x^2}} \]  

(3.15)

where we introduce a variable \( x \equiv \frac{\xi}{R_s} \), and by \( R_s \) we mean \( R_s(\theta_1) \), \( \theta_1 \) being the polar angle at which the LOS enters the shell on the side facing the observer (see Figure 3.12). Obtaining Equation (3.15) from Equation (3.14) requires use of the approximation \( R_s(\theta_\perp) \simeq R_s(\theta_1) \).
The contribution from the second passage through the shell gives a similar term, and the total RM is the sum of these. One simple case is when the two contributions are approximately equal, so we have a rough expression for the total RM

$$RM = \frac{2Cn_0B_0R_s}{3\epsilon} \frac{x}{\sqrt{1 - x^2}}$$

(3.16)

This is one of the equations we shall compare to our data.

There is an interesting opposite extreme for the value of total RM. If the angle $\Theta$ is sufficiently large, one of the shell transits will be in the upper half plane, and the other will be in the lower half plane. In this case, the projection of the shell magnetic field on the line of sight will be the opposite in the two segments. What is happening is that in one half of the shell (say that facing the observer), the ISM magnetic field is rotated towards the observer, while in the opposite shell (e.g. back wall of shell) the ISM field is rotated away. If the magnitude of the RM in the two portions of the LOS are equal, the RM from the facing shell transit will be opposite and equal to that from the rear portion. In this case, the total RM is zero. This fact can be easily grasped by imaging a line of sight with $\Theta = \frac{\pi}{2}$.

The rough model presented in this section can therefore account for one of the striking results from Stil et al. (2009). For his model shell, motivated by the work of Ferrière et al. (1991), the RM is very large when the LOS is parallel to the interstellar magnetic field ($\Theta = 0$). However, when the LOS is perpendicular, ($\Theta = \frac{\pi}{2}$), the RM is small and fluctuates between positive and negative values, presumably due to instabilities in the shell, which can be present in the simulation studies of Stil et al. (2009).

The conditions for this to occur depend on $\Theta$ and $\xi$. It can be shown that there
is a critical value of \( x = \frac{\xi}{R_s(\theta)} \), \( x_c \),

\[
x_c = \sin \Theta.
\] (3.17)

If \( x \geq x_c \), then the RM contributions from the two parts of the shell have the same sign, and Equation (3.16), or a generalization of Equation (3.14) or (3.15) holds. If \( x < x_c \), the RM contributions from the two portions of the shell are of opposite sign, and the total RM could be zero.

### 3.6.2.2 RM for Lines of Sight Entirely Within the Shell

If the “impact parameter” \( \xi \) is sufficiently large, the entire line of sight is within the shell from the point of ingress to that of egress. It can be shown that this occurs for \( x \geq x_{\text{min}} \), with

\[
x_{\text{min}} = 1 - \frac{1}{2} \epsilon \cos \Theta
\] (3.18)

and the circumstances can be visualized by reference to Figure 3.12. In this case, the rough approximation embodied in Equation (3.13) needs to be replaced by an integral over the line of sight from the ingress point to the egress point. This can be done by changing the variable of integration from \( s \) (a coordinate along the line of sight) to the polar coordinate \( \theta \), and the integral in Equation (3.13) is integrated from \( \theta_1 \), the ingress point (see Figure 3.12), to \( \theta_{\perp} \) (point of closest approach of the LOS to the center of the bubble) and then to \( \theta_2 \), the egress point.

After a reasonable amount of algebra, it can be shown the contribution \( RM_1 \), consisting of that part of the RM from \( \theta_1 - \theta_{\perp} \), is given by

\[
RM_1 = \frac{2Cn_0 B_0 \xi}{3\epsilon^2} \int_{\theta_1}^{\theta_{\perp}} \frac{d\theta}{\sin \theta \sin (\Theta - \theta)}
\] (3.19)

The integral in Equation (3.19) depends on \( \Theta \), and via \( \theta_1 = \theta_1(x) \), on \( x \) or \( \xi \). It can
be recast in the form

\[ RM_1 = \frac{2Cn_0B_0R_s}{3e^2} \left[ x \int_{\phi_1(x)}^{\pi/2} \frac{dA}{\sin(A + \Theta) \sin A} \right] \]  

(3.20)

\[ \phi_1(x) = \arcsin \left( \frac{\xi}{R_s(\theta_1)} \right) \]  

(3.21)

If one makes the brutal assumption that the RM on the second half of the integration through the shell, \( RM_2 \) is approximately the same as \( RM_1 \), and adopts a first order approximation that \( R_s \) is independent of \( \theta \), so that \( x = \frac{\xi}{R_s} \), we have the following approximate expression for the total RM in the outer parts of the shell. These are lines of sight for which \( x > x_{\text{min}} \).

\[ RM = \frac{4Cn_0B_0R_s}{3e^2} \left[ x \int_{\phi_1(x)}^{\pi/2} \frac{dA}{\sin(A + \Theta) \sin A} \right] \]  

(3.22)

\[ \phi_1(x) = \arcsin(x) \]  

(3.23)

The integral in Equation (3.22) can be readily evaluated via a Mathematica notebook or a Python code. Equations (3.16) and (3.22) and (3.23) comprise our rough analytic model for the RM(\( \xi \)) throughout the shell. The expression depends on the parameters \( R_s \), \( \epsilon \), \( \Theta \), \( n_0 \), and \( B_0 \). Another parameter that must be specified when comparing the model to data is the background Galactic RM, \( RM_{\text{off}} \).

A plot of the expressions which utilize Equations (3.16), (3.22), and (3.23) is shown in Figure 3.13, for a representative set of parameters for IC 1805 (see Table 3.6). The values of \( \epsilon \) and \( n_0 \) are calculated using Equations (3.9) and (3.12), respectively, given the electron density in the shell, the thickness of the shell, and the outer shell radius from the thick shell approximation in Terebey et al. (2003). We utilize the same value of \( \Theta \), \( B_0 \), and \( RM_{\text{off}} \) as in Section 3.6.1. The dashed line is the expression
in Equation (3.16), and the solid line is Equation (3.22). The dotted line indicates the range of x values for which \( x < x_c \), and a RM of zero is expected. The fact that these 2 expressions do not agree for \( x = x_{\text{min}} \) is an indication of the limits of the approximations used. We do not attempt to refine the model here, because a comparison with the data (discussed below) indicates that such a refinement is unwarranted.

Figure 3.13: Model for the analytic approximation to the bubble model of Ferrière et al. (1991). The solid line indicates the solution for which the LOS is entirely within the shell, the dashed line is for lines of sight which intersect the shell in 2 places, and the dotted line is a range of values of \( x = \frac{\xi}{R_s} \) for which the total RM is close to zero. The dot-dashed line represents the discontinuity between Equations (3.16) and (3.22). The plotted points represent measurements presented in this paper.
3.7 Discussion of Observational Results

3.7.1 Comparison of Models with Observations in the H\textsc{ii} Region

In this section we discuss the results of the two models presented in Sections 3.6.1 and 3.6.2. In both cases, we adopt the Terebey et al. (2003) center for geometric ease and spherical symmetry as well as the parameters given in their Table 3 for a thick shell.

Figures 3.11a and 3.11b show model RM values for lines of sight south of IC 1805 ($b < 0.9\degree$) with the Whiting et al. (2009) model for the RM as a function of distance and the shell parameters from Terebey et al. (2003). Table 3.6 gives the values of the center of the bubble, the shell radii, the electron density, $X$, and $\theta$ for Figure 3.11. Neither model reliably reproduces the observed RM as a function of distance, and as in Costa et al. (2016), the model cannot account for the dispersion of RM values at similar distances. Generally, the lines of sight in the cavity are low and are more consistent with the background RM. In the thin shell approximation, the largest RM values are associated with lines of sight outside the shell. While the model without amplification of the magnetic field in the shell can marginally account for the magnitude of the RM, the model with amplification (Equation 3.5) predicts far too high values for the RM for $\theta = 55\degree$. The analysis contained here mildly supports a result from Costa et al. (2016) for the Rosette Nebula; Faraday rotation values through these H\textsc{ii} regions do not permit a substantial increase in $|B|$ over the general Galactic field.

To reproduce the observed RM in the shell at $\xi \sim 20$ pc, the angle between the magnetic field and the observer would need to be tilted more into the plane of the sky for the $X = 4$ case or into the line of sight for the $X = 1$ case. For the former case, an angle $\sim 75\degree$ would reproduce the magnitude of the RM in the shell; such an
angle is greater than that expected from a geometric argument, even accounting for a magnetic field pitch angle of $\sim 8^\circ$. Also, no one angle can account for the range of the RM values in the cavity.

From our analytical solution for the RM due to a magnetized bubble with solutions from Ferrière et al. (1991), we can approximate the behavior of the RM for different values $\Theta$. As stated in Section 3.6.1, a geometric approach gives $\Theta = 55^\circ$. As seen in Figure 3.13, the model overestimates the value of the RM in the shell and performs poorly in the cavity. To reproduce the magnitude of the observed RMs in the shell, the magnetic field would need to be bent more into the line of sight.

Stil et al. (2009) carried out numerical MHD simulations of the Ferrière bubbles, which are obviously more accurate than our analytic approximations. Furthermore, they specifically consider and calculate the Faraday rotation through their models. However, Stil et al. (2009) only consider $\Theta = 0^\circ$ and $\Theta = 90^\circ$, so the calculations reported in that paper cannot explore the changes in RM structure with $\Theta$. Furthermore, the Faraday rotation calculation of Stil et al. (2009) is done when the outer radius $R_s \sim 200$ pc (see Figure 14 of Stil et al. 2009), which is much larger than the structure we are modeling in Section 3.6.2 of this paper. In what follows, we compare our observations with the results presented in Section 6 of Stil et al. (2009).

If LOS $\parallel \mathbf{B}_{\text{ext}}$, then the highest values of RM will be through the shell closest to the Galactic plane, but the mean RM across the region will be similar to the mean RM exterior to the bubble (see Figure 14 of Stil et al. 2009). Out of the Galactic plane, the RM is 20 – 30% of the mean RM exterior to the bubble. Effectively, the largest RMs will always be found in the Galactic plane, and different lines of sight through the bubble will have varying RM values.

In comparing the simulations of Stil et al. (2009) to our observational results, we find low RM measures for lines of sight through the cavity, though not always low (e.g., W4-I14 and -I17 vs -I15 and -I21). Lines of sight through the shell have
generally large RMs, which is inconsistent with a $\mathbf{B}_{\text{ext}}$ perpendicular to the LOS. The case of LOS $\parallel \mathbf{B}_{\text{ext}}$ is inconsistent as well because far from the bubble, the RM is low (e.g., W4-O26 vs -I24) even at similar latitudes, and the lines of sight at $b > 1^\circ$ are consistent with the background RM instead of being reduced by 70 – 80%. Unsurprisingly, our results indicate a case somewhere between these two predictions. As a reminder, we note that the largest values of the RM are for lines of sight exterior to the shell, which is not a prediction from Stil et al. (2009), most likely due to their simulations modeling the ionized bubble and not a PDR structure.

3.7.2 Magnetic Fields in the PDR

In Section 3.5.3, we examine evidence for a PDR outside the southern loop of IC 1805. Brogan et al. (1999), Troland et al. (2016), and Pellegrini et al. (2007) report large ($\sim 150 \mu G$) magnetic fields in PDRs associated with the Orion Veil and M17. In the analysis that follows, we attempt to understand the large RM values for lines of sight through the IC 1805 PDR.

If we consider the PDR and the H$^\text{II}$ region to be in pressure equilibrium and include magnetic pressure in the PDR, then

$$P_{\text{H}^\text{II}}^\text{th} = P_{\text{th}}^{\text{PDR}} + P_{\text{mag}}^{\text{PDR}},$$

(3.24)

where $P_{\text{H}^\text{II}}^\text{th}$ and $P_{\text{th}}^{\text{PDR}}$ are the thermal pressures in the H$^\text{II}$ region and PDR, respectively, and $P_{\text{mag}}^{\text{PDR}} = \frac{B^2}{8\pi}$ is the magnetic pressure in the PDR. In the H$^\text{II}$ region, $P_{\text{th}}^{\text{H}^\text{II}} = 2n_e^{\text{H}^\text{II}} k T_{\text{H}^\text{II}}$, where $n_e^{\text{H}^\text{II}}$ and $T_{\text{H}^\text{II}}$ are the electron density and temperature, $k$ is the Boltzmann constant, and the factor of 2 accounts for the contribution from both ions and electrons. For $P_{\text{th}}^{\text{PDR}} = N_{\text{PDR}} k T_{\text{PDR}}$, $N_{\text{PDR}}$ and $T_{\text{PDR}}$ are the neutral hydrogen density and the temperature in the PDR.

Near the interface of the PDR and the H$^\text{II}$ region, the electron density in the
PDR is governed by photoionization of carbon (Tielens & Hollenbach, 1985), so we estimate \( n_{e}^{\text{PDR}} \) by

\[
n_{e}^{\text{PDR}} = N_{\text{PDR}} X_{C},
\]

where \( X_{C} \) is the cosmic abundance of carbon given in Table 1.4 of Draine (2011) \( (X_{C} \sim 2.95 \times 10^{-4}) \). Solving for \( B \) in Equation (3.24) gives

\[
B = \sqrt{8\pi k(2n_{e}^{\text{HII}} T_{\text{HII}} - N_{\text{PDR}} T_{\text{PDR}})},
\]

and inserting it into Equation (3.2), we express the RM in the PDR as

\[
RM = 0.81 L X_{C} N_{\text{PDR}} \sqrt{8\pi k(2n_{e}^{\text{HII}} T_{\text{HII}} - N_{\text{PDR}} T_{\text{PDR}})}.
\]

We differentiate Equation (3.26) with respect to \( N_{\text{PDR}} \) to find the value of \( N_{\text{PDR}} \) that maximizes the RM, which is

\[
N_{\text{PDR}} = \frac{4 n_{e}^{\text{HII}} T_{\text{HII}}}{3 T_{\text{PDR}}},
\]

Inserting values of \( T_{\text{HII}} = 8000 \) K, \( n_{e}^{\text{HII}} = 10 \) cm\(^{-3}\) (Terebey et al., 2003), and \( T_{\text{PDR}} = 100 \) K (Tielens & Hollenbach, 1985), gives \( N_{\text{PDR}} \sim 1000 \) cm\(^{-3}\), \( B \sim 14 \) \( \mu \)G (Eq 3.25), and \( RM \sim 100 \) rad m\(^{-2}\). The electron density in the H\(\text{II} \) region is governing the maximum \( B \) expected in the PDR given pressure balance. For the IC 1805 H\(\text{II} \) region, \( n_{e} \) is low compared to M17 \( (n_{e} \sim 560 \) cm\(^{-3}\)) (Pellegrini et al., 2007), which suggests that a high density (pressure) H\(\text{II} \) region is needed to explain large magnetic fields in the PDR.

We now consider what values of the RM we might expect in the PDR given magnetic fields strengths of \( \sim 100 \) \( \mu \)G. Assuming thermal pressure balance \( (P_{\text{th}}^{\text{HII}} = P_{\text{th}}^{\text{PDR}}) \) only, then \( N_{\text{PDR}} = 200 N_{H}^{\text{HII}} \), where \( N_{H}^{\text{HII}} \) is the neutral hydrogen in the dust
shell of the H II region. Using $N_{H II}^{H II} = 4.8 \text{ cm}^{-3}$ from Table 2 of Terebey et al. (2003) for the dust shell, $n_{e}^{\text{PDR}} = 200 N_{H II}^{H II} X_C$. For a 100 $\mu$G field in the PDR, RM $\sim 700 \text{ rad m}^{-2}$, assuming $L \sim 30 \text{ pc}$. This is the same magnitude of the RM we find for W4-I2, -I6, and -I24 in the IC 1805 PDR.

Our analysis suggests that a simple pressure balance analysis predicts low RM values from the PDR that are inconsistent with our observations. Terebey et al. (2003) discuss an extended halo of ionized emission around the southern loop, which may indicate that there are more free electrons present outside the obvious ionized shell as seen in Figure 3.2. This may account for the larger values of the RM we observe. It is clear that knowing the electron density in this region and determining the presence of a PDR through observations, such as carbon radio recombination lines, is necessary to understand how the magnetic field is modified in this complex region.

3.8 A Comparison of IC 1805 and the Rosette Nebula as “Rotation Measure Anomalies”

We are interested in how the Galactic magnetic field is modified by OB associations via their stellar winds and ionizing photons, and we started our study with the Rosette Nebula, where we found large ($\sim 10^3 \text{ rad m}^{-2}$) RM measurements through the ionized shell of the H II region (Costa et al., 2016).

In the case of the Rosette, we find positive RM across the region, and for IC 1805, we find negative values. If the Galactic magnetic field follows the spiral arms in a clockwise direction, then we would expect the LOS magnetic field component to be pointed towards us (positive B) for $\ell > 180^\circ$, and pointed away from us (negative B) for $\ell < 180^\circ$. Except for one line of sight in each nebula, we find that the polarity of the Galactic magnetic field is preserved across each nebula and is consistent with
Table 3.7: Stellar Parameters

<table>
<thead>
<tr>
<th>Star Cluster</th>
<th>Star</th>
<th>Type</th>
<th>( \dot{M} ) ( \text{M}_\odot \text{yr}^{-1} )</th>
<th>( V_\infty ) km/s</th>
<th>( L_W = \frac{1}{2} \dot{M} V_\infty^2 ) a (erg s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 2244</td>
<td>HD 46223</td>
<td>O4V(f)</td>
<td>( 1.6 \times 10^{-6} ) c</td>
<td>3100</td>
<td>( 4.8 \times 10^{36} )</td>
</tr>
<tr>
<td></td>
<td>HD 46150</td>
<td>O5.5V</td>
<td>( 2.0 \times 10^{-6} ) c</td>
<td>3100</td>
<td>( 6.0 \times 10^{36} )</td>
</tr>
<tr>
<td></td>
<td>HD 46202</td>
<td>O9V(f)</td>
<td>( 6.3 \times 10^{-8} ) c</td>
<td>1150</td>
<td>( 2.6 \times 10^{34} )</td>
</tr>
<tr>
<td></td>
<td>HD 46149</td>
<td>O8.5V(f)</td>
<td>( 2.0 \times 10^{-7} ) c</td>
<td>1700</td>
<td>( 1.8 \times 10^{35} )</td>
</tr>
<tr>
<td>OCl 352</td>
<td>HD15570</td>
<td>O4II</td>
<td>( 1.0 \times 10^{-5} ) c</td>
<td>2200</td>
<td>( 1.5 \times 10^{37} )</td>
</tr>
<tr>
<td></td>
<td>HD15558</td>
<td>O4III</td>
<td>( 6.3 \times 10^{-6} ) c</td>
<td>3000</td>
<td>( 1.8 \times 10^{37} )</td>
</tr>
<tr>
<td></td>
<td>HD 15629</td>
<td>O5V</td>
<td>( 2.0 \times 10^{-6} ) c</td>
<td>2900</td>
<td>( 5.3 \times 10^{36} )</td>
</tr>
</tbody>
</table>

a Calculated mechanical wind luminosity based on cited mass loss rates and terminal velocities.

b Massey et al. (1995)
c Howarth & Prinja (1989)
d Chlebowski & Garmany (1991)
e Román-Zúñiga & Lada (2008)
f Garmany (1988)
g Bouret et al. (2012)
h Groenewegen et al. (1989)

In our study of the Rosette, we investigated whether the magnetic field is amplified in the shell of the nebula. We found that the model without amplification was weakly favored over the case when the magnetic field is amplified in the shell. When we applied the same model to IC 1805, however, it is difficult to conclude in favor of either model, but in both cases, the model with an enhanced magnetic field over predicts the RM. From inspection of Figures 3.11a and 3.11b, it seems that the model without amplification better accounts for the magnitude of the observed RMs, the observations do not conform to the model prediction of \( \text{RM}(\xi) \), and the model cannot account for the wide range in observed values of RM at a given \( \xi \).

In the present study, we find the highest RMs for lines of sight outside the obvious shell structure, though one line of sight (W4-I13) does appear to intersect the ionized shell and it has a large RM. These lines of sight may be probing the
magnetic field within the PDR. In the case of the Rosette, we found that the highest RM values were for lines of sight through the bright ionized shell. However with our work on IC 1805 and the PDR associated with it, we have briefly revisited our results in the Rosette, particularly Figure 1 from Costa et al. (2016). There are a few lines of sight with RM of order a few $10^2$ rad m$^{-2}$ that appear to be outside the ionized shell. These lines of sight were included in the background estimate for the Rosette, but if the Rosette also has a PDR, then these lines of sight may actually be probing that material.

Table 3.7 lists spectral type, mass loss rate, terminal wind velocity, and calculated wind luminosity from the literature for O stars with the largest wind luminosities in both NGC 2244, which is associated with the Rosette Nebula, and OCl 352. The sum of the wind luminosities of the three main stars in OCl 352 is $3.8 \times 10^{37}$ ergs s$^{-1}$, while the corresponding number for NGC 2244 (4 stars) is $1.1 \times 10^{37}$ ergs s$^{-1}$. In addition, OCl 352 appears to have more luminous stars. As such, OCl 352 might be expected to produce a more energetic stellar bubble than NGC 2244. Our Faraday rotation measurements show no indication of this, in that the largest RMs observed are similar for the two objects. In fact, higher RMs were measured for the Rosette than for any line of sight through IC 1805. A number of factors can control the impact a star cluster has on the ISM. If some relationship exists between the total wind luminosity of a star cluster and properties of an interstellar bubble that can be measure with Faraday rotation, it will apparently require a large sample of clusters/H$\text{II}$ regions to reveal it.

3.9 Future Research

In the future, we will continue our investigation of H$\text{II}$ regions and how they modify their surroundings and the Galactic magnetic field. An immediate investigation will be centered on observations of the H$\text{II}$ region IC 1396. This will provide a
third $\text{H} \text{II}$ region with different age, stellar content, and Galactic location. The observations are similar to those we have made of the Rosette Nebula and IC 1805. The observations of IC 1396 have been made with the VLA and are awaiting analysis. By adding more $\text{H} \text{II}$ regions to our study, we can begin to address questions such as

1. Since the electron density distributions in $\text{H} \text{II}$ regions are known from radio continuum observations, we can inquire what conditions would result in an RM $\gg 10^3 \text{ rad m}^{-2}$ through the shell of an $\text{H} \text{II}$ region.

2. Is it a general property of $\text{H} \text{II}$ regions and stellar bubbles that the polarity of the Galactic magnetic field is preserved within the region? The answer to this question has implications for the amplitude of MHD turbulence in the ISM on scales of the order of the $\text{H} \text{II}$ regions, $\sim 10 - 30 \text{ pc}$.

3. Do PDRs around other nebulae produce high RM$s$? What is the magnitude of the RM due to the PDR relative to that of the shell of an $\text{H} \text{II}$ region?

In addition to increasing the number of $\text{H} \text{II}$ regions, understanding Faraday complexity and how to interpret the associated RM measurements is important to studies of Galactic magnetic fields, particularly with large polarization surveys like the VLA Sky Survey (VLASS) and Polarisation Sky Survey of the Universe’s Magnetism (POSSUM) with the Australian Square Kilometre Array Pathfinder (ASKAP) in the near future.

### 3.10 Summary and Conclusions

1. We performed polarimetric observations using the VLA for 27 lines of sight through or near the shell of the $\text{H} \text{II}$ region and stellar bubble associated with the OB association OCl 352.

2. We obtain RM measurements for 20 sources using two methods. The first is
through the traditional least-squares fit to $\chi(\lambda^2)$, and the second is using RM Synthesis. Including components that are resolved, we report 27 RM values, and we find good agreement between the two methods. We find the same sign of the RM across the entire region with the exception of one source, W4-I18. We estimate a background RM due to the general ISM of $-145$ rad m$^{-2}$ in this part of the Galactic plane. We measure an excess of RM of $\sim +600$ to $-800$ rad m$^{-2}$ due to W4.

3. Only one line of sight has a positive RM value, W4-I18. It has a RM of $+501 \pm 33$ rad m$^{-2}$, and it is located 5.6 arcminutes from the center of OCl 352. This line of sight may be probing the material close to the massive stars. The orientation of the line of sight component of the magnetic field is directed towards the observer, whereas in the rest of the region, the magnetic field is directed away.

4. We find that some of the lines of sight with the largest RM values occur just outside the obvious ionized shell of IC 1805 and are potentially probing the magnetic field in the PDR. The lines of sight through the cavity of the bubble have lower RM values than those through the shell. In the W4 Superbubble, which is north of OCl 352, we find RM values consistent with the background RM.

5. We discuss two shell models to reproduce the magnitude of the RM and its dependency on distance from the center of the star cluster. We employed the first of these models in Savage et al. (2013) and Costa et al. (2016), and it is based on the Weaver et al. (1977) solution for a stellar bubble, which includes a shock expanding into an ambient medium. The second model uses magnetic flux conservation to describe how the magnetic field is modified in the shell and consists of a simplified analytic approximation to the results presented by Ferrière et al. (1991). Neither of these simplified models accounts for the
dependence of RM on spatial location. Both types of model have a dependence of RM on $\xi$, the separation of the line of sight from the center of the nebula, whereas the observations show a large range of RM for sources with similar values of $\xi$.

6. Because we have independent information on the electron density from radio continuum observations of both IC 1805 and the Rosette Nebula, our observations can limit the magnitude of the magnetic field in the H\textsc{ii} regions. Our RM measurements indicate that the field does not greatly exceed the value in the general ISM.

7. We compare our results from the current study of IC 1805 and our previous study of the Rosette Nebula. Notably, we find the same order of magnitude for the RM for the two nebulae, but the sign of the RM in each region is opposite. Since IC 1805 and the Rosette are at different Galactic longitudes and on either side of $b = 180^\circ$, the sign difference between the two nebula is consistent with a Galactic magnetic field that follows the spiral arm structure in a clock-wise direction, as suggested in models (Van Eck et al., 2011).
CHAPTER 4
SUPPORTING WORK

In this chapter, I present supporting work to the results and conclusions discussed in Chapters 2 and 3.

4.1 Analysis of Spectral Indices for Radio Sources

In Chapter 2, Section 2.3.1.2 and Chapter 3, Section 3.3.2, I introduce RM Synthesis, which utilizes 4 MHz wide images in Stokes \( Q \) and \( U \) as inputs. Because the “channel” mode of CLEAN averages adjacent channels, it is necessary to correct for the frequency dependence that can affect \( Q \) and \( U \). The method I employ in Costa & Spangler (2018) corrects \( Q \) and \( U \) at each frequency, \( \nu \), using the spectral index (\( \alpha \)). The flux densities, \( S_{\nu} \), are determined using the CASA task imfit, which fits a 2-D Gaussian to the intensity distribution of the source. The spectral index is then determined by a least-squares fit to \( \log S_{\nu} \) versus \( \log \nu \). I refer to these data as “corrected” in the discussion that follows.

In general, the sources have spectral indices consistent with an optically thin synchrotron spectrum. Figure 4.2a shows an example of such a spectrum for reference. Tables 4.1 and 4.2 list spectral indices for the lines of sight through IC 1805 and the Rosette, respectively. Figure 4.1 is a histogram of the spectral indices for the sources in Tables 4.1 and 4.2, which has a distribution similar to that of radio galaxies from Kellermann et al. (1969) (see their Figure 6) and more recently Franzen et al. (2014) (see their Figure 5).

W4-I12, W4-O4, R-N4, R-N7, and R-N10L are sources that raise cautionary flags. Figure 4.2 shows the spectra for these sources. W4-I12 and R-N10L have spectral indices that are consistent with being flat, R-N7 is largely scatter, and R-N4 has a negative but small spectral index. W4-O4 has a spectral index that is negative.
Table 4.1: Flux Densities for Lines of Sight through IC 1805

<table>
<thead>
<tr>
<th>Source Name</th>
<th>$S_{1.402,GHz}$ (mJy)</th>
<th>$S_{7.697,GHz}$ (mJy)</th>
<th>$S_{1.4,GHz}$</th>
<th>$\alpha_{7.697,GHz}$</th>
<th>$\chi^2_{\nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W4-I1</td>
<td>77.9 ± 1.4</td>
<td>70.4 ± 0.3</td>
<td>200</td>
<td>0.87</td>
<td>4.97</td>
</tr>
<tr>
<td>W4-I2</td>
<td>10.6 ± 1.1</td>
<td>6.5 ± 0.6</td>
<td>181</td>
<td>0.97</td>
<td>0.17</td>
</tr>
<tr>
<td>W4-I2b</td>
<td>5.5 ± 0.2</td>
<td>3.1 ± 0.2</td>
<td>181</td>
<td>1.01</td>
<td>0.78</td>
</tr>
<tr>
<td>W4-I3</td>
<td>82.6 ± 0.2</td>
<td>52.5 ± 0.1</td>
<td>169</td>
<td>0.86</td>
<td>40</td>
</tr>
<tr>
<td>W4-I4</td>
<td>44.7 ± 2.6</td>
<td>24.9 ± 1.5</td>
<td>161</td>
<td>1.06</td>
<td>0.31</td>
</tr>
<tr>
<td>W4-I4b</td>
<td>41.3 ± 2.7</td>
<td>23.3 ± 1.8</td>
<td>161</td>
<td>1.01</td>
<td>0.27</td>
</tr>
<tr>
<td>W4-I6</td>
<td>11.6 ± 0.7</td>
<td>6.2 ± 0.3</td>
<td>145</td>
<td>1.18</td>
<td>1.65</td>
</tr>
<tr>
<td>W4-I8</td>
<td>47.3 ± 0.6</td>
<td>28.3 ± 0.5</td>
<td>111</td>
<td>0.95</td>
<td>0.3</td>
</tr>
<tr>
<td>W4-I11</td>
<td>39.4 ± 0.4</td>
<td>22.3 ± 0.7</td>
<td>94</td>
<td>1.04</td>
<td>0.28</td>
</tr>
<tr>
<td>W4-I12</td>
<td>77.9 ± 1.4</td>
<td>79.1 ± 2.0</td>
<td>63</td>
<td>-0.03</td>
<td>0.45</td>
</tr>
<tr>
<td>W4-I13</td>
<td>16.5 ± 0.3</td>
<td>9.8 ± 0.3</td>
<td>54</td>
<td>0.96</td>
<td>0.82</td>
</tr>
<tr>
<td>W4-I13b</td>
<td>5.6 ± 0.2</td>
<td>3.5 ± 0.2</td>
<td>54</td>
<td>0.84</td>
<td>0.42</td>
</tr>
<tr>
<td>W4-I14</td>
<td>2.3 ± 0.1</td>
<td>1.3 ± 0.1</td>
<td>54</td>
<td>1.23</td>
<td>2.11</td>
</tr>
<tr>
<td>W4-I14b</td>
<td>2.9 ± 0.1</td>
<td>1.6 ± 0.1</td>
<td>54</td>
<td>1.06</td>
<td>2.56</td>
</tr>
<tr>
<td>W4-I15</td>
<td>27.2 ± 0.2</td>
<td>16.4 ± 0.3</td>
<td>52</td>
<td>0.91</td>
<td>0.21</td>
</tr>
<tr>
<td>W4-I17</td>
<td>9.5 ± 0.4</td>
<td>4.8 ± 0.2</td>
<td>43</td>
<td>1.24</td>
<td>0.28</td>
</tr>
<tr>
<td>W4-I17b</td>
<td>8.9 ± 0.3</td>
<td>5.4 ± 0.2</td>
<td>43</td>
<td>0.96</td>
<td>0.23</td>
</tr>
<tr>
<td>W4-I18</td>
<td>24.5 ± 0.1</td>
<td>15.9 ± 0.2</td>
<td>42</td>
<td>0.77</td>
<td>1.79</td>
</tr>
<tr>
<td>W4-I19</td>
<td>11.8 ± 0.6</td>
<td>10.4 ± 0.8</td>
<td>42</td>
<td>0.23</td>
<td>0.09</td>
</tr>
<tr>
<td>W4-I21</td>
<td>5.7 ± 0.4</td>
<td>2.9 ± 0.3</td>
<td>32</td>
<td>1.09</td>
<td>0.24</td>
</tr>
<tr>
<td>W4-I21b</td>
<td>5.2 ± 0.2</td>
<td>2.6 ± 0.2</td>
<td>32</td>
<td>1.10</td>
<td>0.60</td>
</tr>
<tr>
<td>W4-I21c</td>
<td>16.0 ± 0.9</td>
<td>8.9 ± 0.5</td>
<td>32</td>
<td>1.11</td>
<td>0.12</td>
</tr>
<tr>
<td>W4-I23</td>
<td>3.4 ± 0.4</td>
<td>1.1 ± 0.5</td>
<td>28</td>
<td>1.90</td>
<td>2.39</td>
</tr>
<tr>
<td>W4-I23b</td>
<td>0.8 ± 0.1</td>
<td>0.2 ± 0.1</td>
<td>28</td>
<td>1.40</td>
<td>15.52</td>
</tr>
<tr>
<td>W4-I24</td>
<td>20.4 ± 0.6</td>
<td>12.4 ± 0.9</td>
<td>72</td>
<td>0.94</td>
<td>0.06</td>
</tr>
<tr>
<td>W4-I24b</td>
<td>12.6 ± 0.5</td>
<td>7.5 ± 0.5</td>
<td>72</td>
<td>1.01</td>
<td>0.23</td>
</tr>
<tr>
<td>W4-O1</td>
<td>377.3 ± 0.8</td>
<td>229.8 ± 0.6</td>
<td>923</td>
<td>0.94</td>
<td>29.85</td>
</tr>
<tr>
<td>W4-O2</td>
<td>107.0 ± 0.3</td>
<td>52.3 ± 0.1</td>
<td>406</td>
<td>1.34</td>
<td>29.55</td>
</tr>
<tr>
<td>W4-O4</td>
<td>747.2 ± 4.1</td>
<td>792.1 ± 2.7</td>
<td>364</td>
<td>0.06</td>
<td>4.59</td>
</tr>
<tr>
<td>W4-O5</td>
<td>94.5 ± 0.2</td>
<td>54.6 ± 0.2</td>
<td>245</td>
<td>0.94</td>
<td>2.02</td>
</tr>
<tr>
<td>W4-O6</td>
<td>120.2 ± 0.2</td>
<td>81.7 ± 0.1</td>
<td>222</td>
<td>0.69</td>
<td>6.19</td>
</tr>
<tr>
<td>W4-O7</td>
<td>52.2 ± 0.1</td>
<td>24.0 ± 0.1</td>
<td>196</td>
<td>1.40</td>
<td>6.75</td>
</tr>
<tr>
<td>W4-O8</td>
<td>40.7 ± 0.1</td>
<td>1.3 ± 1.1</td>
<td>195</td>
<td>1.90</td>
<td>0.43</td>
</tr>
<tr>
<td>W4-O10</td>
<td>64.9 ± 0.6</td>
<td>36.7 ± 3.1</td>
<td>174</td>
<td>1.04</td>
<td>0.06</td>
</tr>
</tbody>
</table>

\(a\) Flux densities retrieved from the NVSS catalog (Condon et al., 1998) at a resolution of 45 arcseconds.

\(b\) Reduced Chi-Squared of fit for log $S_\nu$ vs log $\nu$. 

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Table 4.2: Flux Densities for Lines of Sight Through the Rosette Nebula

<table>
<thead>
<tr>
<th>Source Name</th>
<th>$S_{4.402\text{GHz}}$ (mJy)</th>
<th>$S_{7.697\text{GHz}}$ (mJy)</th>
<th>$S_{1.4\text{GHz}}$</th>
<th>$\alpha_{4.402}^{7.697}$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-I9</td>
<td>12.2 ± 0.1</td>
<td>8.1 ± 0.1</td>
<td>24</td>
<td>0.77</td>
<td>1.4</td>
</tr>
<tr>
<td>R-I11</td>
<td>11.2 ± 0.1</td>
<td>7.4 ± 0.1</td>
<td>21</td>
<td>0.75</td>
<td>3.2</td>
</tr>
<tr>
<td>R-I13</td>
<td>4.7 ± 0.2</td>
<td>2.9 ± 0.2</td>
<td>30</td>
<td>0.89</td>
<td>1.6</td>
</tr>
<tr>
<td>R-I13b</td>
<td>4.8 ± 0.3</td>
<td>3.3 ± 0.2</td>
<td>30</td>
<td>0.68</td>
<td>0.3</td>
</tr>
<tr>
<td>R-I17</td>
<td>18.8 ± 0.2</td>
<td>11.4 ± 0.2</td>
<td>63</td>
<td>0.92</td>
<td>2.7</td>
</tr>
<tr>
<td>R-N4</td>
<td>27.4 ± 0.6</td>
<td>25.3 ± 0.4</td>
<td>10</td>
<td>0.16</td>
<td>0.1</td>
</tr>
<tr>
<td>R-N5</td>
<td>4.7 ± 0.2</td>
<td>3.6 ± 0.1</td>
<td>15</td>
<td>0.44</td>
<td>4.0</td>
</tr>
<tr>
<td>R-N5b</td>
<td>2.6 ± 0.2</td>
<td>1.8 ± 0.2</td>
<td>15</td>
<td>0.34</td>
<td>1.9</td>
</tr>
<tr>
<td>R-N7</td>
<td>6.9 ± 0.1</td>
<td>7.2 ± 0.1</td>
<td>21</td>
<td>0.12</td>
<td>14.7</td>
</tr>
<tr>
<td>R-N9</td>
<td>9.8 ± 0.6</td>
<td>4.8 ± 0.3</td>
<td>18</td>
<td>1.3</td>
<td>0.5</td>
</tr>
<tr>
<td>R-N9b</td>
<td>10.2 ± 0.8</td>
<td>7.4 ± 0.9</td>
<td>18</td>
<td>0.58</td>
<td>1.2</td>
</tr>
<tr>
<td>R-N10T</td>
<td>6.8 ± 0.4</td>
<td>5.5 ± 0.3</td>
<td>7</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>R-N10R</td>
<td>4.1 ± 0.2</td>
<td>2.6 ± 0.1</td>
<td>7</td>
<td>0.61</td>
<td>3.5</td>
</tr>
<tr>
<td>R-N10B</td>
<td>1.7 ± 0.1</td>
<td>1.1 ± 0.1</td>
<td>7</td>
<td>0.65</td>
<td>2.1</td>
</tr>
<tr>
<td>R-N10L</td>
<td>3.2 ± 0.3</td>
<td>2.8 ± 0.2</td>
<td>7</td>
<td>0.01</td>
<td>6.5</td>
</tr>
</tbody>
</table>

$^a$ Flux densities retrieved from the NVSS catalog (Condon et al., 1998) at a resolution of 45 arcseconds.

$^b$ Reduced Chi-Squared of fit for log $S_\nu$ vs log $\nu$.

at low frequencies but a rising index at higher frequencies, albeit with large error bars. For these sources, I explored the effects of the spectral index on the RM Synthesis results. I determined the spectral index at low ($\nu < 5.3$ GHz) and high ($\nu > 6.7$ GHz) frequencies independently. For each solution, I applied the spectral index to $Q$ and $U$ maps and perform the RM Synthesis analysis. I also performed multiple iterations of RM Synthesis using the $\nu < 5.3$ GHz data and the $\nu > 6.7$ GHz data. I found that the RM Synthesis values are the same within the errors. In Section 4.2, I discuss an alternative method of correcting for the spectral dependence.
4.2 Faraday Complex Sources

4.2.1 Introduction

In Costa et al. (2016) and Costa & Spangler (2018), I utilize two methods to obtain RM values. The first is a traditional least-squares fit to

\[ \chi = \chi_0 + RM \lambda^2, \]

where RM = \( \phi \) in Equation (1.2). This method assumes that a purely rotating foreground medium of magnetized thermal plasma is responsible for the rotation of the plane of polarization of the radio wave.

The second method is RM Synthesis. In Chapter 2, Section 2.3.1.2 and Chapter 3, Section 3.3.2, I introduce and provide the details of the application of RM Synthesis. Here, I briefly review some details for the discussion presented below.

RM Synthesis takes as inputs the observable Stokes \( I, Q, \) and \( U \) spectra to extract the reconstructed Faraday dispersion function, \( \tilde{F}(\phi) \). \( \tilde{F}(\phi) \) is then deconvolved.

Figure 4.1: Histogram of spectral indices for sources in Tables 4.1 and 4.2.
Figure 4.2: Plot $S_\nu$ vs $\nu$ for (a) W4-I8, (b) W4-I12, (c) W4-O4, (d) R-N4, (e) R-N7, and (f) R-N10L. These sources have a significant ($>7\sigma_{QU}$) peak in $F(\phi)$.

via the rmclean algorithm (Heald, 2009) to recover the Faraday dispersion function, $F(\phi)$. rmclean recovers information lost due to incomplete frequency coverage and is analogous to the clean task utilized in radio data reduction. The outputs of rmclean are clean components, which indicate where $F(\phi)$ is nonzero.
For a purely rotating foreground medium, the behavior of $F(\phi)$ is a delta function at one value of $\phi$, $Q(\lambda^2)$ and $U(\lambda^2)$ are sinusoidal functions of the equal amplitude, $P(\lambda^2)$ is constant, and $\chi$ is a linear function of $\lambda^2$. This is called a *Faraday simple* spectrum or a Faraday thin source (Farnsworth et al., 2011; Anderson et al., 2015). If there are multiple peaks in $F(\phi)$, then it is affected by depolarization, and it is termed *Faraday complex*, or Faraday thick (Farnsworth et al., 2011; Anderson et al., 2015). It is possible to identify Faraday complex sources with RM Synthesis (Brentjens & de Bruyn, 2005), even in low signal/noise cases because RM Synthesis uses the entire bandwidth (2 GHz in this work) to determine $\phi$. If there is depolarization, then $\phi \neq \text{RM}$ and the relationship between $\chi$ and $\lambda^2$ is nonlinear.

![Figure 4.3](image_url)

**Figure 4.3:** Comparison of RM Synthesis results to least-squares fit method for the RM (Equation 4.1). The results from the Rosette Nebula (Costa et al., 2016) are shown in blue and in red for IC 1805 (Costa & Spangler, 2018). The solid line represents perfect agreement between the two measurements.
Generally, I find good agreement for the RM between the two methods. Figure 4.3 compares the results of the RM Synthesis analysis to the least-squares fit to $\chi(\lambda^2)$ method for all sources in this thesis. In Costa et al. (2016) and Costa & Spangler (2018), I do not find any sources with multiple peaks in $F(\phi)$, and I interpret the $\phi$ from the RM Synthesis analysis as a characteristic RM of the source.

With large polarization surveys like the VLA Sky Survey (VLASS) and the Polarisation Sky Survey of the Universe’s Magnetism (POSSUM) with the Australian Square Kilometre Array Pathfinder (ASKAP), there is a lot of interest in identifying and interpreting Faraday complex sources however. Anderson et al. (2015) report in their study of 563 radio sources that at least 12% are Faraday complex, but they warn that the number is likely higher given the limitation of the signal/noise in their sample. Identification of Faraday complex spectra is an important step to extracting magnetic field estimates from $\phi$ in a complex spectrum.

The conclusions of Farnsworth et al. (2011) and O’Sullivan et al. (2012) suggest that RM Synthesis and rmclean alone are insufficient to determine $\phi$ if the spectrum is Faraday complex. It is necessary to model the $Q(\lambda^2)$ and $U(\lambda^2)$ spectra to determine the nature of a multi-peaked $F(\phi)$. The outputs of RM Synthesis and rmclean, however, are useful in identifying Faraday complex spectra (Brown, 2011; Law et al., 2011; Anderson et al., 2015) and as inputs to the $QU$ analysis.

In the analysis that follows, I explore a Faraday complexity analysis for the lines of sight in this thesis. In most cases, I do not expect departures from the conclusions in Costa et al. (2016) and Costa & Spangler (2018), which discuss the modification of the Galactic magnetic field through the shells of HII regions. For some lines of sight, the percent polarization (m) decreases as a function of $\lambda^2$, which is an indicator of depolarization, and these sources raised cautionary flags during the RM Synthesis analysis. Additionally, there are some lines of sight that have multiple rmclean clean components in a single $F(\phi)$ peak. Because $QU$ modeling and RM Synthesis
are relatively new in practice, I explore a QU analysis in the sections below to gain insight into the technique and Faraday complex spectra.

4.2.2 Depolarization Models

For the QU analysis, I utilize a Python program written by C. Purcell\textsuperscript{1} that uses PyMultiNest (Buchner et al., 2014) to determine the best fit depolarization model. PyMultiNest is a Python wrapper for MultiNest (Feroz et al., 2009), which is a Nested sampling algorithm for Monte Carlo parameter estimation, and it utilizes Bayesian analysis tools to perform model selection. The code takes as inputs $Q(\lambda^2)$, $U(\lambda^2)$, and a function describing the depolarization.

Sokoloff et al. (1998) describe depolarization and provide mathematical representations of many of the models discussed below. I briefly describe the models I employ in the QU analysis here.

1. **Faraday Thin.** For a thin layer of a purely rotating medium of magnetized thermal plasma, the complex polarized emission $P$ is

$$P = p_0 \exp[2i(\chi_0 + R M \lambda^2)],$$  \hspace{1cm} (4.2)

where $p_0$ is the intrinsic linear polarization, $\chi_0$ is the intrinsic polarization position angle, and $\lambda$ is the wavelength (Eq 2; Sokoloff et al. 1998). Figure 4.4a is a cartoon illustrating a configuration that produces a Faraday thin spectrum.

2. **External Faraday Dispersion (EFD)/ Beam Depolarization.** EFD occurs when there are variations in the magnetic field (magnitude or direction) in a foreground purely rotating screen within the beam of the antenna or when there are multiple external purely rotating media with turbulent magnetic fields within

\textsuperscript{1}Dr. Cormac Purcell is a lecturer in the Department of Physics and Astronomy at Macquarie University. His RM Synthesis, RM Clean, and QU codes are available on github at https://github.com/crpurcell/RM-tools.
a beam. Both are described by

\[ P = p_0 \exp\left[-2\sigma_{RM}^2 \lambda^4\right] \exp\left[2i(\chi_0 + RM\lambda^2)\right], \tag{4.3} \]

where \( \sigma_{RM} \) is the dispersion about the mean RM (Eq 21 of Burn 1966; Farnsworth et al. 2011). Figure 4.4b is a cartoon schematic of beam depolarization.

3. **Differential Faraday Rotation (DFR, uniform slab).** Generally, DFR is due to an external magnetized medium that is co-spatially emitting and rotating radio waves. The radio waves on the far side of the slab will undergo a different amount of Faraday rotation than those on the near side. This is Eq 3 of Sokoloff et al. (1998)

\[ P = p_0 \frac{\sin(\phi\lambda^2)}{(\phi\lambda^2)} \exp\left[2i(\chi_0 + \frac{1}{2}\phi\lambda^2)\right]. \tag{4.4} \]

Figure 4.4c is an example of a uniform slab.

4. **Internal Faraday Dispersion.** When a turbulent magnetized medium is co-spatially emitting and rotating, then

\[ P = p_0 \exp\left[2i\chi_0\right] \left(1 - \exp\left[2i\phi\lambda^2 - 2\xi_{RM}^2 \lambda^4\right]\right), \tag{4.5} \]

where \( \xi_{RM} \) is the internal Faraday dispersion of the random field (Eq 18 of Burn 1966). Figure 4.4d is an example of internal Faraday dispersion.

Models may be added together to construct multicomponent models that depolarization effects along a line of sight (O’Sullivan et al., 2012). I restrict my \( QU \) analysis to the following models and combinations of models.

1. One Faraday thin component (Equation 4.2).
2. One foreground screen (EFD) (Equation 4.3).
3. Two independent Faraday thin components with a common foreground screen (EFD) in the form of

\[ P = (P_1 + P_2) \times \exp[-2\sigma_{RM}^2 \lambda^4], \]

where \( P_1 \) and \( P_2 \) are defined in Equation (4.2).

4. Two independent Faraday thin components with independent foreground screens (EFD) such that \( P = P_1 + P_2 \), where each contribution is equal to Equation (4.3).

5. Uniform slab (DFR) (Equation 4.4).

6. One Faraday thin component and a Uniform slab such that \( P = P_1 + P_2 \), where
$P_1$ is given by Equation (4.2) and $P_2$ given by Equation (4.4).

7. Internal Faraday dispersion (Equation 4.5).

8. Two Faraday thin components such that $P = P_1 + P_2$, where each is given by Equation (4.2).

While there are many combinations of models that could be tested, I restrict the number of combinations to these models because they test for sources of depolarization that would likely affect extragalactic sources with lines of sight through an H\textsc{ii} region. Additionally, almost all of the sources in this work are extragalactic in nature and are in the anti center of the Milky Way, which should reduce the number of rotating media (e.g., other H\textsc{ii} regions) along the line of sight.

Farnsworth et al. (2011), O’Sullivan et al. (2012), Anderson et al. (2015), and Pasetto et al. (2018) discuss and employ similar QU analyses for a number of lines of sight; however, the lines of sight in this work are among the few that specifically target the plasma shells of H\textsc{ii} regions. In the discussion that follows, I explore QU analysis techniques using fractional and corrected $Q$ and $U$ values (see discussion below) and the choice of channel width (e.g., 4 MHz) of the QU maps.

4.2.3 Results of the QU Analysis

I account for the frequency dependence that can independently affect Stokes $Q$ and $U$ in two ways. The first method utilizes the spectral index of each source, the center frequency $\nu_c$, and the measured value of $Q$ and $U$ at each frequency, $\nu$, to find to find $Q_o$ and $U_o$ using the relationship

$$Q = Q_o \left( \frac{\nu}{\nu_c} \right)^{-\alpha} \quad \text{and} \quad U = U_o \left( \frac{\nu}{\nu_c} \right)^{-\alpha}.$$
I refer to these data as “corrected”. The second method utilizes fractional values of $Q$ and $U$, where $q = Q/I$ and $u = U/I$, as the inputs to the $QU$ analysis, and I refer to these data as “fractional”. The fractional method is used in Farnsworth et al. (2011), O’Sullivan et al. (2012), Anderson et al. (2015), and Pasetto et al. (2018), and I use the corrected method in Costa et al. (2016) and Costa & Spangler (2018).

I also explore the effects of channel widths in the $QU$ analysis. RM Synthesis utilizes the entire bandwidth to calculate $F(\phi)$, and the input images in my analysis have a channel width of 4 MHz, which is related to the maximum detectable Faraday depth before bandwidth depolarization, $|\phi_{\text{max}}|$ (Eq 63, Brentjens & de Bruyn 2005). The modeling procedure may be adversely affected by low signal/noise due to the small channel widths. The 4 MHz data have a maximum signal/noise $\sim 5$ and, at times, $\sim 1$.

The other choice for the channel width is from the original CASA clean maps, which are generated with the “mfs” mode in the clean task over the entire 128 MHz bandwidth. The 128 MHz data typically have a signal/noise $> 25$ for the orthogonal Stokes parameters, and only a few sources have signal/noise $< 10$. In addition, the “mfs” mode takes into account the different $(u, v)$ tracks corresponding to different frequencies, and it determines and applies the spectral index correction to the source during the clean task. Thus, these data are corrected.

The outputs of the $QU$ analysis Python program are best fit parameters for the selected depolarization model, the reduced $\chi^2$, the Bayesian information criterion (BIC), and the Akaike information criterion (AIC). The Python program produces plots of $I$ vs $\nu$, $\chi$ vs $\lambda^2$, $P$, $Q$, and $U$ vs $\lambda^2$, and $Q$ vs $U$. Additionally, it produces confidence plots for the free parameters, which shows the posterior probability $p(M_i|D, I)$, where $M_i$ is the model, $D$ is the data, and $I$ is a statement representing the prior information and confidence intervals. For each free parameter in the model, there is a corresponding histogram of the marginalized likelihood plotted on the diagonal.
Figure C.1 is an example of a triangular grid of the confidence plots.

I use the BIC to distinguish the more likely model; however, I also visually inspect the model against the data and the corner plot to validate the selected model. The results of the $QU$ analysis can be placed into four categories, and Appendix C, Figures C.1 – C.8 are representative plots of these categories using the fractional data.

1. A poor fit (high BIC or AIC) to the data and/or no convergence in the posterior probability (Figures C.1 & C.2).

2. A low BIC or AIC but a large reduced $\chi^2$ value, a poor visual fit, and no convergence in the posterior probability. (Figures C.3 & C.4).

3. A good fit (low BIC or AIC) but multiple peaks in the posterior probability (Figures C.5 & C.6).

4. A good fit (low BIC or AIC) and convergence in the posterior probability on a single peak (Figures C.7 & C.8).

4.2.3.1 Results of 4 MHz Data

Selecting the best model is a difficult task for the 4 MHz data, as different models have similar values of the BIC or AIC. For almost all of the sources, the $Q$ and $U$ data are too compact, and the associated errors too large, to clearly identify a good fit to the model. Figure 4.5a shows an example of this. The corrected and fractional data are similarly affected, and for many parameters in the models, the posterior probability does not converge to a value.

For W4-II, -I3, -I8, and -I12, it is possible to identify at least three preferred models that have a good fit (low BIC) and the posterior probability converges on a single peak for the each parameter. Tables C.1 lists the preferred models, the
Figure 4.5: Plots of $Q$ vs $U$ of the corrected data for (a) W4-I24a and (b) W4-I1. The color of the symbols indicates the value of $\lambda^2$ associated with the $Q$ and $U$ data where blue is the short wavelengths and red is the long wavelengths.

parameters, and goodness of fit parameters for these sources.

For the fractional and corrected data, however, the favored models do not always agree. The corrected data typically have lower BIC scores than the fractional data and favor models with more components, such as a two Faraday thin components+EFD component and two Faraday thin components+two EFD components. The fractional data tend to favor a singular Faraday thin or EFD component. For two Faraday thin components+EFD component, the $\sigma_{RM}$ is of equal magnitude to one of the Faraday thin components, which may not be an accurate description of the media along such a line of sight. In general, the best fit $\phi$ associated with the thin component in any model is consistent with the RM derived from RM Synthesis, but it is not clear what causes the disagreement between the two methods.

4.2.3.2 Results of 128MHz Data

For the 128 MHz data, the $QU$ analysis returns at least two most likely models for almost all of the lines of sight. Tables C.3 and C.4 lists the results of the $QU$ analysis for W4-I1, -I2, -I3, -I4, -I6, -I8, -I13, -I15, -I17, -I18, -I19, -I24, -O6, and -O10. The remaining lines of sight, W4-I11, -I12, -I14, -I21, -O1, -O2, -O4, -O7, and -O8, are
similar to the 4 MHz data in that the error bars are large and the data are compact. A representative confidence plot and best fit parameters for a Faraday thin+EFD model are presented in Appendix C, Figures C.9 & C.10, respectively. Generally, the two Faraday thin components+two EFD components and the Uniform Slab+Faraday thin component models are consistently chosen by the Bayesian analysis as one of the preferred models for the 128 MHz data.

4.2.3.3 Summary of QU Analysis

The QU analysis is a relatively new technique in practice due to GHz bandwidths now available to modern radio instruments, and it could provide a method for characterizing magnetic fields along lines of sight with complex structures. In general, I find that the QU method is inconclusive for low signal/noise sources or when the channel width is small such that each input map has low signal/noise. While a narrow channel width is preferred to maximize $|\phi_{\text{max}}|$, it may not be a suitable choice for a QU analysis. The work presented here is an exploratory comparison between the treatment of the input data. Because the different treatments do not always agree on the same favored model, it is necessary to use simulated data to further explore the treatment of frequency dependent effects (fractional vs corrected) and channel dependent noise.

4.3 Photodissociation Regions

In Chapter 3, Section 3.5.3, I introduce evidence from the literature for a PDR around the southern loop of IC 1805. Figure 4.6 is similar to Figure 3.10 in Chapter 3; however, the two new panels show the WISE emission for the southwest and north portions of IC 1805. It is included here for completeness, though it does not change the conclusions of Chapter 3.
Figure 4.7 is a RGB WISE image with 4.95 GHz radio continuum contours of the Rosette Nebula. The 4.95 GHz map is the original radio map from Celnik (1985) that I utilize in Costa et al. (2016) to probe the electron density in the Rosette. The 12 $\mu$m emission (green) shows the PAH emission, which is a tracer of PDRs. As in the case of IC 1805, the hot dust (red) is enclosed by the radio contours and the PAH emission. Unlike IC 1805 (Figure 4.6), much of the PAH emission is inside the radio contours. This PAH emission may be part of the rear or near bubble wall.

The sources in the PAH emission and outside the ionized shell are R-I1, -I13, -O4, and -O15. These lines of sight have RM values $< +200$ rad m$^{-2}$, which is consistent with the background RM. The lines of sight with the largest RM values are inside the ionized shell, as I concluded in Costa et al. (2016). In general, these lines of sight intersect the red (hot dust) regions and not the PAH emission, however.

For the lines of sight in this work, it appears that the lines of sight through the ionized shell of the Rosette Nebula have larger RM values than those through the PDR around the Rosette. This is in contrast to my observations of lines of sight through the IC 1805 PDR. Perhaps this hints at more interesting properties of PDRs and ionized shells near OB stars, or it may reflect differences in the environment around the Rosette and IC 1805. More lines of sight through both PDRs and through the ionized shell of IC 1805 are necessary to further probe general properties between PDRs, H$\text{ii}$ regions, and the RM anomaly associated with these structures.
Figure 4.6: RGB image of archive WISE data at 4.6 µm (blue), 12 µm (green), and 22 µm (red) with radio continuum contours from CGPS in white. The lines of sight from the present study are shown with circles and are labeled similar to Figure 3.2.
Figure 4.7: RGB image of WISE data at 4.6 $\mu$m (blue), 12 $\mu$m (green), and 22 $\mu$m (red) with radio continuum contours at 4.5 GHz (see Celnik 1985) at 4 and 6 times the background signal in white. The lines of sight from the present study are shown with circles and are labeled similar to Figure 5.1b.
4.4 Assessing Faraday Rotation and Nebular Properties from the Literature

In this section, I explore Faraday rotation results from the literature for H II regions and wind-blown bubbles. Table 4.3 list properties for 4 Faraday rotation studies of H II regions from the literature that report RM values for extragalactic radio sources that have lines of sight that pass through or near to the H II region. These include the H II regions from the study of Harvey-Smith et al. (2011), the Gum Nebula, which Purcell et al. (2015) conclude is a wind-blown bubble and not a supernova remnant, NGC 6334A (Rodríguez et al., 2012), Sh2-232 (Heiles & Chu, 1980), Sh2-117, and Sh2-119 (Heiles et al., 1981).

The RM values in Harvey-Smith et al. (2011) and Purcell et al. (2015) are from the Taylor et al. (2009) catalog, which utilized NVSS data to determine the RM values for extragalactic radio sources. Because the Gum Nebula is \( \sim 36^\circ \) in size, there are many lines of sight. The H II regions in the Harvey-Smith et al. (2011) study have between 5 and 50 lines of sight through the H II region. There are only two lines of sight through Sh2-232, Sh2-117, and Sh2-119, and only one measurement of \( +5100 \pm 900 \text{ rad m}^{-2} \) for NGC 6334A (Rodríguez et al., 2012).

Table 4.4 lists properties of these objects. Column one gives the astronomical object, and columns two and three give the primary star responsible for the nebula and the spectral type (if known), respectively. Column four gives the calculated wind luminosities of the primary star, where \( L_w = \frac{1}{2} \dot{M} v_\infty^2 \). The mass loss rates, \( \dot{M} \), are from Howarth & Prinja (1989), and the reference for the terminal wind velocities, \( v_\infty \), are noted in the table entry for \( L_w \). The fifth column gives the diameter of the nebula. Column six lists the maximum value of \(|\text{RM}|\) for lines of sight through or near to the H II region, and column seven gives the range in RM measurements. Figure 4.8 plots...
the range of RM measures against wind luminosity for the nebulae in Table 4.4. The
entries without $L_w$ are not plotted in Figure 4.8a; however, all entries are included in
Figure 4.8b, which plots the measured RMs against diameter. Figure 4.8c plots RM
vs electron density.

Without the inclusion of the Gum Nebula, which is powered by O stars and a
Wolf-Rayet star, then there is seemingly a correlation between wind luminosity and
RM. However, it is not very strong, and a larger sample of H II regions is needed
to understand it. It is surprising, however, that despite differences between the H II regions, the range and magnitude of the RM doesn’t vary much. The exception
is NGC 6334A, which is very small compared to the other H II regions and also
very dense. If these H II regions represent H II regions of $\sim 10–50$ parsecs in size,
then perhaps the RMs for lines of sight through compact H II regions will be much
larger. If there is a correlation between the range, or magnitude, of the RM and wind
luminosity, diameter, or density, then a larger sample of H II regions is needed.

With this small selection of H II regions, it is difficult to make significant conclu-
sions about general properties between the H II region, star cluster, and RM signature.
One consideration may be that H II regions typically have RM signatures on order
$\sim 1000 \text{ rad m}^{-2}$ or less whereas other ionized, or partially ionized, regions, such as
supernova remnants, PDRs, etc., may have quite different RM signatures. It may also
be that the RM anomaly associated with H II regions is generally driven by density
increases and not due to an increase in the magnetic field, which is consistent with
my results and those of Harvey-Smith et al. (2011).
Table 4.3: H\textsubscript{II} Regions From Literature

<table>
<thead>
<tr>
<th>H\textsubscript{II} Region</th>
<th>(\alpha)</th>
<th>(\delta)</th>
<th>Distance (pc)</th>
<th>(n_e) (\text{cm}^{-3})</th>
<th>Primary Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sh2-27</td>
<td>16:35:56</td>
<td>-12:43:05</td>
<td>146 (^a)</td>
<td>10.6</td>
<td>Harvey-Smith et al. (2011)</td>
</tr>
<tr>
<td>Sh2-264</td>
<td>05:35:18</td>
<td>+09:56:00</td>
<td>386 (^a)</td>
<td>9.7</td>
<td>Harvey-Smith et al. (2011)</td>
</tr>
<tr>
<td>Sivan 3</td>
<td>04:56:12</td>
<td>+65:58:46</td>
<td>1010 (^b)</td>
<td>1.5</td>
<td>Harvey-Smith et al. (2011)</td>
</tr>
<tr>
<td>Sh2-171</td>
<td>00:01:09</td>
<td>+67:25:17</td>
<td>840 (^c)</td>
<td>14.1</td>
<td>Harvey-Smith et al. (2011)</td>
</tr>
<tr>
<td>Sh2-220</td>
<td>04:03:18</td>
<td>+36:25:18</td>
<td>398 (^b)</td>
<td>11.2</td>
<td>Harvey-Smith et al. (2011)</td>
</tr>
<tr>
<td>Gum</td>
<td>07:43</td>
<td>-42:06</td>
<td>450 (^d)</td>
<td>1.4</td>
<td>Purcell et al. (2015)</td>
</tr>
<tr>
<td>NGC 6334A</td>
<td>17:20:19</td>
<td>-35:54:55</td>
<td>17100 (^e)</td>
<td>350</td>
<td>Rodriguez et al. (2012)</td>
</tr>
<tr>
<td>Sh2-232</td>
<td>05:42:30</td>
<td>+36:11:00</td>
<td>2090 (^f)</td>
<td>5</td>
<td>Heiles &amp; Chu (1980)</td>
</tr>
<tr>
<td>Sh2-117</td>
<td>20:58:47</td>
<td>+44:19:48</td>
<td>580 (^g)</td>
<td>9</td>
<td>Heiles et al. (1981)</td>
</tr>
<tr>
<td>Sh2-119</td>
<td>21:18:30</td>
<td>+43:56:00</td>
<td>1130 (^h)</td>
<td>14</td>
<td>Heiles et al. (1981)</td>
</tr>
</tbody>
</table>

\(^a\) Sota et al. (2008)
\(^b\) Shull & van Steenberg (1985)
\(^c\) Blitz et al. (1982)
\(^d\) Woermann et al. (2001)
\(^e\) Quireza et al. (2006)
\(^f\) Foster & Brunt (2015)
\(^g\) Straizys et al. (1993)
\(^h\) Megier et al. (2009). Note: the given distance is to the exciting star, 68 Cyg.
Table 4.4: Comparison of Wind Luminosities and RMs for H\textsc{II} Regions

<table>
<thead>
<tr>
<th>H\textsc{II} Region</th>
<th>Star</th>
<th>Spectral Type</th>
<th>$L_w^a$ (erg s$^{-1}$)</th>
<th>Diameter (pc)</th>
<th>$\text{RM}_{\text{max}}^b$ (rad m$^{-2}$)</th>
<th>$\Delta\text{RM}^b$ (rad m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosette</td>
<td>HD 46150</td>
<td>O5.5V$^c$</td>
<td>$6.0\times10^{36}$ d</td>
<td>38</td>
<td>1400$^e$</td>
<td>1700</td>
</tr>
<tr>
<td>IC 1805</td>
<td>HD15558</td>
<td>O4III$^f$</td>
<td>$1.8\times10^{37}$ g</td>
<td>50</td>
<td>960$^o$</td>
<td>1450</td>
</tr>
<tr>
<td>Sh2-27</td>
<td>HD149757</td>
<td>O9V$^h$</td>
<td>$1.8\times10^{35}$ d</td>
<td>34$^h$</td>
<td>300$^h$</td>
<td>300</td>
</tr>
<tr>
<td>Sh2-264</td>
<td>HD36861</td>
<td>O8III$^h$</td>
<td>$5.7\times10^{35}$ i</td>
<td>60$^h$</td>
<td>300$^h$</td>
<td>300</td>
</tr>
<tr>
<td>Sivan 3</td>
<td>HD30614</td>
<td>O9.5Ia$^h$</td>
<td>$8.1\times10^{35}$ g</td>
<td>166$^h$</td>
<td>150$^h$</td>
<td>150</td>
</tr>
<tr>
<td>Sh2-171</td>
<td>BD+661673</td>
<td>O9.5V$^j$</td>
<td>–</td>
<td>58$^h$</td>
<td>200$^h$</td>
<td>130</td>
</tr>
<tr>
<td>Sh2-220</td>
<td>HD24912</td>
<td>O7.5III$^h$</td>
<td>$6.5\times10^{35}$ g</td>
<td>34$^h$</td>
<td>300$^h$</td>
<td>400</td>
</tr>
<tr>
<td>Gum</td>
<td>$\gamma$ Vel</td>
<td>WC8$^k$</td>
<td>$5.9\times10^{38}$ l</td>
<td>178$^i$</td>
<td>500$^k$</td>
<td>700</td>
</tr>
<tr>
<td>NGC 6334A</td>
<td>–</td>
<td>–</td>
<td>6$^n$</td>
<td>5100$^m$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Sh2-232</td>
<td>HD37737</td>
<td>O9.5II$^n$</td>
<td>–</td>
<td>24$^o$</td>
<td>760$^o$</td>
<td>100</td>
</tr>
<tr>
<td>Sh2-117</td>
<td>HD199579</td>
<td>O6V$^p$</td>
<td>$1.1\times10^{36}$ p</td>
<td>41$^j$</td>
<td>1100$^q$</td>
<td>1040</td>
</tr>
<tr>
<td>Sh2-119</td>
<td>68 Cyg</td>
<td>O7.5III$^n$</td>
<td>$1.6\times10^{36}$ p</td>
<td>26$^n$</td>
<td>2140$^n$</td>
<td>900</td>
</tr>
</tbody>
</table>

Note: Entries with dashes indicate no further information is available.

$^a$ Calculated mechanical wind luminosity based on mass loss rates from Howarth & Prinja (1989) and terminal velocities from citation given at table entry.

$^b$ Approximate range of RM values through the H\textsc{II} region.

$^c$ Román-Zúñiga & Lada (2008)

$^d$ Chlebowski & Garmany (1991)

$^e$ This work.

$^f$ Massey et al. (1995)

$^g$ Garmany (1988)

$^h$ Harvey-Smith et al. (2011)

$^i$ Groenewegen et al. (1989)

$^j$ Wendker (1968)

$^k$ Purcell et al. (2015)

$^l$ De Marco & Schmutz (1999)

$^m$ Rodriguez et al. (2012)

$^n$ Sota et al. (2011)

$^o$ Heiles & Chu (1980)

$^p$ Howarth & Prinja (1989)

$^q$ Heiles et al. (1981)
Figure 4.8: (a) Plot of RM values vs wind luminosity of the principle ionizing star in each H II region. The ranges of the RM values are given in Table 4.4. The H II regions that do not have an estimate of $L_w$ are not shown. (b) Plot of RM vs diameter and (c) RM vs electron density. The inset plot includes NGC 6334A.
I have studied two wind-blown bubbles and H\textsc{ii} regions: the Rosette Nebula and IC 1805. The OB associations associated with these H\textsc{ii} regions are NGC 2244 and OCl 352, respectively. They are located in the Perseus spiral arm. Figures 5.1a and 5.1b show the main results of my research, where the plotted symbols represent the location of the observed lines of sight, and the size of the symbol is directly proportional to $|\text{RM}|$. In either image, the largest symbol corresponds to $|\text{RM}| \sim 10^3$ rad m$^{-2}$, and the color of the symbols gives the sign of the RM, where red is negative and blue is positive.

The first major result from my Faraday rotation study is that the polarity of the RM is generally preserved across each region, suggesting that the H\textsc{ii} region does
not change the polarity of the parallel component of the magnetic field, \( B_{\text{LOS}} \). For the Rosette (\( \ell = 206^\circ \)), I find that \( B_{\text{LOS}} \) is positive (pointed towards the observer) (Savage et al., 2013; Costa et al., 2016), and for IC 1805 (\( \ell = 135^\circ \)), \( B_{\text{LOS}} \) is negative (pointed away). The change in sign of the RM is consistent with a Galactic magnetic field that follows the Perseus spiral arm in a clockwise fashion (Van Eck et al., 2011). My results show that the expected polarity of the large-scale Galactic magnetic field is preserved over large distances (> 3 kpc) in the Perseus arm, and the presence of the \( \text{H} \text{II} \) region does not induce a sign change in \( B_{\text{LOS}} \) within the shell or across the \( \text{H} \text{II} \) region.

The second result is that the observed RMs due to the nebulae are similar in magnitude. The simple shell model described in Savage et al. (2013) predicts RMs for lines of sight through IC 1805 of \( 2 - 5 \times 10^3 \) rad m\(^{-2}\), and from simulations of wind-blown bubbles, Stil et al. (2009) predict similar magnitudes of the RM. Despite OCl 352 having higher wind luminosities than NGC 2244, the range of RM values across the nebulae do not differ dramatically. However, the location of the lines of sight with the largest values of the RM in each nebula is different. For the Rosette, I find that the lines of sight through the ionized shell of the \( \text{H} \text{II} \) region have the largest RM values. In Section 4.3, I present WISE data for the Rosette, which show that the lines of sight with the largest values of the RM are not in a PDR around the Rosette; however for IC 1805, some of the largest RM values are associated with lines of sight outside the ionized shell, potentially in the PDR.

It seems that the Galactic magnetic field is insensitive to the presence of \( \text{H} \text{II} \) regions and stellar bubbles, but \( \text{H} \text{II} \) regions may not be insensitive to the magnetic field. The orientation and magnitude of \( B_{\text{LOS}} \) is not modified inside the shells of the \( \text{H} \text{II} \) regions, but theory and simulations suggest that the morphology of the \( \text{H} \text{II} \) region and stellar bubble is affected by magnetic fields (Ferrière et al., 1991; Stil et al., 2009). To further understand the relationship between \( \text{H} \text{II} \) regions and the Galactic
magnetic field, a larger sample of \text{H\textsc{ii}} regions is needed.
CHAPTER 6
FUTURE RESEARCH

In this chapter, I discuss my potential plans for future work, which includes a Faraday rotation study of IC 1396, observations of compact H\textsc{ii} regions in the southern hemisphere, and further analysis of Faraday complex sources.

6.1 Modification of the Galactic Magnetic Field near IC 1396

![Palomar Sky Survey mosaic of IC 1396 with observed lines of sights represented by filled circles. The blue circles represent lines of sight observed to probe the background RM, and the red are through the ionized shell. The three parallel lines in the lower right corner are image artifacts.]

Figure 6.1: Palomar Sky Survey mosaic of IC 1396 with observed lines of sights represented by filled circles. The blue circles represent lines of sight observed to probe the background RM, and the red are through the ionized shell. The three parallel lines in the lower right corner are image artifacts.

IC 1396 is an H\textsc{ii} region located in the Cepheus OB2 association at Galactic longitude $\ell = 99^\circ$, and it is relatively nearby ($\sim 870$ pc; Nakano et al. 2012) and located in the Perseus arm of the galaxy. IC 1396 is an ideal environment to study stellar feedback. Getman et al. (2007) conclude from a X-ray study with \textit{Chandra}
that IC 1396 is a region with triggered star formation that is the result of a radiation-driven implosion (RDI), which has a characteristic direction oriented age gradient such that older stars are closer to the ionizing star. In their study of pre-main sequence stars, Nakano et al. (2012) also see evidence of triggered star formation as a result of the expansion of the H\textsc{ii} region. To probe properties of the magnetic field in the shell of the H\textsc{ii} region and stellar bubble, I have completed observations of 32 lines of sight through or near to the shell of the H\textsc{ii} region, shown in Figure 6.1.

6.2 Compact H\textsc{ii} Regions at $\ell \sim 328^\circ$

An Evolutionary Map of the Universe (EMU) early science project is to study discrete Galactic radio sources at $\ell \sim 328^\circ$. In late 2017, I and Dr. S. Brown proposed a POSSUM early science project with ASKAP for polarimetric observations in this field to probe the plasma structure of compact H\textsc{ii} regions. The observations will most likely be complete in 2018. In addition to studying the RM structure through compact H\textsc{ii} regions, this project will test analysis techniques and the POSSUM data reduction pipeline.

6.3 Faraday complex Sources Using Simulated Data

I intend to continue my analysis of Faraday complex sources. I will simulate Faraday complex spectra to compare the preferred depolarization model when the choice of channel width (e.g., 128 MHz vs 4 MHz) in $Q$ and $U$ is varied and when the use of fractional vs corrected $Q$ and $U$ spectra is used (see Section 4.2). I already have the Python code that will create the simulated spectra, and I can continue to explore the $QU$ analysis technique.
APPENDIX A

CASA MAPS FOR LINES OF SIGHT THROUGH OR NEAR TO IC 1805

In this appendix, I present radio maps for the sources with lines of sight through IC 1805. In each map, the gray scale is the linear polarized intensity, the line segments are the polarization position angle, and the contours are Stokes $I$ at $-2$, $-1$, 2, 10, 20, 40, 60, and 80% of the peak intensity of the source.
Figure A.1: CASA map at 4913 MHz for (a) W4-I1, (b) -I2, (c) -I3, and (d) -I4.
Figure A.2: CASA maps at 4913 MHz for (a) W4-I6, (b) -I8, (c) -I11, and (d) -I12.
Figure A.3: CASA maps at 4913 MHz for (a) W4-I13, (b) -I14, (c) -I15, and (d) -I16.
Figure A.4: CASA Maps for (a) W4-I17, (b) -I18, (c) -I19, and (d) -I20.
Figure A.5: CASA maps at 4913 MHz for (a) W4-I21, (b) -I23, (c) -I24, and (d) -O1.
Figure A.6: CASA maps at 4913 MHz for (a) W4-O2, (b) -O4, (c) -O5, and (d) -O6.
Figure A.7: CASA maps at 4913 MHz for (a) W4-O7, (b) -O8, and (c) -O10.
APPENDIX B
FIGURES FOR RM SYNTHESIS ANALYSIS

The figures presented in this Appendix are plots of $F(\phi)$ for lines of sight through IC 1805 and the Rosette Nebula. In each image, the amplitude of $F(\phi)$ ($P$) is the solid curve, the real part of $F(\phi)$ ($Q$) is the dashed curve, and the imaginary part of $F(\phi)$ ($U$) is the dotted curve. The clean components from rmclean are also represented by the vertical lines. The amplitude is shown in red, the real part is blue, and the imaginary part is green. If no clean components are shown, it indicates that there was not a significant ($> 7\sigma_{QU}$) peak in $F(\phi)$. The range in $\phi$ has been reduced to show the maximum amount of structure. No other peaks in $F(\phi)$ are seen out to $\pm 10,000$ rad m$^{-2}$. The RMSF is presented in Figures 3.7b and 2.6 for IC 1805 and the Rosette lines of sight, respectively.
Figure B.1: Plot of $F(\phi)$ for (a) W4-I1, (b) -I2, (c) -I2b, (d) -I3, (e) -I4, (f) -I4b, (g) -I6, and (h) -I8.
Figure B.2: Plots similar to Figure B.1 for (a) W4-I11, (b) -I12, (c) -I13, (d) -I13b, (e) -I14, (f) -I14b, (g) -I15, and (h) -I17.
Figure B.3: Plots similar to Figure B.1 for (a) W4-I17b, (b) -I18, (c) -I19, (d) -I21, (e) -I21b, (f) -I21c, (g) -I23, and (h) -I23b.
Figure B.4: Plots similar to Figure B.1 for (a) W4-I23c, (b) -I24, (c) -O1, (d) -O2, (e) -O4, (f) -O5, (g) -O6, and (h) -O7.
Figure B.5: Plots similar to Figure B.1 for (a) W4-O8, (b) W4-O10, (c) R-I9, (d) R-I13, (e) R-I13b, (f) R-I17, (g) R-I17b, and (h) R-N4.
Figure B.6: Plots similar to Figure B.1 for (a) R-N5, (b) -N5b, (c) -N7, (d) -N9, (e) -N9b, (f) -N10T, (g) -N10R, and (h) -N10B.
Figure B.7: Plots similar to Figure B.1 for the source R-N10L.
APPENDIX C

FIGURES AND TABLES FOR QU ANALYSIS

The tables and figures below are discussed in Chapter 4, Section 4.2.3.
### Table C.1: Results of QU Analysis for 4 MHz “Fractional” Data

<table>
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<th>$P_0$ (mJy)</th>
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<th>$\phi_0$ (rad m$^2$)</th>
<th>$P_1$ (mJy)</th>
<th>$\chi_1$ ($^\circ$)</th>
<th>$\phi_1$ (rad m$^2$)</th>
<th>$\sigma_{RM}$ (rad m$^2$)</th>
<th>$\chi^2_{nu}$</th>
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Columns one and two give the source and the RM derived from RM Synthesis. Column three gives the components of the model. When the order of multiple components is important, the first component corresponds to subscript ‘0’. Columns four to ten give the parameters for the depolarization model. Column eleven lists the reduced $\chi^2$, column twelve gives the BIC, and column thirteen gives the AIC.
Table C.2: Results of QU Analysis for 4 MHz “Corrected” Data

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<th>φ₀ (rad m²)</th>
<th>p₁ (mJy)</th>
<th>χ₁ (°)</th>
<th>φ₁ (rad m²)</th>
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<th>χ²</th>
<th>BIC (mJy)</th>
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Columns one and two give the source and the RM derived from RM Synthesis. Column three gives the components of the model. When the order of multiple components is important, the first component corresponds to subscript ‘0’. Columns four to eleven give the parameters for the depolarization model. Column twelve lists the reduced χ², column thirteen gives the BIC, and column fourteen gives the AIC.
Table C.3: Results of $QU$ Analysis for 128 MHz Data

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<th>Source</th>
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<th>$\phi_0$ (rad m$^2$)</th>
<th>$p_1$ (mJy)</th>
<th>$\chi_1$ (°)</th>
<th>$\phi_1$ (rad m$^2$)</th>
<th>$\sigma_{RM_0}$ (rad m$^2$)</th>
<th>$\sigma_{RM_1}$ (rad m$^2$)</th>
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**Columns**

Columns one and two give the source and the RM derived from RM Synthesis. Column three gives the components of the model. When the order of multiple components is important, the first component corresponds to subscript ‘0’. Columns four to eleven give the parameters for the model. Column twelve lists the reduced $\chi^2$, column thirteen gives the BIC, and column fourteen gives the AIC.
Table C.4: Results of $QU$ Analysis for 128 MHz Data Cont.

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Columns one and two give the source and the RM derived from RM Synthesis. Column three gives the components of the model. When the order of multiple components is important, the first component corresponds to subscript ‘0’. Columns four to eleven give the parameters for the model. Column twelve lists the reduced $\chi^2$, column thirteen gives the BIC, and column fourteen gives the AIC.
Figure C.1: Triangular confidence plot for W4-I11 for a 2 Faraday Thin model. From left to right, the gray scale is the probability density of model parameters for $p_0$ (mJy), $p_1$ (mJy), $\chi_0$ ($^\circ$), $\chi_1$ ($^\circ$), and $\phi_0$ (rad m$^{-2}$). On the ordinate from bottom to top is $\phi_1$ (rad m$^{-2}$), $\phi_0$ (rad m$^{-2}$), $\chi_1$ ($^\circ$), $\chi_0$ ($^\circ$), and $p_1$ (mJy). The contours are confidence intervals of 1$\sigma$, 2$\sigma$, and 3$\sigma$. On the diagonal, the histogram is the marginalized likelihood for $p_0$ (column 1, top), $p_1$ (column 2, top), $\chi_0$ (column 3, top), $\chi_1$ (column 4, top), $\phi_0$ (column 5, top), and $\phi_1$ (column 6).
Figure C.2: $Q$ and $U$ data for W4-I11 with best fit values for a 2 Faraday Thin model. In the top left panel is the flux density (mJy) as a function of $\nu$ (GHz). The top right panel is $\chi$ (°) vs $\lambda^2$ (m²). The bottom left is $q$ (blue), $u$ (red), and $p$ (black) vs $\lambda^2$, and the bottom right is $q$ vs $u$. 
Figure C.3: Triangular confidence plot for W4-I8 for a 2 EFD+2 Faraday Thin model. From left to right, the gray scale is the probability density of model parameters for $p_0$ (mJy), $p_1$ (mJy), $\chi_0$ ($^\circ$), $\chi_1$ ($^\circ$), $\phi_0$ (rad m$^{-2}$), and $\sigma_{\text{RM}0}$ (rad m$^{-2}$). On the ordinate from bottom to top is $\sigma_{\text{RM}1}$ (rad m$^{-2}$), $\sigma_{\text{RM}0}$ (rad m$^{-2}$), $\phi_1$ (rad m$^{-2}$), $\phi_0$ (rad m$^{-2}$), $\chi_1$ ($^\circ$), $\chi_0$ ($^\circ$), and $p_1$ (mJy). The contours are confidence intervals of 1$\sigma$, 2$\sigma$, and 3$\sigma$. On the diagonal, the histogram is the marginalized likelihood for $p_0$ (column 1, top), $p_1$ (column 2, top), $\chi_0$ (column 3, top), $\chi_1$ (column 4, top), $\phi_0$ (column 5, top), $\phi_1$ (column 6), $\sigma_{\text{RM}0}$ (column 7, top), and $\sigma_{\text{RM}1}$ (column 8).
Figure C.4: $Q$ and $U$ data for W4-I8 with best fit values for a 2 EFD+2 Faraday Thin model. In the top left panel is the flux density (mJy) as a function of $\nu$ (GHz). The top right panel is $\chi$ (°) vs $\lambda^2$ (m$^2$). The bottom left is $q$ (blue), $u$ (red), and $p$ (black) vs $\lambda^2$, and the bottom right is $q$ vs $u$. 
Figure C.5: Triangular confidence plot for W4-I1 for a Uniform Slab+Faraday Thin model. From left to right, the gray scale is the probability density of model parameters for $p_0$ (mJy), $p_1$ (mJy), $\chi_0$ (°), $\chi_1$ (°), and $\phi_0$ (rad m$^{-2}$). On the ordinate from bottom to top is $\phi_1$ (rad m$^{-2}$), $\phi_0$ (rad m$^{-2}$), $\chi_1$ (°), $\chi_0$ (°), and $p_1$ (mJy). The contours are confidence intervals of 1σ, 2σ, and 3σ. On the diagonal, the histogram is the marginalized likelihood for $p_0$ (column 1, top), $p_1$ (column 2, top), $\chi_0$ (column 3, top), $\chi_1$ (column 4, top), $\phi_0$ (column 5, top), and $\phi_1$ (column 6).
Figure C.6: $Q$ and $U$ data for W4-I1 with best fit values for a Uniform Slab+Faraday Thin model. In the top left panel is the flux density (mJy) as a function of $\nu$ (GHz). The top right panel is $\chi$ (°) vs $\lambda^2$ (m$^2$). The bottom left is $q$ (blue), $u$ (red), and $p$ (black) vs $\lambda^2$, and the bottom right is $q$ vs $u$. 
Figure C.7: Triangular confidence plot for W4-I1 for an EFD model. From left to right, the gray scale is the probability density of model parameters for $p$ (mJy), $\chi$ ($^\circ$), and $RM$ (rad m$^{-2}$). On the ordinate from bottom to top is $\sigma_{RM}$ (rad m$^{-2}$), $RM$ (rad m$^{-2}$), and $\chi$ ($^\circ$). The contours are confidence intervals of 1$\sigma$, 2$\sigma$, and 3$\sigma$. On the diagonal, the histogram is the marginalized likelihood for $p_1$ (column 1, top), $\chi$ (column 2, top), $RM$ (column 3, top), and $\sigma_{RM}$ (column 4).
Figure C.8: $Q$ and $U$ data for W4-I1 with best fit values for an EFD model. In the top left panel is the flux density (mJy) as a function of $\nu$ (GHz). The top right panel is $\chi$ (°) vs $\lambda^2$ (m$^2$). The bottom left is $q$ (blue), $u$ (red), and $p$ (black) vs $\lambda^2$, and the bottom right is $q$ vs $u$. 
Figure C.9: Triangular confidence plot for W4-I1 for a 2 Faraday Thin+EFD model for the 128 MHz data. From left to right, the gray scale is the probability density of model parameters for $p_0$ (mJy), $p_1$ (mJy), $\chi_0$ (°), $\chi_1$ (°), $\phi_0$ (rad m$^{-2}$), and $\phi_1$ (rad m$^{-2}$). On the ordinate from bottom to top is $\sigma_{\rm RM}$ (rad m$^{-2}$), $\phi_1$ (rad m$^{-2}$), $\phi_0$ (rad m$^{-2}$), $\chi_1$ (°), $\chi_0$ (°), and $p_1$ (mJy). The contours are confidence intervals of 1σ, 2σ, and 3σ. On the diagonal, the histogram is the marginalized likelihood for $p_0$ (column 1, top), $p_1$ (column 2, top), $\chi_0$ (column 3, top), $\chi_1$ (column 4, top), $\phi_0$ (column 5, top), $\phi_1$ (column 6, top), and $\sigma_{\rm RM}$ (column 7).
Figure C.10: $Q$ and $U$ data for W4-I1 with best fit values for a 2 Faraday Thin+EFD model for the 128 MHz data. All data have errors; however, the errors are small compared to the symbol size. The top panel is $\chi$ ($^\circ$) vs $\lambda^2$. The bottom left is $Q$ (blue), $U$ (red), and $P$ (Jy) (black) vs $\lambda^2$, and the bottom right is $Q$ vs $U$. 

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