Spring 2018

Essays on the elasticity of intertemporal substitution

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ESSAYS ON THE ELASTICITY OF INTERTEMPORAL SUBSTITUTION

by

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A thesis submitted in partial fulfillment of the
requirements for the Doctor of Philosophy
degree in Economics
in the Graduate College of
The University of Iowa

May 2018

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Iowa City, Iowa

CERTIFICATE OF APPROVAL

PH.D. THESIS

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ACKNOWLEDGEMENTS

I would first and foremost like to thank my advisor, Antonio Galvao, for his guidance, instruction, and unwavering support throughout graduate school. His encouragement and advice made the completion of my dissertation possible and helped me grow as an economist and as an individual. In addition, I would also like to thank all the members of my committee: Anne Villamil, Suyong Song, Alex Poirier, and Luciano de Castro.

I must also thank Renea Jay, who supports all economics graduate students at the University of Iowa. Her devotion to the program is what enables all of us students to succeed.

I am incredibly grateful to all of my family and friends for their support, encouragement, and inspiration over the last four years, without which I would have been lost. I am particularly indebted to my parents for instilling in me a strong work ethic and love of learning. Ever since I was a child, they made education a priority in my life. They also showed me exactly how work ethic translates to a successful career. Lastly, I am thankful for my amazing colleagues and friends, David Enocksson and Chelsea Crain, for their unfailing support of me as not only an economist but also as a person.

ABSTRACT

This dissertation estimates the elasticity of intertemporal substitution (EIS) of consumption using the Nielsen Consumer Panel. The Nielsen Consumer Panel is built from transactional data that follows households in the United States and their grocery purchases from 2004 to 2014. Because of the transactional nature of the dataset, there is a low source of measurement error in consumption, and aggregation bias can be minimized. Due to changes in the economy during this timeframe, the data is examined for structural breaks. The data suggests evidence for two structural changes in the U.S. economy leading to three regimes. The first regime, 2004 to 2006, was a period of economic expansion, while the second regime, 2007 and 2008, was a period of recession. Lastly, during the third regime, 2009 to 2014, the economy exhibited quantitative easing.

Chapter 1 introduces the EIS and provides an overview of the literature. In Chapter 2, the EIS is estimated for each regime using expected utility with linearized Epstein-Zin preferences and by the use of fixed effects and instrumental variables. In order to estimate the EIS, consumption is aggregated weekly, and consumption growth is measured over a four-week time period in order to match four-week Treasury bills. This study adds to the literature by examining individual EIS during different periods of economic activity. With a more complete dataset that has less measurement error and aggregation bias than the existing literature, this study gives evidence of a small and negative EIS during a period of expansion, a small and positive EIS during a period of recession, and a large and positive EIS during quantitative easing.

Lastly, Chapter 3 extends Chapter 2 by assuming quantile utility preferences instead of the expected utility framework. The quantile EIS is estimated by the use of a smooth instrumental variables method of moments estimator. Estimates give evidence of heterogeneity of the EIS in the periods of expansion and quantitative easing. These quantile results can be used to inform the theory behind the EIS and quantile models of rational behavior.

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This dissertation estimates the elasticity of intertemporal substitution (EIS) of consumption using the Nielsen Consumer Panel. The Nielsen Consumer Panel is built from transactional data that follows households in the United States and their grocery purchases from 2004 to 2014. Because of the transactional nature of the dataset, there is a low source of measurement error in consumption, and aggregation bias can be minimized. Due to changes in the economy during this timeframe, the data is examined for structural breaks. The data suggests evidence for two structural changes in the U.S. economy leading to three regimes. The first regime, 2004 to 2006, was a period of economic expansion, while the second regime, 2007 and 2008, was a period of recession. Lastly, during the third regime, 2009 to 2014, the economy exhibited quantitative easing.

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CHAPTER 1
THE ELASTICITY OF INTERTEMPORAL SUBSTITUTION

1.1 Introduction

The purpose of this paper is to estimate the elasticity of intertemporal substitution (EIS) of consumption using disaggregated data from the Nielsen Consumer Panel. The EIS measures the response of consumption growth to the real interest rate. It is important to study the EIS, because it relates decision making between two time periods. It represents the willingness of a consumer to substitute future consumption for present consumption. There is a large literature studying the EIS, but researchers have been unable to agree on an estimate of the EIS. Explanations for these differing estimates in the literature include measurement error, aggregation bias, and weak instruments. I estimate the EIS using household consumption data from the Nielsen Consumer Panel and address each of these issues. This paper provides new information to the EIS literature as it uses a more accurate and complete dataset than previous studies. Because the dataset spans 2004 to 2014, which is before and after the Great Recession in the United States, the EIS is able to be estimated for different periods of economic activity.

The structure of the Nielsen Consumer Panel dataset is what makes this study unique. It has three main advantages. First, this data is not survey data, so there is no estimate of consumption. This minimizes measurement error in consumption, which is a common issue in the EIS. Second, the data is transactional at the consumption level, so it allows for aggregation over a small time period. This helps to lessen aggregation bias, which often affects EIS estimates. Third, the data is a panel, so it follows individuals over time. This allows for the use of fixed effects estimation to remove individual and time effects from the data. Because of these advantages, this dataset provides more accurate estimates than other datasets used in studies of the EIS.

The Nielsen Consumer Panel has not yet been used to study the EIS. Previous literature analyzes annual, quarterly, and monthly survey data, which have high sources of measurement error. However, the Nielsen Consumer Panel is transactional, so there is a low source of measurement error. The data is transactional, as it includes all items purchased from every retail shopping trip. Rather than consumers providing long-range estimates in surveys, they scan each item purchased after each trip. These purchases are organized into shopping trips that can be aggregated accordingly. Since consumers are providing transactional line items rather than estimates, measurement error is diminished. Hence, the data does not suffer from the large measurement error problem that is found in most studies of the EIS, which allows for a more accurate estimate of the EIS.

Another advantage of using a disaggregated micro dataset, such as the Nielsen Consumer Panel, is that it lessens aggregation bias. Micro data is less influenced by serial correlation than aggregated data. Goodfriend (1992) and Pischke (1995) show how aggregate consumption can be smoothed. This consumption smoothing leads to an aggregation bias. By using the Nielsen Consumer Panel, consumption can be aggregated over small time periods in order to lessen the effects of aggregation bias. Micro data also provides a structural look at an individual EIS rather than an aggregate EIS.

The EIS is estimated through an Euler equation. Euler equations result from dynamic behavior as individuals make optimal decisions by equating marginal costs with marginal benefits. Individuals maximize lifetime utility with respect to current and future consumption, and they make consumption decisions according to expected real interest rates. Euler equations provide moment conditions so that parameters can be estimated. This paper derives an Euler equation from time-separable Epstein-Zin preferences, and the interest of the paper lies in the EIS parameter.

Euler equations have been studied extensively in the economics literature, but this study uses a dataset that provides further insight than previous datasets. Original studies use aggregated agents, but this study uses household micro data that follows consumers over a time period, which allows for the use of fixed effects. Other studies use micro data of individuals over time, but they do not aggregate over transactional data as this study does. Because the data is aggregated from an item by item transactional dataset, it provides more detail and more accuracy than other datasets, which allows for better estimation of the EIS. This transactional nature minimizes any source of measurement error in consumption, and it allows for the ability to lessen aggregation bias.

On the other side of the equation is the interest rate. The expected real interest rate is what drives a consumer to make their intertemporal consumption decision. This study uses the four-week Treasury bill, which represents the interest rate of saving. To convert to a real interest rate, it is deflated using the monthly Consumer Price Index (CPI) on all items. The CPI is broken into four regions: northeast, midwest, south, and west. This allows the interest rate to be deflated regionally, so that the final dataset has regionally specific real interest rates.

Timing between consumption and interest rates plays an important role in this study. As consumption is transactional, it must be aggregated over a time period. If consumption is aggregated over a smaller time period, it lessens aggregation bias and provides a closer look at consumption habits. As stated, the interest rate that is used has a four-week maturity. Hence, intertemporal decisions are based on a four-week timespan. When choosing the period over which consumption is aggregated, two considerations are balanced: a short time period of consumption versus matching the timing between consumption and interest rates. This paper chooses a small aggregation time period and matches the timing of consumption growth with the maturity of the interest rates by aggregating weekly and considering consumption growth over four weeks.

It is also important to consider the timespan of the data. The dataset covers the United States from 2004 to 2014. During this time, the U.S. economy had three distinct intervals of economic activity. From 2004 to 2006, the economy was expanding, and real interest rates were increasing. In 2007 and 2008, the Great Recession and financial crisis occurred. This led to declining real interest rates. Lastly, 2009 to 2014 was a period of quantitative easing, where real interest rates were steady around zero. Because each of these three time periods are vastly different and likely lead to different consumer responses, I consider structural breaks in the data. Using Bai and Perron (2003), I find that there are two structural breaks in the real interest rate over time, which leads to three regimes. These regimes represent expansion, recession, and quantitative easing. I present separate estimates of the EIS during each of these regimes.

Before estimating the model, the data was tested for weak instruments. In order to test for weak instruments, this study follows Yogo (2004) and Gomes and Paz (2013). The results suggest that the data does not suffer from the weak instrument problem. This is similar to Yogo (2004) and Dacy and Hasanov (2011) when they use Treasury bills as the rate of return. When Dacy and Hasanov (2011) use a different rate of return, a synthetic mutual fund, they did find evidence for weak instruments. Yogo (2004), Dacy and Hasanov (2011), and my results represent a small sample size, but estimating the EIS with T-Bills as the rate of return appears to be free of the weak instrument problem. Because the data does not suffer from weak instruments, the estimates of the EIS in this paper are not biased.

The EIS is estimated with expected utility in Chapter 2. It is estimated for three regimes. In Period 1 (Feb 1, 2004 - Aug 31, 2006), the estimated EIS is -0.170. During Period 2 (Sep 1, 2006 - Dec 31, 2008), the estimated EIS is 0.129. Lastly, in Period 3 (Jan 1, 2009 - Dec 31, 2014), the estimated EIS is 2.810. All of these estimates are significant at the 1% level. The estimates for Periods 1 and 2 are considered small, but the Period 3 estimate can be considered large. During expansion in Period 1, the negative EIS indicates that the income effect dominates. As real interest rates rise, consumers can receive the same return next period by saving less. Hence, they consume more, and consumption growth decreases. During the recession in Period 2, the positive EIS indicates that the substitution effect dominates. As real interest rates rise, consuming now is relatively more expensive since the rate of return is higher. Hence, consumers save more, and consumption growth increases. The quantitative easing result in Period 3 states that households respond excessively to changes in real interest rates. This estimate is likely affected by a lack of variation in the real interest rate. During this period of quantitative easing, the interest rate is steady near 0. This leads to a much higher standard error in the EIS estimate of Period 3.

This study is the first to examine the EIS at a weekly interval. Many studies have explored the EIS or constant relative risk aversion from a monthly perspective, such as Hansen and Singleton (1983), Hall (1988), and Epstein and Zin (1991). Hall (1988) and Epstein and Zin (1991) find the EIS to be small or close to zero. However, these studies were completed on macro datasets.

Studies completed on micro datasets similar to mine are Attanasio and Weber (1993), Blundell, Browning and Meghir (1994), Attanasio and Browning (1995), Beaudry and van Wincoop (1996), Dynan (2000), Lee (2001), Vissing-Jørgensen (2002), Vissing-Jørgensen and Attanasio (2003), Parker and Preston (2005), Guvenen (2006), and Gruber (2013). However, none of these studies can be directly compared to mine. Attanasio and Weber (1993), Blundell, Browning and Meghir (1994), and Attanasio and Browning (1995) use U.K. data, and they simulate a panel using mean cohort data. Beaudry and van Wincoop (1996) use data from the U.S., but they simulate a panel using state cohorts. Dynan (2000), Lee (2001), and Parker and Preston (2005) are interested in Euler equation parameters other than the EIS.

The Panel Survey of Income Dynamics (PSID) and Consumer Expenditure Survey (CEX) are the datasets most relatable to the Nielsen Consumer Panel, because they are also panels. However, none of the PSID studies are interested in the EIS of consumption. Using the CEX and stock market participation, Vissing-Jørgensen (2002) estimates the EIS to be between 0.8 and 1. Vissing-Jørgensen and Attanasio (2003) extend this to find that the EIS of stockholders is likely above 1. Gruber (2013) also uses the CEX and tax rate movements to estimate the EIS to be around 2. This estimate is closest to my Period 3 estimate.

Many studies had small estimates or estimates close to zero. These studies use different data and mechanisms, but results can still be compared. Hall (1988), Campbell and Mankiw (1989), Epstein and Zin (1991), Campbell (2003), Yogo (2004), Dacy and Hasanov (2011), and Gomes and Paz (2013) all have small estimates or estimates close to zero. The Period 1 and Period 2 estimates are similar in that they are both small and close to 0.

There is no other study in the literature that uses similar data and gets similar results to this study. Because of the transactional nature of the dataset, these estimates have a low source of measurement error and aggregation bias. Additionally, they do not suffer from the weak instrument problem. This study examines individual EIS during different periods of economic activity. It gives evidence of a different EIS depending on the state of the U.S. economy. Using a dataset with minimal measurement error and aggregation bias, this study gives evidence of a small and negative EIS during a period of expansion, a small and positive EIS during a period of recession, and a large and positive EIS during quantitative easing.

In Chapter 3, the quantile EIS is estimated. This chapter follows a similar structure to Chapter 2, except quantile preferences are assumed instead of standard expected utility. The smoothed instrumental variables method of moments estimator from de Castro, Galvao, Kaplan and Liu (2018) is applied to the Nielsen Consumer Panel, and the EIS is estimated along a range off quantiles. The quantile EIS is estimated for each of the three periods as done in Chapter 2.

In Period 3, the EIS increases as the quantile increases. Results give evidence of heterogeneity in Period 1 and Period 3, but Period 2 estimates oscillate around the median. Lastly, we do see that the within EIS and median EIS are similar.

1.2 Literature Review

Euler equations have been studied considerably in the economics literature. Hansen and Singleton (1983) was one of the first papers to study risk aversion through consumption using United States data. They examined consumption through risk aversion and stock returns rather than interest rates. Hall (1988) shows there is no strong evidence that the EIS is positive for twentieth-century United States. He also accounted for time aggregation in his estimates.

Campbell and Mankiw (1989) develop a framework involving the “rule of thumb” where half of the agents consume their current income. They estimate that half of the agents consume their permanent income and the other half consumes their current income. This framework is consistent with data, and the permanent income consumers have an EIS that is close to 0. Beaudry and van Wincoop (1996) use the Campbell and Mankiw (1989) framework and U.S. state panel data to estimate the EIS. They estimate the EIS to be close to 1.

Attanasio and Weber (1993) explore the role of aggregation when estimating the EIS. Using U.K. data, they find that estimates are consistently lower for aggregate data than for average cohort data. Blundell, Browning and Meghir (1994) use U.K. micro data to conclude that demographics and labor market status are significant factors when estimating the EIS. Attanasio and Browning (1995) extend Blundell, Browning and Meghir (1994) to find that the excess sensitivity of consumption growth to labor income disappears when demographic variables are controlled for.

Mankiw and Zeldes (1991) propose that limited participation in asset markets matters for consumption and asset returns. Vissing-Jørgensen (2002) further explores this idea by accounting for limited asset market participation when calculating the EIS. She uses data from the CEX and computes Euler equations for stock index returns and T-Bill returns. For stockholders, EIS estimates are around 0.3 - 0.4, and for bondholders, EIS estimates are around 0.8 - 1. Vissing-Jørgensen and Attanasio (2003) use semi-annual consumption data from the CEX and two assets, T-Bills and stock returns, to calculate the EIS. They concluded that the EIS is likely above 1.

Campbell (2003) gives a Handbook of the Economics of Finance chapter that estimates the EIS for eleven developed countries. He uses stock returns and interest rates and finds many of the EIS estimates to be close to 0. Mulligan (2002) uses aggregated consumption data and uses U.S. national accounts data to serve as interest rates. The resulting EIS estimates are close to 1.

Attanasio and Low (2004) show that log-linearized Euler equations yield consistent estimates of preference parameters. They show that as long as discount rates are not very high, log-linearized estimates are not systematically biased, and non-linear GMM has a similar issue. In all situations that they examine, they find that log-linearized Euler equations do better than non-linear GMM. Parker and Preston (2005) derive the consumption Euler equation into four sources: new information, intertemporal substitution, changes in the preferences for consumption, and incomplete markets for consumption insurance. They find that the economic importance of precautionary savings is similar to that of the interest rate and conclude that low estimates of the EIS in linearized or aggregate models are not due to the models omitting precautionary savings.

Yogo (2004) shows that weak instruments can explain why the EIS is significantly less than 1, but the coefficient of relative risk aversion is not different from 1. He finds that the EIS is not significantly different from 0 when using the dataset of eleven developed countries from Campbell (2003). He tests for weak instruments using Stock and Yogo (2005). Stock and Yogo (2005) provide a definition of weak instruments, explain tests for weak instruments, and give critical values for four different tests. Staiger and Stock (1997) develop asymptotic theory for weak instrument equation models. Lee (2001) focuses on labor supply models, but more importantly, he discusses four approaches to correct for the finite sample bias that is induced by weak instruments.

Instead of using a single asset, Dacy and Hasanov (2011) compile a net real rate of return on a synthetic mutual fund (SMF). Their SMF includes money, Treasury bills, intermediate and long-term government bonds, municipal tax-exempt bonds, corporate AAA bonds, common stocks, and residential real estate. Using this SMF, they estimate the EIS to be about 0.2. Gomes and Paz (2013) combine Yogo (2004) with Dacy and Hasanov (2011) to analyze weak instruments using the SMF rate of return. They find that the SMF rate of return suffers from weak instruments and that EIS estimates are around 0.2 and at the low end of the estimates given by Dacy and Hasanov (2011). They also agree that estimates using T-Bill returns do not suffer from weak instruments, but the estimates are not significantly different than 0.

Guvenen (2006) compares aggregate consumption data that typically finds the EIS to be close to zero with calibrated models that typically find the EIS to be close to 1. Gruber (2013) argues that capital income tax rate movements cause exogenous shifts in the after-tax interest rates. Using the CEX, he estimated the EIS to be near 2.

Havranek, Horvath, Irsova and Rusnak (2015) collect 2,735 estimates of the EIS from 169 published papers that cover 104 countries over different time periods. They find that the most important factors explaining heterogeneity are income and asset market participation. They also find that micro-level studies provide higher estimates of the EIS. Havranek (2015) examines selection bias in the collection of studies done by Havranek, Horvath, Irsova and Rusnak (2015). He finds that researchers tend to discard negative and insignificant estimates, which biases the mean estimate upwards to 0.5. He determines that the corrected mean for macro estimates is 0, and the corrected mean for asset holders in micro estimates is 0.3 - 0.4.

Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999) explore habit formation as a way to solve the equity premium puzzle. These studies of time non-separable preferences use aggregate consumption data, but they do not agree on the strength of habit-formation. Dynan (2000) builds on their habit-formation methods but uses a micro-level dataset, the PSID. With this dataset, she finds no evidence of habit formation at the annual frequency.

In this literature, it is often found that the EIS is small and closer to 0 than to 1. However, as seen above, some authors have argued that the EIS is closer to 1. It can be agreed that there is no consensus on the most accurate estimate of the EIS.

Koenker and Bassett (1978) popularized the work of quantile regression. Quantile preferences were developed by de Castro and Galvao (2018), and a smoothed instrumental variables estimator was developed by de Castro, Galvao, Kaplan and Liu (2018). Both de Castro and Galvao (2018) and de Castro, Galvao, Kaplan and Liu (2018) provide applications of quantile preferences to the EIS using macro datasets. In de Castro, Galvao, Kaplan and Liu (2018), they apply the dataset originally from Campbell (2003) and also used by Yogo (2004). Chapter 3 applies quantile preferences to a more complete dataset, the Nielsen Consumer Panel, which is a micro dataset. These quantile EIS estimates can be compared to results from de Castro and Galvao (2018) and de Castro, Galvao, Kaplan and Liu (2018) as well as estimates discussed above that are given by standard expected utility.

CHAPTER 2
ESTIMATING THE ELASTICITY OF INTERTEMPORAL SUBSTITUTION WITH DISAGGREGATED CONSUMPTION DATA

2.1 Introduction

The purpose of this chapter is to estimate the elasticity of intertemporal substitution of consumption using disaggregated consumption data from the Nielsen Consumer Panel. This chapter examines the EIS under expected utility. The EIS represents the willingness of a consumer to substitute future consumption for present consumption. By using the Nielsen Consumer Panel, I am able to address common issues such as measurement error, aggregation bias, and weak instruments. The EIS has not been estimated with such a complete dataset before.

2.2 Economic Model

In order to examine the EIS, this paper follows the standard asset-pricing model often used in economics. The model gives way for the study of intertemporal decisions, risk aversion, and discount parameters. This paper uses the model to study intertemporal decision making. Cochrane (2005), Mehra (2008), and Ljungqvist and Sargent (2012) detail the asset pricing models.

Typical economic models studying the EIS borrow from the endowment economy brought forth by Lucas (1978). Hansen and Singleton (1982), Mehra and Prescott (1985), and Mehra and Prescott (2008) further examine this economy and the equity premium puzzle. At each time period, an individual makes an intertemporal decision to select how much to consume and save (asset holding) over a finite time period. This decision is made subject to a linear time constraint that includes an endowment. Maximizing utility in this economy brings about a policy function in which parameters of interest can be estimated.

The utility of consumption is often defined as basic time-separable power utility:

$$u(c_{i,t}) = \frac{c_{i,t}^{1-\gamma} - 1}{1-\gamma},$$

where γ represents the coefficient of constant relative risk aversion (CRRA). In this utility, decision making does not depend on scale. In other words, an individual with a large amount of wealth has the same risk aversion as an individual with little wealth, or the choice of consumption as a percentage of wealth is the same for all levels of wealth

Under this utility, the consumer's EIS, ψ, is the reciprocal of risk aversion, γ. It is not obvious why these two coefficients should be reciprocals. They have different meanings, and empirical studies often give two different estimates for γ and ψ. Risk aversion is atemporal, and it relates how a consumer substitutes consumption across different states of the world. On the other hand, the EIS is intertemporal, as it relates how a consumer substitutes consumption between two time periods.

To relax this link, Epstein and Zin (1989), Epstein and Zin (1991), and Weil (1989) expand on the theoretical framework of Kreps and Porteus (1978) to create a more flexible version of the basic power utility model. This model retains the scale-independence of the power utility model, but it removes the reciprocal relation between the EIS and CRRA. In this study, I use Epstein-Zin preferences to estimate the EIS.

Consider an infinitely lived representative agent that receives utility from consumption. In any period t, current consumption, C_t is known, but future consumption is uncertain. Due to the setup of this problem, which will be discussed in more detail in the Data section, future utility will be defined as U_{t+4}, because the maturity of the interest rate, four weeks, is four periods as long as the aggregation of consumption, which is weekly. The Epstein-Zin objective function for a representative agent is then defined recursively by

$$U_t = \left\{(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\left(\mathbb{E}_t U_{t+4}^{1-\gamma}\right)^{\frac{1}{\theta}}\right\}^{\theta/(1-\gamma)},$$

where δ is the discount factor and $\theta = (1-\gamma)/(1-1/\psi)$. If $\gamma = 1/\psi$, then $\theta = 1$, and the recursion can be solved to return the basic power utility model. Thus, Epstein-Zin preferences are a generalized version of power utility.

The intertemporal budget constraint for each agent is

$$W_{t+4} = (1 + R_{W,t+4})(W_t - C_t),$$

where W_{t+4} represents the agent's wealth and $(1 + R_{W,t+4})$ is the gross real return on the portfolio of all invested wealth. Using dynamic programming, Epstein and Zin have shown that the Euler equation takes the form

$$1 = \mathbb{E}_t\left[\left\{\delta\left(\frac{C_{t+4}}{C_t}\right)^{-\frac{1}{\psi}}\right\}^{\theta}\left\{\frac{1}{(1+R_{W,t+4})}\right\}^{1-\theta}(1+R_{i,t+4})\right], \qquad (2.1)$$

where $(1 + R_{i,t+4})$ is the gross real return on any available asset i.

If asset returns and consumption are homoskedastic and jointly log normal conditional on information at time t, the Euler equation of Equation (2.1) can be linearized. Let $\Delta c_{t+4} = \ln C_{t+4} - \ln C_t$, $r_{W,t+4} = \ln(1 + R_{W,t+4})$, and $r_{i,t+4} = \ln(1 + R_{i,t+4})$. Then, the linearized Euler equation is:

$$\mathbb{E}_t r_{i,t+4} = \lambda_f + \lambda_i + \frac{1}{\psi}\mathbb{E}_t \Delta c_{t+4}, \qquad (2.2)$$

where

$$\lambda_f = -\log\delta + \frac{\theta - 1}{2}\operatorname{Var}(r_{W,t+4} - \mathbb{E}_t r_{W,t+4})$$
$$- \frac{\theta}{2\psi^2}\operatorname{Var}(\Delta c_{t+4} - \mathbb{E}_t\Delta c_{t+4})$$

and

$$\lambda_i = -\frac{1}{2}\operatorname{Var}(r_{i,t+4} - \mathbb{E}_t r_{i,t+4})$$
$$+ \frac{\theta}{\psi}\operatorname{Cov}(r_{i,t+4} - \mathbb{E}_t r_{i,t+4}, \Delta c_{t+4} - \mathbb{E}_t\Delta c_{t+4})$$
$$+ (1-\theta)\operatorname{Cov}(r_{i,t+4} - \mathbb{E}_t r_{i,t+4}, r_{W,t+4} - \mathbb{E}_t r_{W,t+4})$$

For a conditionally risk-free asset, $\lambda_i = 0$, and Equation (2.2) reduces to

$$\mathbb{E}_t r_{i,t+4} = \lambda_f + \frac{1}{\psi}\mathbb{E}_t\Delta c_{t+4}. \tag{2.3}$$

As seen in Equation (2.3), expected consumption growth and interest rates are linearly related through the constants λ_f and $1/\psi$. Campbell and Viceira (1999) and Campbell (2003) provide more detail on the linearization of Epstein-Zin preferences. In the next section, the expectations in Equation (2.3) will be removed, and the equation will be rearranged to estimate the EIS, which is ψ. The problem is best viewed with a panel data structure, and a panel dataset will be applied to this model.

Note that I am concerned with the EIS of consumption, which represents the intertemporal decision of consumption between two time periods. Another type of model could explore utility dependent on consumption and leisure. For example, Nevo and Wong (2015) estimate the elasticity of substitution between time and goods to be 1.7

2.3 Estimation

In order to estimate the EIS, the Euler equation of Equation (2.1) has been log-linearized. Log-linearization provides an advantage over GMM. Since such a large dataset is available, log-linearization is much faster than GMM. Because Attanasio and Low (2004) show that log-linearization provides consistent preference parameters and that results are unbiased when the discount factor is not large, accurate estimates can be expected.

The expectation operator needs to be removed, so that the model can be estimated with data. For ease of notation, I remove index i, which corresponds to asset i, because there is only one asset. I also add index h to represent household h, because I'm using household data to estimate the EIS. Note that households have regionally specific real interest rates, so there is an index on interest rate $r_{h,t+4}$ as well as on consumption growth $\Delta c_{h,t+4}$. Now, to remove the expectation operator, define the error term for household h to be

$$\xi_{h,t+4} = \Delta c_{h,t+4} - \mathbb{E}_t \Delta c_{h,t+4} - \psi(r_{h,t+4} - \mathbb{E}_t r_{h,t+4}). \tag{2.4}$$

It represents the error in expectations made by consumers. This is a measurement error for which instrumental variables will be needed to estimate the EIS.

Substitute the error term into Equation (2.3) and rearrange to find:

$$\Delta c_{h,t+4} = \beta + \psi r_{h,t+4} + \xi_{h,t+4}, \tag{2.5}$$

where β is a constant. It is likely that $\xi_{h,t+4}$ is correlated with $\Delta c_{h,t+4}$. In other words, as consumption growth increases, households are more likely to err in their decision making. Thus, for household h, I introduce an individual fixed effect, μ_h. The resulting regression equation used to estimate the EIS is

$$\Delta c_{h,t+4} = \beta + \psi r_{h,t+4} + \mu_h + \xi_{h,t+4}. \tag{2.6}$$

In this equation, it can be seen that expected consumption growth is determined by four things: (i) the time preference located within β, (ii) the expected portfolio return, (iii) the individual fixed effect, and (iv) the effects of uncertainty summarized in the variance term.

Equation (2.6) simply regresses consumption growth on real interest rates. To find the best estimate of this effect, one needs to isolate the causal effect of real interest rates on consumption growth. In order to do this, it is important to account for observable and unobservable changes in tastes for consumption. Due to minimal measurement error in the dataset, this problem becomes much easier. Two methods are used to isolate the causal effect. First, fixed effects estimation accounts for unobservable changes in tastes. Second, instrumental variables remove endogeneity. With little measurement error, the use of fixed effects and instrumental variables allow me to find the causal effect of the real interest rate on consumption growth, which is the EIS.

2.3.1 Fixed Effects

To account for unobservable changes in tastes, fixed effects estimation is used. Because the dataset follows the same individuals over multiple time periods, fixed effects estimation accounts for unobservable changes in tastes for consumption. By using a fixed effects estimator, the unobservable individual and seasonal effects can be removed.

As stated above, an individual fixed effect is introduced to account for the correlation between consumption growth and the error term. When consumption growth is larger, the likelihood for error is larger. The individual effect helps control for this. It removes the idiosyncratic nature of individuals that are time-invariant. This allows me to control for unobserved heterogeneity among individuals. These individual effects vary across individuals but are constant across time. As an economic example, this individual effect captures the ability of an agent to make purchasing decisions. Some individuals are wiser with their money than others, and more mistakes are likely to happen as consumption growth increases.

Along with individual effects, a seasonal time effect can be controlled for. These seasonal effects vary across time but are constant across individuals. Economically, this time effect represents macroeconomic trends that all individuals face as the economy changes over time. I will control for a seasonal effect by using month as a control variable.

Under fixed effects estimation, individual-specific differences in tastes have been removed with the individual effect, and seasonal aggregate taste changes have been removed with the time affect. The remaining error component, $\xi_{h,t+4}$, is orthogonal to the individual and time effects.

2.3.2 Instrumental Variables

Instrumental variables are used to remove measurement error and endogeneity. The Nielsen Consumer Panel has minimal measurement error in the consumption variable, but there is endogeneity and measurement error located in the error term. Endogeneity occurs when there is a correlation between the regressors and errors. As seen in Equation (2.4), the regressor, $r_{h,t+4}$, is located within the error term, $\xi_{h,t+4}$. This error term represents the difference between the expected values and measured values of consumption growth and scaled real interest rates, which is a measurement error. This measurement error is assumed to be a moving average. To correct for this endogeneity and measurement error, an instrument set, $Z_{h,t}$, that directly affects the regressor but does not directly affect the outcome variable is used. In other words, the following must be true:

$$\mathbb{E}_t[Z_{h,t}\xi_{h,t+4}] = 0,$$

Thus, the instrument set, $Z_{h,t}$, must be correlated with $r_{h,t+4}$, but not directly affect $\xi_{h,t+4}$. By using an observable variable as an instrument, the causal effect that a change in the real interest rate has on consumption growth can be found.

Instruments used to control for the difference in individual decision making include the second lags of consumption growth, nominal interest rate, and inflation rate. These are standard instruments commonly found in the literature. The second lag must be used because of the correlation between the first lag of consumption growth and the error term. The moving average measurement error assumption allows lags to be a valid instruments. Also, as Campbell and Mankiw (1989) discuss, the time average of a continuous-time random walk is uncorrelated with all variables lagged more than one period, so the second lags help with aggregation bias.

2.3.3 Weak Instruments

Weak instruments have been important in the Euler equation literature. If measurement error is present in the data, an issue with weak instruments would be expected. As previously stated, Yogo (2004) found that weak instruments can cause the EIS to be different than the reciprocal of the coefficient of relative risk aversion. Weak instruments will be tested in order to show that the estimate of the EIS is not biased.

As shown above, the Euler equation was arranged so that

$$\Delta c_{h,t+4} = \beta + \psi r_{h,t+4} + \xi_{h,t+4}. \tag{2.7}$$

where EIS $= \psi$. However, the equation can be rearranged so that

$$r_{h,t+4} = \tau + \gamma \Delta c_{h,t+4} + \eta_{h,t+4}. \tag{2.8}$$

where γ is the coefficient of relative risk aversion. To discuss weak instruments, I will discuss the testing of weak instruments when estimating both the EIS and risk aversion. However, I will only estimate the EIS in this paper.

In order to use the instrument set, $Z_{h,t}$, it must be uncorrelated with the error term. For Equation (2.7), the necessary moment restriction is

$$\mathbb{E}_t[Z_{h,t}\xi_{h,t+4}] = 0,$$

and for Equation (2.8), the necessary moment restriction is

$$\mathbb{E}_t[Z_{h,t}\eta_{h,t+4}] = 0.$$

These equations state that instruments are exogenous. However, for accurate estimation, exogeneity is not enough. It is also important that instruments are relevant. Instruments that are only marginally relevant can be called "weak" instruments. When instruments are weak, first-order asymptotics do not match actual sampling distributions as well as strong instruments. This can lead to poor estimation.

Stock and Yogo (2005) provide an important guide on weak instruments. They begin by giving two quantitative definitions of weak instruments. The first definition is that "a group of instruments is weak if the bias of the IV estimator, relative to the bias of ordinary least squares (OLS), could exceed a certain threshold b". The second definition is that "instruments are weak if the conventional α-level Wald test based on IV statistics has an actual size that could exceed a certain threshold r".

Using this definition, Stock and Yogo (2005) can test for weak instruments. To explain the test, I follow notation used by Staiger and Stock (1997). Consider the following matrix regression model:

$$y = Y\beta + X\gamma + u, \tag{2.9}$$

$$Y = Z\Pi + X\Phi + V, \tag{2.10}$$

in which Equation (2.9) is the structural equation of interest and Equation (2.10) is the reduced form equation for the n endogenous regressors. In this model, y is a $T \times 1$ vector with T observations, Y is a $T \times n$ matrix of endogenous regressors, X is a $T \times K_1$ matrix of exogenous regressors, Z is a $T \times K_2$ matrix of instruments, u is a $T \times 1$ vector of errors, and V is a $T \times n$ matrix of errors. The errors have serially uncorrelated rows with mean zero, and the covariance matrix is:

$$\mathbb{E} = \left[\begin{pmatrix} u_t \\ V_t \end{pmatrix} (u_t, V_t') \right] = \Sigma = \begin{bmatrix} \sigma_{uu} & \Sigma_{Vu}' \\ \Sigma_{Vu} & \Sigma_{VV} \end{bmatrix}.$$

β, γ, Π, and Φ are unknown parameters.

Let $\bar{Z} = [X, Z]$. In order to identify the parameters of interest, $\mathbb{E}[\bar{Z}_t(u_t, V_t')] = 0$.

Then, the reduced form equation for y is

$$y = Z\Pi\beta + X(\Phi\beta + \gamma) + v,$$

where $v = u + V\beta$.

Stock and Yogo (2005) test the null hypothesis that instruments are weak based on the statistic proposed by Cragg and Donald (1993). If there is only one endogeneous estimator, then weak instruments can be tested through the use of the first-stage F-statistic. This statistic tests whether the instruments do not enter through the first stage regression of TSLS. Even though they use the same statistic, Stock and Yogo (2005) have a different null hypothesis and thus different critical values than Cragg and Donald (1993). Cragg and Donald (1993) test a null hypothesis of underidentification, whereas Stock and Yogo (2005) test the null hypothesis that instruments are weak even though parameters might be identified. Specifically, Stock and Yogo (2005) test using the definition of weak instruments as described above. Stock and Yogo (2005) provide tables of critical values for different estimators, number of endogenous regressors, number of instruments, and different thresholds. These critical values are based off of the weak instrument asymptotic distributions as developed by Staiger and Stock (1997).

The F-statistic that determines the relevance of instruments is based off the concentration parameter

$$\mu^2 = \frac{\Pi' Z^{\perp\prime} Z^{\perp} \Pi}{\Sigma_{VV}},$$

where $Z^{\perp} = M_X Z$, $M_X = (I - P_X)$, and $P_X = X(X'X)^{-1}X'$. Stock and Yogo (2005) propose using the first stage F-statistic to test for weak instruments:

$$F = \frac{\hat{\Pi}' Z^{\perp\prime} Z^{\perp} \hat{\Pi}}{K_2 \hat{\Sigma}_{VV}}, \tag{2.11}$$

where $\hat{\Pi} = [Z^{\perp\prime} Z^{\perp}]^{-1} Z^{\perp\prime} Y^{\perp}$ and $\hat{\Sigma}_{VV} = Y' M_{\bar{Z}} Y/(T - K_1 - K_2)$. This F-statistic is just the sample analog of μ^2 scaled by the number of exogenous instruments, K_2. In an Euler equation, one can find the F-statistic by simply regressing the endogenous regressor, such as the real interest rate, on the exogenous instruments. If the F-statistic is sufficiently small, there may be bias or size distortion. The F-statistic can be compared to the critical values provided by Stock and Yogo (2005) to determine if the null hypothesis that there are weak instruments can be rejected.

Yogo (2004) tested weak instruments for eleven developed countries. Using the same instrument set in both cases, he found that instruments were not weak when estimating ψ through Equation (2.7), but he found evidence of weak instruments when estimating γ through Equation (2.8). Because weak instruments cause the estimate of γ to be biased, Yogo (2004) does not find $\gamma = 1/\psi$. He concludes that the estimate of ψ is close to 0. Similar to Yogo, I test for weak instruments in my data in order to verify that instruments are not biasing my results.

2.4 Data

The novel part of this study is the dataset. The EIS has not been studied with a transactional dataset before. There are three main advantages of using this data. First, consumption does not have to be estimated as the data is not survey data. This minimizes measurement error in consumption, which is a common issue in studies of the EIS. Second, the data is transactional, so consumption can be aggregated into a short time period. This lessens the effects of aggregation bias, which often affects EIS estimates. Third, the data is a panel dataset, so it follows households over time. This allows me to remove unobserved individual and time effects from the data. These advantages provide a better dataset to study the EIS than previous studies have used.

In order to estimate the EIS, I create a dataset that combines three sources of data. These sources include consumption data from the Nielsen Consumer Panel, interest rate data from the Federal Reserve of Economic Data (FRED), and CPI data from the Bureau of Labor Statistics (BLS). The time period of the final dataset will be determined by the Nielsen Consumer Panel, as this data covers the smallest time period.

2.4.1 Nielsen Consumer Panel

The Nielsen Consumer Panel,[1] which contains the consumption data, is available through the Kilts-Nielsen Data Center at the University of Chicago Booth School of Business. It is a longitudinal dataset beginning in 2004 with annual updates. It tracks a panel of $40,000 - 60,000$ households per year in the United States and their purchases from a wide range of retail outlets. Panelists use in-home scanners to record purchases from any outlet that is intended for personal use. Data is available from 2004 - 2014.

The sample of households are randomly selected to create a demographic and geographic representation of the United States. To ensure accuracy of the data, panelists are given incentives and constant reminders to regularly upload their data. Households that do

[1] Data is available at http://research.chicagobooth.edu/nielsen/

not meet minimum reporting standards are dropped from the data, and nearly 80 percent of its respondents are retained each year. The Nielsen Company has a number of systems and processes in place to ensure the quality of the data.

Different types of datasets are located within the panel. I use the panelist, trip, and retailer information datasets. Panelist data includes demographic and geographical information. Trip data includes purchase dates and total money spent for each trip. Retailer data includes retailer types. The panel does have transactional data, but this study does not use this data for computational purposes. It is much too large of a dataset, and that fine of information is not needed. The trip dataset captures everything I need from the transactional data. It aggregates the transactional data into a total amount spent on each trip. The total amount spent serves as a proxy for consumption. There is also another Nielsen dataset named the Retail Scanner Data, which includes weekly pricing, volume, and store environment information generated from point-of-sale systems from 90 retails chains across the United States. I do not use this dataset as all necessary information can be found in the Consumer Panel data.

As stated, the raw panel data is transactional. At every retail trip, each item is recorded and uploaded into the dataset. Each of these items is keyed to a trip, which has a purchase date. From here, I aggregate trips (and therefore, purchases) over the desired time frame, which is weekly.

Using the transactional Nielsen data provides a substantial advantage. There is very low measurement error since the data is aggregated from transactions as opposed to coming from a survey. In surveys, respondents often give vague estimates of their food expenditures. Not many people, if anyone, can give an exact measurement off the top of their head of the amount of money they spent on food in the last year. This can be compared to the Nielsen data where respondents scan every item they purchase from a retail store. This exact measurement minimizes measurement error from the dataset. There is still a small potential for measurement error in the data. For example, a respondent may forget to scan one of the purchased items. As another example, grocery purchases that act as a proxy for consumption could also be a source of measurement error. Goods purchased at a farmer's market would not be included in the data, and grocery data does not fully represent consumption data. However, the potential for measurement error in the Nielsen data is much smaller than estimates given in a survey. Thus, using Nielsen data provides an advantage to survey data. A small disadvantage of this data is that most household characteristics are categorical, but many of these categories have a high number of bins.

Another advantage of using a disaggregated dataset like the Nielsen Consumer Panel is that it lessens aggregation bias. Micro data is less influenced by serial correlation than aggregated data. Aggregate consumption leads to consumption smoothing. By using this dataset, consumption can be aggregated over a small time period, one week, in order to lessen the effects of aggregation bias.

In order to create the dataset required to estimate the EIS, the channel type is restricted to only be grocery. This restricts retailers to being traditional grocery stores as opposed to the Internet or other types of retail outlets. It ensures that non-grocery store purchases are not included in the dataset. Grocery stores are of interest because groceries are non-durable goods. Non-durable goods serve as a proxy to consumption purchases, because they must be regularly bought and consumed. Grocery purchases are commonly used as a proxy for consumption.

2.4.2 Interest Rates

In order to construct real interest rates, four-week Treasury bills are deflated. Treasury bills represent the interest rate of saving as opposed to borrowing. The four-week Treasury bills are recorded in weekly intervals. To clarify, this means that the Treasury bills have a maturity of four weeks, and each week provides a four-week Treasury bill.

The Treasury bill data comes from FRED, and FRED has recorded this data from August 3, 2001, to the present. In order to deflate these nominal interest rates into real interest rates, a regionally specific CPI is used. The CPI data is taken from the BLS. Regions include northeast, midwest, south, and west, and it allows me to approach as close to a household specific interest rate as possible. This CPI involves all urban consumers and all items, and it is unseasonal and monthly. The BLS began tracking this data monthly in 1977 and can be found up to the present. In the end, I have monthly real interest rates for each week, and for each week, I have four real interest rates depending on which region the household resides in. I also use the CPI to deflate consumption in order to measure real consumption growth.

2.4.3 Final Dataset

Aggregation of consumption over time periods must be applied in order to create a dataset that represents consumer purchase timeframes. If consumption is aggregated over a smaller time period, it lessens aggregation bias. Note that the interest rate used has a four-week maturity, so intertemporal decisions are based on a four-week timespan. This paper takes timing into account by choosing a small aggregation time period but matching the timing of consumption growth with the maturity of the interest rates. The result is that consumption is aggregated weekly, but consumption growth is considered over four weeks. Thus, individuals make intertemporal decisions over a four week time period. Under this model, aggregate consumption remains a small time period, but the timing of the intertemporal decision between interest rates and consumption also matches. This explains why variables have an index of $t+4$ and $\Delta c_{t+4} = \ln C_{t+4} - \ln C_t$.

The beginning dataset included every trip. After removing households according to the qualifications above, there were 38 million observations. From here, the EIS was aggregated weekly to match the model discussed. The final dataset tied together the consumption, interest rate, and CPI datasets. It consists of a panel that aggregates consumption weekly, includes characteristics of the household, combines corresponding real interest rates during the time period, and adds necessary instruments relating to the household and time period. The data spans from the sixth full week of 2004 to the last full week of 2014. For ease of notation, I denote the full time span to be Feb 1, 2004 - Dec 31, 2014. The data begins the sixth week of 2004, because I need the fifth lag of weekly consumption in order to calculate the second lag of consumption growth. Weeks start on Monday and end on Sunday.

The size of each of these datasets can be found in Table (2.1). Before aggregating consumption over time, the raw data had 38 million observations with 149,000 unique households. When consumption was aggregated weekly, the number of observations became 18 million with 143,000 unique households.

Table 2.1: Dataset Sizes

	Observations	Unique Households
Raw Data	$38,249,996$	$149,107$
Final Dataset	$18,412,458$	$143,948$

The size of the data proved to be challenging. Although there were not many variables, there was a large number of observations. The folder housing the original raw consumption data was 16.5 GB. After removing non-grocery consumption data, the size of the data was 6.0 GB. Finally, after joining interest rate data and creating instrumental variables, the final dataset was 16.2 GB. I had to use the high performance computing cluster at the University of Iowa to store and manipulate the data as well as estimate the model.

2.4.4 Structural Breaks

The time period of this study is restricted by the Nielsen data. The Nielsen Consumer Panel lasts from 2004 to 2014. This time period provides an interesting economic time to study the EIS in the United States as it encompasses the Great Recession. Along with the Great Recession, there is a period of expansion leading up to the Great Recession, and there is a period of quantitative easing after the Great Recession. This can be seen in Figure (2.1). This figure plots the real interest rate of one example region, the northeast, over time. As can be seen, real interest rates rise, fall, and then remain steady around zero.

This gives evidence of structural breaks in the real interest rate. Because of this, I examine the data for structural breaks in the real interest rate over time.

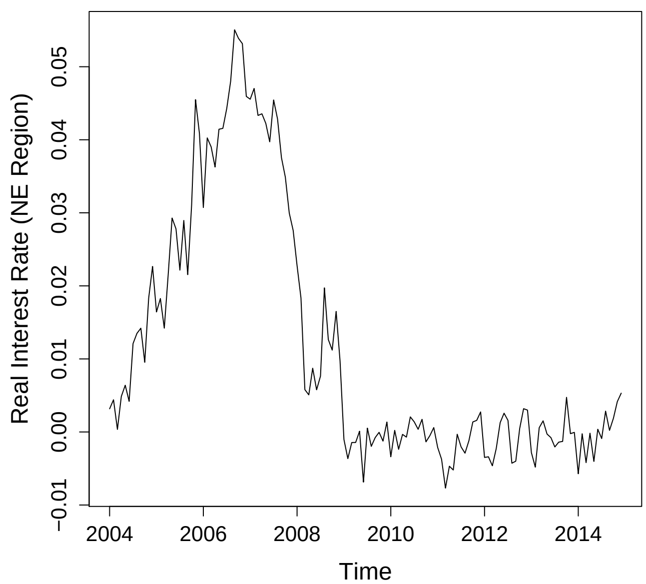

Figure 2.1: Real Interest Rates over Time

In order to find structural breaks in the real interest rate over time, I follow methodology for estimating multiple breakpoints in time series regression models as established by Bai and Perron (1998) and Bai and Perron (2003). Consider the following classical linear regression model:

$$y_t = x_t'\beta + \mu_t.$$

If there are m breakpoints segmenting $m+1$ regimes where the regression coefficients are constants, the model can be written as

$$y_t = x_t'\beta_j + \mu_t \qquad (t = T_{j-1} + 1, \ldots, T_j, \quad j = 1, \ldots, m + 1) \tag{2.12}$$

where j denotes the index of the regime. Bai and Perron (2003) estimate each breakpoint, T_j, by minimizing the residual sum of squares of Equation (2.12). Their algorithm uses the principle of dynamic programming in order obtain global minimizers of the residual sum of squares. The algorithm gives the optimal number of breakpoints in the model.

Visual examination of Figure (2.1) suggests three regimes, but I will use Bai and Perron (1998) to separate the data into an exogenous number of segments. Because individuals are likely to respond differently during economic expansion than during a recession, and they are likely to respond differently during a period of quantitative easing, I expect to find different estimates of the EIS in each time period. Hence, I will provide separate estimates of the EIS for each regime as determined by the exogenous structural breaks.

2.5 Results

Equation (2.6) was estimated to find the EIS. Before estimating, two things were considered. First, the data was examined for structural breaks. Structural breaks are likely, because the dataset covers an unusual time period in the United States economy. Second, the data was tested for weak instruments. By testing for weak instruments, it can be verified that the estimates are not biased due to weak instruments. After finding structural breaks in the data and verifying that instruments are relevant, I can then estimate the EIS using Equation (2.6).

2.5.1 Number of Structural Breaks

Due to the uniqueness of the time period covered in the dataset, I examine structural changes in the real interest rate over time. Following Bai and Perron (2003), I find the optimal number of breakpoints. Panel (A) in Figure (2.2) plots the residual sum of squares (RSS) and Bayesian Information Criterion (BIC) against the number of breakpoints. As can be seen, the RSS and BIC are both minimized when there are two breakpoints splitting the data into three segments.

According to the algorithm, the optimal breakpoints are August 31, 2006 and December 31, 2008. Thus, my data is split into three periods as seen in Table (2.2). Period 1 represents economic expansion from Feb 1, 2004 - Aug 31, 2006. Period 2 represents a recession from Sep 1, 2006 - Dec 31, 2008. Lastly, Period 3 is a time of quantitative easing from Jan 1, 2009 - Dec 31, 2014. Henceforth, I will refer to these regimes as Period 1, Period 2, and Period 3, respectively. Panel (B) in Figure (2.2) shows this segmentation on a plot of the real interest rates. The breakpoints are represented by vertical dotted lines.

Table 2.2: Regimes

Period	Dates
1	Feb 1, 2004 - Aug 31, 2006
2	Sep 1, 2006 - Dec 31, 2008
3	Jan 1, 2009 - Dec 31, 2014

Due to the structural breaks in the real interest rates, I estimate the EIS separately for each of these three periods. Thus, I provide EIS estimates during times of expansion, recession, and quantitative easing.

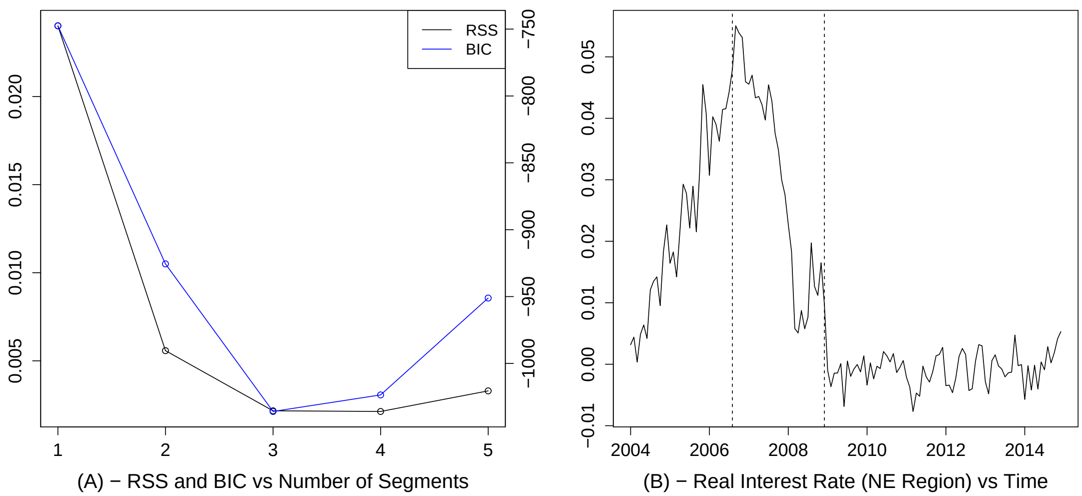

Figure 2.2: Structural Break Figures

2.5.2 Testing for Weak Instruments

Estimates of the EIS could be affected by weak instruments. Yogo (2004) found that weak instruments biased his estimates for the coefficient of relative risk aversion, but they did not affect his estimates for the EIS. Instruments are used during estimation in order to correct for the endogeneity induced by the variable of interest, which is the expected real interest rate. Due to the error term, there is measurement error between the realized values and expected values of consumption growth and real interest rates. By using instruments, the causal effect of the real interest rate on consumption growth can be found.

When estimating the EIS, the second lags of consumption growth, nominal interest rate, and inflation rate were used as instruments. These instruments remove the endogeneity of the expected real interest rates. Second lags must be used due to the correlation between the first lag of consumption growth and the error term. The instrument choice follows Yogo (2004). He also uses the second lag of dividend-price ratio as an instrument, but I do not have access to such a variable.

In order to test for weak instruments, the null hypothesis is that instruments are weak. First, the F-statistic as shown in Equation (2.11) is estimated. This is found by estimating the first stage regression of the endogenous regressor, the real interest rate, on the instruments, which are the second lags of consumption growth, nominal interest rate, and inflation rate. First stage regressions were estimated for each period of interest.

For each of the estimates, the F-statistic is compared to a critical value. This study applies the bias definition from Stock and Yogo (2005), which says that instruments are weak if the TSLS relative bias is greater than 10%. Considering the parameters of the regressions in this paper and using Table 5.1 from Stock and Yogo (2005), the critical value is 9.08. This is due to having one endogenous regressor, three instruments, and bias greater than 10%.

Table (2.3) gives results from the necessary first stage regressions that provide the F-statistic for weak instrument testing. For Period 1, the F-statistic is 110,403,793. During Period 2, the F-statistic is 7,623,733. Lastly, for Period 3, the F-statistic is 1,159,452. Since the F-statistics are greater than the critical value of 9.08 in all three cases, the null hypothesis that there are weak instruments can be rejected in all three cases. Thus, the

Table 2.3: Weak Instrument Testing

	Period 1	Period 2	Period 3
F-Statistic	110,403,793	7,623,733	1,159,452

Note: First stage regression of real interest rate on second lags of consumption growth, nominal interest rate, and inflation. Period 1 = Feb 1, 2004 - Aug 31, 2006; Period 2 = Sep 1, 2006 - Dec 31, 2008; Period 3 = Jan 1, 2009 - Dec 31, 2014. Source: Calculated based on data from The Nielsen Company (US), LLC and marketing databases provided by the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business

instruments are deemed to be strong, and all three estimates of the EIS are not biased due to weak instruments.

Results give evidence that instruments are not weak. This provides confirmation to the accuracy of estimation results. Due to the transactional nature of the data, there is minimal measurement error of consumption biasing the estimates of the EIS, and this section gives evidence that weak instruments do not bias the estimate of the EIS. Therefore, the following estimates should be accurate estimates of the EIS. The next subsection presents results of EIS estimation.

2.5.3 EIS Estimates

After testing for structural breaks and weak instruments, the model was estimated based on Equation (2.6). This equation regresses real consumption growth on real interest rates. Since the dataset is a panel, fixed effects help isolate the effect of the real interest rates on real consumption growth. Both an individual effect and a seasonal effect at a monthly interval were used in this fixed effects regression.

Timing plays an important role in this study. Due to consumption being transactional, it must be aggregated over a time period. When it is aggregated over a smaller time period, it provides a closer look at consumption habits by lessening aggregation bias. The time period of interest rates, however, is more rigid. The Treasury bill with the shortest maturity date, which is four weeks, is used in the data. Thus, interest rates cover a monthly time period, which means intertemporal decisions are made over a four-week timespan.

With the uniqueness of the transactional dataset, one goal is to take advantage of the ability to view consumption at the smallest aggregation level possible. However, it also important for the intertemporal decision making of the consumption decision to match the time period of the maturity of the interest rates. In order to balance this issue, the model aggregates consumption weekly, but considers the growth of consumption over a four-week time period. This allows consumption to be aggregated over a short time period, but it still matches the timing of the intertemporal decision regarding interest rates and consumption choices.

To recap, four-week Treasury bills provide households with an expected real interest rate four periods from the current time. Then, when households make consumption decisions, they are choosing consumption four periods from current time. Thus, I consider consumption growth of weekly aggregated consumption over a four week time period.

Table (2.4) gives results of the regression in Equation (2.6) for each of the three time periods. In Period 1, the estimated EIS is -0.170. During Period 2, the estimated EIS is 0.129. Lastly, for Period 3, the estimated EIS is 2.810. Each of these estimates is

Table 2.4: Estimates of the EIS

	Period 1	Period 2	Period 3
EIS Estimate	-0.170***	0.129***	2.810***
Standard Error	(0.050)	(0.039)	(0.356)
Observations	3,365,010	4,259,667	10,787,781

*** $p < 0.01$

Note: Consumption growth is regressed on real interest rates with fixed effects and instruments. Period 1 = Feb 1, 2004 - Aug 31, 2006; Period 2 = Sep 1, 2006 - Dec 31, 2008; Period 3 = Jan 1, 2009 - Dec 31, 2014. Source: Calculated based on data from The Nielsen Company (US), LLC and marketing databases provided by the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business.

significant at the 1% level. The next section relates results to the existing literature.

2.5.4 Summary of Results

The Nielsen Consumer Panel provided a unique dataset to analyze the EIS. First, the data is examined for structural breaks in the model. Results give evidence of two structural breaks separating three regimes. Period 1 is Feb 1, 2004 - Aug 31, 2006. Period 2 is Sep 1, 2006 - Dec 31, 2008, and Period 3 is Jan 1, 2009 - Dec 31, 2014. Next, weak instruments are tested in each period by following Yogo (2004) and Gomes and Paz (2013). Results state that the data did not suffer from weak instruments. This is similar to Yogo (2004) when he estimates the EIS using Treasury bills as the interest rate. Whereas Yogo (2004) uses a macro dataset and examines eleven developed countries, this study analyzed a micro dataset of the United States. Gomes and Paz (2013) use the micro dataset from Dacy and Hasanov (2011) and find evidence for weak instruments in their regressions when a SMF was used as the real rate of return. When using T-Bills, they do not find evidence for weak instruments. Albeit under a small sample size, estimating the EIS with Treasury bills as the interest rate appears to be free of the weak instruments problem as evidenced by Yogo (2004), Gomes and Paz (2013), and my results.

Using fixed effects and instruments, the data gives estimates for the EIS that are -0.170, 0.129, and 2.810 for Period 1, Period 2, and Period 3, respectively. All of these estimates are significant at the 1% level.

This study is the first to examine the EIS when consumption is aggregated at a weekly interval. Many studies have explored the EIS or constant relative risk aversion from a monthly perspective, such as Hansen and Singleton (1983), Hall (1988), and Epstein and Zin (1991). These three studies are also three of the original studies of the EIS. None of the recent studies examine consumption at a monthly frequency. Hansen and Singleton (1983), Hall (1988), and Epstein and Zin (1991) all study macro datasets in the United States. Hansen and Singleton (1983) study constant relative risk aversion, but both Hall (1988) and Epstein and Zin (1991) find estimates of the EIS to be small or close to zero. Many of the studies examine consumption at a quarterly or yearly interval.

Other studies that use micro datasets similar to this study are Attanasio and Weber (1993), Blundell, Browning and Meghir (1994), Attanasio and Browning (1995), Beaudry and van Wincoop (1996), Dynan (2000), Lee (2001), Vissing-Jørgensen (2002), Vissing-Jørgensen and Attanasio (2003), Parker and Preston (2005), Guvenen (2006), and Gruber (2013). None of these studies can be directly compared to my study. Attanasio and Weber (1993), Blundell, Browning and Meghir (1994), and Attanasio and Browning (1995) all use

U.K. data, and they simulate a panel using mean cohort data. Beaudry and van Wincoop (1996) use data from the U.S., but they simulate a panel using state cohorts. Dynan (2000) uses the PSID in the U.S. but is interested in habit formation as opposed to the EIS. Lee (2001) also uses the PSID but is interested in labor supply. Vissing-Jørgensen (2002) and Vissing-Jørgensen and Attanasio (2003) use the CEX in the U.S. and use limited asset market participation to estimate the EIS. Parker and Preston (2005) also use the CEX, but they are interested in precautionary savings. Lastly, Gruber (2013) uses the CEX and tax rate movements to estimate the EIS.

The PSID and CEX are the datasets most relatable to the Nielsen Consumer Panel, because they are also panel studies. However, neither of the PSID studies are interested in the EIS of consumption. Using the CEX and stock market participation, Vissing-Jørgensen (2002) estimates the EIS to be between 0.8 and 1, and Vissing-Jørgensen and Attanasio (2003) find that the EIS of stockholders is likely above 1. Estimates with stock market participation tend to be higher than those without. Gruber (2013) uses the CEX and tax rate movements and estimated the EIS to be 2, which is the highest of any study mentioned in this paper. Gruber's estimate is closest to my Period 3 estimate of 2.8.

Many studies had small estimates or estimates close to 0. The Period 1 and Period 2 can be considered small and close to 0. These studies can be said to have similar results as this study even if they used different data and mechanisms. Such studies include Hall (1988), Campbell and Mankiw (1989), Epstein and Zin (1991), Campbell (2003), Yogo (2004), Dacy and Hasanov (2011), and Gomes and Paz (2013). Hall (1988) and Epstein and Zin (1991) were previously discussed, as well as Dacy and Hasanov (2011). Campbell and Mankiw (1989), Campbell (2003), Yogo (2004), and Gomes and Paz (2013) all use the same macro datasets of eleven developed countries.

When comparing to the literature, there is no other study that uses similar data and gets similar results as this study. This study adds to the literature because it provides evidence using a unique dataset that gives estimates of the EIS while minimizing measurement error and aggregation bias. It provides, for the first time, the ability to view the EIS during different economic times. The estimates give evidence of a different EIS depending on the state of the U.S. economy.

2.6 Conclusion

The purpose of this paper is to estimate the EIS using disaggregated data from the Nielsen Consumer Panel. In principle, this data should be better than datasets used in previous literature. Previous literature uses survey data at yearly or quarterly intervals. Due to the use of survey data, there are obvious measurement error issues. This study uses the Nielsen Consumer Panel, which tracks grocery purchases for participating households. Because of the transactional nature of this dataset, measurement error is minimized as households scan each of their purchases into a database. It also allows for aggregation of consumption over a small time period, such as weeks, which lessens aggregation bias. The dataset is very large with over 38 million rows of data. Given these advantages, the data should provide better estimates of the EIS than the previous literature has.

To find the EIS, this study aggregates consumption weekly and considers consumption growth over a four-week time period. Consumption growth is over a four-week time period in order to match the maturity of four-week Treasury bills. This allows for small time period aggregation, while matching consumption growth with interest rates.

The dataset covers households in the United States from 2004 to 2014. This is an interesting time period for the United States economy as it encompasses the Great Recession. Period 1 (Feb 1, 2004 - Aug 31, 2006) was economic expansion that had increasing real interest rates. Period 2 (Sep 1, 2006 - Dec 31, 2008) was a financial crisis with decreasing real interest rates. Period 3 (Jan 1, 2009 - Dec 31, 2014) was a period of quantitative easing. During each of these time periods, it can be expected that individuals respond differently. Thus, I examined the data for structural breaks and found evidence of structural breaks separating these three time periods. Additionally, weak instruments have been known to affect EIS estimates, but there is no evidence of weak instruments.

In Period 1, the EIS is estimated to be -0.170, and in Period 2, the EIS is estimated to be 0.129. During Period 3, the EIS is estimated to be 2.810. The estimates for Periods 1 and 2 are considered small, but the Period 3 estimate can be considered large. During expansion in Period 1, the negative EIS indicates that the income effect dominates. As real interest rates rise, consumers can receive the same return next period by saving less. Hence, they consume more, and consumption growth decreases. During the recession in Period 2, the positive EIS indicates that the substitution effect dominates. As real interest rates rise, consuming now is relatively more expensive since the rate of return is higher. Hence, consumers save more, and consumption growth increases. The Period 3 estimate states that households respond excessively to changes in real interest rates. However, this estimate is likely affected by a lack of variation in the real interest rate. During this period of quantitative easing, the interest rate is steady near 0. This leads to a much higher standard error in the EIS estimate of Period 3. The standard error of the estimated EIS is 0.356 in Period 3, which can be compared to smaller standard errors of 0.050 and 0.039 in Periods 1 and 2, respectively.

There is no other study in the literature that uses similar data and gets similar results to this study. The Nielsen Consumer Panel provides a unique dataset that gives small estimates of the EIS. Because of the transactional nature of the dataset, these estimates have a low source of measurement error and aggregation bias. Additionally, they do not suffer from the weak instrument problem. This study examines individual EIS during different periods of economic activity. With a more complete dataset that has much less potential for measurement error than the existing literature, this study gives evidence of a small and negative EIS during a period of expansion, a small and positive EIS during a period of recession, and a large and positive EIS during quantitative easing.

CHAPTER 3
ESTIMATING THE QUANTILE ELASTICITY OF INTERTEMPORAL SUBSTITUTION WITH DISAGGREGATED CONSUMPTION DATA

3.1 Introduction

[1]The purpose of this chapter is to estimate the quantile elasticity of intertemporal substitution of consumption using disaggregated data from the Nielsen Consumer Panel. Previously, in Chapter 2, the EIS was estimated using expected utility and disaggregated data from the Nielsen Consumer Panel. This chapter follows similarly, but agents have a quantile utility preference instead of expected utility. These quantile utility preferences were developed by de Castro and Galvao (2018).

Quantile utility preferences create Euler equations for each quantile of the conditional distribution, and quantile regression is used to estimate these equations. Since the seminal work of Koenker and Bassett (1978), quantile regression has attracted considerable interest in statistics and econometrics. Quantile regression offers an easy-to-implement method to estimate conditional quantiles, and it has provided a valuable method of statistical analysis of the heterogenous effects of policy variables. This is especially true for program evaluation studies, where these methods help to analyze how treatments or social programs affect the outcome's distribution.

Quantile regression has two main advantages compared to mean regression. First, it provides a way to study the heterogeneity of agents. Mean regression provides one

[1]This chapter is joint work with Luciano de Castro (University of Iowa) and Antonio F. Galvao (University of Arizona).

estimate of the parameter of interest, while quantile regression provides multiple estimates along the conditional distribution. If the estimates change along the quantiles, there is evidence of heterogeneity. Understanding heterogeneity can be especially important with consumption, as different individuals tend to have different consumption habits. Second, quantile regression is robust to outliers. In mean regression, large outliers can pull the mean toward the direction of the outlier. In some studies, the median may provide a better estimate than the mean. This is especially true when dealing with consumption.

It has been recently documented in Toda and Walsh (2015) and Toda and Walsh (2017) that the cross-sectional distributions of U.S. consumption and its growth rate obey the power law in both the upper and lower tails with exponents approximately equal to four. There has also been discussion on the constancy of the EIS. Papers such as Crossley and Low (2011), Attanasio and Browning (1995), and Blundell, Browning and Meghir (1994) have allowed the EIS to change. By estimating a quantile EIS, we explore how the EIS changes along the conditional distribution. Quantiles measure riskiness with a larger quantile representing a more risk-seeking agent. Measuring how the EIS changes along the conditional distribution allows us to understand how the EIS changes with riskiness.

Our paper contributes to three literatures. First, we contribute to a large structural literature studying intertemporal substitution in consumption, reviewed by Attanasio and Weber (2010) and Thimme (2017). This literature estimates consumption Euler equations using aggregate data (e.g., Hall (1988); Campbell and Mankiw (1989)) relying on Epstein and Zin (1989, 1991) and Weil (1989) preferences and time series movements in interest rates, producing a wide range of estimates. The Epstein and Zin preferences are able to disentangle the risk aversion from the EIS. The main conceptual differences between our approach and this literature is that we use quantile preferences. The quantile model is important for multiple reasons. First, the quantile utility model allows for separation between risk attitude and intertempotal substitution. Risk aversion describes the consumer's reluctance to substitute consumption across states of the world and is meaningful even in an atemporal setting, whereas the EIS describes the consumer's willingness to substitute consumption over time and is meaningful even in a deterministic setting. Second, the quantile model enjoys the ability to capture heterogeneity thorough the quantiles. Finally, the quantile model has robustness properties. Our estimating equation is a quantile Euler equation, and our estimates show heterogeneity across EIS.

Second, we contribute to literature using micro survey data (e.g., Zeldes (1989); Attanasio and Weber (1993), Attanasio and Weber (1995); Vissing-Jørgensen (2002); Gruber (2013)) to estimate the EIS. We make use of the Nielsen Consumer Panel, which is a large disaggregated dataset. It tracks households and their retail purchases over time. This dataset is advantageous, because it minimizes measurement error and aggregation bias.

Third, this paper contributes to a rich literature on economic models with heterogeneity. Heckman (2001), Blundell and Stoker (2005), Krusell and Smith (2006), and Guvenen (2011) provide reviews of the main ideas on heterogeneity and aggregation. Dynamic models with heterogeneity typically feature individual-specific uncertainty that stems from fluctuations in labor earnings, health status, and portfolio returns, among others. Virtually all of these models rely on the expected utility framework and capture heterogeneity in a variety of ways. Part of the literature allows for heterogeneity of the economic variables and shocks, but restricts the parameters of interest. For example, Krusell and Smith (1998), Dynan (2000), and Heaton and Lucas (2008) restrict parameters that characterize the preference to be homogeneous. Another body of the literature encompasses heterogeneity by allowing the parameters to vary in a small set, such as a binary set. Examples include Mazzocco (2008) and Guvenen (2009). Yet another stream of the literature incorporates more general heterogeneity in the parameters of interest but imposes ad-hoc parametric restrictions on them, such as Herranz, Krasa and Villamil (2015). In this paper, we contribute to this literature by using the quantile preference instead of the expected utility, which allows us to account for heterogeneity through the quantiles.

As stated, this paper follows a similar structure to the estimation of the EIS in Chapter 2. Besides using quantile utility preferences instead of standard expected utility, the structure remains the same. To estimate the quantile EIS, a method of moments estimator is used. More specifically, we estimate using smoothed instrumental variables quantile regression as developed by de Castro, Galvao, Kaplan and Liu (2018).

3.2 The Consumption Quantile Utility Model

This section discusses an economic model of intertemporal allocation of consumption considering the quantile utility framework. The quantile model will be useful because of its robustness properties and its ability to capture heterogeneity. In addition, the corresponding Euler equation has a conditional quantile function representation, and the quantile estimators are well know to be robust to outliers and fat tails.

For each quantile-τ, i.e. each fixed risk aversion index, the model has two main parameters of interest, the discount factor and the EIS. This paper will focus on the EIS. It is a parameter of central importance in macroeconomics and finance. Along with the discussion in Chapter 1, readers can refer to Campbell (2003), Cochrane (2005), Ljungqvist and Sargent (2012), and the references therein, for a comprehensive overview.

The goal of this paper is to employ disaggregated data at the household level to estimate the EIS using the quantile maximization model. This is an important contribution for two reasons. First, the quantile utility model allows for separation between risk and intertempotal substitution. Risk aversion describes the consumer's reluctance to substitute consumption across states of the world and is meaningful even in an atemporal setting, whereas the elasticity of intertemporal substitution describes the consumer's willingness to substitute consumption over time and is meaningful even in a deterministic setting. The τ-quantile captures the riskiness of individuals. Second, the model allows for heterogeneity of the EIS coefficient across the τ-quantiles, and it is robust to outliers and fat tails.

3.2.1 Quantile Function

Before describing the economic model, the quantile function must be defined. Let X be a random variable and F_X (or simply F) denote its cumulative distribution function such that $F_X(\alpha) \equiv \mathrm{P}[X \leq \alpha]$. The quantile function $U : [0,1] \to \bar{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$ is the generalized inverse of F:

$$Q(\tau) \equiv \begin{cases} \inf\{\alpha \in \mathbb{R} : F(\alpha) \geq \tau\}, & \text{if } \tau \in (0,1] \\ \sup\{\alpha \in \mathbb{R} : F(\alpha) = 0\}, & \text{if } \tau = 0. \end{cases}$$

It is clear that if F is invertible, then $Q(\tau) = F^{-1}(\tau)$. Because it is important to highlight the random variable to which the quantile refers, $Q(\tau)$ will be denoted by $Q_\tau[X]$.

3.2.2 Economic Model

We employ a variation of the standard economy model that allows for heterogeneous households as, for example, in Dynan (2000), Parker and Preston (2005), and Heaton and Lucas (2008). The economic agents decide on the intertemporal consumption and savings (assets to hold) over an infinite horizon economy subject to a linear budget constraint. The decision generates an intertemporal policy function, which is used to estimate the parameters of interest for a given utility function.

Let c_{it} denote the amount of consumption good that household i consumes in period t. In period t, household i seeks to maximize the τ quantile of the discounted utility:

$$Q_\tau^\infty \left[\sum_{t=s}^{\infty} \delta_\tau^{t-s} U(c_{it}) \middle| \Omega_s \right],$$

where

$$\begin{aligned} Q_\tau^\infty \left[\sum_{t=0}^{\infty} \delta_\tau^{t} U(c_{it}) \middle| \Omega_0 \right] &\equiv Q_\tau \left[Q_\tau \left[Q_\tau \left[U(c_{i0}) + \delta_\tau U(c_{i1}) + \delta_\tau^2 U(c_{i2}) + \cdots |\Omega_2 \right] \middle| \Omega_1 \right] \middle| \Omega_0 \right] \\ &= U(c_{i0}) + Q_\tau \left[\delta_\tau U(c_{i1}) + Q_\tau \left[\delta_\tau^2 U(c_{i2}) + \cdots |\Omega_1 \right] \middle| \Omega_0 \right]. \end{aligned}$$

In this equation, $\delta_\tau \in (0, 1)$ is the discount factor, $U : \mathbb{R}_+ \to \mathbb{R}$ is the utility function, and Ω_t is the information set at time t.

At the beginning of period t, household i has x_{it} units of the risky asset, which pays dividend z_{it}. The price of the consumption good is normalized to one, while the price of the risky asset in period t is $p(z_{it})$. Then, the consumer decides how many units of the risky asset x_{it+1} to save for the next period and its consumption c_{it}, satisfying:

$$\begin{aligned} c_{it} + p(z_{it})x_{it+1} &\leqslant [z_{it} + p(z_{it})] \cdot x_{it}, \qquad (3.1) \\ c_{it}, x_{it+1} &\geqslant 0. \end{aligned}$$

Let $\mathcal{X} \subset \mathbb{R}^n$ denote the state space and $\mathcal{Z} \subseteq \mathbb{R}^k$, the range of the shocks (random variables) in the model. Let $x_t \in \mathcal{X}$ and $z_t \in \mathcal{Z}$ denote, respectively, the state and the shock in period t, both of which are known by the decision maker at the beginning of the period.

Following the literature, we assume that the holdings must not exceed one unit. In equilibrium, for each household i, we have that $x^*_{itk} = 1, \forall i, t, k$. Let $\bar{x} > 1$ and $\mathcal{X} = [0, \bar{x}]^k$.

In each period, the decision maker cares about utility $u(x_{it}, y_{it}, z_{it})$, where $x_{it} \in \mathcal{X}$ denotes the current state, $c_{it} \equiv y_{it} \in \mathcal{Y}$, the choice in the current state, and $z_{it} \in \mathcal{Z}$, the current shock. Given $x_{it} \in \mathcal{X}$ and $z_{it} \in \mathcal{Z}$, the decision maker has to choose $y_{it} \in \Gamma(x_{it}, z_{it}) \subset \mathcal{Y}$, which is a constraint set determined by x_{it} and z_{it}. Let $\mathcal{Z}^t = \mathcal{Z} \times \cdots \times \mathcal{Z}$ (t-times, for $t \in \mathbb{N}$), $\mathcal{Z}^\infty = \mathcal{Z} \times \mathcal{Z} \times \cdots$ and $\mathbb{N}^0 \equiv \mathbb{N} \cup \{0\}$.

From Equation (3.1), we can determine the consumption entirely from the current and future states, that is, $c_{it} = z_{it} \cdot x_{it} + p(z_{it}) \cdot (x_{it} - x_{it+1})$. Now, for each household i, we denote x_{it} by x, x_{it+1} by y, and z_{it} by z. Then, the above restrictions are captured by the feasible correspondence $\Gamma : \mathcal{X} \times \mathcal{Z} \to \mathcal{Y} = \mathcal{X}$ defined by:

$$\Gamma(x, z) \equiv \{y \in \mathcal{X} : p(z) \cdot y \leqslant (z + p(z)) \cdot x\}.$$

For each pricing function $p : \mathcal{Z} \to \mathbb{R}^k_+$, define the utility function as:

$$u(x, y, z) \equiv U[z \cdot x + p(z) \cdot (x - y)].$$

In order to derive the Euler equation, we apply the results in de Castro and Galvao (2018). They show that the following Euler equation holds for every $t \in \mathbb{N}$:

$$u_y\left(x_t, x_{t+1}, z_t\right) + \delta_\tau Q_\tau[u_x\left(x_{t+1}, x_{t+2}, z_{t+1}\right)|z_t] = 0. \tag{3.2}$$

In Equation (3.2), u_y represents the derivative of u with respect to (some of the coordinates of) its second variable, y, and u_x represents the derivative of u with respect to (some of the coordinates of) its first variable, x.

We assume the following:

Assumption 3.1. *1. $\mathcal{Z} \subseteq \mathbb{R}$ is an interval;*

2. z follows a Markov process with pdf f satisfying the property that $z \leqslant z'$ implies $f(\alpha|z') \leqslant f(\alpha|z), \forall \alpha \in \mathcal{Z}$;

3. $U : \mathbb{R}_+ \to \mathbb{R}$ is given by $U(c) = \frac{1}{1-\gamma}c^{1-\gamma}$, for $\gamma > 0$;

4. $z \mapsto z + p(z)$ is non-decreasing and differentiable, with $(\ln(z + p(z)))' \leqslant \gamma$.

Assumptions 3.1(1)-(3) are standard in economic applications. Assumption 3.1(4), which states that $z \mapsto z + p(z)$ is non-decreasing, is natural. It states that the price of the risky asset and its return are a non-decreasing function of the dividends. Note that it is natural to expect that the price is non-decreasing with the dividends, but Assumption 3.1(4) is even weaker than this, as it allows the price to decrease with the dividend; only $z + p(z)$ is required to be non-decreasing.

Given Assumption 3.1, we can verify the assumptions in de Castro and Galvao (2018) for establishing the quantile utility model in the household intertemporal consumption model context. Thus, we have the following:

Lemma 3.1. *Assumption 3.1 implies Assumptions 1 and 2 in de Castro and Galvao (2018).*

Proof of Lemma 3.1: Since U is strictly concave, it is easy to verify that u is strictly concave in (x, y). The other verifications are straightforward. □

Therefore, Theorems 3.3 and 3.4 in de Castro and Galvao (2018) imply the existence of a value function v, which is strictly concave and differentiable in its first variable, satisfying

$$v(x, z) = \max_{y \in \Gamma(x,z)} Q_\tau[g(x, y, z, \cdot)|z],$$

where

$$g(x, y, z, w) = u\,(x, y, z) + \delta_\tau v(y, w),$$

and $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial x}$. Note that

$$\begin{aligned} \frac{\partial u}{\partial x}(x, y, z) &= U'\left[z \cdot x + p(z) \cdot (x - y)\right](z + p(z))\,; \\ \frac{\partial u}{\partial y}(x, y, z) &= U'\left[z \cdot x + p(z) \cdot (x - y)\right](-p(z))\,. \end{aligned}$$

Because, in equilibrium, the holdings are $x_{it} = 1$ for all (i, t), we can derive the Euler equation as in Equation (3.2) for this particular problem to obtain:

$$-p(z_{it})U'\left(c_{it}\right) + \delta_\tau Q_\tau[U'\left(c_{it+1}\right)\left(z_{it+1} + p(z_{it+1})\right)|\Omega_t] = 0. \tag{3.3}$$

Let us define the return by:

$$1 + r_{it+1} \equiv \frac{z_{it+1} + p(z_{it+1})}{p(z_{it})}.$$

Therefore, the Euler equation in Equation (3.3) simplifies to:

$$Q_\tau\left[\delta_\tau(1 + r_{it+1})\frac{U'(c_{it+1})}{U'(c_{it})}\middle|\Omega_t\right] = 1. \tag{3.4}$$

Note that Equation (3.4) is in the format of Equation (3.8), which is suitable for instrumental variables nonlinear quantile regression.

It is illustrative to compare the Euler equation from the quantile maximization utility model in Equation (3.4) with its counterpart from the expected utility maximization (e.g., Dynan (2000)). It is well known that the Euler equation for the expected utility has the following representation:

$$\mathbb{E}\left[\delta(1+r_{it+1})\frac{U'(c_{it+1})}{U'(c_{it})}\middle|\Omega_t\right]=1. \tag{3.5}$$

When comparing Equations (3.4) and (3.5), one can notice that these equations share similarities. The former model describes the conditional quantile function, while the latter a conditional expectation, but the expressions inside the conditional quantile and conditional expectation are essentially the same. The main difference is that, for the quantile function, the parameters of interest might vary with the quantile, while for the expectation, there is only one set of parameters. Thus, the heterogeneity is captured by the parameters describing the conditional quantile function. Moreover, for a given data set and quantile of interest, one is able to estimate the parameters indexed by τ for the quantile model, that is δ_τ and the τ-parameters that parameterize the utility function. On the other hand, for the expectation model, one is able to estimate only δ and the corresponding parameters of the utility function.

3.2.3 Isoelastic Utility Function

The Euler equation in Equation (3.4) possesses a nonlinear conditional quantile function representation. Thus, for a given utility function, one is able to estimate the parameters of interest using the quantile regression methods described in this paper.

In this application, we follow a large body of the literature, as for example, Dynan (2000) and Campbell (2003), among others, and use an isoelastic utility function of the form

$$U(c_{it}) = \frac{1}{1-\gamma} c_{it}^{1-\gamma},$$

for $\gamma > 0$. The parameter γ is the standard measure of the degree of relative risk aversion that is implicit in the utility function. We are interested in the EIS, ψ, which is the reciprocal of γ in the isoelastic utility function.

The ratio of marginal utilities can be written as

$$\frac{U'(c_{it+1})}{U'(c_{it})} = \left(\frac{c_{it+1}}{c_{it}}\right)^{-\gamma}. \tag{3.6}$$

Finally, from Equations (3.4) and (3.6), the Euler equation can be rewritten as

$$Q_\tau \left[\delta_\tau (1 + r_{it+1}) \left(\frac{c_{it+1}}{c_{it}}\right)^{-\gamma_\tau} - 1 \middle| \Omega_t \right] = 0. \tag{3.7}$$

After deriving the Euler equation in Equation (3.7), we can estimate the parameters of interest (ψ_τ, δ_τ). Given a random sample $\{(r_{it}, c_{it}) : t = 1, ...T\}$, we are able to apply instrumental variable quantile regression methods and for each quantile $\tau \in (0, 1)$, estimate the corresponding parameters (ψ_τ, δ_τ). In this way, we uncover the potential underlying heterogeneity across the quantiles.

Several considerations are in order when estimating the parameters in Equation (3.7). First, Equation (3.7) is an equilibrium condition. It is difficult to obtain a complete characterization of the stochastic equilibrium under weak assumptions about the forcing variables. Thus, in the literature, it has become common to use such an equilibrium condition together with instrumental variables to derive orthogonality conditions that can be used to estimate the parameters of the utility function. Second, it has been recognized in the literature that the presence of a “taste-shock” (or measurement error) might affect the estimation of the parameters of interest when estimating Euler equations (see, e.g., Yogo (2004) and Attanasio and Low (2004)). Therefore, the use of instruments has been essential to recover the parameters of interest. Finally, when bringing Equation (3.7) to the data, rational expectations are an underlying assumption. This means that the conditional quantile function operator in Equation (3.7) coincides with the theoretical given all information available to the consumer at time t. Thus, the conditional quantile function is valid over time.

3.2.4 Quantiles and Risk

Risk attitudes in the quantile model were first studied by Manski (1988) and Rostek (2010). Rostek (2010) shows that in a static model, a decision maker that maximizes a lower quantile is more “risk-averse” than one who maximizes a higher quantile. In other words, risk-attitude can be related to the quantile rather than to the concavity of the utility function. However, the static model does not have a concept of intertemporal substitution. Risk attitudes are shown similarly in the dynamic model by de Castro and Galvao (2018). This allows risk attitudes to be disentangled from the degree of the elasticity of intertemporal substitution, even when using standard isoelastic utility. The EIS can be estimated across a set of quantiles, which represents a risk attitude. As the quantiles increase, we interpret individuals to be more risk-seeking. A further discussion is provided in de Castro and Galvao (2018).

In Chapter 2, I used Epstein-Zin preferences to remove the link between the EIS and coefficient of relative risk aversion. Rather than Epstein-Zin preferences, this chapter uses quantile preferences. As shown in de Castro and Galvao (2018), quantile preferences allow us to disentangle the EIS from the coefficient of relative risk aversion, while also accounting for heterogeneity. Thus, this chapter uses standard isoelastic utility as the utility function in the quantile model.

3.3 Estimation

We apply the smoothed instrumental variables quantile regression model of de Castro, Galvao, Kaplan and Liu (2018) for estimating the parameters of interest. This section briefly describes the estimation procedure.

3.3.1 Econometric Model

The model of interest is described by the following nonlinear conditional quantile function:

$$Q_\tau[\Lambda(Y_{it}, \mathbf{X}_{it}, \boldsymbol{\beta}_{0\tau})|\Omega_{it}] = 0, \tag{3.8}$$

where $\tau \in (0, 1)$ is a quantile of interest, $(Y_{it}, \mathbf{X}_{it})$ are the observable variables, $\boldsymbol{\beta}_{0\tau}$ is a $p \times 1$ vector of parameters of interest, the parameter space is denoted by $\mathcal{B} \subseteq \mathbb{R}^p$, which is a compact set, $\Lambda(\cdot)$ is a "residual function" known up to a finite dimensional parameter $\boldsymbol{\beta}_{0\tau}$, Ω_{it} denotes the σ-field generated by $\{Y_{is}, s \le t\}$ that contains the information up to time t, and $Q_\tau(\cdot|\Omega_{it})$ is the conditional τ-quantile of $\Lambda(\cdot)$ given Ω_{it}. The function $\Lambda(\cdot)$ is known for any particular application, and the parameters of interest are then $\boldsymbol{\beta}_{0\tau}$. We assume that the left hand-side of the Equation (3.8) is increasing with respect to τ.

The model in this paper allows for endogeneity of $\mathbf{X}_{it}$, so we need to conduct estimation and inference for nonlinear quantile regression models under endogeneity. To accomplish this aim, we discuss a general framework for generic moment restriction estimators (Z-estimators). We consider the following moment condition representation of the model in Equation (3.8)

$$\mathbb{E}\left[\tau - \mathbb{1}\left\{\Lambda(Y_{it}, \mathbf{X}_{it}, \boldsymbol{\beta}_{0\tau}) \leq 0\right\} \mid \mathbf{X}_{it}, \mathbf{Z}_{it}\right] = 0. \tag{3.9}$$

Equation (3.9) is a non-smooth moment condition representation of Equation (3.8). This is because $\mathbb{E}\left(\mathbb{1}\left\{\Lambda(Y_{it}, \mathbf{X}_{it}, \boldsymbol{\beta}_{0\tau}) \leq 0\right\} \mid \mathbf{X}_{it}, \mathbf{Z}_{it}\right) = F(\Lambda(Y_{it}, \mathbf{X}_{it}, \boldsymbol{\beta}_{0\tau}) \mid \mathbf{X}_{it}, \mathbf{Z}_{it})$, and when $F(\cdot|\cdot)$ is invertible, one is able to recover Equation (3.8) from Equation (3.9). As will be discussed in Section 3.3.3, Y_{it}, $\mathbf{X}_{it}$, and $\mathbf{Z}_{it}$ are free of fixed effects.

3.3.2 Smoothed IVQR Estimation

A distinctive feature of the standard quantile regression estimation is that the objective function is not differentiable. Therefore, the basic smoothness assumption imposed in the nonlinear estimation literature is not satisfied for the standard quantile regression estimator. In this paper, instead of the standard quantile regression estimator, we use estimation and inference procedures for the estimator defined by a minimizer of a smoothed version of the quantile regression objective function. Smoothing the quantile regression objective function is employed in Horowitz (1998) to study the bootstrap refinement for inference in conditional quantile models. The basic insight of Horowitz (1998) is to smooth over $I(Y_{it} \leq \Lambda(\mathbf{X}_{it}, \boldsymbol{\beta}_{0\tau}))$ by using a kernel function.[2]

Consider the following smoothed estimator developed by de Castro, Galvao, Kaplan and Liu (2018). Let the population map $\mathbf{M}_n : \mathcal{B} \times \mathcal{T} \mapsto \mathbb{R}^{d_Z}$ be

$$\mathbf{g}_{nit}(\boldsymbol{\beta}_\tau, \tau) \equiv \mathbf{g}_{nit}(Y_{it}, \mathbf{X}_{it}, \mathbf{Z}_{it}, \boldsymbol{\beta}_\tau, \tau) \equiv \mathbf{Z}_{it} \left[\tilde{I} \left(\frac{\Lambda\left(Y_{it}, \mathbf{X}_{it}, \boldsymbol{\beta}_\tau\right)}{h_n} \right) - \tau \right],$$
$$\mathbf{M}_n\left(\boldsymbol{\beta}_\tau, \tau\right) \equiv \frac{1}{n} \sum_{i=1}^{n} \mathbf{g}_{nit}(\boldsymbol{\beta}_\tau, \tau),$$

where h_n is a bandwidth (sequence), and $\tilde{I}(\cdot)$ is the smoothing function of the indicator function $\mathbb{1}\left\{\cdot \leq 0\right\}$. Thus, $\tilde{I}(\cdot)$ is an "indicator-like function" or an "integral of a kernel". The triple subscript on $\mathbf{g}_{nit}$ is a reminder that we have a triangular array setup because $\mathbf{g}_{nit}$ depends on h_n in addition to the observed random variables.

The population moment condition is then

$$\mathbf{M}_n(\boldsymbol{\beta}_{0\tau}, \tau) = \mathbf{0}. \tag{3.10}$$

Thanks to smoothing, Equation (3.10) always has a solution. With exact identification $(d_Z = d_\beta)$, the estimator solves the smoothed sample moment conditions

$$\hat{\mathbf{M}}_n(\hat{\boldsymbol{\beta}}_{0\tau}, \tau) = \mathbf{0}. \tag{3.11}$$

This is the method of moments (MM) estimator.

The smoothed sample analog of Equation (3.9) in Kaplan and Sun (2017) is originally from Whang (2006) (albeit only with $\mathbf{X} = \mathbf{Z}$):

$$\mathbf{0} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{Z}_{it} \left[\tilde{I} \left(\frac{\Lambda\left(Y_{it}, \mathbf{X}_{it}, \boldsymbol{\beta}_\tau\right)}{h_n} \right) - \tau \right], \tag{3.12}$$

[2]Horowitz (1998) suggested smoothing the quantile regression criterion function. We work with the Z-estimator directly.

where $h_n \to 0$ is a bandwidth and $\tilde{I}(\cdot)$ is the integral of a usual kernel function over $[-1, 1]$, i.e., $\tilde{I}(v) = 0$ for $v \le -1$, $\tilde{I}(v) = 1$ for $v \ge 1$, and $\tilde{I}(\cdot)$ smoothly transitions from zero to one over $v \in [-1, 1]$. The $\tilde{I}(\cdot)$ used by Horowitz (1998), Whang (2006), and in the code of Kaplan and Sun (2017) is the same,

$$\tilde{I}(u) = \mathbb{1}\{-1 \le u \le 1\}\left[0.5 + \frac{105}{64}\left(u - \frac{5}{3}u^3 + \frac{7}{5}u^5 - \frac{3}{7}u^7\right)\right] + \mathbb{1}\{u > 1\},$$

which is the integral of a fourth-order kernel.

Computationally, Equation (3.12) is easy as long as h_n is not too close to zero, and it can be solved using standard root-finding methods. Results in Kaplan and Sun (2017), such as Table 1, show that a small enough h_n to yield estimates nearly identical to the (unsmoothed) method in Chernozhukov and Hansen (2006) is still large enough to numerically solve Equation (3.12) easily.

Only the Z-estimator in Equation (3.12) is considered in Kaplan and Sun (2017), where $\mathbf{Z}$ and $\mathbf{X}$ have the same dimension. For computational purposes, this MM estimator was used. For overidentification, de Castro, Galvao, Kaplan and Liu (2018) also provide a GMM estimator.

3.3.3 Estimation Procedure

In order to estimate the EIS and discount factor, we follow a two-step procedure. First, we remove fixed effects by performing a separate mean fixed effects regression of each variable on month. This regression uses the within estimator and has an individual effect. Variables used in each regression include consumption growth, real interest rates, and the instruments. The two instruments used are the fitted values of consumption growth and real interest rates regressed on the second lags of consumption growth, nominal interest rate, and inflation rate. The residuals are kept from each fixed effects regression, so that Y_{it}, $\mathbf{X}_{it}$, and $\mathbf{Z}_{it}$ are free of fixed effects. Second, these residuals are applied to the smoothed instrumental variables estimator of de Castro, Galvao, Kaplan and Liu (2018) using the log-linearized version of the Euler equation in Equation (3.7).

The first step allows us to control for fixed effects. Due to the combination of a large dataset, MM estimation, and a lack of fixed effects methods in quantile regression, it is not feasible to estimate fixed effects at each quantile. Instead, we consider an expected fixed effect per household that is applied to create the residuals used in the second step. Based on exploration in Chapter 2, an individual effect is likely to be small anyway.

3.4 Data

There are three main advantages of using disaggregated consumption data from the Nielsen Consumer Panel. First, consumption does not have to be estimated as the data is not survey data. This minimizes measurement error in consumption, which is a common issue in studies of the EIS. Second, the data is transactional, so consumption can be aggregated into a short time period. This lessens the effects of aggregation bias, which often affects EIS estimates. Third, the data is a panel dataset, so it follows households over time. This allows us to remove unobserved time effects from the data. These advantages provide a better dataset to study the EIS than previous studies have used.

In order to estimate the EIS, we create a dataset that combines three sources of data. These sources include consumption data from the Nielsen Consumer Panel, interest rate data from the Federal Reserve of Economic Data (FRED), and CPI data from the Bureau of Labor Statistics (BLS). The time period of the final dataset will be determined by the Nielsen Consumer Panel, as this data covers the smallest time period. Refer to Chapter 2 for a larger discussion on the data.

Aggregation of consumption over time periods must be applied in order to create a dataset that represents consumer purchase timeframes. If consumption is aggregated over a smaller time period, it lessens aggregation bias. Note that the interest rate used is a four-week Treasury bill that is recorded on a weekly frequency. This means that intertemporal decisions are based on a four-week timespan. This paper takes timing into account by choosing a small aggregation time period but matching the timing of consumption growth with the maturity of the interest rates. The result is that consumption is aggregated weekly, but consumption growth is considered over four weeks. Thus, individuals make intertemporal decisions over a four week time period. Under this model, aggregate consumption remains a small time period, but the timing of the intertemporal decision between interest rates and consumption also matches. Rather than variables having an index of $t+1$ as in the previous notation of this chapter, variables actually have an index of $t+4$ and $\Delta c_{t+4} = \ln C_{t+4} - \ln C_t$, as in Chapter 2.

3.5 Results

The smoothed MM estimator of Equation (3.11) was applied to estimate the EIS over a range of quantiles of the condition distribution. This moment condition considers a log-linearized Euler equation that regresses real consumption growth on real interest rates.

Instruments used were the fitted values of consumption growth and real interest rates regressed on the second lags of consumption growth, nominal interest rate, and inflation rate. Mean individual and seasonal effects were applied to take advantage of the panel dataset. It was not computationally feasible to include individual effects in the quantile regression.

This application is similar to the application in de Castro, Galvao, Kaplan and Liu (2018), except we use a disaggregated micro dataset, the Nielsen Consumer Panel, instead of an aggregated macro dataset. The Nielsen Consumer Panel is advantageous, because it is a survey aggregated from transactional data, and it is a panel dataset. These features help to minimize measurement error and aggregation bias, and it allows for the removal of unobserved time effects. These advantages provide a better dataset to study the EIS than standard macro datasets. Results in this section are an extension of Chapter 2, where Chapter 2 estimated the EIS using expected utility and this chapter estimates a quantile EIS using quantile preferences.

The small number of papers that explore whether the EIS varies with the level of consumption (or wealth) seem to reject the constant EIS hypothesis (Blundell, Browning and Meghir (1994), Atkeson and Ogaki (1996), and Attanasio and Browning (1995)). By estimating the quantile EIS, we explore heterogeneity along the conditional distribution of the EIS.

Notice that there are two measures of riskiness in this model. First, for a fixed quantile τ, γ_τ captures the relative risk aversion, for which a larger γ_τ signifies a larger risk aversion. Second, the model also captures the risk across quantiles. The quantile utility model predicts that the agent that maximizes the larger quantile is more risk seeking; thus,

Table 3.1: Quantile Results

	Period 1		Period 2		Period 3	
Tau	EIS	SE	EIS	SE	EIS	SE
0.1	0.160	0.106	0.274***	0.082	2.038***	0.774
0.2	0.174***	0.070	0.150***	0.055	1.564***	0.518
0.3	0.081	0.052	0.099***	0.041	2.462***	0.406
0.4	-0.088**	0.041	0.083***	0.032	2.894***	0.345
0.5	-0.205***	0.084	0.112***	0.036	2.708***	0.389
0.6	-0.276***	0.042	0.145***	0.032	2.780***	0.333
0.7	-0.408***	0.052	0.117***	0.039	3.333***	0.398
0.8	-0.453***	0.070	0.127**	0.055	3.533***	0.499
0.9	-0.509***	0.099	-0.061	0.083	3.794***	0.727
Within	-0.170	0.050	0.129***	0.039	2.810***	0.356

*** $p < 0.01$, ** $p < 0.05$

Source: Calculated based on data from The Nielsen Company (US), LLC and marketing databases provided by the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business.

the theorem suggests that the coefficient of relative risk aversion should decrease over the quantiles, that is $\gamma_{\tau'} < \gamma_{\tau}$ for $\tau' > \tau$. If the coefficient of relative risk aversion should decrease over the quantiles, then the EIS should increase over the quantiles.

Estimates of the EIS and discount factor are shown in Table 3.1. These estimates are given by period as similar to Chapter 2. Descriptions of the time periods can be found in Table 2.2. The results of the within estimator from Chapter 2 are also provided for comparison. Figure 3.1 provides a graphical version of the results in Table 3.1

In Period 1, the United States was in an expansion with increasing real interest rates. The within estimate from Chapter 2 was negative, and from the 0.4 quantile and above, the quantile EIS is negative. As the quantile increases from 0.4, the absolute value of the EIS also increases. If riskiness is considered to be movement away from 0, then

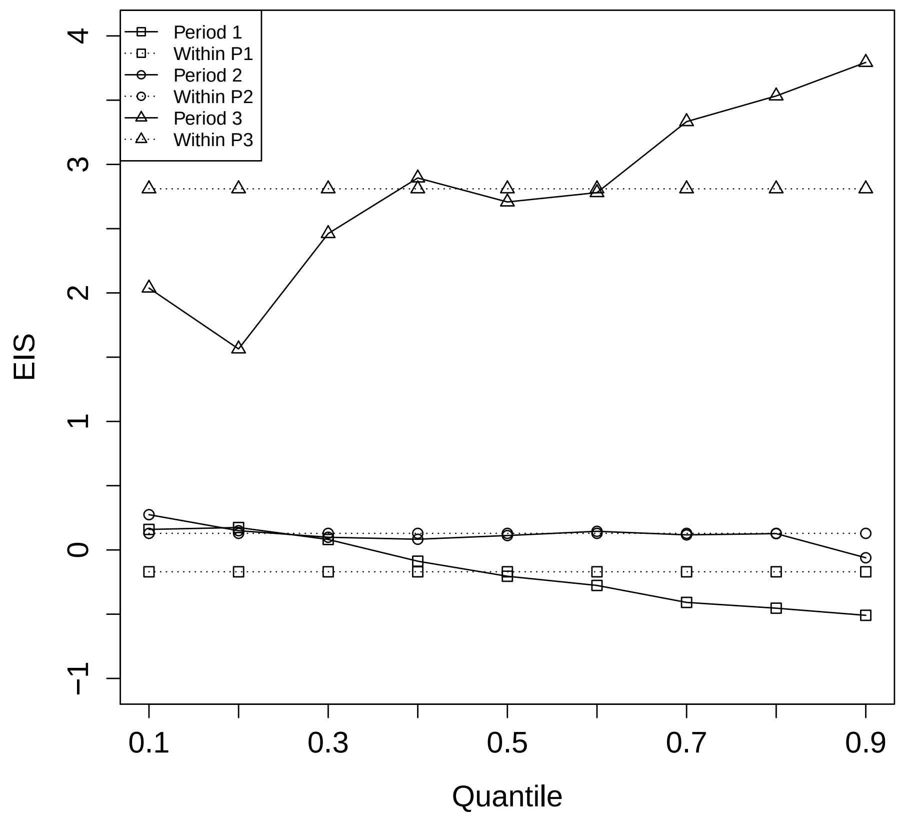

Figure 3.1: Quantile EIS

Source: Calculated based on data from The Nielsen Company (US), LLC and marketing databases provided by the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business.

this follows the theory that individuals are more risky as the quantile increases. Period 1 exhibits evidence of heterogeneity since the EIS has a general downward trend. Lastly, the mean within estimate is slightly larger (closer to 0) than the median estimate, but the within estimate is still closest to the median in Table 3.1

In Period 2, the United States was in a recession with decreasing real interest rates.

Except for the extreme quantiles of 0.1 and 0.9, the EIS was relatively flat. Hence, this period does not provide evidence of heterogeneity. The within estimate was closest to the 0.8 quantile estimate in Table 3.1, but it was relatively close to nearly all of the quantile estimates.

In Period 3, the United States was exhibiting quantitative easing with relatively stable real interest rates near zero. During this timeframe, the EIS was the largest, and the EIS is significantly positive across all quantiles. As seen in Figure 3.1, the quantile EIS has a general upward trend, which follows the theory that the EIS increases as the quantile increases. This period gives evidence of heterogeneity amongst households. Lastly, the within estimate is closest to the 0.4 quantile in Table 3.1.

Summarizing the results, we find that Periods 1 and 3 give evidence of heterogeneity, while Period 2 does not. The quantile estimates tend to follow the within estimates with the median estimate never far from the mean estimate. Also, Periods 1 and 2 have a low standard error in the estimates, but Period 3 has a larger standard error by a factor of 10.

3.6 Conclusion

This paper estimates the quantile elasticity of intertemporal substitution using disaggregated consumption data. Instead of standard expected utility, we assume quantile utility preferences developed by de Castro and Galvao (2018). A smoothed instrumental variables MM estimator from de Castro, Galvao, Kaplan and Liu (2018) is applied to the data in order to estimate the EIS for different quantiles along the conditional distribution. Consumption data was gathered from the Nielsen Consumer Panel, and interest rates were four-week Treasury bills. Estimates were given for three time periods: expansion in Period 1, recession in Period 2, and quantitative easing in Period 3.

Results provide evidence of heterogeneity for household EIS in Periods 1 and 3. If we fix the quantiles, results follow Chapter 2, and each of the median estimates are similar to the within estimates. In expansion, the income effect tends to dominate along the quantiles. As interest rates increase, households trade less consumption today for consumption tomorrow. However, in recession and quantitative easing, the substitution effect dominates. As interest rates increase, households trade more consumption today for consumption tomorrow.

These results can be used to inform the theory behind the EIS and quantile models of rational behavior. Because the theory does not capture the dynamics of the interest rate, Period 3 results best reinforce the theory. This is a period of quantitative easing with steady interest rates around 0. Interest rates are predictable, so consumers are more willing to trade consumption today for consumption tomorrow when they expect an increase in the real interest rate. This leads to a large positive EIS. Period 3 also shows that as the quantiles increase and households become more risky, the EIS also increases. When there is more fluctuation of the interest rates, such as in Periods 1 and 2, it is more difficult to capture the effect of real interest rates on consumption growth. In conslusion, we have reinforced the theory of dynamic quantile models by using micro data to estimate the quantile EIS and finding heterogeneity of households along the conditional distribution.

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