Turbulence in heliospheric plasmas: characterizing the energy cascade and mechanisms of dissipation

J. L. Verniero
University of Iowa

Follow this and additional works at: https://ir.uiowa.edu/etd

Part of the Applied Mathematics Commons

Copyright © 2019 J. L. Verniero

This dissertation is available at Iowa Research Online: https://ir.uiowa.edu/etd/6870

Recommended Citation

Follow this and additional works at: https://ir.uiowa.edu/etd

Part of the Applied Mathematics Commons
TURBULENCE IN HELIOSPHERIC PLASMAS: CHARACTERIZING THE ENERGY CASCADE AND MECHANISMS OF DISSIPATION

by

J. L. Verniero

A thesis submitted in partial fulfillment of the requirements for the Doctor of Philosophy degree in Applied Mathematical and Computational Sciences in the Graduate College of The University of Iowa

May 2019

Thesis Supervisor: Associate Professor Gregory G. Howes
ACKNOWLEDGEMENTS

This material is based upon work supported by the National Science Foundation Graduate Research Fellowship Program under Grant No. 1048957.

I acknowledge that the material in §2.1 (in addition to conclusions in §4.1) is from my original article,


I also acknowledge that the material in §2.2 (in addition to conclusions in §4.1) is from my original article,


Both of these reproduced works adhere to the following Copyright Notice:

The copyright in the material on these pages is owned by or licensed to Cambridge University Press, or reproduced with permission from other third-party copyright owners. It may be downloaded and printed for personal reference, but not otherwise copied, altered in any way or transmitted to others (unless explicitly stated otherwise) without the written permission of Cambridge University Press. Hypertext links to other web locations are for the convenience of users and do not constitute any endorsement or authorisation by Cambridge University Press.
ABSTRACT

In space and astrophysical plasmas, turbulence is responsible for transferring energy from large scales driven by violent events or instabilities, to smaller scales where turbulent energy is ultimately converted into plasma heat by dissipative mechanisms. In the inertial range, the self-similar turbulent energy cascade to smaller spatial scales is driven by the nonlinear interaction between counterpropagating Alfvén waves, denoted Alfvén wave collisions. For the more realistic case of the collision between two initially separated Alfvén wavepackets (rather than previous idealized, periodic cases), we use a nonlinear gyrokinetic simulation code, AstroGK, to demonstrate three key properties of strong Alfvén wave collisions: they (i) facilitate the perpendicular cascade of energy and (ii) generate current sheets self-consistently, and (iii) the modes mediating the nonlinear interaction are simply Alfvén waves. Once the turbulent cascade reaches the ion gyroradius scale, the Alfvén waves become dispersive and the turbulent energy starts to dissipate, energizing the particles via wave-particle interactions with eventual dissipation into plasma heat. The novel Field-Particle Correlation technique determines how turbulent energy dissipates into plasma heat by identifying which particles in velocity-space experience a net gain of energy. By utilizing knowledge of discrete particle arrival times, we devise a new algorithm called PATCH (Particle Arrival Time Correlation for Heliophysics) for implementing a field-particle correlator onboard spacecraft. Using AstroGK, we create synthetic spacecraft data mapped to realistic phase-space resolutions of modern
spacecraft instruments. We then utilize Poisson statistics to determine the threshold number of particle counts needed to resolve the velocity-space signature of ion Landau damping using the PATCH algorithm.
When cream is mixed into coffee, only one or two stirs is required before the cream is evenly mixed. This process is irreversible, meaning the cream cannot be unmixed from the coffee. On the other hand, if one mixes two colors of paint together, it takes quite a few stirs before mixing can occur, and the resultant process is reversible. What is the difference between coffee and paint? Coffee is less viscous than paint, so the force induced by one stir of cream is enough to overcome the force of the viscosity. The fluid consequently becomes turbulent, which unleashes a cascade of ever smaller stirs, until the stirs eventually dissipate. In other words, the energy from that initial stir of cream successively passes on to smaller scales, until that energy converts to heat, which is a statistically irreversible process. One the other hand, a strong viscous medium, such as paint, would not undergo this phenomenon. In coffee, the process that transferred energy to smaller spatial scales was mediated by the interaction between individual stirs. In an astrophysical setting, an analogous process occurs, but instead, the energy is transferred between fluctuations of magnetic field lines. This thesis investigates the nature of how this turbulent energy is transferred to smaller scales and where that energy goes once the turbulent energy is dissipated. In particular, a method is devised for identifying how turbulent energy dissipates in space, based on counting particles. The method aims towards implementation onboard spacecraft as a form of a data compression technique, since the space-Earth internet is insufficient to download all of the information a spacecraft can collect.
1 INTRODUCTION AND BACKGROUND .......................................... 1
  1.1 Hydrodynamic Turbulence .................................................. 3
  1.2 Turbulence in Our Home in the Universe ............................. 8
  1.3 Magnetohydrodynamic Turbulence in the Inertial Range .......... 11
    1.3.1 MHD Background ..................................................... 11
    1.3.1.1 Ideal MHD in Symmetrized Elsasser Form ................ 12
    1.3.1.2 Reduced MHD .................................................... 14
    1.3.2 Early Competing Theories of Weak MHD Turbulence in the Inertial Range ........................................... 15
    1.3.3 Strong MHD Turbulence ............................................ 19
  1.4 Astrophysical Plasma Turbulence in the Dissipation Range ...... 22
    1.4.1 Current Sheets ...................................................... 25
    1.4.2 Astrophysical Turbulence in the Gyrokinetic Limit .......... 35
      1.4.2.1 Perturbation Theory Background .......................... 35
      1.4.2.2 The gyrokinetic Approximation ............................ 37
      1.4.2.3 Qualitative Energy Transfer in the Weakly Nonlinear Limit ........................................ 40
    1.4.3 Kinetic Dissipation Mechanisms .................................. 42
    1.4.4 Observational Constraints ........................................ 51
  1.5 Outline of Dissertation Goals .......................................... 53

2 THE FUNDAMENTAL BUILDING BLOCK OF PLASMA TURBULENCE .......... 55
  2.1 Part I: Alfvén Wave Collisions in the Strongly Nonlinear Limit ... 56
    2.1.1 Overview of Previous Work ....................................... 56
    2.1.2 Simulation .......................................................... 59
    2.1.3 Evolution of the Nonlinear Interaction ........................ 63
      2.1.3.1 The Perpendicular Cascade of Energy .................. 65
      2.1.3.2 Current Sheet Development .................................. 67
      2.1.3.3 Evolution of Energy ......................................... 72
      2.1.3.4 Visualizing Wavepacket Collisions ....................... 74
  2.2 Part II: The Alfvénic Nature of Energy Transfer Mediation ......... 76
2.2.1 Overview of Further Investigation of Energy Transfer Dynamics ........................................... 76
2.2.2 Case Study Setup .................................................................................................................. 79
2.2.3 Evolution of Energy of Secondary (1,1) Mode ....................................................................... 82
2.2.4 Identification of Nonlinearly Generated Modes as Alfvén Waves ........................................... 86
  2.2.4.1 Alfvén Wave Eigenfunction Relation .............................................................................. 87
  2.2.4.2 Alfvén Wave Dispersion Relation ................................................................................... 93
2.2.5 Strong vs. Weak Turbulence .................................................................................................. 96
2.2.6 Current Sheet Development Reconfirmed .............................................................................. 98

3 DIAGNOSING TURBULENT DISSIPATION MECHANISMS
ONBOARD MODERN SPACECRAFT .......................................................................................... 101

  3.1 Motivation ................................................................................................................................ 101
  3.2 History of Wave-Particle Correlators ......................................................................................... 103
  3.3 The Field-Particle Correlation Technique: Background .............................................................. 116
    3.3.1 Comparison to Past Wave-Particle Interaction Measurement Methods ................................ 116
    3.3.2 Derivation Outline ............................................................................................................... 118
    3.3.3 Observational and Theoretical Work Supporting the FPC Method ........................................ 121
  3.4 Determining Limitations of Instrumental Phase-Space Resolution ............................................ 123
    3.4.1 Simulation Parameters .......................................................................................................... 123
    3.4.2 Synthetic Spacecraft Data Creation ....................................................................................... 124
    3.4.3 Velocity-Space Resolution Results ....................................................................................... 128
  3.5 Electrostatic Analyzer Background ............................................................................................. 131
    3.5.1 Top-Hat Design ..................................................................................................................... 132
    3.5.2 Geometric Factor ................................................................................................................... 136
  3.6 New Theory for Onboard Spacecraft Measurements ................................................................... 140
    3.6.1 Demonstration of the PATCH Algorithm .............................................................................. 141
    3.6.2 The Klimontovich-Dupree Equation ...................................................................................... 144
    3.6.3 The Ensemble Average ........................................................................................................... 147
    3.6.4 Redefining the FPC Technique Using Particle Phase-Space Density ...................................... 149
      3.6.4.1 Onboard Correlation Derivation Using Kinetic Theory ..................................................... 150
    3.6.5 Rate of Work Done on the Particles ....................................................................................... 156
  3.7 Testing PATCH with Synthetic Spacecraft Data ....................................................................... 158
    3.7.1 PATCH Implementation Procedure ....................................................................................... 158
    3.7.2 Results Using PATCH ......................................................................................................... 161
    3.7.3 Noise Model .......................................................................................................................... 164

4 SUMMARY AND FUTURE WORK ............................................................................................. 174
LIST OF TABLES

Table

A.1  Convergence of $\alpha$ with increasing number of particle counts. . . . . . . . 215
LIST OF FIGURES

Figure

1.1 Leonardo da Vinci’s drawing of a turbulent flow in 1507. .................. 1
1.2 Cascade à la Richardson. ................................................................. 4
1.3 Kolmogorov scaling in the inertial range. ................................. 7
1.4 Artist’s depiction of Sun-Earth interaction (Credit: NASA). ........... 9
1.5 Turbulent cascade of energy in the near-Earth solar wind. ............ 10
1.6 Schematic of the magnetic energy wavenumber spectrum in the solar wind, showing the form of the spectrum in the energy containing, inertial and dissipation ranges. Scale ranges for reduced MHD (green) and gyrokinetic (red) simulations are depicted, along with typical Larmor radius scales for protons (magenta) and electrons (blue) from Solar Energetic Particle (SEP) events. ................................................................. 24
1.7 Visualization of gyrokinetics as the evolution of ring averages of charged particles moving along magnetic fields. .............................. 38
1.8 Schematic of the initial conditions specifying two perpendicularly polarized, counterpropagating Alfvén waves overlapping within a periodic domain (Howes and Nielson, 2013). ............................................. 40
1.9 Schematic diagram of the Fourier modes in the \((k_x, k_y)\) perpendicular plane arising in the asymptotic solution. The Fourier modes depicted are the primary \(O(\epsilon)\) modes (circles), secondary \(O(\epsilon^2)\) modes (triangles), and tertiary \(O(\epsilon^3)\) modes (squares). Filled symbols denote the key Fourier modes that play a role in the secular transfer of energy to small scales in the Alfvén wave collision. The parallel wavenumber \(k_z\) for each of the modes is indicated by the diagonal grey lines, a consequence of the resonance conditions for the wavevector (Howes and Nielson, 2013). .................... 41
2.1 Schematic of the initial conditions specifying the two perpendicularly polarized, counterpropagating Alfvén wavepackets localized within the periodic domain. Plotted is the $z$-dependence of the normalized amplitudes of the perpendicular magnetic field perturbation $(\delta B_y/B_0)(a_0/\rho_0)$ for the unipolar wavepacket (red) and of the perpendicular magnetic field perturbation $(\delta B_x/B_0)(a_0/\rho_0)$ for the dipolar wavepacket (blue). The unipolar wavepacket has a perpendicular wavevector $\mathbf{k}_-^\perp = (k_x L_\perp, k_y L_\perp) = (1,0)$ and the dipolar wavepacket has $\mathbf{k}_+^\perp = (0,1)$.

2.2 Three-dimensional isocontours of the normalized parallel current density $j_z/j_0$ between Alfvén wavepacket collisions at (a) $t/T_c = 0$, (c) $t/T_c = 1$, and (e) $t/T_c = 2$ and at the midpoint of collisions at (b) $t/T_c = 0.5$, (d) $t/T_c = 1.5$, and (f) $t/T_c = 2.5$.

2.3 Plots of the perpendicular magnetic energy $E_{B_\perp}(k_x, k_y)$ (arbitrary units) on a log scale in the perpendicular Fourier plane $(k_x, k_y)$ (a) at the initial time $t/T_c = 0$, (b) after the first strong Alfvén wavepacket collision at $t/T_c = 1$, and (c) after the second strong Alfvén wavepacket collision at $t/T_c = 2$.

2.4 Plot of the normalized parallel current density $j_z/j_0$ at $z = 0$ during the first collision at (a) $t/T_c = 0.3$ and (b) $t/T_c = 0.5$, at $z = L_z/2$ during second collision at (c) $t/T_c = 1.3$ and (d) $t/T_c = 1.5$, and at $z = 0$ during the third collision at (a) $t/T_c = 2.3$ and (b) $t/T_c = 2.5$.

2.5 Plot of the normalized parallel current density $j_z/j_0$ of colliding Alfvén wavepackets before the first collision at $t = 0$ for (a) the unipolar wave at $z = -L_z/4$ and (b) the dipolar wave at $z = +L_z/4$, after the first collision when the wavepackets have separated for (c) the unipolar wave at $z = +L_z/4$ and $t/T_c = 0.98$ and (d) the dipolar wave at $z = -L_z/4$ and $t/T_c = 0.96$, and after the second collision for (e) the unipolar wave at $z = -L_z/4$ and $t/T_c = 2.1$ and (f) the dipolar wave at $z = +L_z/4$ and $t/T_c = 2.0$.

2.6 Plot of the evolution of energy transfer between $(k_x, k_y)$ modes. Note that the $(1,0)$ mode is plotted as the same line as the cyan (-1,0) mode, indicating they are identical.

2.7 Setup for perpendicularly polarized Alfvén waves in the localized and periodic cases. Note that the blue curve corresponds to the $(k_x, k_y) = (1,0)$ mode and the red curve corresponds to the $(k_x, k_y) = (0,1)$ mode. Note that the blue and red fluctuations are polarized perpendicularly to each other, with $\delta B_x$ (red) and $\delta B_y$ (blue).
2.8 Energy evolution of each case for key \((k_x, k_y)\) modes after 3 collisions, for the (a) localized, strongly nonlinear case LS, (b) periodic, strongly nonlinear case PS, (c) localized, weakly nonlinear case LW, and (d) periodic, weakly nonlinear case PW.

2.9 Full energy evolution of each case for key \((k_x, k_y)\) modes, for the (a) localized, strongly nonlinear case LS, (b) periodic, strongly nonlinear case PS, (c) localized, weakly nonlinear case LW, and (d) periodic, weakly nonlinear case PW.

2.10 Snapshots in time of \(\hat{B}_x\) (blue, dotted), \(\hat{E}_y\) (green, dashed), \(\hat{B}_y\) (red, dotted), and \(\hat{E}_x\) (black, dashed) of select \((k_x, k_y)\) Fourier modes in the LS case. The first, second, and third row corresponds to the primary, secondary, and tertiary modes respectively. All times are normalized to the localized Alfvén collision time, \(T_{c}^{(l)}\). The black arrows indicate the direction of motion of the two colliding wavepackets.

2.11 Snapshots in time of perpendicular Elsasser field components \(z^+_y\) (red, dotted), \(z^-_x\) (green, dashed), \(z^-_y\) (blue, dotted), and \(z^+_x\) (black, dashed) of key \((k_x, k_y)\) modes in the LS case. The first, second, and third row corresponds to the primary, secondary, and tertiary modes respectively. All times are normalized to the localized Alfvén collision time, \(T_{c}^{(l)}\). The black arrows indicate the direction of motion of the two colliding wavepackets.

2.12 Snapshots in time of \(\delta B_{\perp}\) vs. \(z\) of primary modes \((1,0)\) and \((0,1)\) overlapping the secondary mode \((1,1)\) in the LS case. All times are normalized to the localized Alfvén collision time, \(T_{c}^{(l)}\).

2.13 Current sheet formation before and after each collision of case LS.

3.1 (a) The gyrotrropic complementary distribution function \(g_p(v_\parallel, v_\perp)\) at a single point in the \(\beta_p = 1.0\) turbulent simulation, as well as the correlations (b) \(C_{E_\parallel}^{\parallel}(\tau = 0)\) and (c) \(C_{E_\parallel}^{\parallel}(\tau \omega_A = 10.4)\) at time \(t \omega_A = 14.1\). The resonant parallel velocity associated with the maximum proton damping rate is shown as a solid grey vertical line. Note that this figure was adopted from Fig. 6 of Klein et al. (2017).

3.2 Demonstration of synthetic spacecraft data creation using (a) high-resolution \(\delta f_i\) from AstroGK, (b) reconstructed to \(f_i\), (c) transformed to plasma frame-of-reference, (d) down-sampled to PSP resolution (as observed by spacecraft), and (e) transformed back to \((v_\parallel, v_\perp)\)-space for comparison to (b).
3.3 Field-Particle Correlations in \((v_\parallel, v_\perp)\)-space, showing ion Landau damping velocity-space signature in (a) high-resolution AstroGK, comparing to (b) PSP SWEAP resolution \((\Delta E / E \times \Delta \phi = 7\% \times 3.75)\).  

3.4 Realistic instrumental resolutions for various spacecraft missions (b)-(f) to resolve ion Landau damping velocity-space signature, comparing to (a) AstroGK.  

3.5 Physical picture of “top-hat” ESA design, including a side view (left) of the electron particle trajectory (red) crossing the symmetry axis (dotted black line). The Field of View (FOV) spins 360\(^\circ\) around this axis, which also defines the area of the aperture opening. The cutaway view (right) shows the two nested hemispheres responsible for the net voltage difference.  

3.6 Sketch of aperture on the ESA top-hat detector.  

3.7 2D representation of the PATCH procedure in a single 6D phase-space bin \(\Delta r_p, \Delta v_p\). The blue curve, \(N(t)\), represents the number density of particles in this bin. The green vertical lines indicate particles arriving with probabilities dictated by \(N(t)\). The red curve is the electric field, \(E(t)\), with open green circles indicating that the electric field is evaluated at the particle time of arrival. The summation of these values with appropriate normalization yields the correlation, \(C^*_r\), in this bin.  

3.8 Testing the ability of the PATCH algorithm to recover the velocity-space signature of Landau damping, using the normalized correlation \(C^*_\text{norm}\) (given by Eq. (3.81)) in PSP resolution for (a) 15, (b) 16, (c) 25, (d) 39, (e) 50, (f) 75 and (g) 100 particles counted per phase-space bin per sampling time. Comparison to using \(C'(f_{PSP})\) instead of the PATCH method is represented again in panel (h).  

3.9 Noise model power law break in first point \((x, y, z) = (22, 5, 0)\).  

3.10 Noise model power law break in second point \((x, y, z) = (22, 11, -7)\).  

3.11 Noise model power law break in third point \((x, y, z) = (22, 8, 0)\).  

3.12 Noise model power law break in fourth point \((x, y, z) = (22, 1, 0)\).  

4.1 SWEAP/FIELDS interface on PSP.  

4.2 Laboratory calibration of PSP correlator instrumentation at UC Berkeley Space Sciences Laboratory.
A.1 Sketch of Poisson distribution derived from Eq. (A.1), representing the probability density versus time between particle counts. 200

A.2 Representation of discrete particle counts arriving according to the probability distribution $P(t)$. 201

A.3 Binomial approximation with spacing $\Delta t$ in an interval $[0, T]$. 201

A.4 Contour plot of Standard Deviation of the Percent Error for a statistical ensemble of 512 and relative phase shift $\delta = 0.2\pi$. 210

A.5 Over-plot of Standard Deviation of the Percent Error for phase shift $\delta = 0.2\pi, 0.45\pi, 0$ represented by solid, dashed and dotted curves, respectively. 211

A.6 Standard Deviation as a function of $N_0$ for all $\delta N/N_0$. 212

A.7 Model curves C (dashed) and $(\sigma/\mu)(100)$ (solid). 212

A.8 Model curves SNR (dashed) and $\sigma/\mu$ (solid). 213

B.1 Sketch of typical modern electrostatic analyzer “top-hat” design. 217

B.2 Sketch of a data processing unit (DPU) onboard spacecraft receiving time-delayed information from both an electrostatic analyzer and a waves instrument. 219
CHAPTER 1
INTRODUCTION AND BACKGROUND

Figure 1.1: Leonardo da Vinci’s drawing of a turbulent flow in 1507.

“Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to the random and reverse motion,” said Leonardo da Vinci in 1507, referring to their drawing depicted in Fig. 1.1. Leonardo da Vinci called this phenomenon “la turbolenza” and asked the following three questions:
1. *Dove la turbolenza dell’acqua si genera?*  
(Where is turbulence in water generated?)

2. *Dove la turbolenza dell’acqua si mantiene a lungo?*  
(Where does turbulence in water persist for long times?)

3. *Dove la turbolenza dell’acqua si riposa?*  
(Where does turbulence in water come to rest?)

Within the context of turbulence in an astrophysical plasma (rather than water), this thesis will address the latter two questions, posed more modernly as:

1. **What is the nature of the turbulent cascade of energy to smaller perpendicular spatial scales?**

2. **What is the nature of the dissipation mechanisms of turbulent energy that eventually lead to plasma heat?**

As we shall discover, Leonardo da Vinci’s observations from their crafted visualization of *la turbolenza* are still relevant today.¹

Plasma turbulence can best be perceived by first comparing it to hydrodynamic turbulence in §1.1. §1.2 provides context for studying plasma turbulence in the heliosphere. In §1.3, I introduce plasma turbulence in the magnetohydrodynamic (MHD) setting along with several competing theories. §1.4 outlines the theoretical and observational work done to understand the mechanisms for the dissipation of

¹More details can be found by consulting Ecke (2005).
plasma turbulence. Although this literature review appears extensive, the details are 
necessary to appreciate the nuances of the emerging field of plasma turbulence, which 
is a currently active area of research in heliophysics. A reader who is interested in 
starting research in plasma turbulence is encouraged to read every section. However, to 
respect the reader’s time, I have indicated several places where one is free 
to “choose their own adventure,” and skip sections without losing sight of the main 
goals of this dissertation outlined in §1.5.

1.1 Hydrodynamic Turbulence

The study of turbulence started in a hydrodynamic setting, such as water or 
air. In the 19th century, the first mathematical description of a fluid flow, \( u \), with 
constant density, \( \rho \), and viscosity, \( \nu \), appeared through the Navier-Stokes equations:

\[
\rho \left( \frac{\partial u}{\partial t} + u \cdot \nabla u \right) = -\nabla p + \nu \nabla^2 u \tag{1.1}
\]

\[
\nabla \cdot u = 0 \tag{1.2}
\]

where \( p \) is the pressure and the incompressibility of the flow, \( u \), is represented by Eq 
1.2. Note that Eq. 1.1 is simply a force-balance equation using Newton’s first law,

\[
F = ma
\]

since the right-hand side of Eq. 1.1 is the sum of the forces (pressure + viscous 
dissipation) on the fluid, and the left-hand side represents the convective derivative 
of the flow (which is the velocity of the fluid), so the rate of change yields acceleration. 
The nonlinear term in Eq. 1.1, \( u \cdot \nabla u \), dictates how the turbulence transports kinetic
energy to smaller length scales, which we refer to as the *turbulent cascade*. The linear term, $\nu \nabla^2 u$, governs the dissipation of the turbulent energy cascade.

Reynolds (1895) found a way to characterize the nature of the turbulence in a fluid flow by evaluating the ratio of the nonlinear term (inertial forces) to the linear term (viscous forces). For a fluid with characteristic length-scale, $L$, and velocity, $u$, they found

$$Re = \frac{|u \cdot \nabla u|}{|\nu \nabla^2 u|} \sim \frac{uL}{\nu}$$

which is commonly referred to as the Reynolds Number. When $Re > 100$, this means that the nonlinear term is dominant enough over the linear, viscous term, enabling the energy to be transferred from large to small scales along the turbulent cascade.

Figure 1.2: Cascade à la Richardson.
The concept of a cascade of energy to smaller scales was first introduced by Richardson and Shaw (1920) and is depicted in Fig. 1.2. In a book about weather prediction, Richardson (1922) remarked that, “big whorls have little whorls that feed their velocity, and little whorls have lesser whorls and so on to viscosity.” At the onset of a turbulent cascade, there exists an eddy of diameter $L$, which is swirling with velocity, $v$. In the direction of the turbulent energy flux, this large-scale eddy of diameter $L$ then breaks into two eddies, each of diameter $L/2$, which in turn breaks into four eddies, each of diameter $L/4$, and so on, until the eddies eventually dissipate and so does the turbulence. Each of these individual eddies mediate the energy transfer in the turbulent cascade (at least, in the hydrodynamic case).

In 1941, this general concept of a turbulent energy cascade was made mathematically rigorous by Andrey Kolmogorov, who was able to quantify how the energy scales as a function of the turbulent fluctuations in (wavenumber) $k$-space. The idea that an eddy of diameter $L$ transferred its energy to 2 eddies of length $L/2$ (rather than say 100 eddies of length $L/100$) means that the transfer of energy is dominated between eddies of the same scale-length. This hypothesis of local energy transfer, in

\footnote{In 1936, Andrey Kolmogorov and Pavel Alexandrov (another Soviet mathematician who made significant contributions to topology) participated in the political persecution of their same PhD thesis advisor, Nikolai Luzin. Evidence suggests their involvement was forced due to threats by police of publicly announcing their homosexual love affair. They both agreed to testify in the “Luzin Affair,” supporting claims that Luzin consistently plagiarized the work of others in foreign academic journals. Luzin allegedly “felt no shame in declaring the discoveries of his students to be his own achievements,” but ultimately it was the disloyalty to the Soviets that declared him convicted. Curiously enough, his punishment was less severe than others of the same time, but there is speculation that Stalin was more concerned with other mathematicians being condemned as part of the counter-revolutionary group within the Moscow Mathematical Society.}
addition to assuming that the energy cascade rate was constant, led to the derivation
of the famous "5/3" scaling in wavenumber space (Kolmogorov, 1941). Moreover,
they assumed that the turbulent energy cascade was isotropic, meaning that the
energy did not preferentially cascade in a particular direction in k-space.

Using dimensional analysis, the scaling can be derived as follows. Assume an
eddy of length, l, and velocity, v, then the time it takes for an eddy to rotate 2\pi
(turnaround time) is \( \tau \sim l/v \). Since wavenumber, k, has units of 1/l and frequency,
\( \omega \), has units of \( kv \), the frequency of energy transfer from \( k \) to \( 2k \) then has units of
\( kv_k \), where \( v_k = v(k) \). Note that transferring energy to a higher frequency translates
to a smaller spatial scale. Let \( \epsilon \) be the constant energy transfer rate between eddies.

Then
\[
\epsilon \sim \frac{\text{energy}}{\text{time}} \sim (\text{energy})(\text{frequency}) \sim v_k^2(kv_k) \sim kv_k^3
\]

Solving for \( v \) as a function of \( k \), we get \( v_k \sim \epsilon^{1/3}k^{1/3} \). For a 1-d energy spectrum,

\[
E_k = E(k) \sim \frac{\text{Energy}}{k} \sim \frac{v_k^2}{k} \sim \epsilon^{2/3}k^{-5/3}
\]

and since \( \epsilon \) is a constant, this shows \( E_k \propto k^p \) for \( p = -5/3 \), which we will refer to as
the energy spectrum. Fig. 1.3 shows a log-log plot of the energy, \( E_k \), as a function of
wavenumber, \( k \). This region of the turbulent cascade, located right after it is driven
at the scale, \( L \), and before the onset of dissipation at the viscous scale, \( lv \), is called the
inertial range. The fact that there is a constant \(-5/3\) slope implies that the energy
cascades self-similarly, statistically independent from the macroscopic physics above
the driving scale and the microscopic physics below the viscous scale.\footnote{The original self-similarity hypotheses was based on Richardson’s concept of turbulence at all possible scales between some outer, $L$, and inner, $l$, length-scale and an energy transfer from coarser-scale eddies to finer-scale eddies. However, Landau soon noticed that a consequence of the random nature of energy transfer from coarse to fine eddies is that increasing the ratio $L : l$ leads to an unlimited increase in the variation of the energy dissipation, $\epsilon$. Furthermore, there exists asymptotic behavior for the dispersion of $\log(\epsilon)$ (Kolmogorov, 1962).}

The slope, $p$, at which this happens in the case of a conducting fluid in the presence of a magnetic field, was independently discovered by Iroshnikov (1963) and Kraichnan (1965), known as the Iroshnikov-Kraichnan (IK) theory of magnetohydrodynamic (MHD) turbulence. I will shift the discussion to adapting the same ideas from hydrodynamic turbulence to the MHD setting after first providing brief situational awareness of heliospheric plasma.

Figure 1.3: Kolmogorov scaling in the inertial range.
1.2 Turbulence in Our Home in the Universe

Before diving into MHD theory, I will first provide context for plasma turbulence the heliosphere, our home in the universe. For the sake of simplicity, we will define a plasma in this thesis as a conducting fluid whose dynamics is dominated by long-range electromagnetic forces. A plasma is a state of matter that does not exist for very long on Earth – it becomes unstable quickly in our atmosphere, which is why it can only be studied in a laboratory or astrophysical setting. In fact, most of plasma fusion research is dedicated to finding out how to contain a plasma within our atmosphere and subsequently harness energy from the confined plasma that could be a nearly infinite power source for the Earth. The subject of this thesis however does not attempt to address this important question, but rather to understand the plasmas that exist naturally in space, such as stars, stellar winds and astrophysical jets. We focus the work of this thesis is the environment of the sun and its solar wind that engulfs our solar system, also known as the heliosphere. Space plasma turbulence is thus an area of research that attempts to describe the complex dynamics occurring in our home in the universe.

The heliosphere is essentially the sphere of influence that the sun has on the celestial bodies around it, as depicted in Fig. 1.4. The sun is constantly blowing a stream of charged particles at the Earth, called the solar wind (dashed white lines), which defines this sphere of influence. The charged particles that comprise the solar

---

4 Much more rigid definitions of a plasma exist, but it is not essential for the reader to know these technicalities to understand the purpose of this thesis.
wind follow magnetic field lines originating from the sun that extend throughout the heliosphere, called the interplanetary magnetic field. We are protected from the harmful radiation of charged particles in the solar wind by the Earth’s magnetosphere (purple).

Turbulence plays an important role in facilitating particle energization and plasma heating in space and astrophysical plasmas, influencing the macroscopic evolution of many poorly understood systems, such as the solar corona (the sun’s atmosphere) and solar wind, planetary magnetospheres, and black hole accretion disks. The turbulent cascade mediates the transfer of energy from magnetic fields and plasma flows at large scales down to much smaller scales where dissipation mechanisms can effectively remove energy from the turbulent fluctuations, ultimately converting that energy to plasma heat. The sun intermittently launches a piece of itself towards the
Earth, called a Coronal Mass Ejection (CME), which can sometimes penetrate the Earth’s magnetosphere and cause damage such as massive blackouts. CMEs, interplanetary shocks, and other violent events or instabilities, are examples of circumstances that could initialize a large stir of energy causing the turbulence phenomenon to permeate within the solar wind. The dissipation of the energy cascade could then energize particles to harmful levels that can fry electronics, such as GPS satellites. As society is increasingly dependent on these space-based assets, characterizing the nature of turbulence in the solar wind is increasingly relevant.

![Figure 1.5: Turbulent cascade of energy in the near-Earth solar wind.](image)

As seen in Fig. 1.5, the same picture of a hydrodynamic energy cascade, shown in Fig. 1.3, exists in the case of plasma turbulence in space. The self-similar -5/3 scaling region is also called the inertial range, and the region after the onset of dissipation, usually at the length-scale of an ion, is called the dissipation range (see §1.4). The ultimate goal of plasma turbulence is to understand the energy dynamics of the entire cascade depicted in Fig. 1.5. Understanding the details of the nonlinear turbulent
cascade to small scales and of the mechanisms by which the turbulent energy is thermalized represents a grand challenge problem in heliophysics and astrophysics. The aim of this dissertation is to provide one more piece that can ultimately be used to complete\textsuperscript{5} this jigsaw puzzle embedded in the universe.

1.3 Magnetohydrodynamic Turbulence in the Inertial Range

In this section, I will review the first studies of MHD turbulence that led to the modern research area of plasma turbulence, which has a home in both the fusion and astrophysical communities. Background material about MHD is first provided in §1.3.1, before narrating the rather Byzantine history of both weak and strong MHD turbulence theories in §1.3.2 and §1.3.3, respectively.

1.3.1 MHD Background

Turbulence is characterized by an injection of energy that nonlinearly cascades to smaller scales. Hydrodynamic turbulent fluids flow in mediums such as air or water. In plasma turbulence, however, the magnetic field plays an important role, fundamentally changing the nature of the turbulence from the hydrodynamic case (Howes, 2015b). The fundamental difference between hydrodynamic and plasma turbulence is the existence of magnetic tension, which is a restoring force that leads to the propagation of Alfvén waves (Alfvén, 1942). In the ideal MHD regime, the energy

\textsuperscript{5}The notion of a \textit{complete} description of turbulence cannot actually exist due to the inherently nonlinear and stochastic nature of turbulence. Mathematicians speculate that this is because the existence and uniqueness theorem does not hold for the Navier-Stokes equations. Turbulence can thus be alternatively defined as this mathematical consequence of the “unsolvability” of these equations.
cascade is mediated by Alfvén waves, which behave like waves on a string after being
plucked by some outside perturbation, $\delta \mathbf{B}$, that travels along the magnetic field in
the form of an Alfvén wave, with Alfvén velocity, $v_A$.

1.3.1.1 Ideal MHD in Symmetrized Elsasser Form

The ideal MHD equations can be cast into a formulation that describes the
dynamics in terms of the time evolution of Alfvén waves traveling up or down the
magnetic field. This set of incompressible MHD equations in symmetrized Elsasser
form is:

$$\frac{\delta z^\pm}{\delta t} \mp v_A \cdot \nabla z^\pm = -\nabla^\perp \cdot \nabla z^\pm - \nabla P/\rho_0$$

$$\nabla \cdot z^\pm = 0$$

where $\mathbf{B} = B_0 + \delta \mathbf{B}$, and $v_A = B_0/\sqrt{4\pi \rho_0}$ is the Alfvén velocity due to $B_0 = B_0 \hat{z}$.

The variables, $z^\pm(x, y, z, t) = u \pm \delta \mathbf{B}/\sqrt{4\pi \rho_0}$, are the Elsasser fields given by velocity
fluctuation, $u$ and mass density, $\rho_0$. Physically, the Elsasser fields, $z^\pm$, represent
Alfvén waves traveling up (or down) the uniform background magnetic field, $B_0$. The
linear term, $v_A \cdot \nabla z^\pm$, represents the linear propagation of the Elsasser fields along
the mean magnetic field with Alfvén speed, $v_A$. A nonzero linear term requires that
$k_\parallel \neq 0$, so a parallel dimension to the Elsasser field is required. The nonlinear term,
$-\nabla^\perp \cdot \nabla z^\pm$, corresponds to nonlinear counterpropagating Alfvén waves. This term
is nonzero if and only if there exists a 2D plane perpendicular to the Elsasser field.
Hence, plasma turbulence is inherently 3D (Howes, 2015a). Finally, the last term,
$\nabla P/\rho_0$, accounts for the incompressibility of the plasma, where $P$ represents the total
pressure. Since only nonlinear physics are responsible for turbulence, the structure of these equations show that counterpropagating Alfvén wave collisions are the building blocks of plasma turbulence (Iroshnikov, 1963; Kraichnan, 1965).

As shown by many numerical simulations (Maron and Goldreich, 2001), MHD turbulence is *anisotropic*, which means that power preferentially cascades in the perpendicular direction, i.e. $k \perp \gg k \parallel$. Let

$$
\chi \equiv \frac{|z^\pm \cdot \nabla z^\pm|}{|v_A \cdot \nabla z^\pm|} \sim \frac{k_{\perp} v_{\perp}}{k_{\parallel} v_A}
$$

be the nonlinear parameter, i.e. the ratio of the nonlinear term to the linear term. Note the similarity between $\chi$ in plasma turbulence and the Reynold’s number in hydrodynamic turbulence; it characterizes the turbulence in an analogous way, as in they are both a measure of the ratio between the nonlinear and linear terms. The main difference is that in the MHD case, the linear response is not necessarily dissipative, whereas in the Reynold’s number, the linear term represents viscous dissipation. *Weak turbulence* corresponds to $\chi \ll 1$ and *strong turbulence* occurs when $\chi \sim 1$. Eventually, all situations of weak turbulence will approach the strong turbulence limit since MHD turbulence strengthens as it cascades (Sridhar and Goldreich, 1994; Goldreich and Sridhar, 1995). This approximate equality between the nonlinear and linear term is often referred to as *critical balance* (Goldreich and Sridhar, 1995). When this occurs, it is hypothesized that a parallel energy cascade, i.e. an increase in $k_{\parallel}$ to smaller scales, must also occur in order to maintain $\chi \sim 1$ as $k_{\perp}$ increases.
1.3.1.2 Reduced MHD

The full equations of MHD are extremely computationally costly and are often times unable to solve analytically. If we assume that the background magnetic field is uniform and unidirectional, $\mathbf{B} \approx B_z \hat{z}$, then we can greatly simplify the MHD equations to the reduced MHD (rMHD) equations. These widely used equations in space plasma turbulence are useful for studying large-scale incompressible fluctuations, which matches the same environment of the solar wind. This assumption is valid if we are interested in the large-scale dynamics of the problem, since if we “zoom out far enough,” i.e. when the ion Larmor radius\(^6\) becomes infinitesimally small, the background field will indeed be uniform and unidirectional. To see this, let

$$\mathbf{B} = \mathbf{B}_\perp + B_z \hat{z}$$

then since $\mathbf{B}_\perp$ is small compared to $B_z$, we define

$$\frac{B_\perp}{B_z} = \epsilon \ll 1$$

so that each term in the MHD equations can be ordered by some power of $\epsilon$. When all higher order terms are ignored ($\epsilon^2$ or higher), the reduced set of MHD equations are obtained. For a full derivation of these equations, refer to Strauss (1976). The important fact about the derivation is that the pressure terms are negligible (ensuring an incompressible description) since in this ordering, $P \sim \epsilon^2$.

\(^6\)The ion Larmor radius, also referred to as the ion gyroradius, is the small length-scale of the ion gyrating about its magnetic field line.
1.3.2 Early Competing Theories of Weak MHD Turbulence in the Inertial Range

As discussed in §1.3.1, both Iroshnikov (1963) and Kraichnan (1965) used the MHD equations in Elsasser symmetrized form to argue that the magnetic field produces Alfvén waves which affect the energy transfer rate, as opposed to the hydrodynamic case where the mean flow is distortion-free. Analogous to eddies in the hydrodynamic case, Alfvén waves mediate the energy transfer in the inertial range of MHD turbulence. From first principles, they asserted that the turbulent cascade can be thought of as upward and downward propagating Alfvén waves along the magnetic field, where only counterpropagating waves can exchange energy nonlinearly. Assuming isotropy, they both independently derived what is known as the IK spectrum for the inertial range of MHD turbulence:

\[ E_k \sim k^{-3/2} \]

contrasting with the Kolmogorov -5/3 spectrum. Using dimensional analysis, IK theory also shows that nonlinear interactions become weaker at smaller scales since the number of required collisions increases with \( k \), enabling a theory with formalism suggesting an infinite inertial range until dissipation terms become dominant.

Shebalin et al. (1983) questioned the notion of isotropy and addressed how anisotropy could develop in MHD turbulence with an external magnetic field. Using a spectral method in a 2D incompressible MHD simulation\(^7\) and assuming an initially

---

\(^7\)As explained in §1.3.1, turbulence is inherently 3D, so all results reported from 2D models of turbulence must be taken with a grain of salt. Modeling turbulence in 3D, however, is extremely computationally expensive, which is why 3D turbulence simulations did not exist until the use of supercomputers became canon.
isotropic spectrum, they showed that the turbulent energy cascades in the direction perpendicular to the mean magnetic field, i.e. $k_\perp >> k_\parallel$ where $\perp$ and $\parallel$ indicates the direction perpendicular and parallel to the mean magnetic field, respectively; this anisotropy also increases with greater wavenumber. They introduced a 3-wave resonant interaction concept to explain the numerical results that showed only perpendicular and no parallel cascade of energy with respect to the mean magnetic field, i.e. no $k_\parallel$ cascade. Since this concept is still relevant and accepted today (with some caveats), it is worth reviewing here.

Consider two waves with wavenumber $k_1$, $k_2$ and frequency $\omega(k_1)$, $\omega(k_2)$, respectively. Then, the conditions to excite a third wave with wavenumber, $k_3$, and frequency, $\omega(k_3)$, are

$$k_3 = k_1 + k_2$$  \hspace{1cm} (1.4)

$$\pm \omega(k_3) = \omega(k_1) - \omega(k_2)$$  \hspace{1cm} (1.5)

The dispersion relation for Alfvén waves is $\omega = |k_\parallel|v_A$ (in the ideal MHD regime), so Eq. (1.5) becomes

$$|k_\parallel_1| + |k_\parallel_2| = |k_\parallel_3|$$  \hspace{1cm} (1.6)

and the parallel component of Eq. (1.4) is

$$k_\parallel_1 + k_\parallel_2 = k_\parallel_3$$  \hspace{1cm} (1.7)

Thus, the system (1.4)-(1.5) has a solution if both $k_\parallel_1$ and $k_\parallel_2$ have the same sign, implying the waves are traveling in the same direction. However, based on IK theory, the nonlinear interaction is zero since only counterpropagating waves can exchange
energy nonlinearly. The system (1.4)-(1.5) also has a solution if \( k_{\parallel 1} \) and \( k_{\parallel 2} \) have opposite signs, say \( k_{\parallel 1} \geq 0 \) and \( k_{\parallel 2} \leq 0 \) for definiteness. If \( k_{\parallel 2} \neq 0 \), then no solution exists to (1.6)-(1.7), implying that \( k_{\parallel 2} = 0 \). This means that \( k_{\parallel 1} = k_{\parallel 3} \), and hence no parallel cascade.

The work of Sridhar and Goldreich (1994) then sparked enormous controversy challenging the IK theory, leading to decades of harsh debate and ultimately confusing literature littered with erroneous claims that were never corrected or rescinded. A long footnote contained in Lithwick and Goldreich (2003) is the one of the only places in the literature that attempts to clarify this debacle. The rest of this section will also recount this story to appreciate how the landscape of the present-day space plasma turbulence community was shaped. At this point, the disinterested reader can choose their own adventure and skip to §1.4.

Sridhar and Goldreich (1994) argued that Shebalin et al. (1983) was completely wrong because resonant 3-wave interactions do not exist, claiming the solution of \( k_{\parallel 2} = 0 \) waves have no actual power in them. They deduced that the IK theory of weak incompressible turbulence described resonant 3-wave interactions and therefore was also wrong. Instead, they offered an alternative explanation using 4-wave resonant interactions and predicted a different 1-d spectrum of \( E_{k_{\perp}} \sim k_{\perp}^{-2} \). The co-authors of Shebalin et al. (1983) stood their ground, and asserted that 3-wave interactions do not vanish (Montgomery and Matthaeus, 1995). They argued that excitations with \( \mathbf{k} \) perpendicular to \( v_A \) is not actually a wave and misinterpreting Fourier components
as waves\(^8\) leads to missing information about properties of excitation.

By using perturbation theory, Ng and Bhattacharjee (1996) derived detailed analytical solutions to show that 4-wave interactions are subdominant to 3-wave interactions if no \(k_\parallel = 0\) component exists. This proved Sridhar and Goldreich (1994) wrong and supported the claims of Shebalin et al. (1983) using actual mathematical arguments, rather than heuristic ones and concluded that IK theory still holds. In response, Goldreich and Sridhar (1997) admitted 3-wave interactions were realizable, but asserted perturbation theory is invalid to describe it, since the strains in the fluid are so large that no smallness parameter (the essential expansion parameter of perturbation theory\(^9\)) could exist. They tried to explain themselves by defining a new type of “intermediate turbulence” based on the inapplicability of perturbation theory since all order wave couplings have similar magnitude; but they were again completely wrong about their resolution. Galtier et al. (2000) showed that perturbation theory actually does apply to 3-wave interactions, meaning the construction of the so-called “intermediate” turbulence in Goldreich and Sridhar (1997) does not exist. Along with many fascinating mathematical insights, Galtier et al. (2000) derived a detailed closed set of kinetic equations for the energy spectrum and other conserved quantities.

---

\(^8\)The question of if waves can exist in turbulence is still a fervent area of debate, splitting the plasma turbulence community into two camps of interpretation: waves or statistics. Since turbulence has a broadband spectrum, some argue that wave coherence is not possible since there is no dominant wave-mode. Whether or not one should call it a “wave” or a “statistical ensemble” of fluctuations is a purely linguistic thought experiment that has lead to an unnecessary chasm in the plasma turbulence community. A reader who is about to embark in plasma turbulence research should be aware of this fundamental estrangement in the literature, at conferences and sometimes even in job interviews.

\(^9\)See \(\S1.4.2.1\) for an introduction to perturbation theory.
such as magnetic helicity (Woltjer, 1958a,b).

Using more physical arguments than Galtier et al. (2000), Lithwick and Goldreich (2003) derived a different set of kinetic equations to describe weak MHD turbulence and provided another proof that energy only cascades in the perpendicular direction with respect to the mean magnetic field. They also verified that 3-wave interactions do indeed submit to perturbation theory and confirmed “intermediate” turbulence described by Goldreich and Sridhar (1997) was actually just weak turbulence. In addition, they pointed out that the original mistake of Sridhar and Goldreich (1994) was their ignorance about magnetic field line wandering.\(^\text{10}\)

1.3.3 Strong MHD Turbulence

The first theory of strong MHD turbulence was introduced by Goldreich and Sridhar (1995) as a second part of their theory of interstellar turbulence. Although the first paper in this series (Sridhar and Goldreich, 1994) made highly controversial and incorrect claims, it set the stage for them to produce a theory of strong turbulence that has now become canon. Through a renormalization of frequencies, they assumed a finite cascade time exists by a small finite nonlinearity parameter, \(\chi = k_\perp v_\perp k / k_z v_A\) (a “frequency-time uncertainty relationship”). As \(\chi \to 1\) from below, this parameter no longer grows since the \(k_z v_A\) approaches the magnitude of \(k_\perp v_\perp k\) and turbulence

\(^{10}\)Archaic models of the heliosphere depict the magnetic field lines outward from the Sun as an Archemedian spiral. In reality, turbulent plasma that follow these field lines have random fluctuations, and in turn cause the magnetic field lines to follow a random walk (Jokipii and Parker, 1968). This process is stochastic in nature. Note that in the literature, the term magnetic field line wander, magnetic stochasticity, and field line exponentiation are often used interchangeably. The mathematical description of a wandering magnetic field can also be expressed as a magnetic fluctuation with no parallel wavenumber.
becomes strong. As discussed in §1.3.1, a critical balance between the linear wave period and the nonlinear timescale of the energy transfer cascade to smaller scales eventually occurs in shear Alfvénic turbulence when $\chi \sim 1$ (Goldreich and Sridhar, 1995). In this regime, all energy in that scale is transferred within a single Alfvén wave collision. In addition, a scale-dependent anisotropy of $k_z \sim k_{\perp}^{2/3}L^{-1/3}$ exists, meaning there is a correlation between parallel and perpendicular scales of turbulence (Goldreich and Sridhar, 1995).

The $z$-direction is usually the standard choice made for the magnetic field direction, i.e. imposing a mean magnetic field, $\mathbf{B} = B_0 \hat{z}$. Since Alfvén waves travel along the mean magnetic field and mediate the self-similar energy cascade, the direction of constant energy must be parallel to the mean magnetic field direction; hence, $k_z \sim k_{\parallel}$. By assuming a constant energy cascade rate, $\epsilon$, in the $k_{\parallel}$ direction, Goldreich and Sridhar (1995) found that

$$k_z = k_{\parallel} = \epsilon = \frac{k_{\perp}^{2/3}}{L^{1/3}} = \left( \frac{k_{\perp}^2}{L} \right)^{1/3}$$

Taking the limit as $L \to \infty$, observe that

$$\lim_{L \to \infty} \frac{k_{\perp}^2}{L} \sim \epsilon$$

and since $\epsilon$ is constant, $k_{\perp}^2 \to \infty$ as well. This implies that at small length scales, $k_{\perp} \gg k_{\parallel}$. From the physical perspective, this can also be seen by noting that Alfvén waves are perpendicular perturbations in the mean magnetic direction; since all of the energy transfer is mediated by their nonlinear interactions, the energy cascades in the direction perpendicular to the mean field as well. In the strong MHD turbulence
regime, Goldreich and Sridhar (1995) also calculated the 1-d energy spectrum

\[ E_k \sim k_{\perp}^{-5/3} \]

known as the Goldreich-Sridhar (GS) spectrum, which is inherently anisotropic. Even though this was a distinct difference from Kolmogorov’s isotropic prediction, the Kolmogorov -5/3 scaling returned again, consistent with observations from the interstellar medium (Armstrong et al., 1981), also known as the “Great Power Law In the Sky.”

Over the next few years, there were controversies over numerical results showing a \( E(k_{\perp}) \sim k_{\perp}^{-3/2} \) spectrum in strong, incompressible magnetohydrodynamic turbulence (Maron and Goldreich, 2001; Müller et al., 2003; Müller and Grappin, 2005; Haugen and Brandenburg, 2004), rather than the predicted \( E(k_{\perp}) \sim k_{\perp}^{-5/3} \) GS spectrum. Boldyrev (2006) addressed this controversy by using “dynamic alignment” theory from decaying MHD turbulence, that magnetic and velocity field fluctuations tend to become more aligned with their polarization directions. For Alfvén waves propagating down (\( z^+ \)) or up (\( z^- \)) the magnetic field, this means that \( v_{\perp}(r) = b_{\perp}(r) \) or \( v_{\perp}(r) = -b_{\perp}(r) \). This implies that, either \( z^+ \neq 0 \) and \( z^- = 0 \), or \( z^+ = 0 \) and \( z^- \neq 0 \). Both states result in zero nonlinear interaction and consequently no cascade of energy. This means that dynamic alignment cannot be perfectly achieved while maintaining a constant energy flux to small scales, but there exists an optimal angle. Calculating the minimum angle, \( \theta_k \), between \( \mathbf{v} \) and \( \mathbf{b} \) led to deriving the Boldyrev Spectrum, \( E_{k_{\perp}} \sim k_{\perp}^{-3/2} \) (Boldyrev, 2006). In other words, by assuming the theory of dynamic alignment, an additional anisotropy in the other perpendicular direction exists, i.e. turbulence is 3-dimensional with anisotropy between all three axes. In con-
trast, the set-up of Goldreich and Sridhar (1995) assumed both directions, $x$ and $y$, perpendicular to the mean magnetic field in the $z$-direction, had the same anisotropy scaling. Boldyrev (2006) theory predicts $x$ and $y$ having different anisotropies, leading to a more accurate theoretic prediction of $E_{k \perp} \sim k_{\perp}^{-3/2}$. This result mostly settled the controversy between numerical results showing a $k_{\perp}^{-3/2}$ spectrum and the predicted $k_{\perp}^{-5/3}$ GS spectrum, though the spectrum is still debated today.

In critical balance where $\theta_k < 1$, $k_{\parallel}v_A \sim k_{\perp}v_k\theta_k$. The extra factor of $\theta_k$ found by Boldyrev (2006) weakens the nonlinear interactions, requiring a higher value of $k_{\perp}$ to achieve critical balance. This means that dynamic alignment leads to thinning of turbulent structures in the $k_{\perp}$ direction, consistent with observations of current sheets (see §1.4.1) at small scales in MHD turbulence simulations (Biskamp and Müller, 2000; Cho and Vishniac, 2000; Milano et al., 2001; Maron and Goldreich, 2001), rather than small-scale filaments predicted by Goldreich and Sridhar (1995), which is not observed (Boldyrev, 2006).

1.4 Astrophysical Plasma Turbulence in the Dissipation Range

When the turbulent cascade of energy reaches the scale of the ion gyroradius, there is a clear break in the energy spectrum (seen in Fig. 1.5), implying that the self-similar energy cascade in the inertial range starts to dissipate and eventually converts to plasma heat. As discussed briefly in §1.2, the dynamics of energy conversion in this dissipation range is largely an open question; but solar wind observations, numerical simulations and analytical calculations over the last few decades have guided the
plasma turbulence community towards general classes of competing theories. Some suggest no dissipation exists, but instead believe in the onset of a second, dispersive inertial range. Most agree that dissipation does exist and it happens approximately at the ion gyroscale; but the mechanism for that dissipation, and how that energy dissipates to heat both ions and electrons, is the heart of the current debate. Proposed mechanisms for turbulent dissipation include resonant mechanisms such as Landau damping (Landau, 1946; Dobrowolny and Torricelli-Ciamponi, 1985; Leamon et al., 1999; Quataert, 1998; Howes et al., 2008; Schekochihin et al., 2009; Horbury et al., 2012) and ion-cyclotron damping (Coleman, 1968; Marsch et al., 1982; Isenberg and Hollweg, 1983), nonresonant mechanisms such as stochastic heating (Chen et al., 2001; Johnson and Cheng, 2001; Chandran et al., 2010; Chandran, 2010), and intermittent dissipation concentrated in current sheets and magnetic reconnection locations (Dmitruk et al., 2004; Matthaeus and Montgomery, 1980; Karimabadi et al., 2013; Zhdankin et al., 2013; Osman et al., 2014a,c; Zhdankin et al., 2014).

MHD cannot adequately describe small-scale turbulent dynamics in astrophysical plasma since the ion mean free path is greater than one, a much larger length-scale than the parallel wavelengths of the turbulent fluctuations, requiring a kinetic description (Schekochihin et al., 2009). Kinetic plasma theory (Nicholson, 1983) describes the evolution of plasma in phase-space in terms of individual particles and wave-particle interactions. However, at large scales, a fluid reduced MHD (rMHD) (Strauss, 1976) description of undamped Alfvénic fluctuations is appropriate until the cascade reaches the length-scale of the ion gyroradius (Quataert, 1998; Lithwick and
Figure 1.6: Schematic of the magnetic energy wavenumber spectrum in the solar wind, showing the form of the spectrum in the energy containing, inertial and dissipation ranges. Scale ranges for reduced MHD (green) and gyrokinetic (red) simulations are depicted, along with typical Larmor radius scales for protons (magenta) and electrons (blue) from Solar Energetic Particle (SEP) events.

Goldreich, 2001; Schekochihin et al., 2009). When the Alfvénic fluctuations reach the ion Larmor radius-scale, some theories suggest that a portion of their energy channels into kinetic Alfvén waves (KAWs) which continue to cascade to smaller perpendicular scales (Gruzinov, 1998; Quataert and Gruzinov, 1999), while others suggest that the dispersion is mediated by whistler waves (Stawicki et al., 2001; Krishan and Mahajan, 2004; Galtier, 2006). As discussed in §1.3.1.2, the pressure terms are negligible in the rMHD equations, meaning that rMHD simulations do not depend on key plasma parameters such as plasma beta. Thus, comparing an rMHD simulation with a gyrokinetic simulation can reveal whether such plasma parameters play a role in the large-scale dynamics. Fig. 1.6 represents the regions in the magnetic energy
spectrum where the rMHD approximation is valid until the ion gyroscale is reached, and a kinetic description approximation, called gyrokinetics (Howes et al., 2006), is needed.

In §1.4.1, I review the competing theories over the role of current sheets in dissipation and how they are generated. §1.4.2 explains the essential background material needed for understanding astrophysical plasma turbulence in a gyrokinetic framework, which enables discussion of kinetic dissipation mechanisms, such the KAW cascade in §1.4.3. In §1.4.4, I briefly review observational constraints of diagnosing dissipation mechanisms with spacecraft data.

1.4.1 Current Sheets

When MHD turbulence dissipates, intermittent current sheets are found both in numerical simulations and observations. The hydrodynamic analog to current sheets in MHD is intermittent filaments of vorticity (She et al., 1990). Current sheets are coherent structures in the form of elongated currents confined to a surface. In this section, I review the methodologies, results and open questions that arose from attempting to understand their nature both observationally and numerically. At this point, the uninvested reader may skip to §1.4.2, as these findings are summarized in §2.1.1.11

Early MHD turbulence simulations revealed that turbulent energy dissipation

---

11 The more adventurous reader will embark on a journey revealing, yet again, the drama that occurs in the plasma turbulence community over different interpretations of similar observed phenomena.
occurred in a highly intermittent manner at small scales (Matthaeus and Montgomery, 1980), and the observed spatially intermittent magnetic field structures existed for one eddy-turnover time (Meneguzzi et al., 1981). These predictions were consistent with early solar wind and cosmic ray observations revealing small-scale “magnetic field and plasma filaments” (Parker, 1963), “filamentary tubes” (Ness et al., 1966) and “flow tubes” (Thieme et al., 1989; Tu and Marsch, 1990; Marsch, 1991), supporting a “flux tube” picture of the texture of the solar wind (Bruno et al., 2001).

Borovsky (2008) introduced a statistical diagnostic enabling identification of flux tube wall boundaries using 7 years of ACE data at 1 AU. They characterized flux tube boundaries, also known as “magnetic field discontinuities,” as large changes in the magnetic field direction. This was done by calculating the magnetic field angular shifts, \( \Delta \theta \), between temporally different measurements, \( \mathbf{B}_1 \) and \( \mathbf{B}_2 \), by

\[
\Delta \theta = \cos^{-1} \left( \frac{\mathbf{B}_1 \cdot \mathbf{B}_2}{||\mathbf{B}_1|| \cdot ||\mathbf{B}_2||} \right)
\]

Exponential fits of Probability Density Functions (PDFs) of this quantity showed the existence of a distinct small-change population (fitted by \( \exp(-\Delta \theta/9.4^\circ) \)), interpreted to be caused by MHD turbulence, and a large-change population (fitted by \( \exp(-\Delta \theta/24.4^\circ) \)), interpreted as flux tube interface crossings (Bruno et al., 2001). This large-change population, and consequently flux tube boundary locations, were also shown to be coincident with other large changes in the plasma properties of the solar wind, including flow velocity, magnetic field strength, ion entropy density, electron temperature, alpha-to-proton ratio and field-aligned electron flux. It was also found that at 1 AU, the slow solar wind consists of larger flux tubes than in the
fast wind, and that magnetic flux concentrations in flux tubes are identical to the “magnetic carpet” blanketing the solar surface. Furthermore, these flux tubes can be mapped back from 1 AU to a Co-rotating Interaction Region (CIR) to reveal its origin at granule and super-granule scales. In addition, solar wind expansion and flux tube thickness prevents magnetic reconnection during advection of the flux tube to 1 AU. Magnetic reconnection refers to the process of magnetic field lines breaking, releasing a large amount of energy, and reconnecting again. All of the above evidence supported their grand conclusion that flux tubes found at 1 AU are “fossil structures” coming from the surface of the sun, enabling a realization of the inner heliosphere as a “network of entangled flux tubes” (Borovsky, 2008).

Expanding on their previous work, Borovsky (2010) studied the effect of solar wind discontinuities on the power spectrum of the solar wind. Using 8.5 years of ACE magnetic field measurements at 1 AU in the solar wind, an artificial time series was constructed, keeping discontinuity amplitudes and timing preserved. The strong discontinuities were characterized by changes in the magnetic field direction of at least 45° (as defined by Borovsky (2008)). Then, they calculated the Power Spectral Density (PSD) of this series and compared it with the solar wind magnetic field PSD. Through this analysis, Borovsky (2010) determined that in the inertial range, the Kolmogorov -5/3 spectrum is found for the PSD of strong discontinuities, consistent with solar wind magnetic field spectrum observations (Coleman, 1968; Dobrowolny et al., 1980; Tu et al., 1984). Furthermore, these strong discontinuities account for half the power in the solar wind magnetic field, implying that strong discontinuities
need to be considered when modeling solar wind dynamics (Borovsky, 2010).

Many types of discontinuities and their implications about solar wind dynamics were reported since the mid-1980s. Ultimately, this gave rise to the following three main competing theories about how current sheets are generated: (1) highly Alfvénic Tangential Discontinuities (TD) (Neugebauer et al., 1986) defined by plasma boundaries, which are found to dominate in the solar wind (Horbury et al., 2001; Knetter et al., 2004; Riazantseva et al., 2005), and to contribute significantly to Alfvénic properties of the solar wind (Borovsky and Denton, 2010); (2) non-tangential (rotational) discontinuities, which are caused by large-amplitude Alfvén waves steepening from the Sun (Malara et al., 1996; Vasquez and Hollweg, 1999) and being collisionlessly dissipated (Medvedev, 1999); (3) as MHD turbulent energy cascades to smaller length scales, current sheets form (Biskamp and Welter, 1989; Dmitruk et al., 2004; Greco et al., 2008; Leamon et al., 2000; Matthaeus et al., 2003), leading to small-angle discontinuities with ion length-scale thickness (Vasquez et al., 2007).

The interwoven flux tube picture of the solar wind introduced by Borovsky (2008) had many implications for altering interpretations of the dynamics of MHD solar wind turbulence. Many solar wind observations showed heating of protons (Marsch et al., 1982; Schwartz and Marsch, 1983; Freeman and Lopez, 1985; Richardson et al., 1995) and electrons (Pilipp et al., 1990; Phillips et al., 1995), but their heating mechanism remains an open question. Motivated by resistive MHD simulations showing dissipation and heating at current sheets generated by turbulence (Dmitruk et al., 2004; Lehe et al., 2009), it was hypothesized that particle heating occurred within
localized strong current sheets (Leamon et al., 2000; Mattheaus et al., 2003; Greco et al., 2010; Osman et al., 2011). The question of particle heating, along with the flux tube portrait of the solar wind, sparked another contentious debate in the space plasma turbulence community that still remains today. It ultimately started as a “chicken or the egg” style debate, asking whether the fast solar wind causes heating, or heating is a consequence of more flux tubes boundaries (hypothesized to generate current sheets) contained in the fast wind. The link between magnetic reconnection and current sheets was also speculated as a consequence of their findings, which is another area of current active research.

Osman et al. (2011) sought to answer if turbulent-generated current sheets are the sites of ion and electron heating in the solar wind. They used ACE and WIND solar wind observations to compute heating diagnostics such as heat flux, temperature and non-Gaussian contributions. From the spacecraft data, they employed the Partial Variance of Increments (PVI) method and computed

$$PVI = \frac{|\Delta B|}{\sigma}$$

where $\sigma = \langle |\Delta B|^2 \rangle^{1/2}$ represents the ensemble average of the non-centralized standard deviation. The deviation of the PVI from Gaussianity is indicative of a coherent structure, specifically $|\Delta B| > 3\sigma$. Their results from diagnosing current sheets from solar wind observations using this PVI method, along with comparison to numerical MHD simulations, showed evidence that current sheets are “associated” with heating of protons and electrons. In addition, it was found that reconnection sites are located at some of the current sheet locations (Osman et al., 2011).
The paper of Osman et al. (2011) titled as, *Evidence for inhomogeneous heating in the solar wind*, prompted Borovsky and Denton (2011) to publish work titled, *No evidence for heating of the solar wind at strong current sheets*, which aimed to answer the same question of whether solar wind heating occurred at the locations of strong current sheets. They did so by analyzing 12 years of ACE solar wind data at 1 AU containing 194,070 strong current sheets (2-3 sheets per hour), and compared proton temperature and entropy to adjacent plasma. Since no increase in proton entropy or temperature was found, they concluded that there was no localized solar wind heating at strong current sheets. On average, it was determined that plasma with higher proton (and electron) temperature and entropies, called the *fast wind*, contained more strong current sheets. Even though Borovsky and Denton (2011) found the same statistical results as Osman et al. (2011), they reached an entirely different conclusion; the fast wind is more likely to contain current sheets, not that the current sheets cause heating of the wind.

As a rebuttal to the conclusions of Borovsky and Denton (2011), Osman et al. (2012) re-analyzed the solar wind data using the same PVI method as Osman et al. (2011), but instead used non-Poissonian clustering as a diagnostic for plasma heating. They asserted again that locally, enhancements in temperature are “linked” to intermittent dissipation and current sheets in solar wind turbulence. Osman et al. (2012) explained that their results contrast with Borovsky and Denton (2011) since their methodology limited their study to comparing nearby points. Osman et al. (2012), on the other hand, argued that their method was better since it considered the rela-
tionship between the averaged dissipation rate and the local temperature. The local statistics linking these two quantities suggest that intermittent dissipation is driven by kinetic processes through MHD turbulence in the solar wind.

Further investigating the link found (Osman et al., 2011) between magnetic reconnection and intermittent magnetic field structures in solar wind turbulence, Osman et al. (2014b) used WIND data at 1 AU and the same PVI method as previously to detect non-Gaussian behavior indicative of intermittency. The observed intermittency was shown to be concentrated in locations of current sheets and likely reconnection events. In addition, large-amplitude, non-Gaussian magnetic field fluctuations were determined to be “statistically associated” with solar wind reconnection exhausts. This conclusion, however, has the caveat that not all current sheets that are reconnection sites are driven by turbulence, but a fraction of reconnection exhausts are related to intermittent dissipation of turbulence. Therefore, the relationship between current sheets and magnetic reconnection still remains unclear.

Uritsky et al. (2010) investigated the similar link between turbulent intermittency and magnetic energy conversion and dissipation, utilizing data from a 3D MHD decaying turbulence simulation. Instabilities in magnetic topologies often arise in regions of enhanced intermittency, as seen from regions of solar activity (Abramenko et al., 2003). In the Earth’s magnetotail, magnetic reconnection locations have been shown to correspond with MHD intermittency (Klimas et al., 2005; Vörös et al., 2006; Angelopoulos et al., 1999), triggering multi-scale plasma instabilities (Lui, 2001) and particle acceleration in the aurora (Uritsky et al., 2003; Stepanova et al., 2006; Urit-
sky et al., 2007). In MHD simulations, the velocity and magnetic field in localized dissipative structures are well-correlated at large Reynolds numbers (Grappin et al., 1982; Meneguzzi et al., 1996) and small scales (Matthaeus et al., 2008). Magnetic reconnection, through localized dissipative structures, can interact with the turbulence and affect the global dissipation of turbulent energy.

By developing a multi-scale cluster algorithm to diagnose dissipative structures and scaling behavior, Uritsky et al. (2010) determined that the statistical properties of current and vorticity sheets do not depend on Reynolds number or velocity-magnetic-field correlation, but thinner current sheets do correspond to higher Reynolds number. They concluded that inertial-range scale dynamics are governed by MHD, but self-organized criticality (SOC) \(^\text{12}\) may dictate dissipative-range scales.

Using the same method to detect discontinuities as Borovsky (2008) on CLUSTER II magnetometer data, Perri et al. (2012) showed the existence of current sheets at proton to electron length scales. They suggested that current sheets are probably common in solar wind turbulence at ion scales, but current sheet observations do not necessary imply dissipation locations. However, the energy cascade has been observed to be globally scale-invariant at sub-proton scales (Kiyani et al., 2009), which agrees with the findings of Perri et al. (2012), that discontinuities exist from proton

\(^{12}\)SOC is a property of dynamical systems that implies the macro-scale physics is scale-invariant (spatially and/or temporally) to the physics occurring at the phase-transition of that system. Furthermore, the system naturally evolves towards an SOC state independent of the initial conditions. Uritsky et al. (2010) remarked that, “the fact that the dynamics of turbulent flows contains elements of critical phenomena and conformal invariance associated with invariance properties and symmetry groups of the underlying equations points to the need to further our studies of such flows using scaling tools.”
to electron length scales, suggesting that within the sheets, dissipation must also be occurring (Sundkvist et al., 2007). If current sheets are found at dissipative scales, then the natural next question to address is if the observed discontinuities in the solar wind can be explained by MHD turbulence. Again, using a similar procedure as Borovsky (2008), Miao et al. (2011) used Ulysses spacecraft data to also show the existence of two distinct populations, but with slightly different thresholds for $\Delta \theta$. Both studies interpreted the strong discontinuities as boundaries of coronal flux tubes and weak discontinuities as fluctuations due to turbulence. At the same time, other studies (Li, 2008; Neugebauer and Giacalone, 2010; Vasquez et al., 2007) showed that nonlinear interactions dominate the generation of strong discontinuities, enabling a picture of dynamically evolving solar wind discontinuities expanding outward from the Sun.

Zhdankin et al. (2012) investigated magnetic discontinuities and their statistical properties with various scalings of uniform guide field, $B_0$, using data from driven incompressible MHD turbulence, to find

$$P(\Delta \theta) \propto \exp(-\Delta \theta/\theta_*)$$

where $\theta_* \approx (14^\circ)(b_{\text{rms}}/B_0)^{65}$. This is consistent with both the findings of Borovsky (2008) and Miao et al. (2011), but offers the alternative explanation that current sheets could be generated from MHD turbulence with fluctuating guide-field ratios. Dissipative scale structures are observed by interpretation of excess small-angle discontinuities seen in the PDFs as $\Delta x \to \infty$. Zhdankin et al. (2012) concluded that inertial-range scale turbulence corresponds to strong discontinuities and near-
dissipation scale turbulence corresponds to weak discontinuities, in qualitative agreement with solar wind observations.

Now that there was enough numerical and observational evidence that current sheets could indeed exist as a consequence of MHD turbulence, the question shifted to how the turbulence could generate current sheets. TenBarge and Howes (2013) addressed this, in addition to investigating how Landau damping (see §1.4.3) plays a role in both the current sheet generation and dissipation. Using a wave-driven 3D gyrokinetic turbulence simulation of weakly collisional plasma and implementing heating diagnostics derived from analytical calculations (Howes et al., 2006), TenBarge and Howes (2013) showed that turbulence self-consistently generates current sheets at electron scales. Furthermore, there was a strong correlation between electron heating rate and current sheet filling fraction with little correlation between total turbulent energy and electron heating rate, meaning that dissipation in current sheets must dominate electron heating. TenBarge and Howes (2013) were thus able to deduce that electron Landau damping was enough to explain the heating, suggesting resonant wave-particle interactions could govern the current sheet dissipation and their locations are associated with heating enhancement.

Howes (2016) showed how current sheets are generated from first principles, in terms of Alfvén wave collisions. A 3D gyrokinetic turbulence simulation of counter-propagating Alfvén wave collisions revealed natural generation of current sheets. He found that the lifetime, $\tau$, of the current sheet was comparable to the Alfvén crossing time, $T_A$, of the original Alfvén waves, i.e. $\tau = 3T_A/4$. In addition, the parallel and
perpendicular components of the initial Alfvén wavelengths determined the dimensions of the current sheet \((l, w, \delta)\). The thickness, \(\delta\), was on the same spatial-scale of the ion Larmor radius where Alfvén waves become dispersive KAWs. Howes (2016) also gave a conceptual picture of current sheet generation through constructive interference of the nonlinearly generated modes in each Alfvén wave collision. In this picture, the nonlinear modes generated must be of the same order as the original modes, meaning that this concept only holds true in the strongly nonlinear limit.

Both TenBarge et al. (2013) and Howes (2016) were able to achieve their powerful conclusions through the use of the 3D gyrokinetic simulation code, AstroGK (Numata et al., 2010). In general, 3D turbulence is difficult to model due to the high computational cost of running the simulation. Using a gyrokinetic framework for plasma turbulence not only reduced computational cost, but also enabled a kinetic description of the plasma turbulence energy cascade to be realized. This led to novel theoretical predictions about the dissipation mechanisms and eventual heating of the plasma, which were then confirmed with spacecraft observations (Chen et al., 2019).

1.4.2 Astrophysical Turbulence in the Gyrokinetic Limit

1.4.2.1 Perturbation Theory Background

The notion of strong anisotropy, that the turbulent energy fluctuations, \( \mathbf{k} = (k_\parallel, k_\perp) \), preferentially cascades as \( k_\perp \gg k_\parallel \), has strong mathematical consequences. If \( k_\perp \gg k_\parallel \), then we can define the small parameter,

\[
\epsilon = \frac{k_\parallel}{k_\perp} \ll 1
\]
which enables perturbation theory to be valid in analytical calculations using $\epsilon$ as the small expansion parameter. Perturbation theory is one of the most common mathematical methods in a plasma physicist’s toolbox. If one wants to find an approximate solution to a problem that cannot be solved, then one could take an exact solution known from a simplified version of the problem and add a small perturbation to it, leading to an expression that enables the more complex problem to be solvable. The perturbation then leads to a power series formulation of the desired solution and it can be broken up into “solvable” and “perturbed” parts. In plasma physics, this “solvable” part is usually the equilibrium solution, i.e. for a distribution of particles, $F$, the solvable equilibrium, $F_0$, is a Maxwellian and we could construct an approximate solution

$$F = F_0 + \epsilon\delta F + \epsilon^2\delta^2 F + ...$$

that quantifies how much $F$ deviates from its equilibrium. Approximating a polynomial with a power series requires the expansion parameter, $\epsilon$, to be small enough so that in the limit $\epsilon \to 0$, the constructed approximation, $F$, to the unsolvable problem, converges to the exact solution, $F_0$, of the original solvable problem. A simple example is a Taylor series approximation. For a function, $f(x)$, with an infinitely differentiable (exact) solution $f(a)$,

$$f(x) \approx f(a) + (x - a)f'(a) + (x - a)^2f''(a) + ...$$

where here, $(x - a)$ is the small perturbation, $\epsilon$, and the first derivative, $f'(a)$, is the first order perturbed component of $f(x)$. The second term is the second order solution, etc. If $\epsilon$ is small, higher order terms of $\epsilon$ will become smaller by definition, and can
either be neglected or kept based on how small the dynamics are that need to be captured in the constructed approximate solution. In this way, one can use ordering in powers of $\epsilon$ to systematically organize the solution in a way that separates the dynamics occurring at different (smaller and smaller) scales.

1.4.2.2 The Gyrokinetic Approximation

In the gyrokinetic limit of plasma physics (Frieman and Chen, 1982), we assume a strong magnetization condition that restricts the spatial and timescales such that

$$\rho_i \ll L \quad \text{and} \quad \omega \ll \Omega_i$$

where $\rho_i$ is the ion gyroradius, $L$ is the macroscopic length-scale of the equilibrium plasma, $\omega$ is the frequency of the fluctuations, and $\Omega_i$ is the ion gyrofrequency. Gyrokinetics utilizes perturbation theory to employ a formal ordering to manage the disparate length and timescales typical of magnetized plasmas. The small dimensionless parameter is defined by $\epsilon = \rho_i / L \ll 1$. The distribution function is then decomposed into an equilibrium and fluctuating component,

$$f_s = F_{s0} + \delta f_s$$

where the formal gyrokinetic ordering is defined by

$$\frac{\rho_i}{L} \sim \frac{\omega}{\Omega_i} \sim \frac{\delta f}{B_0} \sim \frac{|\delta E|c}{B_0 v_{ti}} \sim \frac{k_{||}}{k_{\perp}} \sim \epsilon \ll 1$$

Here, $k_{||}$ and $k_{\perp}$ are the parallel and perpendicular wave numbers, respectively. The perturbed magnetic field and equilibrium magnetic field are represented by $|\delta B|$ and
Figure 1.7: Visualization of gyrokinetics as the evolution of ring averages of charged particles moving along magnetic fields.

$B_0$, respectively. Finally, $|\delta E|$ represents the perturbed electric field, $c$ is the speed of light, and $v_t$ is the ion thermal velocity. Note that the choice of $\epsilon$ occurs naturally through the magnitude of the anisotropy, $k_\parallel/k_\perp$.

The basic concept of gyrokinetics is depicted in Fig. 1.7. The frequency of the turbulence, $\omega$, is of the same magnitude as the ratio $\delta B/B_0$ which is also proportional to $l_\perp/l_\parallel$, where $l_\perp$ and $l_\parallel$ are the characteristic length scales in gyrokinetic formalism. Since we assume that the frequency, $\Omega_i$, of the ion gyration about the magnetic field is much greater than the frequency of the turbulent fluctuations, $\omega$, the ion gyration timescale is fast enough to be averaged over; this means that the ion gyroradius can be ring-averaged to a guiding center position. The dynamics of a plasma are usually understood in terms of 6-dimensional phase-space of the distribution of particles, $f_s(\mathbf{r}, \mathbf{v}, t)$ for each particle species, $s$, where $\mathbf{r}$ and $\mathbf{v}$ represent the spatial and velocity coordinates, respectively. The gyrokinetic approximation has the enormous advantage
of eliminating one degree of freedom, reducing the velocity dimensions from three ($v = (v_x, v_y, v_z)$) to two ($v = (v_\parallel, v_\perp)$), drastically improving the computational cost of solving the problem. Hence, exploiting the anisotropy reveals an azimuthal symmetry that reduces the problem in phase-space from 6 dimensions (3D,3V) to 5 dimensions (3D,2V), enabling full 3D plasma turbulence simulations to be efficiently run in a reasonable time with present-day computing power.\textsuperscript{13}

Taking advantage of the $\omega \ll \Omega_i$ approximation corresponding to a fundamental timescale separation, Howes et al. (2006) was the first to extend the expansion to second order to obtain the slow evolution of the energy, enabling the gyrokinetic equations to be derived for weakly collisional turbulent plasma in a uniform magnetic guide field. The dynamics are consequently separated into three different timescales: fast (based on $\Omega_i$, order 1), intermediate ($\omega$, order $\epsilon$) and slow (heating rate, order $\epsilon^2$). The fast timescale is the cyclotron frequency and therefore is “ordered” out of this approximation by the gyro-averaging procedure, which means that the cyclotron resonance cannot be resolved. The intermediate timescale corresponds to the timescale of the turbulence, and consequently the Alfvén wave modes which dominate the cascade. The slow timescale corresponds to the heating that leads to slow evolution of the equilibrium distribution function.

\textsuperscript{13}In addition to the assumption of critical balance, the power of gyrokinetics ultimately rests in the power of finding the extra azimuthal symmetry. This begs the question of what other powerful approximations could be made if other symmetries were discovered.
1.4.2.3 Qualitative Energy Transfer in the Weakly Nonlinear Limit

The ordering introduced by the gyrokinetic approximation (Howes et al., 2006) led to an asymptotic solution to be derived for the nonlinear evolution of the interaction between two Alfvén waves shown in Fig. 1.8 (Howes and Nielson, 2013). This approach also leads to a qualitative understanding of the energy transfer by organization into different orders of $\epsilon$ and is essential background knowledge for comprehending Chapter 2.

Consider a uniform equilibrium magnetic field $\mathbf{B}_0 = B_0 \hat{z}$ and a magnetized, incompressible ideal MHD plasma in a triply periodic box. Let the initial conditions of two perpendicularly polarized counterpropagating Alfvén waves be described by the Elsasser variables

\[ z^+ = z_+ \cos(k_+ x - k_+ z - \omega_0 t) \hat{y} \]

\[ z^- = z_- \cos(k_- y + k_+ z - \omega_0 t) \hat{x} \]
Figure 1.9: Schematic diagram of the Fourier modes in the $(k_x, k_y)$ perpendicular plane arising in the asymptotic solution. The Fourier modes depicted are the primary $\mathcal{O}(\epsilon)$ modes (circles), secondary $\mathcal{O}(\epsilon^2)$ modes (triangles), and tertiary $\mathcal{O}(\epsilon^3)$ modes (squares). Filled symbols denote the key Fourier modes that play a role in the secular transfer of energy to small scales in the Alfvén wave collision. The parallel wavenumber $k_z$ for each of the modes is indicated by the diagonal grey lines, a consequence of the resonance conditions for the wavevector (Howes and Nielson, 2013).

for initial constant amplitudes $z_+$ and $z_-$ as shown in Fig. 1.8. Qualitatively, one can understand the transfer of energy to smaller scales by a plot of the wavevector space $(k_x, k_y)$, depicted in Fig. 1.9, which is in the plane perpendicular to $B_0$. Define $\epsilon \sim k_\parallel / k_\perp \ll 1$ as the ordering parameter of the anisotropic limit. The initial (red solid circle) modes, $k_1^-$ and $k_1^+$, correspond to the initial waves, $z^+$ and $z^-$, respectively. Then, these two primary Alfvén waves interact to create a secondary, inherently nonlinear and purely magnetic mode (green solid triangle). Hence, on the order of $\epsilon^2$,

$$k_1^+ + k_1^- = k_2^{(0)}$$

Next, the primary and secondary modes interact to produce a tertiary mode (blue
square). Thus, on the order of $\epsilon^3$,
\[
k_1^+ + k_2^{(0)} = k_3^+
\]

Note that this tertiary mode has the same value of $k_\parallel$ as its respective primary mode, as indicated by the fact that they lie on the same $k_\parallel$ line in Fig. 1.9. Since
\[
|k_3^\pm| = \sqrt{5k_\perp^2 + k_\parallel}
\]

this means that $k_\perp$ has increased. Therefore, this describes the anisotropic cascade of energy to smaller perpendicular scales.

### 1.4.3 Kinetic Dissipation Mechanisms

Howes et al. (2006) also calculated the linear dispersion relation and found a separation of Alfvén and slow waves in the limit of $k_\perp \rho_i \ll 1$. Slow waves can damp via the Landau resonance in $\beta \gtrsim 1$ plasmas, whereas Alfvén waves do not damp until the ion Larmor radius-scale ($k_\perp \rho_i \sim 1$). At $k_\perp \rho_i \gg 1$, Alfvén waves experience non-zero parallel motions due to finite Larmor radius effects which cause Alfvén waves to channel into kinetic Alfvén waves (KAWs), which dictate the low-frequency dynamics until electron Landau damping occurs at $k_\perp \rho_e \sim 1$.

The concept of a kinetic Alfvén wave (KAW) was first introduced by Hasegawa (1976a); Hasegawa and Chen (1976); Hasegawa (1976b, 1977), who modified the MHD equations to include the effects of a finite Larmor radius, enabling its interpretation as the result of an Alfvén wave coupling with an ion-acoustic mode (Lysak and Lotko, 1996; Hollweg, 1999). The term “finite Larmor radius effects” refers to physical
consequences that happen in the situation where the perpendicular component of the turbulent fluctuations, $k_\perp$, is of similar spatial-scale to the radius of the ion gyrating about its magnetic field line. This means that the ion now has more freedom to move transverse with the field. Hasegawa and Uberoi (1982) showed that when the KAW couples with the electrostatic mode, both ion and electron Landau damping can occur. This is because the KAWs become dispersive at $k_\perp \rho_i$ and so the compressive parallel magnetic fluctuations are induced, which has an accompanied parallel electric field. Since KAWs have an associated parallel electric field component, $E_\parallel$, the ions that are now moving more transversely along the magnetic field lines are susceptible to experience an interaction with $E_\parallel$. If the ions are moving with approximately the resonant phase velocity of $E_\parallel$, a wave-particle interaction will occur, enabling a net transfer of energy from the wave to the particle. The term Landau damping refers to this process of particle energization at the expense of the wave, causing its energy to damp. This process can occur for either ions or electrons and is hypothesized to play an important role in plasma heating (Hasegawa and Chen, 1975; Schekochihin et al., 2009), in addition to auroral generation via electron acceleration (Hasegawa, 1976b). A multitude of other geophysical processes are speculated to be a consequence of KAWs and the reader can refer to Hollweg (1999) for an exhaustive list from the solar corona to the ionosphere. The excitation of KAWs is a universal property\textsuperscript{14}

\textsuperscript{14}The concept of universality in statistical mechanics means that the qualitative behavior of a system exists independently from the details of the system dynamics. This implies an overarching symmetry exists and so through the renormalization group, which is the mathematical object associated with scale-invariants (such as self-similar energy transfer in the inertial range), one can classify operators that parameterize near-critical behavior of systems (such as a the transition in phase-space from the inertial to the dissipation range).
of space plasmas and so understanding their dynamics sheds light on many plasma phenomena such as the dissipation of plasma turbulence.

Evidence of KAWs playing a primary role in turbulent energy dissipation is seen both through spacecraft observations and theoretical predictions. For example, using 33 intervals of WIND data at 1 AU, Leamon et al. (1998a,b, 1999) produced seminal work suggesting that the onset of dissipation in solar wind turbulence, which occurred at the ion-cyclotron frequency, was due to damping of KAWS; however, the specific damping mechanisms by either resonant or non-resonant processes remained unclear. Furthermore, (Leamon et al., 1998a) found the dissipation range spectral indices at 1 AU were shallower than other spacecraft investigations within 1 AU, such as Helios (Denskat et al., 1983) and Mariner (Smith et al., 1990), and steeper than observations beyond 1 AU, such as Voyager (Smith et al., 1990). Although the slope of the dissipation range spectrum tends to vary by case study, they all share the common observation that frequencies associated with the dissipation range were comparable to the ion-cyclotron frequency ($\sim 1$ Hz). Solar wind magnetic field spectrum steepening at 1 Hz was first postulated to be due to ion-cyclotron activity (Coleman, 1968; Behannon, 1976; Denskat et al., 1983). Leamon et al. (1998b) tested this hypothesis and sought how much the cyclotron-resonant damping mechanism contributed to the dynamics in the dissipation range. Expanding on their previous analysis on WIND observations (Leamon et al., 1998a), they performed correlation and spectral analysis on the same 33 intervals of WIND data, while making several assumptions including incompressible MHD, Alfvén waves co-propagating in the in-
ertial and dissipation range, and negligible velocity field energy in the dissipation range. Leamon et al. (1998b) ultimately found magnetic helicity signatures in the dissipation range,\textsuperscript{15} consistent with Goldstein et al. (1994), having values explained by resonant ion-outward propagating Alfvén wave coupling leading to ion thermalization. Magnetic helicity is a signature of the cyclotron-resonant mechanism, but the “non-purity” of the observed signature suggested a competing dissipation mechanism must be present. Leamon et al. (1998b) introduced a data-constrained “rate balance theory” that explained how cyclotron-resonant mechanisms could balance with non-resonant mechanisms in the dissipation range. In an attempt to find the dominant mechanism of KAW damping, Leamon et al. (1999) used the linearized Maxwell-Vlasov equations to calculate KAW damping rates in various interplanetary conditions, enabling a model which correctly predicted the onset of dissipation when compared to observations. The role of electron Landau damping, with subsequent electron heating, was found to correspond significantly to the damping of interplanetary magnetic field fluctuations (Leamon et al., 1999).

Through a series of spectral analysis techniques (FFT, PSD, interpolation, filtering, etc) applied to CLUSTER II data, Bale et al. (2005) reported first observations of the electric field fluctuation spectrum in $\beta \geq 1$ plasmas at 1 AU, which showed Kolmogorov -5/3 scaling in the inertial range and spectrum steepening at $k\rho_i \geq 1$.

\textsuperscript{15}See Howes and Quataert (2010) for an argument suggesting that one cannot draw the conclusions of the Leamon et al. (1998a,b, 1999) studies based on magnetic helicity alone.
break at $k_{\perp} \rho_i \sim 1$, they applied the *Taylor hypothesis*.\(^{16}\) In line with Leamon et al. (1999), Bale et al. (2005) reconciled that the dispersion regime transition could be explained by dispersion relation of KAWs, which are hypothesized to isotropize the ion and electrons via damping, explaining the fluid-like characteristics of the solar wind. In addition, they found a strong correlation between the electric and magnetic field fluctuation spectrum up to $k\rho_i = 2.5$, where data in this region became too noisy to analyze.

By analyzing 800 intervals of IMF and solar wind ACE data at dissipation range frequencies $\geq 0.3$ Hz at 1 AU, Smith et al. (2006) sought to answer why the dissipation range contains a broad range of spectral indices. They observed that the inertial range of the spectrum has a narrow range of index, while the dissipation range has a wide index range. In addition, larger energy cascade rates resulted in steeper dissipation slopes, meaning that there must exist a dependence between these two quantities. Differing from hydrodynamic turbulence, dissipation onset does not depend on energy cascade rate, but rather the slope of the dissipation spectrum does depend on the energy cascade rate. Smith et al. (2006) also concluded that steeper spectra correspond to faster heating rates, consistent with Leamon et al. (1998a).

Howes et al. (2008) connected solar wind turbulence observations to an energy cascade model motivated by their previous gyrokinetic formulation (Howes et al., 2006). They offered an explanation of the wide distribution of power law indices

\(^{16}\)The validity of the Taylor Hypothesis applied to spacecraft observations is a current subject of debate which lies beyond the scope of this thesis.
seen in the dissipation range of solar wind turbulence as due to a combination of instrumental resolution noise floors and linear damping. In addition, they showed that ion-cyclotron damping is not an important dissipation mechanism to consider (unless $\beta_i < 0.01$), by using a Vlasov-Maxwell dispersion relation (Quataert, 1998) and comparing to wave phase velocities in Bale et al. (2005). Following advice by Tu and Marsch (1995), careful considerations are required when performing large ensemble averages of solar wind observations, since one may be mixing both slow and fast wind. Both types of solar wind have fundamentally different affects on the turbulent cascade evolution due to their fundamentally different plasma parameters, such as $T_i/T_e$, $\beta_i$ and different origin from the sun. Using measured $\beta_i$ (Leamon et al., 1998a), $T_i/T_e$ (Newbury et al., 1998) and the linear dispersion calculation from gyrokinetics (Howes et al., 2006), Howes et al. (2008) found that slow wind consists of significantly stronger Landau damping than the fast wind. Another difficulty in interpreting solar wind observations is the fact that single-point measurements are used (Fredricks and Coroniti, 1976; Bruno and Carbone, 2005). Reverse modeling from observations therefore may not be an effective approach to understand the governing mechanisms of solar wind turbulence. The gyrokinetic framework developed in Howes et al. (2006) provides computational efficiency to make forward modeling possible.

Schekochihin et al. (2009) generalized turbulence power law spectra characteristics (such as self-similar energy cascades) to kinetic plasma turbulence. They used the gyrokinetic equations derived by Howes et al. (2006) to develop a framework
for astrophysical gyrokinetics in a weakly collisional plasma, which is the regime of interest in heliosphere. The mean free path is 1 AU, so solar wind data from 1 AU is an accurate testbed for collisionless plasmas. Schekochihin et al. (2009) derived rigorous equations for the dynamics separating 4 distinct scales of the turbulent energy spectrum: ion and electron gyroradius, mean free path, and electron diffusion scale. First, the inertial range energy decouples into both an Alfvénic cascade component, governed by the reduced MHD (rMHD) equations (see §1.3.1.2), and a passive cascade of compressive components (density and magnetic field fluctuations), governed by a linear kinetic equation. Together, the inertial range can be described by the hybrid fluid kinetic reduced MHD (KRMHD) equations (Schekochihin et al., 2009). The dissipation range also separates into two cascades: some Alfvén waves channel their energy into KAWs governed by the electron reduced MHD (ERMHD) equations and are eventually Landau damped by electrons to cause electron heating. The other cascade is passive ion entropy that ion Landau damps to heat ions. Through their notion of a generalized entropy cascade, Schekochihin et al. (2009) explained that a kinetic cascade to a collisional scale eventually mixes the energy nonlinearly in phase-space, which then leads to heating. For weakly collisional plasma, a phase-space with a large gradient in both velocity and space is required for collisions to occur and cause irreversible heating.

The essential transition at the ion gyroscale dictates how much heat is deposited into the electrons and ions from the energy in the inertial range. Plasmas with different values of $\beta_i$ may change this transition point, since it is postulated (Smith
et al., 2001) that the spectral break occurs at the ion inertial scale, \( d_i = \rho_i / \sqrt{\beta_i} \). However, other studies have shown that this fact still remains unclear (Leamon et al., 2000; Markovskii et al., 2008). The weak correlation between enhanced ion temperature and steeper spectra found by Leamon et al. (1998a), in addition to a weak correlation between steep dissipation range spectra and inertial range cascade rates found by Smith et al. (2006), support the picture of KAW energy channel suppression. This leads to the decoupling of entropy fluctuations, which dominate and heat the ions. Weak KAW cascades correspond to steep energy spectra in the dissipation range. In addition, the anisotropic cascade \( k^{-7/3} \) scaling prediction by Schekochihin et al. (2009) for KAW turbulence agrees with Bale et al. (2005), further supporting their theoretical framework with observational evidence.

Additional spacecraft data analysis by Sahraoui et al. (2009) and Salem et al. (2012) also reinforced the KAW cascade conjecture of Schekochihin et al. (2009). Sahraoui et al. (2009) computed spectrum from burst mode (450 Hz) Cluster II electric and magnetic field data in the solar wind at 1 AU, which allowed for spectral analysis up to 100 Hz, enabling electron-scale physics to be illuminated.\(^{17}\) Assuming the Taylor hypothesis, Doppler-shifted ion and electron gyroscales identified spectral break points at 0.4 and 35 Hz, respectively. Below the doppler-shifted ion gyroscale, a Kolmogorov \( f^{-1.62} \) scaling fit the spectrum; above it, the spectrum steepened to a scaling of \( f^{-2.3} \) until the fluctuations reached the Doppler-shifted electron gyroscale.\(^{17}\) Note that this study was an interval in the foreshock, and subsequent studies in the pristine solar wind showed steeper dissipation range magnetic energy spectra (closer to \( f^{-2.8} \)).
Beyond this scale, the spectrum steepened further to $f^{-4.1}$ until the instrumental noise floor was reached. This provided the first direct evidence in solar wind turbulence observations of the dissipation range at electron scales. The properties of the observed dissipation range agrees with the KAW cascade from theoretical predictions of Schekochihin et al. (2009) and Howes et al. (2008). Only a small amount of damping was detected at the ion gyroscale which enabled a KAW cascade to be observed in perpendicular direction that scales as $f^{-2.3}$. Furthermore, Sahraoui et al. (2009) showed that the spectrum steepening at electron scales was due to electron Landau damping of KAW modes. They did this by performing numerical calculations of Vlasov-Maxwell equations and including the observed plasma properties as parameters. The assumption of different dissipation mechanisms leads to various theoretical predictions of the scaling in the dissipation range.

Salem et al. (2012) compared Cluster II electric and magnetic field data to theoretical predictions of both KAW and whistler mode cascades to see which mechanism matched the observations. Numerical simulations (Howes et al., 2011) support the validity of applying linear theory to the dissipation regime. Salem et al. (2012) computed spacecraft-frame frequency spectra, $(|\delta E|/|\delta B|)_{s/c}$ and $(|\delta B_{\|}/|\delta B|)_{s/c}$, from linear eigenfunctions of both KAWs and whistler waves. These measurable quantities that can be used to characterize the compressive fluctuations (Chaston et al., 2009; Gary and Smith, 2009), demonstrated that the spacecraft observations of dissipative-scale turbulence showed agreement with the KAW cascade and not whistler dispersion. Sahraoui et al. (2009) found agreement with KAWs as well, but Salem et al.
(2012) argued that Sahraoui et al. (2009) could not rule out whistler waves, since they invoked the Taylor hypothesis in their data analysis.

1.4.4 Observational Constraints

Salem et al. (2012) outlined several reasons why in situ measurements of solar wind measurements are hard to interpret, which coincides with caveats mentioned by Howes et al. (2008). The main challenge is that measurements are only made at a single or a few points; only the angle between $v_{sw}$ and $B_0$ can be obtained from single points. Additional assumptions must be made to characterize the turbulence using this parameter. Single-point measurements cannot separate the spatial fluctuations measured in the super-Alfvénic moving spacecraft-frame and the plasma-frame temporal fluctuations. However, theoretical predictions can assume a model for the turbulence in the plasma-frame and then predict what it would look like in the spacecraft-frame. This procedure then allows for direct comparison to the spacecraft observations to test which theory matches. Another issue is that the dissipation-scale turbulent fluctuations in question approach the instrumental noise floor and could distort the data without careful filtering and knowledge of the noise floor. Salem et al. (2012) took caution by using suggestions by Bale et al. (2005) for a minimum instrumental noise floor. In addition, they used a wavelet notch filtering technique to remove false spikes in the data due to spacecraft effects, such as interferences with other instruments and harmonics at the spacecraft spinning frequency. Through these data analysis techniques, Salem et al. (2012) also showed agreement with Bale et al.
(2005) and Chen et al. (2011), that there is a strong correlation between the electric and magnetic field spectra.

Motivated by the constraint of extracting information from a single point in space, the pioneering work of Klein and Howes (2016) addressed how one could diagnose and distinguish turbulence dissipation mechanisms using single-point measurements from modern spacecraft observations. The Field-Particle Correlation (FPC) Technique is a novel procedure that can diagnose and distinguish various particle energization mechanisms by characterizing their unique velocity-space signature (Klein and Howes, 2016; Howes et al., 2017; Klein et al., 2017). The Lorentz force term of the Vlasov-Maxwell equations dictates the secular transfer of energy between fields and particles (Klein and Howes, 2016). The energy transfer at a single point, \( \mathbf{r}_0 = (x_0, y_0, z_0) \), is determined as a function of velocity-space, \( \mathbf{v} = (v_\parallel, v_\perp) \), by the correlation between the electric field, \( \mathbf{E} = (E_\parallel, E_\perp) \), and particles, quantified by

\[
C_\tau'(q_s \mathbf{v}_s \mathbf{f}_s(\mathbf{r}_0, \mathbf{v}, t), \mathbf{E}(\mathbf{r}_0, t)) = \frac{1}{\tau} \int_{t-\tau/2}^{t+\tau/2} dt \ q_s \mathbf{f}_s(\mathbf{r}_0, \mathbf{v}, t) \mathbf{v} \cdot \mathbf{E}(\mathbf{r}_0, t) \tag{1.8}
\]

over a sufficiently long time period, \( \tau \), so that the oscillating transfer of energy associated with undamped waves averages to zero, thereby isolating the secular energy transfer that leads to particle energization (Howes et al., 2017). The different parallel or perpendicular components of \( \mathbf{v} \) and \( \mathbf{E} \) can be separated to explore different mechanisms. Note that, when integrated over all velocity space, Eq. 1.8 simply gives the rate of work done by the electric field on the particles. There are several caveats for using the FPC technique in practice (Howes et al., 2017). I will later review these caveats in Chapter 3 and provide a more detailed derivation of the FPC technique.
1.5 Outline of Dissertation Goals

The gyrokinetic approximation derived by Howes et al. (2006), and discussed in §1.4.2.2, enabled computation of an analytical solution for the evolution of Alfvén wave collisions (AWCs) in the weakly nonlinear limit (Howes and Nielson, 2013), which has been validated by nonlinear gyrokinetic numerical simulations (Nielson et al., 2013) and verified in the laboratory (Howes et al., 2012, 2013; Drake et al., 2013). The concept of AWCs being the “Fundamental Building Block of Plasma Turbulence,” suggested by the earlier works of Iroshnikov (1963); Kraichnan (1965), motivates the study of AWCs as a way to study plasma turbulence from first principles. Chapter 2 is dedicated to presenting original research expanding on this topic using the 3D gyrokinetic turbulence simulation code, AstroGK. The simulation details of AstroGK are briefly presented in §2.1.2 and data analysis from AstroGK output is performed throughout this dissertation as a tool to understand plasma turbulence. The goal of Chapter 2 is to further characterize the turbulent cascade of energy to smaller perpendicular scales by analyzing the nonlinear dynamics occurring through successive AWCs. In addition, I verify the work of Howes (2016), that current sheets are self-consistently generated by AWCs in the strongly nonlinear limit. I also identify the nonlinearly generated energy mediation modes (between AWCs) as Alfvén waves themselves, thereby simplifying the strongly nonlinear turbulent cascade to a purely Alfvénic representation.

Chapter 3 of this thesis is dedicated to applying the Field-Particle Correlation (FPC) technique, discussed in §1.4.4, to devise an algorithm, called PATCH (Par-
particle Arrival Time Correlation for Heliophysics), to perform the FPC calculations onboard modern spacecraft. Instruments onboard spacecraft can constantly sample high-resolution data, but due to telemetry restrictions, only a small fraction can be transmitted to the ground for analysis. By calculating FPCs onboard, we would use the higher resolution measurements to better characterize small-scale kinetic signatures of dissipation. This post-processed data would then be sent back to Earth as a form of data compression, potentially maximizing the scientific data return by several orders of magnitude. The PATCH algorithm utilizes the discrete arrival times of particles and correlates the field measurements at that time. In order to test this method, AstroGK is again utilized to generate an artificial time-series of Poisson-distributed particle counts as would be seen realistically on modern spacecraft instruments. A Poisson noise model is constructed in order to diagnose threshold particle count rates for resolving ion Landau damping on modern spacecraft such as Parker Solar Probe (PSP).

Conclusions are summarized in Chapter 4, in addition to a discussion of future work. The data downlinked from PSP is an ideal testbed for realizing the PATCH algorithm in a more practical setting. This would then allow for exploration of possible plans for implementation of the PATCH algorithm to build an onboard wave-particle correlator on a future mission where diagnosing particle energization mechanisms via wave-particle interactions is of the main science goals.
CHAPTER 2
THE FUNDAMENTAL BUILDING BLOCK OF PLASMA TURBULENCE

This chapter investigates the nature of the turbulent cascade of energy to smaller perpendicular spatial scales, the first question posed in the introduction. Leonardo da Vinci’s drawing in Fig. 1.1 may have been the first use of a scientific visualization as a tool to understand turbulence. With similar intent, the first part of this chapter reveals insights about the turbulent energy cascade from producing a 3D visualization of successive Alfvén wave collisions, the fundamental building block of plasma turbulence. For the more realistic case of the collision between two initially separated Alfvén wavepackets, we use a nonlinear gyrokinetic simulation to show here that these key properties persist: strong Alfvén wavepacket collisions indeed facilitate the perpendicular cascade of energy and give rise to current sheets. Furthermore, the evolution shows that nonlinear interactions occur only while the wavepackets overlap, followed by a clean separation of the wavepackets with straight uniform magnetic fields and the cessation of nonlinear evolution in between collisions, even in the gyrokinetic simulation presented here which resolves dispersive and kinetic effects beyond the reach of the MHD theory. The second part of this chapter unveils results from several case studies of Alfvén wave collisions with different initial conditions to see how the energy transfer is mediated between successive collisions. Our results reveal that in the more realistic case of localized Alfvén wave collisions (rather than the periodic case), all nonlinearly generated fluctuations are Alfvén waves, which
mediates nonlinear energy transfer to smaller perpendicular scales.

2.1 Part I: Alfvén Wave Collisions in the Strongly Nonlinear Limit

2.1.1 Overview of Previous Work

Early research on incompressible magnetohydrodynamics (MHD) turbulence in the 1960s (Iroshnikov, 1963; Kraichnan, 1965) suggested that nonlinear interactions between counterpropagating Alfvén waves—or Alfvén wave collisions—support the turbulent cascade of energy from large to small scales. Following significant previous studies on weak incompressible MHD turbulence (Sridhar and Goldreich, 1994; Montgomery and Matthaeus, 1995; Ng and Bhattacharjee, 1996; Galtier et al., 2000), recent work has elucidated the mechanism of energy transfer in Alfvén wave collisions in the weakly nonlinear limit by computing an asymptotic analytical solution using incompressible MHD (Howes and Nielson, 2013), verifying that solution using nonlinear gyrokinetic simulations in the MHD limit of perpendicular scales larger than the Larmor radius, $k_{\perp} \rho_i \gg 1$ (Nielson et al., 2013), and confirming the results experimentally in the laboratory (Howes et al., 2012, 2013; Drake et al., 2013, 2014, 2016). The derivation of the analytical solution was possible using the idealized initial conditions of two overlapping, perpendicularly polarized Alfvén waves in a periodic geometry, and solving for the nonlinear evolution of the system. As discussed in §1.4.2.3, for initial plane Alfvén waves with wavevectors $\mathbf{k}_1^+ = k_{\perp} \hat{x} - k_{\parallel} \hat{z}$ and $\mathbf{k}_1^- = k_{\perp} \hat{y} + k_{\parallel} \hat{z}$, the nonlinear energy transfer is mediated by a nonlinearly generated, purely magnetic mode with wavevector $\mathbf{k}_2^{(0)} = k_{\perp} \hat{x} + k_{\perp} \hat{y}$, which can be interpreted as an oscillating
shear in the magnetic field along which the Alfvén waves propagate (Maron and Goldreich, 2001; Howes and Bourouaine, 2017). The nonlinear interaction between \( k_1^\pm \) and \( k_2^{(0)} \) yields a secular transfer of energy from the \( k_1^+ \) Alfvén wave to an Alfvén wave with \( k_3^+ = 2k_\perp \hat{x} + k_\perp \hat{y} - k_\parallel \hat{z} \), and from the \( k_1^- \) Alfvén wave to an Alfvén wave with \( k_3^- = k_\perp \hat{x} + 2k_\perp \hat{y} + k_\parallel \hat{z} \). Since the energy is transferred to an Alfvén wave with a higher perpendicular wavenumber, \( |k_3^\pm| > |k_1^\perp| \), this interaction represents the fundamental mechanism by which turbulence transfers energy from larger to smaller scales.

Another important discovery about plasma turbulence followed from the finding that the nonlinear evolution of MHD turbulence simulations leads to the development of intermittent current sheets (Matthaeus and Montgomery, 1980; Meneguzzi et al., 1981), and that the dissipation of turbulent energy is found to be largely concentrated in the vicinity of these intermittent current sheets (Uritsky et al., 2010; Osman et al., 2011; Zhdankin et al., 2013). This finding has motivated significant recent efforts to seek evidence of the spatial localization of plasma heating by the dissipation of turbulence in current sheets through statistical analyses of solar wind observations (Osman et al., 2011; Borovsky and Denton, 2011; Osman et al., 2012; Perri et al., 2012; Wang et al., 2013; Wu et al., 2013; Osman et al., 2014c) and numerical simulations (Wan et al., 2012; Karimabadi et al., 2013; TenBarge and Howes, 2013; Wu et al., 2013; Zhdankin et al., 2013). Although these works clearly demonstrate a connection between current sheets and plasma heating, the origin of these current sheets in plasma turbulence remains unknown: do they represent advected
flux tube boundaries (Borovsky, 2008, 2010), or are they generated dynamically by
the turbulence itself (Boldyrev et al., 2011; Zhdankin et al., 2012)? A significant
breakthrough on this question was the discovery that Alfvén wave collisions in the
strong turbulence limit naturally generate current sheets (Howes, 2016), making a
connection for the first time between the nonlinear mechanism governing the transfer
of energy to small scales and the self-consistent development of intermittent current
sheets.

The analytical solution for energy transfer in weak Alfvén wave collisions
(Howes and Nielson, 2013) and the simulations showing that strong Alfvén wave
collisions naturally generate current sheets (Howes, 2016) were based on an ideal-
ized initial condition in which two finite-amplitude, plane Alfvén waves are initially
overlapping in a periodic geometry before they begin to interact nonlinearly. Here
we eliminate the unrealistic aspect of those studies by simulating the strong nonlin-
ear interactions between two initially separated Alfvén wavepackets, with the aim to
determine whether the general properties of Alfvén wave collisions found in the ide-
alized case persist in this more realistic case of colliding wavepackets. Specifically, we
focus here on answering two questions: (i) Do collisions between Alfvén wavepackets
still mediate the transfer of energy to small perpendicular scales?; and (ii) Do Alfvén
wavepacket collisions in the strongly nonlinear limit still lead to the development of
intermittent current sheets?

In §2.1.2, we describe the setup of this strong, localized Alfvén wavepacket
collision simulation. The nonlinear evolution of this simulation is analyzed in §2.1.3,
with particular emphasis on the perpendicular cascade of energy in §2.1.3.1, current sheet development in §2.1.3.2, evolution of the energy in §2.1.3.3, and general qualitative properties of localized Alfvén wavepacket collisions in §2.1.3.4. §2.2 will investigate whether the nonlinearly generated \((k_x/k_\perp, k_y/k_\perp, k_z/k_\parallel) = (1, 1, 0)\) mode still mediates the energy transfer in the weakly collisional limit of Alfvén wavepacket collisions.

### 2.1.2 Simulation

Here we employ the Astrophysical Gyrokinetics code **AstroGK** (Numata et al., 2010) to perform a gyrokinetic simulation of the nonlinear interaction between two initially separated, counterpropagating Alfvén wavepackets in the strongly nonlinear limit.

**AstroGK** evolves the perturbed gyroaveraged distribution function \(h_s(x, y, z, \lambda, \varepsilon)\) for each species \(s\), the scalar potential \(\varphi\), the parallel vector potential \(A_\parallel\), and the parallel magnetic field perturbation \(\delta B_\parallel\) according to the gyrokinetic equation and the gyroaveraged Maxwell’s equations (Frieman and Chen, 1982; Howes et al., 2006). Velocity space coordinates are \(\lambda = v_\perp^2/v^2\) and \(\varepsilon = v^2/2\). The domain is a periodic box of size \(L_\perp^2 \times L_z\), elongated along the straight, uniform mean magnetic field \(B_0 = B_0\hat{z}\), where all quantities may be rescaled to any parallel dimension satisfying \(L_z/L_\perp \gg 1\). Uniform Maxwellian equilibria for ions (protons) and electrons are chosen, with a realistic mass ratio \(m_i/m_e = 1836\). Spatial dimensions \((x, y)\) perpendicular to the mean field are treated pseudospectrally; an upwind finite-difference
Figure 2.1: Schematic of the initial conditions specifying the two perpendicularly polarized, counterpropagating Alfvén wavepackets localized within the periodic domain. Plotted is the $z$-dependence of the normalized amplitudes of the perpendicular magnetic field perturbation $(\delta B_y/B_0)(a_0/\rho_0)$ for the unipolar wavepacket (red) and of the perpendicular magnetic field perturbation $(\delta B_x/B_0)(a_0/\rho_0)$ for the dipolar wavepacket (blue). The unipolar wavepacket has a perpendicular wavevector $k_{-\perp} = (k_x L_{\perp}, k_y L_{\perp}) = (1, 0)$ and the dipolar wavepacket has $k_{\perp}^+ = (0, 1)$.

scheme is used in the parallel direction, $z$. Collisions employ a fully conservative, linearized collision operator with energy diffusion and pitch-angle scattering (Abel et al., 2008; Barnes et al., 2009).

The functional forms along $z$ of the initial Alfvén wavepackets used in this simulation are shown in Fig. 2.1. Initially at $z < 0$ is a wavepacket with a magnetic field perturbation $\delta B_y$ polarized in the $y$ direction, a unipolar variation in $z$ (red), and a perpendicular structure with wavenumber $k_{-\perp} = (k_x L_{\perp}, k_y L_{\perp}) = (1, 0)$. The eigenfunction dictating the different field components and perturbed distribution functions for this Alfvén wavepacket is determined by solving the linear, collisionless, gyrokinetic dispersion relation (Howes et al., 2006) for an Alfvén mode with the chosen perpendicular Fourier wavevector $(k_x, k_y)$ to obtain the complex eigenfrequency $\omega$, the complex Fourier coefficients for the eigenfunctions of the electromagnetic poten-
tials $\hat{\phi}$, $\hat{A}_\parallel$ and $\delta \hat{B}_\parallel$, and the complex perturbed gyrokinetic distribution functions for the ions $\hat{h}_i(v_\parallel, v_\perp)$ and electrons $\hat{h}_e(v_\parallel, v_\perp)$, where the hat symbol denotes the $(k_x, k_y)$ Fourier coefficient (Nielson et al., 2013). The procedure for localizing this Alfvén wavepacket in the $z$ direction is described in the Appendix of Verniero et al. (2018). Through this procedure, this unipolar Alfvén wavepacket in Fig. 2.1 propagates in the $+z$ direction. The other Alfvén wavepacket, initially at $z > 0$, has a magnetic field perturbation $\delta B_x$ polarized in the $x$ direction, a dipolar structure in $z$ (blue), and a perpendicular structure with wavenumber $k_+^\perp = (k_x L_\perp, k_y L_\perp) = (0, 1)$; the eigenfunction specified by the same procedure dictates that this dipolar wavepacket propagates in the $-z$ direction. Note that these unsymmetric initial conditions were chosen to further test the complexities in Alfvén wave collisions. In particular by having a unipolar wavepacket, the collision does not shear and unshear in an oscillatory manner as it does in the interaction between two dipolar wavepackets.

The plasma parameters for this strong Alfvén wavepacket collision simulation are ion plasma beta $\beta_i = 1$ and ion-to-electron temperature ratio $T_i/T_e = 1$. To study the nonlinear evolution of this Alfvén wavepacket collision in the limit $k_\perp \rho_i \ll 1$, we choose a perpendicular simulation domain size $L_\perp = 40\pi \rho_i$ with simulation resolution $(n_x, n_y, n_z, n_\lambda, n_\epsilon, n_s) = (64, 64, 128, 32, 32, 2)$. Therefore, the initial Alfvén wavepackets have perpendicular wavevectors $k_\perp^- = (k_x \rho_i, k_y \rho_i) = (0.05, 0)$ for the unipolar wavepacket and $k_\perp^+ = (k_x \rho_i, k_y \rho_i) = (0, 0.05)$ for the dipolar wavepacket, so both waves have the same initial perpendicular wavenumber $k_\perp^+ \rho_i = 0.05$, but are polarized perpendicular to each other. The fully resolved perpendicular range
in this dealiased pseudospectral method covers $0.05 \leq k_\perp \rho_i \leq 1.05$. Here the ion thermal Larmor radius is $\rho_i = v_{ti}/\Omega_i$, the ion thermal velocity is $v_{ti}^2 = 2T_i/m_i$, the ion cyclotron frequency is $\Omega_i = q_iB_0/(m_ic)$, and the temperature is given in energy units. The parallel length of the simulation domain is $L_z$, extending over the range $[-L_z/2,L_z/2]$. Note that the simulation domain is triply periodic, so when a wavepacket exits the domain at $z = \pm L_z/2$, it re-enters at the opposite end at $z = \mp L_z/2$, enabling these two wavepackets to undergo successive collisions with each other. The linearized Landau collision operator (Abel et al., 2008; Barnes et al., 2009) is employed with collisional coefficients $\nu_i = \nu_e = 10^{-3}$, yielding weakly collisional dynamics with $\nu_s/\omega \ll 1$.

The amplitude of the initial wavepackets is parameterized by the nonlinearity parameter (Goldreich and Sridhar, 1995), defined by taking the ratio of the magnitudes of the linear to the nonlinear terms in the incompressible MHD equations (Howes and Nielson, 2013; Nielson et al., 2013). In terms of Elsasser variables, defined by $z^\pm = u \pm \delta B/\sqrt{4\pi(n_0im_i + n_0em_e)}$, the nonlinearity parameter is defined by $\chi^\pm \equiv |z^\pm \cdot \nabla z^\pm|/|v_A \cdot \nabla z^\pm|$, where $\chi^\pm$ characterizes the strength of the nonlinear distortion of the $z^\pm$ Alfvén wave by the counterpropagating $z^\mp$ Alfvén wave. For the particular initial Alfvén wavepackets shown in Fig. 2.1, the nonlinearity parameter simplifies to $\chi^\pm = 2k_\perp \delta B^\pm_\perp/(k_\parallel B_0)$. With the $z^\pm$ wavepackets having parallel wavenumbers of approximately $k_\parallel a_0 = \mp 3$, where $a_0 = L_z/2\pi$, the amplitude of the unipolar wavepacket $(\delta B^-_\perp/B_0)(a_0/\rho_i) \simeq 60$ gives $\chi^+ = 2$ and the amplitude of the dipolar wavepacket $(\delta B^+_\perp/B_0)(a_0/\rho_i) \simeq 40$ gives $\chi^- = 1.3$. Strong, critical balanced
Figure 2.2: Three-dimensional isocontours of the normalized parallel current density $j_z/j_0$ between Alfvén wavepacket collisions at (a) $t/T_c = 0$, (c) $t/T_c = 1$, and (e) $t/T_c = 2$ and at the midpoint of collisions at (b) $t/T_c = 0.5$, (d) $t/T_c = 1.5$, and (f) $t/T_c = 2.5$.

turbulence (Goldreich and Sridhar, 1995) corresponds to a nonlinearity parameter of $\chi \sim 1$, so this simulation falls into the desired limit of strong Alfvén wavepacket collisions.

2.1.3 Evolution of the Nonlinear Interaction

The basic evolution of this strong Alfvén wavepacket collision simulation is illustrated by three-dimensional contour plots of the parallel current density $j_z$ asso-
ciated with each of the interacting wavepackets, shown in Fig. 2.2 and displayed as a movie embedded in Fig. 3 of Verniero et al. (2018). Time is normalized in terms of the time for a single Alfvén wavepacket collision, $T_c$, during which the initially separated Alfvén wavepackets approach each other along $z$ (with the $+z$ direction from left to right in Fig. 2.2), overlap and interact nonlinearly, and then move away from each other after the collision. At $t = 0$, the unipolar wavepacket is centered at $z = -L_z/4$, and the dipolar wavepacket at $z = +L_z/4$; during a collision time $T_c$, each wavepacket propagates at the Alfvén velocity $v_A$ over a parallel distance $L_z/2$. The midpoint of each collision occurs at $t/T_c = 0.5, 1.5, 2.5$.

In Fig. 2.2(a) are plotted isocontours of the normalized parallel current density $j_z/j_0$ for each of the wavepackets at $t = 0$, where the current density is normalized by $j_0 = n_0 q_i v_t L_\perp / L_z$. It is clear that the unipolar wavepacket (at $z < 0$, left side) initially has only perpendicular variation in the $x$ direction, while the dipolar wavepacket (at $z > 0$, right side) initially has only perpendicular variation in the $y$ direction. Fig. 2.2(b) shows the midpoint of the first collision occurring at $z = 0$ and $t/T_c = 0.5$, showing a significantly more complicated perpendicular structure parallel current density $j_z$.

After the first collision at $t/T_c = 1.0$ in Fig. 2.2(c), the unipolar wavepacket, now at $z = +L_z/4$, has gained some variation in the $y$ direction, and the dipolar wavepacket, now at $z = -L_z/4$, has developed variation in the $x$ direction. Each wavepacket has been distorted by passing through, and interacting nonlinearly with, the counterpropagating wavepacket. Mathematically, when expressed in terms of
Fourier modes, the strong Alfvén wavepacket collision has mediated the nonlinear transfer of energy from the two initial perpendicular Fourier modes to other Fourier modes with larger values of $k_\perp$, as shown quantitatively in §2.1.3.1. Therefore this visualization clearly shows the nonlinear cascade of energy to smaller scales in strong Alfvén wavepacket collisions, the fundamental building block of astrophysical plasma turbulence, a key result of this study.

Because the simulation domain is periodic in the $z$ direction, the Alfvén wavepackets undergo a second collision at the boundary of the domain $z = \pm L_z/2$ at $t/T_c = 1.5$, shown in Fig. 2.2(d), followed by a third collision at $z = 0$ at $t/T_c = 2.5$, shown in Fig. 2.2(f). Below we explore in more detail the cascade of energy to smaller perpendicular scales, the development of current sheets, the evolution of the energy in perpendicular Fourier modes, and key properties of localized Alfvén wavepacket collisions.

### 2.1.3.1 The Perpendicular Cascade of Energy

To explore the nonlinear cascade of smaller perpendicular scales in this strong Alfvén wavepacket collision simulation, we plot in Fig. 2.3 the perpendicular magnetic energy of fluctuations in perpendicular Fourier space integrated over $z$,

$$ E_{B\perp}(k_x, k_y) \equiv \int_{-L_z/2}^{L_z/2} dz |\delta B_{\perp}(k_x, k_y)|^2 / 8\pi $$

in arbitrary units. In Fig. 2.3(a) is the perpendicular magnetic energy of the initial Alfvén wavepackets at $t = 0$, showing all of the energy is contained within the three perpendicular Fourier modes $(k_x \rho_i, k_y \rho_i) = (1, 0), (0, 1), \text{ and } (-1, 0)$. Note that AstroGK uses a reality condi-
Figure 2.3: Plots of the perpendicular magnetic energy $E_{B\perp(k_x,k_y)}$ (arbitrary units) on a log scale in the perpendicular Fourier plane $(k_x, k_y)$ (a) at the initial time $t/T_c = 0$, (b) after the first strong Alfvén wavepacket collision at $t/T_c = 1$, and (c) after the second strong Alfvén wavepacket collision at $t/T_c = 2$. 
tion imposed on the complex Fourier coefficients such that the magnetic field can be described using only Fourier modes in the upper half-plane \( k_y \geq 0 \). The reality condition requires \( \hat{\delta B}(k_x, k_y) = \hat{\delta B}^*(-k_x, -k_y) \), so if there is power in the \((1, 0)\) mode, there must be equivalent power in the \((-1, 0)\) mode, as seen in Fig. 2.3(a).

After the first collision at \( t/T_c = 1 \), Fig. 2.3(b) shows clearly that the strong nonlinear interactions have transferred significant energy to many other modes in the perpendicular Fourier plane \((k_x, k_y)\). Note that the perpendicular wavenumber is given by \( k_\perp = \sqrt{k_x^2 + k_y^2} \), so that modes further away from the origin represent smaller scale fluctuations in the perpendicular plane. The plot after the second collision at \( t/T_c = 2 \) in Fig. 2.3(c) shows that successive collisions continue to facilitate the perpendicular cascade of energy. These results demonstrate that the finding from the idealized periodic case—that Alfvén wave collisions mediate the nonlinear transfer of energy to smaller perpendicular scales—indeed persists in the more realistic case of localized Alfvén wavepacket collisions, answering the first key question posed in the introduction of this chapter.

### 2.1.3.2 Current Sheet Development

For plane Alfvén wave collisions in the idealized periodic case, plots of the parallel current density \( j_z \) in the \((x, y)\) plane perpendicular to the equilibrium magnetic field demonstrate that strong Alfvén wave collisions self-consistently generate current sheets (Howes, 2016). These current sheets extended the full parallel length of the original Alfvén waves, with widths in the perpendicular plane of approximately
Figure 2.4: Plot of the normalized parallel current density $j_z/j_0$ at $z = 0$ during the first collision at (a) $t/T_c = 0.3$ and (b) $t/T_c = 0.5$, at $z = L_z/2$ during second collision at (c) $t/T_c = 1.3$ and (d) $t/T_c = 1.5$, and at $z = 0$ during the third collision at (a) $t/T_c = 2.3$ and (b) $t/T_c = 2.5$. 
the perpendicular wavelength of the original interacting Alfvén waves, but with a much smaller thickness in the perpendicular plane. Here we use plots of the \((x,y)\) plane perpendicular to the equilibrium magnetic field to determine whether this current sheet formation persists in the more realistic case of strong collisions between localized Alfvén wavepackets.

First, we examine the development of the parallel current density \(j_z\) in the \((x,y)\) plane during the course of each collision. In Fig. 2.4, we plot the normalized parallel current density \(j_z/j_0\) at \(z = 0\) (a) early in the first collision at \(t/T_c = 0.3\) and (b) later in the same collision at \(t/T_c = 0.5\), where we remind the reader that the midpoint of each collision occurs at \(t/T_c = 0.5, 1.5, 2.5\). The nearly circular \(j_z\) pattern in Fig. 2.4(a) is largely due to the linear superposition of the current of each of the initial Alfvén wavepackets (the top row of Fig. 2.5 shows the current patterns of each of the initial Alfvén wavepackets). The nonlinear interaction of the Alfvén wavepackets during the first collision leads to the thinning of these initial circular current patterns into a more sheet-like morphology in Fig. 2.4(b). The second row of Fig. 2.4 shows the evolution of the second collision in the plane \(z = L_z/2\) (at the parallel boundary of the periodic domain) at (c) \(t/T_c = 1.3\) and (d) \(t/T_c = 1.5\). Here we see the current sheets becoming yet more elongated and intense through the course of the second collision. The third row shows plots of \(j_z/j_0\) during the third collision at \(z = 0\), with further thinning and intensification of the current sheets. Note that the three-dimensional isocontours of \(j_z\) in Fig. 2.2 shows that the parallel extent of the current sheets is the same as the parallel length of the original wavepackets.
Figure 2.5: Plot of the normalized parallel current density $j_z/j_0$ of colliding Alfvén wavepackets before the first collision at $t = 0$ for (a) the unipolar wave at $z = -L_z/4$ and (b) the dipolar wave at $z = +L_z/4$, after the first collision when the wavepackets have separated for (c) the unipolar wave at $z = +L_z/4$ and $t/T_c = 0.98$ and (d) the dipolar wave at $z = -L_z/4$ and $t/T_c = 0.96$, and after the second collision for (e) the unipolar wave at $z = -L_z/4$ and $t/T_c = 2.1$ and (f) the dipolar wave at $z = +L_z/4$ and $t/T_c = 2.0$. 

(a) $t/T_c = 0.0$

(b) $t/T_c = 0.0$

(c) $t/T_c = 0.98$

(d) $t/T_c = 0.96$

(e) $t/T_c = 2.1$

(f) $t/T_c = 2.0$
These results show clearly that strong collisions of localized Alfvén wavepackets indeed self-consistently generate intermittent current sheets, answering the second question in the introduction of this chapter.

Another important question, however, is whether the current sheets that develop during the wavepacket collisions persist within each wavepacket after the collision. In Fig. 2.5, we show the initial pattern at $t = 0$ of the normalized parallel current density $j_z/j_0$ for (a) the unipolar wave at $z = -L_z/4$ with only variation in the $x$ direction and (b) the dipolar wave at $z = +L_z/4$ with only variation in the $y$ direction. After the first collision, we show $j_z$ for (c) the unipolar wave at $z = +L_z/4$ and $t/T_c = 0.98$ and (d) the dipolar wave at $z = -L_z/4$ and $t/T_c = 0.96$. At this time when the wavepackets are no longer overlapping in $z$, one can clearly see the distortion of current density pattern, due to the previous collision, has lead to a thinning and intensification of the current density into a more sheet-like morphology that persists after the wavepackets have separated. After the second collision, we plot (e) the unipolar wave at $z = -L_z/4$ and $t/T_c = 2.1$ and (f) the dipolar wave at $z = +L_z/4$ and $t/T_c = 2.0$. Here we can see that the current sheets that develop as a consequence of the strong Alfvén wavepacket collisions indeed persist within the wavepackets after the collision is over, showing that these interactions in the more realistic case may explain the ubiquitous observations of intermittent current sheets in plasma turbulence.
2.1.3.3 Evolution of Energy

A simple physical interpretation of the evolution of the energy in this localized Alfvén wavepacket collision can be developed using the equations of incompressible MHD for guidance. Recalling §1.3.1, these equations, written in terms of Elsasser variables, take the form

\[ \frac{\partial z^\pm}{\partial t} \mp \mathbf{v}_A \cdot \nabla z^\pm = -z^\mp \cdot \nabla z^\pm - \nabla P/(n_0i_m + n_0e_m), \] (2.1)

and \( \nabla \cdot z^\pm = 0 \). Here \( \mathbf{v}_A = B_0/\sqrt{4\pi(n_0i_m + n_0e_m)} \) is the Alfvén velocity due to the equilibrium field \( B_0 = B_0\hat{z} \) where \( \mathbf{B} = B_0 + \delta \mathbf{B} \), \( P \) is total pressure (thermal plus magnetic), and \( n_0i_m + n_0e_m \) is mass density. Recall that the Elsasser variables are defined by \( z^\pm = u \pm \delta \mathbf{B}/\sqrt{4\pi(n_0i_m + n_0e_m)} \), representing waves that propagate up or down the mean magnetic field. The nonlinear term, \( z^\mp \cdot \nabla z^\pm \), governs the nonlinear interactions of the counterpropagating Alfvén wave collisions.

In Fig. 2.6, we plot the temporal evolution of the magnetic energy \( E_{B_\perp}(k_x,k_y) \) in some of the lowest perpendicular Fourier modes \((k_x,k_y)\), illustrating some of the key properties of Alfvén wavepacket collisions. The first main point is that there is no nonlinear transfer of energy among Fourier modes until the two wavepackets begin to overlap in space along \( z \) (first vertical dashed line). As expected from theoretical considerations of incompressible MHD turbulence (Iroshnikov, 1963; Kraichnan, 1965; Sridhar and Goldreich, 1994; Montgomery and Matthaeus, 1995; Ng and Bhattacharjee, 1996; Galtier et al., 2000; Howes and Nielson, 2013), the nonlinear interaction is zero unless both \( z^- \) (the unipolar wavepacket) and \( z^+ \) (the dipolar wavepacket) are both non-zero at the same point in space, as can be seen by inspection of the
Figure 2.6: Plot of the evolution of energy transfer between \((k_x, k_y)\) modes. Note that the \((1,0)\) mode is plotted as the same line as the cyan \((-1,0)\) mode, indicating they are identical.
nonlinear term $z^\mp \cdot \nabla z^\pm$. Between collisions, when the wavepackets have no overlap in $z$, there is no nonlinear transfer of energy among different $(k_x, k_y)$ modes, as clearly seen in Fig. 2.6. The second main point is that the perpendicular Fourier modes $(k_x L_{\perp}, k_y L_{\perp})$ of the initial Alfvén wavepackets, $(1, 0)$ and $(0, 1)$ generally lose energy to nonlinearly generated perpendicular Fourier modes during each collision. This is the transfer of energy to smaller perpendicular scales, illustrated by the plots of perpendicular magnetic energy in $(k_x, k_y)$ Fourier space in Fig. 2.3. A more quantitative examination contrasting the nonlinear transfer of energy between the idealized plane Alfvén wave collisions in a periodic geometry and localized Alfvén wavepackets collisions, in both the weakly and strongly nonlinear limits, will be presented in §2.2.

2.1.3.4 Visualizing Wavepacket Collisions

The three-dimensional visualization of this localized Alfvén wavepacket collision in Fig. 2.2 presents a concise overview of many of the properties examined above. The series of panels (a) through (c) show clearly the distortion of the original wavepackets that arises during the collision and persists after the wavepackets have separated again. This is a physical visualization of the nonlinear cascade of energy to smaller scales, the key characteristic of turbulence in space and astrophysical plasmas. Although a little difficult to see clearly in the 3D projection in Fig. 2.2, the nonlinear evolution of this strong Alfvén wavepacket collision also leads to the self-consistent generation of a sheet-like morphology for the regions of intense parallel current density $j_z$. This thinning and intensifying of the current into sheets is more clearly seen
in the perpendicular cross section at \( z = L_z/2 \) (the right-hand \( z \) boundary of the simulation domain) in Fig. 2.2(d); the same cross section at \( z = L_z/2 \) is also shown in Fig. 2.4(d).

In addition to the isocontours of the normalized parallel current density \( j_z/j_0 \), we also plot in Fig. 2.2 the paths of a sample of magnetic field lines as they traverse the domain. Beginning at points on a \( 5 \times 5 \) grid at \( z = -L_z/2 \), we trace the field lines in the \( +z \) direction. Although the perpendicular magnetic field fluctuation associated with each Alfvén wavepacket is small compared to the equilibrium magnetic field, \( \delta B_\perp \ll B_0 \), one can still see the distortion of the field lines as they pass through each wavepacket. In the space between wavepackets, both before and after each collision, the magnetic field lines are straight and uniform, further illustrating the point that nonlinear evolution ceases when the wavepackets are separated, even in a gyrokinetic simulation where dispersive and kinetic effects are resolved.

It is worthwhile also noting the important point that, even after each strong Alfvén wavepacket collision, the individual wavepackets continue to propagate along the equilibrium magnetic field and they remain localized in \( z \). There is a very small spreading of the wavepacket, hardly noticeable in Fig. 2.2, due to the fact that Alfvén waves become dispersive at \( k_\perp \rho_i \gtrsim 1 \), with an increasing parallel group velocity. Therefore, some of the higher \( k_\perp \) modes that are non linearly generated by the strong Alfvén wavepacket collision in this gyrokinetic simulation—a numerical approach that resolves these dispersive kinetic effects at \( k_\perp \rho_i \gtrsim 1 \)—will propagate a little faster than Alfvén waves in the MHD limit \( k_\perp \rho_i \ll 1 \) that have a parallel group velocity at the
Alfvén speed $v_{g||} = v_A$, causing a slight spreading out of the localized wavepacket.

2.2 Part II: The Alfvénic Nature of Energy Transfer Mediation

2.2.1 Overview of Further Investigation of Energy Transfer Dynamics

The previous idealized work on Alfvén wave collisions (Howes and Nielson, 2013; Nielson et al., 2013) explored the nonlinear interactions between two perpendicularly polarized, counterpropagating plane Alfvén waves under periodic boundary conditions. These two plane Alfvén waves were initially overlapping before they began to interact non-linearly, an unrealistic, idealized set up that enabled an asymptotic analytical solution to be obtained in the weakly nonlinear limit. A depiction of the initial conditions in this case is shown in Fig. 2.7(a), where the variation along the direction $z$ (parallel to the equilibrium magnetic field) for each of the two initial, perpendicularly polarized Alfvén waves is plotted. The upward propagating Alfvén wave has a $\delta B_y$ polarization with a perpendicular Fourier mode $(1,0)$ (blue) and the downward propagating Alfvén wave has a $\delta B_z$ polarization with a perpendicular Fourier mode $(0,1)$ (red). Note these initial plane Alfvén wave modes fill the simulation domain and are periodic in both the perpendicular plane as well as the parallel direction. We refer to this Alfvén wave initialization as the periodic case. Note that the periodic boundary conditions are not what makes this scenario unrealistic, but rather the fact that the two waves started on top of each other and consequently did not arrive in those positions while undergoing a self-consistent nonlinear interaction.

An important question is whether the key properties of the nonlinear evolution
of Alfvén wave collisions found in this idealized periodic case persists for the more realistic case of the interaction between two initially separated Alfvén wavepackets. To answer this question, we perform nonlinear kinetic simulations of the interaction between two localized Alfvén wavepackets that do not initially overlap, as shown in Fig. 2.7(b). Here, the upward propagating Alfvén wave has a $\delta B_y$ polarization but the wavepacket is localized along the field parallel direction around $z = -L_z/4$. Note that this Alfvén wavepacket remains periodic in the perpendicular plane, with its variation given by the Fourier mode $(1, 0)$ (blue). The downward propagating Alfvén wave has a $\delta B_z$ polarization, is localized in $z$ around $z = L_z/4$, and corresponds to a perpendicular Fourier mode $(0, 1)$ (red). Although the simulation domain itself is periodic in the $z$ direction, such that a wave propagating in the $+z$ direction will exit the domain at $z = L_z/2$ and re-enter the domain at $z = -L_z/2$, the localization of the wavepackets along $z$ means that these two wavepackets will not interact nonlinearly until they come together and overlap along $z$, a more realistic situation. We refer to this initially separated Alfvén wavepacket initialization as the *localized case*.

Our previous study in §2.1 of strongly nonlinear, localized Alfvén wave collisions (Verniero et al., 2018) found that indeed nonlinear interactions between initially separated wavepackets facilitate the cascade of energy to smaller perpendicular scales relative to the background magnetic field and self-consistently give rise to current sheets, just as found in the periodic case. But that study employed asymmetric initial Alfvén wavepackets (see Fig. 2.1), where one of the wavepackets had a significant $k_\parallel = 0$ component initially relative to the background magnetic field. Since it is the
secondary mode with \( k_{\parallel} = 0 \) that plays the key role in mediating the secular transfer of energy to smaller perpendicular scales in the periodic case, it is important to ensure that the non-zero \( k_{\parallel} = 0 \) component of the wavepacket in §2.1 does not affect the results in a fundamental way. To address this issue, we pursue here a detailed comparison of periodic Alfvén wave and localized Alfvén wavepacket collisions, where the initial wavepackets are symmetric and neither wavepacket has a significant \( k_{\parallel} = 0 \) component. This study will enable us to determine the nature of the nonlinearly generated modes that mediate the cascade of energy to smaller perpendicular scales relative to the background magnetic field in the localized case and to ensure that the non-zero \( k_{\parallel} = 0 \) component in the §2.1 study did not qualitatively alter the resulting cascade by artificially initializing a mode that dominates the nonlinear energy transfer.

In the second part of this chapter, we aim to answer two primary questions: (i) What is the nature of the nonlinearly generated secondary mode that mediates the cascade of energy in localized Alfvén wave collisions?; and (ii) How does the localization of the interacting Alfvén waves into separated wavepackets affect the qualitative and quantitative evolution of the perpendicular cascade of energy and the development of current sheets?

In §2.2.2, we describe the setup of the simulation for each of the four cases being compared. The nonlinear energy evolution of each case is presented in §2.2.3. Our results in §2.2.4 show that the secondary (1,1) mode is an Alfvén wave mode. The strongly and weakly nonlinear limits are compared in §2.2.5. Current sheet
development is confirmed in §2.2.6.

2.2.2 Case Study Setup

The nonlinear interaction between two counterpropagating localized Alfvén wavepackets or periodic Alfvén waves is simulated using the Astrophysical Gyrokinetics code AstroGK (Numata et al., 2010), as described in §2.1.2.

To reveal details of the turbulent transfer of energy through the interaction of Alfvén waves, we directly compare four simulations runs:

1. Localized Alfvén wavepacket collisions in the strongly nonlinear limit, LS
2. Periodic Alfvén wave collisions in the strongly nonlinear limit, PS
3. Localized Alfvén wavepacket collisions in the weakly nonlinear limit, LW
4. Periodic Alfvén wave collisions in the weakly nonlinear limit, PW

For all cases, the plasma parameters are ion plasma beta $\beta_i = 1$ and ion-to-electron temperature ratio $T_i/T_e = 1$. We choose a perpendicular simulation domain size $L_\perp = 40\pi\rho_i$ with simulation resolution $(n_x, n_y, n_z, n_\lambda, n_\varepsilon, n_s) = (32, 32, 128, 32, 16, 2)$ such that our initial Alfvén waves fall into the MHD limit, $k_\perp\rho_i \ll 1$. The fully resolved perpendicular range in this dealiased pseudospectral method covers $0.05 \leq k_\perp\rho_i \leq 0.5$. As before, the parallel length of the simulation domain is $L_z$, extending over the range $[-L_z/2, L_z/2]$, and is triply periodic.

The initial Alfvén wavepackets have perpendicular wave vectors
(a) Periodic case in strongly nonlinear limit at $t/T_c=0$

(b) Localized case in strongly nonlinear limit at $t/T_c=0$

Figure 2.7: Setup for perpendicularly polarized Alfvén waves in the localized and periodic cases. Note that the blue curve corresponds to the $(k_x, k_y) = (1,0)$ mode and the red curve corresponds to the $(k_x, k_y) = (0,1)$ mode. Note that the blue and red fluctuations are polarized perpendicularly to each other, with $\delta B_x$ (red) and $\delta B_y$ (blue).
\[ \mathbf{k}_{\perp}^{+}\rho_{i} = (k_{x}\rho_{i}, k_{y}\rho_{i}) = (0.05, 0) \] for the upward \((z^-)\) wavepacket and
\[ \mathbf{k}_{\perp}^{-}\rho_{i} = (k_{x}\rho_{i}, k_{y}\rho_{i}) = (0, 0.05) \] for the downward \((z^+)\) wavepacket, so both waves have the same initial perpendicular wavenumber \(k_{\perp}^{\pm}\rho_{i} = 0.05\), but are polarized perpendicular to each other. For brevity, we will refer to modes normalized to the domain scale perpendicular wave vector \(k_{\perp0} \equiv 2\pi/L_{\perp}\), giving \(k_{\perp}/k_{\perp0} = (k_{x}/k_{\perp0}, k_{y}/k_{\perp0}) = (1, 0)\).

Fig. 2.7 illustrates the initial conditions for both the (a) periodic and (b) localized cases. In panel (a), we plot the waveforms for the periodic cases, which are exactly the same as the localized case but without the application of the windowing function in \(z\), so that the localized and periodic cases are directly comparable. Here we plot the waveforms along the parallel direction \(z\) at \(t = 0\) of the perpendicular Fourier mode \((k_{x}/k_{\perp0}, k_{y}/k_{\perp0}) = (1, 0)\) of \(\delta B_{y}\) (blue) and of the perpendicular Fourier mode \((0, 1)\) of \(\delta B_{x}\) (red) for the localized Alfvén wavepacket case in panel (b). Fig. 2.7 shows the amplitudes for the strongly nonlinear (a) periodic and (b) localized cases; the weakly nonlinear cases have the same initial waveforms but smaller amplitudes.

For the particular initial Alfvén wavepackets shown in Fig. 2.7, the nonlinearity parameter simplifies to \(\chi^{\pm} = 2k_{\perp}\delta B_{\perp}^{\mp}/(k_{\parallel}B_{0})\). With the \(z^{\pm}\) wavepackets having parallel wavenumbers of approximately \(k_{\parallel}a_{0} = \mp 3\), where \(a_{0} = L_{z}/2\pi\), the amplitude of the wavepackets in the strongly nonlinear case \((\delta B_{\perp}^{\pm}/B_{0})(a_{0}/\rho_{i}) \approx 60\) gives \(\chi^{\pm} = 2\) and the amplitude of the wavepackets in the weakly nonlinear case \((\delta B_{\perp}^{\pm}/B_{0})(a_{0}/\rho_{i}) \approx 4\) gives \(\chi^{\pm} = 0.13\). Critically balanced, strong turbulence corresponds to a nonlinearity parameter of \(\chi \sim 1\) (Goldreich and Sridhar, 1995), and weak turbulence corresponds to \(\chi \ll 1\), so these simulations fall into the desired limits of strong and weak nonlin-
earity, respectively.

2.2.3 Evolution of Energy of Secondary (1,1) Mode

The nonlinear evolution of the localized and periodic strong and weak Alfvén wave collisions during the first few collisions is concisely illustrated by a plot of the evolution of the energy in particular perpendicular Fourier modes in Fig. 2.8. A meaningful quantitative comparison between the localized cases and the periodic cases is made possible by selecting comparable energies for each Fourier mode and a suitable definition of the Alfvén wave collision timescale in each case.

First, because the waveform in the \( z \) direction differs between the localized and periodic cases, we choose to integrate the energy of each perpendicular Fourier mode \((k_x/k_{\perp 0}, k_y/k_{\perp 0})\) along the \( z \) direction to facilitate comparison.

Second, we choose to normalize our timescales to the appropriate timescale of a single complete Alfvén wave collision in both the localized and periodic cases. In the localized case, the wavepackets collide twice during the time it takes for an Alfvén wave to propagate the parallel length of the domain, defined by \( T_{L_z} \equiv L_z/v_A \). By comparison, each wavelength in the periodic case passes through three wavelengths of the counterpropagating waves during one wave-crossing period \( T_{L_z} \). Therefore, we define the collision time as \( T_{c}^{(l)} = T_{L_z}/2 \) for the localized Alfvén wavepacket collision case and \( T_{c}^{(p)} = T_{L_z}/3 \) for the periodic Alfvén wave collision case. To further illustrate the evolution in the localized case, note that the first collision begins when the counterpropagating wavepackets begin to overlap in \( z \) at \( t/T_{c}^{(l)} = 1/6 \) and ends
at $t/T_c^{(l)} = 5/6$. Subsequently, the second collision begins at $t/T_c^{(l)} = 7/6$ and ends at $t/T_c^{(l)} = 11/6$.

The temporal evolution of energy of select $(k_x, k_y)$ modes for the first three collisions is shown in Fig. 2.8 while Fig. 2.9 shows the full time evolution of the simulations. To illustrate differences between the periodic and localized cases, we first focus on the weakly nonlinear limit. From panel (d), in the periodic case, the evolution agrees with the analytical solution from Howes and Nielson (2013), as described qualitatively above in §2.1.1. Notice that the secondary (1,1) mode, which mediates the secular transfer of energy to the tertiary (1,2) and (2,1) modes, does not experience a net gain in energy. This (1,1) mode corresponds to the inherently nonlinear fluctuation that does not propagate, as described in §2.2. In contrast, the secondary (1,1) mode of the localized case in panel (c) clearly does gain energy, which is the most consequential difference among all the curves. This means that in the localized case, this secondary mode gains energy like all other nonlinearly generated modes.

One other major distinction between LW and PW is that in LW, energy is only transferred during periods when the wavepackets overlap in $z$, giving the energy evolution curve a stair-step appearance. In contrast, PW has persistent energy transfer since the wavepackets never separate.

Note that convergence studies have been done to verify that the results in this $(n_x, n_y) = (32,32)$ resolution are accurately resolved by the grid in AstroGK. We initially started this experiment using a resolution of $(n_x, n_y) = (10,10)$ and replicated the same results using $(n_x, n_y) = (16,16)$. For the $(n_x, n_y) = (32,32)$ case,
Figure 2.8: Energy evolution of each case for key \((k_x, k_y)\) modes after 3 collisions, for the (a) localized, strongly nonlinear case \(\text{LS}\), (b) periodic, strongly nonlinear case \(\text{PS}\), (c) localized, weakly nonlinear case \(\text{LW}\), and (d) periodic, weakly nonlinear case \(\text{PW}\).
Figure 2.9: Full energy evolution of each case for key \((k_x, k_y)\) modes, for the (a) localized, strongly nonlinear case LS, (b) periodic, strongly nonlinear case PS, (c) localized, weakly nonlinear case LW, and (d) periodic, weakly nonlinear case PW.

- \((1,0), (-1,0), (0,1), (1,2), (2,1), (-2,1), (-1,2), (-1,1), (1,1)\)
we followed the evolution of energy until it deviated from the (16,16) resolution case and ceased the simulation at that point. The results presented in Fig. 2.9 follow the evolution of energy up until the time step of this deviation point for each of the localized and periodic cases in the weakly and strongly nonlinear limit. At the end of the time evolution shown in Fig. 2.9, about 13% of the initial magnetic energy has been transferred nonlinearly to higher $k_\perp$ modes (not shown in the figure) for case LS and about 17% of the initial magnetic energy for case PS.

2.2.4 Identification of Nonlinearly Generated Modes as Alfvén Waves

In the periodic case, as reviewed in the §2.2.1, the secondary (1,1) mode mediates the secular transfer of energy from the primary Alfvén waves to the tertiary Alfvén waves, and this mode is an inherently nonlinear fluctuation that satisfies neither the linear eigenfunction relation nor the linear dispersion relation for an Alfvén wave. For the more realistic case of localized Alfvén wavepacket collisions, we aim to determine here the nature of the secondary (1,1) mode. Specifically, we ask whether this secondary (1,1) mode is an Alfvén wave. A linear Alfvén wave must satisfy two conditions (Howes and Nielson, 2013): (i) it satisfies the linear eigenfunction relation for an Alfvén wave, $u_\perp/v_A = \pm \delta B_\perp/B_0$; and (ii) it has a frequency given by the linear Alfvén wave dispersion relation, $\omega = \pm k_\parallel v_A$. The strongly nonlinear localized case LS is the most relevant to the case of heliospheric plasma turbulence, so we focus strictly on this case below.
2.2.4.1 Alfvén Wave Eigenfunction Relation

To confirm that the fluctuations that are nonlinearly generated by the LS Alfvén wave collisions have the character of linear Alfvén waves, we first verify that the electric and magnetic field fluctuations are related by the following linear eigenfunction relation (Howes and Nielson, 2013)

$$\frac{B_\perp}{B_0} = \pm \frac{cE_\perp}{v_A B_0} \times \hat{z}$$  \hspace{1cm} (2.2)

where the positive sign corresponds to an Alfvén wave travelling down the magnetic field in the $-z$ direction, and the negative sign corresponds to an Alfvén wave travelling up the magnetic field in the $+z$ direction.

Separating the two components perpendicular to the equilibrium magnetic field $B_0 = B_0 \hat{z}$ given by Equation 2.2, we note that Alfvén waves travelling up the magnetic field in the $+z$ direction will satisfy the relations

$$\frac{B_x}{B_0} = -\frac{cE_y}{v_A B_0} \quad \frac{B_y}{B_0} = +\frac{cE_x}{v_A B_0} $$  \hspace{1cm} (2.3)

and that Alfvén waves travelling down the magnetic field in the $-z$ direction will satisfy the relations

$$\frac{B_x}{B_0} = +\frac{cE_y}{v_A B_0} \quad \frac{B_y}{B_0} = -\frac{cE_x}{v_A B_0} $$  \hspace{1cm} (2.4)

For notational simplicity, we use a hat to denote these dimensionless magnetic and electric field components, $\hat{B}_j \equiv B_j/B_0$ and $\hat{E}_j \equiv cE_j/(v_A B_0)$. Note that the propagation direction of the Alfvén wave is easily determined by computing the Poynting flux, $S = (c/4\pi)E \times B$. 
(a) Primary $t/T_c^{(l)}=0$

(b) Primary $t/T_c^{(l)}=1$

(c) Primary $t/T_c^{(l)}=2$

(d) Secondary $t/T_c^{(l)}=0$

(e) Secondary $t/T_c^{(l)}=1$

(f) Secondary $t/T_c^{(l)}=2$

(g) Tertiary $t/T_c^{(l)}=0$

(h) Tertiary $t/T_c^{(l)}=1$

(i) Tertiary $t/T_c^{(l)}=2$

(j) Legend

Figure 2.10: Snapshots in time of $\hat{B}_x$ (blue,dotted), $\hat{E}_y$ (green,dashed), $\hat{B}_y$ (red,dotted), and $\hat{E}_x$ (black,dashed) of select $(k_x, k_y)$ Fourier modes in the LS case. The first, second, and third row corresponds to the primary, secondary, and tertiary modes respectively. All times are normalized to the localized Alfvén collision time, $T_c^{(l)}$. The black arrows indicate the direction of motion of the two colliding wavepackets.
In Fig. 2.10, we present normalized $\hat{E}$ and $\hat{B}$ field components of the primary, secondary, and tertiary modes in the (first, second, and third row, respectively) at times $t/T_c^{(l)} = 0, 1, 2$ (in the first, second, and third columns, respectively). In the first column of Fig. 2.10 at $t/T_c^{(l)} = 0$, we have only (a) the primary Alfvén wavepackets. The upward propagating Alfvén wave has a perpendicular variation given by the $(1,0)$ Fourier mode and has a magnetic field polarization in the $y$ direction. This wavepacket satisfies the normalized eigenfunction for an upward propagating Alfvén wave, $\hat{B}_y = \hat{E}_x$ (red/black). The downward propagating Alfvén wave has a perpendicular variation given by the $(0,1)$ Fourier mode and has a magnetic field polarization in the $x$ direction. This wavepacket satisfies the normalized eigenfunction for a downward propagating Alfvén wave, $\hat{B}_x = \hat{E}_y$ (blue/green). In addition, at $t/T_c^{(l)} = 0$, (d) the secondary $(1,1)$ Fourier mode and (g) the tertiary $(1,2)$ Fourier mode are zero.

In the second column of Fig. 2.10, we show the primary, secondary, and tertiary modes after the first collision at $t/T_c^{(l)} = 1$. In Fig. 2.10(b), the primary Alfvén waves have passed through each other completely and still satisfy the same linear Alfvén wave eigenfunction relations as before the first collision in (a). Shown in panel (e), energy has been transferred to the secondary $(1,1)$ Fourier mode, in two separate localized wavepackets, each with magnetic field components in both the $x$ and $y$ direction. At $z < 0$, the downward propagating wavepacket satisfies the eigenfunction relations $\hat{B}_y = -\hat{E}_x$ (red/black) and $\hat{B}_x = \hat{E}_y$ (blue/green), as expected for a downward travelling Alfvén wave. At $z > 0$, the upward propagating wavepacket satisfies the eigenfunction relations $\hat{B}_y = \hat{E}_x$ (red/black) and $\hat{B}_x = -\hat{E}_y$ (blue/green),
as expected for an upward travelling Alfvén wave. This confirms that this secondary (1,1) mode satisfies the linear Alfvén wave eigenfunction. Shown in panel (h), the tertiary (1,2) Fourier mode also involves two separate localized wavepackets with magnetic field components in both the $x$ and $y$ direction. A close inspection of the curves confirms that this tertiary (1,2) mode also satisfies the linear Alfvén wave eigenfunction.

In the third column of Fig. 2.10, we show the primary, secondary, and tertiary modes after the second collision at $t/T_c^{(l)} = 2$. In panel (c), the upward and downward moving Fourier wavepackets have developed a component of polarization perpendicular to their original polarizations. For instance, the upward wavepacket, which initially (at $t/T_c^{(l)} = 0$) consisted of only a (1,0) Fourier mode with magnetic field polarized in the $y$ direction (red), now has a smaller (0,1) Fourier mode contribution moving in the $+z$ direction that has a magnetic field polarized in the $x$ direction (blue). Similarly, the downward moving wavepacket, originally solely involving a (0,1) Fourier mode polarized in the $x$ direction (blue), now also includes a smaller contribution from a (0,1) mode polarized in the $y$ direction (red). These newly generated contributions to the upward and downward moving wavepackets gained energy through nonlinear energy transfer from other modes during the second collision. The secondary and tertiary modes at $t/T_c^{(l)} = 2$ in panels (f) and (i) also show an increase in amplitude relative to $t/T_c^{(l)} = 1$, showing that nonlinear interactions in the second collision have further transferred energy to those modes from the primary Alfvén wavepackets.
Another way to visualize the upward and downward propagating Alfvén waves is to compute the Elssaser fields, $z^\pm$. Specifically, we write the components of the normalized Elssaser variables for the upward ($z^-$) and downward ($z^+$) Alfvén waves as

$$\hat{z}^\pm_x \equiv \frac{z^\pm_x}{v_A} = \frac{cE_y}{v_A B_0} \pm \frac{\delta B_x}{B_0}$$

and

$$\hat{z}^\pm_y \equiv \frac{z^\pm_y}{v_A} = -\frac{cE_x}{v_A B_0} \pm \frac{\delta B_y}{B_0}.$$  \hspace{1cm} (2.5)

In Fig. 2.11, we plot the downward travelling Elsasser components $z^+_x$ (black) and $z^+_y$ (red) and the upward travelling Elsasser components $z^-_x$ (green) and $z^-_y$ (blue) for the same primary (first row), secondary (second row), and tertiary (third row) modes shown in Fig. 2.10. Note that in each of the two separate, counterpropagating wavepackets, the downward moving components (red/black) are always together in the same wavepacket localized in $z$, and likewise the upward moving components (blue/green) are always together, confirming the fact that these wavepackets remain localized in their extent along the equilibrium magnetic field.

The main message from Fig. 2.10 and Fig. 2.11 is that, in the localized, strongly nonlinear Alfvén wavepacket collision (LS) case, all of the nonlinearly generated components of the Alfvén wavepackets satisfy the linear Alfvén wave eigenfunction condition given by (2.2). This includes the secondary (1,1) Fourier mode, which does not satisfy this eigenfunction condition in the periodic case (Howes and Nielson, 2013; Nielson et al., 2013). Note that this characteristic of the difference between the periodic Alfvén wave and localized Alfvén wavepacket collisions is true in both the weakly
Figure 2.11: Snapshots in time of perpendicular Elssaser field components $z_y^+$ (red,dotted), $z_x^-$ (green,dashed), $z_y^-$ (blue,dotted), and $z_x^+$ (black,dashed) of key $(k_x, k_y)$ modes in the LS case. The first, second, and third row corresponds to the primary, secondary, and tertiary modes respectively. All times are normalized to the localized Alfvén collision time, $T_{c_l}$. The black arrows indicate the direction of motion of the two colliding wavepackets.
and strongly nonlinear limits (not shown).

### 2.2.4.2 Alfvén Wave Dispersion Relation

In the MHD limit $k_\perp \rho_i \ll 1$, the Alfvén wave satisfies the linear dispersion relation $\omega = |k_\parallel|v_A$, where we adopt the convention that $\omega \geq 0$, so the sign of $k_\parallel$ indicates the direction of propagation of a plane Alfvén wave along the equilibrium magnetic field, $B_0 = B_0 \hat{z}$. This simple dispersion relation indicates that Alfvén waves are non-dispersive. The parallel phase velocity is given by $v_{p\parallel} = \omega/k_\parallel = \pm v_A$ and indicates that wave crests of constant phase propagate up or down the equilibrium magnetic field at the Alfvén speed, $v_A$. The parallel group velocity is given by $v_{g\parallel} = \partial \omega / \partial k_\parallel = \pm v_A$, meaning that the envelope of an Alfvén wavepacket will propagate up or down the equilibrium magnetic field at the Alfvén speed, $v_A$.

A brute-force determination of whether any nonlinearly generated mode satisfies the linear Alfvén wave dispersion relation requires a decomposition of the fluctuation into plane-wave modes to enable a comparison between the parallel wavenumber $k_\parallel$ of each constituent plane-wave mode and its linear frequency $\omega$. Such a task is complicated for the case of collisions between localized Alfvén wavepackets, which necessarily contain a broad spectrum of parallel wavenumbers to accomplish localization in $z$. But the non-dispersive nature of Alfvén waves makes an alternative approach possible: if the nonlinearly generated modes propagate along the equilibrium field direction together with the original Alfvén wavepackets at the Alfvén speed, then collectively they describe a localized wavepacket propagating non-dispersively.
In Fig. 2.12, we overplot the perpendicular magnetic field perturbation \( \delta B_\perp \) of the secondary (1,1) Fourier mode with that of the primary (0,1) and (1,0) Fourier modes at times \( t/T_c^{(l)} = 1, 2, 3 \), showing that the nonlinearly generated (1,1) mode does indeed propagate up or down along \( z \) with the primary modes at the Alfvén speed. Furthermore, as predicted from the analytical solution for Alfvén wave collisions (Howes and Nielson, 2013; Howes et al., 2013), the (1,1) mode is phase shifted by \( \pi/2 \) relative to the primary mode from which it gained energy. For example, in Fig. 2.12(a), the downward (0,1) mode (red) passes through zero at the same position in \( z \) at which the downward propagating secondary (1,1) mode (black) reaches a peak. The crucial point of Fig. 2.12 is that, in the localized Alfvén wavepacket collision, the nonlinearly generated, secondary (1,1) Fourier mode satisfies the linear Alfvén wave dispersion relation, propagating along the equilibrium magnetic field non-dispersively.

It is worthwhile noting that the gyrokinetic simulations performed here indeed capture the physics of the finite-ion-Larmor-radius corrections that cause the Alfvén wave solution to become dispersive at \( k_\perp \rho_i \to 1 \), transitioning to the dispersive kinetic Alfvén wave. Therefore, there is a very slight spreading of the wavepackets after nonlinear interactions have transferred energy into modes with \( k_\perp \rho_i \gtrsim 1 \). This behavior is noticeable in Fig. 2.6 in §2.1.3.3 and is discussed in more detail in §2.1.3.4.

In summary, the results presented in Fig. 2.10 and Fig. 2.11 show that, in the more realistic strong, localized Alfvén wavepacket collision case, the secondary (1,1) mode satisfies the linear Alfvén wave eigenfunction condition. The results presented in Fig. 2.12 show that this mode also satisfies the linear Alfvén wave dispersion relation.
Figure 2.12: Snapshots in time of $\delta B_\perp$ vs. $z$ of primary modes (1,0) and (0,1) overlapping the secondary mode (1,1) in the LS case. All times are normalized to the localized Alfvén collision time, $T_c^{(l)}$. 
Therefore, we conclude that this secondary (1,1) Fourier mode, which plays a key role in the nonlinear transfer of energy to smaller perpendicular scales, is simply an Alfvén wave. Note that one may interpret this (1,1) mode of the Alfvén wave as a shear that propagates along the magnetic field at the Alfvén speed (Howes and Bourouaine, 2017). This finding leads to a simplification of the picture of the nonlinear cascade of energy in plasma turbulence relative to the idealized (but analytically soluble) periodic case. In the periodic case, the nonlinear energy transfer to smaller scales was mediated by an inherently nonlinear (1,1) Fourier mode. In the more realistic localized case, the mode that mediates the energy transfer is simply an Alfvén wave itself, both gaining energy from the nonlinear interaction and mediating further energy transfer to smaller scales.

2.2.5 Strong vs. Weak Turbulence

Although the primary aim of this study is to understand how the physics of Alfvén wave collisions changes in the more realistic case of localized Alfvén wavepacket collisions, it is also worthwhile to explore the differences between the weak and strong cases in both the periodic and localized cases.

In Fig. 2.8 and Fig. 2.9, comparing the (d) weakly and (b) strongly nonlinear periodic cases, the most obvious difference is that the energy of the primary Alfvén waves is significantly diminished in the strongly nonlinear case, whereas in the weakly nonlinear case, the loss of energy by the primary Alfvén waves is negligible, even over the long time scale shown in Fig. 2.9(d), as expected. What is not necessarily ex-
pected is that the evolution between the strongly and weakly nonlinear periodic cases is qualitatively similar, with the secondary (1,1) mode and the tertiary (1,2) and (2,1) Alfvén waves as the dominant recipients of the energy nonlinearly transferred from the primary Alfvén waves. The physics governing the nonlinear cascade of energy to smaller scales appears to be similar in the weakly and strongly nonlinear limits, suggesting that physical intuition from the weakly nonlinear limit provides a useful framework for the interpretation of the strongly nonlinear dynamics. Such an approach, in fact, underlies the recent discovery that strong Alfvén wave collisions naturally develop current sheets (Howes, 2016). A final qualitative feature of the long term evolution in the PS case, shown in Fig. 2.9(b), is that the primary Alfvén waves lose energy up to \( t/T_c^{(p)} \sim 5 \), and then their amplitudes begin to rise again. This curious behavior arises from the dispersive nature of kinetic Alfvén waves in the limit \( k_\perp \rho_i \rightarrow 1 \). The nonlinearly generated tertiary Alfvén waves in the gyrokinetic system have slight dispersive increase in their frequency due to finite Larmor radius averaging, and over time begin to shift out of phase with the primary modes, eventually transferring some of their energy back to the primary waves (Nielson, 2012).

Comparing the (c) weakly and (a) strongly nonlinear localized cases in Fig. 2.8 and Fig. 2.9, we observe the same qualitative similarity between the weakly and strongly nonlinear dynamics, with a more significant fraction of energy lost by the primary Alfvén wavepackets in the strongly nonlinear case, again as expected. In contrast to the periodic cases, in both weakly and strongly nonlinear localized cases, all nonlinearly generated modes gain energy secularly over time. Because all of these
smaller perpendicular scale modes are gaining energy, there is a substantially greater loss of energy from the primary Alfvén wavepackets in the LW case relative to the loss from the primary Alfvén waves in the PW case, clearly shown by comparing Fig. 2.9(c) and (d). The strongly nonlinear LS and PS cases in Fig. 2.9(a) and (b) show a similar relation, where the energy loss from the localized case is much more significant than in the periodic case. Therefore, it appears that localized Alfvén wavepacket collisions are much more effective in mediating the nonlinear cascade to smaller perpendicular scales. This is a key result because the localized, strongly nonlinear LS case, the primary focus of this chapter, is the most physically relevant case for application to particular space and astrophysical environments, such as the solar wind and solar corona.

2.2.6 Current Sheet Development Reconfirmed

The final aim of this chapter is to determine whether current sheets naturally develop in the localized case of a strongly nonlinear collision between two symmetric Alfvén wavepackets, where neither initially has a substantial \( k_{||} = 0 \) component. Fig. 2.13 shows plots of the normalized parallel current density \( j_z/j_0 = (j_z/n_0 q_i v_{ti}) (a_0/\rho_0) \) in the \((x, y)\) plane perpendicular to the equilibrium magnetic field.

The left column of Fig. 2.13 follows the evolution of the upward propagating \( z^- \) Alfvén wavepacket, while the right column shows the downward propagating \( z^+ \) Alfvén wavepacket. Note that the waves collide at the midpoint of the simulation box \( z = 0 \) and periodically at the end points \( z = \pm L_z/2 \). We plot the perpendicular cross
section of the parallel current density \( j_z \) of each wavepacket at \( z = \pm L_z/4 \) when the wavepackets are not overlapping at \( t/T_c^{(l)} = 0 \) in (a) and (b), after the first collision at \( t/T_c^{(l)} = 1 \) in (c) and (d), and after the second collision at \( t/T_c^{(l)} = 2 \) in (e) and (f). In (c) and (d), we see that the nonlinear distortion of the original current pattern persists after the first collision, leading to a narrowing and intensification of the current sheet. After the second collision in (e) and (f), the current density has further thinned and intensified into a sheet-like morphology. Note that the amplitude of the color scale increases with later snapshots, making it clear that the current sheets are becoming increasingly intense and narrow over time. Therefore, the result first shown in §2.1.3.2, that strong localized Alfvén wavepacket collisions naturally lead to the development of current sheets, is not dependent on the nonzero \( k_\parallel = 0 \) component of one of the colliding Alfvén wavepackets in that study. We may therefore conclude that the development of current sheets in strong, localized Alfvén wavepacket collisions is a robust result that is not dependent on any particular forms of the initial wavepackets, further extending the impact of the initial discovery that strong Alfvén wave collisions self-consistently generate current sheets (Howes, 2016), providing a first-principles explanation for the ubiquitous observations of current sheets in turbulent space and astrophysical plasmas.
Figure 2.13: Current sheet formation before and after each collision of case LS.
CHAPTER 3
DIAGNOSING TURBULENT DISSIPATION MECHANISMS
ONBOARD MODERN SPACECRAFT

3.1 Motivation

One of the greatest conundrums we have in modern spacecraft missions is the fact that instruments are capable of sampling high-resolution data 24 hours a day, but due to telemetry limitations between the spacecraft and the Earth, the poor bandwidth can afford to downlink only a few minutes of it. If we know \textit{a priori} what kind of measurements we seek, data compression can be done by performing data analysis onboard the spacecraft, and then sending back the post-processed data. That way, we would be able to gain insight on a much larger fraction of the high-resolution data collected, and not limit our scientific return to a few minutes of it. In particular, we present a prescription for processing onboard measurements of turbulence-driven energy transfer from fields to particles.

Plasma turbulence is a widespread phenomenon in our universe, such as in the solar wind, solar corona, and black hole accretion disks. Understanding how turbulent energy is transferred from large to small scales and is eventually dissipated into plasma heat, or some other form of particle energization, is a grand challenge problem at the forefront of space and astrophysical plasma physics. In the evolution of turbulent plasmas under typical conditions in weakly collisional space environments, such as the heliosphere and planetary magnetospheres, the collisionless interactions between electromagnetic fields and individual plasma particles dictate the removal of energy
from turbulent fluctuations. The recently devised Field-Particle Correlation (FPC) technique can quantify the energy transfer in this wave-particle interaction by measuring the correlation between the fields and particles (Klein and Howes, 2016; Howes et al., 2017; Klein et al., 2017). This chapter describes how we can apply this method to develop an onboard wave-particle correlator using a new algorithm called Particle Arrival Time Correlation for Heliophysics (PATCH), enabling calculation of field-particle correlations using discrete particle times of arrival and field measurements at that time. This would allow computation of field-particle correlations onboard in real time, utilizing high-resolution capabilities of modern spacecraft instruments, such as the suite on the recently launched Parker Solar Probe (PSP). For onboard implementation of wave-particle correlators, it is easiest to focus on resonant interactions, such as ion Landau damping, where particles moving about the resonant phase velocity of a nearby kinetic Alfvén wave are accelerated at the expense of the wave energy. The FPC will thus produce a velocity-space signature featuring a zero-crossing from negative to positive about the resonant velocity, $v_{\parallel}/v_i$. Hence, we devote most of this chapter to resolving the velocity-space signature of ion Landau damping, a test case for the implementation of the PATCH algorithm. Successful implementation of the PATCH method, enabling onboard computation of energy transport, will drastically improve the quality of downlinked in situ measurements of turbulent dissipation signatures at kinetic scales.

PSP will unprecedentedly collect data samples from the solar corona, a poorly understood region of the heliosphere that has stirred controversial debate in the he-
liophysics community for decades. One of the primary goals of PSP is to answer the long-standing question concerning the heating of the solar corona: the surface of the Sun is about 6000 Kelvin, but the solar corona is about a million Kelvin. This drastic increase cannot be completely answered using existing theory and simulations, so PSP must return the highest quality in situ measurements possible back to Earth to test competing theories and discover new ones. One of the leading theories is that energy disturbances originating from the Sun’s surface mostly manifest in the form of Alfvén waves. When they arrive at the corona, those waves can become turbulent. As they collide with other counterpropagating Alfvén waves, they transfer that energy to smaller scales, enabling energy removal from those fluctuations via wave-particle interactions and converting that energy to plasma heat. Diagnosing signatures of wave-particle interactions such as these will therefore shed light on one of the most relevant mysteries in heliophysics.

In §3.2, I detail the history of attempts to build a wave-particle correlator to measure wave-particle interactions in situ. The reader may skip to §3.3 since §3.3.1 provides a concise summary.

### 3.2 History of Wave-Particle Correlators

Observations of wave-particle interactions have been sought over 4 decades, due to their mediating role in collisionless energy transfer from waves to particles in weakly collisional plasmas such as the solar wind, planetary magnetospheres, and the solar corona. The earliest experiments flown on sounding rockets launched from
Poker Flat, Alaska into the aurora, in hopes to find evidence of Langmuir waves or whistler chorus emissions, coincident with energetic electrons “bunched” around the cyclotron-resonance frequency. The theory behind this wave-particle interaction can be understood by following the linear theory of Nicholson (1983), who demonstrates that particles in resonance with waves of frequency $\omega$ can be seen by measuring the power spectrum of the perturbed distribution and then comparing with the power spectrum of the wave perturbation. This would then test if there exists a modulation at a specific frequency in the particle distribution function.

A very faint modulation of electrons was measured by Spiger et al. (1974), who developed an electron detector to observe electrons “bunched” in the 50 kHz-10 MHz range with an usually large geometric factor in order to achieve high enough count rates for statistically significant measurements. An onboard spectrum analyzer was utilized due to downlink bandwidth needed to process the large range of frequencies. Spiger et al. (1976) showed that the observed 5% modulation, which was dangerously close to the instrumental noise floor, was indeed significant ($\geq 2\sigma$) and not corrupted due to calibration or interference by other instruments on the payload. The significance was determined by a ground-based cross-correlation technique which looked at correlations of successive frames and compared them to random data. They suspected that Langmuir oscillations in the 4-6 MHz range were the most probable culprit of the modulation. In addition, it was determined that the events occurred in the region marking the boundary between two Birkeland current sheets (determined using magnetometer data), which demonstrates a possible connection between plasma
wave generation and sharp electron flux gradients. Lin (1974) showed that parallel velocity distribution bumps occurred during the modulation time and were enough to cause linear Langmuir wave growth. However, the low energy observed during this ‘bump-on-tail’ instability was not enough to explain modulation over the wide pitch angle range.

Gough (1980) suggested that Spiger et al. (1974) used a flawed method of electron bunching detection because it required a high count rate ($10^7$) cps and subsequently large geometric factor on the electron detector. Gough (1980) instead devised a new “buncher technique” for rocket-borne detection of electron bunching that could be suitable for existing cylindrical/toroidal analyzers that have $10^5$ cps rates. The technique was based on the widely-accepted assumption that if energetic electrons are randomly distributed in position-space before the wave-particle interaction, then the instrument would measure them according to a Poisson distribution. The electron detection time was then measured in terms of cycles on a 10 MHz crystal oscillator per second and frequency, $F$. Subsequently, the probability of measuring two electrons that are separated by $M$ cycles of the crystal oscillator clock was determined to be $P(m) = P_0 \exp^{-MF_0}$. Then, after the wave-particle interaction, $P(m)$ was approximated to quantized peaks at $M = nTF$. Onboard the rocket, histograms of electron measurements, $N(m)$, were made, separated by $M$ crystal oscillator clock units. Applying Fourier analysis to histograms on the ground could then ultimately resolve the bunching. This marked the birth of spectral analysis methods since this information was then Fourier transformed to obtain the power spectra and deduce
where the bunching frequency occurred. The main limitation of the method was that it could not produce accurate results with a low signal of electrons.

On the original experiment testing this method, the count rate was too low to perform the statistical analysis (Gough, 1980), but on many other later flights, it was successfully implemented. In the first of these, bunching of 8-10 keV auroral electrons were detected in a "mother-daughter" experiment where the sounding rocket had a 'mother' (main payload) that measured 0.2-3.6 MHz waves and 8-10 keV energetic particles accompanied with a 'daughter' (sub payload) equipped with an artificial electron gun (Gough et al., 1980). Later, the bunching technique also successfully detected electron modulation with similar active experiments STS-46 and STS-75 which flew the SPREE instrument, measuring 10 eV-10 keV electrons and a 1 keV electron gun which stimulated wave-particle interactions (Gough et al., 1995, 1997, 1998a,b; Rubin et al., 1999). Natural auroral beam experiments also yielded promising results of the buncher technique including ESB (Gough and Urban, 1983) and CAESAR (Gough et al., 1990), which observed modulation energies just below the velocity distribution maximum and a section with a positive velocity derivative in the particle distribution function. The technique was also featured on the WIND spacecraft (Lin et al., 1995) and more recently part of the Digital Wave-Processing (DWP) Experiment on Cluster II (Woolliscroft et al., 1997). The DWP experiment intended to provide a coordination of the Wave-Experiment Consortium (WEC) measurements and direct wave-particle interactions could theoretically be measured with the onboard data-handling system. To synchronize instrument sampling, the DWP
provided electrical signals and complex WEC modes could be formed by the ability of DWP to time-tag data consistently by means of an internal 900 Hz clock (Woolliscroft et al., 1997). In addition, the PEACE instrument could provide raw electron pulses to the DWP particle correlator and subsequently calculate autocorrelation functions (ACF) (Johnstone et al., 1993). Data compression methods such as averaging the ACFs could be performed to mitigate telemetry bandwidth issues and improve signal-to-noise ratios (Woolliscroft et al., 1997).

On Cluster II, the DWP experiment was implemented on all 4 satellites, making multi-point correlations possible for the first time. Unfortunately, the time-sampling resolution was too low to perform inter-spacecraft ACF cross-correlation; this meant that the wave frequency and wave number corresponding to the wave-particle interaction in question could not be determined (Gough et al., 2003). Even though the DWP was not able to achieve the original goal of directly measuring wave-particle interactions onboard, the information from the data products it computed enabled new statistical plasma diagnostics to be developed. For example, the DWP can compute the zero-lag of the ACF onboard, which reveals the variance of the particle counts. For \( N \) particle counts in some sampling time, the index of dispersion

\[
Idisp = \frac{Var(N)}{Mean(N)}
\]

equals unity (independent of count rate) for a Stationary Poisson Process (SPP) (Cox and Isham, 1980). Gough et al. (2003) measured this quantity for the duration of orbit 191 of Cluster II spacecraft 3; deviations from 1 were indicative of a deviation from an SPP, suggesting a more interesting process such as a wave-particle interaction
occurred. They found that most measurements yielded a value of 1, but some were less than 1 which were most likely a consequence of SPP perturbations by waves. During a magnetopause crossing, the value of $I_{disp}$ dropped from 1 to 0.6 and back to 1 which was seen on all 4 spacecraft, making randomness not an option. This means that when Cluster II crossed the magnetopause boundary, it observed the electrons temporarily in a more ordered state. However, this was not always observed after all magnetopause crossings. Another advantage of calculating $I_{disp}$ was using it to distinguish between natural electrons and photoelectrons from spacecraft origin, contaminating the data (Gough et al., 2003). The success of ground-based statistical diagnostic techniques, born from the failure of onboard methods, such as ones planned on Cluster II, highlight the importance of knowledge of statistical processes as means for understanding wave-particle interactions when carefully designed hardware, such as the DWP, does not perform as intended in practice.

In another example supporting this claim, Watkins et al. (1998) conveyed that the original failure of Gough (1980) prompted the implementation of the first ground-based cross-correlation method, where modulation at a bunching frequency was measured by comparing particle count time series with square waves in a wide range of frequencies. For each frequency, $\omega$, high count rates, $C_H$, and low count rates, $C_L$, were determined to enable the evaluation of the cross-correlation function (CCF) significance given by

$$Z(v, \omega) = \frac{|C_H - C_L|}{\sqrt{C_H + C_L}}$$

which is known as the zero-lag of the cross-correlation function. $Z$ values greater
than 3 were considered highly significant since the probability of obtaining $Z > 3$ out of a Poisson distribution is 0.0014. When the count rate is of the same magnitude of the sampling frequency, then the numerator of $Z$ could be calculated onboard, yielding information about which frequency mode possesses the most power, enabling \textit{in situ} estimation of the resonant frequency. As discussed in Watkins et al. (1998), the CRRES satellite attempted to implement computation of $Z(v, \omega)$ onboard but fell short because its duty cycle (ratio of time spent recording high to low number of particles) deviated from exactly 50\%, which is the criterion for assigning the correct statistical weight to the CCF. A duty cycle of exactly 50\% gives active control of the square wave and was first successfully implemented by Ergun et al. (1991a) and later on Freja (Boehm et al., 1994) and WIND (Lin et al., 1995). An \textit{in situ} estimation of the quantities $C_H - C_L$ and $C_H + C_L$ were made, with the former obtained from a 90$^\circ$ out-of-phase input wave – coining the phrase “phase-correlator.” This allowed for the percentage bunching

$$PB = \frac{C_H - C_L}{C_H + C_L}$$

and $Z(z, \Omega)$ values to be determined onboard for each in-phase COS and out-of-phase SIN square wave input component. This mitigated statistical error of broadband waves because it did not depend on the existence of a dominant frequency in the power spectrum of the wave. The method maximized counts in phase with a narrow-band wave signal. It is important to note however, that it did not actually calculate the CCFs directly due to difficulty in measuring the normalization factor required (Watkins et al., 1998). Therefore, they were not able to directly measure a physical
quantity, only interpret their results as physical through their definition of percentage bunching.

The aforementioned refinement of the buncher method by Ergun et al. (1991a) was flown as the 1988 UC Berkeley wave-particle correlator which launched from Poker Flat, Alaska on March 4, 1988. The primary goal of the experiment was to find the energies of resonant electrons so that the source of Langmuir wave growth could be determined. High-amplitude Langmuir waves were measured in coincidence with low energy (300 eV-3 keV) dispersive bursts and field-aligned electrons. It was determined that these waves were a result of positive velocity derivatives in the electron distribution function which continued for 100 ms time intervals (Ergun et al., 1991a). The UC Berkeley wave-particle correlator was able to detect electron bunching with a large statistical significance ($> 5\sigma$) by correlating the phase of the waves with the electron time of arrival. By using rudimentary energy analysis arguments, it was concluded by Ergun et al. (1991b) that Landau damping (or growth) could explain the electron bunching observed on the UCB 1988 sounding rocket. They also argued that the expected particle trapping did not occur in the observations either due to small wavepackets or finite frequency bandwidths on the electrostatic analyzer. The large geometric factor on the electrostatic analyzer provided high enough count rates for the statistics needed, but was a trade-off for lower energy resolution that may have missed particle trapping. By solving for the distribution function linear perturbation as the sum of its resistive (in-phase or 180° out-of-phase with electric field) and reactive components (90° in-phase), the phase relationships of Gaussian Langmuir wavepack-
ets were prescribed by Muschietti et al. (1994). Their analytical model diagnosed the nature of the wave-particle interaction to interpret the correlator data from the UCB 1988 sounding rocket, accounting for the finite wave packet size. It was shown that the observed finite wavepacket leads to resonance broadening which causes a bounded perturbed distribution function over all phase-space. The wave growth and decay then set the bounds for observed sizes of packets. The wavelength could then be determined since it was shown that its resistive component is at a maximum for \( \nu = \omega/k \). It is important to note that applying kinetic theory to experimental data is difficult because of the relative motion between the rocket and plasma. Only estimates of wave growth (decay) rate can be obtained from simultaneous measurements of particle distributions and waves. Nevertheless, these difficulties were circumvented by the UCB 1988 wave-particle correlator which captured the relative phase between the waves and particles.

The same wave-particle correlator technology was also flown on the FAST (Fast Auroral SnapshoT) satellite to diagnose particle energy in pitch angles participating in wave-particle interactions (Ergun et al., 1998). Identical to the method of Ergun et al. (1991a), the distribution function and wave evolution calculation was accomplished by measuring the particle oscillations and phase relationship with the waves. The novelty of this mission was the existence of a large solid state onboard memory capability which allowed for data acquisition several times the maximum telemetry rate. The electric and magnetic field sensors sent a signal to a central signal processing system for several types of processed data including survey data,
which consisted of continuous lower resolution measurements over the auroral zone, and burst mode data,\textsuperscript{1} which was a high-resolution selection of data lasting a few minutes and triggered by high auroral activity (Ergun et al., 2001). The concept of downlinking both higher resolution triggered-burst-mode and lower resolution survey data still exists today as a means of adapting scientific data return to strict telemetry limitations.

The quest for observing wave-particle interactions continued with the University of Iowa (UI) wave-particle correlator flown on the Rocket Auroral Correlator Experiment (RACE) which launched from Poker Flat, Alaska on February 6th, 2002 into an active aurora (Kletzing et al., 2005). The UI wave-particle correlator used a phase-locked loop (PLL) and master voltage-controlled oscillator (VCO) which operated at 16 times the frequency of the measured waveform. The wave was subdivided by the 16 phase bins using the VCO and a phase bunching map was acquired by sorting electron counts into these bins. Contrasting with the observations made by Ergun et al. (1991b), RACE made the first observations of electron bunching at 90° with respect to the wave field (also known as the reactive component), accompanied with Langmuir waves of high amplitude. The wave-particle correlation event was shown to have a significance of over $4\sigma$. At the same time and energy of the correlated electrons, a small electron beam parallel to the magnetic field was observed which was not present before and after the correlation, suggesting that the beam caused the

\textsuperscript{1}FAST was not the first mission to use burst-mode data, but among the earliest to implement the concept.
correlation. Subsequently, a Langmuir wavelength of 8.2 m was derived from the wave frequency (1.6 MHz) and correlated electron energy (468 eV). In addition, Kletzing et al. (2005) offered an explanation to the contrasting Ergun et al. (1991b) observations by suggesting that the relatively crude energy resolution of the UCB correlator instrument could only see the resistive component because the single positive polarity adds over the whole energy range. The reactive components are naturally bipolar, so a narrow energy response of less than 10% is required to be detected. Due to the narrow energy response on RACE, Kletzing et al. (2005) claimed the UI correlator probably saw one side of the reactive component and the other side did not average to zero (which is what likely happened on the UCB correlator). Kletzing et al. (2005) also argued that RACE may have observed a long, coherent train of Langmuir waves, while UCB saw shorter wavepackets as Ergun et al. (1991b) suggested.

The latest attempt to directly observe wave-particle interactions onboard spacecraft has been with the Wave-Particle Interaction Analyzer (WPIA) developed by Fukuhara et al. (2009), who argued that Ergun et al. (1991a,b) and Kletzing et al. (2005) did not actually obtain physical quantities because they did not account for the phase relation between the electric field vector and the particle velocity vector. Fukuhara et al. (2009) claim that the newly designed WPIA directly quantifies kinetic energy flow between waves and particles by using 3D properties of observed waveforms to derive a measure of inner product between instantaneous wave and particle velocity vectors,

\[ I = q \sum_i E_{w_i} \cdot v_i \]
over some amount of time samples indexed by \( i \), incorporating their phase difference for the first time. Calculation of this inner product comes at a large computational cost, however, because the accompanied waveform calibration requires knowledge about both Fast Fourier Transforms (FFTs) and their inverses. They use a Field-Programmable Gate Array (FPGA), so that processing the inner products can be done in real time with minimal consumption of power. The accumulated data of inner product calculations is processed onboard via a Data Processing Unit (DPU) that collects pulses from the wave-receiver and particle detector; only the results are downlinked for optimal telemetry conditions. Fukuhara et al. (2009) also mention that the previous methods for observing wave-particle interactions did not have a high enough time resolution since the time-scale for the phenomena in question is much smaller than the instrument sampling time. The Japanese spacecraft GEOTAIL, which had the Wave-Form Capture receiver, was able to show the importance of high time resolution by using a high-sampling frequency analog-digital converter to observe phase information about the waveforms in question (Matsumoto et al., 1994). The absence of data accumulation, unlike previous spectral methods, allowed for their ability to observe fast changes in wave features. The WPIA concept is implemented on the JAXA satellite mission, Arase, formally known as Exploration of energization and Radiation in Geospace (ERG) project, whose primary goal is to investigate chorus emissions interacting with relativistic electrons (Katoh et al., 2013) in the Van Allen radiation belts. Using simulations of whistler-mode chorus emissions, Katoh et al. (2013) verified that the WPIA can achieve the statistical significance necessary
to directly quantify wave-particle interactions in space, specifically acceleration of relativistic electrons in the magnetosphere.

The WPIA method achieved recent success on a ground-based analysis of MMS observations to diagnose a wave-particle event in the Earth’s magnetosphere (Kitamura et al., 2018). This was largely due to the wide field-of-view and ultra-fast time resolution on MMS to sufficiently resolve kinetic-scale energy transfer. Instead of calculating the inner product between the velocity and the electric field, they calculate \( \mathbf{J} \cdot \mathbf{E}_w \) over time, which is essentially the same method used in Fukuhara et al. (2009), except it has the direct interpretation of the work done on the fields by the particles (or vice versa). A nonzero quantity of \( \mathbf{J} \cdot \mathbf{E} \) averaged over a time period longer than the wave period was used as a diagnostic for agyrotropy, which is an observed asymmetry of ion gyration indicative of an ion-cyclotron resonance. The WPIA method was thus ultimately used as the measure of particle non-uniformity about magnetic field lines and the observed waveform to calculate energy transfer. The current density, \( \mathbf{J} \), was calculated over several pitch angles and energies to directly observe hot ions in cyclotron resonance with spontaneously generated chorus emissions, which gave energy from the particles to the wave. Then, the excited wave exchanged energy non-resonantly with cold ions that were phase-bunched at 90 degrees (rotating in-phase with the wave) to accelerate them to 2 keV. They were able to directly measure the 2 wave-particle events that ultimately led to net energy transfer by using the diagnostic of a nonzero \( \mathbf{J} \cdot \mathbf{E}_w \) averaged over one wave-period. They found that the non-resonant
particles, rather than the particles in cyclotron resonance, were the culprits of net energy transfer, which has never been self-consistently simulated.

### 3.3 The Field-Particle Correlation Technique: Background

The Field-Particle Correlation (FPC) technique is a ground-based method for diagnosing and distinguishing turbulent dissipation mechanisms by quantifying the secular transfer of energy as a function of velocity (Klein and Howes, 2016; Howes et al., 2017). In §3.3.1, we will explain how this method compares to previous ones described in §3.2. The derivation of the method is summarized in §3.3.2, followed by a discussion of successful applications of the method in §3.3.3.

#### 3.3.1 Comparison to Past Wave-Particle Interaction Measurement Methods

The earliest experiments flew on sounding rockets launched from Poker Flat, Alaska into the aurora, due to early speculation that resonant electrons preferentially “bunch” at a specific phase in occurrence with large-amplitude Langmuir waves (Spiger et al., 1974, 1976; Gough and Urban, 1983; Gough et al., 1990, 1995, 1997, 1998a,b; Rubin et al., 1999; Lin et al., 1995; Woolliscroft et al., 1997). However, these were examples of particle auto-correlators which yielded information about the particle count rate statistics, and not necessarily about energy transfer.

In addition, the early aforementioned correlators were designed to study electrons accelerated by Langmuir waves in the magnetosphere, but their results did not directly capture causal energy transfer dynamics in the aurora. Furthermore, the frequency of the Langmuir waves were at or above the counting rate on the
particle detector. Solar wind turbulence however, is dominated by Alfvénic fluctuations (Kraichnan, 1965; Howes and Nielson, 2013; Verniero et al., 2018; Verniero and Howes, 2018) having much lower frequencies than Langmuir waves. The modern spacecraft instrumentation on PSP has particle count rates greater than the characteristic frequencies of wave-particle interactions,\(^2\) enabling new strategies for detecting wave-particle interactions of interest to the turbulence community.

The cross correlation of two signals, as approximated by Ergun et al. (1991a) and Kletzing et al. (2005), tells how much information was exchanged between the two signals over some time (also known as a Cross Correlation Function (CCF) method). It does not, however reveal how the two signals are causally related, also known as their coherence. In a turbulent spectrum, the coherence is time-varying and the dynamical system is not guaranteed to satisfy the ergodic hypothesis. Therefore, CCF methods in turbulence would not be feasible because there is no coherence in the severely broadband nature of turbulence. Techniques emerging from signal processing, such as the cross wavelet transform, may be able to discern a coherent signal first, and then correlate the phase signal with the particle time arrival time. Future work will investigate this claim.

Finally, the FPC method differs from the WPIA technique by its unique ability to resolve the energy transfer as a function of velocity space, so the WPIA method cannot be used in the same way to distinguish different energization mechanisms in

\(^2\)This fact may not always be true as PSP flies closer to the Sun, but adapting the FPC methodology to the case where count rates may be less than the wave frequencies is currently under consideration.
the heliosphere. While it was true that Kitamura et al. (2018) was able to diagnose a particle energization event, they may not be able to use their same method to discern various other mechanisms.

3.3.2 Derivation Outline

The Field-Particle Correlation (FPC) Technique is a novel procedure that can diagnose and distinguish various particle energization mechanisms by characterizing their unique velocity-space signature (Klein and Howes, 2016; Howes et al., 2017; Klein et al., 2017). Together with Maxwell’s equations, the Boltzmann equation describes the evolution of the six-dimensional (3D 3V) velocity distribution function, $f_s(r, v, t)$, for each species, $s$, through a weakly collisional plasma,

$$\frac{\partial f_s}{\partial t} + v \cdot \nabla f_s + \frac{q_s}{m_s} \left[ E + \frac{v \times B}{c} \right] \cdot \frac{\partial f_s}{\partial v} = \left( \frac{\partial f_s}{\partial t} \right)_{\text{coll}}$$

(3.1)

where $c$ is the speed of light, $v$ is the velocity of the bulk plasma flow, $E$ is the electric field, and $B$ is the magnetic field. The third term, the Lorentz force term, is responsible for the interactions between the fields and particles. On the timescale of the field-particle energy transfer of interest, the collisions are negligible, so the right-hand side of Eq. (3.1) vanishes, leaving the Vlasov equation,

$$\frac{\partial f_s}{\partial t} + v \cdot \nabla f_s + \frac{q_s}{m_s} \left[ E + \frac{v \times B}{c} \right] \cdot \frac{\partial f_s}{\partial v} = 0$$

(3.2)

Summarizing the more detailed derivation by Howes et al. (2017), the second (energy) moment of Eq. 3.2 is computed to obtain an equation for the instantaneous rate of
change of particle energy $W_s$,

$$\frac{\partial W_s}{\partial t} = \int d^3r \int d^3\mathbf{v} \frac{1}{2m_s v^2} \frac{\partial f_s(r, \mathbf{v}, t)}{\partial t}$$  \hspace{1cm} (3.3)

and thus find that measuring the change in particle energy can be achieved by using measurements of the change in the distribution function. However, Eq. (3.3) is integrated over all velocities and all space so it is not observationally accessible on a spacecraft. Hence, the strategy of the FPC technique is to focus instead on single-point observations and time average the correlation long enough so the oscillating energy transfer averages to zero, leaving the kinetic physics that leads to particle energization. Now, define the phase-space energy density as,

$$w_s(r, \mathbf{v}, t) = \frac{1}{2} m_s v^2 f_s(r, \mathbf{v}, t)$$  \hspace{1cm} (3.4)

At a single spatial point $r_0$, we multiply Eq. 3.2 by $\frac{1}{2}m v^2$, this time without integrating over all phase-space, and get

$$\frac{\partial w_s(r_0, \mathbf{v}, t)}{\partial t} = -\mathbf{v} \cdot \nabla w_s - q_s \frac{v^2}{2} \mathbf{E} : \frac{\partial f_s}{\partial \mathbf{v}} - \frac{q_s v^2}{c} \frac{1}{2} (\mathbf{v} \times \mathbf{B}) : \frac{\partial f_s}{\partial \mathbf{v}}$$  \hspace{1cm} (3.5)

To isolate the physics responsible for the energy transfer, we note upon integration in velocity-space, the first term (ballistic term) cancels when integrating over physical space with appropriate boundary conditions (either periodic or over all space).\(^3\) When integrated by parts, the third term contains $\mathbf{v} \cdot (\mathbf{v} \times \mathbf{B}) = 0$ in the integrand, meaning that the magnetic field will do no work on the particles. We instead focus on the

\(^3\) Even when the boundary conditions do not lead to a vanishing ballistic term, it still only represents advection of particles (and their energy) from one position to another, not a change of particle energy.
second term in Eq. 3.5 which is the Lorentz force due to the electric field. The energy transfer at a single point, $r_0 = (x_0, y_0, z_0)$, is determined as a function of velocity-space, $v = (v_x, v_y, v_z)$, by the correlation between the electric field, $E = (E_x, E_y, E_z)$, and particles, quantified by

$$C_r \left\langle -\frac{q_s v^2}{2} \frac{\partial f_s(r_0, v, t)}{\partial v}, E(r_0, t) \right\rangle = \frac{1}{\tau} \int_{t-\tau/2}^{t+\tau/2} dt q_s \frac{-v^2}{2} \frac{\partial f_s(r_0, v, t)}{\partial v} \cdot E(r_0, t)$$

(3.6)

over a sufficiently long time period, $\tau$, so that the oscillating transfer of energy associated with undamped waves averages to zero, thereby isolating the secular energy transfer that leads to particle energization (Howes et al., 2017). Note that, when integrated over all velocity space, Eq. 3.6 simply gives the rate of work done by the electric field on the particles. However, this quantity requires information about the velocity derivative of the distribution function, which cannot directly be measured in space. Instead, we can integrate by parts in velocity-space to obtain

$$\int_{-\infty}^{\infty} d^3v q_s \frac{-v^2}{2} \frac{\partial f_s}{\partial v} \cdot E = \left[ \int_{-\infty}^{\infty} d^3v q_s f_s \right] \cdot E = J \cdot E$$

(3.7)

where $J = \sum_s J_s$ is the total current density. Note that when integrated over all velocity, both quantities yield the same result: the inner product of the total species current with the electric field, which is the work done on the particles by the field. Thus, we use the alternative correlation

$$C'_r \langle q_s v f_s(r_0, v, t), E(r_0, t) \rangle = \frac{1}{\tau} \int_{t-\tau/2}^{t+\tau/2} dt q_s f_s(r_0, v, t) v \cdot E(r_0, t)$$

(3.8)

which is an equivalent measure of energy transfer that does not require derivatives. Therefore, for a discrete number of time slices, $M$, in the correlation interval, $\tau$, the
quantity
\[
C'_\tau \left\langle q_s f_s(r_0, \mathbf{v}, t) \mathbf{v} \cdot \mathbf{E}(r_0, t) \right\rangle_\tau \approx \frac{1}{M} \sum_{j=1}^{M} q_s f_s \mathbf{v} \cdot \mathbf{E}_j
\] (3.9)
can be directly measured \textit{in situ} on a spacecraft. Focusing on isolating the kinetic-scale transfer of energy through the second term on the RHS of Eq. 3.5 and combining with Eq. 3.9, we find an alternative estimate of the rate of change of phase-space energy density as
\[
\left( \frac{\partial w_s(r, \mathbf{v}, t)}{\partial t} \right)_E \approx q_s f_s(r, \mathbf{v}, t) \mathbf{v} \cdot \mathbf{E}(r, t)
\] (3.10)
which is the prescription for onboard small-scale, secular energy transfer rates leading to particle energization.\footnote{The subscript $E$ indicates this is only the contribution due to the electric field term, not the total phase-space energy density change. This approximation is only strictly true when integrated over all velocity-space.}
The different parallel or perpendicular components of $\mathbf{v}$ and $\mathbf{E}$ can be separated to explore different mechanisms.

3.3.3 Observational and Theoretical Work Supporting the FPC Method

Chen et al. (2019) has recently shown the feasibility of applying the FPC technique to MMS spacecraft observations and consequently provided the first direct evidence of electron Landau damping in a space plasma. In their application of the method, they high-pass filtered the electric field at 1 Hz (well below the point in which the electric field measurement hit the instrumental noise floor at 100 Hz) to isolate the small-scale component that leads to net energy transfer. Then, they subtracted off the mean electron velocity distribution to isolate the perturbed distribution, $\delta f$, and Lorentz transformed the filtered electric field to the plasma frame-of-reference...
(the frame with no bulk plasma flow). Using Eq. 3.9 to correlate $\delta f$ and the filtered, Lorentz transformed $E_{||}$, they were able to observe the bi-modal velocity-space signature of electron Landau damping. Direct energy transfer was measured by first reducing to a 1D measure in $v_{||}$-space,

$$C'_{E_{||}}(v_{||}) = \int C'_{E_{||}}(v)d^2v_{\perp}$$

and calculating

$$C_{E_{||}}(v_{||}) = -\frac{v_{||}}{2} \frac{\partial C'_{E_{||}}(v_{||})}{\partial v_{||}} + \frac{C'_{E_{||}}(v_{||})}{2}$$

When compared to predictions of the turbulent cascade rate, the high correlation value corresponding to energy transfer suggested that electron Landau damping was the dominant mechanism, consistent with the idea that the electrons were undergoing significant energization.

Chen et al. (2019) performed several quality checks to ensure they were not observing false-positive results. First, they affirmed that the symmetric Landau damping pattern was coherent over time. Second, they demonstrated signal distortion evolution as the phase of $E$ was randomized, as suggested by Howes et al. (2017). In addition, they used the PLUME KAW dispersion solver to confirm that the damping for the observed plasma parameters was predicted to occur at the velocity where the change of sign of the correlation was seen (Klein and Howes, 2015), which was observed to be relatively broad (i.e. greater than the thermal speed). The pioneering work of Chen et al. (2019) confirms that the FPC method can indeed diagnose turbulent dissipation mechanisms in space plasma using actual spacecraft observa-
tions. It is therefore worthwhile to explore further applications of this method, such as implementing the FPC method onboard a spacecraft (see §3.6.1).

3.4 Determining Limitations of Instrumental Phase-Space Resolution

This section and beyond presents original research about application of the Field-Particle Correlation technique to diagnosing energy transfer dissipation mechanisms onboard spacecraft. In §3.4.1, I briefly describe the simulation parameters employed with AstroGK to create synthetic spacecraft data in §3.4.2. I present results of discerning the velocity-space signature of ion Landau damping using realistic instrumental resolutions from modern spacecraft in §3.4.3.

3.4.1 Simulation Parameters

To implement the synthetic spacecraft data creation, we utilized the same AstroGK output data described in §4 of Klein et al. (2017). The general code description of AstroGK can be found in §2.1.2 of this thesis. This particular simulation was employed on a grid defined by \((n_x, n_y, n_z, n_\lambda, n_E) = (64, 64, 32, 64, 32)\) over a \(k_\perp \rho_i \in [0.25, 5.5]\) domain span. The driven simulation (with Langevin antenna described in TenBarge et al. (2014)) reached a strongly turbulent saturated state and was run up to \(t \omega_A = 20\). Here, \(\omega_A = k_\parallel v_A\), where \(v_A\) is the Alfvén velocity and \(k_\parallel = 2\pi/L_\parallel\), with \(L_\parallel\) defined as the length of the periodic box domain \(L_\perp^2 \times L_\parallel\), elongated along the mean magnetic field direction \(B_0 = B_0 \hat{z}\). Since we aim to model solar wind conditions, we set \(\beta = 1\) and ion collision frequency \(v_i/k_\parallel v_{ti} = 2 \times 10^{-4}\).

In order to replicate the Landau damping signature shown in Fig. 6(c) of
Klein et al. (2017), we used the same correlation interval, $\tau_\omega A = 10.4$ centered at time $t_\omega A = 14.1$, and single spatial point $(x, y, z) = (22, 5, 0)$. For ease of reference, Fig. 6(c) of Klein et al. (2017) is shown in Fig. 3.1, representing the correlation $C$, given by Eq. (3.6). The cadence for the FPC diagnostic output$^5$ is $\Delta \hat{t} = 0.16$ with time normalization $\hat{t} = t_\omega A$.

Figure 3.1: (a) The gyrotropic complementary distribution function $g_p(v_\parallel, v_\perp)$ at a single point in the $\beta_p = 1.0$ turbulent simulation, as well as the correlations (b) $C_{E\parallel}(\tau = 0)$ and (c) $C_{E\parallel}(\tau_\omega A = 10.4)$ at time $t_\omega A = 14.1$. The resonant parallel velocity associated with the maximum proton damping rate is shown as a solid grey vertical line. Note that this figure was adopted from Fig. 6 of Klein et al. (2017).

3.4.2 Synthetic Spacecraft Data Creation

We now describe our procedure for finding the minimum phase-space resolution necessary to resolve the velocity-space signature of particle energization mechanisms on realistic instrumental energy grids. This resolution depends on energy and angle,$^6$ i.e. $\delta E/E \times \Delta \phi$. First, we set up an energy vs. angle grid numerically, and then av-

$^5$Note that the lowest period of Alfvén waves in the simulation corresponds to $\hat{T} = 2\pi$, so this sampling is 39 slices per period.

$^6$Note that in general, the velocity-space resolution on spacecrafts instruments have one energy dimension and two angle dimensions, however here we simplify to two dimensional velocity-space of energy and angle.
verage the high-resolution results in \((v_{\parallel}, v_{\perp})\) coordinates from the gyrokinetic plasma turbulence simulation, \texttt{AstroGK}, into each of the instrumental bins. We apply the field-particle correlation (using Eq. (3.9)) to the downsampled, instrumental resolution data and map the instrumental field-particle correlation results back to \texttt{AstroGK} coordinates for direct comparison. This procedure highlights how one can utilize a plasma simulation to design spacecraft instrumentation.

As discussed in §2.1.2, \texttt{AstroGK} evolves the perturbed gyroaveraged distribution function \(h_s(x, y, z, \lambda, \epsilon_v)\) for each species \(s\) (Numata et al., 2010). The velocity-space coordinates are \(\lambda = v_{\perp}^2/v^2\) and \(\epsilon = v^2\). A field-particle correlation diagnostic implemented in \texttt{AstroGK} outputs particle data at single points in space. We choose one of these, namely \((x, y, z) = (22, 5, 0)\), where we know from previous analysis by Klein et al. (2017), that there exists a velocity-space signature of Landau damping.

\texttt{AstroGK} is a gyrokinetics code (see §1.4.2), which assumes a small, dimensionless parameter defined by the ratio of the ion gyroradius to the macroscale length of the equilibrium plasma, \(\epsilon = \rho_i/L \ll 1\) (Howes et al., 2006; Numata et al., 2010). The distribution function is then decomposed into an equilibrium and fluctuating component,

\[ f_s = F_{s0} + \delta f_s \]

where \(\delta f_s/F_{s0}\) is of the same magnitude as \(\epsilon\). In \texttt{AstroGK}, the perturbed distribution function used is \(g_s\), which is the complement of \(h_s\) with respect to the equilibrium distribution, \(F_{s0}\). With the gyrokinetic ordering of §1.4.2 in mind, we can interpret
\(g_s\) as \(\delta f_s\) and construct the original distribution function\(^7\) as

\[
f_s = (1 + \epsilon g_s)e^{-\left(v_\perp^2 + v_\parallel^2\right)}
\]

Note in constructing \(f_s\) from \(g_s\), we also employ a floor value of \(10^{-16}\) if \(f_s < 0\) at any point which ensures that \(f_s\) remains a physical quantity and \(f_s \geq 0\) everywhere.\(^8\)

For the strongly nonlinear\(^9\) regime of interest, such as the solar wind, a reasonable value of \(\epsilon = 0.1\) was employed.

We then execute the following coordinate-space transformation:

\[
f_s(v_\parallel, v_\perp) \rightarrow f_s(E, \phi)_{sw} \rightarrow (f_s(E, \phi)_{sw}) \Delta E/E \times \Delta \phi = f_{sc}
\]

\[\text{(3.11)}\]

The distribution function is transformed from \texttt{AstroGK} \((v_\parallel, v_\perp)\)-space to instrumental \((E, \phi)\)-space, boosted to the chosen solar wind velocity, which converts the system to the plasma rest frame. Then, this function is down-sampled to a given velocity-space resolution, \(\Delta E/E \times \Delta \phi\), which we define as \(f_{sc}\). This transformed and down-sampled distribution function then represents what the actual instrument will be able to measure in the spacecraft frame-of-reference.

We now demonstrate how the synthetic spacecraft data is created from \texttt{AstroGK} and down-sampled to instrumental resolution, utilizing the transformation represented by 3.11. For this specific test-case, we use the instrumental specifications

---

\(^7\)In relation to the original gyro-averaged \(h_s\), \(g_s\) is \(O(\epsilon)\) and the magnitude of \(\delta f_s\) is \(O(\epsilon^2)\).

\(^8\)This expression for \(f_s\) is only an approximation. The Boltzmann term was neglected since it does not depend on \(v\) and therefore would not affect our results.

\(^9\)In this limit, the nonlinear term in the equation of evolution is \(O(1)\), but the amplitude of the perturbed \(\delta f_s/F_{s0}\) is small.
Figure 3.2: Demonstration of synthetic spacecraft data creation using (a) high-resolution $\delta f_i$ from AstroGK, (b) reconstructed to $f_i$, (c) transformed to plasma frame-of-reference, (d) down-sampled to PSP resolution (as observed by spacecraft), and (e) transformed back to $(v_\parallel, v_\perp)$-space for comparison to (b).
relevant to the PSP particle instrument suite, SWEAP (Kasper et al., 2016), with
\( \frac{\Delta E}{E} \times \Delta \phi = 7\% \times 3.75 \) as our down-sampled resolution for ions. In Fig. 3.2, we plot
(a) \( \delta f_i \) on the original AstroGK coordinates \((v_\parallel, v_\perp)\), normalized to the ion thermal velocity, \( v_t \), and (b) the constructed \( f_i \) distribution on the AstroGK coordinate grid. Note that we imposed a floor value on \( f_i \), ensuring that it does not un-physically become negative. As a consequence, \( f_i \) is plotted on a log-scale so that fluctuations can be more discernible for the reader. Panel (c) of Fig. 3.2 represents the high-resolution, \( f_i \), distribution function data from AstroGK in instrumental \((E, \phi)\)-space, boosted to the solar wind velocity, \( v_{sw} \), at an angle, \( \alpha \), with respect to the magnetic field.\(^{10}\) In this study, we set \( \alpha = 30^\circ \) and \( v_{sw} = 400 \text{ km/s} \), indicated by the diagonal magenta line. Panel (d) shows the down-sampled distribution in PSP resolution, i.e. \( f_{i,sc} = f_{i,PSP} \). The general structure of \( f_{i,PSP} \) is still maintained, but as one can see, fine-scale fluctuations are smoothed out by the down-sampling procedure. Finally, panel (e) of Fig. 3.2 shows the down-sampled distribution function, \( f_{i,PSP} \), mapped from \((E, \phi)\)-space (black dots) back to \((v_\parallel, v_\perp)\)-space to facilitate comparison to panel (b). Note that Fig. 3.2 does not show the field-particle correlations, only the distribution function data on various grids.

### 3.4.3 Velocity-Space Resolution Results

This section will present results on resolving the velocity-space signature of ion Landau damping, where particles moving about the resonant phase velocity of a

\(^{10}\) Although solar wind flow is always nearly radial, the angle of the magnetic field about the Parker spiral value can vary nontrivially, so this assumption is also an approximation.
nearby kinetic Alfvén wave are accelerated at the expense of the wave energy. The FPC given by Eq. (3.6) will thus produce a velocity-space signature featuring a zero-crossing from negative to positive about the resonant velocity, \(v_\parallel/v_i\). The alternative correlation \(C'\), given by Eq. (3.9), will yield a signature featuring a (maximum) peak at the resonant velocity. Fig. 3.3 (a) shows the FPC in high-resolution AstroGK \((v_\parallel, v_\perp)\) coordinates, noting the aforementioned maximum peak about the resonant velocity which is indicative of Landau damping. This characteristic signature of Landau damping, a correlation change from negative (blue) to a positive (red), is clearly resolvable with PSP instrumental resolution, seen by the bi-modal correlation signature featuring a peak about the resonant velocity in Fig. 3.3 (b). These results, one of the main findings of this chapter, indicate that in situ PSP observations will have enough velocity-space resolution for successful application of the FPC technique to diagnose particle energization signatures such as Landau damping.
Figure 3.4: Realistic instrumental resolutions for various spacecraft missions (b)-(f) to resolve ion Landau damping velocity-space signature, comparing to (a) AstroGK.
Utilizing (3.11), we also constructed $f_{sc}$ using various realistic instrumental resolutions $(\delta E/E \times \Delta \phi)$ from several other past, present and future missions. Fig. 3.4 shows the FPC calculations using Eq. 3.9 and the down-sampled distribution functions, highlighting the importance of implementing finer velocity-space resolutions on future missions. In panel (a), we again show the high-resolution $C'$ correlation from AstroGK in $(v_\parallel, v_\perp)$ coordinates, facilitating comparison to other mission resolutions. Panels (b)-(f) show the down-sampled $C'$ correlation transformed from $(E, \phi)$-space (black dots) to $(v_\parallel, v_\perp)$-space in (b) PSP, (c) Solar Orbiter, (d) MMS, (e) CLUSTER II and (f) WIND resolutions, with the energy and angular resolution for ion measurements on each mission specified. Observe from panels (b)-(d) that PSP indeed has enough resolution to resolve the Landau damping velocity-space signature, as well as Solar Orbiter and MMS. However, once $\Delta E/E$ is greater than 10%, as in CLUSTER II (e) and WIND (f), the signature starts to become indiscernible and less localized. The signal is also fainter, indicated by the decrease of the color scale representing the correlation amplitude. These results indicate that (i) one can use the FPC method with modern instrumentation to resolve velocity-space signatures of turbulent dissipation mechanisms, and (ii) only recent instrumentation has sufficient phase-space resolution to resolve the signatures.

### 3.5 Electrostatic Analyzer Background

Before deriving the PATCH method for FPC implementation onboard spacecraft, we must first define a particle arrival time. As we shall discover in §3.6.1,
the PATCH algorithm is based on time-tagging particle counts; the definition of the “time” a particle is “counted” on a spacecraft must be defined clearly, to link the theory behind PATCH to physical observations and make improvements based on instrumental specifications. This section departs briefly from original research to describe background material about spacecraft particle detectors, facilitating a more precise definition of a particle arrival time.

3.5.1 Top-Hat Design

The particle detector on a spacecraft is generally of a type known as an Electrostatic Analyzer (ESA), with typical modern configurations as a “top-hat” design, shown in Fig. 3.5. Physically, it is a cylindrically symmetric configuration consisting of a particle entrance aperture on top of two nested hemispheres. The aperture entrance is a cylindrical figure of revolution, also known as the Field of View (FOV), which rotates 360° about an axis of symmetry (see left side view in Fig. 3.5). The

Figure 3.5: Physical picture of “top-hat” ESA design, including a side view (left) of the electron particle trajectory (red) crossing the symmetry axis (dotted black line). The Field of View (FOV) spins 360° around this axis, which also defines the area of the aperture opening. The cutaway view (right) shows the two nested hemispheres responsible for the net voltage difference.
axis of this rotation is symmetric about the set voltage difference of the two nested hemispheres, (see right cutaway view in Fig. 3.5).

The energy, $E$, for a particle of charge, $q$, is related to the voltage difference, $\delta V$, by the expression,

$$E = q\delta V$$

where the units of $E$, $q$ and $\delta V$ are joules, volts and coulombs, respectively. Therefore, a set voltage will select particles of charge, $q$, with energy, $E$.

Figure 3.6: Sketch of aperture on the ESA top-hat detector.

The place where the particles physically enter the ESA, the aperture of the detector, is shown in Fig. 3.6 presenting a horizontal close-up (left) of a particle passing through the surface of the rotating FOV (right), sketched as a 3D pancake shape. The *particle arrival time* is defined as the time, $t_p$, that the particle crosses the surface area of this aperture, a curvilinear surface approximated locally as a 2D

---

11 For electrons and ions, the voltage difference is set to positive ($+V$) and negative ($-V$), respectively.

12 Note that the energy is usually converted to units of eV.
rectangle with an infinitesimal slab of area, $A$, in a plane parallel to the azimuthal axis of symmetry. For more details on the trajectory of particles flying into the aperture, in addition to other instrumental nuances, the reader can refer to Appendix B.1.

The ESA then collects these particles which arrive at an angle of incidence,\(^{13}\) approximately normal to the surface of the aperture plane, with a specific time, position and velocity. This information at the particle arrival time, $t_p$, is then recorded into 5 measurement bins encompassing 6-dimensional phase-space: (1) 1D azimuthal angle $\phi$, (2) 1D polar angle $\theta$, (3) 1D energy $E$, (4) 1D velocity $v$ and (5) 2D position area $A$. The width of these bins, $\Delta$, determines the data resolution; a smaller $\Delta$ enables a finer measurement grid to yield better resolution, and vice versa. The finiteness of physical hardware invokes a natural upper bound on the instrumental resolution quality. In addition, any prediction made from kinetic plasma theory must account for breaking the assumption that measurements are sampled continuously. True continuity does not exist in the physically observable world since the concept of infinity was invented by the human mind. The PATCH algorithm specifically utilizes the discreteness of particle count measurements to optimize accuracy of calculating quantities existing in the physical universe.

The aperture encircles the axis of symmetry, which determines the relative azimuthal angle, $\phi$, where the particle arrived, and the relative polar angle, $\theta$, the

\(^{13}\)In reality, a multidirectional range of particle angles could be detected in velocity-space, for example, $\mathbf{v} = (\theta \pm 3^\circ, \phi \pm 10^\circ)$. This finite range of uncertainty in both azimuthal and polar angles can lead to particle count distortion into a single angle bin in velocity-space. Accounting for this error goes beyond the scope of this dissertation, but it important to recognize the gargantuan complexities of measuring space plasma.
small vertical tilt. Both $\phi$ and $\theta$ can be combined to form a 2D solid angle of rotation, $\Omega$, which would describe the general angle of incidence that the particle arrived at time $t_p$, normal to the aperture plane. As explained further in §3.5.2, canonical 3-dimensional ‘position-space’ is described by the coordinates $(v, A)$, the velocity and position of the particle arriving at time $t_p$, passing through the plane incident normal to the aperture. Standard 3-dimensional ‘velocity-space’ is described by the coordinates $(E, \phi, \theta)$. The ESA samples particles of charge, $q$, for a certain sampling time interval, $dt_p$, and counts how many particles enter each of the 5 $(v, A, E, \phi, \theta)$ measurement bins. After each sampling time interval is completed, the ESA is then set to a different voltage, and the whole process repeats. When the ESA samples particles from every voltage in the range of interest, it completes an energy sweep, and the time it takes to make an entire energy sweep is generally referred to as the time cadence. Thus, the faster the time cadence, the faster the instrument can acquire full 3-dimensional velocity-space information about the plasma particles; but, a faster sweep could mean lower particle counts. Therefore, a faster time cadence does not necessarily imply a better one.

Since kinetic-scale energy dissipation mechanisms occur at time-scales in the near-Earth solar wind ($\sim 1$ s for ions, $\sim 0.02$ s for electrons), gaining explicit knowledge about data acquisition on the instrument is essential for any theorist inclined to model the dynamics of the universe based on spacecraft observations. In addition, accounting for the time delays in the electronics is important for the field-particle correlation calculations, so careful calibration is essential for any proposed correla-
tor instrumentation. The reader can refer to Appendix B.2 for a discussion of time calibration error considerations.

3.5.2 Geometric Factor

On a spacecraft, the standard phase-space \((r, v)\) distribution function, \(f\), is defined as,

\[
f = \frac{N}{d^3r \, d^3v}
\]  

where \(N\) is the number of particles with mass, \(m\), counted in some infinitesimal region of phase-space, \(d^3r \, d^3v\). For the sake of theoretical argument, we assume \(f\) is continuous and can be defined locally such that \(f\) is constant with respect to the same infinitesimal region of phase-space, i.e. \(f\) can be restricted to an open set, \(U\), of radius \(\Delta \in (r, v)\). Note that \(\Delta\) can be thought of as the smallest bin width on the phase-space measurement grid. So \(f\) is locally defined as

\[
\frac{df}{d\Delta} d\Delta = df = f\big|_{U_\Delta} = f_{loc}
\]  

Substituting Eq. (3.13) into Eq. (3.12), we get

\[
f_{loc} = \frac{dN}{d^3r \, d^3v} = \frac{dN}{dr_1 \, dr_2 \, dr_3 \, dv_1 \, dv_2 \, dv_3}
\]  

where the subscripts correspond to each general 3-dimensional direction of position and velocity. For a particular \(r_1\) and \(v_1\), note that Eq. (3.54) implies

\[
\frac{dr_1}{dt} \, dt = v_1 dt
\]

Let \(A\) be the 2D surface area of the particle detector aperture such that

\[
dA = dr_2 \wedge dr_3
\]
Invoking azimuthal symmetry about the \( v_1 \) axis and converting to polar coordinates,

\[
d^3 \mathbf{v} = v_1^2 \, dv_1 \, d\Omega
\]  

(3.17)

where \( \Omega \) is the solid angle of rotation about the \( v_1 \) axis, i.e. \( \Omega = dv_2 \wedge dv_3 \). Then, substituting Eq. (3.15)-(3.16) into Eq. (3.14),

\[
f_{\text{loc}} = \frac{dN}{v_1^2 \, dv_1 \, d\Omega \, dA \, v_1 \, dt}
\]  

(3.18)

Also observe by the chain rule,

\[
\frac{d}{dv_1} \left( \frac{v_1^2}{2} \right) dv_1 = v_1 \, dv_1
\]  

(3.19)

Now, substituting Eq. (3.19) into Eq. (3.18) and rearranging, we get

\[
f_{\text{loc}} = \frac{dN}{\frac{d}{dv_1} \left( \frac{v_1^2}{2} \right) dv_1 \, v_1^2 \, d\Omega \, dA \, dt} = \frac{2 \, dN}{\frac{d}{dv_1} (v_1^2) \, dv_1 \, v_1^2 \, d\Omega \, dA \, dt}
\]  

(3.20)

Further massaging Eq. (3.20),

\[
f_{\text{loc}} = \frac{2 \, dN}{\frac{d}{dv_1} (v_1^2) \, dv_1 \, v_1^2 \, d\Omega \, dA \, dt} \cdot \frac{1}{2} m^2 = \frac{m^2 \, dN}{mv_1^2 \, \frac{d}{dv_1} \left( \frac{1}{2} mv_1^2 \right) \, dv_1 \, d\Omega \, dA \, dt}
\]  

(3.21)

Since the kinetic energy, \( E \), of a particle with velocity, \( v_1 \), is defined as \( E = \frac{1}{2}mv_1^2 \), we can invoke the chain rule again to obtain

\[
\frac{d}{dv_1} \left( \frac{1}{2} mv_1^2 \right) dv_1 = \frac{dE}{dv_1} dv_1 = dE
\]  

(3.22)

Substituting Eq. (3.22) into Eq. (3.21),

\[
f_{\text{loc}} = \frac{m^2 \, dN}{2 \, E \, dE \, dA \, d\Omega \, dt}
\]  

(3.23)

Now define the local differential particle count flux, \( j \), as

\[
j = \frac{dN}{dE \, dA \, d\Omega \, dt}
\]  

(3.24)
Then substituting Eq. (3.24) into Eq. (3.23), we obtain

\[ f_{loc} = \frac{m^2 j}{2E} \quad (3.25) \]

Neglecting the error due to systematic counting efficiency of the instrument, the particle count measurement, \( C_p \), is over a finite phase-space region, \( \Delta E \Delta t \Delta \Omega \Delta A \), so that the actual measurement of a particle count is

\[ C_p = \int_{A-\frac{\Delta A}{2}}^{A+\frac{\Delta A}{2}} \int_{t-\frac{\Delta t}{2}}^{t+\frac{\Delta t}{2}} \int_{\Omega-\frac{\Delta \Omega}{2}}^{\Omega+\frac{\Delta \Omega}{2}} \int_{E-\frac{\Delta E}{2}}^{E+\frac{\Delta E}{2}} j \, dEd\Omega dt dA \quad (3.26) \]

Assuming \( j \) is constant over these intervals,\(^{14}\) then Eq. (3.26) simplifies to

\[ C_p = j \Delta E \Delta \Omega \Delta t \Delta A \]

which implies that

\[ j = \frac{C_p}{\Delta E \Delta \Omega \Delta t \Delta A} \quad (3.27) \]

The energy response on an instrument is defined as

\[ \frac{\Delta E}{E} = \gamma \]

where \( \gamma \) is a constant, so we obtain the relation

\[ \Delta E = \gamma E \quad (3.28) \]

If we incorporate a systematic counting efficiency error \( \varepsilon \), then Eq. (3.27) becomes

\[ j = \frac{C_p}{\varepsilon \Delta E \Delta \Omega \Delta t \Delta A} \quad (3.29) \]

\(^{14}\)This is true as we have defined it locally which involved the notion of continuity. In reality, the assumption that \( j \) is constant has questionable validity.
where $\mathcal{E} > 1$, meaning the local differential counting flux will be less than expected due to this error. Exploring the definition of this error is beyond the scope of this dissertation, but is worthwhile including for understanding the relationship of further realistic instrumental constraints. The geometric factor, $g$, on the ESA is defined as

$$ g = \Delta A \wedge \Delta \Omega $$ (3.30)

Now, let the particle count rate be defined as

$$ \dot{C}_p = \frac{C_p}{\Delta t} $$ (3.31)

Then, substitute Eq. (3.30)-(3.31) into Eq. (3.29) to get

$$ j = \frac{\dot{C}_p}{\mathcal{E}g\gamma E} $$

which implies

$$ \dot{C}_p = \mathcal{E}jg\gamma E $$ (3.32)

Solving for $j$ in Eq. (3.25),

$$ j = \frac{2E}{m^2}f_{loc} $$ (3.33)

So substituting Eq. (3.33) into Eq. (3.32) yields

$$ \dot{C}_p = \frac{\mathcal{E}g\gamma 2E^2}{m^2}f_{loc} $$

Rearranging,

$$ f_{loc} = \frac{\mathcal{E}\dot{C}_pm^2}{g \gamma 2E^2} = \frac{\dot{C}_pm^2}{2G_{tot}E^2} $$ (3.34)

where $G_{tot}$ is the total $G$ factor including the dependance on the energy response $\Delta E/E$ in addition to the counting efficiency factor, $\mathcal{E}$. However, in practice, the
geometric factor does not include this error term and is referred to as

\[ \mathcal{G} = g\gamma = \frac{\Delta E}{E} \Delta A \Delta \Omega \]  \hspace{1cm} (3.35)

Hence, in terms of \( \mathcal{G} \), Eq. (3.34) becomes

\[ f_{\text{loc}} = \frac{\dot{C}_p m^2}{2\mathcal{G} E^2} = \frac{jm^2}{2E} \]  \hspace{1cm} (3.36)

which relates the distribution function to the count rate, geometric factor, and differential energy flux. An exploration of the connection between the PATCH algorithm and these three quantities is found in Appendix B.3.

### 3.6 New Theory for Onboard Spacecraft Measurements

Instruments onboard spacecraft can constantly sample the plasma at full high-resolution cadence, but due to telemetry restrictions, only a small fraction of the highest resolution data can be transmitted to the ground for analysis. Onboard correlation calculations enables the most efficient use of the highest resolution measurements of an instrument, maximizing scientific data return by only downlinking time-averaged energy transfer information as a form of data compression.\(^{15}\) This section serves as proof-of-concept that the proposed, prototypical methodology could indeed achieve this goal. A simple way to perform FPC calculations onboard spacecraft has been developed, called the Particle Arrival Time Correlation for Heliophysics (PATCH) algorithm. We demonstrate the basic intuition in §3.6.1 and show it is consistent with more rigorous kinetic theory developed in §3.6.4.

\(^{15}\)This type of data compression is distinct from lossless compression.
3.6.1 Demonstration of the PATCH Algorithm

Onboard the spacecraft, the detector counts particles with discrete arrival times, defined as the time the particle enters the aperture, described in §3.5. Thus, the observed number of particles in the single phase-space bin $\Delta r_p \Delta v_p$ is given by

$$N_{obs}(t) = \sum_{j=1}^{N_{tot}} \delta(t - t_j)$$  \hspace{1cm} (3.37)

at a sampling time $dt_p$. The total number of particles counted, centered at time $t_p$ over the appropriate correlation time, $\tau$, is

$$N_{\tau} = \int_{t_p-\frac{\tau}{2}}^{t_p+\frac{\tau}{2}} dt_p \, N_{obs}(t)$$  \hspace{1cm} (3.38)

which is interpreted as the count rate over the correlation time interval, $\tau$.

Traditionally, kinetic theory uses the ensemble averaged distribution function over all possible states of the system in phase-space to describe the evolution of plasma. §3.6.3 discusses why an ensemble averaged distribution function is unphysical and not well-justified for real-world applications. The act of making an observation on a spacecraft automatically chooses one plasma state in the ensemble. Therefore, we can define a specific physical distribution function observed by the spacecraft particle detector with an aperture geometry-defined phase-space to derive the PATCH method intuitively, based on the observed distribution function defined by Eq. (3.12). In a single phase-space bin, a fixed point in space $r_p$ and velocity $v_p$, $f(r_p, v_p, t) = \frac{N(t)}{\Delta r_p \Delta v_p}$ \hspace{1cm} (3.39)

where $N(t)$ is the number of particle counts observed as a function of time given by Eq. (3.37). By substituting Eq. (3.39) into Eq. (3.10), the rate of change of
phase-space energy density due to the electric field is approximated as

\[
\left( \frac{\partial w(r_p, v_p, t)}{\partial t} \right)_E \approx q_s v_p \frac{N(t)E(r_p, t)}{\Delta r_p \Delta v_p}
\]

Eq (3.8) says the correlation over some time, \( \tau \), determines the rate of change of particle energization:

\[
C^*_\tau \langle q_s N(t)v_p \cdot E(t) \rangle = \frac{1}{\tau} \int_{t-\tau/2}^{t+\tau/2} dt' \ q_s N(t')v_p \cdot E(t')
\]

Substituting Eq. (3.37) for \( N(t') \), the correlation, \( C^*_\tau \), constructed from discrete particle arrival times (in a single phase-space bin) is,

\[
C^*_\tau = \frac{1}{\tau} \int dt \ \sum_{j=1}^{N_r} \delta(t - t_j)q_s v_p \cdot E(t)
\]

\[
= \frac{1}{\tau} \sum_{j=1}^{N_r} \int dt \ \delta(t - t_j)q_s v_p \cdot E(t)
\]

\[
= \frac{1}{\tau} \sum_{j=1}^{N_r} q_s v_p \cdot E(t_j)
\]

(3.40)

The PATCH algorithm, based on Eq. (3.40), is simply the summation of the electric field evaluated at the particle arrival times (with the possible time delay calibration given by Eq. (B.3) in Appendix B.2) over the correlation interval, \( \tau \). As a simple demonstration shown in Fig. 3.7, consider a sinusoidal electric field (red) with a known time evolution at the point of measurement \( r_p \) and sinusoidal particle number density \( N(t) \) (blue) within a single \( (\Delta r_p, \Delta v_p) \) phase-space bin. \( N(t) \) can be interpreted as the number of particle counts within a sampling time, \( dt_p \), that enter the aperture as a function of time. Hence, \( N(t) \) is simply the count rate in each phase-space bin. The in-phase component between \( N(t) \) and \( E(t) \) is what leads
Figure 3.7: 2D representation of the PATCH procedure in a single 6D phase-space bin $\Delta r_p, \Delta v_p$. The blue curve, $N(t)$, represents the number density of particles in this bin. The green vertical lines indicate particles arriving with probabilities dictated by $N(t)$. The red curve is the electric field, $E(t)$, with open green circles indicating that the electric field is evaluated at the particle time of arrival. The summation of these values with appropriate normalization yields the correlation, $C^*_\tau$, in this bin.

to net energy transfer. The particles arrive in a Poisson-distributed manner with probabilities dictated by $N(t)$. The reader is encouraged to learn about Poisson counting statistics, specifically applied to the PATCH concept, in Appendix A.1.

To find the correlation over the interval, $\tau = 3$ periods, we perform the following procedure. If a particle arrives at some time, $t = t_j$, represented by the green vertical lines in Fig. 3.7, then the electric field is evaluated at that time, $E(t_j)$, represented by the green circles. We then multiply $E(t_j)$ by the velocity, $v_p$, and compute the summation given by Eq. (3.40) over a sufficiently long time interval, $\tau$. Statistical
models of this simplified case are presented in Appendix A.2.

3.6.2 The Klimontovich-Dupree Equation

In modern theoretical plasma physics, the kinetic description of a plasma is adopted from the widely accepted theory transcribed in Nicholson (1983).\textsuperscript{16} The following description follows Chapter 3 of Nicholson (1983). It is based on the notion that we describe point particles in a plasma existing in a 3D configuration-space $x$ each with an orbit $X(t)$, which is defined by Nicholson (1983) as “the set of positions $x$ occupied by the particle at successive times $t$. “Likewise, the point particles exist in a 3D velocity-space $v$ with an orbit $V(t)$. Then we combine the two spaces to define the 6D phase-space, $(x,v)$, of the dynamical system of point particles, with each particle having a density in this phase-space as

$$N(x,v,t) = \delta[x - X_1(t)]\delta[ v - V_1(t)]$$ (3.41)

where $\delta[x - X_1] = \delta(x - X_1)\delta(y - Y_1)\delta(z - Z_1)$ (see Eq. 3.1 of Nicholson (1983)). They note, “$(X_1, V_1)$ are the Lagrangian coordinates of the particle itself, whereas $(x, v)$ are the Eulerian coordinates of the phase space." Lagrangian coordinates represent dynamics in the frame of reference of the moving particles, whereas the Eulerian coordinates corresponds to the dynamics in a stationary frame of reference that is observing the moving particles. For each species $s$ having $N_0$ particles in the system,

\textsuperscript{16}The theory was developed well before 1983, but Nicholson explained it in their broadly read textbook.
the density $N_s$ is

$$N(x, v, t) = \sum_i^{N_0} \delta [x - X_i(t)] \delta [v - V_i(t)]$$

(3.42)

and so the total density $N$ of all species is

$$N(x, v, t) = \sum_{s,i} N_s(x, v, t)$$

(3.43)

The time derivative of the position orbit of particle $i$, denoted as an over-dot is

$$\dot{X}_i(t) = V_i(t)$$

(3.44)

and the time derivative of the velocity orbit of particle $i$ satisfies the Lorentz force equation

$$m_s \dot{V}_i(t) = q_s E^m [X_i(t), t] + \frac{q_s}{c} V_i(t) \times B^m [X_i(t), t]$$

(3.45)

where the superscript $m$ denotes the self-consistently generated microscopic electric and magnetic fields produced by the point particles themselves, in addition to the externally applied fields. Maxwell’s equations then describe the microscopic fields

$$\nabla \cdot E^m(x, t) = 4\pi \rho^m(x, t)$$

(3.46)

$$\nabla \cdot B^m(x, t) = 0$$

(3.47)

$$\nabla \times E^m(x, t) = -\frac{1}{c} \frac{\partial B^m(x, t)}{\partial t}$$

(3.48)

$$B^m(x, t) = \frac{4\pi}{c} J^m(x, t) + \frac{1}{c} \frac{\partial E^m(x, t)}{\partial t}$$

(3.49)

where the microscopic charge density is

$$\rho^m(x, t) = \sum_{s, i} q_s \int d\nu N_s(x, \nu, t)$$

(3.50)
and the microscopic current is

\[ \mathbf{J}^m(x, t) = \sum_{s,i} q_s \int d\mathbf{v} \mathbf{v} N_s(x, \mathbf{v}, t) \]  \hspace{1cm} (3.51)

Equations (3.44)-(3.45) determine the exact particle orbits in terms of exact fields, while equations (3.46)-(3.51) determine the exact fields in terms of the exact particle orbits, ensuring that the system (3.44)-(3.51) is a closed set of equations (Nicholson, 1983).

An exact description of the time evolution of the distribution of each species is then derived as

\[ \frac{\partial N_s(x, \mathbf{v}, t)}{\partial t} + \mathbf{v} \cdot \nabla_x N_s + \frac{q_s}{m_s} \left( \mathbf{E}^m + \frac{\mathbf{v}}{c} \times \mathbf{B}^m \right) \cdot \nabla_\mathbf{v} N_s = 0 \]  \hspace{1cm} (3.52)

called the Klimontovich-Dupree equation (Klimontovich, 1967; Dupree, 1963). Together with Maxwell’s equations, Eq. (3.52) yields an exact deterministic description of the plasma (Nicholson, 1983). The total time derivative of a particle moving along an orbit in phase space, also known as the convective derivative, is then defined as

\[ \frac{D}{Dt} N_s(x, \mathbf{v}, t) = 0 \]  \hspace{1cm} (3.56)
which is a statement about conservation of particle number density (Nicholson, 1983).

3.6.3 The Ensemble Average

In the next step of deriving the plasma kinetic equations, Nicholson (1983) introduces

\[ f_s(x, v, t) = \langle N_s(x, v, t) \rangle \]  \hspace{1cm} (3.57)

as the ensemble average over an “infinite number of realizations of the plasma,” as rigorously defined by Reif (1965). This distribution function is then substituted for \( N_s \) in the Klimontovich-Dupree Equation (3.52) and the rest of kinetic plasma theory as we know it is derived from this critical stage (for example, the Vlasov equation is Eq. (3.52) with \( f_s \) substituted for \( N_s \)). It was justified that this was done to deduce information about the average properties of the plasma (Nicholson, 1983).

The ensemble average represents the average number density of particles in all possible states of the system, which is an abstract mathematical notion to cast the spiky delta functions of \( N_s(t) \) as a smooth distribution function. As suggested by Erofeev (2014), this notion of the ensemble average is not physically informative since it represents time evolutions of probabilities that do not naturally exist in the universe. In particular, these probabilities represent the statistics of all possibilities of any \( f_s \) realization, which is equivalent to saying that all \( f_s \) distribution functions are indistinguishable, meaning that plasma physicists refer to \( f_s \) as if they were all the same. By definition of the ensemble average, indeed they are all the same; they are the average of every possible solution, which is certainly not observationally ac-
cessible knowledge. In a provocative article, Erofeev (2004) mentions in a footnote that, “as a first important system ensemble study we can mention [the] renown[ed] equation by Boltzmann. His concept of [a] distribution function supposes indirectly the substitution of a real mixture of discrete gas atoms by a continuous probabilistic ensemble of such mixtures. Boltzmann himself had no perception of this fact, although afterwards he explicitly spoke about the desirability of substituting system ensembles for real systems (Boltzmann, 1896).”

Obtaining statistics about the number density of particles in any possible state obscures the specific physical problem at hand by including too much information that does not actually exist. To derive an actual physical result, substituting an unphysical $f_s$ should be avoided (Erofeev, 2014). To smooth out the delta functions defined by $N_s$, Erofeev (2014) suggests that one should instead start with the Klimontovich-Dupree equation, and then take an appropriate phase-space or time average for the specific problem at hand. That way, the distributions will actually contain the much smaller subset of statistically relevant information to the physical problem, not the average statistics of every possible solution to the problem. As an example, Erofeev (1997) started with the Klimontovich definition of a particle density function (which is also a distribution function) in canonical position $\mathbf{r}$ and momentum $\mathbf{p}$ coordinates:

$$N_\alpha(\mathbf{r}, \mathbf{p}, t) = \sum_i \delta^3 [(\mathbf{r} - \mathbf{r}_i(t)) \delta^3 [\mathbf{p} - \mathbf{p}_i(t))]$$

(3.58)

where $\alpha$ is the particle species, the subscript $i$ denotes the number of particles for each species, and the position and momentum particle trajectories are represented by $\mathbf{r}_i(t)$ and $\mathbf{p}_i(t)$, respectively. In this more general formalism, the Klimontovich-Dupree
The equation for the time evolution for the microdistribution of particles is

$$\frac{\partial N_\alpha(r,p,t)}{\partial t} + \mathbf{v} \cdot \nabla N_\alpha + e_\alpha \left[ \mathbf{E}' + \frac{1}{c} (\mathbf{v} \times \mathbf{B}') \right] \cdot \frac{\partial N_\alpha}{\partial \mathbf{p}} = 0 \quad (3.59)$$

where $e_\alpha$ represents the species charge and $\mathbf{E}'$ and $\mathbf{B}'$ represents the electric and magnetic field acting on the particle, respectively. Erofeev (1997) demonstrated how to avoid the ensemble average substitution when they derived a three-wave interaction equation using Eq. (3.58)-(3.59) by defining a distribution function in a way that accurately described the statistics of the physically observable situation.

### 3.6.4 Redefining the FPC Technique Using Particle Phase-Space Density

In this section, I show how to circumvent the ensemble average to redefine the FPC technique with Eq. (3.43), enabling an expression to be obtained for the instantaneous rate of change in particle kinetic energy, using only information about the particle number density, rather than the abstract and unphysical $f_s$ (see §3.3.2 for the derivation of the field-particle correlation technique). Ultimately, redefining the widely accepted approach of describing plasma evolution using the ensemble-averaged $f$ and instead obtaining an expression in terms of particle counts (an observable quantity), will be more accessible information for instruments relevant for implementation of the PATCH method onboard spacecraft. Note that for derivation consistency with Klein and Howes (2016) and Howes et al. (2017), the Vlasov equation must also be redefined carefully (beyond the scope of this dissertation) and will be left for future work.
3.6.4.1 Onboard Correlation Derivation

Using Kinetic Theory

Since our ultimate goal is to implement this technique onboard a spacecraft, we must first restrict infinite 3D-3V phase-space to a finite 3D-3V phase-space instrumental measurement grid, \((r \times v)_G\). Note that the subscript, \(G\), suggests that the phase-space resolution of the particle instrument (ESA) is dependent on the geometric factor described by Eq. (3.35) in §3.5.2. Then \((r \times v)_G\) is comprised of approximately infinitesimal (smallest measurement bin size) 3D-3V phase-space volumes \(\Delta r_j \Delta v_k\) centered at one phase-space coordinate, \((r_j, v_k)\), where

\[
\sum_{j=1}^{J} \sum_{k=1}^{K} \Delta r_j \Delta v_k = (r \times v)_G
\]

As defined, \(J\) and \(K\) correspond to the total number of spatial and velocity-space measurement bins on the instrument, respectively. The ‘×’ notation refers to the Cartesian product, defining \((r \times v)_G\) as the set

\[
(r \times v)_G = \{(r_j, v_k) | r_j \in \Delta r_j \ \forall \ j \in J \ and \ v_k \in \Delta v_k \ \forall \ k \in K\}
\]

Also note that since each volume of phase-space is 3D-3V,

\[
r_j = (r_{1j}, r_{2j}, r_{3j}) \ \forall \ j \in J \ and \ v_k = (v_{1k}, v_{2k}, v_{3k}) \ \forall \ k \in K
\]

Using Eq. (3.43) for \(N_s\), the microscopic kinetic energy of a collection of \(N_0\) particles for each species, \(s\), arriving at the detector aperture of the instrument is defined as\(^{17}\)

\[
W_s^*(r, v, t) = \int_{(r \times v)_G} d^3r \ d^3v \ \frac{1}{2} m_s v^2 N_s
\]

\(^{17}\)In terms of statistical moments, this quantity can also be interpreted as the variance of the particle velocities as they arrive in phase-space with time.
\[
\int_{(r \times v)\mathcal{C}} d^3r d^3v \frac{1}{2} m_s v^2 \sum_{i=1}^{N_0} \delta^3 [r - r_i(t)] \delta^3 [v - v_i(t)]
\]

To convert this to a phase-space energy density, define

\[
w_s^* (r_j, v_k) \equiv \frac{W_s^*}{(\Delta r_j \Delta v_k)} \tag{3.60}
\]

where \((\Delta r_j \Delta v_k)\) is one single volume element of phase-space. We explicitly define the spatial limits of integration as

\[
\int_{\Delta r_j} d^3r = \int_{r_{1j} - \frac{\Delta r_{1j}}{2}}^{r_{2j} + \frac{\Delta r_{2j}}{2}} \int_{r_{2j} - \frac{\Delta r_{2j}}{2}}^{r_{3j} + \frac{\Delta r_{3j}}{2}} \int_{r_{3j} - \frac{\Delta r_{3j}}{2}}^{r_{4j} + \Delta r_{4j}} \, dr_1 \, dr_2 \, dr_3 \tag{3.61}
\]

and the velocity-space integration limits as

\[
\int_{\Delta v_k} d^3v = \int_{v_{1k} - \frac{\Delta v_{1k}}{2}}^{v_{2k} + \frac{\Delta v_{2k}}{2}} \int_{v_{2k} - \frac{\Delta v_{2k}}{2}}^{v_{3k} + \frac{\Delta v_{3k}}{2}} \int_{v_{3k} - \frac{\Delta v_{3k}}{2}}^{v_{4k} + \Delta v_{4k}} \, dv_1 \, dv_2 \, dv_3
\]

By Eq. (3.60), the microscopic kinetic energy within each approximately infinitesimal volume of phase-space for each species, \(s\), is

\[
w_s^* = \frac{1}{(\Delta r_j \Delta v_k)} \int_{\Delta r_j} d^3r \int_{\Delta v_k} d^3v \frac{1}{2} m_s v^2 N_s
\]

\[
= \frac{1}{(\Delta r_j \Delta v_k)} \int_{\Delta r_j} d^3r \int_{\Delta v_k} d^3v \frac{1}{2} m_s v^2 \sum_{i=1}^{N_0} \delta^3 [r - r_i(t)] \delta^3 [v - v_i(t)]
\]

Since we are in a finite space, we use the notion of a Dirac measure to test if a particle, \(p_i\), is counted in \(\Delta r_j \Delta v_k\):

\[
\delta_{p_i}(\Delta r_j \Delta v_k) = \begin{cases} 
1 & \text{if } p_i \in \Delta r_j \Delta v_k \\
0 & \text{if } p_i \notin \Delta r_j \Delta v_k
\end{cases}
\]

This means the summation, over the \(\delta\)-functions within a phase-space volume \(\Delta r_j \Delta v_k\), commutes with (and is moved in front of) the integral when the upper
summation limit is defined as the total number of particles, $N_{tot}$, counted within the 6D phase-space volume $\Delta r_j \Delta v_k$. Thus,

$$w_s^* = \frac{1}{(\Delta r_j \Delta v_k)} \sum_{i=1}^{N_{tot}} \int_{\Delta r_j} d^3r \, \delta^3[r - r_i(t)] \int_{\Delta v_k} d^3v \, \frac{1}{2} m_s v^2 \delta^3[v - v_i(t)]$$

(3.62)

Note the summation in Eq. (3.62) is over the index $i$ where $(r_i(t), v_i(t)) \in \Delta r_j \times \Delta v_k$.

Now, we restrict our coordinates to a single-point spacecraft measurement at the ESA detector aperture with 2D surface area $A$ depicted in Fig. 3.6. Define $\Delta r_p$ for some index $p \in J$ as the single 3D spatial volume element where a particle of species $s$, with velocity $v_p$, arrives at the aperture plane at some arrival time $t_p$, during a sampling time $dt_p$. In the summation over $N_{tot}$ particles, the particle indexed by $i$ with arrival time $t_{pi}$, occupies the fixed phase-space coordinate $(r_p(t_{pi}), v_p(t_{pi}))$ centered at $\Delta r_p$, at an angle incident normal to the plane perpendicular to the detector aperture, within an approximately infinitesimal area $dA_p$. This suggests we are defining a local particle count in phase-space, similar to the notion of the local distribution function, $f_{loc}$, discussed in §3.5.2. We also assume only 1 particle passes through\textsuperscript{18} the aperture in the small local area $dA_p$ at the instantaneous particle arrival time $t_p$, but multiple particles (indexed by $i$) can be counted in the sampling time $dt_p$. Recalling Eq. (3.15) from §3.5.2, we redefine $\Delta r_p$ as

$$\Delta r_p = v_{1p} \Delta t_{1p} \Delta A_{(2,3)p}$$

(3.63)

where the bin width, $\Delta$, is the approximated infinitesimal width, $d$. Hence, $dr_{1p} =$

\textsuperscript{18} In reality, this assumption could be invalid in some cases. For this dissertation, we accept it as true to enable development of formal theory using Dirac measures and Poisson counting statistics.
\[ d^3 \mathbf{r}_p = dr_1 \wedge dr_2 \wedge dr_3 = v_1 p dt_1 \wedge d\mathbf{A}_{(2,3)p} = v_1 p dt_1 p d\mathbf{A}_{(2,3)p} \] (3.64)

To avoid cumbersome notation, we will omit the subscripts identifying the dimension in the 3D coordinate \( \mathbf{r}_p = (r_{1p}, r_{2p}, r_{3p}) \), since they are already fixed and defined by Eq. (3.64). Thus, the spatial integration coordinates from Eq. (3.61) become

\[ \int_{\Delta r_p} d^3 \mathbf{r}_p = \int_{t_p - \frac{\Delta t_p}{2}}^{t_p + \frac{\Delta t_p}{2}} \int_{A_p - \frac{\Delta A_p}{2}}^{A_p + \frac{\Delta A_p}{2}} v_p dt_p d\mathbf{A}_p \] (3.65)

Substituting Eq. (3.65) for Eq. (3.61) (and using the property \( \delta(x) = \delta(x) \sum \delta(x) \)), we find

\[ \int_{\Delta r_p} d^3 \mathbf{r}_p \delta^3 [\mathbf{r}_p - \mathbf{r}_{pi}(t)] = \int_{t_p - \frac{\Delta t_p}{2}}^{t_p + \frac{\Delta t_p}{2}} \delta [v_p t_p - v_{pi} t_{pi}(t)] v_p dt_p \int_{A_p - \frac{\Delta A_p}{2}}^{A_p + \frac{\Delta A_p}{2}} \delta^2 [A_p - A_{pi}(t)] d\mathbf{A}_p \]

\[ = \int_{t_p - \frac{\Delta t_p}{2}}^{t_p + \frac{\Delta t_p}{2}} \delta [v_p t_p - v_{pi} t_{pi}(t)] v_p dt_p \]

\[ = \int_{t_p - \frac{\Delta t_p}{2}}^{t_p + \frac{\Delta t_p}{2}} \delta [t_p - t_{pi}(t)] v_p dt_p \] (3.66)

Substituting this result back into Eq. (3.62),

\[ w_s^* = \frac{1}{(\Delta r_p \Delta v_k)} \sum_{i=1}^{N_{tot}} \int_{t_p - \frac{\Delta t_p}{2}}^{t_p + \frac{\Delta t_p}{2}} \delta [t_p - t_{pi}(t)] dt_p \int_{\Delta v_k} d^3 \mathbf{v} \frac{1}{2} m_s v^2 \delta^3 [\mathbf{v} - \mathbf{v}_i(t)] \]

\[ = \frac{1}{(\Delta r_p \Delta v_k)} \sum_{i=1}^{N_{tot}} \int_{t_p - \frac{\Delta t_p}{2}}^{t_p + \frac{\Delta t_p}{2}} \delta [t_p - t_{pi}(t)] dt_p \left[ \frac{1}{2} m_s [\mathbf{v}_i(t)]^2 \right] \] (3.67)

For each particle count indexed by \( i \),

\[ t_{pi}(t) = \{ t \in \Delta t_p | t = t_{pi} \} \] (3.68)
so $t_{pi}(t) = t|_{\Delta t_p}$ and $[t_p - t_{pi}(t)] = 0$ when $t = t_{pi}$. Since the integration variable in Eq. (3.67) is $dt_p$, the $\delta$-functions are nonzero only when $t \in \Delta t_p$ for each particle indexed by $i$. Therefore, each term in the summation of Eq. (3.67) is nonzero only at times $t = t_{pi}$, the instantaneous particle arrival times, meaning

$$w_s^* = \frac{1}{(\Delta r_p \Delta v_k)} \sum_{i=1}^{N_{tot}} \frac{1}{2} m_s [v_i(t)]^2$$

(3.69)

where $t \in \Delta t_p$ for each particle count indexed by $i$. The instantaneous change in microscopic phase-space kinetic energy density is found by calculating the instantaneous time derivative of Eq. (3.69) to get

$$\frac{\partial w_s^*}{\partial t} = \frac{\partial}{\partial t} \left[ \frac{1}{(\Delta r_p \Delta v_k)} \sum_{i=1}^{N_{tot}} \frac{1}{2} m_s [v_i(t)]^2 \right] = \frac{1}{(\Delta r_p \Delta v_k)} \sum_{i=1}^{N_{tot}} \frac{1}{2} m_s \frac{\partial}{\partial t} [v_i(t)]^2$$

$$= m_s \frac{1}{(\Delta r_p \Delta v_k)} \sum_{i=1}^{N_{tot}} v_i(t) \cdot \dot{v}_i(t)$$

where the over-dot implies the time derivative given by Eq. (3.45). By substituting Eq. (3.45) for $\dot{v}_i(t)$, we get

$$\frac{\partial w_s^*}{\partial t} = \frac{1}{(\Delta r_p \Delta v_k)} \sum_{i=1}^{N_{tot}} \frac{1}{m_s} q_s v_i(t) \cdot \left[ \mathbf{E}(r_p(t),t) + \frac{q_s}{c} v_i(t) \times \mathbf{B}(r_p(t),t) \right]$$

(3.70)

Note that the magnetic field term vanishes since $v_i(t) \cdot (v_i(t) \times \mathbf{B}) = 0$. The term $v_i(t) \in \Delta v_k$, is interpreted as the 3D velocity orbit of particle $i$ measured in the velocity-space bin $\Delta v_k$. Thus, the summation terms in Eq. (3.70) are only nonzero when $i = k$, so

$$\frac{\partial w_s^*}{\partial t} = \frac{1}{(\Delta r_p \Delta v_k)} \sum_{i=1}^{N_{tot}} q_s v_k(t) \cdot \mathbf{E}(r_p(t),t)$$

(3.71)
By definition of the set of particle arrival times given by (3.68), $v_k(t)$ and $r_p(t)$ are only nonzero when evaluated at $t = t_{pi}$. Therefore,

$$\frac{\partial w_s^*}{\partial t} = \frac{1}{(\Delta r_p \Delta v_k)} \sum_{i=1}^{N_{tot}} q_s v_k(t_{pi}) \cdot E(r_p(t_{pi}), t)$$ \hspace{1cm} (3.72)

The electric field term will have both an oscillating component ($osc$) contributing to oscillatory energy transfer and a secular component ($net$) leading to net energy transfer. To isolate the component that leads to net energy transfer, we now redefine the field-particle correlation over some correlation time interval, $\tau$, that is much longer than the particle sampling time $dt_p$, and chosen to be long enough so that the large-scale electric field contributing to the oscillating energy transfer averages to 0. Hence, we choose $\tau$ centered at a time, $t_0$, such that

$$\frac{1}{\tau} \int_{t_0 - \frac{\tau}{2}}^{t_0 + \frac{\tau}{2}} dt \cdot E_{osc}(r_p(t_{pi}), t)$$

$$= \langle E_{osc}(r_p(t_{pi}), t) \rangle_\tau$$

$$= 0$$

and so $\langle E(r_p(t_{pi}), t) \rangle_\tau = E_{net}(r_p(t_{pi}))$, the time-averaged electric field over a sufficiently long $\tau$ that leads to net energy transfer, such as particle energization. The field-particle correlation technique is hence redefined for onboard spacecraft implementation as the time-averaged rate of change of phase-space energy density:

$$C_\tau^* = \langle \frac{\partial w_s^*}{\partial t} \rangle_{\tau} = \frac{1}{\tau (\Delta r_p \Delta v_k)} \sum_{i=1}^{N_r} q_s v_k(t_{pi}) \cdot E(r_p(t_{pi}))$$ \hspace{1cm} (3.73)

where $N_r$ represents the total number of particles counted in a phase-space volume $\Delta r_p \Delta v_k$ over the suitable correlation time, $\tau$, of Eq. (3.72) that isolates the net energy transfer between the fields and particles.
Note this is consistent with the intuition prescribed in §3.6.1, with total number of particles, \(N_\tau\), given by Eq. (3.38). Using, Eq. (3.73) the correlation (in a single phase-space bin \(\Delta r_p \Delta v_p\)) is

\[
C^*_\tau = \frac{1}{\tau} \frac{1}{(\Delta r_p \Delta v_p)} \sum_{j=1}^{N_\tau} q_s v_p \cdot E(r_p, t_j)
\]

(3.74)

which is in agreement with Eq. (3.40) per phase-phase bin. Note that \(v_p\) was substituted for \(v_j(t_j) \in \Delta v_p\) since this quantity is interpreted as the velocity (in spacecraft \((E, \Omega)\) coordinates) that each particle arrives in the bin \(\Delta v_p\). This velocity is fixed for each particle arrival time in local phase-space, since on the ESA, it corresponds to a fixed voltage difference and solid angle of rotation.

### 3.6.5 Rate of Work Done on the Particles

In this section, we calculate the rate of work done on the particles from the fields, \(J \cdot E\), in a phase-space volume. This quantity, in addition to the energy flux leaving that volume, yields the rate of change of energy per unit volume. In differential form, this statement is summarized as Poynting’s Theorem:

\[
-\frac{\partial u}{\partial t} = \nabla \cdot S + J \cdot E
\]

(3.75)

where \(u\) is the phase-space energy density of the fields and \(S = \frac{1}{\mu_0}(E \times B)\) is the Poynting vector, which indicates the direction of electromagnetic energy flux.

From Eq. (3.51),

\[
J(r, t) = \sum_{s,i} q_s \int d^3v \nu N_s(r, v, t) = \sum_{s,i} q_s \int d^3v \nu \delta^3[r - r_i(t)] \delta^3[v - v_i(t)]
\]
Repeating the previous procedure in §3.6.4, we restrict the current density locally within a spatial volume \( \Delta r_p \) where particles arrive on the aperture, within in a single velocity-space bin \( \Delta v_k \). To obtain a quantity for the work done on the particles in the phase-space volume \( \Delta r_p \Delta v_k \), we compute the inner product of \( \mathbf{J}(\mathbf{r}, t) \) with the electric field in the spacecraft frame-of-reference (at the ESA aperture). Integrating over the locally defined phase-space volume, we get

\[
\mathbf{J} \cdot \mathbf{E} \bigg|_{(\Delta r_p \Delta v_k)} = \sum_{s,i} q_s \left( \frac{1}{(\Delta r_p \Delta v_k)} \right) \int_{\Delta r_p} d^3 \mathbf{r} \mathbf{E}(\mathbf{r}_p(t), t) \delta^3 [\mathbf{r} - \mathbf{r}_i(t)] \int_{\Delta v_k} d^3 \mathbf{v} \mathbf{v} \delta^3 [\mathbf{v} - \mathbf{v}_i(t)]
\]

\[
= \frac{1}{(\Delta r_p \Delta v_k)} \sum_{i} N_{tot} \int_{t_p - \frac{\Delta t_p}{2}}^{t_p + \frac{\Delta t_p}{2}} \mathbf{E}(\mathbf{r}_p(t), t) \delta [t_p - t_{pi}(t)] \, dt_p \cdot [\mathbf{v}_k(t)]
\]

\[
= \frac{1}{(\Delta r_p \Delta v_k)} \sum_{i} N_{tot} \mathbf{E}(\mathbf{r}_p(t_{pi}), t) \cdot [\mathbf{v}_k(t)]
\]

(3.76)

where again, \( t_{pi} \) indicates the time of arrival for each particle indexed by \( i \) contained in the small phase-space volume \( \Delta r_p \Delta v_k \). But since \( t = t_{pi} \ \forall t \in \mathbf{v}_k(t) \), Eq. (3.76) becomes

\[
\mathbf{J} \cdot \mathbf{E} \bigg|_{(\Delta r_p \Delta v_k)} = \frac{1}{(\Delta r_p \Delta v_k)} \sum_{s,i} q_s \mathbf{v}_k(t_{pi}) \cdot \mathbf{E}(\mathbf{r}_p(t_{pi}), t)
\]

\[
= \frac{\partial u_s^*}{\partial t}
\]

(3.77)

which matches Eq. (3.72). Time-averaging Eq. (3.77) over a long enough correlation interval, \( \tau \), to isolate the net transfer of energy by the electric field,

\[
\left< \mathbf{J} \cdot \mathbf{E} \bigg|_{(\Delta r_p \Delta v_k)} \right>_{\tau} = \frac{1}{\tau} \frac{1}{\Delta r_p \Delta v_k} \sum_{s,i} q_s \mathbf{v}_k(t_{pi}) \cdot \mathbf{E}(\mathbf{r}_p(t_{pi}))
\]

\[
= \left< \frac{\partial u_s^*}{\partial t} \right>_{\tau}
\]

\[
= C^*_{\tau}
\]
which verifies that $C_{\tau}^*$ is indeed measuring the work done on the particles by the electric field over a long-time average $\tau$.

### 3.7 Testing PATCH with Synthetic Spacecraft Data

In this section, I describe how the PATCH method for diagnosing ion Landau damping was implemented using synthetic spacecraft data created from AstroGK output (utilizing the same simulation parameters described in §3.4.1). I prescribe the procedure for implementing PATCH with the simulation output data in §3.7.1, along with results demonstrating that the ion Landau damping velocity-space signature can be resolved with this novel method in §3.7.2. A noise model and statistical diagnostic tool for identifying the threshold number of particle counts needed to resolve the ion Landau damping velocity-space signature is described in §3.7.3.

#### 3.7.1 PATCH Implementation Procedure

To test the PATCH method in a more realistic scenario than in §3.6.1, we applied the algorithm to synthetic spacecraft data created in Parker Solar Probe (PSP) spacecraft energy resolution. We invoked the same procedure for using AstroGK to create synthetic data for specific instrumental velocity-space resolutions in §3.4.2.

Let $f(E, \phi)$ be the down-sampled distribution function in PSP $(E, \phi)$-space, and $\Delta t_\rho$ be the courser time resolution of the distribution function dictated by the FPC diagnostic output ($\Delta t = 0.16$). The parallel electric field output was linearly interpolated over a finer time resolution with time step $\hat{t}_{\text{fine}} = 0.001$ in order to allow up to $N_{max} = 160$ particles to arrive per FPC diagnostic output time step. Note that
\( \Delta \hat{t}_p \) corresponds to the particle sampling time bin, \( \Delta t_p \), described in §3.6.4. Thus, there are

\[
N_{\text{max}} = \frac{\Delta \hat{t}_p}{t_{\text{fine}}}
\]

possible particle arrivals per sampling time bin, \( \Delta \hat{t}_p \), which is also the bin size of the correlation time slices. Now define the probability of counting \( N \) particles within a time period comprised of \( N_{\text{max}} \) possible particle arrivals as:

\[
P(\text{particle arrives}) \equiv \frac{N \cdot f(E, \phi)}{N_{\text{max}}} \quad (3.78)
\]

Based on this probability in each sampling time bin \( \Delta \hat{t}_p \), the particle arrival times, \( t_k \), were determined; an artificial time series for the parallel electric field component was created analogous to the green vertical lines in Fig. 3.7 as:

\[
\sum_{t_k=1}^{N_{\text{tot}}} E_{\parallel}(r_p, t_k)
\]

where \( r_p \) represents a single spatial volume (described in §3.6.4) where the particles arrive at the ESA aperture. Note that we are focusing on the parallel contribution to the energy transfer due to Landau damping.

The PATCH algorithm computes the correlation, \( C^* \), using Eq. (3.73) with particle arrival times created by probabilities defined in Eq. (3.78), and evaluating the electric field at those times using Eq. (3.79). The PSP velocity-space bins are in \((E, \phi)\) coordinates\(^\text{19}\) indexed by \( ie, i\phi \), respectively. Over a correlation interval

\(^\text{19}\)Note that the small polar-angle tilt, \( \theta \), relative to the azimuthal symmetry axis on the ESA was neglected.
comprised of $\tau = N_{\text{corr}}$ time-slices in one velocity-space bin, $C^*_\tau$ was computed as

$$\frac{1}{N_{\text{corr}}} \sum_{ct=1}^{N_{\text{corr}}} \frac{\Delta t_p}{N_{\text{tot}ct}} \sum_{t_k=1}^{N_{\text{tot}ct}} q_sv_\parallel(ie,i\phi)E_\parallel(ie,i\phi,t_k)$$

The total number of particles counted per sampling time $N_{\text{tot}ct}$, corresponds to the correlation time index $ct$. For consistency with the theory developed in §3.6.4, the total number of particles in the whole correlation time interval, $\tau$, is defined as

$$N_\tau = \sum_{ct=1}^{N_{\text{corr}}} N_{\text{tot}ct}$$

Eq. (3.72) tells us that the rate of energy transfer per phase-space bin (centered at a single spatial point $r_p$) is,

$$\frac{\partial w^*_s}{\partial t} = \frac{1}{\Delta r_p \Delta v_k} \sum_{t_k=1}^{N_{\text{tot}}} q_s v(t_k) E(r_p(t_k),t)$$  \hspace{1cm} (3.80)

The velocity-space in PSP resolution has $e_{\text{tot}}$ total energy ($E$) bins and $\phi_{\text{tot}}$ total azimuthal ($\phi$) bins. Thus, there are $e\phi_{\text{tot}} = (e_{\text{tot}})(\phi_{\text{tot}})$ total bins in the down-sampled ($E,\phi$) PSP instrumental velocity-space grid. A normalized time-averaged correlation in each velocity-space bin of ($E,\phi$) is

$$C^*_{\text{norm}}(E,\phi) = \frac{\partial w^*_s}{\partial t}_{\text{norm}} (E,\phi) = \frac{(\Delta t_p)(e\phi_{\text{tot}})}{(N_{\text{corr}})(N_\tau)} q_i v_\parallel(E,\phi)E_\parallel(E,\phi)$$  \hspace{1cm} (3.81)

where $v_\parallel$ is in the spacecraft frame-of-reference and $E_\parallel$ is evaluated at each particle arrival time, with possible calibration frame delay defined by Eq. (B.3). Here, we specifically take the $\parallel$ contribution to $v$ and $E$, setting $s = i$ to diagnose ion Landau damping.

Notice the similarities between Eq. (3.81) and Eq. (3.73), repeated here:

$$C^*_\tau = \left\langle \frac{\partial w^*_s}{\partial t} \right\rangle_\tau = \frac{1}{\tau (\Delta r_p \Delta v_k)} \sum_{i=1}^{N_\tau} q_s v_k(t_{pi}) \cdot E(r_p(t_{pi}))$$
Comparing to Eq. (3.81), \( N_{\text{corr}} \) corresponds to \( \tau \), since this was defined as the total number of time-slices comprising the correlation interval, \( \tau \). Since \( C^*_{\tau} \) describes the calculation performed in each velocity-space bin, \( \Delta r_p \Delta v_k \), and time sampling bin, \( \Delta \hat{t}_p \), the normalized correlation over all of velocity-space, \( C^*_{\text{norm}}(E, \phi) \), must be multiplied by the total number of these bins \((\Delta \hat{t}_p)(e\phi_{\text{tot}})\). Similarly, \( C^*_{\text{norm}}(E, \phi) \) must be divided by the total number of particles counted over \( \tau \), \( N_\tau \). The particle arrival times in \( C^*_{\tau} \) were \( t_{pi} \), while the particle arrival times \( t_k \) in Eq. (3.80) were summed in each velocity-space bin per sampling time to produce \( N_\tau \) total particles in \( C^*_{\text{norm}}(E, \phi) \).

Thus, the normalization factor in \( C^*_{\text{norm}}(E, \phi) \),

\[
A = \frac{(\Delta \hat{t}_p)(e\phi_{\text{tot}})}{(N_{\text{corr}})(N_\tau)}
\]

is defined such that \( C^*_{\text{norm}}(E, \phi) = C^*_{\tau} \) when \( C^*_{\tau} \) is evaluated locally in each \((E, \phi)\) bin, enabling direct comparison to physical quantities of energy transfer. Note that the goal of the FPC technique is to characterize dissipation mechanisms based on their velocity-space signature, so \( C^*_{\tau} \) must be evaluated in each bin to produce a complete picture.

### 3.7.2 Results Using PATCH

The quantity \( N \) in Eq. (3.78) is effectively a number density sampling rate over \((E, \phi)\)-space, which reveals the total number of particles counted in each velocity-space bin per sampling time. It is essentially the same as the mean number density, \( N_0 \), in Fig. 3.7 and \( N_{\text{tot}} \) in Eq. 3.80. Generally, if \( N \) becomes larger, so does the particle count rate, \( \dot{N} \), (discussed in §3.5.2), thereby yielding better statistics.
To see the relationship between $N$ and $\dot{C}$, one can observe from Eq. (3.78) that the quantity $N/N_{\text{max}}$ can be interpreted as the average number of particles per $\Delta \hat{t} = 0.16$ time bin, which defines an average count rate. Thus, changing $N$ is actually changing the average count rate per sampling time interval, $\Delta \hat{t}_p$, which is the amount of time the instrument would spend counting particles arriving at an angle $\theta$ incident with the normal surface area $A$ of the detector aperture, with a fixed energy $E$, in one azimuthal angle bin $\phi$. Note that $\theta$ and $\phi$ would combine to form the solid angle $\Omega$. Therefore, the discussion in Appendix B.3 could reveal useful insight to the required geometric factor $G$ to count the threshold number of particles, $N$, to resolve the velocity-space signature of Landau damping.\textsuperscript{20}

The background spatial number density of the plasma is a fixed quantity that we cannot change. Note that, if the spatial number density increases, the magnitude of the energy transfer will increase. Physically, to change $N$, an onboard correlator can instead vary the sampling interval for each phase-space bin. For a fixed spatial number density, doubling the sampling time will generally double the number of particles counted. So, by “changing” $N$, we are changing the sampling time, $\Delta \hat{t}_p$, such that the number of particles counted in a given energy and angle bin is $N$. In other words, the ESA described in §3.5 will sample $N$ particles in a given $\Delta \hat{t}_p$ before the ESA sweeps to the next energy channel. The interpretation of $N$ in relation to the sampling time and ESA geometry is explored in Appendix B.3.

\textsuperscript{20}Note that we assume the Landau damping signature is time-stationary over the samples, which may be questionable in practice.
Figure 3.8: Testing the ability of the PATCH algorithm to recover the velocity-space signature of Landau damping, using the normalized correlation $C_{\text{norm}}^*$ (given by Eq. (3.81)) in PSP resolution for (a) 15, (b) 16, (c) 25, (d) 39, (e) 50, (f) 75 and (g) 100 particles counted per phase-space bin per sampling time. Comparison to using $C'(f_{\text{PSP}})$ instead of the PATCH method is represented again in panel (h).
Fig. 3.8 shows snapshots from a visualization depicting the ability of the PATCH algorithm to recover the velocity-space signature of energy transfer with improved counting statistics using the normalized correlation quantity $C_{\text{norm}}^*$, given by Eq. (3.81), for each $N$. Eq. (3.81) includes a normalization, $A$, of the (linear) number density dependence of the energy transfer. This facilitates comparison to the velocity-space signature patterns obtained when different numbers of particles are counted (varying $N$). Fig. 3.8 (h) shows the FPC of the down-sampled distribution function, $C'(f_{PSP})$ computed by Eq. (3.10), which is the correlation without incorporating Poisson sampling statistics. This is the velocity-space signature we are trying to recover, so the goal of Fig. 3.8 is to see how the difference between this pattern (panel (h)) and recovered patterns using PATCH (panels (a)-(f)) decreases with increased $N$. As we let the number of particles, $N$, per correlation sampling time, $\Delta \hat{t}_p$, vary from 1 to 100 particles in panels (a)-(g) of Fig. 3.8, we can see how the signal slowly patches together. Starting around $N = 16$ particles shown in panel (b), the velocity-space signature of Landau damping is steady, i.e. the bi-modal signature featuring a peak about the resonant velocity, is maintained. In §3.7.3, we demonstrate why $N = 16$ is indeed the threshold number of particle counts per phase-space bin per sampling time needed to resolve the signature.

### 3.7.3 Noise Model

We now justify our observation of 16 particles (or time-sampling bins) as our threshold particle count number for resolving the velocity-space signature of Landau
damping by quantifying the statistical noise as the error varies with $N$. Note that this threshold value of $N = 16$ particles counted in a given phase-space bin per sampling time can be used to determine how long one needs to sample in each velocity-space bin on the spacecraft. Thus, this study enables the determination of particle instrument specifications using synthetic data derived from AstroGK simulations.

To calculate the error for each $N$, we define our ‘exact’ answer as the correlation values of $C'(f_{PSP})$ produced by Eq. (3.10) and shown in Fig. 3.8 (h). The correlation quantities produced by $C_{norm}^*(E, \phi)$ using the PATCH algorithm, calculated by Eq. (3.81), is then defined as the ‘experimental’ energy transfer quantity evaluated at each of the $(E, \phi)$ values (represented by the black dots in Fig. 3.8).

In theory, the ‘experimental’ values should converge to the ‘exact’ values as $N \to \infty$ using the PATCH algorithm. For each $N$, we produce an array of correlations over $(E, \phi)$-space calculated by Eq. (3.81), defined as $Corr(N)$. For a given value of $N$, the correlation has an array of real values at each point in $(E, \phi)$-space. Using $i$ and $j$ as indices, the correlation has values $C_N^*(E_i, \phi_j)$ where $i = 1, \ldots, e_{tot}$ and $k = 1, \ldots, \phi_{tot}$ are the indices that denote the different bins in $(E, \phi)$-space. For each index $(i, j)$, the error is calculated as the Euclidean length of the difference between $C_{f_{PSP}}'$ and $C_N^*$ for each index, normalized by the Euclidean length of $C_{f_{PSP}}'$:

$$\text{Error}(N) = \frac{\sqrt{\sum_{i,j} [C_{f_{PSP}}'(i, j) - C_N^*(i, j)]^2}}{\sqrt{\sum_{i,j} [C_{f_{PSP}}'(i, j)]^2}}$$

In other words,

$$\text{Error}(N) = \frac{||\text{exact} - \text{corr}(N)||}{||\text{exact}||} \quad (3.82)$$
for each $N$, where $|||$ denotes the 2-norm of the matrix comprised of the correlation quantities indexed by $(i, j)$. Fig. 3.9 (a) shows a log-log plot of the error versus $N$; using standard data fitting techniques, we find a power law break at 15 particles. This is consistent with our findings in Fig. 3.8, that 15 particles was not able to fully resolve the bimodal velocity-space signature of ion Landau damping, suggesting that statistical fluctuations were dominant for the first 15 particles.

The error has a nearly Poissonian slope of $-0.44 \pm 0.05$ for the first 15 particles, indicated visually by the randomness of the statistical fluctuations in Fig. 3.9 (b). The drastic change of slope at beyond 15 particles shown in Fig. 3.9 (c) could be a diagnostic that the particles have sufficiently deviated from a stationary Poisson process, indicating an interesting physical phenomenon is occurring, i.e. a wave-particle interaction, which is certainly not Poissonian. Therefore, we propose that a power law break in the noise model could be a useful diagnostic for identifying the threshold particle counts needed for a sufficiently patched signal. Another interpretation is that for the first 15 particles, the signal-to-noise ratio approximately scales as $\sqrt{N(t)}$ and is dominated by Poisson noise; once the velocity-space signature of Landau damping is sufficiently resolved, the signal-to-noise ratio increases to a $N(t)^{0.30}$ scaling and the statistical fluctuations are dominated by the turbulence. This could unveil insight about the particle statistics in the dissipation range of the turbulent energy cascade. However, this possibility will require significantly more study, beyond the scope of this dissertation, to determine if one can use these statistical diagnostics to discern meaningful interpretations of the physical behavior.
(a) Noise model for $(x, y, z) = (22, 5, 0)$, $N = 1$-100 Particles

(b) 1-15 Particles,
Slope $\approx -0.44 \pm 0.05$

(c) 15-100 Particles,
Slope $\approx -0.30 \pm 0.04$

Figure 3.9: Noise model power law break in first point $(x, y, z) = (22, 5, 0)$. 
(a) Noise model for second test point \((x, y, z) = (22, 11, -7)\)

(b) 1-17 Particles,
Slope \(\approx -0.51 \pm 0.04\)

(c) 17-100 Particles,
Slope \(\approx -0.20 \pm 0.04\)

Figure 3.10: Noise model power law break in second point \((x, y, z) = (22, 11, -7)\).
(a) Noise model for third test point \((x, y, z) = (22, 8, 0)\)

\[
\begin{align*}
\text{Slope} & \approx -0.42 \pm 0.05 \\
\text{Slope} & \approx -0.26 \pm 0.04
\end{align*}
\]

Figure 3.11: Noise model power law break in third point \((x, y, z) = (22, 8, 0)\).
(a) Noise model for fourth test point \((x, y, z) = (22, 1, 0)\)

(b) 1-16 Particles,  
Slope \(\approx -0.39 \pm 0.05\)

(c) 16-100 Particles,  
Slope \(\approx -0.15 \pm 0.05\)

Figure 3.12: Noise model power law break in fourth point \((x, y, z) = (22, 1, 0)\).
Turbulence is spatially non-homogeneous, however, so we should not expect quantitatively the same noise model for different points in space. Nevertheless, this noise model was tested with three other points and all have quantitatively different slopes on either side of the power law breaks, shown in Fig. 3.10 - Fig. 3.12. Qualitatively, all the tested points featured a negative slope dominated by “Poisson noise randomness” in the range 0.39-0.51 and then sharply decreased to a “more ordered” process with a negative slope in the range 0.15-0.30. In the domain defined by $N = 1, \ldots, N_{\text{break}}$, where $N_{\text{break}}$ indicates the location of the power law break, the log-log plot of $\text{Error}(N)$ has a random Poissonian slope closest to -0.50. Thus, the boundary of this region, the location of $N_{\text{break}}$, indicates the transition where the threshold number of particles produces a statistically more stable Landau damping signature.$^{21}$ In addition, the power law break occurred in the range $N_{\text{break}} = 14 - 17$ particles, yielding similar threshold particles counts for each test-point case. These findings suggest possible methodology for identifying regions of wave-particle interactions and subsequently trigger burst-mode sampling on the spacecraft by measuring the particle statistics and a determining a definitive threshold for particle fluctuations in a “more ordered” state.

Also included in Fig. 3.9 - Fig. 3.12 are “Goodness of fit” parameters: Sum of Squares Due to Error (SSE), R-Square and Root Mean Square Error (RMSE). For clarity, the terms ‘noise’ and ‘data’ refer to the noise model itself computed using Eq. 21. This is shown in Fig. 3.8 for the first point. The other test points have also been verified but not presented for brevity.
Note also that the randomness of the fit with respect to the data does not refer to the same randomness that is being measured by the noise model itself. The SSE is a measure of the total deviation from the fit due to random errors, i.e. SSE values closer to zero means the data has less errors due to randomness. The R-Square measures how close the fit matches the data and ranges from 0 (worst) to 1 (best). The RMSE, also known as the ‘standard error,’ measures the variance in the randomness of the data and similar to the SSE, a better fit would correspond to an RMSE value closer to 0. As can be seen in all cases from panel (a) in Fig. 3.9 - Fig. 3.12, all three “Goodness of fit” parameters are better for the quadratic fit rather then the linear fit for modeling the noise ranging from 1-100 particles. Even though adding more fit parameters will in general yield better results, Fig. 3.10 shows a more pronounced quadratic trend, suggesting that a power law break for the linear error trend indeed exists.

Panel (b) from all cases in Fig. 3.9 - Fig. 3.12 show that in the regions before the power law break, defined as the $\text{Error}(N)$ in the domain $N = 1, ..., N_{\text{break}}$, the SSE and R-Square measures are better than any fit from panel (a). On the other hand, panel (c) from all cases in Fig. 3.9 - Fig. 3.12 show that in the region after the power law break, defined as the $\text{Error}(N)$ in the domain $N = N_{\text{break}}, ..., 100$, the RMSE values are better than any fit from panel (a). This is because the regions before the power law break occurs represent systematic Poissonian noise, so we would not expect the values to oscillate too far from the near-Poissonian linear fit with
negative slopes close to 0.5. However, in the regions after power law break occurs, the data is extremely oscillatory with respect to their mean values, but these mean values are well-approximated by the linear fit.\textsuperscript{22} Low values of RMSE indicate that the amplitude in the error oscillation between the linear fit and the data is consistent. Thus, the error fluctuates at a nearly constant amplitude about their mean values, and these mean values are well-fitted by a linear trend. In the regions after the power law break, the R-Square values are worse in all test-point cases, but the RMSE values are better. Therefore, in the regions after the power-law break, the noise model is systematically non-Poissonian, but the noise oscillates systematically about a mean value, meaning that the noise is statistically steady in this region. This fact justifies the interpretation of the Landau damping velocity-space signature as a more statistically steady signal after 15 particles are counted in Fig. 3.8, discussed in §3.7.1.

\textsuperscript{22}For clarity, I am not saying that the ‘best fit’ for large \( N \) values obeys a power law, but rather the mean error (ignoring fluctuations about the mean) follows a power law.
4.1 Alfvén Wave Collisions: Conclusions From First Principles

Our results in the first part of Chapter 2 (§2.1) show that many of the fundamental properties of Alfvén wave collisions, originally characterized analytically and numerically in the idealized case of a periodic Alfvén wave collision, persist under the more realistic conditions of a localized Alfvén wavepacket collision. Specifically, we have demonstrated that strong localized Alfvén wavepacket collisions are effective in mediating the nonlinear cascade of energy to small perpendicular scales, as demonstrated in Fig. 2.3. Furthermore, we have shown that strong localized Alfvén wavepacket collisions also self-consistently generate current sheets that persist even between collisions, as shown in Figures 2.4 and 2.5, confirming a robust mechanism to explain the ubiquitous current sheets observed in numerical simulations of plasma turbulence (Wan et al., 2012; Karimabadi et al., 2013; TenBarge and Howes, 2013; Wu et al., 2013; Zhdankin et al., 2013) and inferred from measurements of solar wind turbulence (Osman et al., 2011; Borovsky and Denton, 2011; Osman et al., 2012; Perri et al., 2012; Wang et al., 2013; Wu et al., 2013; Osman et al., 2014c).

The overall evolution of localized Alfvén wavepacket collisions involves strong nonlinear interactions occurring while the wavepackets overlap, followed by a clean separation of the wavepackets with straight uniform magnetic fields in between and the cessation of nonlinear evolution, as visualized in Fig. 2.2. The wavepackets re-
main localized along the equilibrium magnetic field before and after each wavepacket collision. It is important to emphasize that these characteristics are predicted based on the properties of the incompressible MHD equations, but are found even in the gyrokinetic simulation presented here which resolves dispersive and kinetic effects beyond the reach of the MHD theory. That these important properties of Alfvén wave collisions persist in the realistic localized wavepacket case further supports the contention that Alfvén wave collisions represent the fundamental building block of astrophysical plasma turbulence.

Previous analytical and numerical work in the limit of weakly nonlinear, periodic Alfvén wave collisions, (Howes and Nielson, 2013; Nielson et al., 2013) has shown that a nonlinearly generated mode, which is not a solution of the linear dispersion relation, serves to mediate the energy transfer to small perpendicular scales. The second part of Chapter 2 (§2.2) addresses the question of whether this nonlinearly generated mode still plays a key role in the more realistic case of localized Alfvén wavepacket collisions (Verniero and Howes, 2018).

The results presented in §2.2 settles the issue of the nature of the nonlinearly generated secondary mode – the mode that mediates the nonlinear transfer of energy in Alfvén wave collisions – in a more realistic setting than the idealized periodic case that was used in previous work to enable an analytical solution to be computed. In §2.2.1, we posed two questions: (i) Do collisions between Alfvén wavepackets still mediate the transfer of energy to small perpendicular scales?; and (ii) Do Alfvén wavepacket collisions in the strongly nonlinear limit still lead to the development of
intermittent current sheets?

Addressing question (i), we conclude that these secondary modes are indeed Alfvén modes in the case of localized Alfvén wavepacket collisions. This fact was confirmed by showing (i) the eigenfunction condition, that there is the correct relationship between the $E$ and $B$ fields described by Equation 2.2 and shown in Fig. 2.10 and Fig. 2.11 and (ii) the correct frequency condition, that the (1,1) mode travels at the Alfvén speed in accordance with the rest of the energy modes as shown in Fig. 2.12.

Observing Fig. 2.8 and Fig. 2.9, we found that in the periodic cases, only the tertiary (1,2) and (2,1) modes experience a secular gain of energy after successive collisions, while in the localized cases, the secondary (1,1) mode gains energy in addition to the tertiary modes. This means that in the case of localized wave collisions in both the strongly and weakly nonlinear limit, energy transfer to smaller perpendicular scales is more efficient than in the periodic case. We also saw, by comparison between the weakly and strongly nonlinear cases, that the primary modes in the strongly nonlinear limit lost significantly more energy than the weakly nonlinear cases. This saturation is the most discernible quantitative difference between strong and weak turbulence, while most other key features remain qualitatively similar such as overall evolution of the energy of different perpendicular Fourier modes in time. We conclude that the localized, strong (LS) case is the most effective way to transfer energy to smaller perpendicular scales. This particular case of localized, strong turbulence, the focal point of §2.2, is the most applicable case to space and astrophysical
plasmas. Hence, this case is crucial for understanding the various turbulent energy
cascades within our universe such as black hole accretion disks, the solar wind, and
planetary magnetospheres.

From Fig. 2.13, we have also demonstrated that for the strong, localized LS
case, self-consistent current sheets are generated after successive collisions and persist
in between collisions, consistent with previous findings on strong turbulence simula-
tions for both the periodic (Howes, 2016) and initially asymmetric localized cases in
§2.2. This particular finding shows that Alfvén wavepacket collisions in the strongly
nonlinear limit are a robust mechanism for current sheet development, regardless of
initial waveform. In turbulent space and astrophysical plasmas, current sheets are
commonly observed and have been proposed to play a key role in the conversion of
turbulent energy into plasma heat. The quest to understand how a plasma becomes
heated is currently an active topic of research in the plasma physics community. For
example, the Parker Solar Probe, launched in August 2018, will investigate how the
solar corona becomes heated to temperatures of more than a million Kelvin, a topic
that has been debated for decades. The result presented in §2.2 – that localized,
strongly nonlinear Alfvén wave collisions naturally produce current sheets – means
that the observations of current sheets in many space and astrophysical plasma sys-
tems can be explained from first principles.

We conclude that in the most physically applicable case of localized, strongly
nonlinear interactions, the fundamental properties of plasma turbulence still persist:
energy cascades nonlinearly to smaller perpendicular scales and intermittent current
sheets are self-consistently generated, answering the question (ii) posed in §2.2.1, about how the localization of the interacting Alfvén waves into separated wavepackets affect the qualitative and quantitative evolution of the perpendicular cascade of energy and the development of current sheets. In §2.1, we analyzed the case of localized, strongly nonlinear Alfvén wavepacket collisions with asymmetric initial waveforms. The symmetric conditions presented in §2.2 demonstrate that the effect of a nonzero $k_\parallel$ component does not alter the main characteristics of the Alfvén wave collisions that govern plasma turbulence.

Our findings of the Alfvénic nature of the key (1,1) mode in the localized, strongly nonlinear case is a satisfying simplification of the picture of the nonlinear energy transfer to small scales in plasma turbulence. It is important to emphasize the fact that an Alfvén wave collision is the fundamental unit of interaction in plasma turbulence (Kraichnan, 1965; Howes and Nielson, 2013), and a turbulent plasma would contain many such nonlinear interactions among upward and downward propagating Alfvén wavepackets. Such an ab initio approach to this subject allows for a clearer picture to be painted and consequently enables deeper insight about the nonlinear dynamics. The results presented §2.2 highlight the central role played by Alfvén waves in the nonlinear cascade of energy. The generation of the secondary mode mediates the transfer of energy from the primary to tertiary modes. The secondary mode is essentially a shear in the magnetic field that propagates along the magnetic field as an Alfvén wave, shearing the perpendicular waveform of counterpropagating Alfvén wavepackets and thereby nonlinearly transferring their energy to smaller perpendic-
ular scales (Howes and Bourouaine, 2017). In contrast to the idealized periodic case, this secondary \((1,1)\) mode gains energy secularly along with all of the other nonlinearly generated modes. The striking difference between the periodic case with two initially overlapping plane Alfvén waves and the localized Alfvén wavepacket case raises the question of whether the non-Alfvénic “beat” modes that arise in the periodic case will alter the statistics of the turbulence. For decaying turbulence simulations, in which the initialized Alfvénic fluctuations are already overlapping as in our periodic case, this is an issue that merits further investigation.

A follow up study could investigate the role of the Alfvénic propagating shear, discussed in §2.2, on magnetic field line wander, enabling a more atomistic description of the tangling of magnetic field lines within the framework of Alfvén wave collisions. Our analysis of the more realistic case of localized Alfvén wave collisions brings us closer to understanding the fundamental characteristics of plasma turbulence from first principles.

### 4.2 PATCH: Summary of New Theory and Preliminary Results

The technological advances of data acquisition onboard spacecraft are far exceeding the rate of improvements of telemetry downlink bandwidths. Until Space-Earth telemetry methods (such as the Deep Space Network) become more developed and affordable, the solution to this excess data problem will need to include a method of compression. Data compression can take on many forms, but usually involves an

1 Lossless compression algorithms, distinct from averaging methods, are commonly used on modern spacecraft.
element of averaging or redundancy removal in order to transmit less data with minimal loss in the quality of the original signal.

Understanding the entire cascade of turbulent energy with eventual dissipation into plasma heat is a grand challenge problem in heliophysics. The recent launch of Parker Solar Probe (PSP) provides direct measurements in the inner heliosphere to determine the mechanisms of particle energization in weakly collisional plasmas. Proposed mechanisms for turbulent dissipation include resonant mechanisms such as Landau damping (Landau, 1946; Dobrowolny and Torricelli-Ciamponi, 1985; Leamon et al., 1999; Quataert, 1998; Howes et al., 2008; Schekochihin et al., 2009; Horbury et al., 2012) and ion-cyclotron damping (Coleman, 1968; Marsch et al., 1982; Isenberg and Hollweg, 1983), nonresonant mechanisms such as stochastic heating (Chen et al., 2001; Johnson and Cheng, 2001; Chandran et al., 2010; Chandran, 2010), and intermittent dissipation concentrated in current sheets and magnetic reconnection locations (Dmitruk et al., 2004; Matthaeus and Montgomery, 1980; Karimabadi et al., 2013; Zhdankin et al., 2013; Osman et al., 2014a,c; Zhdankin et al., 2014).

Each of these mechanisms has a distinct velocity-space signature, dictated by the wave-particle coupling in the Lorenz force term in the Vlasov equation (Howes et al., 2017). The Field-Particle Correlation (FPC) technique developed by Klein and Howes (2016); Howes et al. (2017); Klein et al. (2017) is a novel procedure that can diagnose and distinguish turbulence dissipation mechanisms, producing their unique velocity-space signature. The derivation outlined in §3.3.2 showed that the instantaneous time rate of change in microscopic kinetic energy of particles can be character-
ized by the time rate of change of the particle distribution function. The goal of the FPC technique is to isolate the kinetic physics that leads to net energy transfer, using observationally accessible single spatial-point spacecraft measurements. It was shown that this can be achieved by performing a correlation, $C$, given by Eq. (3.6), between the fields and particles over a long enough time interval so that the large-scale oscillating energy transfer averages to zero, thereby isolating the small-scale component that leads the net particle energization.

In order to adapt the FPC technique for onboard implementation, we first determined the minimum spacecraft velocity-space resolution to resolve the kinetic signature of ion Landau damping. We stress that this method can be generalized to diagnose any mechanism, but for the sake of simplicity, we chose this particular resonant mechanism since its signature is already well understood (Klein and Howes, 2016; Howes et al., 2017; Klein et al., 2017). We created synthetic spacecraft data by using the fine-resolution distribution function output from the gyrokinetics code, AstroGK. The distribution function was first down-sampled to realistic instrumental energy grids and then transformed to the spacecraft frame-of-reference, boosted to the solar wind velocity. Fig. 3.2 revealed results from reproducing the ion Landau damping signature using velocity-space resolutions from various past, present and future missions. Fig. 3.3 showed that the PSP SWEAP resolution exceeds the threshold for sufficiently resolving the signature. As discussed in §4.3, this fact enables the validity of plans to use the FPC technique to investigate turbulent dissipation mechanisms with PSP observations, as was done recently with MMS observations (Chen et al.,
In §3.6.1, I introduced an original algorithm called Particle Arrival Time Correlation for Heliophysics (PATCH) which can diagnose and distinguish various mechanisms of turbulent dissipation onboard modern spacecraft. Developing the new theory of diagnosing energy transfer mechanisms onboard spacecraft was based on time-tagging particles, which required explicitly defining a particle arrival time. In §3.5, I described a typical particle detector with a ‘top-hat’ design, called an electrostatic analyzer (ESA), to facilitate an explicit definition; the particle time of arrival was defined as the moment the particle crossed the surface of the entrance aperture. 3D position space was then recast in terms of the velocity, \( v_p \), of the particle at arrival time, \( t_p \), at an angle of incidence normal to the aperture plane of 2D area. The 3D velocity space was redefined in terms of the rotating aperture about the azimuthal axis of symmetry which defined the voltage difference (energy) with a relative 2D solid angle. The geometric factor of the instrument was then derived in §3.5.2 on a locally defined distribution function, enabling a physical connection between instrumental hardware design and algorithm improvement.

We adapted the FPC concept to measuring the net energy transfer by quantifying the statistical change in discrete particle counts over time. Utilizing spacecraft observations of discrete particle arrival times eliminates the need for calculating the phase-space bin-average of the distribution of particles. We instead showed that the distribution of the field measurements evaluated at the particle arrival times can yield the same information about the particle distribution function itself. The validity of
this concept was justified in §3.6, where we re-imagined kinetic theory by starting with the Klimontovich-Dupree equation (§3.6.2) and refraining from substituting the number density of particles, \( N_s \), with the unphysical, ensemble averaged (§3.6.3) distribution \( f_s \), as is traditionally done following Nicholson (1983).

By instead treating the evolution of the plasma as the evolution of a sum of \( \delta \)-functions representing individual particles in phase-space, we showed in §3.6.4, that we can achieve an equivalent measure of net energy transfer (given by Eq. 3.72). We computed this expression for the work done on the particles by the fields by directly evaluating the instantaneous rate of change of kinetic energy of the particle number density, which is only the first term in the Klimontovich-Dupree equation (3.52). In the derivation of the FPC method detailed by Howes et al. (2017) (summarized in §3.3.2), they showed that this term is in fact equal to the Lorentz force term, but by using the Vlasov equation in an infinite phase-space which describes the evolution of the collisionless plasma in terms of the smooth particle distribution function. Since we instead manipulate the equation in terms of a distribution of \( \delta \)-functions over a finite phase-space, replicating the derivation procedure by Howes et al. (2017) requires careful consideration of non-zero boundary integrals that result from both the convective (ballistic) term in a finite-space and integrating the Lorenz term by parts to acquire the more observationally accessible \( C'' \) (Eq. (3.10)) correlation, which eliminates the need for noisy velocity-derivative knowledge. Transforming from the direct energy transfer measure correlation, \( C \) to \( C' \), by integrating the Lorenz force term by parts (Eq. 3.7) introduces a possibly nonzero boundary term which means that \( C' \) must be
carefully interpreted in terms of direct energy transfer. Future work will address this nuance, but for the purpose of the general concept introduced in Chapter 3, we make an implicit assumption that the nonzero-boundary integrals resulting from the integration by parts step (and the convective term) will cancel or be negligible. Taking averages over time and space makes this a reasonable approximation because we do not expect to have an accumulation of particles in any phase-space volume boundary element over time. Thus, averaging over a long enough time will reveal that, on average, the particles arriving on the boundaries will cancel, or become sufficiently negligible.

The PATCH algorithm was tested with synthetic spacecraft data produced by AstroGK by creating an artificial time series of particle counts using the numerically computed distribution function as the probability for counting a particle. The parallel electric field was then evaluated at those times and a time correlation was performed and summed over velocity-space. In the spirit of Leonardo da Vinci, Fig. 3.8 presented a visualization of the improved statistics of the new correlation $C_x^*$, given by Eq. 3.72, as the number density of particles increases with time, also interpreted as the sampling time for counting particles on the spacecraft. While the visualization itself could not definitively determine the threshold number of particle counts needed to resolve the velocity-space signature of ion Landau damping, the Poisson noise model constructed in §3.7.3 determined that over the appropriate correlation time, 16 particles need to be counted in each phase-space bin per instrumental sampling time interval. In Fig. 3.9, we showed that fact by observing a power law break in the noise model
at 15 particles. The region before the break (1-15 particles) exhibited nearly Poissonian behavior, indicating the particle statistics were dominated by Poission counting randomness. After the break, the region (15-100 particles) showed a sharp deviation from a Poisson process, indicating a more interesting physical phenomenon occurred, since the particles were in a more ordered state (such as a wave-particle interaction). We confirmed our findings by testing 3 other spatial points, displayed in Fig. 3.10 - Fig. 3.12, and showing that each case had qualitatively similar behavior. Note that since turbulence is spatially non-homogeneous, other points in space would not agree quantitatively, so we can only make qualitative comparisons.

4.3 Developing an Onboard Wave-Particle Correlator

4.3.1 Overview of Proposed Future Project

Chapter 3 aimed towards theoretical development of an onboard wave-particle correlator to diagnose turbulent dissipation mechanisms and efficiently transmit that information to the ground. PATCH is ultimately a data compression algorithm which time averages correlation calculations of onboard fine-resolution measurements and downlinks data products to Earth in the form of unique velocity-space signatures which can characterize the mechanism of turbulent dissipation. PATCH only requires particle arrival times and the field measurements evaluated at the time, which is simple enough for implementing a wave-particle correlator onboard spacecraft. In this section, we propose a project that will use data from both the particle (SWEAP) and field (FIELDS) instruments on Parker Solar Probe (PSP) to first compute FPCs
on ground-based measurements to identify the mechanism of particle energization in at least one time interval. After identification of that interval, we will test, refine and optimize the PATCH algorithm to potentially increase sampling time on future missions by several orders of magnitude, maximizing scientific data return. The primary goal is to understand energization of particles due to damping of turbulent fluctuations. This proposed project is beyond the scope of this thesis, but represents a natural extension of the work that can provide an important foundation for the operation of existing instruments as an onboard correlator.

The PATCH method was conceived with the new capabilities of PSP in mind. It has been tested and analyzed with numerical simulations, but now it must be refined using actual spacecraft data downlinked from PSP. I will compute field-particle correlations with ground-based measurements from PSP to diagnose velocity-space signatures of various dissipation mechanisms using the established methods of Klein and Howes (2016); Howes et al. (2017); Klein et al. (2017). Chen et al. (2019) has recently shown the feasibility of applying the FPC technique to spacecraft observations and consequently provided the first direct evidence of a specific energization mechanism, electron Landau damping, in a space plasma.

Adapting the methodology of Chen et al. (2019) using PSP (rather than MMS) observations would lead to eventual development of the novel PATCH algorithm for implementation on a future field-particle correlator instrument. The FPC will be a joint venture between the particle instrument suite, SWEAP (Kasper et al., 2016), and the electromagnetic fields instrument suite, FIELDS (Bale et al., 2016), which in-
Fig. 4.1 shows a schematic of a single interface cable (red) that was placed between the particle electrostatic analyzer, SWEAP, and fields instrument, FIELDS. The existence of this cable enables the opportunity for PATCH algorithm refinement and further development for a future mission where exploring wave-particle interactions is the primary science goal. The clock signal from FIELDS to SWEM provides a master clock so that FIELDS and SWEAP measurements can be made simultaneously with minimal time delays. Wave-particle correlations could be seen up to 1 MHz, so we could potentially determine how energy is exchanged between electromagnetic fields and selected portions of the particle phase space density. Furthermore, the particle pulses can be selected from any combination of anodes from the electrostatic analyzers, called the
Solar Probe ANalyzers (SPAN), so both electron and ion-scale kinetic physics can be studied. The primary goal is to understand the dissipation mechanisms that cause heating in both particle populations.

4.3.2 Tasks

The major tasks of this project are:

Task #1: Analysis of PSP ground-based spacecraft observations: The field-particle correlation technique will be applied to SWEAP and FIELDS data to diagnose at least one particle energization mechanism. The successful initial use of FPCs to provide the first direct evidence of electron Landau damping in the magnetosheath (Chen et al., 2019) indicates that this is a feasible goal.

Task #2: Use the SWEAP/FIELDS connection data to test the PATCH algorithm: Using the same time interval of the observed field-particle correlation from Task 4.1, I will recreate the velocity-space signature using the PATCH algorithm to test if an onboard correlator using this new method would see a resolvable signature.

We ultimately want to find the first definitive signatures of turbulent particle energization mechanisms in the inner heliosphere observed by PSP.

4.3.2.1 Analysis of PSP Ground-Based Spacecraft Observations

The first step in this task is to develop IDL scripts to properly conduct the required spacecraft data analysis and calculate field-particle correlations. In order to
find hints of where field-particle correlations may be occurring, I will then observe the particle counts from SWEAP in tandem with the electric field measurements from FIELDS. Next, I will inspect particle distribution functions for events that may be well-correlated with the receiver. Features of the particle distribution function such as quasilinear flattening are indicative of resonant wave-particle interactions. The emergence of such prominent features synchronous with field measurements will allow for a more detailed investigation of correlation events, specifically identifying the correct correlation time interval. I will compute field-particle correlations using the developed IDL scripts to determine the velocity-space signature of the energization mechanism responsible for the shape of the distribution function.

One of the biggest challenges is filtering out the large-scale electric field fluctuations to isolate the physics occurring at kinetic scales. From the theory of field-particle correlations, one can sum over an integral number of wave-periods to solve this issue; turbulence is broadband in nature, though, so it is thus critical to implement a suitable frequency filter to isolate the physics in question for both ion and electron scales. To resolve electron Landau damping in MMS observations, Chen et al. (2019) applied a standard high-pass filter to the electric field to observe electron-scale energy transfer. However, extreme caution will be considered when filtering since it could cause a phase delay, a potential source of error for highly time-sensitive measurements. If standard filters reveal themselves to be unsuitable, I will design a new one by experimenting with multiplying the electric field by a suitable windowing function, thereby reducing Fourier leakage and obtaining a more precise measurement.
For example, the Hanning window acts as a Finite Impulse Response (FIR) filter that, when applied to the electric field, introduces a linear phase delay. Since the delay is linear, we could easily calibrate with the particle data. The cross-wavelet transform, briefly mentioned in §3.3.1, will also be investigated for application to the case where particle count rates may be lower than the wave frequencies. Using this method, wave coherence could be discerned and reveal a dominant frequency of the turbulent spectrum. In relation to this dominant frequency, the particles could then be binned by phase instead of time, producing a more statistically significant signal.

4.3.2.2 Testing PATCH Using the SWEAP/FIELDS Connection

Using the same correlation time interval where a specific mechanism can be identified in §4.3.2.1, I will recreate the velocity-space signature of the observed particle energization event using the PATCH algorithm. This would test if an onboard correlator would see a resolvable signature using this novel technique. First, I will determine if we see the correlation in the particle arrival data from the SPAN ion or SPAN electron anodes sent across the correlator cable. Next, I will test various ground-based filtering algorithms from §4.3.2.1. I will then experiment with other noise filtering techniques that could be implemented onboard. Filters are typically applied to ground measurements, where data have been sampled for a long enough time to recognize the definition of noise and subsequently discard those points. There is another type of filter, however, called the Kalman filter that can perform “on-
the-fly” noise reduction through an iterative process. The disadvantage of its high computational expense could be mitigated with new technologies offered by FPGAs, where almost anything is possible as long as we collaborate carefully with the hardware engineers.

Figure 4.2: Laboratory calibration of PSP correlator instrumentation at UC Berkeley Space Sciences Laboratory.

Once we determine that the correlator data can recreate the particle energization signature, we can start to consider how to choose the correlation interval automatically onboard. I will devise and test statistical diagnostic tools for identifying which regions of velocity-space are experiencing a wave-particle interaction or some other process worthy of triggering full burst-mode data collection. This will ex-
pand on the preliminary noise model discussed in §3.7.3, where we found the threshold particle number density where the particles deviated from a stationary Poisson process, indicating more interesting physical phenomena occurred. The particle count pulses from the SWEAP/FIELDS interface will make a similar investigation possible, utilizing better time resolution on PSP than previous missions. Fig. 4.2 shows an example of real data from laboratory calibration, conducted at UC Berkeley Space Sciences Laboratory, of the flight instrumentation on the PSP FIELDS/SWEAP particle correlator. During this measurement, the particle flux from an (ion) gun was modulated by a sinusoidal voltage applied to a modulation grid. Individual particles detected by the PSP/SPAN-Ion sensor were transmitted as short voltage pulses to the FIELDS/TDS instrument and recorded at extremely high time resolution. The same modulation signal was also provided to FIELDS/TDS instrument, digitized and shown in the top panel. This is a real world example of the simulated data shown in Fig. 3.7, and represents another proof of concept test of the PATCH methodology applied on the actual correlator system on PSP.

4.3.3 Future Onboard Wave-Particle Correlator Mission Plan

When the tasks in §4.3.2.1 and §4.3.2.2 are completed, we will have enough information to devise an optimal approach to implement a future onboard wave-particle correlator mission. This will involve determining the most effective sweeping patterns to observationally identify energy transfer. Once the data from both SWEAP and FIELDS instruments are understood, we will learn more about how to command
them to utilize the full-burst mode capabilities of PSP and detect correlation events.
The PATCH algorithm has demonstrated its potential for uploading onboard PSP; however, due to the budget, timeline, and priority science goals for PSP, the PATCH method will be more suitable to be implemented on a future dedicated mission in the heliosphere. Since PSP has better velocity-space resolution than any other mission, the data analyzed from PSP will yield the best insight possible for designing a mission specifically for PATCH implementation. The proposed project outlines the crucial groundwork needed towards building onboard wave-particle correlator instrumentation for efficiently diagnosing particle energization mechanisms in the heliosphere, thereby mitigating telemetry limitations on resolving kinetic-scale velocity-space signatures of dissipation.

Although the methodology in Chapter 3 focused on diagnosing Landau damping using the field-particle correlation technique, we stress that other particle energization mechanisms that lead to heating can also be distinguished using a similar procedure. Ion-cyclotron damping, for example, is also another contender for the coronal heating mechanism that can be diagnosed with this technique. In addition, we could also characterize equally popular theories of non-resonant heating such as magnetic reconnection and stochastic ion heating. The theory behind the field-particle correlation technique is also general enough that we would not be confined to low plasma beta environments such as the corona and magnetosphere, but also order unity beta environments such as the solar wind, another primary PSP data collection domain. Understanding the fundamental physics of turbulent dissipation and plasma
heating contributes to the fundamental knowledge needed to understand the physics in the near-Sun environment.

While PSP will not be sampling the Earth’s magnetosphere, wave-particle interactions have historically been studied here due to the nature of solar wind-magnetosphere coupling. After performing the tasks outlined in §4.3.2, I will have enough experience with in situ measurements of wave-particle interactions to work towards designing my own mission diagnosing particle energization and unravelling fundamental knowledge about kinetic-scale physics in the heliosphere. The proposed project will employ theoretical modeling specifically for the betterment of future spacecraft instrumentation to progress towards more predictive space weather forecasting, protecting astronauts and other space-based assets.

4.4 A Macroscale Discussion About the Microscale Concepts in This Dissertation

We started this dissertation with talking about visualizing turbulence in water in 1507. This was before René Descartes started thinking about his own existence. Once he realized, “I think, therefore, I am,” his mind was free to wonder about other things, such as a way to map out the trajectory of a fly landing on his ceiling. After his subsequent establishment of the “Cartesian” coordinate system in 1637, thinkers around the globe could suddenly draw visualizations of functions. The ability to visualize continuously changing functions naturally led to the invention of Calculus, independently by Newton and Leibniz. The physicist Isaac Newton used the new
coordinate system visualization tool to map out trajectories of moving objects, which led to the development of what we know today as ‘classical mechanics.’ His concept of an instantaneous rate of change of a point, a ‘fluxion’ of ‘fluent,’ was published in 1665 in his mathematical treatise, *Method of Fluxions*. The mathematician and philosopher Gottfried Leibniz used the new coordinate system to generalize the notion of sums and differences, enabling him to conceptualize an *infinitesimal* increment, $dx$, a notation that first appeared in his 1684 article, *Nova Methodus pro Maximis et Minimis*. In 1686, his first publication including the $\int$ integral notation appeared in *De Geometria Recondita et analysi indivisibilium atque infinitorum* (On a hidden geometry and analysis of indivisibles and infinites). Ultimately, in 1804, English mathematicians felt that Newton’s dot notation for a derivative was too cumbersome, and Leibniz notation was adopted instead as we generally know it today.

Once calculus was established, systems partial differential equations could be formally defined; in 1822, the Navier-Stokes equations for motion in a viscous fluid were derived. This enabled formal turbulence studies through the Reynold’s number, the visualization of the transfer of energy with the cascade à la Richardson and finally, the self-similar Kolmogorov -5/3 rate of cascade. Employing new visualizations and expressing abstract concepts through inventive notations are both ways of communicating complex information. Notational simplicity allows for ideas to be conveyed more efficiently, leading to acceleration of scientific discovery. A visual depiction of

---

$^2$The derivative notation was generalized further in the 19th century by Weierstrass and Cauchy who defined these concepts more rigorously in terms of limits, leading to the development of modern analysis in *Cour d’Analyse*. 

a physical process enables understanding of a poorly understood mechanism when equations cannot be written down.

Chapter 2 of this thesis contained visualizations of turbulence in cases where analytical solutions do not exist, i.e., one cannot simply write the exact solutions in a closed form. Through these visualizations, we addressed the first question in the introduction posed by Leonardo da Vinci, *Doue la turbolenza dell’acqua si mantiene a lungo?* In the context of plasma turbulence, we saw ‘where the turbulence persists for long times’ by characterizing the nature of the turbulent cascade of energy to smaller perpendicular scales.

Chapter 3 of this thesis contained both equation derivations and visualizations to conceptualize the onset of turbulent energy dissipation through wave-particle interactions, a process we cannot see with the naked-eye. However, as humans, we have learned how to attain more knowledge about the universe by constructing devices to help observe the universe beyond the capabilities of human senses. Spacecraft instrumentalists have created sensors which collect information and communicate it down to Earth. But spacecraft can collect this information at a rapidly higher rate than can be downlinked to the ground; the ability to visualize the universe more clearly depends on methods of communicating this information more efficiently. We addressed the second question posed by Leonardo da Vinci in the introduction, *Doue la turbolenza dell’acqua si riposa?* By creating an algorithm designed to diagnose turbulent dissipation mechanisms onboard modern spacecraft, we developed a method based on counting particles in space to discern ‘where the turbulence comes to rest,’
eventually leading to plasma heat.

The task of understanding the universe reduces to developing methods of observing objective truths – whether through paintings, notations, computer generated movies, or spacecraft hardware. As Sir Karl Popper said in 1962, “the more we learn about the world, and the deeper our learning, the more conscious, specific, and articulate will be our knowledge of what we do not know, our knowledge of our ignorance. For this, indeed, is the main source of our ignorance — the fact that our knowledge can be only finite, while our ignorance must necessarily be infinite.”
APPENDIX A
MODELING POISSON PROCESSES

This appendix chapter is intended to give the reader additional insights about modeling Poisson processes, specifically applied to the PATCH algorithm. The formalism described in A.1 provides a theoretical basis for the Poisson discrete particle counting statistics in space plasma. The statistical modeling described in A.2 presents an analysis of the simple PATCH demonstration prescribed in §3.6.1. The information gleaned from these appendix sections provided key foundation for how to implement PATCH in the more realistic scenario described in §3.7.1, in addition to familiarity with Poisson noise quantification that inspired the exploration in §3.7.3.

A.1 Poisson Statistics in Space

In this appendix section, we prove that the spacecraft particle detector will count particles according to a Poisson distribution. Let $P(\xi)$ be the probability that the detector counted a particle within the time interval $t$ to $t + \Delta t$. Then for $\Delta t \to 0$,

$$P(\xi) \propto \Delta t$$

and so

$$\lim_{\Delta t \to 0} \frac{P(\xi)}{\Delta t} = \lambda$$
where $\lambda$ is some constant. Using the property $P(A \& B) = P(A | B) \cdot P(B)$ for two events $A$ and $B,$

\[
P(1\text{st particle counted in } t \text{ to } t + \Delta t)
= P(\text{particle counted in } t \text{ to } t + \Delta t \mid \text{no particle counted before } t) \cdot P(\text{no particle counted before } t)
= P(\xi) \cdot P(\text{no particle counted before } t)
= \lambda \cdot \Delta t \cdot P(\text{no particle counted before } t)
\]

Now,

\[
P(\text{no particle counted before } t + \Delta t) = 1 - P(1\text{st particle counted before } t + \Delta t)
= 1 - [P(1\text{st particle counted in } t \text{ to } t + \Delta t) + P(1\text{st particle counted before } t)]
= P(\text{no particle counted before } t) - P(1\text{st particle counted in } t \text{ to } t + \Delta t)
= P(\text{no particle counted before } t) - \lambda \cdot \Delta t \cdot P(\text{no particle counted before } t)
= (1 - \lambda \cdot \Delta t) \cdot P(\text{no particle counted before } t)
\]

Therefore,

\[
\lim_{\Delta t \to 0} \frac{P(\text{no particle counted before } t + \Delta t)}{\Delta t} \cdot -P(\text{no particle counted before } t)
= -\frac{\lambda \cdot \Delta t}{\Delta t} \cdot P(\text{no particle detected before } t)
= -\lambda \cdot P(\text{no particle counted before } t)
\]

Hence we obtain the differential equation

\[
\frac{d}{dt} P(\text{no particle counted before } t) = -\lambda \cdot P(\text{no particle counted before } t)
\]

Solving, we get that

\[
P(\text{no particle counted before } t) = e^{-\lambda t}
\]
So the probability density function is

$$-\frac{d}{dt} e^{-\lambda t} = \lambda e^{-\lambda t}$$

Thus, the number of particles counted in the interval $[0, T]$ has a Poisson distribution, shown as a sketch in Fig. A.1:

$$P(k \text{ particles counted in } [0, T]) = \frac{(\lambda T)^k}{k!} e^{-\lambda T} \quad (A.1)$$

The discreteness of the particle counts is depicted in Fig. A.2 to describe the probability distribution with time, $P(t)$, where the number of particles $n$ within an interval $b - a$ is given by

$$n = \int_a^b P(t) dt$$

Given Eq. (A.1), for some initial $n_0$,

$$P(n = n_0) = \frac{(\lambda(b - a))^{n_0}}{n_0!} e^{\lambda(b-a)}$$
So our average number of particle counts within some interval $b - a$ will be

$$\lambda \sim \frac{n}{b - a}$$

Let $\mathbb{E}[n]$ denote the expectation value of counting $n$ particles within some interval $[0, T]$. Then,

$$\mathbb{E}[n] = \lambda(b - a)$$

We can use the binomial approximation, to truncate the Poisson process of counting particles to such an interval $[0, T]$, with spacing $\Delta t$, shown in Fig. A.3.
The variance related to counting \( n \) particles in \([0, T]\) is given by

\[
Var[n] = \mathbb{E}[(n - \mathbb{E}n)^2]
\]

We will show that \( \mathbb{E}[n] = Var[n] \), as expected for a Poisson process. Now, the chance of a particle count at \( t = t_n \) is \( \lambda \cdot \Delta t \). So, we define the variable \( c_k \):

\[
c_k = \begin{cases} 
0, & \text{if no particle counted at } t = t_k \\
1, & \text{if particle counted at } t = t_k
\end{cases}
\]

Then we have that

\[
P_{\Delta t}(t) = \sum_{k=0}^{N-1} c_k \delta(t - t_k)
\]

and

\[
n_{\Delta t} = \sum_{k=0}^{N-1} c_k
\]

Calculating the expectation value, and noting that \( \mathbb{E} \) is a linear operator that can be distributed inside the summation,

\[
\mathbb{E}[n_{\Delta t}] = \mathbb{E}\left[ \sum_{k=0}^{N-1} c_k \right]
\]

\[
= \sum_{k=0}^{N-1} \mathbb{E}[c_k]
\]

\[
= \sum_{k=0}^{N-1} \left( (0 \cdot P(c_k = 0) + 1 \cdot P(c_k = 1)) \right)
\]

Substituting \( P(c_k = 1) = \lambda \cdot \Delta t \), we get

\[
\mathbb{E}[n_{\Delta t}] = N \lambda \Delta t = \lambda T
\]
Before calculating the variance, we will show the following equality using properties of \( E \). For any random variable \( x \),

\[
E[(x - Ex)^2] = E[x^2 - 2x \cdot Ex + (Ex)^2]
\]

\[
= E[x^2] - 2 \cdot E[x \cdot Ex] + (Ex)^2
\]

Since \( Ex \) is no longer a random variable, we can withdraw it out of the argument of \( E \) in the second term to get

\[
= E[x^2] - 2Ex \cdot Ex + (Ex)^2
\]

\[
= E[x^2] - 2(Ex)^2 + (Ex)^2
\]

\[
= E[x^2] - (Ex)^2
\]

Using this result to calculate the variance,

\[
Var(n_{\Delta t}) = E[(n - En)^2]
\]

\[
= E(n^2) - (En)^2 \tag{A.3}
\]

Computing the first term,

\[
E(n^2) = E \left( \left( \sum_{k=0}^{N-1} c_k \right) \left( \sum_{l=0}^{N-1} c_l \right) \right)
\]

\[
= E \left[ \sum_{k,l=0}^{N-1} c_k c_l \right]
\]

\[
= E \left[ \sum_{k,l=0,k\neq l}^{N-1} c_k c_l + \sum_{k=0}^{N-1} c_k^2 \right]
\]

Noting that \( E[c_k^2] = E[c_k] \) by definition of \( c_k \),

\[
= \sum_{k,l=0,k\neq l}^{N-1} E[c_k]E[c_l] + \sum_{k=0}^{N-1} E[c_k]
\]
Substituting Eq. (A.2),
\[
\sum_{k,l=0,k\neq l}^{N-1} (\lambda \cdot \Delta t)(\lambda \cdot \Delta t) + \lambda \cdot \Delta t
\]
\[
= (N^2 - N)\lambda^2(\Delta t)^2 + N \cdot \lambda \Delta t
\]
\[
= N^2\lambda^2(\Delta t)^2 - N\lambda^2(\Delta t)^2 + N\lambda \Delta t
\]
\[
= \lambda^2T^2 - \lambda^2 \Delta \cdot T + \lambda T
\]
Inserting this answer back into Eq. (A.3),
\[
Var(n_{\Delta t}) = \lambda^2T^2 - \lambda^2 \Delta \cdot T + \lambda T - (\lambda T)^2
\]
\[
= \lambda^2T \cdot \Delta t + \lambda T
\]
Taking the limit \( \Delta t \to 0 \), we get \( \lambda T \), which matches Eq (A.2). This equality further illustrates that the counting of particles is indeed a Poisson process. However, is the field-particle correlation, a procedure based on particle counts, a Poisson process? For the present time, we consider the case where \( \lambda \) is constant. In a more realistic situation, \( \lambda \) will be time varying. The field-particle correlation within an interval \( b-a \) for the probability distribution \( p_{\Delta t}(t) \) and electric field \( E(t) \) is given by
\[
\mathcal{E}_{\Delta t}
\]
\[
= \int_{a}^{b} p_{\Delta t}(t)E(t)
\]
\[
= \sum_{k=0}^{N-1} c_k \int_{a}^{b} \delta(t - t_k)E(t)dt
\]
\[
= \sum_{k=0}^{N-1} c_k E(t_k)
\]
Calculating the expectation value,
\[ E[C_{\Delta t}] = \sum_{k=0}^{N-1} E[c_k] E(t_k) \]
\[ = \sum_{k=0}^{N-1} \lambda \Delta t E(t_k) \]

Taking the limit as \( \Delta t \to 0 \), this Reimann sum equates to the Reimann integral:
\[ = \lambda \int_a^b E(t) dt \quad \text{(A.4)} \]

Calculating the variance,
\[ Var(C_{\Delta t}) = E(C_{\Delta t}^2) - (E(C))^2 \]
\[ = E \left[ \left( \sum_{k=0}^{N-1} c_k E(t_k) \right) \left( \sum_{l=0}^{N-1} c_l E(t_l) \right) \right] - \lambda^2 \left( \int_0^T E(t) dt \right)^2 \quad \text{(A.5)} \]

Evaluating the first term,
\[ E \left[ \left( \sum_{k=0}^{N-1} c_k E(t_k) \right) \left( \sum_{l=0}^{N-1} c_l E(t_l) \right) \right] \]
\[ = \sum_{k,l=0}^{N-1} E(t_k) E(t_l) E[c_k] E[c_l] \]
\[ = \sum_{k,l=0, k \neq l}^{N-1} E(t_k) E(t_l) E[c_k] E[c_l] + \sum_{k=0}^{N-1} E(t_k)^2 E[c_k]^2 \]
\[ = \sum_{k,l=0, k \neq l}^{N-1} E(t_k) E(t_l) (\lambda \Delta t)(\lambda \Delta t) + \sum_{k=0}^{N-1} E(t_k)^2 (\lambda \Delta t) \]
\[ = \lambda^2 (\Delta t)^2 \left[ \sum_{k,l=0, k \neq l}^{N-1} E(t_k) E(t_l) \right] + \lambda \Delta t \sum_{k=0}^{N-1} E(t_k)^2 \]
\[ = \lambda^2 (\Delta t)^2 \left[ \sum_{k=0}^{N-1} E(t_k) E(t_l) - \sum_{k=0}^{N-1} E(t_k)^2 \right] + \lambda \Delta t \sum_{k=0}^{N-1} E(t_k)^2 \]
\[ = \lambda^2 (\Delta t)^2 \left( \sum_{k=0}^{N-1} E(t_k) \right) \left( \sum_{l=0}^{N-1} E(t_l) \right) - \lambda^2 (\Delta t)^2 \sum_{k=0}^{N-1} E(t_k)^2 + \lambda \Delta t \sum_{k=0}^{N-1} E(t_k)^2 \]
\[ = \lambda^2 \left( \sum_{k=0}^{N-1} E(t_k)(\Delta t) \right) \left( \sum_{l=0}^{N-1} E(t_l)(\Delta t) \right) - \lambda^2 (\Delta t) \sum_{k=0}^{N-1} E(t_k)^2 (\Delta t) + \lambda \sum_{k=0}^{N-1} E(t_k)^2 (\Delta t) \]
Taking the limit as $\Delta t \to 0$ turns the Reimann sums to integrals and so we get

$$= \lambda^2 (\text{avg}E) \cdot (\text{avg}E) - \lambda^2 \Delta t \int_0^T E(t)^2 dt + \lambda \int_0^T E(t)^2 dt$$

where $\text{avg}E$ denotes the average of $E$ over $[0,T]$. Noting that the second term goes to 0 as $\Delta t \to 0$, we obtain

$$= \lambda^2 (\text{avg}E)^2 + \lambda \int_0^T E(t)dt$$

Inserting this result back into Eq. (A.5) for the first term,

$$\text{Var}(C_{\Delta t})_{\Delta t \to 0} \to \lambda^2 (\text{avg}E)^2 + \lambda \int_0^T E(t)^2 dt - \lambda^2 (\text{avg}E)^2$$

$$= \lambda \int_0^T E(t)^2 dt$$

Comparing with our result from Eq. (A.4), and setting $a = 0$ and $b = T$, we get that the $E(C_{\Delta t})_{\Delta t \to 0} \to \lambda \int_0^T E(t)dt$ which differs from the variance by a factor of $E(t)$ inside the integrand. Since the expectation value does not equal the variance, the field-particle correlation from poisson random particle counts, does not result in a Poisson process. Hence, extra precautions must be made to understand the complexity of the statistics of the PATCH method in relation to the number of counts needed to acquire statistical “significance.” Future work, beyond the scope of this Appendix, will redo this calculation assuming a time-varying $\lambda = \lambda(t)$ which would be more realistic in space if we need to account for a sudden increase in average number of counts, in a given interval due to, for example, a solar flare or shock.

### A.2 Statistical Modeling Results

This appendix section details results from statistical modeling of the simplified PATCH demonstration depicted in §3.6.1. Note that in the initial of development of
the PATCH algorithm, we normalized the result (in each phase-space bin) by the total number of particles counted within the correlation interval. However, when we applied PATCH to a fully turbulent system using synthetic data produced from AstroGK, we realized that low count rates yielded divergent results when normalized per bin. Therefore, in the more refined version described in §3.7.1, normalization by the total number of particle counts over all velocity-space bins is performed at the end of the correlation. The alternatively normalized PATCH method in this appendix section, referred to as \( \langle E_j \rangle \), is still valid if one devises a clever procedure for removing noisy correlation data points from bins with insufficient count rates.

Recalling the scenario in §3.6.1 and depicted in Fig. 3.7, consider an electric field with a known time evolution at the point of measurement \( r_0 \) (in 1D for simplicity), given by

\[
E(t) = E_0 \sin \omega t
\]

and a number of particles within a single \((\Delta r_0, \Delta v_0)\) phase-space bin given by

\[
N(t) = N_0 \left[ 1 + \frac{\delta N}{N_0} \sin(\omega t + \delta) \right]
\]

For this particular demonstration, we set the relative phase shift to \( \delta = 0.2\pi \), and frequency \( \omega = 2\pi \). The total number of periods was chosen as \( T = 2 \) and the fine time step (representing the higher electric field sampling time) was \( t_{fine} = 0.0001 \). Each coarser time bin was size \( \Delta \hat{t} = 0.02 \), allowing a maximum, \( N_{max} \), of 200 particles counted per sampling \( \Delta \hat{t} \) time bin (representing the coarser time resolution of distribution function measurements). In the numerical routine, the particle counts are sampled from a Poisson distribution. We set \( N_0 = 30 \), as the mean number of
particles counted per coarse sampling time bin, and normalized variation from the mean as $\delta N/N_0 = 0.3$.

Note that these values of $t_{fine}$, $\Delta \hat{t}$ and $N_{max}$ are analogous to the variables defined in §3.7.1, since this study laid essential groundwork for adapting the PATCH methodology to fully turbulent synthetic spacecraft data. In addition, $N_0$ has the same interpretation as $N$, and $\delta N/N_0 = 0.3$ is related to the meaning of $\delta f_s/F_{s_0}$, as discussed in §3.4.2.

We compare the statistics of the PATCH method using $\langle E_j \rangle$ versus the theoretical “exact” value $\langle NE \rangle/\langle N \rangle$. In this context, the PATCH methodology is defined (in a single phase phase bin) as

$$\langle E_j \rangle = \frac{\int dt \ N_{obs}(t) E(t)}{\int dt \ N_{obs}(t)} \quad \text{and} \quad N_{obs}(t) = \sum_{j=1}^{N_{tot}} \delta(t - t_j)$$

so

$$\langle E_j \rangle = \frac{\int dt \ \sum_{j=1}^{N_{tot}} \delta(t - t_j) E(t)}{\int dt \ \sum_{j=1}^{N_{tot}} \delta(t - t_j)} = \frac{\sum_{j=1}^{N_{tot}} \int dt \ \delta(t - t_j) E(t)}{\sum_{j=1}^{N_{tot}} \int dt \ \delta(t - t_j)} = \frac{\sum_{j=1}^{N_{tot}} E(t_j)}{N_{tot}}$$

Performing a 512 statistical ensembles, we get

$$\langle NE \rangle/\langle N \rangle = 0.121$$

$$\langle E_j \rangle = 0.122 \pm 0.013 \quad \text{with} \quad \%Error = 0.9\% \pm 10.5\%$$

Note that $\langle E_j \rangle$ is of the form

$$\langle E_j \rangle = \mu \pm \sigma$$

where $\mu$ is the mean value of $\langle E_j \rangle$ and $\sigma$ is the standard deviation of the mean. If we perform the correlation over an integral number of periods, then the theoretical
result is

$$(E_0)(\delta N/N_0)(0.5 \cos(\delta))$$

The standard deviation of the percent error (in this case 10.5%) measures the precision of the measurement. This is equivalent to the quantity,

$$CV(100) = \frac{\sigma}{\mu}(100)$$

(A.6)

defined as the Coefficient of Variation, also known as the relative standard deviation. Fig. A.4 is a contour plot, where the contours are the standard deviations of the percent error (normalized by $\cos(\delta)$). The $x$-axis represents different values of $\delta N/N_0 \in [0.01, .5]$ and the $y$-axis represents varying values of the mean number of particles $N_0 \in [5, 250]$.

Although various other variables need to be considered, plots such as these can give a crude estimation of the spreading of the error for a given number of particles, $N_0$ and (normalized) variation from the mean, $\delta N/N_0$. For example, if one wanted to know how many particles were needed to achieve a 10% spreading of the error given the variation was $\delta N/N_0 = 0.2$, the answer from looking at Fig. A.4 would be around 50 particles. These results will be useful to instrument builders for operational constraints in constructing the wave-particle correlator (as discussed in §4.3). We see by Fig. A.5 that as long the correlation is normalized by $\cos(\delta)$, the precision of the measurement is the same for the phase-shift cases $\delta = 0.45\pi$ and $\delta = 0$, respectively. Hence, we can hypothesize at the present time that the measurement precision will be independent of the phase shift, $\delta$, between the number density of particles, $N(t)$ and the electric field, $E(t)$. 
Figure A.4: Contour plot of Standard Deviation of the Percent Error for a statistical ensemble of 512 and relative phase shift $\delta = 0.2\pi$.

Fig. A.6 shows a plot of the standard deviation $\sigma$ for all values of $\delta N/N_0$ (rainbow curves). Observe that, the Standard Deviation is independent of $\delta N/N_0$, but clearly is dependent on $N_0$. Using standard fitting techniques, we can approximate the Standard Deviation as a function of $N_0$ as

$$\sigma(N_0) \sim e^{-2.78}/\sqrt{N_0} = 0.062/\sqrt{N_0}$$  \hspace{1cm} (A.7)

Similarly, we find that

$$\mu \sim 0.5(\delta N/N_0)$$  \hspace{1cm} (A.8)

If we did not normalize by the phase shift $\cos(\delta)$, then $\mu \sim 0.5(\delta N/N_0) \cos(\delta)$. Since the mean value scales theoretically with $\cos(\delta)$, then the precision of the measurement
Figure A.5: Over-plot of Standard Deviation of the Percent Error for phase shift $\delta = 0.2\pi, 0.45\pi, 0$ represented by solid, dashed and dotted curves, respectively.

will also scale with $\cos(\delta)$. Therefore, this result further justifies that the impact of the phase shift, $\delta$, can be scaled out of the problem under suitable normalization.

Knowing that the original precision curves, $P$, in Fig. A.4, are represented by $\sigma/\mu$, we find that $P$ has a simple model. Substituting Eq. (A.7) for $\sigma$ and Eq. (A.8) for $\mu$, we get

$$P(N_0, \delta N/N_0) = \frac{12.4}{\sqrt{N_0(\delta N/N_0)}} \approx \frac{4\pi}{\sqrt{N_0(\delta N/N_0)}} \quad (A.9)$$

Fig. A.7 shows the modeled curve (dashed) with the curves in Fig. A.4, noting that the model is a reasonably good fit; hence, it can be used to extrapolate the information about precision of measurement for any number of particles.

This model can also be utilized to estimate the signal-to-noise ratio, which is
Figure A.6: Standard Deviation as a function of $N_0$ for all $\delta N/N_0$.

Figure A.7: Model curves C (dashed) and $(\sigma/\mu)(100)$ (solid).
Figure A.8: Model curves SNR (dashed) and $\sigma/\mu$ (solid).

defined as

$$SNR = \frac{\mu}{\sigma}$$

Hence, we obtain the reciprocal of Eq. (A.9), representing the curves depicted in Fig. A.7, and multiply by 100 to eliminate the percentage interpretation to derive our signal-to-noise model as

$$SNR(N_0, \delta N/N_0) = \frac{100}{P(N_0, \delta N/N_0)} = \frac{25 \sqrt{N_0} (\delta N/N_0)}{\pi}$$

which is represented in Fig. A.8.

To interpret these findings (is the factor of $4\pi$ in Eq. (A.9) a coincidence?), we can first determine what the factor 0.062 means in Eq. (A.7) for $\sigma$. Does 0.062 change when we alter anything else in the code? Substituting an unknown parameter
\( \alpha \) for 0.062 in Eq. (A.7),

\[
\sigma = \frac{\alpha}{\sqrt{N_0}}
\]

We then assigned \( N_0 = 50 \) and the number of summing bins over the correlation, \( Sum_\tau = 100 \). The total number of particle counts, \( N_\tau \), equated to 5010 after a single ensemble. We then altered the values of frequency, \( \omega = 4\pi \) and number of periods, \( T = 4 \), to get the following relationship:

\[
\alpha \sim e^{1.678} \sqrt{N_\tau} \approx 5.35 \sqrt{N_\tau}
\]

and

\[
\alpha \sim e^{-0.327} \sqrt{Sum_\tau} \approx 0.761 \sqrt{Sum_\tau}
\]

Setting both equations for \( \alpha \) equal to each other and solving for the ratio, \( N_\tau : Sum_\tau \),

\[
\frac{N_\tau}{Sum_\tau} \approx \left(\frac{5.35}{7.61}\right)^2 \approx 49.42
\]

which is sufficiently close to 50. This confirms our methodology, since we assigned \( N_0 = 50 \). Choosing \( \omega = 16\pi \) and \( T = 16 \), we extend to more possible particle counts to see if there exists a lower bound on \( \alpha \). We find by Table (A.1) that, indeed, \( \alpha \) converges to the value of 0.025. Thus, as \( N_\tau, Sum_\tau \to \infty \),

\[
\sigma \to \frac{0.025}{\sqrt{N_0}}
\]

It is logical that the spread of error decreases with increasing number of summing bins. The “theoretical result” is based on the multiplication of two continuous quantities. In our PATCH method, we approximate an integral based on a discrete number of summing bins. Hence, increasing the number of summing bins will yield more accurate
<table>
<thead>
<tr>
<th>$N_\tau$</th>
<th>$Sum_\tau$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>23884</td>
<td>480</td>
<td>0.036</td>
</tr>
<tr>
<td>27905</td>
<td>560</td>
<td>0.0306</td>
</tr>
<tr>
<td>31783</td>
<td>640</td>
<td>0.0263</td>
</tr>
<tr>
<td>35748</td>
<td>720</td>
<td>0.0261</td>
</tr>
<tr>
<td>39764</td>
<td>800</td>
<td>0.0251</td>
</tr>
</tbody>
</table>

Table A.1: Convergence of $\alpha$ with increasing number of particle counts.

results, with a lower bound of $0.025/\sqrt{N_0}$. Still, the question remains as to how we can predict this from first principles, using similar formalism described in §A.1.
This appendix chapter serves as an extension to §3.5. The purpose is for the reader to expand awareness of the difficulties that may arise in implementing any method derived from theory, such as PATCH, to an actual spacecraft instrument, such as an Electrostatic Analyzer (ESA). Note that the particle trajectories explored in B.1 and the time delays described in B.2 barely scratch the surface of possible nuances requiring additional considerations in practice. In B.3, we discuss possibilities for improving the PATCH algorithm by connecting definitions of particle count rate to the geometric factor on the detector, described in §3.5.2. Although the conclusions made might seem trivial, they may serve as fundamental initial steps towards critically thinking about onboard implementation of a future wave-particle correlator, described in §4.3, in terms of physical flight hardware.

### B.1 Particle Trajectories

In this appendix section, we aim to define the ESA particle arrival time more precisely by providing additional details about how the particles enter the aperture of the detector. Fig. B.1 depicts a rudimentary sketch of Fig. 3.5, where the axis of symmetry (green), $V$, corresponds to the fixed energy, $E$. The particles\(^1\) with this energy will be collected for some period of time, $\Delta t$. Depending on the initial energy

\(^1\)For electrons and ions, the voltage difference is set to positive (+$V$) and negative ($-V$), respectively.
and angle of the particle entering the surface of the pancake-shaped opening (black),
the particle could have various possible trajectories, depicted in Fig. B.1. Note that
this opening corresponds to the FOV in Fig. 3.5, the ‘top’ of the ESA where the
particles enter; it rotates azimuthally 360°, which are divided into azimuthal bins
comprising one angle, $\phi$, in velocity-space. The symmetry of the nested hemispheres
produce images all around the 2D pancake surface where the particles enter the
detector. The particle trajectories, shown in Fig. B.1, are then projected onto a 2D
circular plane (purple) below the ESA, which portrays the paths of particles as lines.
The azimuthal bin widths can be adjusted on the projection plane, while the polar
angle bin widths can be adjusted at the top.

In Fig. B.1, particle 1 (blue) flew in from the left of the symmetry axis, en-
tering the aperture of the pancake with a corresponding position and velocity. Then, the 3D trajectory is projected onto the 2D circular plane (see dashed grey lines for approximate projection coordinates) at initial position \( x_i \) and final position \( x_f \) in a time interval \( \Delta t \). Therefore, the velocity, \( \mathbf{v}_{p_1} \), of particle 1 is computed as

\[
\mathbf{v}_{p_1} = \frac{x_f - x_i}{\Delta t}
\]

which is negative since \( x_f < x_i \). Particle 2 (red) flew in from the right, so the ESA will record a positive velocity, instead. Furthermore, since the red arrow on the projection plane is longer than the blue arrow, the magnitude of the velocity for particle 2 will be recorded as greater than the magnitude of \( \mathbf{v}_{p_1} \). Particle 3 (pink), however, flew into the aperture with either too much energy or at an angle too steep with respect to the pancake opening. Therefore, it crashed into the boundary of the hardware (pink X) and would not be counted. For the sake of simplicity, we will omit the case of particle 3 for the rest of this dissertation appendix, but the reader should be aware of the various kinds of errors the instrument can make when counting particles.

The arrival time, \( t^* \), is then defined for a particle with coordinates \((x_1(t^*), v_1(t^*))\) on the projection plane in Fig. B.1 as the time the initial position, \( x_i \), was recorded. Since the transit-time through the detector is negligible (\( \sim 1 \) ns), we can assume the arrival time, \( t^* \), occurs when \( x_1 \) is incident normal to the aperture, i.e. \( [x_1] \cdot \hat{n} = n_d \), where \( n_d \) is the position on the 2D aperture detector plane. Therefore, at the aperture entrance position, \( n_d \), the particle arrives at time \( t^* \), in a fixed 5D space of set voltage difference \( V \), \( \Delta E \) and geometric factor \( G \) (described in §3.5.2).
Figure B.2: Sketch of a data processing unit (DPU) onboard spacecraft receiving time-delayed information from both an electrostatic analyzer and a waves instrument.

B.2 Time Delay Calibration

Since our ultimate goal, outlined in §4.3, will be to correlate the particle time of arrival with simultaneous measurements of waves (such as the electric field), we must also be aware of additional required time calibration. Fig. B.2 depicts a (highly simplified) sketch of an onboard data processing unit (DPU), recording information from both a wave receiver (left) and ESA (right). The DPU, like the one on PSP shown in Fig. 4.1, is the physical hardware on the spacecraft that processes information; it is the location for implementing data processing algorithms, such as PATCH. The waves instrument, depicted in Fig. B.2, receives a wave (green) at some time, $t_\omega$, and is then filtered within the electronics, which will cause a delay both in time and phase. Preflight instrument testing determines the calibration factor of these two important parameters that will be delayed due to pure hardware realities. Assume
that the phase and time delay calibration is done carefully and successfully. Then,
there is still a time delay, $\Delta t_E$, between the time the wave enters the receiver and the
time it passes through all of the electronics to arrive at the DPU, recorded at time,
$t_{rE}$. Similarly, there is a time delay, $\Delta t_p$, between the particle time of arrival at the
detector aperture and the transit to the DPU at time, $t_{rp}$.

The time delay through the electronics, for either the particles or the waves,
can sometimes be negligible. For example, the choice of omitting the wave time
delay, $\Delta t_E$, will depend on the frequency of the wave in question and the desired
accuracy. For example, if the instrument receives a wave at or below 100 Hz, then
the period of that wave is at least 10 ms and a 1% time accuracy$^3$ would correspond
to 100 $\mu$s. Therefore, if the desired calculation performed in the DPU requires a time
resolution on the order of 100 $\mu$s, this time calibration cannot be ignored. However,
if the calculation does not require this fine of a time resolution by several orders of
magnitude, then the delay can be safely neglected.

Since the field-particle correlation technique is extremely sensitive to time ac-
curacy, future implementation towards a wave-particle correlator (described in §4.3)
should proceed with careful definitions of the particle arrival times in terms of these
additional time delays. As was shown in §3.6.1, onboard implementation of the
field-particle technique through the PATCH algorithm requires simultaneous mea-
surements of both the particles and waves. This correlation will be performed on the

---

$^2$This notation for the particle time delay should not be confused with the particle
sampling time on the instrument.

$^3$This is usually the best we can do, according to reputable instrumentalists.
DPU, so the recorded times will not be the same as the actual times that the particles and waves arrived at their respective instruments. By Fig. B.2, the recorded time of particle arrival on the DPU is

\[ t_{r_p} = t_p + \Delta t_p \] (B.1)

and the recorded time of the wave is similarly

\[ t_{r_E} = t_E + \Delta t_E \] (B.2)

Solving Eq. (B.1) for \( t_p \), we find that the particle time of arrival at the aperture is

\[ t_p = t_{r_p} - \Delta t_p \]

The next task is to find the recorded time of the wave arrival, \( t_{r_E} \), when \( t_p = t_\omega \). Solving for \( t_\omega \) from Eq. (B.2),

\[ t_\omega = t_{r_E} - \Delta t_E \]

Therefore, correlating the particle time of arrival, \( t_p \), with simultaneous wave measurements requires a time calibration, such that

\[ t_p = t_\omega = t_{r_E} - \Delta t_E \] (B.3)

to equate the time the particle actually arrived at the aperture with the time that the wave arrived at the instrument receiver. A multitude of other instrumental nuances, beyond the scope of this dissertation appendix, will need to be considered for actual onboard implementation.
B.3 Connecting PATCH to the ESA Geometry

The theory developed in §3.6.4 for the PATCH algorithm was based on approximating infinitesimal phase-space to finite bin sizes. Connecting to the physical particle detector described in §3.5, the width of these bins will be closely related to the $G$ factor, meaning that the success of the onboard correlation itself can be optimized according to the ESA geometry.

As discussed in §3.5.2, the observed distribution of $N$ particles with species, $s$, by the spacecraft particle detector with an aperture geometry-defined phase-space $(r \times v)_G$, is defined in Eq. (3.12) as

$$f(r, v, t) = \frac{N(r, v, t)}{d^3r d^3v}$$

In a single 3D spatial phase-space bin, $\Delta r_p$ (described in §3.6.4), a particle arrives at time $t_{1p}$ with velocity $v_{1p}$, within a sampling time $dt_p$, at the particle detector aperture within a fixed small area $dA_p$. Similarly, in a single 3D velocity-space bin, $\Delta v_p$, a particle arrives at a fixed energy, $E_p$ and solid angle of rotation, $\Omega_p$. Thus, $f$ is locally defined as, $\Delta f = f_p$, in the approximately infinitesimal bin width of phase space, $\Delta r_p \Delta v_p \in (r \times v)_G$, as

$$f(r, v)\bigg|_{(\Delta r_p \Delta v_p)} = f_p = \frac{\Delta N}{\Delta r_p \Delta v_p} \quad (B.4)$$

Assuming the axis of symmetry on the ESA (see §3.5) is $v_1$, we substitute Eq. (3.15)-(3.16) for $\Delta r_p$, and Eq. (3.17) for $\Delta v_p$, to get

$$f_p = \frac{\Delta N}{v_{1p} \Delta t_p v_{1p}^2 \Delta v_{1p} \Delta \Omega_p} \quad (B.5)$$
Following the same procedure as Eq. (3.15)-(3.36),

\[ f_p = \frac{\Delta N}{\Delta t_p} \cdot \frac{m_s^2}{2E_p^2} \cdot \frac{1}{\frac{\Delta E_p}{E_p} \Delta A_p \Delta \Omega_p} \]  

(B.6)

where \( \Delta N/\Delta t_p \) is interpreted as the particle count rate, \( \dot{C}_p \), in a sampling time, \( \Delta t_p \), and \( G_p = \frac{\Delta E_p}{E_p} \Delta A_p \Delta \Omega_p \) is the locally defined \( G \) factor, given by Eq. (3.35). This says that the local geometric factor of the particle detector, \( G_p \), is inversely proportional to the number of particles counted per sampling time in a single phase-space bin \( \Delta r_p \Delta v_p \).

This is logical since if \( \dot{C}_p \to 0 \), then \( G_p \to \infty \), meaning that a higher geometric factor is required for lower count rates. Conversely, if \( \dot{C}_p \to \infty \) then \( G_p \to 0 \), meaning that for higher count rates, the geometric factor can be smaller and still achieve statistically significant measurements.

Locally, the quantity \( N(t) \), defined in §3.6.1 over the correlation time, \( \tau \), is linearly proportional to the count rate, \( \dot{C}_p \), defined in §3.5.2. In a single phase-space bin, the number of particles counted, \( \Delta N \), per sampling time, \( \Delta t_p \), over a correlation interval, \( \tau \), is related by the expression:

\[ \frac{\Delta N}{\Delta t_p \tau} = \dot{C}_p \tau \]  

(B.7)

Since the local count rate is inversely proportional to the local geometric factor, Eq. (B.7) says that counting particles over a longer correlation time in each phase-space bin will allow for a smaller \( G_p \)-factor, and vice versa. Although this statement may seem tautological, it suggests that if the correlation time interval is long enough, implementation of the PATCH method onboard spacecraft could be independent of the
bin sizes defined by the aperture geometry and subsequent velocity-space resolution. For example, for low count rates and a fixed geometric factor, one can keep the count rate-to-geometric factor ratio
\[ \frac{\dot{C}_p}{G_p} \]
constant by counting particles for a longer period of time, i.e. by multiplying the numerator by a linear factor corresponding to a counting time interval. For example, if \( \dot{C}_p \) decreased by two orders of magnitude, then by Eq. (B.7), the correlation time interval, \( \tau \), should also increase by two orders of magnitude to sustain this local \( \dot{C}_p : G_p \) ratio. In addition, Eq. (B.6) says that the local count rate is inversely proportional to the square of the set voltage difference on the ESA. Consequently, the count rate decreases nonlinearly with increasing energy. This suggests that the correlation interval, \( \tau \) should increase to \( \tau^2 \), with each successive energy bin in the sweep.

Alternatively, the local differential counting flux (described in §3.5.2), \( j_p \), which is the particle density flux incident to a surface per unit energy, may be a more beneficial quantity to explore PATCH implementation instead of \( f_p \). Substituting the definition of count rate into Eq. (3.27),
\[ j_p = \frac{\dot{C}_p}{\Delta E_p \Delta \Omega_p \Delta A_p} = \frac{\dot{C}_p}{G_p E_p} \]  
(B.8)
If \( j_p \) has a more equivalent interpretation to \( \frac{\partial n^+}{\partial t} \) rather than \( f_p \), the PATCH method can count significantly more particles in higher energy bins. Since Eq. (B.8) says that \( \dot{C}_p \) is now inversely proportional to \( E \) in a linear fashion, the correlation time, \( \tau \), would increase linearly with each increasing energy step. Therefore, interpreting
the PATCH method as a differential counting flux averaged over a correlation time, \(\tau\), would drastically improve the time needed to keep the \(\dot{\mathcal{C}}_p : G_p\) ratio constant with increasing energy step through the sweep.

These simple observations may yield powerful insights when implementing PATCH onboard spacecraft in bins with too low count rates to obtain significant measurements. Signal-to-noise ratios could therefore be notably improved by solely adjusting \(\tau\) and averaging the correlation over a longer period of time, which directly interprets to counting more particles. Future work will devise a strategy for keeping this \(\dot{\mathcal{C}} : \mathcal{G}\) ratio constant in each bin, by automatically adjusting \(\tau\) based on low count-rate detection and higher energy bin.

By definitively connecting theoretical algorithms, such as PATCH, to physical flight hardware geometry, we can initialize razing the language barriers between instrumentalists and theorists. Communication linking these two disparate groups may be the key to significant scientific progress, “to explore strange new worlds, to seek out new life, and new civilizations, to boldly go where no one has gone before.”

---

4This quote is from the Star Trek: The Next Generation theme song, originally called, “Where No Man Has Gone Before,” by Alexander Courage.
REFERENCES


