A STUDY OF THE PERMEABILITY OF SAND

BY

Frederic Theodore Mavis, C.E., Ph.D.
Associate Director in Charge of Laboratory,
Iowa Institute of Hydraulic Research,
and Acting Head,
Department of Mechanics and Hydraulics,
The State University of Iowa

AND

Edward Franklin Wilsey, Ph.D.
Professor and Head, Department of Physics,
Robert College, Istanbul, Turkey

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I. INTRODUCTION

1. Synopsis.—This paper presents the results of a study of tests made on unigranular sands and on blended samples with the view to determining the effect upon permeability of water temperature, size and shape of grain, porosity, and mechanical analysis of sand samples. Formulas are proposed on the basis of the tests and comparisons are made with other formulas for flow of water through sands.

2. Acknowledgments.—A thesis by E. F. Wilsey submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Hydraulic Engineering at the State University of Iowa is the basis of this paper. The thesis was prepared under the supervision of the senior author.

The tests were conducted in the laboratory of the Iowa Institute of Hydraulic Research—an integral part of the College of Engineering of which B. J. Lambert is Acting Dean. Dr. C. C. Williams was Dean of the College of Engineering while the laboratory studies were in progress. Prof. F. T. Mavis is Associate Director in Charge of the Laboratory.

Acknowledgment is gratefully made to D. L. Yarnell, Senior Drainage Engineer of the Bureau of Agricultural Engineering, U. S. Department of Agriculture, and to Adolph F. Meyer, Consulting Engineer and Professor of Hydraulic Engineering at the State University of Iowa during the academic year 1934-1935, for helpful suggestions and cooperation in the experimental investigations. Edwin Thomas, Laboratory Technician, assisted in making and assembling the laboratory apparatus.

Edward Soucek, Research Assistant Engineer, Iowa Institute of Hydraulic Research, read the final manuscript and offered helpful suggestions.

3. Notation.—The following notation is used in this study:

\( A \) = cross-sectional area of sand column.
\( C \) = a constant.
\( d \) = diameter of sand grains, in mm.
\( d_e \) = equivalent grain diameter, in mm., for varigranular sand.
\( d_n \) = Hazen’s effective size, viz., the diameter of sand grains, in mm.,

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such that ten per cent by weight of a representative sample is
of smaller grains and 90 per cent is of larger grains.

\[ d_e = \text{Slichter's effective [equivalent] size, viz., the diameter of sand}
\text{grains, in mm., such that if all grains were of that diameter the}
\text{sand would have the same transmission capacity that it actually}
\text{has at a given temperature and porosity.} \]

\[ d_m = \text{median diameter of sand grains, in mm., such that 50 per cent by}
\text{weight of a representative sample is of larger and 50 per cent of}
\text{smaller grains.} \]

\[ d_{a1}, d_{a2}, \ldots = \text{diameters of openings of two adjacent sieves in a series.} \]

\[ G = \text{mean specific gravity of sand grains.} \]

\[ h = \text{hydraulic gradient, the ratio of head loss in feet of fluid to the}
\text{length of sand column in which loss occurs.} \]

\[ k = \text{permeability of sand in cu. ft. per day per sq. ft. of sand column}
\text{(measured normal to flow) per unit hydraulic gradient.} \]

\[ L = \text{total depth of sand column} \]

\[ p = \text{porosity (usually) in per cent of volume of sand.} \]

\[ Q = \text{rate of discharge (usually) in cu. ft. per day.} \]

\[ s = \text{surface area of sand grains in sq. cm. per cc. sand.} \]

\[ t = \text{temperature of water in degrees Fahr.} \]

\[ \mu = \text{coefficient of viscosity of water in dyne sec. per sq. cm.} \]

\[ v = \text{apparent velocity of flow in ft. per day} = \frac{Q}{A} . \]

4. Historical Introduction.—H. Darcy is generally credited with
having first proposed in 1856\(^1\) a formula, often referred to as
Darcy's Law, expressing the relation between filter velocity and
hydraulic gradient. According to Engels,\(^2\) Darcy's apparatus con­
sisted of a cylinder approximately 14 in. in diameter and 11.5 ft.
high. Three grills having openings respectively 7, 5, and 2 mm.
were fixed 8 in. above the bottom of the cylinder. Pressures were
observed by means of mercury manometers connected to piezometers
near the top and bottom of the apparatus, respectively.

Materials used in the tests were described as follows: gravel,
mussel shell fragments, etc., 17 per cent; 2.00 mm. grains, 12 per
cent; 1.00 mm. grains, 13 per cent; 0.77 mm. grains, 58 per cent.
After being thoroughly stirred and compacted under water the
porosity of the material was 38 per cent.

The twelve tests summarized by Engels were made at operating
heads ranging from 1.5 to 13.2 times the heights of sand columns
—at hydraulic gradients between 1.5 and 13.2. Water flowed

\(^1\) Darcy, H., "Les Fontaines Publques de la Ville de Dijon," (Paris, 1856) (Abstract,
Annales des Ponts et Chaussees, 1856 (2) pp. 122-125).

\(^2\) Engels, H., "Handbuch des Wasserbaues," (Wilhelm Engelmann, Leipzig and Berlin,
1914) v. 1, pp. 41-43.
downward through columns of unwashed sand 23 in. deep in six
tests, and 45 in. deep in three tests. Washed sand in a layer 67 in.
depth was used in three tests.

Darcy’s conclusion that for a given filter the flow is directly
proportional to the head loss is usually expressed by the formula

\[ v = k \frac{h}{l} \]  

(1)

Mean values of the coefficient of permeability based on abstracts
of Darcy’s data range from about 50 to 90 ft. per day.

Allen Hazen,\(^3\) in 1892, published the results of tests of filter
sands and proposed the following formula in which the paren­
thetical term is the optional Fahrenheit factor:

\[ v = c \cdot d_{e}^{2} \cdot \frac{h}{l} \left( \frac{t + 10}{60} \right) \]  

(2)

in which

\[ c = \text{a constant which “rarely falls below 1300, even for old and} \]
dirty sand, and rarely rises above 4,000, and, in a majority
of ordinary sands, falls between 2,300 and 3,300.”\(^4\)

\[ d_{e} = \text{Hazen’s effective size in mm.—(Limited between 0.10 and} \]
3.00 mm.)

As an index of the gradation of the sand Hazen introduced the
term “uniformity coefficient” which he defined as the ratio of
the size of grain which has 60 per cent of the sample finer than
itself to the size which has 10 per cent finer than itself. The for­
mula is restricted to sands having uniformity coefficients less than
5.0.

F. H. King,\(^5\) in 1898, proposed a method for determining the
grain size of samples of sand using air instead of water as a fluid.
The aspirator proposed by King does not appear to have been
widely used. His observations\(^6\) have been correlated with Slichter’s
mathematical analysis.

\(^3\) Hazen, Allen, “Experiments upon the Purification of Sewage and Water at the
Lawrence Experiment Station, Nov. 1, 1889 to Dec. 31, 1891,” Massachusetts State
Board of Health, Twenty-third Annual Report, p. 431 (1892).

Hazen, Allen, “Some Physical Properties of Sands and Gravels with Special Reference
to Their Use in Filtration,” Massachusetts State Board of Health, Twenty-fourth Annual
Report, p. 541 (1893).

\(^4\) Hazen, Allen, Discussion on Dams on Sand Foundations, Transactions, Am. Soc. C. E.,
v. 73, pp. 199-203 (1911). Metric units converted to feet per day.

\(^5\) King, F. H., “A New Method for the Mechanical Analysis of Soils,” Wisconsin Uni­
versity Agricultural Experiment Station, Fifteenth Annual Report, p. 123 (1898).

C. S. Slichter, in 1899, published a mathematical analysis of the flow of a fluid through a column of soil made up of spherical grains of uniform size and proposed a formula which may be written as follows:

\[ Q = 290 \frac{d^2 s}{K} \frac{h}{l} A \]  

(3)

in which \( \frac{1}{K} \) is a function of the porosity which may be represented by

\[ \frac{1}{K} = 0.0492 \left( \frac{p}{40} \right)^{3.3} \]  

(3a)

Slichter's formula is substantially equivalent to

\[ v = 1096 d^2 s \frac{h}{l} \left( \frac{t + 10}{60} \right) \left( \frac{p}{40} \right)^{3.3} \]  

(4)

The use of the term "effective size" for diameters used in both Hazen's and Slichter's formulas has led to confusion. For the purposes of this paper the following terms will be used:

- **Effective Size.**—The diameter of sand grains, in mm., such that 10 per cent by weight of a representative sample is of smaller grains and 90 per cent is of larger grains.

- **Equivalent Size.**—The diameter of sand grains, in mm., such that if all the grains of a given representative sample were of that size, the calculated permeability of the sample would be unchanged for the same temperature and porosity.

W. R. Baldwin-Wiseman, in 1906, reviewed briefly earlier ground-water observations and basic studies, presented the results of his own experiments on the permeability of rocks, and compiled certain statistics of pumping and filtration plants. In 1910 he reported his experiments on the flow of water through sand filters. Tests were conducted under both low and high pressures. The apparatus for the low-pressure tests consisted of a cylinder 4 in. in diameter and 18 in. high. Sand thicknesses varied from 3 in.

---


to 15 in. by increments of 3 in. and heads varied from 3 in. to 15 in. by 3 in. increments. The apparatus for high-pressure tests was 6 in. in diameter and 4 ft. 3 in. long with sand thicknesses 4 ft. 3 in. and 2 ft. 0 in. Initial water pressures were 60 to 80 lb. per sq. in. with the view to compacting the sand. Hydraulic gradients in the tests were greater than 0.2.

Baldwin-Wiseman proposed a formula of the type

$$Q = z \frac{e^{\frac{1}{s}}}{s} \left( \frac{k}{l} \right) \left( \frac{t + 10}{60} \right)$$

(5)
in which

$$z = a \text{ term which varies inversely as the square root of the hydraulic gradient (approximately).}$$

Other formulas proposed for the permeability in terms of surface area of sand grains per unit volume are ascribed to Krüger (1918)\(^{10}\)

$$k = \frac{c}{\mu} \frac{p}{s^2}$$

(6)

and to Zunker (1920)\(^{11}\)

$$k = \frac{c}{\mu} \frac{p}{(1-p)^2 s^2}$$

(7)

Terzaghi’s formula\(^{12}\) appeared in America in 1925, introducing into the coefficient, \(c\), the effect of grain form:

$$k = \frac{c}{\mu} \left[ \frac{p \cdot 0.13}{\sqrt{1-p}} \right]^2 (\frac{a^2}{\mu})$$

(8)

Kozeny\(^{13}\) integrated Poiseuille’s partial differential equation and obtained the solution:

$$k = \frac{c}{\mu} \frac{\mu^3}{(1-p)^2} \frac{\mu^2}{a^2}$$

(9)

A factor for shape is included in the constant \(c\). The term \(a^2\) is defined in the following equation:


\[
\frac{1}{d'_{n}} = \frac{1}{3} \left[ \frac{1}{d_{n}} + \frac{2}{d_{n} + d_{n+1}} + \frac{1}{d_{n+1}} \right]
\] (11)

in which \( \Delta g_{n} \) is the ratio of the weight of sand contained between two adjacent sieves to the total weight of the sample, and \( d'_{n} \) is the weighted harmonic mean size of sand grain between these two sieves calculated according to the formula:

\[
\frac{1}{\Delta g_{n}} = \sum_{1}^{n} \frac{\Delta g_{n}}{d'_{n}}
\] (10)

N. D. Stearns,\(^{14}\) in 1927, presented the results of laboratory tests intended primarily to obtain quantitative data on permeability and on specific yield of more than fifty permeable materials. Most of the tests were conducted in a permeameter 8 in. high and 3 in. in diameter, the water flowing upward through a 10 cm. column of sand. Some tests, however, were conducted in a long-cylinder apparatus 3 in. in diameter and 48 in. high in which the sand column tested was 100 cm. in height.

Stearns' tests of sand obtained from Fort Caswell, N. C., and having an effective (10 per cent) size of 0.14 mm., a uniformity coefficient of 1.9 and a porosity of 49 per cent "gave consistent results and support Darcy's law for hydraulic gradients ranging from 270 feet down to 5 feet to the mile. The tests made with the long column checked closely with those made with the short column and indicated that tests with the more convenient short-column apparatus are trustworthy. The field determination of rate of movement by means of dye showed a permeability that agreed substantially with that obtained in the laboratory tests. [The coefficient of permeability, defined as the rate of flow in gallons per day through a square foot of its cross section, under a hydraulic gradient of 100 per cent, at a temperature of 60° F., was 415 for the long-cylinder, 379 for the short cylinder, and 306 for the dye measurements.] Moreover, the permeability as computed by Slichter's formula agreed closely with the permeability derived by the laboratory and field tests . . . ."\(^{15}\)

In 1929, Donat\(^{16}\) described several tests, conducted in a glass


permeameter about 1.3 in. in diameter and 4.8 in. high, which led him to conclude that the observations were in agreement with Kozeny's formula.

In 1933, Fair and Hatch,\footnote{Fair, G. M. and Hatch, L. P., "Fundamental Factors Governing the Stream-Line Flow of Water Through Sand," \textit{Journal Am. Water Works Assoc.} v. 25, pp. 1551-1565, November 1933.} using methods of dimensional analysis, derived the following formula which closely resembles Kozeny's:

\begin{equation}
k = \frac{c}{S^2} \frac{p^3}{(1 - p)^2} \left( \sum \frac{1}{\sqrt{d_n \cdot d_{n+1}}} \right)^2 \tag{12}\end{equation}

in which \(S\) is the shape factor.

In the same year Hulbert and Feben\footnote{Hulbert, R. and Feben, D., "Hydraulics of Rapid Filter Sand," \textit{Journal Am. Water Works Assoc.}, v. 25, pp. 19-65, January 1933.} published a formula, based on tests of filter sands, which may be expressed as follows:

\begin{equation}\begin{split}v = 1054 & \quad d^{1.89} \left( \frac{t + 20.6}{69.43 - p} \right) \frac{h}{l} \tag{13}\end{split}\end{equation}

in which \(d\) is the 50 per cent grain size in mm.

H. H. Hatch,\footnote{Hatch, H. H., "Tests for Hydraulic-fill Dams," \textit{Transactions Am. Soc. C. E.}, v. 99, pp. 206-284 (1934).} in 1934, presented the results of tests of fine materials used in the construction of Cobble Mountain hydraulic-fill dam. The results of permeability tests of materials whose effective size ranged from 0.003 to 0.012 mm. are represented approximately by the formula

\begin{equation}\begin{split}v = 240 & \quad d^{2.4}_H \left( \frac{p}{40} \right)^6 \left( \frac{t + 10}{60} \right) \frac{h}{l} \tag{14}\end{split}\end{equation}

II. EXPERIMENTAL INVESTIGATION

5. \textit{General}.—The permeability of a granular material to water depends upon the temperature of the water and upon the porosity, size, shape and gradation of the sand grains. If the variables are independent it should be possible to relate them by a function

\begin{equation}k = f_1(t) \cdot f_2(p) \cdot f_3(d) \cdot f_4(S) \cdot f_5(g) \tag{15}\end{equation}

The plan of investigation was first to keep the porosity and the size, shape and gradation of sand grains constant and to determine
the temperature function. With this function determined, only the porosity and temperature were allowed to vary during the second series of tests. The data were reduced to the common temperature of 60 degrees Fahrenheit and the porosity function was found. Then, for unigranular samples separated from a given sand the size function was determined after the permeability observations had been corrected to a constant temperature and porosity on the basis of the functions previously determined. Finally tests were made to determine permeabilities of varigranular sands and sands of different grain shapes. General functions of shape and gradation have not been suggested on the basis of these tests.

6. Apparatus.—Fig. 1 shows a sketch of the apparatus. Hot and cold water from the laboratory supply were mixed to the desired temperature in a 30-gallon hot water tank. Water from the mixing tank entered a small constant level tank which was mounted on a screw jack. From the constant level tank the water flowed through a \( \frac{3}{4} \)-in. garden hose to the bottom of the permeameter.

The permeameter was made of a piece of 6-in. cast iron pipe 3 ft. long, mounted vertically and fitted with a blank flange at the bottom. A \( \frac{3}{4} \)-in. pipe nipple was fitted through the center

**Fig. 1. Diagram of Apparatus**
of the flange and the opening in the small pipe was covered with 100-mesh copper screen. Two ¼-in. brass nipples were screwed through the side-walls of the permeameter 18 in. apart with the lower nipple 3.5 in. above the bottom. The nipples were filed off flush with the inside wall of the permeameter and the ends were covered with 100-mesh copper screen.

A pipe 1 in. in diameter, serving as an overflow under normal operating conditions of upward flow, was fitted into the side wall of the permeameter 28 in. above the bottom.

Water leaving the overflow pipe was collected in a bucket which was supported on a balance.

7. Materials.—The water used throughout these tests was obtained from the University distribution system.

Washed sand from the Iowa River at Iowa City was used in all but one series of tests. In that series the sample consisted of standard Ottawa sand. The Iowa River sand is angular while the Ottawa sand is smooth and well rounded. The specific gravity of six sets of unigranular samples of Iowa River sand ranging in size between 0.23 mm. and 1.80 mm. was 2.63 while the specific gravity of samples ranging between 0.16 mm. and 0.23 mm. was 2.67.

Although the Ottawa sand was nearly uniform in size the sample used in the tests was graded between the No. 20 and No. 28 sieves in the laboratory. The size of grains ranged from 0.68 to 0.93 mm. and the specific gravity, obtained from six determinations, was 2.64.

The sieves used in the tests were rated by the count-and-weight method. A sample of sand was sieved 5 minutes by a mechanical shaker. The grains which passed through the sieve upon further shaking were collected. For the smaller sizes groups of 1000 of these particles were weighed and the mean diameter was computed assuming the grains to be spheres.

Table 1 shows a comparison of the mean of two adjacent sieve sizes and the mean grain diameter of corresponding unigranular samples of Iowa River sand determined by counting and weighing 1000 grains taken from each sample.
Table 1
Comparison of Measured Average Diameter with Average of Sieve Openings

<table>
<thead>
<tr>
<th>Limiting Sieve Numbers</th>
<th>Mean Diameter from Weight of 1000 Grains mm.</th>
<th>Mean of Limiting Sieve Sizes mm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8–10</td>
<td>1.98</td>
<td>2.16</td>
</tr>
<tr>
<td>10–14</td>
<td>1.50</td>
<td>1.53</td>
</tr>
<tr>
<td>14–20</td>
<td>1.08</td>
<td>1.10</td>
</tr>
<tr>
<td>20–28</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>28–35</td>
<td>0.55</td>
<td>0.56</td>
</tr>
<tr>
<td>35–48</td>
<td>0.39</td>
<td>0.38</td>
</tr>
</tbody>
</table>

8. Procedure.—The sample to be tested was heated to dryness, weighed, and placed in the permeameter while it was still hot. Hot water was then allowed to percolate slowly upward through the sand expelling most of the air contained in the pores. The hot water was allowed to course through the sand under a head of 0.1 ft. for 30 minutes. The water supply was then shut off and the permeameter was allowed to cool over night.

During a test the temperature of the water in the constant level tank was kept slightly above room temperature. Water temperatures ranged, during the year, from approximately 52 degrees to 76 degrees Fahrenheit. During the tests no fixed routine was set for varying the head. In some series of tests the head was increased from test to test, while in others it was decreased.

The data were obtained in the following manner: Water at the desired temperature was allowed to flow through the sand and at intervals the loss of head was noted. When the loss of head became constant, indicating that steady flow had been established, the discharge observations were made. The water which flowed through the sand was collected in a can which rested on one pan of the trip scales. Sufficient weight was placed on the other pan to over-balance the weight of the can. As the water rose in the can the pointer of the scale moved across the zero mark. At this instant the stop watch was started. The hook gages and temperature were read and recorded. Known weights were then added to the scale to over-balance the weight of the can and water and when the scale pointer again moved across the zero mark the watch was stopped and read. The hook gages and temperature in the cylinder were also read at the end of each test. The head was then changed by turning the jack screw under the constant head tank a number of quarter turns. The loss of head and the total discharge for the previous test were then computed.
and plotted. When the head had again become constant the process was repeated.

The water level in the hook gage stilling wells was slow to become steady after the head was changed and the following procedure was found to expedite the tests. After two tests had been completed, the amount each hook gage reading had changed per quarter turn of the jack screw was computed and it was possible to estimate the hook gage readings for any setting of the jack screw. At the conclusion of the second test the water in each stilling well was brought to its estimated level by either adding or removing water. The head was then allowed to adjust itself over a period of about six minutes, before the next test was begun. The time required for each test using this method was about ten minutes.

Tests in which the initial and final hook gage readings differed by more than 0.001 ft. and in which temperature changes during any one test exceeded 0.5 degrees Fahrenheit were discarded.

The height of the sand column was measured at the conclusion of each series of tests.

9. Preliminary Tests.—It had been suggested that some of the differences in the results reported by various experimenters might be accounted for by the fact that in some tests the flow was upward through the sand while in others the flow was downward. Preliminary tests were made subjecting five samples of uniform sand first to upward flow, then to downward flow. Although there was little difference in general, somewhat smaller discharge was obtained with downward flow in a few of the tests. After reversing the direction to upward flow the second time, the original relation between head loss and discharge was obtained. No difference in the height of the sand column was detected in these tests, indicating that the porosity of the sample remained constant.

Apparent differences in permeability under conditions of upward and downward flow might be expected since air is continually being forced out of the sample when flow is upward while it may be trapped within the sand column when flow is downward, increasing the resistance to flow. Water flowed upward through the sand in all subsequent tests.

If water was allowed to percolate through the sand overnight, it was found that results could not be duplicated from day to day. To study the effect of continuous filtration upon the per-
meability of sand a sample was left in the cylinder for five days and a series of tests was made each day. At the conclusion of each day’s tests the head was set at 0.052 ft. \( h/l = 0.035 \) and the water was allowed to pass through the sand over-night.

![Graph showing variation of permeability during 5-day test](http://ir.uiowa.edu/uisie/7)

**Fig. 2. Variation of Permeability During 5-Day Test**

Fig. 2 shows the variation of discharge through the sample from day to day. The head was so low that no movement of the sand grains was detected; the height of the sand column remained the same from day to day; the sand showed no visible evidence of being air-bound. The change in permeability seems to be due to the clogging of the sand. If the clogging were progressive along the cylinder, some time would elapse before the first hook gage would be affected since it was connected to the cylinder 3.5 in. above the bottom. In order that the data be comparable for all other tests the samples were allowed to remain in the cylinder not more than one day.
10. **Constants for Apparatus.**—Porosity and permeability constants for the apparatus and materials used in these tests follow from elementary considerations.

The porosity of a sample of sand is the ratio of the volume of pore spaces to the bulk volume. The mean specific gravity of the Iowa river sand used in the tests was 2.63 and the cross-sectional area of the permeameter was 29.2 sq. in. (188.7 sq. cm.) If \( w \) represents the weight of sand in the cylinder, in pounds, and if \( L \) represents the corresponding depth of sand column, in centimeters, it can be shown that the per cent porosity, \( p \), is given by

\[
p = 100 - 91.6 \frac{w}{L}
\]  

(16)

In these tests the head loss was measured for a sand column 1.5 ft. long having a cross-sectional area of 29.2 sq. in.

11. **Effect of Temperature on Permeability.**—This series of tests was conducted under a relatively high head in order to reduce the temperature gradient within the length of the sand column. When warm water was used, there was considerable radiation from the cylinder, and with low velocities there was a marked difference
in temperature between the water entering and leaving the cylinder.

Fig. 3 shows the ratio of the discharge at 60 degrees Fahrenheit to the discharge at a temperature \( t \) plotted as ordinates and the corresponding temperature \( t \) plotted as abscissas. The points fit the curve obtained by plotting the temperature as abscissas and the ratio of the viscosity\(^{20}\) at 60 degrees Fahrenheit to the viscosity at any other temperature \( t \) as ordinates. Hazen’s relation modified for comparisons to 60 degree temperatures is in agreement with these tests for the lower temperatures ordinarily expected in ground waters.

In all other experiments the observed discharge, \( Q_t \), at the corresponding temperature, \( t \), was corrected to the discharge at 60 degrees F., by multiplying it by the ratio of viscosity coefficients \( (\mu_t/\mu_{60}) \).\(^{21}\)

12. **Effect of Porosity on Permeability.**—The porosity of a granular material depends on the gradation and shape of the grains and on the compaction of the sample. Unigranular sands of geometrically similar grain shapes, when compacted in the same manner, have approximately the same porosity regardless of grain size. The porosities of six unigranular samples of Iowa River sand compacted by pouring dry sand slowly from a scoop into the permeameter cylinder ranged from 40.4 to 40.8 per cent while mean observed grain sizes ranged from 2.0 mm. for the largest to 0.4 mm. for the smallest sample. A bank-run sample of Iowa River sand ranging in grain-size from less than 0.16 mm. to more than 6.7 mm., compacted in the same way, had a porosity of 34.6 per cent.

The chief difficulty in determining a relation between permeability and porosity lies in obtaining a reasonably wide range of constant porosities with a sand of given size and shape. Attempts to vary the porosity by tamping the material in layers did not prove to be satisfactory. The following procedure was finally adopted: Hot, dry sand was slowly poured into the cylinder of the permeameter and the apparatus was prepared for a series of

\[^{20}\]“International Critical Tables,” v. 5, p. 10.

\[^{21}\]For temperatures between 32 and 105 degrees F., the coefficient of viscosity of water in dyne-sec. per sq. cm. may be represented by

\[ \mu = \frac{0.78}{t + 10} \]

The maximum deviation from Bingham and Jackson’s (1917) values is 5 per cent at extremes of temperature range given.
permeability tests in the manner described under *Procedure*. After one series of tests had been completed the outer shell of the cylinder was tapped lightly with a hammer, the blows being distributed over the surface. Another series of permeability tests was made and material in the permeameter was further compacted by tapping the shell of the cylinder. The height of sand column was observed as a part of each series of tests. It was found that in this way porosity of the materials could be varied over a range of about five per cent.

![Graph](http://ir.uiowa.edu/uisie/7)

**Fig. 4. Flow Through Uni-granular Iowa River Sand of Different Porosities**

Fig. 4 shows the results of tests of unigranular Iowa River sand having a mean diameter of 0.80 mm. (graded from 0.68 to 0.93 mm.) and porosities from 38.5 to 43.5 per cent. Hydraulic gradients are plotted as ordinates and the corresponding apparent velocities of flow, \( Q/A \), in feet per day as abscissas. All velocities have been corrected to a common temperature of 60 degrees F.

Fig. 5 shows the results of similar tests of standard Ottawa
sand having the same mean diameter and range of sizes as the Iowa River sand shown in Fig. 4. The grains of Ottawa sand are well rounded and the porosities ranged from 35.9 to 41.2 per cent.

Fig. 6 shows the relation between apparent velocity and hydraulic gradient indicated by tests of bank-run Iowa River sand having an effective (10 per cent) size of 0.37 mm., a uniformity coefficient of 3.2, and porosities ranging from 29.6 to 34.5 per cent. The mechanical analysis of the sample used in these tests is shown as sample No. 1 in Fig. 10.

Fig. 7 shows as ordinates the permeability (at 60 degrees F., and unit hydraulic gradient) of each of the three samples of sand just described and as abscissas the corresponding porosity. On the basis of these tests the permeability of the Iowa River sand varied as the sixth power of the porosity while the permeability of the Ottawa sand varied as the fifth power. The data do not appear to warrant great precision in the value of the exponent of the porosity and no especial significance is attached to the difference in exponents for angular and rounded grains on the basis of these tests.

![Fig. 5. Flow Through Ottawa Sand](http://ir.uiowa.edu/uisie/7)
Fig. 6. Flow Through Pit-run Iowa River Sand

Fig. 7. Relation Between Permeability and Porosity

Fig. 8 shows a comparison of four formulas which have been proposed for the effect of porosity upon the flow of water through granular materials. The data summarized from the tests of Ottawa
sand and Iowa River sands are also shown. Much has been written in defense of one or more of these formulas and little will be added here. Suffice it to say that for any given sample extreme differences of porosity obtainable in the laboratory were approximately five per cent despite the demonstrability that four significant figure porosities from 25.95 to 47.64 per cent—a difference of 21.69 per cent!—are mathematically possible for spheres of uniform diameter. Although the data obtained in the laboratory appear to be consistent in showing that for the particular materials tested the relative permeability varied with the sixth (or fifth) power of the porosity the range of porosity variable could not be made sufficiently large for a given material to warrant broadly general conclusions.

13. Effect of Grain Shape on Permeability.—The shape of sand may be described by such qualitative terms as rounded, angular, or sub-angular, but no convenient quantitative method for measuring the shape of sand particles is known to the writers. It seems reasonable that the permeability of a granular material should be a function of the shape of its particles even if the grain size and porosity of the sample remain constant.
Fig. 7, showing the permeability of Standard Ottawa sand and Iowa River sand of the same size and porosity, indicates that at 40 per cent porosity the permeability of the rounded Ottawa sand was 50 per cent greater than that of the angular Iowa River sand. This difference may be due, at least partially, to the effect of differences in shape of grains comprising the two materials.

14. Effect of Grain Size on Permeability.—Six unigranular samples and twelve blended samples of Iowa River sand were tested to determine the variation of permeability with grain size. The analysis of the laboratory data may be outlined as follows:

1. Correlation of permeability with mean diameter of unigranular samples.
2. Correlation of permeability with Hazen’s (10 per cent) effective size for mixtures.
3. Correlation of permeability of unigranular materials and mixtures to determine the “equivalent grain size” of the mixture.

(a) Unigranular Materials

Strictly, all samples of sand are mixtures and the term unigranular is applied for convenience to a material made up of an assortment of grains, the largest of which nominally is \( \sqrt{2} \) times the diameter of the smallest. A mixture, therefore, is made up of a sequence of unigranular materials whose limiting sizes progress in a constant geometric ratio of \( \sqrt{2} \).

The mean diameter of a unigranular sample whose limiting sizes, \( d_n \) and \( d_{n+1} \), are in a constant geometric ratio would reasonably appear to be the geometric mean of the two limiting sizes

\[
d_{GM} = \sqrt{d_n \cdot d_{n+1}}.
\]  

(17)

Commonest of the averages, perhaps, is the arithmetic mean or

\[
d_{AM} = \frac{d_n + d_{n+1}}{2}.
\]  

(18)

Other definitions of average grain size for unigranular materials have been proposed including a weighted harmonic mean

\[
d_{WHM} = \frac{3}{\left(\frac{1}{d_n} + \frac{2}{d_n + d_{n+1}} + \frac{1}{d_{n+1}}\right)}.
\]  

(19)

If the series of sieves progresses in a geometric ratio of \( \sqrt{2} \),
the arithmetic mean of two adjacent limiting sieve sizes is approximately \textit{one per cent} larger than the geometric mean or the weighted harmonic mean.

If the diameters of the largest and smallest grains in a unigranular sample differ in the order of 30 per cent of their mean—as they do when materials are separated by sieves progressing in geometric ratio of $\sqrt{2}$—the choice of a definition for mean grain size may about as well be made on the basis of convenience as on the basis of philosophical speculation. Mean diameters shown in the tables, therefore, are arithmetic means of nominal limiting sieve openings and the precision indicated in terms of significant figures is likely to be more apparent than real.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig9}
\caption{Flow Through Unigranular Iowa River Sand of Different Grain Sizes}
\end{figure}

Fig. 9 shows the data obtained from tests of six unigranular samples of Iowa River sand. Ordinates show the hydraulic gradient.
A STUDY OF THE PERMEABILITY OF SAND

and abscissas show the corresponding apparent velocity of flow through the sand column in feet per day at a temperature of 60 degrees F. For the six samples, the greatest porosity was 40.8 per cent and the least was 40.4 per cent.

Table 2 shows the relation between permeability and diameter for these samples. Observed permeability coefficients, \( k \), were adjusted from Fig. 9 to a common porosity of 40 per cent on the basis of the previous finding that the permeability varied with the sixth power of the porosity. The permeability coefficient in feet per day at 60 degrees F., and 40 per cent porosity according to these tests may be computed by

\[ k = 1140 d^2 \]  

with a mean absolute error of approximately 2 per cent.

Table 2

<table>
<thead>
<tr>
<th>Diam. ( d ) ( \text{mm.} )</th>
<th>Observed ( k ) ( \text{Ft. per day}^* )</th>
<th>Computed ( k = 1140 d^2 ) ( \text{Ft. per day}^* )</th>
<th>Deviation of Mean Per cent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.98</td>
<td>4300</td>
<td>1100</td>
<td>4460</td>
</tr>
<tr>
<td>1.50</td>
<td>2610</td>
<td>1160</td>
<td>2560</td>
</tr>
<tr>
<td>1.08</td>
<td>1330</td>
<td>1140</td>
<td>1330</td>
</tr>
<tr>
<td>0.80</td>
<td>758</td>
<td>1180</td>
<td>730</td>
</tr>
<tr>
<td>0.55</td>
<td>338</td>
<td>1120</td>
<td>345</td>
</tr>
<tr>
<td>0.39</td>
<td>171</td>
<td>1120</td>
<td>174</td>
</tr>
</tbody>
</table>

mean = 1140 ± 20

* Permeability coefficient in ft. per day at 60° F., and 40 per cent porosity.

(b) Mixtures

Figs. 10 and 11 show mechanical analysis curves for the twelve blended samples of Iowa River sand used in the tests. Fig. 6 shows the results of permeability tests of sample No. 1, a pit-run sand, and Fig. 12 shows the results of permeability tests for the other samples. All data shown in the curves have been adjusted to correspond to a common temperature of 60 degrees F.

Table 3 shows an analysis of the data to determine the coefficient \( c \) in Hazen’s formula expressed in the following terms:

\[ v = c \ d^2 \left( \frac{t + \frac{10}{60}}{k} \right) \text{ feet per day.} \]

The mean value of \( c \) according to these tests is 2,300 with a mean absolute error of 220 or ten per cent.
Table 3

RELATION BETWEEN PERMEABILITY OF BLENDED IOWA RIVER SAND AND HAZEN’S EFFECTIVE SIZE

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Effective Size $d_{50}$</th>
<th>Uniformity Coefficient</th>
<th>Observed $k$ at 60° F.</th>
<th>Observed $c$ in Hazen’s Formula (21)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.38</td>
<td>3.2</td>
<td>394</td>
<td>2330</td>
</tr>
<tr>
<td>2</td>
<td>1.80</td>
<td>1.8</td>
<td>9150</td>
<td>2430</td>
</tr>
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<td>3</td>
<td>1.27</td>
<td>1.9</td>
<td>4470</td>
<td>2380</td>
</tr>
<tr>
<td>4</td>
<td>0.93</td>
<td>1.9</td>
<td>2400</td>
<td>2380</td>
</tr>
<tr>
<td>5</td>
<td>0.68</td>
<td>1.7</td>
<td>1200</td>
<td>2350</td>
</tr>
<tr>
<td>6</td>
<td>0.45</td>
<td>1.8</td>
<td>512</td>
<td>2170</td>
</tr>
<tr>
<td>7</td>
<td>0.32</td>
<td>1.8</td>
<td>233</td>
<td>1960</td>
</tr>
<tr>
<td>8</td>
<td>0.23</td>
<td>1.8</td>
<td>138</td>
<td>2250</td>
</tr>
<tr>
<td>9</td>
<td>0.32</td>
<td>2.5</td>
<td>193</td>
<td>1620</td>
</tr>
<tr>
<td>10</td>
<td>0.45</td>
<td>2.8</td>
<td>544</td>
<td>2310</td>
</tr>
<tr>
<td>11</td>
<td>0.45</td>
<td>4.0</td>
<td>533</td>
<td>2260</td>
</tr>
<tr>
<td>12</td>
<td>0.32</td>
<td>5.6</td>
<td>394</td>
<td>3300</td>
</tr>
</tbody>
</table>

Mean = 2300 ± 220

Table 4 summarizes an analysis of the data to determine the “equivalent size” of grain in each sample which would bring the
permeability observations on pit-run materials and blended sands into agreement with the observations on unigranular materials. First, the observed permeabilities were adjusted to permeabilities at 40 per cent porosity on the basis of their variation with the sixth power of the porosity as indicated by the foregoing tests. Second, the adjusted permeability coefficient, \( k \), in feet per day at 60 degrees F. and 40 per cent porosity, was divided by 1140, giving the square of the "equivalent size" of grain according to formula (20). Third, the percentage of material finer than the "equivalent size" was determined from the mechanical analysis curves (Figs. 10 and 11) for each blended sample. It was found that the largest grain size of the finest 34 per cent (±5 per cent) by weight of the sample represented the "equivalent size." Column 5 in Table 4 shows the equivalent size computed by formula (20) while Column 7 shows the diameter of the largest grain size of the smallest one-third by weight of blended sample. The agreement may not be considered too unsatisfactory if one does not expect precision to several significant figures.
**FIG. 12. FLOW THROUGH SAND MIXTURES**

**TABLE 4**

**EQUIVALENT GRAIN SIZE FOR MIXTURES**

**OF IOWA RIVER SAND**

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Observed Porosity</th>
<th>Observed $k$ Ft. per day at 60° F.</th>
<th>Adjusted $k$</th>
<th>Equivalent $d_e = \sqrt{\frac{k}{1140}}$ mm.</th>
<th>Per cent finer than $d_e$</th>
<th>34 Per cent finer than $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34.5</td>
<td>394</td>
<td>960</td>
<td>0.92</td>
<td>52</td>
<td>0.7</td>
</tr>
<tr>
<td>2</td>
<td>42.2</td>
<td>9150</td>
<td>6630</td>
<td>2.42</td>
<td>25</td>
<td>2.6</td>
</tr>
<tr>
<td>3</td>
<td>40.8</td>
<td>4470</td>
<td>3990</td>
<td>1.87</td>
<td>34</td>
<td>1.9</td>
</tr>
<tr>
<td>4</td>
<td>41.0</td>
<td>2400</td>
<td>2070</td>
<td>1.35</td>
<td>39</td>
<td>1.3</td>
</tr>
<tr>
<td>5</td>
<td>41.2</td>
<td>1200</td>
<td>1000</td>
<td>0.94</td>
<td>32</td>
<td>1.0</td>
</tr>
<tr>
<td>6</td>
<td>40.4</td>
<td>512</td>
<td>484</td>
<td>0.65</td>
<td>27</td>
<td>0.7</td>
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<tr>
<td>7</td>
<td>40.0</td>
<td>233</td>
<td>233</td>
<td>0.45</td>
<td>30</td>
<td>0.5</td>
</tr>
<tr>
<td>8</td>
<td>40.8</td>
<td>138</td>
<td>122</td>
<td>0.33</td>
<td>34</td>
<td>0.3</td>
</tr>
<tr>
<td>9</td>
<td>36.9</td>
<td>193</td>
<td>315</td>
<td>0.53</td>
<td>32</td>
<td>0.5</td>
</tr>
<tr>
<td>10</td>
<td>37.0</td>
<td>544</td>
<td>870</td>
<td>0.87</td>
<td>32</td>
<td>0.9</td>
</tr>
<tr>
<td>11</td>
<td>35.0</td>
<td>533</td>
<td>1180</td>
<td>1.02</td>
<td>36</td>
<td>1.0</td>
</tr>
<tr>
<td>12</td>
<td>33.2</td>
<td>394</td>
<td>1200</td>
<td>1.03</td>
<td>40</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Mean $= 34 \pm 5$

http://ir.uiowa.edu/uisie/7
15. Summary and Conclusions.—

(1) At small hydraulic gradients the rate of flow of water through sand is inversely proportional to the absolute viscosity of the water.

(2) The permeability of Iowa River sand varied as the sixth power of the porosity and that of Ottawa sand as the fifth power.

(3) For a given porosity of 40 per cent, the permeability of Ottawa sand was 50 per cent greater than that of Iowa River sand of the same size (0.68 to 0.93 mm.)

(4) The permeability of sand varies as the square of the diameter of its grains.

(5) The following formulas represent the data obtained from tests of Iowa River sand fairly satisfactorily:

(a) Unigranular sands (water temperature 60° F.)

\[ v = 1140 \, d^2 \left( \frac{p}{40} \right)^6 \frac{h}{l} \] 

(±2 per cent)

(b) Blended sands based on effective (10 per cent) size:

\[ v = 2300 \, d_n^2 \left( \frac{t + 10}{60} \right) \frac{h}{l} \] 

(±10 per cent)

(c) Blended sands based on “equivalent” (34 per cent) size:

\[ v = 980 \, d_e^2 \left( \frac{p}{40} \right)^6 \left( \frac{t + 10}{60} \right) \frac{h}{l} \] 

(±15 per cent)