Behaviour of continuous reinforced concrete beams subjected to shrinkage potential

Chandrakant A. Patel

University of Iowa

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BEHAVIOUR OF CONTINUOUS REINFORCED CONCRETE BEAMS SUBJECTED TO SHRINKAGE POTENTIAL

by

Chandrakant A. Patel

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the Department of Civil Engineering in the Graduate College of the University of Iowa

June, 1966

Chairman: Dr. Dan E. Branson
ACKNOWLEDGEMENTS

The author takes this excellent opportunity in expressing his grateful wishes to his advisor, Dr. D. E. Branson, for his continued encouragement and valuable suggestions in the course of this investigation. He further expresses his gratitude to Mr. B. J. Dave, a graduate student and friend for his valuable guidance during the making of this report.

Grateful acknowledgement is made to Iris Bauerbach and Carole Ann Phillips for the excellent typing, Richard Lindley for nice sketches and graphs, and authorities of Stanley Engineering Company for providing typing and duplicating services.
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CHAPTER I
INTRODUCTION

1.1 General

Reinforced concrete structures are normally exposed to atmospheric conditions which results in drying of the concrete. The hydrated cement in the concrete contains porous calcium silicate gels which contract upon drying. This contraction of the gels constitutes the primary shrinkage of the concrete. Besides contraction of the gels, there are other constituents of cement which may contribute to the shrinkage of concrete, but such contribution is believed to be small.

Over and above the chemical constituents of the cement, the type of coarse and fine aggregate, the amount of cement paste and water cement ratio, and the humidity and temperature during casting and curing are some of the other factors which influence the drying shrinkage of concrete. Of these various factors influencing concrete shrinkage, the most important is the amount of water placed in the mix per unit volume of the concrete. Hence, it can be accepted that shrinkage of concrete is mainly associated with expulsion or absorption of water to or from the surrounding atmosphere which is a function of the surrounding temperature and humidity.
In statically determinate reinforced concrete members, the reinforcement resists the shortening of the concrete. The position of reinforcement is generally eccentric in structural member which leads to warping of the member. In the case of statically indeterminate member, the warping produced by eccentric steel is partially restrained and leads to a redistribution. Hence, it is of importance to study the deformational behaviour of both statically determinate and indeterminate members as influenced by shrinkage.

1.2 Object and Scope of Study

The scope of this investigation includes the analytical and experimental studies of shrinkage warping of both statically determinate and indeterminate reinforced concrete members. The present investigation is organized to study the effects of three different percentages of reinforcement on shrinkage warping behaviour. Hence, all the shrinkage specimens were cast from the same batch and were kept in similar atmospheric conditions. Effect of the presence of compressive steel on shrinkage warping was studied by casting companion specimens with compressive steel equal to approximately 50 percent of the tensile steel.

In the case of statically indeterminate members, the principal objective is to experimentally measure the shrinkage warping and
to compare this observed warping with the computed curvature. A new procedure for computing shrinkage warping of two-span continuous beam is outlined in Chapter III.

As regards statically determinate beams, the objective is to determine experimentally the shrinkage warping and compare it with shrinkage warping computed by several different approaches. Three different approaches are mentioned in Article 1.3.

1.3 Review of Literature and Discussion of Existing Methods for Computing Shrinkage Warping.

At present, at least three methods are available for computing shrinkage warping of unrestrained specimens. A brief discussion of these three methods is outlined below.

(a) Equivalent Tensile Force Method

The ACI Committee 435 Report on "Deflections of Reinforced Concrete Flexural Members," August, 1965, (scheduled for publication in ACI Journal, June or July, 1966) under Section 304.2.1, mentions the equivalent tensile force method. It suggests the possibility of using a convenient modified expression for shrinkage warping. Such a modification can be accepted because of the very indefinite nature of the problem of shrinkage warping. In this investigation, it is proposed to use this modified shrinkage warping
expression, namely:

$$\phi_{sh} = \frac{T_s \cdot e_g}{E_c \cdot I_g \cdot \frac{1}{2}}$$

where

$$\phi_{sh} = \text{Shrinkage warping}$$

$$T_s = \text{Tensile force in steel equal to } A_s \cdot \varepsilon_{sh} \cdot E_s$$

$$\varepsilon_{sh} = \text{Free shrinkage of concrete, in/in } \times 10^{-6}$$

$$A_s = \text{Area of steel in the cross section}$$

$$E_s = \text{Modulus of elasticity of steel}$$

$$e_g = \text{Distance between center of gravity of steel and center of gravity of concrete section}$$

$$E_c = \text{Modulus of elasticity of concrete}$$

$$I_g = \text{Gross moment of inertia of the section}$$

Whenever reference is made to computed warping by the tensile force method, it is meant that the shrinkage warping is computed by equation (1).
(b) **Miller's^2 Empirical Method for Computing Shrinkage Warping of Singly Reinforced Concrete Members.**

The following paragraph is reproduced from reference 2, which indicates the basic assumption involved in this approach.

"Concrete shrinkage is restricted by reinforcement within a limited range and beyond this limit-free shrinkage occurs. In Miller's^2 Method, it is assumed that shrinkage strains of the concave face of a warped specimen never exceed free shrinkage and will equal free shrinkage when outside the restrictive range of the reinforcement."

Such an assumption has lead to shrinkage curvature given by the expression

\[
\frac{\varphi_{sh}}{d} = \frac{\varepsilon_{sh} - \varepsilon_s}{d} = \frac{\varepsilon_{sh}}{d} \left( 1 - \frac{\varepsilon_s}{\varepsilon_{sh}} \right) \tag{2}
\]

where \(\varepsilon_{sh} = \) Free shrinkage  
\(\varepsilon_s = \) Strain in concrete at the level of steel  
\(d = \) Distance from outer face to steel

Miller suggests empirical value of the quantity \(\frac{\varepsilon_s}{\varepsilon_{sh}} = 0.1\) for heavily reinforced members and 0.3 for moderately reinforced members. Formulae (2) is applicable to singly reinforced beams only.

(c) **Branson's^3 Empirical Method for Computing Shrinkage Warping.**

Branson suggested the following empirical expressions for computing shrinkage warping of both singly and doubly reinforced
concrete beams. It provides closer agreement with the existing
test data than the Equivalent Tensile Force Method and Miller's
Method. These shrinkage warping expressions are:

For \((p - p') \leq 3.0\%\)

\[
\Theta_{sh} = (0.7) \frac{\varepsilon_{sh}}{D} \left( p - p' \right)^\frac{1}{3} \left( \frac{p - p'}{p} \right)^\frac{1}{2}
\]

\[\text{------------------------}(3)\]

For \((p - p') > 3.0\%\)

\[
\Theta_{sh} = (1.0) \frac{\varepsilon_{sh}}{D}
\]

\[\text{------------------------}(4)\]

Where

\(p\) = Tensile percentage of steel; \(p = \frac{As}{bd} \times 100\%\)

\(p'\) = Compressive percentage of steel; \(p' = \frac{As}{bd} \times 100\%\)

\(\varepsilon_{sh}\) = Free Shrinkage

\(D\) = Total depth of test specimen
To the knowledge of the investigator, the shrinkage warping computed by all the three methods, outlined above, have been compared with the test data obtained on unrestrained specimens only. Such a comparison is shown in reference 3. All the three methods outlined above, for a given free shrinkage potential, estimates the shrinkage warping curvature to be the same as long as the amount of steel and its location remains the same. In order to get some idea as to how these three methods explain the behaviour of restrained shrinkage specimens, two continuous beams were cast on their sides and shrinkage curvatures were measured at every foot of the test specimens. These observed curvatures are then compared with the computed shrinkage warping. The method for computing shrinkage warping for the continuous beam is outlined in Chapter III.
### 1.4 Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$A_s$</td>
<td>area of tensile steel</td>
</tr>
<tr>
<td>$A'_s$</td>
<td>area of compressive steel</td>
</tr>
<tr>
<td>$b$</td>
<td>width of beam at the compression side</td>
</tr>
<tr>
<td>$D$</td>
<td>total depth of beam</td>
</tr>
<tr>
<td>$d$</td>
<td>effective depth of concrete section</td>
</tr>
<tr>
<td>$EI$</td>
<td>flexural rigidity</td>
</tr>
<tr>
<td>$E_c$</td>
<td>modulus of elasticity of concrete, short duration of loading</td>
</tr>
<tr>
<td>$E_s$</td>
<td>modulus of elasticity of steel</td>
</tr>
<tr>
<td>$e$</td>
<td>distance between the centroids of the uncracked transformed section (using $n_{ct}$) and the steel area</td>
</tr>
<tr>
<td>$e_g$</td>
<td>distance between the centroids of the gross concrete section and the steel area</td>
</tr>
<tr>
<td>$f_c$</td>
<td>compressive stress in concrete</td>
</tr>
<tr>
<td>$f'_c$</td>
<td>concrete compressive strength at age 28 days, or other age if specified</td>
</tr>
<tr>
<td>$f_{cb}$</td>
<td>modulus of rupture of concrete</td>
</tr>
<tr>
<td>$H$</td>
<td>relative humidity</td>
</tr>
<tr>
<td>$I_g$</td>
<td>moment of inertia of the gross concrete section (neglecting all steel)</td>
</tr>
<tr>
<td>$L$</td>
<td>span length</td>
</tr>
<tr>
<td>$M_{cr}$</td>
<td>moment corresponding to flexural cracking</td>
</tr>
</tbody>
</table>
p -- tensile steel percentage defined herein as
\( \left( \frac{A_s}{bd} \right) \times 100 \)%

p' -- compressive steel percentage defined herein
as \( \left( \frac{A'_s}{bd} \right) \times 100 \)%

\( T_s \) -- total compressive force induced in steel by
shrinkage

y -- distance from extreme compressive fibre
to neutral axis

\( \Delta \) -- maximum deflection

\( \Delta_{cr} \) -- computed maximum deflection using the
cracked transformed section moment of inertia

\( \varepsilon_s \) -- steel strain

\( \varepsilon_{sh} \) -- free shrinkage strain

\( \varphi_{sh} \) -- curvature due to shrinkage warping
CHAPTER II
TEST ARRANGEMENT
and
RECORDING OF EXPERIMENTAL RESULTS

2.1 Details of Test Specimens and Test Arrangements

As mentioned in Chapter I, the current investigation includes the measurement of changes in the top and bottom strains in a two-span continuous reinforced concrete beam due to shrinkage only. A study is then made of the changes in strains of these two restrained specimens with the corresponding strains of identical unrestrained specimens. Two continuous beams (continuous over one support, each span 9 ft.) and fourteen unrestrained specimens (each 2.5 ft. long) were the principal test specimen. Both restrained and unrestrained test specimens have 4" x 5" cross section. The nomenclature, geometry and reinforcement details of all the above-mentioned specimens studied in the current investigation are indicated in Table 1. No web reinforcement is used, and all main reinforcement is cut 1'-0" beyond the elastic inflection point.
**FIG. 1 — DETAILS AND GAGE POINT Locations OF CONTINUOUS (RESTRAINED) SPECIMENS**

**a)** PLAN VIEW OF RESTRAINED SPECIMEN

- Concrete Support
- Steel Plate Cap
- Specimen
- Steel Rod - 1" Dia.

**b)** SUPPORT DETAILS

- Top Gage: 3/4"
- Bottom Gage: 1"

**c)** GAGE POINT LOCATIONS

- 10"
- 1'-0"
a) **SINGLY REINFORCED UNRESTRAINED SPECIMEN**

b) **DOUBLY REINFORCED UNRESTRAINED SPECIMEN**

C) **LOCATION OF GAGE POINTS (Gages are on both faces)**

**FIG. 2 - DETAILS AND GAGE POINT LOCATIONS FOR UNRESTRAINED SPECIMENS**
Fig. 3  Whittemore Mechanical Strain Gage
Stainless steel plugs, 1/4" in diameter and 3/8" long, with 1/32" diameter holes in the center, were used as gage points for measuring strains in the top and bottom of the test specimens. These plugs were glued to the concrete with epoxy resin. The gage points were located 3/4" from the top face and 1" from the bottom face of the test specimens. The gage points were glued on only one face of the continuous beam, while in the case of the unrestrained specimens, the plugs were glued on both faces. Details regarding gage point locations and support conditions are shown in Figure 1. Similar details for unrestrained specimens are given in Figure 2.

A Whittemore Mechanical Strain Gage (shown in Figure 3) was used to measure the strains. In order to eliminate the effect of dead-load on the specimens, all the test beams were laid on one side on a plain level platform covered with a galvanized tin sheet. The galvanized tin sheet provided a nearly frictionless surface and allowed free movement of the test specimens. In the case of the fourteen unrestrained specimens (2'-6" long) it was possible to change the face of contact with the platform on alternate days. In the case of the restrained specimens, such changing of the contact face was not possible.
<table>
<thead>
<tr>
<th>Serial No.</th>
<th>Nomenclature</th>
<th>Length</th>
<th>Cross Section</th>
<th>Reinforcement</th>
<th>Sketch and Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Unrestrained Specimens SP₁ = Plain Conc. (Original)</td>
<td>2'-6&quot;</td>
<td>4&quot; x 5&quot;</td>
<td>--</td>
<td>No supports, (Unrestrained)</td>
</tr>
<tr>
<td>2</td>
<td>SP₂ = Plain Concrete (Duplicate)</td>
<td>2'-6&quot;</td>
<td>4&quot; x 5&quot;</td>
<td>--</td>
<td>No supports, (Unrestrained)</td>
</tr>
<tr>
<td>3</td>
<td>SA₁O = Original Shrinkage Specimen (A₁ Type)</td>
<td>2'-6&quot;</td>
<td>4&quot; x 5&quot;</td>
<td>1-1/2&quot; ø = 0.1963 in²</td>
<td>No supports, (Unrestrained)</td>
</tr>
<tr>
<td>4</td>
<td>SA₁D = Duplicate Shrinkage Specimen (A₁ Type)</td>
<td>2'-6&quot;</td>
<td>4&quot; x 5&quot;</td>
<td>1-1/2&quot; ø = 0.1963 in²</td>
<td>No supports, (Unrestrained)</td>
</tr>
<tr>
<td>5</td>
<td>SA₂O = Original Shrinkage Specimen (A₂ Type)</td>
<td>2'-6&quot;</td>
<td>4&quot; x 5&quot;</td>
<td>1-3/8&quot; ø = 0.1963 in²</td>
<td>No supports, (Unrestrained)</td>
</tr>
<tr>
<td>6</td>
<td>SA₂D = Duplicate Shrinkage Specimen (A₂ Type)</td>
<td>2'-6&quot;</td>
<td>4&quot; x 5&quot;</td>
<td>1-3/8&quot; ø = 0.1963 in²</td>
<td>No supports, (Unrestrained)</td>
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</tbody>
</table>
| 7 | SB₁₀ = Original Shrinkage Specimen (B₁ Type) | 2'-6" | 4" x 5" | 2-1/2" ø | 0.3926 in² | Nil | No supports, (Unrestrained) | Cross Section 4" x 5"
| 8 | SB₁Ｄ = Duplicate Shrinkage Specimen (B₁ Type) | 2'-6" | 4" x 5" | 2-1/2" ø | 0.3926 in² | Nil | No supports, (Unrestrained) | Cross Section 4" x 5"
| 9 | SB₂₀ = Original Shrinkage Specimen (B₂ Type) | 2'-6" | 4" x 5" | 2-1/2" ø | 1-1/2" ø | 0.1963 in² | No supports, (Unrestrained) | Cross Section 4" x 5"
| 10 | SB₂Ｄ = Duplicate Shrinkage Specimen (B₂ Type) | 2'-6" | 4" x 5" | 2-1/2" ø | 1-1/2" ø | 0.1963 in² | No supports, (Unrestrained) | Cross Section 4" x 5"
| 11 | SC₁₀ = Original Shrinkage Specimen (C₁ Type) | 2'-6" | 4" x 5" | 2-5/8" ø | 0.6136 in² | Nil | No supports, (Unrestrained) | Cross Section 4" x 5"
| 12 | SC₁Ｄ = Duplicate Shrinkage Specimen (C₁ Type) | 2'-6" | 4" x 5" | 2-5/8" ø | 0.6136 in² | Nil | No supports, (Unrestrained) | Cross Section 4" x 5"
| 13 | SC₂₀ = Original Shrinkage Specimen (C₂ Type) | 3'-0" | 4" x 5" | 2-5/8" ø | 1-5/8" ø | 0.3068 in² | No support, (Unrestrained) | Cross Section 4" x 5"
| 14 | SC₂Ｄ = Duplicate Shrinkage Specimen (C₂ Type) | 3'-0" | 4" x 5" | 2-5/8" ø | 1-5/8" ø | 0.3068 in² | No support, (Unrestricted) | Cross Section 4" x 5"

Top Cover: 3/4"; Bottom Cover: 1"
<table>
<thead>
<tr>
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<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
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<tr>
<td>15</td>
<td>C₁S = Continuous Shrinkage Specimen (C₁ Type)</td>
<td>18'-0&quot; (two spans, 9'-0&quot; each)</td>
<td>4&quot; x 5&quot;</td>
<td>2-5/8&quot; ø</td>
<td>Nil</td>
<td>0.6136 in²</td>
<td>Cross Section: 4&quot; x 5&quot;</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>=0.6136 in²</td>
<td></td>
<td>0.3068 in²</td>
<td>Top Cover: 3/4&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Bottom Cover: 1&quot;</td>
</tr>
<tr>
<td>16</td>
<td>C₂S = Continuous Shrinkage Specimen (C₂ Type)</td>
<td>18'-0&quot; (two spans, 9'-0&quot; each)</td>
<td>4&quot; x 5&quot;</td>
<td>2-5/8&quot; ø</td>
<td>1-5/8&quot; ø</td>
<td>0.6136 in²</td>
<td>Cross Section: 4&quot; x 5&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>=0.6136 in²</td>
<td>=0.3068 in²</td>
<td></td>
<td>Top Cover: 3/4&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Bottom Cover: 1&quot;</td>
</tr>
</tbody>
</table>
2.2 Material Details

The concrete mix design per cubic yard of concrete was as follows:

- Cement - Type I: 400 lbs.
- Sand: 1335 lbs.
- Stone: 1900 lbs.
- Water: 20 gal.

The concrete slump was 1". The 28-day cylinder strength of the concrete averaged 6,020 psi and the corresponding modulus of elasticity was $4.5 \times 10^6$ psi. The tensile yield point strength of the reinforcing steel averaged 48,000 psi.

2.3 Recording of Experimental Results

A comprehensive schedule of top and bottom strains of the specimens was maintained for the entire test period ranging from 36 days to 120 days. In the case of the fourteen unrestrained specimens, it was possible to measure strains on both sides so that average readings could be used. In the case of the two-span continuous beams, duplicate specimens were not used. Hence, it was not possible to obtain strain readings on both the faces. However, since both spans of the continuous beam specimen are identical, it was possible to compare strains in the corresponding sections of the continuous beams.
Temperature and humidity data were also obtained. The temperature ranged from 76° F to 88° F throughout the test period, with an average temperature of 82° F. Relative humidity was measured with a dry and wet bulb thermometer, with an average value of 32%. All the test data mentioned above are furnished in Appendix 1.
CHAPTER III

THEORY FOR COMPUTING SHRINKAGE WARPLING IN TWO-SPAN CONTINUOUS BEAMS

3.1 Procedure for Computing Shrinkage Warping in Continuous Beams

It was proposed in Article 1.3, Chapter I, to present a step-by-step procedure for computing shrinkage warping for a two-span continuous reinforced concrete beam. The procedure is outlined below:

(1) Divide the continuous beam in regions in which the section properties are the same and then compute the shrinkage warping for each region using any of the three methods discussed in Article 1.3, Chapter I.

(2) Compute the cracking moment of the section using the relation

\[ M_{cr} = f'_{cb} \cdot \frac{I_g}{y} \]

\( f'_{cb} = 7.5 \sqrt{f'_{c}} \) was used in this study.

Also compute the moment corresponding to the curvature obtained in Step 1, using the relation

\[ \phi_{sh} = \frac{-M}{E_c I_g} \]

where \( \phi_{sh} \) = Curvature computed in Step 1

\( E_c = 57,700 \sqrt{f'_{c}} \)

\( I_g = \frac{1}{12} \times b \times D^3 \)

Determine the regions in which the cracking occurs due to the moment computed by Equation (5).
(3) If the cracking occurs in some of the regions of the beam, treat it as a nonprismatic member. In such a case, it is recommended to use the effective moment of inertia of the cracked section given by either Branson's Method or Yu and Winter's Method.

(4) In the case of the two-span continuous beam (with no cracking in Step 2), remove the center support and make the beam determinate. Subject this determinate beam to curvatures obtained in Step 1. One has to be careful in noting the algebraic sign of the curvature diagram because of the fact that different locations of steel produces curvatures of opposite sign.

(5) Determine the central deflection of the determinate beam obtained in Step 4.

(6) Compute the magnitude and the direction of a single concentrated load $P$ applied at the center of the determinate beam obtained in Step 4 to produce the same deflection (but opposite in direction) as obtained in Step 5.

(7) Determine the curvature all along the length of a determinate beam due to force $P$, obtained in Step 6.

(8) Algebraic sum of curvatures obtained in Step 1 and Step 7 is the theoretical curvature in two-span continuous
(9) Subject determinate beam (obtained by removing the center support) to curvatures obtained in Step 1. Knowing curvatures all along the length of determinate beam compute deflections at various points by either "Moment Area Method", "Conjugate Beam Method", or "Newmark's Numerical Procedure". Also compute the deflections of this determinate beam due to single concentrated load computed in Step 6, and acting at the center of the beam using any standard procedure of structural analysis, such as Moment Area Method or Conjugate Beam Method. Algebraic sum of the deflections obtained from the two cases as outlined above shall be the overall deflection due to shrinkage effect.

The entire procedure as outlined above applies to specimens which are not cracked by transverse loads. The current investigation deals with the test specimens which are not subjected to transverse loads. However, the following discussion is presented to formulate the background for considering the effect of shrinkage as far as the warping of cracked sections (cracked by transverse loading) are concerned.
Consider a portion of the beam in which cracking has occurred due to transverse loads as shown in Fig. 6(a). Reasonably accurate methods are available for computing the width and spacing of flexural cracks which is the function of strength of concrete, strength of steel, bond stress, perimeter of steel bars, etc.

![Fig. 6(a). A Portion of a Cracked Flexural Member](image)

Such a cracked portion of the beam can be idealized as Fig. 6(b). Thus, major effect of flexural cracking will be to replace a portion of length "l" by blocks of smaller lengths such that $l_1 + l_2 + l_3 + l_4 = l$. With smaller lengths of the specimen, the surface-volume ratio is increased resulting in comparatively more shrinkage strains at bottom in the cracked zones.

![Fig. 6(b). Idealized Portion of a Cracked Portion of a Flexural Member](image)
While due to the absence of cracks in the top and with an increase of shrinkage strains at the bottom, it can be expected that the shrinkage curvature will be smaller as compared to an uncracked specimen.

Note: From the above steps, it can be seen that the effect of continuity of test specimen and restraints on shrinkage warping are obtained by the principle of consistent deflections. If it is realized during Step 2 that the section has cracked in some regions, it is desirable to use Newmark's Numerical Procedure along with the effective section properties for computing deflections and curvatures, instead of using the principle of consistent deflections. The numerical procedure is not outlined here.

3.2 Example Calculation for Computing Shrinkage Warping of Continuous Beams.

The Nomenclature, geometry and section details for the two restrained shrinkage specimens are given in Table 1, Chapter II. For the purpose of illustration for computing shrinkage warping, specimen $C_1S$, Serial No. 15 in Table 1 is selected. At this stage, reference is made to the procedure outlined in Section 3.1 for computing shrinkage warping of continuous beam specimens. For the other restrained specimen, $C_2S$, similar computations are given in Appendix II.
Example Calculation:

The following observed data are used in the example calculation and are reproduced below for easy reference.

(a) Observed free shrinkage between 36 days and 120 days, as measured from the plain concrete specimens $SP_1$ and $SP_2$ ($\varepsilon_{sh}$) $= 160 \times 10^{-6}$ in/in

(b) Average modulus of elasticity of steel used as reinforcement ($E_s$) $= 30 \times 10^6$ psi

(c) Average modulus of elasticity of concrete ($E_c$) $= 4.5 \times 10^6$ psi

(A) Equivalent Tensile Force Method

Longitudinal section of beam $C_1S$.

b. Cross section in region $AB$ and $EF$. Length of region $AB$, $EF$ $= 5.75\text{'}$.

c. Cross section in region $BC$ and $DE$. Length of region $BC$, $DE$ $= 2.0\text{'}$.

d. Cross section in region $CD$. Length of region $CD$ $= 2.5\text{'}$.

Fig. 4 - Longitudinal and Cross Section Details of Specimen $C_1S$. 
Geometry and Cross Sectional Details of Beam C₁S.

Refer Section 3.1 of this chapter for steps to compute theoretical shrinkage warping.

Step 1

\( \phi_{sh} \) in region AB, CD and EF (refer Fig. 3).

\[
\phi_{sh} = \frac{Ts \cdot e_{g}}{Ec/2 \cdot I_{g}}
\]

\( Ts = (As + A's) \varepsilon_{sh} \cdot \varepsilon_{s} = (0.6136 + 0) (160 \times 10^{-6}) (30 \times 10^{6}) = 2,942 \text{ lbs.} \)

\( e_{g} = 1.5'' \)

\( I_{g} = \frac{1}{12} \times 4 \times 5^{3} = 41.66 \text{ in}^{4} \).

\[
\phi_{sh} = \frac{(2,942) (1.5)}{4.5 \times 10^{6} \times 41.66} = 47.2 \times 10^{-6} \text{ 1/in.}
\]
Step 2

\[ M_{cr} = f'_{cb} \times I_g / y \]

expression for cracking moment in equation 11 of reference 1.

\[ f'_{cb} = 7.5 \sqrt{f'_c} \]

\[ = 7.5 \sqrt{6020} \]

\[ = 584.0 \text{ psi} \]

\[ M_{cr} = 584.0 \times \frac{41.66}{2.5} \]

\[ = 9710.0 \text{ lb. in.} \]

Theoretical curvature computed in Step 1 is \(47.2 \times 10^{-6}\) 1/in.

\[ \frac{M}{E_c I_g} = 47.2 \times 10^{-6} \text{ 1/in.} \]

\[ M = 47.2 \times 10^{-6} \times E_c I_g \text{ lb. in.} \]

\[ = 47.2 \times 10^{-6} \times 4.5 \times 10^6 \times 41.66 \]

\[ = 8,850 \text{ lb. in.} \leq 9710 \text{ in. lb. (}M_{cr}\text{ of Test Beam)} \]

The conclusion is that the section has not cracked due to shrinkage warping.
Step 3

Since cracking has not occurred, the entire beam will be treated as a prismatic beam.

Step 4

Center support of continuous beam of Fig. 3 is removed and the determinate beam so obtained is subjected to curvature as follows:

\[ R_A = 54.0 \, \phi_{sh} = R_F \]

\[ \Delta \phi = \text{moment in conjugate beam at centre.} \]

\[ = (54 \, \phi_{sh} \times 108) - (69 \, \phi_{sh} \times 73.5) + (15 \, \phi_{sh} \times 7.5) \]

\[ = 867.5 \phi_{sh} \text{ inches} \quad \text{(A)} \]

Step 5

Therefore:

\[ \Delta \phi = 867.5 \times 47.2 \times 10^{-6} \]

\[ = 0.041 \text{ in.} \]
Step 6

The magnitude of a single concentrated load "P" to be applied at the center of determinate beam to produce the same deflection (but opposite in direction) is

\[
\frac{PL^3}{48EI} = \Delta \xi; \quad E_{sus} = \frac{Ec}{2} = \frac{4.5 \times 10^6}{2}
\]

\[
P = \frac{48 E_{sus} I_g}{L^3} \cdot \Delta \xi = 2.25 \times 10^6 \text{ psi}
\]

\[
= \frac{(48) (2.25 \times 10^6) (41.66)}{(216)^3} \times 0.041,
\]

\[
[ I_g = 1/12 \times 4 \times (5)^3 = 41.66 \text{ in}^4 ]
\]

= 18.29 lbs.

Step 7 and Step 8

Compute the moment and curvatures due to Load "P" obtained in Step 6. Find the algebraic sum of curvatures obtained in Step 1 and curvatures obtained in due to load P. Such calculations are self-explanatory and are directly entered in Table 2. Line G in Table 3 indicates the observed curvatures at various sections of the test specimen. One set of illustrative calculations for computing observed value of shrinkage warping curvatures from observed top and bottom gage readings are given below.

Say for example Section 4 on the beam (Table 2) is selected.
Top gage reading at 36 days = 0.00304
Top gage reading at 120 days = 0.00115 (reduced)
Top strain = 189 x 10^-6 in/in.
Bottom gage reading at 36 days = 0.03036
Bottom gage reading at 120 days = 0.02922
Bottom strain = 114 x 10^-6 in/in.
Difference of Top and Bottom strain = (189 - 114) x 10^-6
= 75 x 10^-6 in/in.
Vertical distance between the top and bottom gage points = 3.25 inches

(See Fig. 1A.)

Curvature $\theta_{sh} = \frac{75 \times 10^{-6}}{3.25} = 23.1 \times 10^{-6} \text{ 1/in.}$

This figure 23.1 x 10^-6 1/in. is entered in Line G under Section 4 of Table 3. All other curvatures of Section 1 through 18 are computed in the same manner and are entered in Line G of Table 2.

Step 9

In this step deflections are calculated using computed shrinkage curvatures in Step 8, all along the beam. Newmark's Numerical Procedure has been used. The example calculation is presented in Appendix III showing detailed procedure.

The final deflections are listed in Column H of Tables 2, 3, and 4 computed from the computed curvatures by three methods, respectively.
### TABLE 2
COMPUTED SHRINKAGE WARping OF SPECIMEN C1S COMPUTED BY EQUIVALENT TENSILE FORCE METHOD

<table>
<thead>
<tr>
<th>Section</th>
<th>A</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dist. from Left End in Inches (x)</td>
<td>B</td>
<td>6</td>
<td>18</td>
<td>30</td>
<td>42</td>
<td>54</td>
<td>66</td>
<td>78</td>
<td>90</td>
<td>102</td>
<td>114</td>
<td>126</td>
<td>138</td>
<td>150</td>
<td>162</td>
<td>174</td>
<td>186</td>
<td>198</td>
<td>210</td>
</tr>
<tr>
<td>Moment Due to Load P, Computed in Step 6 M = P/2 x X</td>
<td>C</td>
<td>54.80</td>
<td>164.50</td>
<td>274</td>
<td>384</td>
<td>494</td>
<td>604</td>
<td>712</td>
<td>823</td>
<td>933</td>
<td>933</td>
<td>823</td>
<td>712</td>
<td>604</td>
<td>494</td>
<td>384</td>
<td>274</td>
<td>165</td>
<td>54.80</td>
</tr>
<tr>
<td>Curvature Computed by Equivalent Tensile Force Method from Step 1 (C\text{sh})</td>
<td>E</td>
<td>+47.20</td>
<td>+47.20</td>
<td>+47.20</td>
<td>+47.20</td>
<td>+47.20</td>
<td>+35.30</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>+35.30</td>
<td>+47.20</td>
<td>+47.20</td>
<td>+47.20</td>
<td>+47.20</td>
<td>+47.20</td>
</tr>
<tr>
<td>Computed Shrinkage Curvature (Algebraic Sum of Lines D and E)</td>
<td>F</td>
<td>+46.46</td>
<td>+45.00</td>
<td>+43.55</td>
<td>+42.07</td>
<td>+40.61</td>
<td>+27.25</td>
<td>-9.50</td>
<td>-22.73</td>
<td>-59.65</td>
<td>-59.65</td>
<td>-22.73</td>
<td>-9.50</td>
<td>+27.25</td>
<td>+40.61</td>
<td>+42.07</td>
<td>+43.55</td>
<td>+45.00</td>
<td>+46.46</td>
</tr>
<tr>
<td>Observed Curvature</td>
<td>G</td>
<td>28.90</td>
<td>23.70</td>
<td>22.48</td>
<td>23.10</td>
<td>20.00</td>
<td>--</td>
<td>3.70</td>
<td>9.85</td>
<td>32.00</td>
<td>29.50</td>
<td>9.55</td>
<td>3.08</td>
<td>--</td>
<td>20.90</td>
<td>24.00</td>
<td>21.50</td>
<td>24.60</td>
<td>27.70</td>
</tr>
<tr>
<td>Deflection Computed Using Curvatures in Column F, (inch)</td>
<td>H</td>
<td>0.021</td>
<td>0.036</td>
<td>0.046</td>
<td>0.047</td>
<td>0.044</td>
<td>0.034</td>
<td>0.015</td>
<td>0.009</td>
<td>0.006</td>
<td>0.009</td>
<td>0.015</td>
<td>0.034</td>
<td>0.044</td>
<td>0.047</td>
<td>0.046</td>
<td>0.036</td>
<td>0.021</td>
<td></td>
</tr>
<tr>
<td>Deflection Computed Using Curvatures in Column G, (inch)</td>
<td>J</td>
<td>0.012</td>
<td>0.020</td>
<td>0.024</td>
<td>0.025</td>
<td>0.023</td>
<td>0.018</td>
<td>0.011</td>
<td>0.005</td>
<td>0.004</td>
<td>0.003</td>
<td>0.005</td>
<td>0.011</td>
<td>0.019</td>
<td>0.024</td>
<td>0.024</td>
<td>0.020</td>
<td>0.020</td>
<td>0.011</td>
</tr>
</tbody>
</table>

1. Test specimen C1S of 18.0 ft. length is divided into 18 sections, each of 1.0 ft. length.
2. \( \phi = \frac{M/E_c}{1/12 \times B \times D} \)
3. Curvatures are not observed but are computed from observed top and bottom strains.
(B) Miller's Empirical Method.

The procedure for computing shrinkage warping curvature of continuous beams on the basis of Equivalent Tensile Force Method has been demonstrated. In adopting Miller's approach as a basis for the same procedure, the only difference occurs in Step 1 while rest of the steps are identical. Hence, without presenting great detail, the following self-explanatory calculations are presented.

\[ \varphi_{sh} \text{(by Miller's Method)} = \frac{\varepsilon_{sh}}{d} \left(1 - \frac{\varepsilon_{s}}{\varepsilon_{sh}}\right) \]

\[ = \frac{160 \times 10^{-6}}{4.0} (1 - 0.1) \]

\[ = 36.0 \times 10^{-6} \text{ in./in.} \]

\[ = 867.5 \varphi_{sh} \text{ (refer Equation 5)} \]

\[ = 867.5 \times 36 \times 10^{-6} \text{ in.} \]

\[ = 0.0312 \text{ in.} \]

\[ P = 13.93 \text{ lbs.} \]

Table 3, similar to Table 2 but using Miller's approach as the basis for computing \( \varphi_{sh} \) in Step 1 is prepared.
**TABLE 3**

**SHRINKAGE WARPING OF SPECIMEN C15 COMPUTED BY MILLER'S EMPIRICAL METHOD**

| Section | A | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| Dist. from Left End in Inches (x) | B | 6 | 18 | 30 | 42 | 54 | 66 | 78 | 90 | 102 | 114 | 126 | 138 | 150 | 162 | 174 | 186 | 198 | 210 |
| Moment Due to Load P, Computed in Step 6 \( M = P/2 \times X \) | C | 41.70 | 125 | 208 | 292 | 362 | 460 | 543 | 626 | 710 | 710 | 626 | 543 | 460 | 362 | 292 | 208 | 125 | 41.70 |
| Curvature Computed by Empirical Method Computed in Step 1 \( E = \frac{3}{4} \times 36.0 \) | E | +36.00 | +36.00 | +36.00 | +36.00 | +36.00 | +27.00 | -9.00 | -36.00 | -36.00 | -9.00 | -- | +27.00 | +36.00 | +36.00 | +36.00 | +36.00 | +36.00 | +36.00 |
| Computed Warping Curvature (Algebraic Sum of Line D and E) | F | +35.45 | +34.34 | +33.23 | +32.11 | +31.17 | +20.88 | -7.24 | -17.36 | -45.47 | -45.47 | -17.36 | -7.24 | +20.88 | +31.17 | +32.11 | +33.23 | +34.34 | +35.45 |
| Observed Curvature 3 | G | 28.90 | 23.70 | 22.48 | 23.10 | 20.00 | -- | 3.70 | 9.85 | 32.00 | 29.50 | 9.55 | 3.08 | -- | 26.90 | 24.00 | 21.50 | 24.60 | 27.70 |
| Deflection Computed Using Curvatures in Column F, \( (\text{inch}) \) | H | 0.016 | 0.026 | 0.032 | 0.033 | 0.031 | 0.024 | 0.015 | 0.006 | 0.005 | 0.005 | 0.006 | 0.015 | 3.024 | 0.031 | 0.033 | 0.032 | 0.026 | 0.016 |
| Deflection Computed Using Curvatures in Column G, \( (\text{inch}) \) | J | 0.012 | 0.020 | 0.024 | 0.025 | 0.023 | 0.018 | 0.011 | 0.005 | 0.004 | 0.003 | 0.005 | 0.011 | 0.019 | 0.024 | 0.024 | 0.020 | 0.020 | 0.011 |

1. Test Specimen C1S of 18.0 ft. length is divided into 18 sections, each of 1.0 ft. length.
2. \( E_c^2 = \frac{M}{E_c \cdot I_g} ; \quad E_c = 57,700 \quad f'c ; \quad I_g = \frac{1}{12} \times B \times D^3 \)
3. Curvatures are not observed, but are computed from observed top and bottom strains.
(C) Branson's Empirical Method

Similar calculations as given above can be carried out for shrinkage warping of continuous beams but using Branson's Empirical Method for computing shrinkage warping in various regions of the continuous beam. (Refer Step 1 of the procedure outlined in Section 3.1, Chapter III).

\[ \Psi_{sh} = (0.7) \left( \frac{\varepsilon_{sh}}{D} \right) (p-p')^{1/3} \left( \frac{p-p'}{p} \right)^{1/2} \]

(by Branson's Empirical equation No. 30 reference 3)

\[ = (0.7) \left( \frac{160 \times 10^{-6}}{5} \right) (3.0) \]

\[ = 32.2 \times 10^{-6} \text{ 1/in.} \]

\[ \Delta_t = 867.5 \Psi_{sh} \quad \text{(refer Equation 5)} \]

\[ = 867.5 \times 32.2 \times 10^{-6} \]

\[ = 0.02792 \text{ inch.} \]

\[ P = \frac{48 \times E_{sus} I_g}{L^3} \]

\[ = \frac{(48) \times (2.25 \times 10^6) \times (41.66)}{(216)^3} \times 0.02792 \]

\[ = 12.46 \text{ lbs.} \]

Table 4 similar to Table 2 but using Branson's approach as basis for computing \( \Psi_{sh} \) in Step 1 is prepared.
<table>
<thead>
<tr>
<th>Section</th>
<th>A</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dist. from Left End in Inches (x)</td>
<td>B</td>
<td>6</td>
<td>18</td>
<td>30</td>
<td>42</td>
<td>54</td>
<td>66</td>
<td>78</td>
<td>90</td>
<td>102</td>
<td>114</td>
<td>126</td>
<td>138</td>
<td>150</td>
<td>162</td>
<td>174</td>
<td>186</td>
<td>198</td>
<td>210</td>
</tr>
<tr>
<td>Moment Due to Load P, Computed in Step 6 M = P/2 x X</td>
<td>C</td>
<td>37.40</td>
<td>112.20</td>
<td>187</td>
<td>262</td>
<td>337</td>
<td>412</td>
<td>486</td>
<td>562</td>
<td>636</td>
<td>636</td>
<td>562</td>
<td>486</td>
<td>412</td>
<td>337</td>
<td>262</td>
<td>187</td>
<td>112.20</td>
<td>37.40</td>
</tr>
<tr>
<td>Curvature Due to Moment Obtained in Line C (M/EI) D</td>
<td>-0.51</td>
<td>-1.50</td>
<td>-2.492</td>
<td>-3.492</td>
<td>-4.49</td>
<td>-5.50</td>
<td>-6.49</td>
<td>-7.49</td>
<td>-8.49</td>
<td>-8.49</td>
<td>-7.49</td>
<td>-6.49</td>
<td>-5.50</td>
<td>-4.49</td>
<td>-3.492</td>
<td>-2.492</td>
<td>-1.50</td>
<td>-0.51</td>
<td></td>
</tr>
<tr>
<td>Curvature Computed by Branson’s Empirical Method E from Step 1</td>
<td>+32.00</td>
<td>+32.00</td>
<td>+32.00</td>
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<td>0.030</td>
<td>0.032</td>
<td>0.029</td>
<td>0.022</td>
<td>0.013</td>
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1. Test Specimen C1S of 18.0 ft. length is divided into 18 sections, each of 1.0 ft. length.
2. $Q = \frac{M}{E_I} \cdot \frac{1}{I_g}; \ E_I = 57,700 \ f^'c, \ I_g = \frac{1}{12} \times B \times D^3$
3. Curvatures are not observed, but are computed from top and bottom strains.
CHAPTER IV
DISCUSSION OF TEST RESULTS

4.1 General

The entire discussion that follows in this chapter pertains to the shrinkage specimens which are either restrained or unrestrained, but whose section is not cracked by external transverse loads or eccentric compressive loads resulting from shrinkage. The test results for free shrinkage and the top and bottom strains of unrestrained and restrained specimens are presented in Figures A-1 through A-11 in Appendix 1. In this chapter, it is intended to discuss these test results in the following sequence and indicate briefly the possible conclusions that can be drawn from such a discussion.

1. Free shrinkage.
2. Shrinkage warping of unrestrained specimens.
3. Shrinkage warping of restrained specimens.

4.2 Free Shrinkage

Free shrinkage values during the test period of 34 days to 120 days averaged $160 \times 10^6$ in/in. This value is a little higher than the value obtained by Miller during the same interval ($120 \times 10^{-6}$ in/in). This is possibly due to lower value of relative humidity and higher ambient temperature encountered during the current investigation which accelerates the drying of concrete.
4.3 Shrinkage Warping of Unrestrained Specimens

A. Equivalent Tensile Force Method

The Equivalent Tensile Force Method is based on the principles of mechanics. In spite of that, the shrinkage warping computed by this method indicates wide variation when compared with test data. This can be seen in Column K of Table 5 and Fig. 6, which shows the ratio of observed curvature to computed curvature by the Equivalent Tensile Force Method.

In the case of singly reinforced specimens, the agreement of the test data with the computed shrinkage warping is closer for specimens having 1.23% steel as compared to the specimens with 2.45% and 3.83% steel. This can be seen from Column K of Table 5 and Fig. 6, which indicates that for specimens having 1.23% steel, the average agreement ratio is 0.84, while similar ratios for specimens having 2.45% and 3.83% steel are 0.48 and 0.45, respectively.

Considering other test data available in the literature, for example, the test data for shrinkage specimens of the Reference 3 have 0.69% and 2.07% steel, and corresponding ratios of 0.66 and 0.49, respectively. Furthermore, the
## TABLE 5
COMPUTED SHRINKAGE WARPING USING TENSILE FORCE METHOD COMPARED WITH TEST DATA

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Fig. 6 - Comparison of measured and computed shrinkage warping using Equivalent Tensile Force Method (for singly and doubly reinforced unrestrained specimens).
test data of reference 2 have test specimens with 3.0%, 2.08%, 1.6%, 1.29%, 1.5%, 1.4%, 0.80%, and 0.65% steel, and the corresponding agreement ratios are 0.91, 0.91, 0.80, 0.99, 0.93, 0.99, 1.02, 0.97, and 0.97, respectively. From this discussion, it may be concluded that the Equivalent Tensile Force Method, in general, overestimates shrinkage warping, and provides closer agreement with the test data for specimens containing tensile steel in the range of 0.6% to 1.60% steel compared to specimens containing more than 2.0% steel. The spread of the results of the current investigation when compared with computed $\phi_{sh}$ on the basis of the Equivalent Tensile Force Method is shown in Figure 6.

As regards doubly reinforced specimens of the current investigation having 50% compressive steel, the observed curvatures were, in general, less than the corresponding specimens having tensile steel only. The reduction in observed shrinkage warping due to compressive steel varied from 30% to 50% for test specimens of this investigation.
B. Miller's Empirical Method

Miller's Method is applicable to specimens having tensile steel only. Computed shrinkage warping of the unrestrained specimens of the current investigation and observed shrinkage warping are shown in Table 6. The average ratios of observed $\phi_{sh}$ are 0.45, 0.45, and 0.57 for specimens containing 1.23%, 2.45%, and 3.83% steel, respectively. This agreement is rather poor. However, it can be observed from Column K of Table 6 and Fig. 7 that Miller's and Washa's data are in close agreement with computed curvatures by Miller's Method. This may be due to the high strength concrete involved in the current investigation. It is concluded that Miller's Method should be used with caution for high strength concretes. The spread of the results of the current investigation when compared with computed $\phi_{sh}$ on the basis of Miller's Method is shown in Fig. 7.
Fig. 7 - Comparison of observed and computed shrinkage warping using Miller’s Empirical Approach (singly reinforced unrestrained specimens).
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C. Branson's Empirical Method

Column K in Table 7 indicates the agreement of observed curvature with computed curvature. This method has better agreement with test results in all cases than Miller's Method. However, study of Column K in Tables 5 and 7 indicates that for lower percentage of steel such as 1.23%, tensile force method has a better agreement with test results than this method, and for higher percentages of steel such as 2.45% and 3.83% steel, this method has a better agreement with test results than the Equivalent Tensile Force Method. The spread of results of the current investigation when compared with computed $\Phi_{sh}$ on the basis of Branson's Method is shown in Fig. 8.

4.4 Shrinkage Warping of Restrained Specimens

The first step of the procedure for computing shrinkage warping of two-span continuous beams is outlined in Section 3.1 and includes the use of any of the three methods being discussed for computing shrinkage warping.

Shrinkage warping using the three methods is computed and entered in Columns B, C, and D in Table 8, for specimen $C_1S$, and in Columns B and C of Table 9 for specimen $C_2S$. Study of
### TABLE 7
COMPUTED SHRINKAGE WARPING USING BRANSON'S EMPirical METHOD COMPARED WITH TEST DATA

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Fig. 8 - Comparison of observed and computed shrinkage warping using Branson's Empirical Method (singly and doubly reinforced unrestrained specimens)
Columns F, G, and H of Table 8, and Column F of Table 9, indicate that all three methods overestimate the shrinkage warping. Branson's Method provides comparatively closer agreement with test results than either Tensile Force Method or Miller's Method. This can be seen by observing that average agreement ratios are 0.74, 0.64, and 0.50, respectively, by Branson's Method, Miller's Method, and Tensile Force Method in the case of specimen C<sub>1</sub>S. Average agreement ratio in the case of specimen C<sub>2</sub>S is 0.69 by both Branson's Method and Tensile Force Method.

The effect of continuity on shrinkage test specimens can be studied by noting the observed curvatures of specimens SC<sub>1</sub>O, SC<sub>1</sub>D, and C<sub>1</sub>S, because of the fact that SC<sub>1</sub>O and SC<sub>1</sub>D are identical in section and reinforcement details to specimen C<sub>1</sub>S, except that specimens SC<sub>1</sub>O and SC<sub>1</sub>D are unrestrained and represent 2.5 ft. length from maximum positive and maximum negative moment regions of continuous beam C<sub>1</sub>S. The average observed curvature of unrestrained specimens SC<sub>1</sub>O and SC<sub>1</sub>D is 20.63 x 10<sup>-6</sup> 1/in.
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Average Ratios: 0.50, 0.640, 0.740
### TABLE 9

**COMPUTED SHRINKAGE WARping OF RESTRAINED SPECIMEN C2S USING TENSILE FORCE-METHOD\(^1\) AND BRANSON'S\(^3\) METHOD COMPARED WITH TEST DATA**

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<th>Branson's Empirical Method</th>
<th>Observed Curvature</th>
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**Average** 0.69
Fig. 9 - Comparison of observed and computed shrinkage warping computed using three methods in positive moment region of restrained specimen C1S (average of 1 ft. interval)
Fig. 10 - Comparison of observed & computed shrinkage warping computed using three methods in negative moment region of restrained specimen C1S (average of 1 ft. interval values along the beam.)

Diagonal lines refer to percentage-spread for each type of data point.
In the case of the continuous beam $C_1S$, the observed curvature in the positive moment region varied uniformly from $20.0 \times 10^{-6}$ 1/in. to $28.9 \times 10^{-6}$ 1/in., giving an average value of $23.63 \times 10^{-6}$ 1/in. This indicates that the average observed curvature in the positive moment region of the continuous beam is 15% higher than the observed curvature of the corresponding unrestrained specimens. For the negative moment region, the average observed curvature of the restrained specimen was $30.75 \times 10^{-6}$ 1/in. This further indicates that the average curvature in the negative moment region is 53.85% higher than the average observed curvature of the corresponding unrestrained specimens. In the case of doubly reinforced specimen $C_2S$, the behaviour is similar to $C_1S$.

The average observed curvatures in positive and negative moment regions were respectively 11.75% and 46% higher than the corresponding average curvatures of the unrestrained specimens $SC_2O$ and $SC_2D$. 
Fig. 11 - Comparison of observed and computed shrinkage warping computed using two methods in the positive moment region of restrained specimen C2S.
Diagonal lines refer to percentage-spread for each type of data point.

○ Tensile Force Method and Branson's Empirical Method

Fig. 12 - Comparison of observed and computed shrinkage warping computed using two methods in negative moment region of restrained specimen C2S.
CHAPTER V
CONCLUSION

5.1 General

The present study indicates the relative merits of the three methods, viz., Equivalent Tensile Force Method, Miller's Method, and Branson's Method for computing shrinkage warping of unrestrained and restrained specimens. In general, there are a number of variables such as change in temperature, humidity, type of concrete and the size of specimens that affect the shrinkage warping. However, all the test specimens of the current investigation were made out of the same concrete mix and were kept in the same environmental conditions. Hence, the following conclusions refer mainly to the variations in percentage of steel and its effect on shrinkage warping.

5.2 Free Shrinkage

Low relative humidity and high average temperature accelerates drying of concrete and thereby increases the free shrinkage. In the current investigation, the observed free shrinkage was relatively higher than free shrinkage observed during the same interval by Miller and Branson, because of the friction and the higher quality concrete in the current investigation.
5.3 Shrinkage Warping of Unrestrained Specimens

The Equivalent Tensile Force Method for singly reinforced, unrestrained specimens overestimated the shrinkage warping. Observed values of $\phi_{sh}$ were about 80% of the computed values of $\phi_{sh}$ for specimens containing 1.23% steel, while in the case of specimens having 2.45% and 3.83% steel, the agreement between observed $\phi_{sh}$ and computed $\phi_{sh}$ is relatively poorer. (Ratio of observed and computed $\phi_{sh}$ is about 0.50; refer Fig. 6 and Table 5 to note the above fact.) The conclusion is that for unrestrained singly reinforced specimens, the Equivalent Tensile Force Method can be used without too much error for steel percentages only up to about 1.60%. In the case of doubly reinforced sections with $A'_s/A_s$ ratio of about 0.5, the observed curvature was in general less than the observed curvature of the corresponding singly reinforced specimens. The closer agreement between observed $\phi_{sh}$ and computed $\phi_{sh}$ by the Equivalent Tensile Force Method up to 1.60% steel and poorer agreement for higher percentages of steel (such as 2.45% and 3.83%) is true for doubly reinforced specimens also.

The agreement of observed $\phi_{sh}$ with the computed $\phi_{sh}$ by Miller's Empirical Method was poor for all the percentages of tensile steel involved in current investigation. (Refer to Fig. 7 and Table 6).
It seems that Miller's Method should be used with caution for high strength concrete such as used in current investigation.

In the case of both singly and doubly reinforced specimens, the agreement of the observed and computed $\Theta_{sh}$ using Branson's Method is found better in the case of the specimens having higher percentage of steel (such as 3.83%. Refer to Fig. 8, Table 7), rather than specimens having 1.23% and 2.45% steel.

5.4 Shrinkage Warping of Restrained Specimens

The procedure for computing shrinkage warping for continuous beams was outlined in Section 3.2. In Step 1, any one of the three methods being investigated can be used to compute shrinkage warping in various regions of the continuous beam (regions having the same section properties).

There are nine sections (each one foot in length) in one span of the continuous beam. The shrinkage warping was observed at all sections. The average ratio of observed $\Theta_{sh}$ to computed $\Theta_{sh}$ by the above-mentioned three methods are 0.50, 0.64, and 0.74, respectively. (Refer to Fig. 9, Fig. 10 and Table 9). The conclusion is that Branson's Method, along with the procedure of Section 3.2, estimates $\Theta_{sh}$ of two-span continuous beam closer than the other two methods. So far as the effect of continuity of the test specimens are concerned, the test results of the current investigation have indicated that the average observed curvature
in the positive moment region is 15% higher than the average observed curvature of the corresponding unrestrained specimens. For the negative moment region, the average observed curvature is about 53% higher than the observed curvature of the unrestrained specimens.

Shrinkage deflections were computed using the curvatures computed by the procedure described in Chapter III. Newmark's Numerical Procedure was used for computation of deflection because the section of the beam in maximum positive or maximum negative regions was not cracked. Review of results in Table 10B shows the magnitude was small during the period of experimentation (i.e. 120 days). This is not true for longer life of the structure. The deformations due to shrinkage are considerably higher and need to be taken into account for the design, especially for the design of canopies where they are constantly subjected to extreme atmospheric changes.

Computed deflections have shown a better agreement using curvatures computed with Branson's Method (Refer Table 4, Chapter III) compared to that by Tensile Force Method or Miller's Method. The average agreement ratio of deflections computed from observed curvatures and computed curvatures is 0.80.

The average observed curvature in the position moment region of doubly reinforced continuous beam \((p = 3.83\%; p' = 1.91\%)\) is about 11% higher than the average observed curvature
of corresponding unrestrained specimens. In the negative moment region, the average observed curvature is 46% higher than the average observed curvature of corresponding unrestrained specimens.

5.5 Concluding Remarks

In the current investigation, the poorer agreement between measured curvature and computed curvature has been reflected mainly due to the abnormal environmental conditions and higher strength of concrete used. It is probable in these respects that the specimens which were used to evaluate the quantity of free shrinkage ($\varepsilon_{sh}$) a major parameter in computation of curvature (Refer eqn. (1), (2), (3) and (4) - have not shown the uniform behaviour.

The higher value of $\varepsilon_{sh}$ will obviously affect the computed value of $\theta_{sh}$, leading to lower agreement. The author believes that with the controlled environments and friction-free surface between specimens and surface on which it is resting, may give better correlation for predicting shrinkage warping with the use of existing methods.

Moreover, it is considered improper to comment on existing methods for evaluation of shrinkage warping because of the smaller number of test specimens used in current investigation.
APPENDIX - I

TEST - DATA
Fig. A-1 - Concrete shrinkage versus time for specimens SP₁ - SP₂ (No Steel)
Fig. A-2 - Concrete shrinkage versus time for specimens SA₁₀ - SA₁₄
Fig. A-3 - Concrete shrinkage versus time for specimens SA_20 - SA_2D

- Top Gage
- Bottom Gage

- $p = 1.23\%$
- $p' = 0.62\%$
Fig. A-4 - Concrete shrinkage versus time for specimens SB_{10} - SB_{1D}
Fig. A-5 - Concrete shrinkage versus time for specimens SB20 - SB2D
Fig. A-6 - Concrete shrinkage versus time for specimens $SC_1^0 - SC_1^D$. 
Fig. A-7 - Concrete shrinkage versus time for specimens SC20 - SC2D
Fig. A-8 - Concrete shrinkage versus time in specimen C1S - Positive moment region.
Fig. A-9 - Concrete shrinkage versus time in specimen C₁S - Negative moment region.
Fig. A-10 - Concrete shrinkage versus time in specimen C2S - Positive moment region.
Fig. A-11 - Concrete shrinkage versus time in specimen \( C_2S \) - Negative moment region.

- Top Gage
- Bottom Gage

\( p = 3.83\% \)
\( p' = 1.92\% \)
Fig. A-12 - Relative humidity and temperature versus time curve.
APPENDIX II

COMPUTATIONS FOR SHRINKAGE WARping
OF RESTRAINED AND UNRESTRAINED SPECIMENS

I. Unrestrained Specimens

(A) Computed Shrinkage Warping:

(1) Equivalent Tensile Force Method.

(2) Miller's Empirical Method.

(3) Branson's Empirical Method.

(1) \( \varphi_{sh} \) Using Equivalent Tensile Force Method:

\[
\varphi_{sh} = \frac{Ts \cdot \varepsilon_g}{Ec/2 \cdot Ig}
\]

Where:

\[
Ts = (A_s+A's)\varepsilon \quad sh. \quad Es
\]

\[
Ig = \frac{1}{12} b x d^3
\]

\[
\begin{align*}
\varphi_{sh} &= \frac{(0.1963)(160 \times 10^{-6})(30 \times 10^6)(1.5)}{(4.5/2 \times 10^6)(41.66)} \\
&= 15.1 \times 10^{-6} \text{ 1/in}
\end{align*}
\]

\[
\begin{align*}
\varphi_{sh} &= \frac{(0.1963 + 0.11)(160 \times 10^{-6})(30 \times 10^6)(0.33)}{(4.5/2 \times 10^6)(41.66)} \\
&= 5.18 \times 10^{-6} \text{ 1/in}
\end{align*}
\]

\[
\begin{align*}
\varphi_{sh} &= \frac{(0.3926)(160 \times 10^{-6})(30 \times 10^6)(1.5)}{(4.5/2 \times 10^6)(41.66)} \\
&= 30.2 \times 10^{-6} \text{ 1/in}
\end{align*}
\]
\[
\Phi_{sh} = \frac{(0.3926 + 0.1963)(160 \times 10^{-6})(30 \times 10^6)(0.42)}{(4.5/2 \times 10^6)(41.66)} \\
= 12.7 \times 10^{-6} \text{ 1/in}
\]

**SC₁O - SC₁D**

\[
\Phi_{sh} = \frac{(0.6136)(160 \times 10^{-6})(30 \times 10^6)(1.5)}{(4.5/2 \times 10^6)(41.66)} \\
= 47.2 \times 10^{-6} \text{ 1/in}
\]

**SC₂O - SC₂D**

\[
\Phi_{sh} = \frac{(0.6136 = 0.3068)(160 \times 10^{-6})(30 \times 10^6)(0.42)}{(4.5/2 \times 10^6)(41.66)} \\
= 19.8 \times 10^{-6} \text{ 1/in}
\]

(2) Miller's Empirical Method: (for singly reinforced beams only)

\[
\Phi_{sh} = \frac{\epsilon_{sh}}{d} \left(1 - \frac{\epsilon_s}{\epsilon_{sh}}\right)
\]

Value of:

\[\frac{\epsilon_s}{\epsilon_{sh}} = 0.3 \text{ selected for specimens SA₁O - SA₁D}\]

\[\frac{\epsilon_s}{\epsilon_{sh}} = 0.3 \text{ selected for specimens SB₁O - SB₁D}\]

\[\frac{\epsilon_s}{\epsilon_{sh}} = 0.1 \text{ selected for specimens SC₁O - SC₁D}\]

**SA₁O - SA₁D**

\[
\Phi_{sh} = \frac{(160 \times 10^{-6})}{4} \left(1 - 0.3\right) \\
= 28.0 \times 10^{-6} \text{ 1/in}
\]

**SB₁O - SB₁D**

\[
\Phi_{sh} = \frac{(160 \times 10^{-6})}{4} \left(1 - 0.2\right) \\
= 32.0 \times 10^{-6} \text{ 1/in}
\]
\( \Phi_{sh} = \frac{(160 \times 10^{-6})}{4} (1 - 0.1) \)

\[ = 36.0 \times 10^{-6} \text{ 1/in} \]

(3) Branson's Empirical Method:

\[ \Phi_{sh} = (0.7) \left( \frac{\varepsilon_{sh}}{D} \right) (p-p')^{1/3} \left( \frac{p-p'}{p} \right)^{1/2} \]

for \( (p-p') \leq 3.0\% \)

and \( \Phi_{sh} = (1.0) \left( \frac{\varepsilon_{sh}}{D} \right) \) for \( (p-p') \geq 3.0\% \)

\( \Phi_{sh} \) for \( \text{SA}_1 \text{O} - \text{SA}_1 \text{D} \)

\[ \Phi_{sh} = (0.7) \left( \frac{160 \times 10^{-6}}{5} \right) (1.23)^{1/3} \]

\[ = 24.0 \times 10^{-6} \text{ 1/in} \]

\( \Phi_{sh} \) for \( \text{SA}_2 \text{O} - \text{SA}_2 \text{D} \)

\[ \Phi_{sh} = (0.7) \left( \frac{160 \times 10^{-6}}{5} \right) (1.23 - 0.685)^{1/3} \left( \frac{1.23 - 0.685}{1.23} \right)^{1/2} \]

\[ = 5.60 \times 10^{-6} \text{ 1/in} \]

\( \Phi_{sh} \) for \( \text{SB}_1 \text{O} - \text{SB}_1 \text{D} \)

\[ \Phi_{sh} = (0.7) \left( \frac{160 \times 10^{-6}}{5} \right) (2.45)^{1/3} \]

\[ = 29.8 \times 10^{-6} \text{ 1/in} \]

\( \Phi_{sh} \) for \( \text{SB}_2 \text{O} - \text{SB}_2 \text{D} \)

\[ \Phi_{sh} = (0.7) \left( \frac{160 \times 10^{-6}}{5} \right) (2.45 - 1.23)^{1/3} \left( \frac{2.45 - 1.23}{2.45} \right)^{1/2} \]

\[ = 16.95 \times 10^{-6} \text{ 1/in} \]
SC1O - SC1D

\[ \delta_{sh} = \frac{160 \times 10^{-6}}{5.0} \]  

in this case (p-p') 3.0%

\[ = 32.0 \times 10^{-6} \text{ 1/in} \]

SC2O - SC2D

\[ \delta_{sh} = (0.7) \left( \frac{160 \times 10^{-6}}{5.0} \right) \left( 3.83 - 1.91 \right)^{1/3} \left( \frac{3.83 - 1.91}{3.83} \right)^{1/2} \]

\[ = 19.8 \times 10^{-6} \text{ 1/in} \]

(B) Observed Shrinkage Warping:

For the purpose of illustration for computing shrinkage warping of specimen SB1O, Serial No. 7, in Table 1 is selected. For all other unrestrained specimens similar computations are made and curvatures are listed in Column J, Tables 5, 6 and 7 under the heading "Observed Curvature".

Example Calculation:

A. Observed Data:

(1) Free shrinkage between 34 days and 120 days

(Refer Fig. A-1) \(-\) \(160 \times 10^{-6} \text{  in/in} \)

(2) Top and bottom gage readings:

<table>
<thead>
<tr>
<th>Location</th>
<th>Age</th>
<th>Temp.</th>
<th>Std. Bar Reading</th>
<th>Wittemore Left</th>
<th>Gage Readings Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>34</td>
<td>80°F</td>
<td>0.03431</td>
<td>0.03210</td>
<td>0.03420</td>
</tr>
<tr>
<td>Top</td>
<td>120</td>
<td>86°F</td>
<td>0.03469</td>
<td>0.03082</td>
<td>0.03283</td>
</tr>
<tr>
<td>Bottom</td>
<td>34</td>
<td>80°F</td>
<td>0.03431</td>
<td>0.03450</td>
<td>0.02630</td>
</tr>
<tr>
<td>Bottom</td>
<td>120</td>
<td>86°F</td>
<td>0.03469</td>
<td>0.03381</td>
<td>0.02540</td>
</tr>
</tbody>
</table>
Left Face:

Top gage reading at 34 days 0.03210
Top gage reading at 120 days -0.03053
Top strain on Left Face 0.00157 in/10 in
= $157 \times 10^{-6}$ in/in

Bottom gage reading at 34 days -0.03450
Bottom gage reading at 120 days 0.04464
Bottom strain on Left Face 0.00097 in/10 in
= $97 \times 10^{-6}$ in/in

Right Face:

Top gage reading at 34 days 0.03420
Top gage reading at 120 days 0.03250
Top strain on Right Face 0.00170 in/10 in
= $170 \times 10^{-6}$ in/in

Bottom gage reading at 34 days 0.02630
Bottom gage reading at 120 days 0.02500
Bottom strain on Right Face 0.00130 in/10 in
= $130 \times 10^{-6}$ in/in

Average top strain on Left and Right Faces
= $\frac{157 + 170}{2}$
= $160 \times 10^{-6}$ in/in

Average bottom strain on Left and Right Faces
= $\frac{130 + 97}{2}$
= $113 \times 10^{-6}$ in/in
Difference between top and bottom strains = \((163 - 113) \times 10^{-6}\) in/in
= \(50 \times 10^{-6}\) in/in

Vertical distance between the top and bottom gages --- 3.25 in

Curvature due to shrinkage \(\left(\phi_{sh}\right) = \frac{50 \times 10^{-6}}{3.25}\)
= \(15.41 \times 10^{-6}\) 1/in

II. Restrained Specimens

(A) Computed Shrinkage Warping:

(1) Equivalent Tensile Force Method.
(2) Miller's Empirical Method.
(3) Branson's Empirical Method.

(1) \(\phi_{sh}\) Using Equivalent Tensile Force Method: (Refer Procedure Outline in Chapter III, Article 3-1)

a) Restrained Specimen \(C_{1S}\)

The detailed computations for computing theoretical warping of specimen \(C_{1S}\) has already been presented in Article 3-2, Chapter III.

b) Restrained Specimen \(C_{2S}\)

\[\phi_{sh} = \frac{Ts \cdot Eg}{Ec/2: Ig}\]

Where:

\[Ts = (As + A's) \phi_{sh} \cdot Es\]
\[Ts = (0.6136 + 0.3068) (160 \times 10^{-6}) (30 \times 10^6)\]
\[= 4,420.0 \text{ lbs.}\]

\[\phi_{sh} = \frac{4420 \times 0.42}{4.572 \times 10^6 \times 41.66}\]
\[= 19.8 \times 10^{-6}\] 1/in
Therefore, section of specimen is not cracked. Hence it will be treated as prismatic beam.

\[ M_{cr} = \frac{f' cb \cdot I_g}{Y_t} \]

\[ = \frac{583.4 \times 41.66}{2.5} \]

\[ = 9,700.0 \text{ lb. in.} \]

\[ = 9.70 \text{ k-in.} \]

Now \[ \phi_{sh} = \frac{M}{E_c \cdot I_g} \]

\[ M = \phi_{sh} \times E_c \times I_g \]

\[ = (19.8 \times 10^{-6}) (4.5 \times 10^6) (41.66) \]

\[ = 3,710 \text{ lb. in.} \]

\[ = 3.71 \text{ k in.} < 9.70 = M_{cr} \]

Therefore, section of specimen is not cracked. Hence it will be treated as prismatic beam.

Conjugate structure loaded with curvature diagram.

\[ R_A = R_F = 54.0 \phi_{sh} \]

Central deflection of determinate structure

\[ \Delta_{CL} = 867.5 \phi_{sh} \]

\[ = 867.5 \times 19.8 \times 10^{-6} \]

\[ = 0.017180 \text{ inch} \]
Redundant Force $P$ necessary to bring central deflection to zero to restore boundary conditions:

$$P = \frac{48 \, E_{sus} \, I_g}{L^3} \times \Delta \, \epsilon \, L \quad E_{sus} = \frac{E_c}{2}$$

$$= \frac{(48)\,(2.25 \times 10^6)\,(41.66)}{(216)^3} \times 0.01718 \quad = 4.5 \times 10^6$$

$$= 2.25 \times 10^6 \text{ psi}$$

$$= 7.667 \text{ lbs.}$$

Table 10 A is prepared to compute theoretical shrinkage warping of continuous beam $C_2S$. 
### TABLE 10A

Computations of Theoretical Shrinkage Warping for Restrained Specimen C\textsubscript{2}S Using Tensile Force Method

<table>
<thead>
<tr>
<th>A</th>
<th>Section</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>17</td>
<td>16</td>
<td>15</td>
<td>14</td>
<td>13</td>
<td>12</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>Moment due to force P = p/2 x X</td>
<td>23.0</td>
<td>69.0</td>
<td>115</td>
<td>161</td>
<td>207</td>
<td>.253</td>
<td>295</td>
<td>345</td>
<td>391</td>
</tr>
<tr>
<td></td>
<td>Curvature due to moment in Line B</td>
<td>0.0307</td>
<td>0.92</td>
<td>1.535</td>
<td>2.15</td>
<td>2.762</td>
<td>3.38</td>
<td>3.93</td>
<td>4.60</td>
<td>5.22</td>
</tr>
<tr>
<td></td>
<td>( \phi = \frac{M}{E\text{sus} \cdot I_g} )</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>C</td>
<td>Curvature computed by Tensile Force Method in Step 1</td>
<td>19.8</td>
<td>19.8</td>
<td>19.8</td>
<td>19.8</td>
<td>19.8</td>
<td>3/4 x 19.8</td>
<td>14.85</td>
<td>--</td>
<td>1/4 x 19.8</td>
</tr>
<tr>
<td>D</td>
<td>Final curvature superimposing Lines C and D</td>
<td>19.493</td>
<td>18.88</td>
<td>18.275</td>
<td>17.65</td>
<td>17.048</td>
<td>11.47</td>
<td>3.93</td>
<td>9.55</td>
<td>25.02</td>
</tr>
</tbody>
</table>
(2) Miller's Empirical Method:

This method is applicable to singly reinforced beams only; hence, in this case, it is not applicable.

(3) Branson's Empirical Method:

a) Restrained Specimen $C_1S$

The detailed computations for computing theoretical shrinkage warping of specimen $C_1S$ has already been presented in Article 3-2, Chapter III.

b) Restrained Specimen $C_2S$

\[
\Theta_{sh} = (0.7) \left( \frac{\epsilon_{sh}}{D} \right) (p-p')^{1/3} \left( \frac{p-p'}{\bar{p}} \right)^{1/2}
\]

for $(p-p') < 3.0\%$

and \[
\Theta_{sh} = (0.7) \left( \frac{\epsilon_{sh}}{D} \right), \quad \text{for} \quad (p-p') \geq 3.0\%
\]

The specimen under consideration has $p = 3.83\%$ and $p' = 1.91\%$, hence

\[
\Theta_{sh} = (0.7) \left( \frac{160 \times 10^{-6}}{5} \right) (3.83 - 1.91)^{1/3} \left( \frac{3.83 - 1.91}{3.83} \right)^{1/2}
\]

\[
= 19.8 \times 10^{-6} \quad 1/\text{in}
\]

since the theoretical shrinkage warping curvature using Branson's Empirical Method is same as that computed by Tensile Force Method (in this particular case only), the computations for theoretical shrinkage warping do not change and Table 10B holds good for this method also.
APPENDIX - III
### TABLE 10B

**COMPUTATION OF DEFLECTIONS DUE TO SHRINKAGE, USING NEWMARK'S NUMERICAL METHOD FOR RESTRAINED SPECIMEN C1S**

#### a) From Curvatures ($\Phi_{sh}$) Computed by Equivalent Tensile Force Method

<table>
<thead>
<tr>
<th>Description</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Section</strong></td>
<td>a</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td><strong>Computed $\Phi_{sh}$</strong></td>
<td>a</td>
<td>+46.5</td>
<td>+45.0</td>
<td>+43.5</td>
<td>+42.1</td>
<td>+40.6</td>
<td>+27.3</td>
<td>-9.5</td>
<td>-23.0</td>
</tr>
<tr>
<td><strong>Equiv. Conc. Value $\Phi$</strong></td>
<td>a</td>
<td>+510</td>
<td>+540</td>
<td>+522</td>
<td>+505</td>
<td>+475</td>
<td>+304</td>
<td>-90.7</td>
<td>-299</td>
</tr>
<tr>
<td><strong>Ave. Slope</strong></td>
<td>a</td>
<td>-2192</td>
<td>-1682</td>
<td>-1142</td>
<td>-620</td>
<td>-115</td>
<td>+360</td>
<td>+664</td>
<td>+573</td>
</tr>
<tr>
<td><strong>Trial Deflection</strong></td>
<td>a</td>
<td>+2192</td>
<td>+3874</td>
<td>+5016</td>
<td>+5637</td>
<td>+5752</td>
<td>5392</td>
<td>4726</td>
<td>4153</td>
</tr>
<tr>
<td><strong>Linear Correction</strong></td>
<td>a</td>
<td>-430</td>
<td>-860</td>
<td>-1290</td>
<td>-1720</td>
<td>-2150</td>
<td>-2580</td>
<td>-3010</td>
<td>-3440</td>
</tr>
<tr>
<td><strong>Deflection</strong></td>
<td>a</td>
<td>+1762</td>
<td>+3014</td>
<td>+3826</td>
<td>+3916</td>
<td>+3602</td>
<td>+2812</td>
<td>+1216</td>
<td>+713</td>
</tr>
<tr>
<td><strong>Deflection (inch)</strong></td>
<td>a</td>
<td>0.021</td>
<td>0.036</td>
<td>0.046</td>
<td>0.047</td>
<td>0.044</td>
<td>0.034</td>
<td>0.015</td>
<td>0.0085</td>
</tr>
</tbody>
</table>

#### b) From Curvatures ($\Phi_{sh}$) Computed by Miller's Empirical Method

| **Computed $\Phi_{sh}$**         | a | +35.5 | +34.3 | +33.2 | +32.1 | +31.1 | +20.8 | -7.24 | -17.26 | -45.5 | |
| **Deflection (inch)**            | a | 0.015 | 0.026 | 0.032 | 0.031 | 0.030 | 0.024 | 0.015 | 0.006 | 0.004 | |

#### c) From Curvatures ($\Phi_{sh}$) Computed by Branson's Empirical Method

| **Computed $\Phi_{sh}$**         | a | +31.5 | +30.5 | +29.5 | +28.5 | +27.5 | +18.5 | -6.49 | -15.5 | -40.5 | |
| **Deflection (inch)**            | a | 0.014 | 0.025 | 0.029 | 0.031 | 0.028 | 0.022 | 0.013 | 0.0046 | 0.003 | |

### NOTES ON TABLE 10C

- a) The continuous Specimen C1S has 18 sections, each 1.0 ft. long.
- b) Curvatures due to shrinkage are computed in Col. F of Table 2.
- c) $\Phi = a/12 (7/2 \Phi_a + 6/2 \Phi_b - 1/2 \Phi_d)$
  $\Phi_{sh} = a/12 (\Phi_a + 10 \Phi_b + \Phi_d)$
- d) Deflections in inches are tabulated in Column H of Table 2.
- e) Curvatures due to shrinkage are computed in Column F of Table 3.
- f) Deflections in inches are tabulated in Column H of Table 3.
- g) Curvatures due to shrinkage are computed in Column F of Table 4.
- h) Deflections in inches are tabulated in Column H of Table 4.
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1. ACI Committee 435 Report, August, 1965, "Deflections of Reinforced Concrete Flexural Members."


7. Ferguson, P. M., Discussion of "Warping of Reinforced Concrete Due to Shrinkage" by A. L. Miller, ACI Journal Proceedings V. 54, No. 6, December, 1958, pp. 1393-1402.