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Teachers' use of reasoning-based questions in procedural and conceptual lessons

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TEACHERS' USE OF REASONING-BASED QUESTIONS IN PROCEDURAL AND CONCEPTUAL LESSONS

by

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CERTIFICATE OF APPROVAL

PH.D. THESIS

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To the late Dr. Vicki Burketta, the professor who became my friend, my family, and my inspiration. Thank you for showing me the power of my own thinking, and helping me see that no dream is too big. You lit a flame in so many people's lives, now it is our turn to light the flames of others.

Time given to thought is the greatest time saver of all.

Norman Cousins

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ABSTRACT

Recent research shows that teachers' level of Mathematical Knowledge for Teaching (MKT) and their beliefs about teaching and learning effect teaching practices and student achievement. Higher levels of MKT typically lead to more effective teaching abilities in terms of helping students make meaning of mathematical concepts, but beliefs seem to be a mediating factor in this relationship. One specific teaching practice that can help guide students through this meaning making is questioning. Although it is known that MKT and beliefs play an important role in outcomes of teacher practices, the effects of these factors on teachers' ability to ask meaningful questions have not yet been explored. This mixed methods study uses descriptive data of teachers' questioning patterns with a cross-case analysis of five elementary mathematics teachers to investigate how the nature of elementary teachers' questioning changes between procedural and conceptual mathematics lessons, and how teachers' level of MKT and their beliefs about teaching and learning aid in or inhibit their ability to ask questions that engage students in mathematical reasoning and sense making. High levels of alignment with rule-based beliefs about teaching mathematics were found to be a major inhibitor to teachers' ability to ask meaningful questions in the classroom. While high MKT is helpful in creating reasoning-based dialogue in the classroom, high rule-based beliefs limit the potential effects of high MKT on teacher questioning practices. Relationships between MKT, beliefs, and questioning are further dissected, and implications for teacher development efforts are discussed.

PUBLIC ABSTRACT

Teacher questioning is known as a useful tool in engaging students in deep thinking about mathematical ideas, however, many teachers find it difficult to ask meaningful questions. Two factors known to affect a teacher's ability to ask meaningful questions in a mathematics classroom are their mathematical knowledge for teaching, which includes both mathematical knowledge and knowledge about teaching mathematics, and their beliefs about how students learn. This study explores the ways in which these factors of knowledge and beliefs affect teachers' ability to ask meaningful questions that engage their students in the process of learning. Six teachers were studied in order to answer two questions: 1) Do teachers' questioning practices change depending on the type of lesson they are teaching, and 2) How do teachers' mathematical knowledge for teaching and beliefs about teaching and learning affect their ability to ask meaningful questions? If teachers are not asking meaningful questions is it because they do not have the knowledge to do so, or because they do not find questions to be an important factor in student learning? This study found that the biggest hindrance to meaningful teacher questioning is a belief that rules and procedures are needed in order for students to succeed in mathematics. Teachers with this belief ask questions that simplify mathematical problems and do not provide students the opportunity to reason and build deep connections. The relationship between mathematical knowledge for teaching, beliefs, and questioning is further dissected, the implications of these results on teacher development are discussed.

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Chapter 1: Introduction

What it means to know and understand mathematics has been discussed by many researchers. Thompson (1992) eloquently noted that "knowing mathematics is making mathematics," meaning to truly understand mathematics in a meaningful way, we must first be immersed in its never-ending patterns and "generative processes" (p. 128), allowing ourselves to make the jump from a known idea to an unknown idea through our own mathematical logic and reasoning. Similarly, Fennema and Franke (1992) discuss how "expert knowledge is better integrated and more accessible than is the knowledge of novices" (p. 152). They state that within this knowledge "connections exist between ideas, the relationship between ideas can be specified, the links can differ among ideas, and the manner in which the knowledge is organized is relevant to understanding and application" (p. 152). This deep level of understanding cannot come about through passive transmission of the teacher's ideas. Constructivists agree that all mathematics lessons should involve meaning making of this kind through student engagement and participation (Windschitl, 2002), including procedural lessons, which are typically more rule based. Yet we, as researchers, teachers, and policy makers all need to ask ourselves if our schools are giving students the opportunity to learn mathematics in this meaningful and connected way.

Many studies have looked at teachers' ability to support meaning making in the mathematics classroom (Charalambous & Hill, 2012; Fennema & Franke, 1992; Hill et al., 2008; Sleep & Eskelson, 2012), however, the focus of these studies has been on teachers doing the meaning making for students. This meaning making includes the teacher taking time to make connections, discuss underlying concepts, and use multiple representations along with more elaborate explanations of ideas (Hill et al., 2008); all of which are based on teacher actions that convey their own conceptions. There is a need to shift focus from teachers as meaning makers to students as meaning makers through engagement in their own complex thinking and reasoning (Hill & Charalambous, 2012; CCSSI, 2010). Teacher actions are still vitally important in this shift, but instead of focusing on the teacher's meaning making, we shift our focus to actions that teachers take to provide opportunities for student engagement in their own meaning making.

If we subscribe to this belief that learners must be active participants in the construction of their knowledge and understanding (Pines & West, 1986; Windschitl, 2002), why have we not focused more efforts on teacher practices that engage students in the practice of their own meaning making? One reason might be that creating opportunities for students to construct mathematical understanding is a complex process that requires various abilities from the teacher. One of those abilities is the effective use of questioning. Questioning is the main tool teachers can use to engage students in meaningful thinking and classroom dialogue (Franke et al., 2009; Ni, Zhou, Li, & Li, 2013). Through deep thought and discussion, students are given the opportunity to sort out what they do and do not understand, and subsequently, how to reason their way to a deeper understanding.

High cognitive demand tasks have been found to help teachers ask more in depth and meaningful questions to engage students in complex reasoning practices (Ni et al., 2013), but rigorous tasks are not enough (Sleep & Eskelson, 2012). In order to be able to ask questions that push students' thinking forward, teachers must have a certain level of mathematical understanding. Hill and Charalambous (2012) found that teachers with higher levels of Mathematical Knowledge for Teaching (MKT) are better able to support meaning making. Similarly, high MKT teachers tend to conceptualize all lessons, whether their lesson goals be conceptually focused or procedurally focused. Low MKT teachers, on the other hand, tend to proceduralize all lessons (Hill et al., 2008). However, this relationship is not straight forward. Teachers with more mathematical knowledge still differ in the "richness of the mathematics available for the learner" (Fennema & Franke, 1992, pp. 149-150), but what causes this variation among these teachers?

Teachers' beliefs about teaching and learning have a very complex relationship with their level of MKT and teaching practices, and this complication makes it more difficult for researchers and teacher educators to help teachers create opportunities for student meaning making. Orientation towards certain beliefs are found to affect "differences in the richness of the mathematical work and in students' participation in the development of the mathematics" (Sleep & Eskelson, 2012, p. 554). In this context, beliefs seem to be a mediator of knowledge. A teacher can theoretically have the highest level of mathematical knowledge, but without an understanding of how students learn, their knowledge could be untapped by students. To complicate matters more, MKT and beliefs are also seen as dependent upon one another, with development in each individually supporting development in the other (Sleep & Eskelson, 2012). For this reason, it is pivotal to look at both teachers' MKT and beliefs about teaching and learning while examining teachers' ability to engage students in mathematical meaning making. Research without both would paint an "incomplete picture" (Thompson, 1992, p. 131).

Without proof that student engagement in reasoning and meaning making leads to increased learning, the complexity of factors that enable teachers to ask questions that push students' thinking further might not be worth the effort; but that is simply not the case. When students are given the opportunity to construct their own understanding and engage in the process of meaning making they begin to see mathematics as the interconnected world of patterns that it truly is (Polly et al., 2013). This gives students a more in depth understanding of the concepts and is also linked to high levels of student achievement (Fennema & Franke, 1992). Moreover, when students are exposed to more conceptually oriented problems and discussions, students start to gain a sense of empowerment that understanding is important in mathematics, and it is not a simple game of memory (Boaler, 2016; Carter & Norwood, 1997).

Students must learn the process of how one comes to understand a concept or idea. This process helps students build critical thinking and problem solving skills, which, when paired with high levels of knowledge, leads to students who are successful problem solvers and critical thinkers (Fennema & Franke, 1992). In order for this to be a more prevalent occurrence in our public-school systems we must give our students opportunities to think for themselves with their own methods, and help teachers ask meaningful questions that engage students in the thinking that needs to occur for them to come to deep levels of mathematical understanding.

Purpose of the Study

Although it is known that high levels of MKT and certain beliefs are helpful or harmful in aiding teachers' creation of opportunities for student meaning making through teacher questioning (Hill & Charalambous, 2012; Hill et al., 2008), we still do not have a good understanding of two things. First, it is not yet known whether teachers' mode of questioning changes based on the type of lesson they are teaching (conceptually based or procedurally based). Second, the affordances that allow teachers to continue to engage students in meaning-making during procedurally based lessons have not yet been explored. While procedurally based tasks or lessons typically lead to lower level questions (Ni et al., 2013), what is it that allows some teachers to continue to focus on meaning making during these tasks (Hill et al., 2008), and others not? Do tasks without referral to conceptual underpinnings represent a teacher's belief of mathematics as memorization? Are teachers not asking students to focus on building a conceptual understanding because they, themselves are unaware of the conceptual connections that could be made? The previous two questions represent teacher beliefs or MKT levels that do not support research and standards based teaching practices. These questions, along with current research, have led me to the following research questions:

1) Does the nature of teachers' questioning change between conceptually based lessons and procedurally based lessons?
2) How do teachers' Mathematical Knowledge for Teaching (MKT) and beliefs about teaching and learning relate to the nature of teacher questions during procedurally based lessons?

These research questions were answered through a mixed methods approach that analyzed the dependencies of teachers' cognitive demand of questioning, during both conceptually and procedurally based lesson, on teachers' level of MKT (Ball, Thames, & Phelps, 2008), and teachers' beliefs about teaching and learning (Drageset, 2010). Through the use of videotaped lessons, teachers' questions were coded with levels of cognitive demand. Analysis of the data were completed by comparing rates of teachers' questioning practices. Semi-structured interviews were used to triangulate the data and also get a more in depth look at teachers' beliefs and conceptual understanding of topics discussed during their lessons (Merriam, 2009). Charalambous, Hill, and Mitchell (2012) noted that some inconsistencies have been found between teachers' professed beliefs and practices. They stated that these inconsistencies could "be manifestations of espoused teaching ideals that cannot be realized because the teachers do not possess the skills and knowledge necessary to implement them" (p. 138). Since this remark is simply speculation, individualized semi-structured interviews allowed for a more concrete understanding of the relationships between the variables.

Significance and Implications

While MKT has been a major focus of research in mathematics education throughout the past decade, Charalambous et al. (2012) note that "one area to which MKT appears to contribute weakly is in engaging students in high-level thinking and reasoning" (p. 511). As previously stated, high-level thinking and reasoning is a key practice in aiding students' deep understanding of mathematical concepts (Fennema & Franke, 1992). Since MKT has been used as an important measure in predicting teachers' practices and student achievement (Hill & Charalambous, 2012; Hill et al., 2008), it is important to explore the misalignment between MKT and engaging students in higher-level thinking and reasoning by means of teacher questioning. This study will advance our understanding of what teachers need in order to improve their ability to engage students in these thinking and reasoning practices.

By answering the research questions stated in this chapter, we will have a better understanding of the relationship between teachers' MKT and beliefs about teaching and learning, which has been called for throughout the past two decades of mathematics education research (Hill & Charalambous, 2012; Thompson, 1992). Although there have been various studies that have looked at this relationship, this study specifically seeks to look at the impact of MKT and beliefs on student engagement in meaning making through teacher questioning practice. With a better understanding of these relationships, researchers and teacher educators will know where to focus professional development and pre-service teacher development efforts in order to allow for maximum engagement of students in mathematical development. This is a critical step towards creating a new generation of critical thinkers who are empowered to take on the challenges of the 21st century.

Chapter 2: Literature Review

The complex relationship between beliefs and MKT in regards to the practice of teacher questioning calls for an in depth look into the research of past decades. In this chapter I seek to build a solid foundation for my research in terms of a theoretical framework for the study, as well as an in-depth review of the literature that has added to the understanding of these factors and their interdependence on each other. The chapter begins with an explanation of the theoretical framework, which is constructivism, and then leads into an explanation of what knowledge, learning, and teaching look like through this lens. Following the discussion of this specific theory, or belief, is a more general description of the role that beliefs play in mathematics education. MKT and the role that it plays in the development of meaningful mathematical understanding is then discussed, followed by the complexity of the relationship between MKT and beliefs. Finally, I will discuss the practice of teacher questioning, and how MKT and beliefs enable teachers to use questioning to open up opportunities for students to think more critically about mathematical ideas.

Constructivism

Regardless of one's epistemological views, teaching plays a pivotal role in the learning process. However, upon delving into the various beliefs that are held among researchers and educators, the aspects of teaching that we focus on begin to differ. For example, in the majority of mathematics classrooms around the United States, you will see teaching that is aligned with the transmission view of learning (Schoenfeld, 2014). The main idea that supports this view is that the teachers' words and actions allow meaning and understanding to be transmitted to the learner (Cobb, 1988). Through this belief, direct instruction through lecture makes complete sense; therefore, enhancement of teacher lectures might be of interest to the researcher.

Mathematics education has continuously shifted back and forth between practices that align with a dichotomy of beliefs about how people learn. However, decades of research support a theory of learning that requires the active involvement of the learner constructing their knowledge upon previously learned ideas and understandings (Cobb, 1988; Simon, 1995; Windschitl, 2002). This belief, which is also the theoretical belief that guides this research, is called constructivism. Constructivists believe that humans do not have access to an objective reality. Reality is independent to the learner because our reality, or knowledge, is constructed based on our past experiences (Simon, 1995). Through this belief, we can assert that constructivists do not study reality, but the act of constructing an independent reality (Steffe & Kieren, 1994). Construction of this reality happens actively through experiences, and reflections on those experiences (Steffe & Kieren). The way in which we interpret a situation is dependent upon the accumulation of experiences we have had throughout our entire life. Consequently, since no two people have the exact same lived experiences, we can no longer believe that the words a teacher says will convey the exact same meaning, or knowledge, to each individual within the class. We can then see that the concept of understanding, as Von Glasersfeld (1989) states, is more "a matter of fit rather than match" (pp. 133-134).

One might ask why constructivism guides the given study if the majority of teachers in the U.S. show a tendency towards acts of teaching that align with the transmission view of teaching and learning. It is important to note that constructivism is not a theory of teaching, but a theory of learning (Windschitl, 2002). Constructivism does not pertain to a specific style of teaching, but the way in which teaching engages the learner in the process of constructing their own knowledge. To fully understand how this theory guides the research, a deeper understanding of the constructivist perspective of knowledge, learning, and teaching is needed.

Constructivist view of knowledge. How does a teacher know when a student has gained the knowledge or understanding they have aimed to develop in their students? There are many instances where students finish listening to a lesson and they are able to complete the mathematics problems the teacher has given them. Does this mean that the student has gained an understanding? Cobb (1988) notes, "students who complete tasks successfully in the context of direct instruction cannot be said to have taken in the knowledge the teacher believes he or she has transmitted. Rather, the students have found a way of acting that is compatible with the teacher's expectations about the outcomes of instruction" (p. 91). From the constructivist perspective, knowledge is not seen as discrete pieces of information or memorized algorithms, but conceptual structures that are deeply connected to each other and rooted in the learner's experiences (Driver & Oldham, 1986). During the development of understanding, conceptual structures are built and reorganized on top of previous knowledge, or structures (Ernest, 2010).

When a constructivist considers the knowledge that has been built, they are not looking for memorized facts that will soon be forgotten. Knowledge and understanding, in this case, are synonymous. When a student has created a strong conceptual structure, they will not only be able to apply that understanding to the current problem, but use it to solve problems in various domains, in a wide variety of situations, and they will continue to be able to do these things long after the construction of said knowledge (Cobb, 1988).

Constructivist view of learning. If knowledge is considered a deep conceptual structure, learning must be a much more involved process than many consider it to be. Although building knowledge is a complex endeavor, learning is a natural part of human activity. When a human adapts to the environment around them, that process of adaptation is learning (Simon, 1995). The person experiences a problem or dilemma that they want to avoid, they make changes based on their current understanding of the situation, they reflect on those changes and decide whether those changes led them to a desirable point. Learning in the classroom traverses an identical path.

Whether in a classroom or any other environment, learning is what Von Glasersfeld (1989) refers to as "the product of self-organization" (p. 136). When a student is taught a lesson in a classroom, the learner's experience of the events of the lesson interact with his or her current ideas or conceptions to potentially result in learning (Posner, Strike, Hewson, & Gertzog, 1982). Two different types of learning may occur if certain conditions are met. These forms of learning are called assimilation and accommodation. Posner and colleagues state that assimilation occurs when the learner uses "existing concepts to deal with new phenomena" and accommodation occurs when the learner must "replace or reorganize his central concepts" (p. 212).

These events do not occur easily. In order for consideration of these events to occur, the learner must experience a state of disequilibrium. Disequilibrium occurs when we experience something that differs from what we would expect the experience to be. Once this state is reached, the adaptive process mentioned above is triggered, and the learner works to sort through the disequilibrium to build upon a current conceptual structure (Simon, 1995). However, disequilibrium alone is not enough to cause learning to occur. First, the learner must be dissatisfied with their existing conceptions (Driver & Oldham, 1986). If the learner believes that the new knowledge is not useful in the larger scheme of things, they may not choose to engage in learning, but if they are exposed to numerous situations, or a large enough situation, where their prior understanding is shown to be insufficient or incompatible with new lines of thinking they are more likely to engage in the learning process (Posner et al., 1982). If this first condition is met and the learner feels dissatisfied with their current understanding, the learner must then find the new conception to be "intelligible, plausible and fruitful in offering new interpretations" (Driver & Oldham, p. 108).

This process of learning is an iterative process of generating cognitive schemas to represent experiences and guide future actions, and then testing the new schema to analyze how it fits with other conceptual structures and lived experiences. When a schema appears to fit into the learner's world, it is temporarily adopted until it is challenged by a new state of disequilibrium (Ernest, 2010). In fact, those new schemas will be used in further acts of construction, but in order for it to be used as a foundation of future knowledge, the student must construct a coherent and meaningful representation of the concept (Posner et al., 1982). With this in mind, we see how active the process of learning must be, and the role of the teacher, along with the practices they employ, must shift with this demand.

Constructivist view of teaching. With the goal of helping students construct such deep knowledge and understandings, the role of the teacher becomes much more complex. Von Glasersfeld (1989) noted this difference in complexity. He stated, "teachers have known that it is one thing to bring students to acquire certain ways of acting… but quite another to engender understanding. The one enterprise could be called 'training', the other 'teaching'" (p. 131). To engender understanding requires much more of the teacher (Cobb, 1988). The constructivist view of teaching is not focused solely on the teacher's actions, as it might be for a behaviorist, but on the teacher's interaction with the learner (Simon, 1995). This layer of interaction adds another element to the dimension of teaching.

The interaction between two individuals, whether it be the teacher with a student, or a student with other students, becomes a very interesting concept to research when one aligns themselves with the constructivist belief of the void of an objective reality. How does communication work when no two people will ever have the exact same understanding of a concept? This is where the job of the teacher shifts from a "constructive organizer" to a "deconstructive organizer" (Pines & West, 1986). In order for a teacher to create opportunities for student learning, they must create opportunities for students to experience disequilibrium. Therefore, one of the main acts of the teacher is to pose problems or tasks and encourage students to reflect on the concepts addressed within the task by questioning student thinking (Simon, 1995). Instead of focusing on the teacher's method of solving a problem, the teacher would be more interested in "how students see the problem and why their path towards a solution seemed promising to *them*" (Von Glasersfeld, 1989, p. 137). While many may see the goal of solving a problem as finding the answer, constructivists would view the goal of the task to be learning, or building conceptual structures that align themselves with students' prior structures (Wheatley, 1991).

To focus on all students' processes and levels of understanding is an intense job that requires the teacher to be a very astute and responsive listener (Le Cornu & Peters, 2009). If the teacher is not aware of students' previous beliefs, then they do not know how to lead students to opportunities for assimilation or accommodation. When this is the case, "the best that can be achieved will be that students will continue to hold their previous beliefs about the world and the way it works while rotely learning the formal content" (Pines & West, 1986, p. 589). As educators, our goals are much higher than this. The goal of mathematics teaching should be to help students restructure their conceptual organizations to make them more complex, powerful, and abstract (Cobb, 1988). The only way to do this is through experience with cognitively demanding instruction, which is a major focus of this study. Regardless of the task at hand, be it conceptual or procedural, a teacher needs to be able to help students restructure their conceptual understanding of the content and connect procedural knowledge to these conceptual structures. The state of disequilibrium needed to enable students to restructure their understanding is often associated with the task the teacher has arranged for the lesson. However, it is important to note that neither the teacher, nor the task creates the disequilibrium within the student. Disequilibrium is dependent on the existing conceptual structures of each student. Therefore, a teacher's actions and a specific activity may be enough to engage some students in the restructuring process, but not others (Lerman, 1996). In this case, the teacher must have a sufficient understanding of various student conceptions related to the topic to create optimal opportunities for students to engage in disequilibrium (Simon, 1995). This makes the dialogical aspect of teaching vitally important.

Role of dialogue in learning. The dialogical interactions between teacher and student or student and student(s) play two important roles in the teaching and learning process. First, it gives the teacher insight into students' current beliefs and understandings. For this reason, one of the teacher's main tasks would be to create a classroom environment where students discuss ideas, even if they are not fully formed (Windschitl, 2002). Each of these discussions or comments inform the teacher on how to best guide the lesson toward a point of student disequilibrium.

Secondly, these interactions give students an opportunity to negotiate meaning with others. As mentioned earlier, understanding cannot be transferred directly through words, but these communications are strengthened when the listener has conceptions that are compatible with the explanations the speaker is giving (Von Glasersfeld, 1989). Opening up the dialogue with the class allows multiple views and ideas to develop in both the private and collective sense (Ernest, 2010). This development happens through the negotiation process where students and the teacher discuss mathematical concepts in a back and forth manner that ideally allows members of the learning process to reach a similar understanding (Wheatley, 1991), in this case, one that is compatible with understandings deemed acceptable by the mathematics community. The teacher's role in this process is to mediate learning by "seeding students' conversations with new ideas or alternatives that push their thinking" (Windschitl, 2002, p. 147).

This mediation of learning can also occur between students, without the teacher's involvement (Sfard & Kieran, 2001). When students are given opportunities to negotiate ideas through discussions with each other they have an enhanced opportunity to work through disequilibrium because they are forced to face their level of understanding head on (Stigler & Hiebert, 2004). The need to be able to communicate about ideas is one way to create the dissatisfaction needed to make engaging in learning an enticing option (Driver & Oldham, 1986). Through these options, we begin to see the teacher as one voice of many in the co-construction of knowledge (Von Glasersfeld, 1989).

Beliefs

Research and theories grounded in constructivism paint a clear picture of the process of learning and the role that teachers and students both play in that process. However, this belief is not held by all educators. In fact, many educators have not taken the opportunity to reflect on their beliefs and build a solid foundation about their beliefs on teaching and learning (Peterson, Fennema, Carpenter, & Loef, 1989). Teachers' actions and decision making practices are affected by their varying beliefs and levels of depth and clarity about those beliefs.

Cross (2009) defines beliefs as “embodied conscious and unconscious thoughts about oneself, the world, and one’s position in it, developed through membership in various social groups; these ideas are considered by the individual to be true” (p. 326). These conscious and unconscious beliefs are thought to be a strong predictor of human behavior, including in the mathematics classroom (Cross, 2009; Hill & Charalambous, 2012; Sleep & Eskelson, 2012). Although beliefs affect the majority of instructional decisions teachers make throughout the day, they are completely personal and “often reside at a level beyond the individual’s immediate control or knowledge” (Cross, 2009, p. 326). For this reason, beliefs are an important aspect to address before shifts in instructional practices take place (Cross).

The interactions of numerous belief systems come into play when an educator takes on the act of teaching. Beliefs in mathematics education are typically centered around three major belief structures; the nature of mathematics, how to teach mathematics, and how students learn mathematics (Cross, 2009). Beliefs in each of these structures influence each other in various ways that are not always straightforward. Part of the complexity in these relationships is due to the varying strength of beliefs that teachers hold. Numerous researchers (Cross, 2009; Green, 1971; Thompson, 1992) have distinguished the difference between central beliefs, which are the most strongly held beliefs, and peripheral beliefs, which are “the most susceptible to change or examination” (Thompson, 1992; p. 130). If measuring teachers’ alignment with a specific belief on a Likert scale, central beliefs would show up at the two extremes, while peripheral beliefs would fall closer to the middle (Thompson, 1992). In past research, the majority of teachers’ pedagogical content beliefs fell in the middle of the Likert scale, meaning that the majority of teachers’ beliefs about teaching and learning are peripheral (Peterson et al., 1989).

Effects of beliefs on teaching. Researchers commonly accept the notion that teachers’ beliefs about teaching and learning guide their instructional decision-making (Cross, 2009; Hill & Charalambous, 2012; Sleep & Eskelson, 2012), but what specific actions are affected by beliefs? Peterson et al. (1989) stated, “teachers’ pedagogical content beliefs and teachers’ pedagogical knowledge may be importantly linked to teachers' classroom actions and, ultimately, to students' classroom learning in mathematics" (p. 36). Different beliefs among teachers have been linked to a difference in the framing of problems and structuring of tasks (Cross, 2009). This includes the role that students play in the development of mathematical ideas through these tasks, and the richness of their mathematical work (Sleep & Eskelson, 2012). Teachers with beliefs that are aligned with constructivist perspectives choose curriculum that are mathematically richer, and the way they interpret and implement the curriculum provides more opportunities for student development of knowledge through problem solving (Peterson et al., 1989; Sleep & Eskelson, 2012; Thompson, 1992). Teachers with this belief alignment and corresponding instructional actions showed higher levels of student achievement. These students scored significantly better on problem solving tests, and similarly on procedural knowledge even though these teachers focused less on procedural processes (Peterson et al., 1989).

One of the major beliefs that affects what a teacher believes in terms of teaching mathematics, is the beliefs that the teacher has about the nature of mathematics itself (Cross, 2009). If teachers see mathematics as an interconnected field of patterns that we can use to create new understandings, then that teacher will most likely attempt to help their students access the knowledge on a level where they can engage in sense making and reasoning with the interconnected patterns. Therefore, if a teacher's beliefs about the nature of mathematics are modified, those beliefs might also shift that teacher's beliefs about teaching mathematics. Additionally, teachers' beliefs about learning mathematics have also been shown to affect students' beliefs about the nature of mathematics. Carter and Norwood (1997) noted, "problems that are conceptually oriented kindle beliefs [for students] that understanding is important in mathematics" (p. 65). Therefore, if we want students to be able to become critical thinkers and learn through problem solving, teachers need to empower students and help them see the interconnectedness of the subject. Through these actions, students will begin to become more independent learners.

Inconsistent beliefs and actions. Regardless of the strength of a core belief, teacher actions are sometimes influenced by constraints imposed on teachers by a school or district. Thompson (1992) notes that teachers' beliefs about teaching and learning are not in a simple cause-and-effect relationship with their instructional practices. Instead, they forge "a complex relationship, with many sources of influence at work; one such source is the social context in which mathematics teaching takes place, with all the constraints it imposes and the opportunities it offers" (p. 138). If a teacher feels the pressure of time, or knows that student scores on a standardized test will be publically tied to teacher names, they may choose to ignore what they believe is best practice to ensure that students get through all content before the exam is given.

This pressurized decision-making tends to appear more emotionally driven, rather than logically driven. This amalgamation of emotional and logical beliefs was explored by Elbaz (1983) when he conceptualized the process of instructional decision-making through beliefs. Elbaz stated that the structure of teachers' knowledge, or what I would call teachers' beliefs, include three dimensions: rules of practice, practical principles, and images. Rules and principles are made up of instructional knowledge, while images use this knowledge, along with emotions and morality to direct the decision-making process. Rules and principles are chosen through images, but if they conflict with the image a teacher holds about a particular situation, different rules and principles will be chosen.

The process of choosing new rules and principles through images shows how a teacher's beliefs might change over time through both logical and emotional considerations. As if this process was not complex enough, researchers have also noted that an individual can easily hold incompatible or inconsistent beliefs if they are never provided opportunities to think about the beliefs concurrently whether they are core or peripheral beliefs (Cross, 2009). Because of the way beliefs are clustered, people can hold two incompatible core beliefs and never have a problem with it because of another mediating belief. Cross gives the following example of this mediation of beliefs: A teacher may believe that students should be able to come to school to excel, while also believing that only tag students should be placed in advanced mathematics classes. These beliefs are conflicting since the first belief makes it apparent that school should enable students to excel, regardless of what means it takes to make that happen, but the second belief removes an avenue that might be needed to help a non-tag student excel. However, both of these conflicting beliefs could be held by a teacher without tension if the teacher also believes that ability is fixed.

Teachers' beliefs about mathematics education are sometimes found to change within different mathematical domains. This could be explained by the conflicting belief phenomena mentioned above. In a study by Cross (2009), one teacher believed algebra was very formula driven, while geometry was more about teaching students how to think. Cross mentions that these differences in beliefs about the different subjects "shaped how they designed their instructional activities, the tasks they engaged their students in, the quality of interaction they encouraged in the classroom, the types of evaluation methods they employed, and the fidelity with which they incorporated and facilitated the reform-oriented materials and practices" (p. 336). This study also found a distinct difference in the types of questions the teacher asked during the different mathematics subjects. Since many other factors are held constant when comparing a teacher to herself, this example is a powerful indicator of how influential a teacher's beliefs can be in the enactment of teaching. Additionally, since teacher beliefs may be content specific, this study will focus on the specific strand of mathematics known in the Common Core State Standards as "Numbers and Operations" (CCSSI, 2010).

Frameworks for belief measurement. Now that the formation of beliefs and factors that mediate the use of beliefs in instructional decision-making have been explored, it is important to look at various ways researchers have measured beliefs in the field of mathematics education. Peterson et al. (1989) developed a framework for analyzing teachers' pedagogical content beliefs. This framework consisted of four constructs, which include the following: 1) Children construct their own mathematical knowledge, 2) mathematics instruction should be organized to facilitate children's construction of knowledge, 3) children's development of mathematical ideas should provide the basis for sequencing topics for mathematical instruction, and 4) mathematical skills should be taught in relation to understanding and problem-solving. Alignment with these four constructs shows an orientation towards constructivism.

Carter and Norwood (1997) created a similar framework, which includes five constructs. Three of those constructs align with Peterson et al.'s (1989) first three constructs. The fourth and fifth constructs focus on the role of the teacher and students in the learning environment. Both the Carter and Norwood (1997) and Peterson et al. (1989) frameworks break belief structures down into constructs that align with the various pedagogical actions that a teacher sets up based on their beliefs or other previously mentioned factors.

Drageset (2010) simplified the analysis by looking at whether teacher actions aligned with a rules-based construct or a reasoning-based construct. Teachers whose beliefs align with rule-based instruction "emphasize formal mathematics and the learning of rules as most important, without focusing on explanations or connections" (p. 37). Those whose beliefs align with reasoning-based instruction represent "a belief that reasoning, argumentation and justification are more important than the answer" (pp. 37-38). The Drageset framework was chosen as the method of measuring teachers' belief alignment for the current study for its ability to distinguish whether a teacher finds reasoning and understanding as an important aspect of all mathematical learning. Teachers who ask conceptual or high level questions during both procedural and conceptual lessons show their belief that it is always important to engage students in sense making. Although other frameworks include important constructs that affect instructional decisions, the Drageset framework is especially useful for analysis of belief alignment with teacher questioning practices since it aligns well with the coding framework chosen for the study, as I will discuss at a later point in time.

Mathematical Knowledge for Teaching

Regardless of what beliefs a teacher holds, if they do not have a certain level of mathematical knowledge, they may not be able to enact certain practices that their beliefs align with. Similarly, a teacher can have a wealth of mathematical knowledge, but if their beliefs align with a transmission view of teaching and learning, they may not be able to reach optimal student achievement levels. The relationship between teachers' beliefs and mathematical knowledge are cyclical in nature, each influencing the other in a very complex and interactive nature (Hill et al., 2008; Thompson, 1992). This section will focus specifically on Mathematical Knowledge for Teaching (MKT). The relationship between MKT and beliefs will be discussed in more depth in later sections.

'Mathematical Knowledge for Teaching' or MKT is the mathematical knowledge that is needed throughout the entire process of teaching mathematics. It has become well known that the knowledge needed for teaching mathematics is quite different from the knowledge needed to use mathematics in any other situation (Ball et al., 2008). While a mathematician spends their time working through mathematics problems as efficiently as possible, a teacher must unpack the mathematics in a way that is digestible to their students. In this sense, "the teacher need not only understand *that* something is so; the teacher must further understand *why* it is so, on what grounds its warrant can be asserted, and under what circumstances our belief in its justification can be weakened or denied" (Ball et al., p. 319). All of these components together are considered MKT.

Development of MKT measure. The measurement of teachers' knowledge has not always been focused on the ideas mentioned above. In the 1880's teacher assessments were approximately 95% content driven, and in an act of over correction, the assessments of the 1980's were almost 100% focused on the person's capacity to teach (Schulman, 1986). Schulman called for a new and more balanced approach to measuring teachers' knowledge. He suggested that developers think about three sets of content knowledge: (1) subject matter content knowledge, meaning the amount and organization of knowledge in the mind of the teacher; (2) pedagogical content knowledge, which "includes the most regularly taught topics in one's subject area, the most useful forms of representations of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others" (p. 9); and (3) curricular knowledge, which is "represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindication for the use of particular curriculum or program materials in particular circumstances" (p. 10). This more dynamic view of teachers' understanding would give a better measurement of whether or not teachers were prepared to help students reorganize their understanding and build stronger conceptual structures.

Based on the ideas of Schulman (1986), the Study of Instructional Improvement worked to design a measurement for MKT for elementary mathematics teachers (Hill, Rowan, & Ball, 2005). They broke up the broader topic of MKT into more specific domains. The two larger domains are subject matter knowledge and pedagogical content knowledge. As one would assume, subject matter knowledge has to do with knowledge of the mathematics needed, while pedagogical content knowledge is focused more on the interaction of knowledge of teaching practices and mathematics. Ball et al. (2008) note, "pedagogical content knowledge, with its focus on representations and conceptions/misconceptions, broadened ideas about how knowledge might matter to teaching, and suggests that it is not only knowledge of content, on the one hand, and knowledge of pedagogy, on the other hand, but also a kind of amalgam of knowledge of content and pedagogy that is central to the knowledge needed for teaching" (p. 392). Each of these two larger domains is broken up into three smaller subdomains that speak to the specific types of knowledge needed to teach mathematics.

MKT subdomains. The three subdomains of subject matter knowledge are common content knowledge (CCK), specialized content knowledge (SCK), and horizon knowledge (Ball et al., 2008). The first subdomain of this category, common content knowledge, refers to mathematical knowledge and skills that are used across all mathematical settings, not just specifically teaching. On the other hand, specialized content knowledge is the understanding of mathematical knowledge that is specific to teaching. For example, an accountant may know that multiplying a number by 10 results in adding a zero to the whole number, or moving the decimal one place to the right (common content knowledge), while a teacher must have a strong understanding of *why* this happens, and what the effects of multiplication by 10 are on a numbers place value (specialized content knowledge). While a mathematician might have understandings aligned with those of the specialized content knowledge subdomain, they certainly do not consider them on a regular basis. This level of unpacking would only slow them down.

Finally, the third subdomain of subject matter knowledge, horizon knowledge, is also specific to the teaching of mathematics. Horizon knowledge is an understanding of the broader spectrum of mathematics, and how topics relate over the span of the entire curriculum. This includes an understanding of what students were supposed to have learned before entering the teacher's classroom, and what mathematics their current learning will need to connect to later in the curriculum.

The three subdomains of pedagogical content knowledge are knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). All of these subdomains include content, but have an added characteristic of some type of pedagogical characteristic. For example, knowledge of content and students refers to the combination of knowledge of mathematics and knowledge of students. Teachers strong in this subdomain understand mathematics, but also understand how students think about mathematics. This could include knowing how to predict what students will find difficult, and anticipating student errors. Similarly, knowledge of content and teaching combines a teacher's knowledge about mathematics and knowledge about teaching. This type of knowledge allows teachers to effectively choose an appropriate sequence for the lesson, choose the best representations for the given content, and which examples or problems to choose that will push students to delve deeper into their understanding of the concepts. Lastly, knowledge of content and curriculum is the combined knowledge of mathematics and the layout of the curriculum.

Teachers with high levels of knowledge of content and curriculum are able to determine which concepts are appropriate for students at a given time, and which concepts should be saved for a later date. They know how to challenge students, while also staying within the realm of the student's current level of understanding.

All of these subdomains help researchers explain the complex practices and knowledge needed for the profession of mathematics education. Since teachers are required to make split-second decisions about numerous situations, such as how to explain certain concepts, how to respond to students' questions, whether a students' method is mathematically sound, how to correct a student's error, etc., it is important for them to be able to have the knowledge to make these decisions rapidly, as to not waste valuable class time (Ball et al., 2008). Teachers with high MKT must be able to determine a way to help students build sound mathematical conceptions, and it is for this reason that I hypothesize that teachers with higher levels of MKT will ask more conceptually based questions in both procedural and conceptual tasks.

Effect of MKT on student achievement. An important area of research on MKT is how MKT affects student achievement. Hill et al. (2005) found MKT to be significantly related to student achievement in first and third grade students. This relationship remained significant even after controlling for numerous student variables such as SES, race, and parent educational background. A study by Campbell et al. (2014) explored a similar avenue and looked at "the relationship between teachers' mathematical and pedagogical knowledge, teachers' perceptions, and their students' achievement, controlling for student demographics, teaching experience, and teaching assignment" (p. 421). They also found a statistically significant positive relationship between teachers' content knowledge and students' performance on standardized mathematics achievement tests for both upper-elementary and middle school students. This relationship was stronger for middle school students than upper elementary grades. With a one standard deviation increase in teachers' content knowledge, the estimated mathematics achievement scores for middle school students increased 16.6%, and 7.1% for upper elementary students.

Without the inclusion of subject matter knowledge, the relationship between teachers' pedagogical content knowledge and student achievement was not statistically significant for upper elementary students, but was greatly significant for middle grades teachers'(Campbell et al., 2014). The predicted achievement scores of middle school students increased by 22.1% with each standard deviation increase in teachers' pedagogical content knowledge. While the relationship between student achievement and MKT may depend on the grade level of students, "middle-grades teachers' mathematical knowledge, both of content and pedagogy, was directly and positively predictive of their students' level of mathematical achievement" (Campbell et al., p. 454).

Campbell et al. (2014) comment that more qualitative studies are needed to gain a more in depth understanding of how MKT supports student achievement. Although previous research shows that MKT affects a teachers' ability to create opportunities for students to reason and think about mathematics on a deeper level, studies have not directly looked at how MKT effects teachers' ability to prompt students to think at higher cognitive levels. With higher cognitive thinking leading to more powerful constructions of mathematics understanding, it is an important factor to consider in increasing student achievement.

Relationships between MKT and teaching practices. Researchers have only been working on the development of a clearer structure of MKT for the past decade, yet numerous studies have been published in an attempt to understand the relationship between MKT and its role in teaching practices and student achievement. If MKT is found to have positive effects on teaching practices and student achievement, MKT could have a profound impact on professional development or teacher education programs. Although the results of numerous studies have shown a positive correlation between MKT and effective teaching practices, the relationship is a bit more complex than a simple cause-effect relationship (Hill et al., 2008; Hill & Charalambous, 2012). Some teaching practices have a stronger relationship with MKT than others, and even still, this relationship varies within similar levels of MKT.

One study by Hill et al. (2008) found “a substantial link between strong MKT and high mathematical quality of instruction (p. 457). High mathematical quality of instruction includes numerous facets of teaching such as avoidance of mathematical errors, better analysis and use of student mathematical ideas, and encouragement of denser, and more rigorous explanations and student reasoning. This evidence is supported in a study by Sleep and Eskelson (2012), who had similar findings of lowered occurrence of mathematical errors, but also found that high levels of MKT supported teachers’ precise use of mathematical language.

Curriculum materials and MKT. According to the research mentioned, teaching practices may look different for teachers with varying levels of MKT, but it is not yet clear how MKT affects specific teaching practices, or what other factors interact with MKT to produce quality teaching. One educational factor that has been studied is the effects of MKT on the use of various curriculum materials. Sleep and Eskelson (2012) and Hill and Charalambous (2012) found that both MKT and curriculum materials make a difference in instructional practices. Curriculum materials set the stage for certain instructional practices, such as students’ level of participation, explanations offered, and mathematical thinking and reasoning, however, these practices were moderated by MKT (Hill & Charalambous). While good curriculum materials helped all teachers include effective teaching practices, those teaching practices amplified the demands on the teacher. Teachers with high MKT were better able to meet the demands of these practices, but low-MKT teachers met the challenge with mathematical errors and slip-ups (Hill & Charalambous). Sleep and Eskelson’s findings align with the previously mentioned study in that while curriculum materials afford richer mathematical opportunities for students, MKT was needed to productively implement these opportunities.

Although high levels of MKT support the use of effective curriculum materials, this does not mean that teachers with low MKT should not use such materials. Campbell et al. (2014) explained that more procedural and scripted lessons would not be an effective way for teachers with low MKT to teach. In fact, Hill and Charalambous (2012) found that one way that teachers with low-MKT can effectively implement curriculum materials is through closely following curriculum materials that have built in supports for teachers. Unlike low MKT teachers, high MKT teachers were able to use their mathematical knowledge to compensate for what certain curriculum materials might lack and offer high-quality instruction to their students with or without those curriculum supports (Hill & Charalambous). With the combined results from these studies, research shows that although curriculum materials can have a positive effect on teacher practices, it is not enough to make up for the effects of MKT. Campbell et al. (2014) explain, "if students' mathematics achievement is the intended outcome, this investigation finds no evidence to support the assumption that emphasizing mathematical procedure and limiting instructional context to a sequential routine—demonstrate or model, guided practice, and independent practice—will compensate for a middle-grades teacher's weak understanding of mathematics content or pedagogy" (p. 454).

Proceduralization vs. conceptualization. Wood's (1998) study discussed the idea of a specific teacher questioning practice that he called "funneling," which refers to a series of leading questions that the teacher asks in order to guide students through a predetermined process to get them to the correct solution. This practice can also be called proceduralization, and during the process of proceduralization the teacher assumes much of the mathematical work, leaving the students to solve simple computational problems. For example, a teacher may put a problem up on the board that involves the use of the order of operations to simplify a numerical expression. If the teacher is proceduralizing the process, they will ask a question at each step in the process. This may include questions such as "what is the first step?", "what is the next step after we've solved all multiplication?", etc. On the other hand, a teacher could also conceptualize a lesson by addressing conceptual aspects in all lessons, even if the lesson is covering a procedural topic. For example, order of operations problems are fairly procedural, but a teacher might ask students why they would combine the quantities inside of the parentheses before multiplying them by a factor outside of the parentheses. They might also ask if there are multiple ways to simplify a problem, if those methods of simplifying would always work, and additionally, why they each work.

Regardless of the type of curriculum given, teachers with lower levels of MKT tend to constantly proceduralize important mathematical ideas, while teachers with high MKT tend to conceptualize all lessons (Hill et al., 2008). Hill and Charalambous (2012) discussed that one teacher with high levels of MKT provided more opportunities for meaning making and regularly discussed brief snippets of meaning making during procedural lessons. However, much of the meaning making that has been discussed in Hill et al.'s (2008; 2005) studies tends to be focused on teacher talk and teacher meaning making. Based on constructivist beliefs, it is the students who need to be doing the meaning making, and one way that this can occur is through teacher prompting.

The difference between the current study and the studies mentioned throughout this section is that the current study seeks to find ways that teachers engage students in their own meaning making, and whether higher levels of MKT enable teachers to engage students in this thinking process. Hill and Charalambous (2012) suspect that the relationship between teachers' MKT and their ability to involve students in meaning making may be weak. Since the relationship between MKT and teaching practices is not straight forward, it is expected that MKT and the specific teaching practice of teacher prompting to engage students in meaning making will not be straight forward either. One of the major factors that is expected to interact with MKT and affect teachers' prompting practices is teachers' beliefs.

Beliefs and MKT

Beliefs as knowledge. MKT is discussed far more often than beliefs in research on mathematics education and one proposed reason for this is the difficulty in distinguishing the difference between beliefs and knowledge (Thompson, 1992). Some researchers as far back as Dewey have thought of beliefs as components of knowledge (Lloyd & Wilson, 1998), which aids in explaining the lack of ability in differentiating between the two terms. Fennema and Franke

(1992) listed four components of teacher knowledge: content knowledge, knowledge of learning, knowledge of mathematical representations, and pedagogical knowledge. While some of these components may be more content driven, other components, such as knowledge of learning and pedagogical knowledge are largely driven by the belief systems adopted by the individual. Therefore, the adoption of certain beliefs, which have been scientifically tied to student achievement, would increase your mathematical knowledge for teaching, and are thus a component of your knowledge.

Drageset (2010) studied this connection between growth in beliefs and MKT. He found that emphases on different beliefs were connected to different aspects of mathematical knowledge. His study, which focused on teachers' alignment with reasoning-based beliefs and rule-based beliefs showed that "an emphasis on reasoning is an affordance for the learning of specialized content knowledge, and that the learning of specialized content knowledge is an affordance for an emphasis on reasoning" (p. 45). Conversely, those who emphasized rules tended to have lower scores in both common content knowledge and specialized content knowledge, as well as less emphasis on reasoning, argumentation and justification. However, a lack of emphasis on rules has differing relationships with specialized content knowledge and common content knowledge. Scoring low on the rules construct acted as "an affordance for the learning of specialized content knowledge and as a barrier against the learning of common content knowledge" (p. 46). This makes sense, because teachers who do not focus on rules are typically exposed to more student methods, strengthening their connections and understanding of mathematical concepts, while focusing less on procedurally driven mathematics that common content knowledge encompasses. Moreover, learning more specialized content knowledge *and* common content knowledge acts as an affordance for not emphasizing rules. Drageset (2010) described this relationship between beliefs and knowledge to be not only connected, but elements that strengthen each other.

This cyclical and interactive nature between growth in MKT and beliefs shows the complexity of the relationship between the two factors. In fact, Thompson (1992) noted that “it is not useful for researchers to search for distinctions between knowledge and belief, but, rather, to search for whether and how, if at all, teachers’ beliefs—or what they may take to be knowledge—affect their experience” (p. 129).

Interaction of knowledge and beliefs in decision-making. The interpretation and implementation of curricula by educators is significantly influenced by both teachers’ knowledge and beliefs (Thompson, 1992). While professional development efforts are often aimed at helping teachers increase their knowledge, Cross (2009) states that beliefs are “considered to be very influential in determining how individuals frame problems and structure tasks” (p. 326). MKT is a strong predictor of effective practice, but alignment with certain belief structures is a necessary, but not sufficient, condition that must first be addressed in order for changes in MKT to play a role in instructional decision-making (Cross).

This relationship between beliefs and MKT also flows in the opposite direction. Similar beliefs do not always lead to the same enacted practices. Thompson (1992) acknowledged these inconsistences between teachers’ “professed beliefs” and their practices and offered that they may not be able to take actions that align with their beliefs because of a lack of knowledge needed to do so. Charalambous (2015) noticed a similar pattern and articulated this relationship as beliefs being the “inclination” to teach in a standards-based manner and the knowledge as being the “toolkit” needed to do so.

Beliefs as a mediator of MKT. Many researchers have found beliefs about learning to be a mediator for teachers’ MKT (Cross, 2009; Lloyd & Wilson, 1998; Sleep & Eskelson, 2012), even though teachers’ beliefs were often shaped by their knowledge (Hill et al., 2008). Although there is typically a positive correlation between low MKT and proceduralization of mathematical ideas, as well as high MKT and conceptualization of mathematical ideas (Hill et al.), a difference in beliefs can complicate this otherwise straightforward relationship. In a case study by Cross, one teacher's differing beliefs about the nature of algebra and geometry resulted in very different methods of teaching these two classes. This teacher believed that algebra was very formula driven, while also believing that geometry was more about teaching students how to think. One of the major disparities in her teaching that resulted from these differing beliefs was her questioning. During geometry lessons, more probing questions were asked and the teacher persisted in asking questions until students "produced valid justifications for their responses" (p. 333).

A similar case study by Lloyd and Wilson (1998) described a teacher who had a very deep, conceptual understanding of mathematics. Although this teacher had a knowledge set that would appear to be optimal for teaching mathematics, he kept these understandings separate from his traditional beliefs about how to teach mathematics. MKT, beliefs, and curriculum are not independent factors. Each of these components are interrelated and come together to contribute to a teacher's mathematical quality of instruction (Sleep & Eskelson, 2012).

Questioning

As previously mentioned, dialogue plays an important role in students' construction of knowledge. Teacher questioning is a practice that guides this dialogue in a manner that enables students to think more deeply about mathematics. In order to push for this depth, teachers must first believe that this depth in understanding is important, and they must also have the knowledge themselves to know how to guide meaningful dialogue in a productive manner. With suitable beliefs and MKT, teacher questioning can be an extremely effective tool for reaching higher levels of student achievement (Boerst, Sleep, Ball, & Bass, 2011).

Purpose of questioning. The act of teacher questioning has multiple purposes in shaping the classroom learning environment and is an important aspect of instruction in mathematics education (Almeida, 2010; Course, 2014; Cullen, 2002; Franke et al., 2009). Questioning includes teacher acts that are used "to structure the conversations in ways that allow for all students to participate in meaningful mathematical work" (Franke, Kazemi, & Battey, 2007, p. 233). Harel (2008) stated, "there will always be a difference between what one can do under expert guidance or in collaboration with more capable peers and what he or she can do without guidance" (p. 894). While this statement may seem obvious, it also points out the importance of the role that discourse plays in engaging students in mathematical thinking and learning.

Students' opportunities to learn are shaped by the questions teachers ask, both through the content teachers focus on and the various types of participation they make available (Boerst et al., 2011). Classroom discourse presents the largest opportunity for the teacher to interact with students during the learning process (Yackel, Cobb, & Wood, 1998). While teachers can gain some insight into students' thinking through written work and assessments, Heid, Wilson, and Blume (2015) note that this information is highly inferential. They state, "it is through a particular kind and quality of discourse that implicit mathematical ideas are exposed and made more explicit" (p. 26). When a teacher prompts students to engage in discourse about this kind of mathematics, it allows them to see the unique ways that students interpret and solve problems (Gall & Gillett, 1980), as well as give teachers multiple opportunities to assess students level of understanding and location in the development of mathematical concepts (Henning, McKeny, Foley, & Balong, 2012).

Posing worthwhile tasks and navigating classroom discourse in a productive way are two of the most important aspects of teaching and creating opportunities for student engagement in learning (Ni et al., 2013). However, this statement relies on the belief that mathematical learning is "a constructive, interactive, problem solving process" (Gall & Gillett, 1980, p. 99). Teacher follow-up questions should take the mathematical task a step further by seeking out clarification on ambiguous explanations, reasoning underlying student errors, further elaboration on problem-solving strategies, and important underlying mathematical ideas (Franke et al., 2009). This probing allows teachers to elicit higher levels of cognitive demand in students' thinking (Smart & Marshall, 2013), particularly when students are not used to practices of mathematical sense making (Øystein, 2011). Probing sequences of specific mathematical questions have many benefits to help both the teacher and the students understand the content in more depth.

Additionally, student thinking can be clarified and other students can make connections to the mathematical concepts that are discussed amongst peers (Franke et al., 2009).

Engaging students in deep mathematical thought. The simple act of being in a classroom where a lesson is being taught does not directly lead to student engagement in learning. Wood, Williams, and McNeal (2006) define engagement in mathematical learning as “the mental activity involved in the abstraction and generalization of mathematical ideas” (p. 226). Mathematical learning can be considered an individual act of building up one’s own cognitive structures, or a process of creating a shared mathematical meaning with a group of learners (Gall & Gillett, 1980). Students must be engaged in mathematical learning in order for deep understanding of a mathematical concept to be constructed. Stein, Grover, and Henningsen (1996) state that complete understanding includes “the capacity to engage in the processes of mathematical thinking, in essence doing what makers and users of mathematics do; framing and solving problems, looking for patterns, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging, and so on” (p. 456). In order for students to fully engage in mathematical practices, they must see themselves as doers of mathematics (Franke et al., 2007).

Even with students’ active engagement in demanding tasks, teacher questioning is needed to support students’ high-level thinking (Henningsen & Stein, 1997). The advantage gained through questioning, that cannot be controlled as well through mathematical tasks, is that teachers are able to extend students’ thinking in real time (Cengiz, Kline, & Grant, 2011). Questioning is a major tool that teachers can use to help fill the gap between students’ current level of understanding and the teacher’s desired level of student understanding (Brodie, 2010). Unlike tasks, that are not able to adapt to students current and ever-changing level of understanding, the teacher is able to meet students at their current cognitive level through dialogical exchanges and move students beyond their initial thinking to make connections and consider various complex ideas (Cengiz et al., 2011). Cengiz et al. called this event an ‘extending episode’ and found three main categories that these episodes fell into; encouraging mathematical reflection, going beyond initial solution methods, and encouraging mathematical reasoning. While these episodes are helpful in strengthening students' ability to reason about mathematics, the study also found that a certain level of teacher knowledge was needed to effectively carry out extending episodes. When teachers have a higher level of knowledge about possible student solution methods and ways of thinking, there are better able to orchestrate the discussion that is involved in maintaining cognitively demanding tasks and thinking (Henningsen & Stein, 1997).

Two additional studies have also found the importance of teachers' questioning in pushing students' thinking. In a study by Øystein (2011), students immediately looked for a formula or algorithm when initiating the problem solving process. The process of reasoning did not come naturally without teacher intervention. When prompted to think about the meaning of the mathematics involved in the problem, students were able to focus more on the underlying mathematical concepts and work their way through the problem more independently.

Similarly, Franke et al. (2009) reported, in "segments in which teachers did not ask questions about students' explanations, students did not provide elaboration. When teachers did ask questions, however, students were much more likely to provide elaboration" (p. 385). Elaboration is a powerful tool that solidifies student thinking and understanding. Students need the opportunity to both hear ideas from the teacher and have the opportunity to articulate their own understanding, while negotiating meaning with their classmates (Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008). If students are unable to elaborate on their thoughts and ideas, this should be considered an indicator of a lack of true understanding (Henning et al., 2012).

Researchers in the area of science education have recently made a large link between the cognitive level of student thinking and teacher questioning (Smart & Marshall, 2013). Smart and Marshall (2013) noted, "teachers have the unique opportunity to facilitate higher cognitive levels in their students by the questions they ask during instruction and the communication pattern they establish in their classrooms" (p. 265). Their results indicated that 45% of the variance in the cognitive level of student thinking was accounted for by the linear combination of various discourse factors. The two factors that had the largest effect on students' cognitive levels were Questioning Level and Communication Pattern. Questioning Level refers to the intended level of cognitive demand of the question, and Communication Pattern refers to the ways in which teachers engage students in discourse, i.e. mainly teacher talk, versus involving student ideas in the process of learning. Although it is unclear whether these results transfer to mathematics education, they are a good indicator of possible factors that affect the cognitive levels of student thinking in mathematics classes.

Teacher prompting is at the heart of maintaining cognitive demand, and it directly affects the types of experiences students have in mathematics classes (Cullen, 2002). However, in order to maintain high cognitive demand, teacher questioning should move beyond low level tactics and aim at developing students' explanations of mathematical concepts by building off of students' ideas (Franke et al., 2009). Teachers have a crucial role in encouraging students to explore justifications, explanations, and meaning through their questions, comments, and feedback (Stein, Engle, Smith, & Hughes, 2008). Resnick and Zurawksy (2006) suggest that teachers should be encouraging students to solve problems that they have not been exposed to, with multiple methods and representations, and they should be required to justify their work through communication with the class. These practices involve the higher-level practice of reasoning. Lithner (2007) defines reasoning as "the line of thought adopted to produce assertions and reach conclusions in task solving" (p. 257). Therefore, reasoning, a higher level of cognitive demand, does not refer to the end product, but the thought that goes into reaching some kind of conclusion. In mathematics, reasoning is a very useful skill for tasks such as communicating about mathematical ideas, proving and constructing theories, and verifying the truth of a mathematical statement (Lithner, 2007).

Meaning making in conceptual and procedural lessons. Regardless of whether intended learning is procedural or conceptual, mathematical meaning is an important underlying factor that should always be considered part of the learning goals. Hess, Jones, Carlock, and Walkup (2009) stated, "students need a greater depth of understanding to explain how or why a concept or rule works, to apply it to real-world phenomena with justification or supporting evidence, or to integrate a given concept with other concepts or other perspectives" (p. 5). Student engagement in learning at low cognitive levels has not been found to build powerful knowledge or understanding. If students are engaged at low cognitive levels, the most that can happen in terms of their learning is routine memorization, no matter how advanced the mathematics. Similarly, in order for students to build true mathematical understanding they must be engaged in high cognitive levels, even when the content is basic (Resnick & Zurawsky, 2006).

Although all lessons should involve meaning making, certain types of lessons typically lead to specific questioning practices. A study by Ni et al. (2013) found that the implementation of high cognitive demand tasks lead to higher level questions from the teacher. Similarly, Henning et al. (2012) found that longer and more mathematically complex tasks lead to more conceptual discussions. However, during these conceptual discussions, the teacher tended to provide higher levels of guidance, which lead to lower levels of student participation. These studies both address Stein and Lane's (1996) call for research that analyzes "the kinds of discourse that are or are not enabled by various kinds of tasks, and the classroom-based factors that either support or inhibit student engagement at high levels with tasks that are intended to be complex and challenging" (p. 76). However, there are still factors left to explore between these two interactive practices, such as how MKT and beliefs about teaching and learning affect the teacher's ability to ask high level questions during various types of tasks.

Types of questioning. The types of questioning practices that teachers embrace can raise or lower intended levels of cognitive demand through their use in mathematical classroom discourse. Several types of prompts have been noted as conducive for higher levels of intended cognitive demand. These prompts are typically open questions, which lead to more elaborate answers by students, or prompts that engage students in making connections or generalizations

(Almeida, 2010; Course, 2014). Adhami (2001) notes, prompts that are based on student ideas, and call for "negotiation of meaning, handling of misconceptions, and attention to minute and idiosyncratic steps of reasoning" (p. 28) are also of benefit to both teachers' and students' depth of mathematical understanding.

Concurrently, prompts that end up diminishing high levels of intended cognitive demand typically include closed, short response questions (Almeida, 2010; Course, 2014) and leading questions where the "teacher assumed much of the mathematical work while supporting students when moving them through correct and complete explanations" (Franke et al., 2009, p. 390). Furthermore, when the teacher exhibits control over the conversation, student participation is largely based on the teacher's agenda and the teacher's line of questioning is reduced in complexity to narrow in on the desired method to be learned (Emanuelsson & Sahlstrom, 2008).

A framework by Stein et al. (1996) classified the cognitive demand of mathematical tasks into four different categories. Since a task may have multiple levels of cognitive demand at various points throughout the lesson, shifts in the demand can be seen through teacher questioning. Therefore, these four levels of cognitive demand can also be used to code a teacher's line of questioning about the task as students work through them. The four categories are: 1) memorization, 2) formulas, algorithms, or procedures *without* connections to concepts, 3) formulas, algorithms, or procedures *with* connections to concepts, and the highest level, 4) "doing mathematics". Doing mathematics involves "complex mathematical thinking and reasoning… such as making and testing conjectures, framing problems, looking for patterns, and so on" (Stein et al., p. 466). Tasks that were considered to demand high cognitive levels were "doing mathematics" and "procedures with connections to concepts". The remaining two categories were considered to exhibit low-levels of cognitive demand. The two low levels, memorization and formulas, algorithms, or procedures without connections to concepts, are what are primarily seen throughout mathematics classrooms in the United States. Since students need to experience mathematical meaning making during both procedural and conceptual lessons, this framework will be used in this study to capture the amount of high level demand student are experiencing in various types of lessons.

Difficulties in questioning. The above-mentioned research paints a clear picture of the importance of good teacher questioning. However, many teachers have a hard time using this practice to maintain high levels of student thinking and engagement. It is important to remember that providing the opportunity for students to participate in meaningful, reflective discourse only supplies the conditions for learning. It is the student who must choose to engage in the deep thinking that enables one to build sound cognitive structures (Cobb, Boufi, McClain, & Whitenack, 1997). Even if the student is engaged in the discussion, it is not an easy task for the teacher to continue to guide students to deeper levels of understanding. The teacher must be able to follow various lines of student thinking and also keep students from getting lost in the talk (Silver & Smith, 1996). Successfully guiding meaningful class discussions is somewhat of a juggling act. The teacher must help keep students cognitively engaged in complex mathematical thinking, while also making discourse moves that are attentive to the thoughts and ideas that students are bringing to the discussion (Boerst et al., 2011).

Challenging students to continuously think deeply about mathematics can be somewhat counterintuitive to a teacher's view of "helping" students. Studies have shown that many teachers have a difficult time maintaining these high levels of cognitive demand, and teacher moves commonly deprive students of these opportunities by guiding students towards a correct solution or method of solving a problem (Lithner, 2007). Similarly, Stein et al. (2008) and Henningsen and Stein (1997) state that teachers reduce the level of cognitive demand when they do not value the accuracy of students' explanation and focus on simply completing the tasks with correct answers. In doing so, the teacher may be doing most of the cognitive work and requiring students to do little more than single step basic arithmetic or recall of vocabulary (Lithner, 2007 Yackel et al., 1998). This is especially true when teachers ask a question and there is a lack of student response. In these cases, teachers often respond by answering the question themselves

(Emanuelsson & Sahlstrom, 2008). This common response trains students to have an overreliance on procedures and algorithms, as well as teacher guidance through any problem that might be seen as difficult (Silver & Smith, 1996).

Setting up an environment that is conducive to meaningful dialogical processes is not an easy task for teachers. Unless the teacher creates an environment where students feel safe to share their ideas or questions with the class, meaningful discussions will not develop (Silver & Smith, 1996). By establishing social norms, teachers can create a space for students to build a shared meaning of mathematical concepts and procedures. Unfortunately, these kinds of processes are not commonplace in U.S. classrooms, and even with a good task, many teachers have difficulties moving student explanations beyond procedural processes (Silver & Smith).

Teaching in this manner requires teachers to have a certain level of MKT. Wilhelm (2014) found that teachers who scored in the top quartile for MKT were able to maintain cognitively demanding tasks three times more often than those in the bottom quartile. Moreover, Stylianides, Stylianides, and Shilling-Traina (2013) found that reasoning and proof is not often found in elementary level mathematics classes, partially because elementary teachers have weak mathematical subject knowledge, and also have beliefs about mathematics that are counterproductive in a cognitively demanding environment. However, even upon removal of these issues, teachers still had difficulty keeping students engaged at high levels of cognitive demand, partially because students were not used to the practice of mathematical sense making.

Questioning and student achievement. When students engage in cognitively demanding mathematical tasks and discussions they are more likely to see larger gains in their mathematics understanding and achievement (Boaler & Staples, 2008; Stein & Lane, 1996), and this relationship is even stronger in classrooms where high levels of cognitive demand are maintained (Kessler, Stein, & Schunn, 2015). Promoting opportunities for cognitively demanding discussions in the classroom can help students develop a deeper understanding of mathematical concepts (Boston & Smith, 2009). Franke et al. (2007) note that “one of the most powerful pedagogical moves a teacher can make is one that supports making detail explicit in mathematical talk, in both explanations given and questions asked" (p. 232). If students are not being presented with tasks that are cognitively demanding and engaging, "there is little reason to expect that scores on measures of learning outcomes will reflect enhanced understanding or increased ability to think and problem solve" (Stein et al., 1996, p. 457). Stein et al. found that students actually used multiple solution strategies, multiple representations, and mathematical explanations and justifications in the majority of cases when teachers appropriately set up tasks. It would be expected that student success would be more likely in tasks that allowed students to solve problems in multiple ways, particularly ways that made sense to the individual student.

Discussions are also the largest known factor in promoting conceptual understanding among students (Boerst et al., 2011). If teachers are able to ask focused questions, these questions can be used to help students focus and make sense of their ideas, as well as push their thinking further than it would otherwise go (Franke et al., 2009). The ways in which teachers involve students in social interactions will affect how students construct mathematical knowledge. Wood et al. (2006) found, interaction patterns that required greater involvement from the participants were related to higher levels of expressed mathematical thinking by children" (p. 248).

Stein and Lane (1996) found that the greatest gains in students' mathematics achievement were related to the use of cognitively engaging tasks that created the opportunity for students to participate in the process of 'doing mathematics'. Henningsen and Stein (1997) confirmed these results and noted that the "instructional practices that have been found to support high levels of cognitive engagement for students include; "scaffolding, modeling high-level performance, and consistently pressing students to provide meaningful explanations" (p. 534). Classrooms that had the greatest proportion of these practices also showed the greatest gains in student achievement.

To strengthen the understanding of this relationship student gains were also much smaller in classrooms where tasks were focused on more procedural activities with little to no mathematical communication (Stein & Lane, 1996). Similarly, Wilhelm (2014) found that mathematics tasks that had low levels of cognitive demand only required memorization or reproduction of mathematical skills or facts, while tasks with high levels required students to make connections and consider the conceptual ideas that lie beneath the surface of those facts and skills.

Although some may believe that high level cognitive engagement is only appropriate after mastery of basic skills, “these relationships between reform instructional practices and positive student learning gains were found to hold true for a population of students who, most likely, had not completely mastered the basic skills and facts of the elementary curriculum” (Stein & Lane, 1996, p. 75). When students are given an opportunity to think about mathematics at higher cognitive levels, they build knowledge that is more transferable to new or more complex situations (Hess et al., 2009). Franke and Kazemi (2001) refer to this idea as “Generativity”. They note, “Generativity refers to individuals’ abilities to continue to add to their understanding. When individuals learn with understanding, they can apply their knowledge to learn new topics and solve new and unfamiliar problems” (p. 105). This kind of knowledge base and transferability is what is sought after in applicants for many of the 21st century jobs (Resnick & Zurawsky, 2006). When a student learns to think critically, they are better able to “see clearly the relationship between evidence and conclusion, and to be proficient at providing reasons in support of one’s beliefs” (Mulnix, 2012, p. 473). These cognitive skills allow for meaningful discussion that will aid students in acquiring even more powerful conceptual structures. Although students learn some reasoning skills through their life experiences, they do not learn mathematical reasoning or critical thinking without being given repeated opportunities to do so in a classroom environment. This speaks to the importance of the use of cognitively demanding experiences in mathematics classes (Øystein, 2011).

In summary, we know that beliefs and MKT play a pivotal role in the actions that teachers take and the levels of achievement that students are able to reach (Hill et al., 2008), however, the effects of MKT and beliefs are not straight forward (Sleep & Eskelson, 2012).

These effects are specifically weak in engaging students in reasoning and meaning making, which is the main purpose of questioning students. By looking at the ways in which MKT and beliefs affect teachers questioning practices, this study will allow for a closer look at why this relationship may be weak, and what factors are allowing some teachers to ask students questions that promote opportunities for critical thinking and conceptual development. The following chapter will guide the reader through the methodological framework that supports this inquiry.

Chapter 3: Methodology

Given the complex and interactive nature of teacher questioning, MKT, and beliefs on learning discussed in earlier chapters, a multi-step, mixed methods approach was used to seek a deeper understanding of how the quantitative results play out in individual classrooms. Quantitative methods were used to look for patterns across all teachers in the sample, while qualitative methods were used to look for evidence within and across cases that supports or contradicts quantitative findings. Cross-case analysis methods were used to get an in-depth understanding of the practice of teacher questioning within a real-life context and setting (Creswell, 2013).

Theoretical Framework

As discussed in chapter 2, this study is guided by constructivist perspectives, which view learning as a process that happens when the learner, or student, experiences an event in the lesson, and that experience causes the student to engage in thinking critically about how these new ideas fit with their current ideas or conceptions. The teacher's role in this case is to engage with the student by questioning them to think deeply about the concepts at hand. The teacher's level of MKT and their beliefs about how students learn are believed to affect this questioning practice (Hill et al., 2008). As the teacher asks questions to the student, the student's responses give the teacher a clearer picture of the student's current level of understanding, which informs the teacher's future questions. This is an iterative process that is intended to eventually result in the expansion of the student's realm of current knowledge. This view of the learning process through interaction between teacher and student is shown in Figure 1 and can be viewed as the larger theoretical framework that guides this study. This particular study investigates the role of the teacher in this process, which focuses on the ways that MKT and Beliefs inform and affect a teacher's questioning practices.

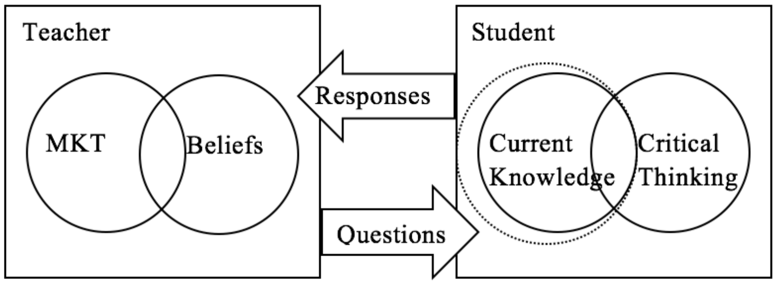

Figure 1: Theoretical Framework and Focus for Study

Participants

To select participants for the study, a series of surveys and assessments were given to a sample of 20 third through fifth-grade teachers from four elementary schools within a rural Midwestern town. The town is approximately 85% white and primarily middle to lower-middle class. The teachers in this sample were involved in professional development designed and led by a research team that includes the author at the time of the study. To add the effects of the professional development as a variable, teachers from the district that had not previously attended the workshops, but planned to join, were included for comparison purposes.

From this sample of 20 teachers, six teachers were chosen for further investigations. These teachers were chosen based on their unique profile of MKT scores, belief survey results for both the reasoning and rule-based constructs, teaching experience, and years of involvement in PD at the beginning of the study. Table 1 shows the selection data for the six teachers that were chosen. All names presented in the study are pseudonyms.

Table 1: Teacher Selection Data

Name	MKT Z-Score	Reasoning Belief Z-Score	Rule Belief Z-Score	Years of Teaching Experience	Years of PD Involvement

Han	+1.64	+0.64	-0.13	6	3
Sue	-0.79	-0.29	-0.74	13	2
Alesha	+0.08	+1.30	+0.93	5	0
Wendy	-0.84	-0.29	+0.78	18	2
Coach	+0.27	-2.11	+0.17	6	0
May	-0.16	+0.61	+1.08	3	1

Table 1—Continued

The values for MKT, reasoning, and rule belief scores are presented as Z-scores for easier comparison. Z-scores represent the number of standard deviations a score is from the mean. For example, Han's MKT score of +1.64 can be interpreted as 1.64 standard deviations above the mean. This is equivalent to the 95th percentile when compared to the national norm for this assessment. Han's Z-scores for reasoning beliefs and rule beliefs are +0.64 and -0.13, respectively. This shows that his belief in reasoning is 0.64 standard deviations above the mean of the group of 20 teachers assessed, and his belief in rules is 0.13 standard deviations below the mean of the 20 teachers assessed.

Han was chosen for analysis because of his high reasoning beliefs and high MKT score, which was the highest out of all 20 teachers in the original sample. Additionally, Han had been involved in PD for 3 years, which is longer than any other participants. Sue was chosen for her Low MKT score and her low belief in rules. Alesha was chosen as an interesting case because she had the highest belief in reasoning, as well as a high belief in rules. She also had not been exposed to PD at the beginning of the study, as she was new to the district. Wendy was chosen as the teacher with the lowest MKT score out of all 20 teachers, as well as the teacher with the most years of experience in teaching. Coach was chosen as the lowest reasoning score out of all 20 teachers, and like Alesha, Coach was also new to the district and had not had any experience with the PD group. Lastly, May was chosen as a case that was closest to average MKT and rule aligned. These six teachers represented a balanced representation of these variables within the original sample of 20 teachers.

The six teachers that were chosen for further analysis were located within two K-5 elementary school buildings. The buildings are similar in size and make-up, with two teachers at each grade level. Both schools share May, who is an English as a Second Language (ESL) teacher, for their increasing Spanish speaking population. All classrooms of teachers in the study had students sitting in groups at tables, with numerous whiteboards available for the teacher or students to write on during class. The majority of students in the classes come from families that are lower to middle class in terms of SES, and a large portion of students come from the surrounding farming communities.

Data Description

Quantitative measures. In order to analyze how MKT and beliefs affect teachers' questioning practices, two quantitative measures were used. The Mathematical Knowledge for Teaching Assessment (Ball et al., 2008) was used to measure MKT, and a belief survey developed for Drageset's (2010) study on rule-based and reasoning-based beliefs was used to measure teachers' beliefs about teaching and learning.

MKT. Teacher's knowledge was measured through the Mathematical Knowledge for Teaching Assessment developed by Ball and colleagues (2008) through the University of Michigan's Learning Mathematics for Teaching (LMT) project. Teachers take a computer adaptive test that focuses on the content domain of Numbers and Operations. The measure also offers tests in the domains of Geometry and Measurement, and Patterns, Functions, and Algebra. The test uses items that reflect real situations that could occur in a mathematics classroom including, but not limited to, assessing students' work or ideas, using various representations, and explaining mathematical rules or procedures. The assessments were scored by the LMT project using Item Response Theory (IRT), which reports an estimated score of the actual teaching ability of the given teacher. National norms were computed from a sample size of 438 teachers representing teachers from each state (Blunk, Hill, & Phelps, 2005). These norms fall along a standard normal distribution, therefore, a score of 0 represents an average teacher, while a score of 1 or -1 represents a score of a teacher one standard deviation above or below the mean, respectively. The reliability for the Numbers and Operations MKT assessment that was used for this study is > .80 (Hill, Schilling, & Ball, 2004). Investigation of the validity of this measure is ongoing, but both measurements of the validity of the assessment and validity of the assessment content in regards to the *Principles and Standards of School Mathematics* support the use of this assessment (Hill & Ball, 2004).

Belief. Teachers' beliefs about teaching and learning were measured through a 21-item survey given to teachers at the end of the first year of the project. The survey from Drageset's (2010) study is built around two constructs entitled "rules" and "reasoning". The rules construct measures teachers' alignment with an emphasis of formal mathematics and "the learning of rules as most important, without focusing on explanations or connections" (p. 37). The reasoning construct measures teachers' alignment with the belief that "reasoning, argumentation and justification are more important than the answer" (pp. 37-38).

Teachers answered all survey questions on a four-point Likert scale. Twelve questions in the rules and reasoning constructs focus on teachers' agreement with statements. The four point Likert scale asked teachers to choose whether they (1) disagree entirely, (2) disagree somewhat, (3) agree somewhat, or (4) agree entirely with each statement. The higher the score in each construct, the more the teacher's beliefs align with that construct. The remaining nine questions of the rules and reasoning constructs focused on teachers' perceived importance on the survey statements. For these questions, the Likert scale points are; (1) not very important, (2) somewhat important, (3) important, and (4) very important. Again, the higher a score in a construct, the more aligned the teacher is with that construct. The reported reliability for the ten survey statements in the rules construct is 0.71, and 0.81 for the eleven survey statements in the reasoning construct. The survey questions for each construct can be found below in tables 2 and 3. The survey given to teachers intermixed statements from both constructs.

Table 2: Statements for the Rules Construct (Drageset, 2010)

Please indicate the extent to which you agree with the statements below (disagree entirely, disagree somewhat, agree somewhat, agree entirely)	
1	The more important aspect of mathematics is to know the rules and to be able to follow them
2	Mathematics means finding the correct answer to a problem
3	The best way to learn mathematics is to see an example of the correct method for a solution, either on the blackboard or in the textbook, and then to try to do the same yourself
4	If you cram and practice enough, you will get good at mathematics
5	Those who get the right answer have understood
6	Mathematics should be learned as a set of algorithms and rules that cover all possibilities
7	What you are able to do you also understand
8	In mathematics, it is more important to understand why a method works than to learn rules by heart [opposite]
Please indicate how important you think each element below is (not very important, somewhat important, important, very important)	
9	Learning rules and methods by heart
10	Learning formal aspects of mathematics (e.g. the correct way to write out calculations) as early as possible

Table 3: Statements for the Reasoning Construct (Drageset, 2010)

Please indicate the extent to which you agree with the statements below (disagree entirely, disagree somewhat, agree somewhat, agree entirely)	
1	The pupils learn more mathematics from problems that do not have a given procedure for solution, where instead they have to try out solutions and evaluate answers and procedures as they go
2	It is important to be able to argue for why the answer is correct
3	Solving mathematical problems often entails the use of hypotheses, approaches, tests, and re-evaluations
4	The pupils learn from seeing different ways to solve a problem, either by pupils presenting their solutions or by the teacher presenting alternative solutions
Please indicate how important you think each element below is (not very important, somewhat important, important, very important)	
5	The ability to explain their answers
6	The ability to argue for their procedures and answers

7	Being able to explain their reasoning
8	Being able to evaluate other procedures than their own
9	Being able to follow the reasoning of another pupil
10	The ability to solve complex problems where the pupils have to use several aspects of mathematics
11	Teaching must focus on understanding as much as possible so that the pupils can explain methods and connections

Table 3—Continued

Qualitative measures. Video recordings of mathematics classes were collected and individual interviews were completed in order to gain a deeper understanding of the questioning practices that teachers enact during their teaching. These measures also allowed the researcher to gain insight into teachers' beliefs and knowledge of mathematics, which aided in answering the research questions.

Videos. Four videos of whole mathematics lessons were self-recorded by teachers with iPads, along with audio-recording bracelets to ensure that all teacher questions could be clearly heard in order for them to be transcribed and coded. Teachers recorded two lessons that they considered to be procedural, and two that they considered conceptual. These considerations were based on definitions of procedural and conceptual lessons that were given to teachers (these definitions will be discussed further in the data collection procedures). Two of the four videos for each teacher were used in data analysis, the rest of the videos were not considered for the current study. The two videos chosen consisted of one procedural lesson and one conceptual lesson. While some lessons provided by the teachers loosely fit definitions, selection of one of each type of lesson ensured that the analysis would be focused on lessons that more closely aligned with each lesson type.

Interviews. These interviews consisted of two sections of questions: 1) beliefs and questioning, and 2) conceptual knowledge. The purpose of these semi-structured interviews was to gain a more in depth insight into the teachers' beliefs about learning and questioning practices, and their knowledge of the conceptual aspects and connections involved in the topics discussed in their recorded lessons. The majority of questions on beliefs and questioning were designed by the researcher prior to analysis, and based on ideas within the review of the literature that may provide insights to the research questions. Interview questions that probed conceptual knowledge were designed by the researcher after viewing each individual teacher's lessons and coding the data. The guide used for the development of these interview questions can be seen in the Appendix under section IV. Questions were designed similar to the format of the MKT assessment questions, but probed teachers' knowledge of the specific content within their two lessons. These questions were an assortment of open-ended questions, analysis of student work and methods, and situational questions where the teacher decides whether the application of the concept is appropriate. The list of questions and guidelines for creating individual questions can be seen in the Appendix.

Data Collection Procedures

The first year of the professional development project has focused on content development in numbers and operations as well as methods on engaging students in meaningful mathematical thought and action. Since research suggests MKT is localized to specific content domains (Hill et al., 2005), all data collected for this study are focused on the numbers and operations domain.

Data collection and analysis was carried out over three phases. Phase 1 consisted of administration of the measures and selection of the participants, Phase 2 involved data collection, coding, and analysis of teacher question patterns, and Phase 3 included a cross-case analysis, interviews, and member checking. Figure 2 shows the flow of these three phases and the remaining sections of this chapter will discuss these phases in more detail.

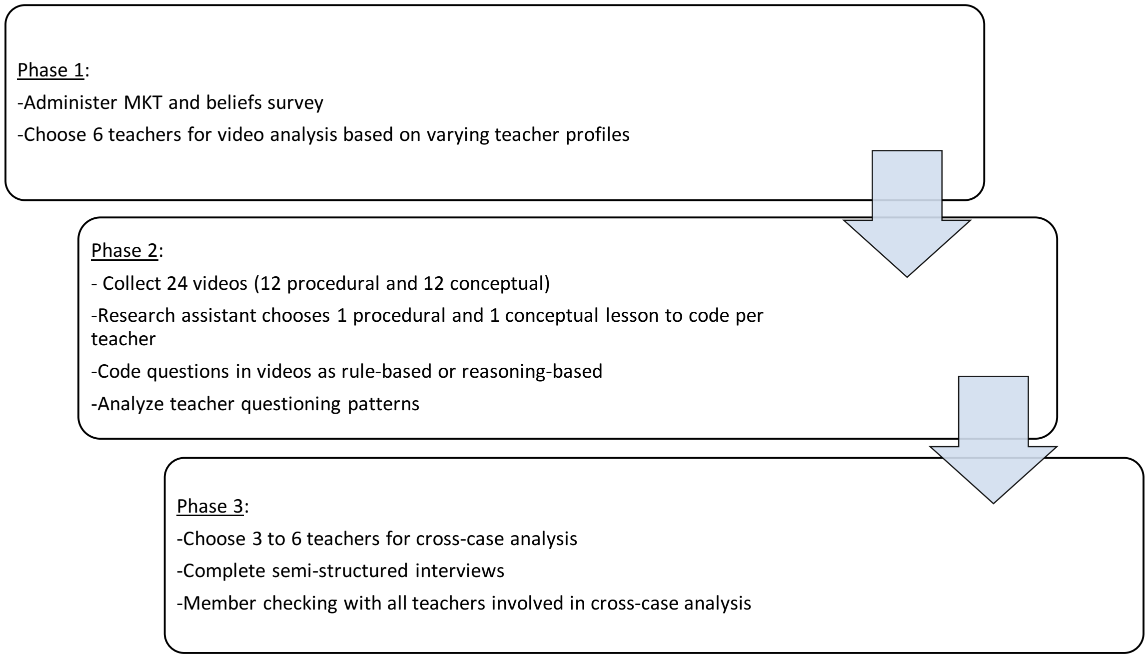

Figure 2: Data Collection and Analysis Phase Flow Chart

Phase 1: MKT and belief. Both the MKT and teacher belief survey were given to all 20 teachers in order to find varying levels and mixtures of MKT and beliefs. Teaching experience and status of participation in professional development were also used as variables to help find distinct cases for further study (Cross, 2009). Adding teaching experience allowed the researcher to evaluate whether experience in teaching, or more specifically, repeated teaching of the same level of mathematics classes allows teachers to refine their questioning practices.

After the MKT assessment and belief survey were administered and scored, six teachers were chosen for further participation in the study based on their unique MKT and reasoning and rules profile, previously gathered from the MKT assessment and beliefs survey. Since the rules and reasoning scores are two different constructs within the belief survey, and therefore may not be compiled into one score, reasoning scores were used for initial distribution of teacher profiles, and rule scores were used to add variation in teacher profiles. The reasoning score was chosen because it was predicted to be a stronger indicator of meaningful questioning, since the construct favors reasoning and justification over answers and memorized procedures (Drageset, 2010). The process for choosing teachers for further analysis included placing all teachers in a matrix of MKT and reasoning belief levels. MKT placement was based on IRT national averages to aid in generalizations of the results. MKT scores were divided into high, medium, and low, based on a cut score of one standard deviation above and below the average. Scores equal to or above 1 standard deviation will be considered high, between 1 and -1 considered medium, and equal to or below -1 considered low.

Due to a lack of national averages and use of the belief survey to group teachers based on belief scores, a mean and standard deviation was calculated from the sample data for both the reasoning and rules construct. Similar to MKT data, reasoning scores were divided into high, medium and low based on the same cut scores of one standard deviation above and below the average. This matrix can be seen in Table 4. Each teacher was placed in one of the nine appropriate cells based on his or her scores. Since teacher scores clustered in four cells, rule construct scores, teaching experience measured in both years teaching the current grade level mathematics, and total years teaching, along with status of participation in professional development, were used to choose between teachers with similar placement in the matrix. The six teachers chosen provide as unique of an array of teacher profiles as the data allowed. The location of the six teachers chosen for analysis can be seen in the cells within Table 4.

Table 4: Matrix of Teacher MKT and Belief Scores

	MKT Scores		
Reasoning Belief Scores	High	Medium	Low
High		1 Teacher	
Medium	1 Teacher	3 Teachers	
Low		1 Teacher	

Phase 2: Videotaping lessons. Each of the six teachers were asked to videotape four lessons, two of which they deemed primarily procedural lessons, and two that they deem primarily conceptual. A research assistant, who was trained by the researcher, chose one lesson that best fit the definition of a procedural lesson, and one lesson that best fit the definition of a conceptual lesson from each teacher's four recorded lessons. The selection of two lessons allowed the researcher to ensure that the lessons chosen for analysis aligned closely to the definitions of procedural and conceptual, while also eliminating possible bias of the researcher. Procedural lessons were defined as lessons built around "tasks which ask students to perform a memorized procedure in a routine manner" (Stein & Lane, 1996, p. 54). Typical procedural lessons are focused on students performing mathematical procedures that are not tied to any real-life context. The majority of class time is spent either learning a step-by-step procedure, or practicing one or more given procedures. The research assistant looked for these characteristics when choosing between procedural lessons. Conceptual lessons were defined as lessons built around "tasks that demand engagement with concepts and that simulate students to make connections between and among ideas" (p. 54). Lessons fitting in this category typically involve problem-solving or tying mathematics into real-world context. These lessons allow students to think about the meaning behind different mathematical ideas, for example, what does it mean to divide and how does this idea apply to a particular situation. Other conceptual lessons might focus on the connections or similarities between multiple solution methods. These were the characteristics that were sought out during selection of conceptual lessons. This left the researcher with 12 lessons to analyze for the nature of teachers' questioning; six conceptual, and six procedural.

Phase 3: Interviews. One semi-structured interview was conducted individually with each teacher, and lasted approximately 30-45 minutes per teacher. All six teachers chosen for analysis were interviewed individually after the video analysis and coding was completed. The semi-structured interview guide can be seen in the Appendix. The researcher followed this list of predetermined questions, but asked additional questions to probe teachers' answers more thoroughly if their answers began to provide insights relevant to the research questions, and therefore needed more exploration. The audio of these interviews were recorded and transcribed for analysis purposes.

Information gathered in the interviews helped the researcher understand if a lack of conceptual questioning during procedural lessons was due to a lack of knowledge or a lack of standards-based beliefs about learning. Interviews also helped the researcher determine whether the MKT scores account for the more specific conceptual knowledge needed to question students in a way that helps them make sense of the mathematical ideas and concepts presented in the given lessons.

Table 5 gives a brief overview of the data sources, their purpose within the study, their supporting sources, the research question they support, and the phase that they will be used in during the study.

Table 5: Data Sources and Their Purpose

Data Collection Phase	Datum Source	Purpose	Research Question
1	Mathematical Knowledge for Teaching (MKT) Assessment	Measurement of teachers' MKT (Ball et al., 2008; Hill et al., 2005)	2
1	Beliefs about Teaching and Learning Survey	Measurement of teachers' beliefs on teaching and learning (Drageset, 2010)	2
1	Teaching experience and professional development participation status	Add variation in teachers' profiles to help explain varying questioning patterns (Cross, 2009; Drageset, 2010)	2
2	Videotaped lessons	Analyze teachers' questioning practices in procedural and conceptual lessons (Miles, Huberman, & Saldaña, 2013)	1, 2
3	Semi-structured interviews	Triangulate data on beliefs and MKT (Merriam, 2009)	1, 2

Data Analysis

Each research question requires varying methods of data analysis; therefore, each question's analysis methods will be discussed separately below.

Research Question 1: Does the nature of teachers' questioning differ between conceptually based lessons and procedurally based lessons? To answer this question, teachers' questions from each of the two chosen lessons were identified within the transcript and coded using Stein et al.'s (1996) framework. The researcher identified questions within the lessons while watching through the videos and reading through the transcripts simultaneously. Questions were only pulled for coding if they were mathematical in nature. Questions that referred to students' feelings about mathematical ideas were not counted unless the teacher included an opportunity for students to reason about why they had those particular feelings. For example, a question such as "which is your favorite method" would not be included in analysis, but "why is that your favorite method" would be included in analysis since it requires students to analyze the differences between multiple methods. If a question was repeated without a student response in between repetitions, the question was only counted for analysis the first time it was asked. If a question was repeated after a student response was given, then that question was counted again, as it provided students the opportunity to provide another solution path or explanation. Once all questions were pulled from the data they were compiled in an excel document in preparation for coding. Each individual lesson was placed in its own spreadsheet.

Each question pulled for analysis was coded using the above-mentioned framework of Stein et al. (1996). This framework gives four levels at which a task or question can be coded: 1) Memorization or recall of a fact, 2) Use of procedures and algorithms <u>without</u> attention to concepts or understanding, 3) Use of procedures and algorithms <u>with</u> attention to concepts or understanding, and 4) "Doing Mathematics," which includes employment of complex thinking and reasoning strategies such as conjecturing, justifying, interpreting, etc. Levels 1 and 2 can be grouped together to form low-level procedural or memorization based questions, which align with a more rule-based view of teaching and learning. These questions ask students to either recall a memorized fact or procedure, such as the standard algorithm for long division, or ask students to follow a procedure in a step-by-step manner. For example, a teacher might be walking students through a problem and ask what the next step would be, without referring to the reason for taking that step. Levels 3 and 4 can be grouped together to form high level questions, which align with a more reasoning-based view of teaching and learning. These questions might ask students to make connections between multiple representations, or ask students why a method works. In order for the coding procedure to align with the beliefs survey, questions falling under levels 1 and 2 of the framework were coded as "rule-based," while questions aligning with levels 3 and 4 were coded as "reasoning-based".

A group of three coders, including the researcher, met to "code by committee," which allowed for intensive discussion and refining of the codes. The three coders met for an initial training session to code a practice lesson as a whole group. During this time, a common understanding of the codes was gained and future guidelines for coding were discussed. By the end of the coding practice session, coders were able to reach an inter-coder agreement rate of 86%, and reach agreement through further discussion on 100% of the codes. After the practice session, the researcher coded all lessons, while the other two coders were assigned independent coding for all of the procedural videos or all conceptual videos. Each of the two coders met separately with the researcher and differences in the codes were discussed until agreement was reached on 100% of the codes. Initial inter-coder agreement rate was 88%, therefore 12% of the codes had to be discussed. Coders reached an agreement on 100% of the codes through further discussion. The coding of all videos by the researcher created consistency in coding between the two sets of videos. Miles, Huberman, and Saldaña (2013) note, "team coding not only adds definitional clarity but also is a good reliability check" (p. 84).

Once all questions from the 12 videos were transcribed and coded, totals were tallied for individual teachers in the following four categories: a) rule-based questions in procedural lessons,

b) rule-based questions in conceptual lessons, c) reasoning-based questions in procedural lessons, and d) reasoning-based questions in conceptual lessons. Teachers' frequencies were then converted into rates of the average number of questions of a particular type asked per 10 minutes of teaching. The categorization of these rates can be seen in Table 6. The period of 10 minutes was chosen in order to have a sufficient segment of time to show numerous questions by each teacher. This allowed for comparison between teachers since each lesson was a different length. Each teacher had an individual rate within each of the four categories, then all six teachers' rates were averaged within each cell in order to analyze whether the nature of teachers' questioning changes between procedural and conceptual lessons.

Table 6: Categorization of Coded Data for Individual Teachers and Combined Averages

	Lesson Type		
Question Type	Procedural Lessons	Conceptual Lessons	Total
Rule-based			
Reasoning-based			
Total			

Teachers' combined rates were used to determine whether the nature of teachers' questioning differs during procedurally versus conceptually-based lessons. These combined rates show, on average, whether the types of questions that teachers ask are dependent upon the type of mathematics lesson the teacher is teaching. The rates placed in table 6 will allow the researcher to determine if teachers are asking more questions in procedural or conceptual lessons, if teachers ask more rule or reasoning-based questions, and if the types of questions that teachers ask differ between the two lesson types.

Research Question 2: How do teachers' MKT and beliefs about teaching and learning relate to the nature of teacher questions during procedurally based lessons? The second research question was used to investigate the reasons behind the results of the first research question. Are teachers not asking conceptual questions because of their lack of knowledge about the mathematical concepts? Do teachers not frequently ask conceptual questions if they do not have reasoning-based beliefs about learning? The data analysis methods for research question 2 are as follows.

This research question was explored through two different analyses: analysis of teachers' rates of questioning, and a qualitative cross-case analysis of five teachers who either strongly support or contradict the questioning patterns found within the data (Merriam, 2009).

In order to analyze differences in teacher questioning practices based on beliefs and MKT, coded data were reorganized into a new table. The coded data from the six teachers were placed in the appropriate quadrant of the table according to their belief alignment and MKT scores. Belief alignment was decided based upon the construct of which each teacher had a higher standardized score. Teachers' MKT categorization was assigned as low for teachers below the 50th percentile and high for teachers at or above the 50th percentile. This categorization of the data can be seen in Table 7. Each teacher's rate of reasoning-based questions in procedural lessons were placed in one of the four categories. For example, if a teacher had high MKT and had a higher alignment with rule-based beliefs than reasoning-based beliefs, their rate of reasoning-based questions in procedural lessons would go in the top left quadrant of Table 7.

Table 7: Reorganization Teachers' Rates for Reasoning-based Questions in Procedural Lessons

	Teachers' Level of MKT		
Belief Alignment	High	Low	Totals
Rule-based			
Reasoning-based			
Totals			

Teacher rates within each of these cells were averaged and multiplied by 6 to get the average rate of reasoning-based questions per hour of teaching. These rates were totaled by column and row for comparison purposes. The results of this analysis show whether the amount of reasoning-based questions teachers ask during procedural lessons are dependent upon MKT and beliefs.

After the completion of this analysis, data were categorized into two additional tables to aid in the analysis of the relationship between MKT and specific belief constructs in terms of questioning practices. The categorization of data placed teachers in a two by two matrix of high or low MKT, based on the top and bottom 50th percentiles, and by high or low rule-based belief score, which is separated by standard scores above or below zero. This matrix can be seen in Table 8.

Table 8: Questioning Rates Based on Rule-based Belief Scores and MKT

	Level of MKT		
Belief Alignment	High	Low	Totals
Rule score < 0			
Rule score > 0			

The second categorization is similar to the previous table, but separates teachers on their reasoning-based belief score, instead of rule-based beliefs. This matrix can be seen in Table 9.

Table 9: Questioning Rates Based on Reasoning-based Belief Scores and MKT

	Level of MKT		
Belief Alignment	High	Low	Totals
Reasoning score < 0			
Reasoning score > 0			

This organization of the data allowed the researcher to explore the relationships between specific belief constructs, MKT, and questioning habits. The data is focused on reasoning-based questions in procedural lessons because this allowed the researcher to see how MKT and beliefs relate to teachers' ability to give students opportunity for meaning making during procedural lessons.

The second part of the analysis of research question 2 was carried out through a cross-case analysis of five teachers. These teachers were chosen based on the unique perspective that they added to the practice of questioning and the relationships between this practice and their MKT and beliefs. Merriam (2009) suggests that multiple cases is a good strategy for "enhancing the external validity or generalizability" of the findings (p. 50). All data for teachers involved in the cross-case analysis, including MKT scores, belief survey results, coded questions, videos transcripts, and individualized memos were made into an individual portfolio for the last stage of analysis. These portfolios aided in organization and accessibility of the data. Qualitative data were read over as a whole in order to get a general feel for the data before analyzing more specific details (Creswell, 2013). After all data were explored, semi-structured interview questions were added to the original list of questions to personalize the interview to fit the topics taught by each teacher. The researcher asked each teacher a number of conceptual mathematics questions during their interview that evoked any conceptual knowledge that could have been used to ask meaningful questions within the topic that was taught during their videotaped lessons. Any mathematical errors that surfaced during the lessons were also probed through conceptual questions in the interview. Since MKT can differ depending on the domain of the mathematical content (Hill et al., 2005), this allowed the researcher to triangulate data for each teachers' MKT in a way that is more closely aligned to the topics in the recorded lessons.

The interview also focused on questions about teachers' beliefs about learning, which allowed the researcher to explore individual teacher's beliefs more specifically in order to aid in answering research question 2. While the majority of the questions were the same for all teachers who were interviewed, any questions about teachers' beliefs that were specific to their choices during their videotaped lessons were also asked during this time. Interviews were approximately 30-45 minutes in length, and were recorded and transcribed. Interview transcriptions were also added to teacher data portfolios. After the interviews, teacher portfolios were inspected and all data collected were considered in the selection of the five teachers who were chosen for the cross-case analysis. These teachers were chosen based on the unique characteristics the teacher's profile or videos offered to the study. For example, teachers that were able to ask conceptual questions, regardless of their MKT and belief scores, were of particular interest in the study. I also sought out teachers that provided evidence as to why they were unable to ask meaningful questions during the recorded lessons. Five of the six teachers displayed unique data that confirmed or gave a counterexample to statistical results, and therefore, these five teachers were included in the cross-case analysis. Wendy and May both provided evidence of low MKT and questioning practices that reduce the complexity of mathematical tasks. Since these two teachers painted a similar picture of the interaction of MKT, beliefs, and questioning, Wendy, the teacher with the most extreme examples of these characteristics was chosen for descriptive purposes.

Portfolios, including data from MKT assessments, belief surveys, video recorded lessons, coded questions, and transcribed interviews were used to create descriptive cases of each of the four teachers. During the cross-case analysis, each teacher profile was first inspected individually and descriptive codes were written in the margins of the interview and lesson transcripts. These codes helped the researcher look for patterns and also locate instances of patterns or themes that were developed later in the analysis process. Throughout the coding and analysis process intensive memos were taken, along with a list of themes and developing theories. A comparison of the cases was used to analyze the level to which the themes and theories were generalizable across the five cases. Patterns within these cases and patterns within teacher questioning rates were used by the researcher to establish any generalizations that were supported by multiple cases, along with atypical aspects of cases that are deviant from the norm. This ensured that themes and theories were backed up by multiple sources of evidence.

After the cross-case analysis was complete, a second meeting was set up with each of the five teachers for member checking and further questions. These individual meetings were approximately 10 to 15 minutes in length and consisted of a general explanation of the findings, as well as teacher input on whether they felt these findings were explanatory of their questioning habits. Teachers were given the opportunity to discuss any disagreements or alternate explanations for the results.

Chapter 4: Results

This section will begin with a description of each of the five cases chosen for the cross-case analysis to help the reader understand the results in a more complete manner. Following the description of the five cases, results for the first research question will be reported with both quantitative data and descriptive data from the case studies: The first research question focuses on how the nature of teacher's questioning changes between conceptually-based and procedurally-based lessons. Next, the results for the second research question will be discussed, which focuses on the relationships between teachers' MKT, beliefs, and the nature of their questioning practices. The results of this question will be split into three sections: (a) MKT and questioning, (b) beliefs and questioning, and (c) the interaction of MKT, beliefs, and questioning. Each section will report statistical results as well as descriptive data from the cross-case analysis that either supports the theories generated from this study, or provides a counterexample to these theories.

Description of Cases

The original sample of six teachers were chosen based on their unique profiles of MKT scores, belief survey results, years of teaching, and participation in PD. Five teachers were chosen from the group of six teachers for cross-case analysis. Han, Sue, Alesha, Coach and Wendy were each chosen for the unique nature of their questioning practices within the recorded lessons. Han was originally chosen for his high MKT, but was included in the cross-case analysis for his unique ability to ask questions that focus on student ideas and push students' thinking to a deeper level. Sue's case was chosen, because of her strong reasoning-based questioning practices in spite of her low MKT scores. Alesha's case was particularly interesting because of her focus on procedural or rule-based questions even though her reasoning score was the highest out of all teachers in the sample. Alesha's case in contrast to other cases brought light to the results within this chapter that focus on the role that rule-based beliefs play in teachers' questioning practices. Coach's case was originally chosen for her lack of reasoning-based beliefs, but was included in the cross-case analysis for her ability to ask more reasoning-based questions than that of the typical rule aligned teacher. The questioning practices of May aligned very closely with Wendy, in a sense that May and Wendy both tended to focus on questions that guided students through procedures in a step-by-step manner without making connections to concepts. The content that both May and Wendy chose lacked in rigor, and the questions that they asked rarely pushed for depth in student understanding. Wendy's case provided a thicker description of the data than May's, with her lower MKT and frequent proceduralization of the content, and therefore, she was chosen as the representative case for these two teachers. Choosing fewer teachers for the cross-case analysis allowed for a more in depth analysis of the chosen teachers (Creswell 2013). Each teachers' case individually brings light to the statistical results that will be presented in the next sections of this chapter.

The purpose of the case descriptions below is to provide the reader with an understanding of the unique styles and characteristics of each teacher involved in the cross-case analysis so that further results discussed in this chapter can be more fully understood. By understanding the complex nature of each teacher, the reader should be able to better understand the results, which address how the numerous variables interact with each other to produce differing questioning practices across teachers.

The case of Han. Han's teaching style and classroom environment are markedly different from other teachers in the study. While other teachers' lessons are focused on a specific activity, his lessons are focused on concepts. Han gives his students ample time to think about how they might go about solving problems for which they have not yet learned a procedure. His 5th grade students spent an entire class period in groups, brainstorming about where they see decimal division in real-life, coming up with a decimal division problem from that scenario, and attempting to solve the problem using any prior knowledge of related concepts. Introducing decimal division in this way allowed students to attempt to solve problems that were much more complex than typical 5th grade division problems, but Han's mathematical knowledge and six years of teaching experience allowed him to traverse this unpredictable path smoothly. Having the highest MKT (+1.64SD) in the group, and high reasoning beliefs (+0.64SD) along with low rule beliefs (-0.13), Han values time that is spent on finding problems and coming up with creative solutions. He believes it is important for students to think through the mathematics problems they see in real-life and consider how they could go about solving them. He states, “it just goes along with being an active participant in society—being able to solve problems,” an idea that guides his philosophy of teaching. This group struggle is a frequent occurrence in Han’s classroom. He mentions, “I’ve had no apprehensions with letting them struggle for 30 minutes. If I can see they’re making some headway with it, I’ll have them keep going… It’s about quality as opposed to quantity.” When walking into Han’s classroom it is easy to see that his students are used to this level of academic rigor. Students are continuously using academic language and productively debating with their group members while focusing on making sense of the mathematical ideas. Since Han’s students work in groups, most of the rule-based questions are asked and discussed by students themselves. This opens up room for Han to ask students reasoning-based questions, such as why they were making the procedural moves they chose to make.

The case of Sue. Sue was chosen for the cross-case analysis for her low MKT score (-0.79SD) and low reasoning (-0.29SD) and rule-based (-0.74SD) belief scores. Sue has been teaching for 13 years, and is very reflective within her profession. Her beliefs about teaching and learning are beginning to shift to more of a focus on student reasoning and meaning making. She attributes this change to her Master’s program and professional development which focuses on reasoning and modeling in mathematics classrooms. Sue teaches the 3rd grade “higher ability” math group, which consists of the top 50% of third-grade mathematics students in her building. She is very adaptable in her teaching and frequently discusses how each student has unique needs. She adjusts various aspects of her teaching to meet each students’ unique needs in hope of helping them “reach their potential.” Some of these adjustments appear in her teaching through the length of time she allows students to struggle with a problem, allowing for various methods of solving a problem, and the level of precision she requires in student responses. A major focus for Sue is allowing students to solve problems in a way that make sense to them. Students' often explain their solution paths to the class and Sue asks students questions that require them to make sense of each other's methods. After one student drew out their representation of a problem that involved 4 elephants each eating 500 pounds of forage per day over the course of three days, Sue asked, "So why did she write a five-hundred above her circles? What is that representing?" This type of question that requires students to make connections between the representation, computation, and problem situation are very common for Sue. This can be seen in her high frequency of reasoning-based questions in procedural lessons, which happened at a rate of approximately 10.58 reasoning-based questions per ten minutes of teaching, or a little over one reasoning-based question per minute. This was the highest frequency of reasoning-based questions in procedural lessons out of all teachers in the study. Meaning making happens frequently in her class, but often the meaning making is done by Sue, herself, by asking questions that give enough detail that the sense making that students' must do in order to answer the question is reduced in cognitive load. She mentioned in her interview that teachers often want to help students at the expense of the students' learning. She stated, "as the year goes on, I am less and less nurturing in the struggle, because that becomes a crutch, too. An enabler. Teachers can be enablers without meaning to be. I mean, our heart is in the right place, I think, but sometimes we have to shake them off our leg and tell them to go." The joint task of helping students get to the answer and helping students learn is a balancing act that many teachers have trouble navigating.

The case of Alesha. Alesha is an interesting case for her high beliefs in both reasoning (+1.30SD) and rules (+0.93), and her average MKT (+0.08SD). Her belief survey, lessons, and interviews all show a guiding belief that meaningful, real-world tasks are important, but her 4th grade students need a lot of scaffolding to get them through such tasks. This is Alesha's fifth year of teaching, and first year with the district, making her the one of the two cases that were not exposed to professional development on reasoning and modeling in the mathematics classroom prior to the beginning of the study. While other teachers are buying into the idea that mistakes are okay and making mistakes is how we learn, Alesha's fear of letting her students fail drives many of the decisions she makes in her classroom. Alesha spent one-third and one-half of her conceptual and procedural lessons, respectively, setting up the task and showing an example to students. She is aware that this is a large portion of time to be spent on showing students how to solve a problem, but her desire for her students to succeed holds her back from allowing students to struggle with the mathematics, themselves. When asked how important she thinks it is for students to struggle in math she stated, "I think I should be giving them more of a chance to do that. I think I'm worried if I let them do that and they do something wrong, that's how they'll think they're supposed to do it. So, I need to work on giving them more of a chance to do that, but then bringing it back to show what you did is okay, but here's maybe what we can do to improve it. Make it more of a positive experience instead of me panicking like, oh my God! That's wrong! I can't let them do that!"

Her time spent setting up the problem consisted of a large amount of questions recalling individual details about the task, and students are rarely given the opportunity to sort through the information and set up a plan themselves. In result of this style of questioning and guidance, Alesha has set up a dependent environment where students are constantly asking her what they are supposed to be doing next to solve the problem.

The case of Coach. Coach is a special case in that her reasoning-based belief alignment (-2.11SD) was the lowest out of all 20 teachers who were given the belief survey, however, her rule-based belief alignment (0.171SD) was also relatively low. Like Alesha, Coach was new to the district at the beginning of the study, and had not yet been exposed to the reasoning and modeling PD. This, along with being a relatively new teacher with five years' experience, and teaching 5th grade for the first time, left her feeling less confident about strongly agreeing or disagreeing with many of the statements on the belief survey. Her time with the district this year has increased her confidence in teaching and she has spent a lot of the year reflecting on changes within her teaching, such as giving students more of an opportunity to explain their thinking, and the importance of process over product.

Coach has the second highest MKT score (+0.27SD) out of the five teachers chosen for the cross-case analysis, however, she struggled with many of the mathematics questions that she was asked during her interview. For example, when asked where the decimal would be placed in the product of $7.85 \times 0.4 = 314$, she stated that it would go in front of the 3, giving a product of 0.314. This answer stemmed from the common procedural rule of moving the decimal the same number of times as the number of digits to the right of the decimals in the original problem. This method works when using the standard algorithm, however, it does not account for the fact that there could be zeros after the digits 3, 1, and 4. When asked if her answer was reasonable, she claimed it was, because her answer should be smaller than 7.85, and it was. Coach did not have a general understanding of decimal multiplication as taking part of a set of a number, which would have helped her estimate her answer to be approximately half of 8, giving an estimate that would have warranted the decimal to be placed in the correct position of 3.14.

This lack of depth in her mathematical knowledge may be the reason for Coach's simplicity of her lessons. Coach's teaching style is self-described as "laid back with structure." The environment is very collaborative, with students often working in pairs and joking with their teacher, but the lessons that she teaches lack in rigor. One lesson of Coach's had students arranging Unifix Cubes into rectangles to find pairs of numbers that could be multiplied to find a product equal to the total number of cubes. Students could easily complete this task and did not seem to grow in their knowledge. Coach admits that the majority of her lessons are too easy and that her weakness as a teacher is knowing how to challenge more advanced students. Although Coach is beginning to value the practice of questioning students to explain the reasoning behind various methods and procedures, she is often very quick to accept incomplete answers from students without pushing for more depth in their explanations. For example, Coach asked students why they would get “a smaller answer” when multiplying by a decimal, and she accepted a student response of “because the decimal is not a whole number.” This answer does not show any conceptual understanding, nor does it explain the reasoning behind the results of this type of procedure, yet Coach moved on with the conversation. Due to her interview responses, it is likely that she accepts these responses because she does not know how to accurately describe the reasoning behind this result, herself.

The case of Wendy. Wendy is a unique case in that her MKT score (-0.84SD) was the lowest out of the twenty teachers who were assessed. At the time of her belief survey, her rule-based belief alignment was very high (+0.78SD) and her reasoning-based alignment was below average (-0.29SD). However, Wendy, like May, the teacher that was not included in the cross-case analysis, was showing rapid changes in her teaching style due to her involvement in PD. Her teaching style is self-described as “moving toward a more student-centered [approach],” and she can often be found joking and laughing with students multiple times per class. Wendy has been teaching for 17 years, and has taught mathematics at the 5^{th} grade level for 14 years. She acknowledges the constant shifts in the educational pendulum; “We’re old school. We’ve been at this longer. You know, our education piece, it goes back to we had to be right or we had to be wrong. It was, there was no shade of grey. Um, and we’ve always kind of learned that with math. You know, it was either black or white. So, learning now that it’s ok to have… multiple ways, I think I’m more comfortable asking questions that have that shade of grey, just because now I completely understand. I don’t care how you got the answer. If you can tell me how you got there and it makes sense to you, you can explain it to us… I don’t care. I just want you to be able to tell me how you got there.”

Using multiple methods, generating prior knowledge, and understanding relationships between mathematical ideas have become a major focus of Wendy’s teaching, however, students are not typically asked to push their knowledge further than their initial level of prior knowledge. Although Wendy is aware of the importance of mathematical relationships, her own lack understanding of the relationships often gets in the way of creating coherent and meaningful lessons. During one lesson, Wendy had a small group of students dividing a certain number of cubes into equal sized groups, the purpose of which was to "understand groupings" and get back to "the basics of division." Students were clearly using the concept of division to see if a certain number of equal sized groups could be made, however, the number sentences Wendy was asking students to write were multiplication sentences. This inability to differentiate between concepts and understand the relationships between them was iterated throughout her lessons and the content portion of her interview. Wendy asks numerous questions that get students to explain their thinking, but her lack of understanding typically causes her to focus on low-level questions and concepts. In a survey, Wendy admitted that her lack of knowledge can sometimes make it difficult to teach certain lessons.

Research Question 1 Results

Does the nature of teachers' questioning change between conceptually-based lessons and procedurally-based lessons?

Teachers' frequency of rule-based and reasoning-based questions in procedural and conceptual lessons were tallied for four different categories: (a) rule-based questions in procedural lessons, (b) reasoning-based questions in procedural lessons, (c) rule-based questions in conceptual lessons, and (d) reasoning-based questions in conceptual lessons. Since each lesson was a different length of time, average rates were calculated for the number of questions of a specific type per 10-minute lesson segment. Reporting teacher frequencies as rates allows for easier comparisons across teacher data. Table 10 shows the individual rates of the six teachers for each question type in both procedural and conceptual lessons. Table 11 shows the average rate of each of the four categories for all six teachers combined. These average rates for each category were calculated by adding together all six teacher rates within the category and dividing the total by six.

Table 10: Individual Teacher Questioning Rates

		Lesson Type	
Question Type	Teacher	Procedural Lessons	Conceptual Lessons
Rule-based	Alesha	9.52	18.13
	Coach	20.15	18.13
	Han	10.58	5.59
	May	13.87	21.52
	Sue	9.52	17.34
	Wendy	8.58	19.59
Reasoning-based	Alesha	1.41	2.61
	Coach	2.99	9.06
	Han	4.53	10.87
	May	2.90	9.75
	Sue	10.58	8.92
	Wendy	2.78	1.96

Table 11: Overall Teacher Questioning Averages

	Lesson Type	
Question Type	Procedural Lessons	Conceptual Lessons
Rule-based	12.85	15.65
Reasoning-based	4.19	7.20

Within the sample, teachers' questioning habits tended to be the similar across both observed lessons. By looking at the rule-based questioning rates in Table 12 we can see that on average, teachers are asking 12.85 rule-based questions per ten minutes during procedural lessons, and 15.65 rule-based questions per ten minutes in conceptual lessons. Teachers such as Coach, who ask a lot of rule-based questions in procedural lessons, also ask a high amount of rule-based questions in conceptual lessons. In fact, by looking at the data in Table 10, it can be noted that four of the six teachers ask more rule-based questions in their conceptual lessons than they do for procedural lessons. The only teachers this is not true for are Coach, who has similar rule-based questioning patterns in both lesson types, and Han, who asks almost half as many rule-based questions in his conceptual lessons as compared to his procedural lessons.

The average number of rule-based questions per ten-minute teaching segment in procedural lessons is 4.19, which is less than the 7.20 reasoning-based questions per ten-minute segment in conceptual lessons. On average, teachers ask about six more questions per ten-minutes of teaching during conceptual lessons than they do per ten-minute segment of procedural lessons. This increase is equal amongst rule and reasoning-based questions, with an increase of 3 rule-based and 3 reasoning-based questions per ten minutes of conceptual lessons, as compared to procedural lessons.

The sample showed a higher frequency of questioning in conceptual lessons than procedural lessons, regardless of the questioning type. This means that teachers are typically asking more mathematics questions in their conceptual lessons than their procedural lessons. Interestingly, rule-based questions in conceptual lessons had the highest rate of frequency for four out of the six teachers. This pattern shows that the majority of teachers tend to continue to focus on rules and recall even when the purpose of the lesson is to build conceptual understanding of the content. Alesha, Coach, May, and Wendy all showed patterns of proceduralizing conceptual lessons to make the reasoning more attainable for their students. This means that these teachers are breaking down more complex or conceptual tasks into a step-by-step process that simplifies the students' reasoning process into nothing more than short answer computational problems. This idea will be discussed further during the results for research question 2. Coach's rate of questioning was slightly higher for rule-based questions in procedural lessons, but rule-based questions were still common in her conceptual lessons as well. Han's questioning patterns, on the other hand, followed a different pattern when compared to the rest of the teachers. His recordings showed a focus on reasoning-based questions in conceptual lessons, and rule-based questions in procedural lessons; a pattern that was expected for a higher number of teachers based on the nature of the lesson types.

Reasoning-based questions were asked far less frequently than rule-based questions for four out of the six teachers: Alesha, Coach, May, and Wendy. These four teachers were the same teachers that tended to proceduralize conceptual lessons, therefore, it would naturally be expected that they would ask reasoning-based questions less frequently in procedural lessons. If a teacher is proceduralizing tasks that are meant to involve reasoning and a conceptual focus, it is of no surprise that they would also proceduralize tasks that are of a procedural nature. These teachers are breaking down both lesson types in a way where the teacher assumes most of the mathematical thought, yet keeps students engaged by making them answer simple recall or computational questions.

Sue's questioning patterns for reasoning-based questions differed from the four previously mentioned teachers. She asked more reasoning-based questions in procedural lessons than rule-based questions in the same procedural lesson. This came from her focus of having students make sense of students' methods and representations while they solved the elephant word problem mentioned in her case description. This questioning style can be seen in the following dialogue that Sue was having with her whole class as they shared their methods after working in groups to solve the problem.

> Sue: Did anyone solve it differently, using circles, or drew a picture? Sarah, can you show us the picture that you drew?
>
> Sue: Oh, so what did she use to represent elephants in her problem solving? Andy, what's Sarah using?...
>
> Student: Circles?
>
> Sue: Yep. So why did she write a five-hundred above her circles? What is that representing? Juan?
>
> Student: The elephant?
>
> Sue: The circle represents the elephant. And the second row, Sarah, for the three-thousand five-hundred. Right? What's that represent?
>
> Student: (Inaudible)
>
> Sue: Oh! There's three five-hundreds per elephant, per day.

Student: Five hundred to each of them, so one-thousand five-hundred. So, I did one-thousand five-hundred to each of them, so one-thousand five-hundred times four equals six-thousand.

This dialogue shows that Sue values having her students make sense of the mathematics problems they are solving. While other teachers use their questioning habits to turn more complex tasks and problems into simple practice of basic facts, Sue does the opposite by requiring students to keep track of the meaning of the numbers they are working with in the problem scenario.

Like Sue, Han regularly asked for students to make meaning of the mathematics at hand regardless of lesson type. Han's frequency of reasoning-based questions was higher in conceptual lessons, but even more notable was the way that his reasoning-based questioning technique changed from procedural lessons to conceptual lessons. During his procedural lessons, Han's reasoning-based questions typically focused on getting students to make conceptual sense of smaller details within procedures. For example, while students were solving decimal division problems, he would ask them why they could add a zero onto the end of the decimal value of the dividend. These questions were typically relatively short questions that required students to quickly make sense of the procedures they were practicing. During his conceptual lessons, Han's reasoning-based questions focused on more general ideas that were tied to real-life scenarios. He constantly questioned students about their ideas and attempted to push the student further along in their own line of thinking rather than redirect them to an explanation or solution that is more in line with his own thinking. For example, during Han's conceptual lesson, students were to come up with a real-life problem that uses decimal division. One group of students wanted to divide the price of a jar of peanut butter, but were unsure how this situation applied to the concept of division. Han asked the student "what is division? Within a word or two, what is division?" After the student responded that division is "breaking down the number," Han began a line of questions to help this student use his understanding of this concept to tie his decimal example to the larger concept of division.

Han: So, if we have one jar of peanut butter for $2.99, how could we break that down?

Student: I'm thinking of breaking it down by $1.99.

Han: But why are we just choosing $1.99?

Student: Because it's half of that.

Han: I'm not necessarily asking a price, but what's a real-life example? How do we break down a jar of peanut butter? How could someone break down a jar of peanut butter to need division?

Student: Oh! Cut it in half!

Han: Why? Cut it in half or use half?

Student: Use half.

Han: I was going to say, I don't remember cutting in half a jar of peanut butter. When would we use half?

Student: When we're like, when we actually need, like need a recipe that needs half.

Han: Ok, now where could we apply this to this? Half of two ninety-nine. What could be a possible problem that we might want to use?

During this discussion, Han continued on the path of the student without giving the student answers or changing the course of their line of thinking. He kept the cognitive demand of the work intact as he used his questions to get students to reason about the concept of division and specify where we see this concept of division happening in life.

Han's conceptual lesson was focused on generating prior knowledge of related concepts and helping students begin to bridge the gap between what they currently know and the new content they will be learning. The purpose of his reasoning-based questions in this lesson were to help students get their line of thinking as close to those new concepts as they can with their own thinking and reasoning. This differs from the purpose of many of his reasoning-based questions in procedural lessons that ask students to focus in on specific details of a procedure to maintain their conceptual understanding of the mathematics involved in the given procedure. Respectively, one can be seen as pushing for growth in understanding, while the latter can be seen as maintaining understanding.

The purpose of research question one was to see if the nature of teachers' questioning changes between procedurally-based lessons and conceptually-based lessons. In summary, the data from the sample provides evidence that the nature of teachers' questioning does not change between conceptually-based lessons and procedurally-based lessons. The majority of the teachers tended to ask similar questions in both lesson types. Rule-based questions were more common than reasoning-based questions in both lesson types, with rule-based questions in conceptual lessons having the highest frequency of all four questioning categories. Teachers such as Alesha, Coach, May and Wendy, who focus on rules and recall maintain this focus regardless of the lesson type. Results for research question 2 will delve into theories on the reasons for these outcomes.

Research Question 2 Results

How do teachers' MKT and beliefs about teaching and learning relate to the nature of teacher questions during procedurally-based lessons?

This section will be reported in three sections: (a) MKT and questioning, (b) beliefs and questioning, and (c) the interaction of MKT, beliefs, and questioning. Multiple sections will refer to the results in Table 12, which show teacher rates of reasoning-based questions in procedural lessons, separating teachers by level of MKT and belief alignment. A teacher's MKT is considered low if it is equal to or below the mean standard score of 0, and High if it is above the standard score of 0. Belief alignment was assigned based on the belief alignment that had the higher standard score. Therefore, the rate in the table of 2.99 reasoning-based questions per 10 minutes belongs to Coach, a teacher whose MKT is above average and has a stronger belief alignment with rules than reasoning.

Table 12: Rate of Reasoning Questions in Procedural Lessons Based on MKT and Beliefs

	MKT	
Belief Alignment	High	Low
Rule-based	2.99(Coach)	2.78(Wendy) 2.9(May)
Reasoning-based	1.41(Alesha) 4.53(Han)	10.58(Sue)

The values in each quadrant of Table 12 were averaged and multiplied by six, which gave the average values for the number of reasoning-based questions per hour of a procedural lesson. These data, along with the frequencies per row and column can be seen in Table 13. These rates were used to compare patterns in questioning habits between teachers with differing levels of MKT and belief alignment, and will also be referred to throughout the results sections.

Table 13: Combined Hourly Rates of Reasoning Questions Based on MKT and Beliefs

	MKT		
Belief Alignment	High	Low	Row Totals
Rule-based	17.94	17.04	34.98
Reasoning-based	17.82	61.5	79.32
Column Totals	35.76	78.54	114.30

MKT and questioning.

Surprisingly, teachers with high MKT asked under half as many reasoning-based questions in procedural lessons than teachers with low MKT. Table 13 shows high MKT teachers asking a sum of 35.76 reasoning-based questions when combining both rule (17.94) and reasoning (17.82) aligned, high MKT teachers' hourly rates. This number is very low when compared to the 78.54 reasoning-based questions that were totaled through adding the hourly rates of rule (17.04) and reasoning (61.5) aligned low MKT teachers.

All four quadrants show similar rates between 17 and 18 reasoning-based questions per hour except for the quadrant representing Sue, the teacher with low MKT and reasoning-based beliefs. As shown in Table 12, two teachers in each level of MKT asked less than three reasoning-based questions per 10-minute segment of their procedural lesson. Han asked slightly more reasoning-based questions (4.53 questions/10-min.) than the previous four teachers, while Sue asked over three times as many reasoning-based questions as the previous four teachers. Sue’s case provides evidence that high MKT is not necessary in order to ask a higher frequency of reasoning-based questions in procedural lessons, being that her MKT score was almost one standard deviation below the national norm.

While high MKT may not be necessary to ask meaningful questions during procedural lessons, it can certainly be a hindrance to the practice. Many of the segments that lacked conceptual correctness or clarity in the observed lessons were cause by gaps in the teacher’s mathematical knowledge. Sue and Wendy were the two lowest scoring teachers on the MKT assessment out of the chosen sample, with Wendy being the lowest scoring out of all 20 teachers that were originally assessed. Although Sue asked a much higher rate of reasoning-based questions in procedural lessons, both her and Wendy had moments within their lessons that were compromised based on their mathematical knowledge of the content in the given lesson.

Sue’s conceptual lesson had her third-grade students working on telling time on both digital and analogue clocks. During this lesson, Sue was encouraging her students to report the time in numerous ways. This conversation lead to a discussion of quarters on a clock, which Sue connected to quarters in a dollar. This is a very useful connection to make if done accurately, however, students were confusing the correct connection of a fourth dividing a quantity into four equal segments, with the incorrect connection of the value of a quarter being equal to 25 cents. Sue did not know how to address this misconception in the conversation.

Sue: Now, how many quarters does it take to make a dollar?

Student: Four quarters.

Sue: So, that's why we say quarter after or quarter to. So, from one to one fifteen would be quarter after one. Now what do I get when I put two twenty-fives together?

Student: Fifty.

Sue: But I don't say like twelve thirty, I don't say twelve fifty, but I could say half past, because I'm using half of my clock. So that can be called half past. What do I call it when I have that much of the clock used up? (colors in three-fourths) I'm going to describe it by the amount of time that I still have left to go through. How much time do I still have left to go through?

Student: Twenty-five.

Sue: Or a quarter 'til the hour. This one is called quarter.

The student who answered twenty-five obviously interpreted the connection of a quarter of the clock to a quarter of a dollar to mean that the values of these two quarters are the same. Sue never addressed this student's misconception and moved on with the lesson. This student, and probably others in the class needed to understand that we call it a quarter because it is split into four equal parts, but quarters do not always have the same value. During Sue's content portion of her interview, I asked her if one-fourth is always the same size or value. She did not have an answer to this question. I continued to prod her understanding of this connection and asked "what is the difference between quarters of a clock versus quarters of a dollar? Or are they exactly the same?" She responded, "they are not exactly the same. Space-wise they are, but really a quarter is fifteen minutes on a clock and a quarter is one fourth of a dollar. Sue was aware that students might make the error of equating the values of the two different types of quarters, but she did not have a deep enough understanding of this concept to address the misconception on the spot.

Wendy showed a larger gap in understanding when teaching a lesson to her students about division and factors. Students in this lesson were asked to take a group of 36 cubes and attempt to split them into equal size groups. Wendy would let students choose what number of groups they wanted to try and then the students would individually sort the cubes into groups to see if there was a remainder or not. Wendy said the purpose of this lesson was to understand groupings and “the basics of division,” but during her interview when she was asked to write an equation for 36 cubes split into 9 equal groups with four in each group, she said “giving them the 36, then you’re hoping that they are coming up with nine times four.” After she stated this equation, I asked her to give me a scenario with the blocks that would be the inverse operation to the equation she wrote. She struggled with this and stated “see, that’s when I think I always have issues trying to figure out how I want to word my questions.” She then came up with a real-life scenario of 36 students needing to take busses that fit 9 people and asks how many busses they need to take. Although this problem accurately describes a division problem, the inverse of a multiplication problem, both of these scenarios involved the concept of division, or splitting a quantity into equal sized groups. Wendy’s confusion between the distinguishing characteristics of division and multiplication, and the relationship between them, can be seen multiple times in her interview and her teaching segments. In result of these gaps in understanding, her questions lacked clarity and purpose, leading students to see the activity as nothing more than manipulating blocks on a table.

While high MKT did not generally lead to a higher frequency of reasoning-based questions, Han, the teacher with the highest MKT did hold his students more accountable during their mathematical talk than those teachers with lower MKT. When Han asks students questions and they respond, he continues to ask them questions that require them to think deeper than their original line of thinking. This questioning helps maintain high levels of cognitive demand throughout the lesson. An example of this questioning practice can be seen in the peanut butter example discussed under the results of research question 1. Han did not accept that the students wanted to simply divide the price of peanut butter. He continued to question them about why you would divide the price of a jar of peanut butter, and what this would look like mathematically. Similarly, in Han’s procedural lesson he pushed students to explain their procedural work conceptually. When a student stated they added a zero onto the dividend, he asked "why'd you add a zero there," pushing the student to explain why these procedural moves made sense.

Han's pattern of using questions to hold students accountable for precise mathematical talk is in stark contrast to other teachers with lower MKT. For example, if students answer a question incorrectly in Wendy's class, she corrects the student instead of asking them questions to help them build a better understanding of why their answer was incorrect. During her conceptual lesson, Wendy asked students to show how many cubes are in each of the six groups if the total is twenty-four, but students created groups of six instead of six groups. She did not address this error or question students about the differences and similarities between these two representations.

Similarly, many of Alesha's questions that ask students to reason can still be answered with a 'yes' or 'no' from students, and she typically does not ask them to elaborate on their one word responses. In one of Alesha's lessons a student comes up with an incorrect answer and she asks, "So when you get a hundred and two and you're repeatedly adding ten, does that even make sense?" The student answers "no." This question is good for getting students to think about the reasonableness of their answer, but her questioning stops here while she proceeds to tell the student what they should do to solve the problem. Although the student correctly answered no, Alesha does not know whether the student understands why their answer is unreasonable, or why they got an incorrect answer in the first place. Questioning students for mathematical clarity and correctness allows the teacher to access a more precise representation of the students' level of understanding, but without a certain level of MKT, teachers might not have the knowledge to push for this level of depth in students' understanding.

In summary, low MKT teachers asked over two times as many reasoning-based questions in procedural lessons than high MKT teachers. However, because of the dependent relationship between MKT and beliefs, there are more factors to explore behind this result, such as how the effects of MKT are altered by differing belief alignments. While low MKT teachers asked more reasoning-based questions in procedural lessons, qualitative results showed low MKT to be tied to more frequent errors in mathematical thinking by the teacher. This caused teachers to be unable to ask questions that probed for more depth in mathematical thinking. Similarly, Han, the teacher with the highest level of MKT was able to push for more precision in language, and a deeper mathematical understanding from his students in a way that teachers with lower levels of MKT were unable to achieve.

Beliefs and questioning. The data from Table 13 shows that teachers aligned with reasoning-based beliefs asked a higher frequency of reasoning-based questions in procedural lessons than teachers that were more aligned with rule-based beliefs. Rule aligned teachers asked 34.98 reasoning-based questions when adding their hourly rates of 17.94 and 17.04 together. This is under half the amount of questions asked by reasoning aligned teachers who asked 79.32 questions when adding their hourly rates of 17.82 and 61.5. By looking at the rows in Table 12, it is apparent that both teachers who asked a higher rate of reasoning-based questions in procedural lessons are more aligned with reasoning beliefs. These teachers are Han and Sue, who asked 4.52 and 10.58 reasoning-based questions per 10-minute segment of teaching, respectively. However, Table 12 also shows that Alesha, one of the reasoning aligned teachers, had the lowest rate of reasoning-based questions in procedural lessons with a rate of 1.41 questions per 10-minute segment. Although Alesha is more aligned with reasoning-based beliefs, her lower rate of reasoning-based questioning does not fit this pattern of higher alignment with reasoning-based beliefs leading to higher levels of reasoning-based questions.

During the cross-case analysis a clear pattern emerged that explained the difference between these high and low questioning frequencies within the reasoning aligned teachers. Han, Sue, and Alesha all have a higher reasoning belief alignment than rule belief alignment, but the difference between Alesha and the other two teachers is her similarly high rule-based belief alignment. Although Alesha has the highest reasoning-based belief alignment in the group (+1.30 SD), she also has a high rule-based belief (+0.93 SD) as well. This is much higher than Sue (-

0.741 SD) and Han (-0.13 SD), who are both below average in rule-based belief alignment. This led the researcher to question the interplay between rule and reasoning-based beliefs. It appeared that teachers with high reasoning-based beliefs and low rule-based beliefs were able to ask a higher rate of reasoning-based questions, while teachers with high reasoning and rule-based beliefs were not. To test for generalizability, the data were categorized into two new tables. Table 14 shows the rates of reasoning-based questions asked during procedural lessons with the frequencies categorized based on the teacher's level of alignment with rule-based beliefs. Teachers were divided into two groups, those with above average rule-based belief alignment, and those below average rule-based belief alignment.

Table 14: Rate of Reasoning Questions in Procedural Lessons Based on Rule Score

	MKT	
Rule belief SD score	High	Low
Below average (< 0)	4.53(Han)	10.58(Sue)
Above average (> 0)	1.41(Alesha) 2.99(Coach)	2.9(May) 2.78(Wendy)

Within the sample, it is clear that teachers scoring below average on the rule-based belief construct asked more reasoning-based questions during procedural lessons than those scoring above average for their rule-based belief alignment. What appears to be important in getting teachers to ask reasoning-based questions in procedural lessons is not that a teacher has high reasoning-based belief alignment, but that they have a low rule-based alignment. To support this theory of rule-based beliefs having more of an impact on teachers' reasoning-based questioning than reasoning-based beliefs, the frequencies were categorized by below and above average reasoning-based belief alignment. Table 15 shows this data.

Table 15: Rate of Reasoning Questions in Procedural Lessons Based on Reasoning Score

	MKT	
Reasoning belief SD score	High	Low
Below average (< 0)	2.99(Coach)	10.58(Sue) 2.78(Wendy)
Above average (> 0)	1.41(Alesha) 4.53(Han)	2.9(May)

Table 15 shows no clear pattern in teacher questioning, with the teachers with higher frequency of reasoning-based questions in opposing belief categories. Although Sue's belief score in reasoning is below average, her reasoning-based questioning practices have a higher rate than any of the other teachers in the study. Low levels of rule-based beliefs tend to account for this high level of reasoning-based questioning frequency, while low reasoning-based beliefs do not account for this pattern.

Alesha's case shows a clear picture of how a high belief in rules can overpower a higher belief in reasoning. Her belief profile, lessons, and interview all show a belief that harder, more meaningful tasks are important, but that students need a lot of scaffolding to get them through the tasks. This is why she spent over a third of each of her lessons questioning students about the details in the given task and showing them an example of how they could go about solving the problem. This focus on rules is heightened by her desire for her students to succeed and a belief that reducing the lesson to recall and procedures is the only way to get students to the end of the task successfully. The enactment of her reasoning-based beliefs can be seen in her task selection, which incorporates tasks that are tied to real-life situations and involve problem-solving, while her rule-based beliefs can be seen in her questioning and teaching practices, which focus on walking her students through the problem-solving process in a step-by-step manner. Although Alesha has reasoning-based intentions, these intentions do not come to fruition because her rule-based actions overpower her reasoning-based actions by simplifying the reasoning process to a procedure that students must follow.

Sue's low reasoning does not show up in her teaching practices or tasks, possibly because her reasoning beliefs have gotten stronger over the course of time since the belief survey was taken, as indicated in her interview. Her low belief in rules can be seen through her choice of letting students work in groups to solve problems with whatever solution path they prefer. This low belief in rules makes it easier for her to apply reasoning-based practices that she learns about during her master's program and professional development work because these ideas are compatible with a low rule belief. For example, allowing for multiple solution paths and discussing the connection between different representations and methods are practices that are easy for Sue to enact in her classroom, since her students are not following one set procedure, as might be done in a classroom led by a high-rules teacher. For Alesha, her efforts to impose reasoning-based practices in her classroom do not result in the same student success that she is hoping for because her rule-based beliefs hinder her ability to enact reasoning-based practices with integrity. For this reason, high rule-based beliefs can be seen to impede on the effects of reasoning-based beliefs and practices, and low rule-based beliefs can be seen to encourage the use of reasoning-based practices.

In summary, teachers with higher reasoning-based beliefs were found to ask more reasoning-based questions in procedural lessons than teachers with higher rule-based beliefs. However, this relationship did not hold true for Alesha, who had the highest reasoning-based beliefs and lowest reasoning-based questioning frequency. Further results showed that higher frequencies in reasoning-based questions were better explained by a below average alignment with rule-based beliefs than a high alignment with reasoning-based beliefs. High levels of rule-based beliefs were found to be a major obstruction to the enactment of reasoning-based beliefs.

Interaction of MKT, beliefs, and questioning. The previous two sections have shown how MKT and beliefs about teaching and learning can independently affect a teacher's questioning practices, however, the relationship between these factors is very intertwined. The effects of similar levels of MKT may differ for teachers with different belief alignments, and visa versa.

Positive effects of high MKT on questioning practices of rule-based teachers. Teachers that were rule aligned tended to ask lower frequencies of reasoning-based questions, but high levels of MKT allowed for a higher frequency of reasoning-based questions in procedural lessons than expected. Previous results indicated that high MKT allowed for more meaning making through teacher questioning. Additionally, high levels of rule-based beliefs were found to be a hindrance to high levels of reasoning-based questions. Based on these results, a likely explanation for this increase in reasoning-based questions is that high levels of MKT aid rule aligned teachers in increasing their typically low frequency of reasoning-based questions.

One example of this increase can be seen in the case of Coach, a teacher that was rule aligned with above average MKT. Since Coach was rule-aligned, her expected frequency of reasoning-based questions was low, but her high levels of MKT likely allowed her to overcome this expectancy and ask more reasoning-based questions than she would have, had she been a low MKT teacher. This can be seen through Coach's questioning, where she pushed students to find patterns within multiplication to understand why multiplying a number by a decimal value between zero and one would result in a product that was less than the original factor. Coach knew that one times a number would result in the original number, and zero times that same number would result in zero, and therefore, multiplying that same number by a number between one and zero would result in a product that was between that original number and zero. This relationship is how Coach understood the reasoning behind why a number times a decimal between one and zero would result in a value less than the original number. This knowledge allowed her to ask questions that pushed for this depth of understanding with her students.

On the other hand, during her interview, Coach showed she did not have an understanding of multiplication as repeated addition, or a certain number of sets of another number. This lack of knowledge hindered her from making the connection that 9 times .5 could be seen as half of the set of nine. This connection would have been useful for students when she asked them to estimate products within decimal multiplication problems. Since she did not have this knowledge herself, she could not utilize this idea as a tool to push for depth of understanding, and students continued to find estimation of these products difficult. These two scenarios within the same lesson show how high levels of MKT can interact with high levels of rule-based beliefs to produce more meaningful questioning practices.

The effects of rule-based beliefs on questioning practices of teachers with varying MKT. One of the most apparent interactions between MKT and beliefs that showed up during qualitative analysis was the relationship between low MKT and low rule-based beliefs. While teachers with low MKT often proceduralized conceptual tasks by turning them into a step-by-step process, low rule-based beliefs could counteract this tendency. The next section will provide examples of the questioning habits of low to average MKT teachers with varying levels of rule-based belief alignment. Those examples will then be compared to show the positive effects that low rule-based beliefs can have on the questioning practices of teachers with low to average MKT.

Wendy, the teacher with the lowest MKT out of the original sample of 20 teachers, provided a conceptual lesson that used manipulatives. Two other teachers within the sample also provided conceptual lessons that utilized manipulatives to get at the conceptual aspects of mathematical ideas. In fact, interview data from these three teachers suggest that teachers believe that lessons that involve manipulatives are always conceptual. Although manipulatives can easily bring out a more conceptual focus in a lesson, if connections are not made between the manipulatives and the concept, the lesson becomes another memorized procedure without meaning. This can be seen in Wendy's lesson when students were working on finding factors of a number by separating a set of cubes into equal sized groups. After students had been working on the task for approximately 15 minutes, it became apparent that students were not making connections between the concepts of division and factors, and the act of separating the cubes into equal sized groups. This can be seen in the following conversation between Wendy and a student.

> Wendy: I want you to decide what you want to divide them (the set of cubes) by. You decide. We're going to see how many ways, just like we did with the twenty-four.
>
> Student: Yeah.
>
> Wendy: So, you decide what you want to do. And you can both do something completely different.
>
> Student: Ok. We divide?
>
> Wendy: Isn't that what we're doing?

Clearly this student was not aware that the manipulatives were being used to represent the process of division and that equal groups of cubes without a remainder represented values that were factors of the original number of cubes. The manipulatives provided the opportunity for strong conceptual connections, but Wendy's own lack of conceptual understanding and reasoning-based questioning made this activity nothing more than the shuffling of cubes. The students were able to tell whether or not a quantity could be split into equal sized groups, but it is unlikely that they could apply this knowledge to a straight forward mathematics question about whether a number is a factor of another number. The effects of Wendy's lack of mathematical knowledge of the concepts was only compounded by her focus on rules over reasoning. The fact that Wendy did not have the knowledge or beliefs to push students deeper hindered her ability to move this activity past a mindless act of manipulating cubes.

Low MKT and high rule-based belief alignment both seem to have a negative effect on teachers' reasoning-based questioning practices. However, this same act of proceduralizing conceptual tasks can be seen in a lesson of Alesha's, who has a high belief in rules, even though she has average MKT. During this lesson, Alesha gave her students a task that required students to find a way to plant 48 trees in equal rows. The students were supposed to find the number of possible rows and the number of trees within each of those rows. One of Alesha's students claimed that they would plant the trees in a 4 by 6 array. Instead of questioning the student to think about the reasonableness of their claim, Alesha's questioning leads the student to remove the meaning of the task and think only about basic multiplication facts by asking "4 times 6 is what?" This type of basic fact questioning is common for Alesha, and it strips the problem of any reasoning that would be required for the student to determine whether a four by six array could have a total of 48 trees. Alesha's high rule-based beliefs cause her to proceduralize conceptual tasks in order to make them more attainable for students.

Alesha's knowledge about the relationships between factors, arrays, and multiplication seemed to be quite strong during the content portion of her interview. In fact, her answers to conceptual questions were more accurate and precise than all of the other teachers in this study, with the exception of Han, yet her willingness to apply this knowledge to her questioning was hindered by her high alignment with rule-based beliefs. Alesha's case, in contrast to Wendy's, shows that the reasoning-based questioning practices of teachers with average MKT can also be hindered by rule-based beliefs. If rule-based beliefs are strong enough, teachers will proceduralize any task, regardless of their knowledge of the content and connections.

In contrast to both of these cases is the case of Sue. Sue's MKT is nearly one standard deviation below the mean, and close to the level of MKT of Wendy. As previously discussed, her interviews did show a lack of understanding of some of the concept and bigger ideas within her lessons, however, this act of proceduralizing conceptual lessons does not happen in Sue's classroom. In fact, Sue allows her students to make sense of the concepts through their own methods, making sure that her students are exposed to multiple solution paths and solution representations. Additionally, she purposefully conceptualizes her procedural tasks by asking questions that require students to consider the meaning behind their selected methods and representations. Although Sue's level of MKT is similar to Wendy's and lower than Alesha's, her lessons include numerous questions that allow students to consider the varying ways to view different mathematical ideas, and the connections between them. The major difference between Alesha and Wendy, and Sue, is that Sue's level of alignment with rule-based beliefs is notably low. Even with low MKT, Sue's low level of rule based beliefs allows her to avoid the typical trend of proceduralizing conceptual lessons. Since Sue does not believe that procedures are needed for students to understand mathematics, she is more likely to allow students to make sense of mathematical ideas in a way that fits with their own understanding.

These three cases of Wendy, Alesha, and Sue provided an example of how low rule-based beliefs can have a positive effect on the questioning habits of teachers with lower levels of MKT. Prior to these cases, Coach's case was discussed, which provided evidence that high levels of MKT can negate some of the effects of high levels of rule-based alignment. The comparison between Alesha and Coach's case is necessary, since their results can seem contradictory in nature. Alesha's case gives an example where high MKT was not enough to make up for the negative effects of high levels of rule-based beliefs. Coach's case provides an example where high MKT was enough to make up for some of the negative effects of high levels of rule-based beliefs. Coach's MKT score of 0.27 is only slightly higher than Alesha's score of 0.08, yet Coach's rule-based belief score of 0.17 is over half a standard deviation lower than Alesha's rule based score of 0.93. This suggests that above average MKT may aid in counteracting some of the negative effects of rule-based beliefs only if the level of rule-based beliefs is relatively low. Although other factors may be at play, Alesha's rule-based beliefs were too ingrained to be counteracted by above average MKT. Due to Alesha's ability to answer conceptual questions during her interview, it is unlikely that her lower MKT score was the reason behind the difference between her questioning practices and Coach's.

The purpose of research question two was to understand how teachers' MKT and beliefs relate to the nature of teacher questions during procedurally-based lessons. The evidence provided in teacher lessons and interviews show that teacher questioning practices are influenced by both their MKT and beliefs about teaching and learning. Furthermore, this relationship is complex, in that the effects of MKT and beliefs are dependent upon one another. High MKT alone does not ensure a higher frequency of reasoning-based questions, but low MKT can hinder a teachers' ability to ask meaningful and rigorous questions. Moreover, Han, the teacher with the highest MKT in this study was able to hold students accountable for mathematical accuracy and clarity in a way that teachers with lower MKT were unable to enact.

The negative effects of low MKT can also be overcome by low rule-based beliefs. Teachers with low MKT who had below average levels of rule-based beliefs were able to ask meaningful question more frequently than teachers with low MKT who had high rule-based beliefs. Evidence in this study also suggests that rule-based beliefs are more influential on meaningful questioning practices than reasoning-based beliefs. Low rule-based beliefs create more of an affordance for meaningful questioning practices than high reasoning-based beliefs. In other words, teachers are more likely to ask reasoning-based questions if they devalue rules than if they simply value reasoning. Similarly, low rule-based beliefs allow teachers to avoid the common practice of proceduralization of conceptual tasks that typically happen within the classrooms of low MKT teachers.

This study shows that high levels of MKT enhance the reasoning-based questioning abilities of teachers. However, it is also important that teachers place less emphasis on rules and procedures. Emphasis on rules and procedures typically lead to the simplification of reasoning-based tasks, which remove opportunities for students to build a deep understanding of the connections between mathematical concepts. The implications of these results will be discussed in chapter 5.

Chapter 5: Discussion

The purpose of this study was to explore patterns in teachers' questioning practices and gain an understanding of the affordances and hindrances to meaningful teacher questioning in both procedural and conceptual lessons. Similar to Ni et al.'s (2013) study, teachers were found to ask less reasoning-based questions in procedural lessons. However, teachers questioning practices did not change in a significant way between procedural and conceptual lessons. In fact, teachers asked more rule-based questions in conceptual lessons than in procedural lessons. This shows that the teachers' questioning habits were not dependent on the type of lesson they were teaching. Furthermore, while high MKT did allow for more in depth questioning, and low MKT acted as a hindrance to meaningful questioning, the biggest factor in a teacher's ability to ask meaningful questions is their level of alignment with rule-based beliefs. High levels of rule-based beliefs acted as a hindrance to meaningful questioning regardless of the level of alignment with reasoning-based beliefs, or level of MKT.

While the results of this study continue to show the complex and interactive nature of MKT and beliefs (Hill et al., 2008), a clearer picture of the relationships between these factors begins to emerge. This chapter will focus on three main themes that surfaced during analysis of the data: 1) high rule-based beliefs lead to proceduralization of all tasks, 2) high MKT and low rule-based beliefs allow for student accountability in precise mathematical talk, and 3) reducing rule-based beliefs is important in developing meaningful questioning practices in teachers. Implications of these results will be discussed along with each of these themes. Finally, limitations of the study and suggestions for future research will be given.

Proceduralizing Tasks

Similar to Hill et al.'s (2008) study, which found that high MKT teachers tend to conceptualize all tasks, while low MKT teachers tend to proceduralize all tasks, the current study found that low MKT did increase the level of proceduralization through questioning, but this proceduralization only happened for teachers with high rule-based beliefs. Proceduralization is synonymous with Wood's (1998) term, "funneling," which can be thought of as a process where the teacher asks leading questions that walk students through a predetermined process to get students from the start of a problem to the correct solution. While low MKT and rule-based beliefs tend to frequently come hand in hand (Cross, 2009), this data provided evidence that the proceduralization may be caused by the rule-based belief alignment, and not the low level of MKT. Sue, who has an MKT score almost 1 standard deviation below the mean, did not proceduralize tasks, similarly to Han, who had an MKT score over 1 standard deviation above the mean. The difference between these two teachers of varying MKT levels whom did not proceduralize tasks, and those who did proceduralize tasks was their alignment with rule-based beliefs. While other teachers with higher rule-based alignment asked questions that simplified the task for students, both Han and Sue asked questions that required students to think more deeply about the concepts behind the mathematics being studied.

One common form of proceduralization for high rule teachers was through their use of manipulatives. Teachers in the study who used manipulatives tended to show a lack of ability to make connections between the task and the concept that the manipulatives were supposed to make accessible to the students. For example, Wendy's students were dividing a quantity of cubes into equal sized groups to try to help students understand how to know when a number is a factor of another number. However, halfway through the lesson Wendy tells students that they can decide what they are going to divide by and a student responded, "we divide?" Similarly, when a student wanted to divide 37 cubes into 5 equal groups, no questions were asked about what students know about multiples of five, or how we can know whether five is a factor of a number. Moments like this provide evidence that students are not always able to generalize patterns and make connections between concepts without prompting by the teacher, and teachers with high rule-based beliefs are not making the dialogical moves to make these connections less abstract for students.

Numerous teachers in this study considered their conceptual lessons to be conceptual because of the use of manipulatives. Manipulatives can create a more concrete representation of abstract concepts, but unless the manipulatives are used meaningfully, “students may view the use of the physical objects as the goal instead of reaching an understanding” of how these manipulatives allow us to make sense of a concept such as factors or division (National Council of Teachers of Mathematics, 2014, p. 26). In the case of Wendy, her students were able to take cubes, place them into equal groups, and state whether there was a remainder of cubes left over, but it is not likely that these students would be able to apply this knowledge outside of the current context, or even to a straight forward mathematics question about whether a number is a factor of another number. Instead of proceduralizing a task involving manipulatives by giving students a process to follow, teachers need to explore ways in which they can ask focused questions to help students make sense of their ideas and push their understanding past physical manipulation of materials (Franke et al., 2009).

When manipulatives are only meant for computational purposes they make the task easier for students, and when students are engaged at low cognitive levels, the learning that occurs is simple rote memorization (Resnick & Zurawsky, 2006). For deep mathematical understanding to occur, students must be engaged at high-cognitive levels, even with content as basic as dividing cubes into equal sized groups. Manipulatives should allow us to discuss deeper conceptual ideas by making connections between the physical act of manipulation and the abstract concept that the manipulation is representing. The only way to make this connection is by prompting students to communicate about these ideas during their transition from concrete to abstract understanding.

Another way that teachers proceduralized conceptual lessons was through working out examples similar to the conceptual task before students were given the opportunity to explore a solution path on their own. This was highly apparent in Alesha’s classroom when she spent one-third of her conceptual lesson making sure students understood the problem, and then showed them an example of how to solve the problem with a different number. The questions that Alesha asked invited her students to follow a procedural path that was set out by the teacher. The connections between rectangular arrays and factors were glossed over as Alesha shifted the focus of the lesson from a real-life scenario of planting 48 trees in equal rows to a recall of a single basic fact resulting in a product of 48. Research has shown that students need to be exposed to problems for which they do not already have a clear solution path, and they need to be required to justify their work through means of communication (Resnick & Zurawsky, 2006). When teachers use large portions of class time to proceduralize a lesson by showing students their own solution path for a problem, they not only lower the cognitive demand of the lesson, but they take time away from students that could be used to reason about mathematics and make connections between concepts.

One way to encourage discussions that involve connections to concepts and generalization of more abstract ideas is through the production of activities that include discussion questions. Teachers in this study did discuss questions that were included in the activities or worksheets that were given to students. When these worksheets included conceptual questions, conversations between the teacher and students were drawn in a more cognitively demanding direction. Hill and Charalambous (2012) note that built in supports are useful in helping low-MKT teachers effectively implement curriculum materials. High rule-based teachers can also benefit from these supports in the form of pre-designed discussion questions. It is important to note that curriculum should include conceptual discussion questions, not just conceptual mathematics questions, as these teachers have already shown a tendency towards proceduralization of mathematical problems. While conceptual mathematics questions might help teachers discuss concepts while walking through a procedure, discussion questions would require students and teachers to take a step away from procedures and explore the big ideas and connections behind the mathematics at hand.

Student Accountability

MKT has been found to affect a teacher's ability to create opportunities for students to reason about and make sense of mathematical ideas on a deeper level (Campbell et al., 2014; Hill et al., 2008). However, Campbell et al. (2014) called for more qualitative studies to explore how MKT affects teachers' abilities to support student achievement. One particular gap in the research was the effect of MKT on teachers' ability to prompt students to think at higher cognitive levels. Confirming findings by Hill et al. (2008), the results of this study show that Han, the teacher with significantly higher MKT, was able to question students about their own mathematical ideas and push for more rigorous explanations, while other teachers typically asked questions that required simple yes or no answers. Moreover, these teachers that did ask less complex questions accepted student answers that were less accurate and precise, while Han continued questioning students until a more precise or accurate explanation could be given. However, Han and Alesha both have above average MKT, but their questioning practices differ dramatically. Alesha's questions that ask students to reason can usually be answered with a yes or no, and students are not typically asked to elaborate or explain their thinking. Han, on the other hand, phrases his questions in a way that requires students to go beyond yes or no answers, and often follows up student responses with additional questions that push for students to consider their ideas on a deeper level. Although this difference in questioning could be caused by the difference in MKT scores, Sleep and Eskelson (2012) found that beliefs that show value in computational procedures can hinder the potential effects of MKT on instructional practice. This appears to be the case with Alesha since her interview showed no lack of MKT on the given content, and her rule-based belief score was exceptionally high. MKT does afford teachers the ability to hold students accountable in their mathematical talk, but low levels of rule-based beliefs were needed in order for teachers to enact these practices.

Results from this study also show that a relationship might be found between a teacher's level of MKT and the level to which they hold their students accountable during mathematical talk. There is a difference between asking students yes or no questions that require them to reason, and asking students questions that require them to explain their reasoning through in-depth responses. Future research should focus on the relationship between teachers' MKT and their ability and willingness to hold students accountable for rigorous explanations by means of teacher questioning. Rule-based belief alignment should be considered as a potential mediating factor.

Reducing Rule-Based Beliefs

The past two sections have explained how high levels of rule-based beliefs cause teachers to proceduralize all lessons, and counteract the positive effects of high MKT on teacher questioning. All results in this study show the negative effects that high rule-based beliefs can have on teacher questioning practices. Therefore, the reduction of rule-based beliefs should be of top priority in professional development efforts. Cross (2009) also mentions that beliefs are an important aspect to address before shifts in instructional practices take place, but Sue is a perfect case to explain why it is the reduction of rule-based beliefs that should be of focus, as opposed to an increase in reasoning-based beliefs. Although Sue is low in both rule-based beliefs and reasoning-based beliefs, she chooses tasks that are fairly conceptual or tied to real-life. Since her rule-based beliefs are low, students are free to solve problems in a way that makes sense to them and she gives them time to think deeply about the mathematics. The conversations that are caused by the production of multiple solution paths are typically geared towards reasoning and sense making, since procedural conversations would only address one solution path. Sue's de-emphasis on rules creates a natural opportunity for meaning making even though her reasoning-based beliefs are low. Therefore, a reduction in rule-based beliefs results in an increase in reasoning-based practices.

Comparably, high rule-based beliefs are a hindrance to teacher questioning practices even for teachers like Alesha, who have abnormally high alignment with reasoning-based beliefs. The true nature of mathematics is interconnected and does not require large amounts of memorization or procedures since these ideas can be derived from various viewpoints. If teachers like Alesha do not trust in this process and find rules and procedures to be vital in the learning of mathematics, they will focus on rules and simplify the reasoning process into a procedure. Cross' (2009) study confirms this idea by mentioning that a teacher's beliefs about the nature of mathematics is one of the major factors affecting their beliefs about the teaching of mathematics. By changing teachers' beliefs about the nature of mathematics, teacher educators may be able to reduce teachers' alignment with rule-based beliefs. Drageset (2010) claims that learning more specialized content knowledge and common content knowledge acts as an affordance for not emphasizing rules. Through the development of this knowledge, teachers are led to better understand the interconnected nature of mathematics, and may therefore, place less emphasis on rules and procedures in the classroom.

Enactment of beliefs. The enactment of teacher beliefs is not always consistent, even within the same teacher. It is important to note that beliefs can be enacted through various teaching practices, with some practices aligning with seemingly contradictory beliefs. For example, Alesha is very high in both belief constructs. Her belief in reasoning can be seen through her task selection, which involves real-life context and opportunities for multiple solution paths. However, her belief in rules can be seen in her discourse patterns by asking students questions that walk them through a pre-determined solution path in a step-by-step manner. Her rule-based beliefs are also manifested in her classroom management, where students are led to believe that it is acceptable to depend on the teacher for direction and answers. Conversely, Han's classroom management requires independence of his students. If they ask questions about what they should be doing he requires them to reference the activity or directions, which trains his students to depend less on the teacher's thinking and more on their own. If rule-based beliefs are to be reduced, it is important to identify the avenues through which the rule-based beliefs are enacted.

Competing beliefs. Cases like Alesha's that show high belief in both reasoning and rules, or cases like Sue that show lower beliefs in both constructs are intriguing in that they seem to have incompatible beliefs. This study has shed light on the way that teachers' beliefs can be enacted through various teaching practices, and how conflicting beliefs can expose themselves simultaneously through different teacher moves. Elbaz (1983) mentions that teachers' beliefs are made up of rules of practices, practical principles, and images. It is images, along with emotions and morality, that direct the decision-making process. This description fits numerous teachers in this study, particularly Alesha, whose fear-based emotions guide her towards stronger rule-based beliefs. Although she views reasoning as important, these emotions override her reasoning beliefs. In order for Alesha to enact reasoning-based practices, she would need to feel secure in the idea that her students would succeed without such a high level of guidance.

The understanding that teachers can have conflicting or incompatible beliefs is not new knowledge. Cross (2009) suggests that competing beliefs can coexist within a person because of a mediating belief. While Alesha believes it is important for students to be exposed to reasoning in mathematics, she also believes that students need rules and scaffolding to successfully complete a reasoning-based task. These are competing beliefs since rules and scaffolding simplify the reasoning task to a point where students are no longer reasoning about mathematical ideas. However, both of these beliefs can coexist because Alesha believes that students' exposure to her own reasoning will increase the students' reasoning abilities and knowledge.

We cannot assume that a teacher with high reasoning beliefs will act in accordance to those beliefs. High rule-based beliefs can, and often do, offset reasoning-based belief enactment in the classroom. The goal of PD should be to get these beliefs to align in a coherent way that supports meaningful mathematics learning. Many elementary school teachers attend multiple threads of PD, and sometimes the agendas of these various sessions send conflicting messages to teachers. Every teacher in the sample mentioned at least two PD initiatives that they were involved in during the time of the study. One of which was the reasoning and modeling PD lead by a team of researchers, including the author of this study, and the other was focused on the Gradual Release of Responsibility (GRR) model. While the first PD initiative focuses on engaging students in complex reasoning and modeling practices with minimal teacher guidance, the GRR model encourages teachers to slowly release the responsibility of the thinking onto the students in four steps. The first step involves the teacher solving a problem in front of the students while sharing their thinking out loud (I do), during the second step the teacher guides students through the problem together as a class (We do), the third step has students solving a problem together without teacher guidance (You do together), and the fourth step has students solving a similar problem alone (You do alone). Without the opportunity to consider how these initiatives can be situated within the same belief framework, teachers of this sample tended to cling to aspects of PD that aligned with their previously formed beliefs. Wendy and Alesha both mentioned the GRR methods, and were particularly drawn to the "think aloud," or "I do" portion of the model. However, they were viewing these practices through a rule-based lens, which aided in the strengthening of their rule-based beliefs. The GRR model can easily be seen as a rule-based initiative, since the teacher is modeling how to solve a particular type of problem, but the model can also become more reasoning aligned if the teacher's voice is seen as one voice of many in the "co-construction of knowledge" (Von Glasersfeld, 1989). Furthermore, if teachers understand that the model does not need to be enacted in the I do, we do, you do together, you do alone order, the model can be seen as a useful tool that allows students to see a concept from multiple perspectives, and reason about the connections between various methods and representations.

Many educators do not naturally take the opportunity to build a solid theoretical foundation for their work, which leaves many of their decisions emotionally driven, or based on inconsistent belief structures (Peterson et al., 1989). It is essential that school PD initiatives provide numerous opportunities for teachers to reflect on their beliefs, and consider conflicting habits within their practices. Teachers should be asked to discuss how the different PD initiatives align with each other, and whether there are any competing ideas. These ideas should be tied to a core theoretical perspective of how students learn. By building a solid belief framework and instilling reflective habits within teachers, the results of student learning from reasoning-based practices will allow teachers to see the value in student reasoning, and not be overcome by conflicting rule-based beliefs.

Limitations and Future Research

The results of this study indicated a strong effect of rule-based beliefs on teacher questioning practices. Regardless of the level of reasoning-based beliefs, high rule-based beliefs are a hindrance to meaningful teacher discourse practices. Furthermore, high MKT and low rule-based belief alignment allowed for more rigorous mathematical discussions amongst teachers and students. While numerous patterns were found through this mixed methods approach, results are limited in generalizability due to the small sample size. A small sample size was chosen to allow for a more in depth analysis of each individual case. Studies focusing on statistical analysis of data from a larger sample of teachers would assess the generalizability of the results found within this study. Furthermore, the sample within this study was from a small rural town, and by the end of the study all teachers in the sample had some exposure to PD focused on reasoning and modeling. For this reason, results may be slightly skewed in favor of more reasoning-based practices. Further analysis should consider teacher samples from multiple sites and differing communities. Since this study would be used to generalize the qualitative results that addressed the role of MKT and beliefs, similar methods such as the coding of questions would be used, along with statistical tests such as the McNemar Dependent Chi Square Test. A larger sample size would be needed to gain higher power for statistical results, however, the study would only need to collect data on rule and reasoning-based beliefs, MKT scores, and video recorded lessons from each teacher.

Another limitation of this study was the small number teachers with high MKT scores. Han was the only teacher with an MKT score well above average (+1.64 SD), with the next highest score falling 0.27 standard deviations above the mean. This limited analysis of patterns within high MKT teachers. Although triangulation of the data through multiple data sources allowed for confirmation of certain findings within the sample, a larger sample of high MKT teachers would allow for a clearer picture of the effects of rule-based beliefs on high MKT teachers.

The coding scheme used in this study was originally designed to code the level of cognitive demand of mathematical tasks. While the coding scheme does align itself with the cognitive demand of teachers questioning, the creation of a refined coding scheme may allow for analysis of more minute changes in teachers' questioning practices based on teacher beliefs and level of MKT. Development of a more coherent coding scheme should consider the following aspects of teacher questions: 1) does the question push students' thinking further, or ask them to consider a previously formed idea, 2) does the question ask students to connect mathematical ideas, 3) does the question guide student thinking in a particular direction or leave students room to follow their own line of thinking, and 4) does the teacher provide ample time for student engagement and perseverance in thinking about the question, or do they answer the question themselves? All of these aspects do not necessarily address events within a single question, but a new coding scheme should consider the various elements that teacher questions can include in order to push for student discourse that is centered around meaning making.

Lastly, the results of this study provide a clear picture of the interference that high levels of rule-based beliefs can cause in the movement towards standards-based teaching. While teachers' MKT and understanding of reasoning-based practices are important, these factors will not allow teachers to reach their full potential if high rule-based beliefs are still in place. Efforts must be made to understand how PD can work to decrease teacher alignment with rule-based beliefs.

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Appendix

Semi-structured Interview Guide

I. Introduction

   a. Explain that participant was selected because we believe we can learn from their questioning practices.

II. Focus

   a. To gain insight into their beliefs about learning

   b. To gain insight into their understanding of the concepts covered in the videotaped lessons

III. Interview questions on beliefs and questioning

   a. How long have you been attending the IMaP^2 professional development?

   b. Do you attend summer professional development sessions?

   c. How many years have you been teaching?

   d. How many years have you taught mathematics?

   e. What grade level do you currently teach?

      i. How many years have you taught mathematics at this grade level?

   f. How would you describe your teaching philosophy?

   g. What do you believe students need in order to have an optimal learning environment?

      i. Are there any constraints that can make it challenging for you to teach in the way you believe is most beneficial for students?

   h. What are the roles of the teacher and the students in your classroom?

   i. What experiences have influenced how you teach or your views about how students learn?

   j. What do you know about teacher questioning as an instructional practice?

k. What prompts you to ask questions during your mathematics lessons?

    i. How do you gage whether a question was effective or not?

    ii. Do you ever prepare your questions in advance?

        1. If so, what goes into this preparation?

l. What experiences have influenced how you ask questions in the classroom?

m. In looking at your responses to the beliefs survey, are their any of your answers that you would like to elaborate on?

n. Would you like to add anything else that might help me understand your process of teaching or questioning in the classroom?

(Questions were developed by the researcher and influenced by articles mentioned in the literature review.)

IV. Interview guide for probing conceptual knowledge (The researcher will prepare the interview questions after viewing each lesson. The interview will be developed using the following questions).

a. What concept does the lesson cover?

b. What procedures are being used, if any?

    i. Does the teacher understand *why* the procedure works?

c. What connections could be made to other concepts?

    i. Is the teacher aware of these connections?

    ii. Can the teacher decipher whether these connections would be helpful to students?

d. Are their various methods that a student might use to solve these problems?

    i. Is the teacher able to understand these methods or decipher whether they will work for all problems?

e. Are their common misconceptions with this concept?

i. If so, what are they, and does the teacher know what concepts to focus in on to address the error?

f. Can the teacher decipher whether the concept would apply to a specific application?

V. Closing Remarks

a. Ask the participant if there is anything else they would like to add before we conclude the interview.

b. Thank the participant for their time.

(Questions were developed by the researcher and influenced by the MKT assessment design discussed in Hill et al. (2004).)